Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC4010 - Assignment 04

45 minutes **07-12-2023**

Important instructions:

- **Answer all the questions (1-5). If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.**
- **If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, <u>without</u> any exceptions or considerations.**

1. In the digital realm where emails abound, a substantial 50% are deemed spam, creating a need for effective filtration. To address this, a specific brand of software has emerged, asserting an impressive 99% accuracy in identifying spam emails. However, no solution is without its quirks, and in this case, a 5% probability of false positives exists—legitimate emails mistakenly flagged as spam. Now, the pivotal question emerges: if this software flags an email as spam, what is the probability that it is, in reality, not spam?

2. Suppose that 20% of all engineering students at the University of Jaffna have completed internships at top technology companies. The university decides to survey a random sample of 30 engineering students to gather information about their internship experiences.

   (a) What is the probability that none of the 30 surveyed engineering students have completed internships at top technology companies?

   (b) Find the probability that exactly one third of the surveyed engineering students have completed internships at top technology companies.

   (c) Find the probability that at least four of the surveyed engineering students have completed internships at top technology companies.

3. As part of the E-Week program at the Faculty of Engineering, scheduled for December 16-22, a team of students is organizing a hackathon. Historical data suggests that, on average, 10% of the participating teams in the hackathon successfully complete their projects within a given four-hour time frame. Considering the probability distribution for the number of teams successfully completing their projects:

   (a) What distribution is appropriate for modeling the number of teams successfully completing their projects within the allocated four-hour time frame? Identify the parameter representing the average rate.

   (b) What is the expected number of teams that will successfully complete their projects within the allocated four-hour time frame?

   (c) What is the probability that exactly two teams successfully complete their projects in this time frame?

   (d) If a faculty member plans to visit the hackathon for 30 minutes, what is the probability that they witness at least one team successfully completing their project during this time?

4. In a Computer Engineering MC4010 class with 60 students, attendance is categorized into two levels: Satisfactory (70% and above) and Unsatisfactory (below 70%). The class is excited about participating in a prestigious coding competition that will contribute to the BMICH (Bandaranaike Memorial International Conference Hall) event in Colombo. The competition requires the formation of a team of 5 students, chosen randomly without replacement. What is the probability that the team selected consists of exactly 3 students with satisfactory attendance and 2 with unsatisfactory attendance?

5. (a) Show that if X and Y are two independent random variables on a sample space S, then $Var(X+Y) = Var(X) + Var(Y)$.

   (b) A fair coin is flipped three times. Let S be the sample space of the eight possible outcomes, and let X be the random variable that assigns to an outcome the number of heads in this outcome. What is the expected value of X?

   (c) Show that if X and Y are independent random variables, then

$$Var(XY) = E(X)^2 Var(Y) + E(Y)^2 Var(X) + Var(X)Var(Y)$$

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Some useful formulas

1. If X follows binomial with parameters n, p. Then, the probability mass function is given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}; \ x = 0, 1, 2, \cdots n$$

2. If X follows Poisson with parameter λ. Then, the probability mass function is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}; \ x = 0, 1, 2, \cdots\cdots$$

3. If X follows Hyper-geometric distribution with parameters N, r, n. Then, the probability mass function is given by

$$P(X = x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

   where

   N - the total number of elements, r - number of successes in the N elements,n- number of elements drawn and x - the number of successes in the n elements

4. $\mu = E(X) = \sum_{\text{for all x}} xP(X = x)$ and $\sigma^2 = Var(X) = E(X^2) - (E(X))^2$
   or $Var(X) = E(X-\mu)^2 = \sum_{\text{for all x}} (x-\mu)^2 P(X = x)$

5. Let us consider that a sample space S is divided into two mutually exclusive partitions S_1 and S_2. An event H has occurred, and $P(S_1|H)$ can be written as

$$P(S_1|H) = \frac{P(S_1)P(H|S_1)}{P(S_1)P(H|S_1) + P(S_2)P(H|S_2)}$$

—————*Good Luck with the End Examination!*——————