**Department of Inter Disciplinary Studies,**
**Faculty of Engineering,**
**University of Jaffna, Sri Lanka**
**MC4010 - Assignment 03**

**40 minutes** **22-11-2023**

**Important instructions:**

- **Answer all the questions (1-4).**
- **If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.**
- **If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, without any exceptions or considerations.**



1. In a discrete mathematics class within the engineering faculty at University of Jaffna, there are 10 students, including a pair of students—Alice and Bob. Unfortunately, due to the eye infection spread on the premises, many students were unable to participate in the class that week. The lecturer randomly selects six students to participate in a special problem-solving activity during the class. Your task is to calculate the number of ways the selected engineering students can be arranged under different conditions:

   (a) Determine the number of arrangements when Alice must be part of the selected group of 6 engineering students.

   (b) Calculate the arrangements when both Alice and Bob must be together in the selected group of 6 engineering students.

   (c) Find the arrangements when exactly one of the pair (Alice or Bob) is included in the selected group of 6 engineering students.

   (d) Calculate the number of ways to form a group of 3 students, including Alice and Bob, from the remaining students who attended the class that week.

2. In a computer engineering lab, there are a dozen USB drives, half of which are designed for data storage (Data Drives) and half for software development (Dev Drives). A computer engineering student randomly selects USB drives in the dark because of an unscheduled power failure. Your task is to answer the following questions:

   (a) How many USB drives must the student take out to be sure that they have at least two USB drives of the same type (either Data Drive or Dev Drive)?

   (b) How many USB drives must the student take out to be sure that they have at least two USB drives designed for software development?

3. (a) Let $n$ and $k$ be positive integers with $n \geq k$. Then prove that

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

(b) Suppose that $k$ and $n$ are integers with $1 \leq k < n$. Prove the **hexagon identity**

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}$$

which relates terms in Pascal's triangle that form a hexagon.

(c) Let $n$ be a positive integer. Then prove that

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$$

4. Nine women and eleven men are on the E-week committee formed for the E-week 2023 program in the Faculty of Engineering at the University of Jaffna.

   (a) How many ways are there to select a sub committee of seven members of the E-week committee if at least one woman must be on the committee?

   (b) How many ways are there to select a sub committee of seven members of the E-week committee if at least one woman and al least one man must be on the committee?

**Some useful formulas**

1. Let $x$ and $y$ be variables, and let $n$ be a non-negative integer. Then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

2. $P(n,r) = nPr = \dfrac{n!}{(n-r)!}$.

3. $C(n,r) = nCr = \binom{n}{r} = \dfrac{n!}{(n-r)!r!}$.