Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 4010 - Assignment 01

40 minutes **04-10-2023**

Important instructions:

- **Answer all the questions (1-7).**
- **If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, <u>without</u> any exceptions or considerations.**

1. Let p, q, and r represent the following propositions in the context of computer engineering:

$$p: \text{The CPU executes instructions correctly.}$$
$$q: \text{The memory is error-free.}$$
$$r: \text{The program passes all test cases.}$$

Write these propositions using p, q, and r and logical connectives (including negations).

a) You passed all test cases, but the memory is error-prone.

b) The CPU executes instructions correctly, the memory is error-free, and the program passes all test cases.

c) To pass all test cases, it is necessary for the CPU to execute instructions correctly.

d) The CPU executes instructions correctly, but you didn't pass all test cases; nonetheless, the program passes all test cases.

e) Executing instructions correctly in the CPU and having error-free memory is sufficient for the program to pass all test cases.

f) You will pass all test cases if and only if you either have error-free memory or the CPU executes instructions correctly.

2. Let $N(x)$ be the statement "x has visited Colombo Port City," where the domain consists of the students in your batch (E21). Express each of these quantifications in English.

(a) $\exists x N(x)$

(b) $\forall x N(x)$

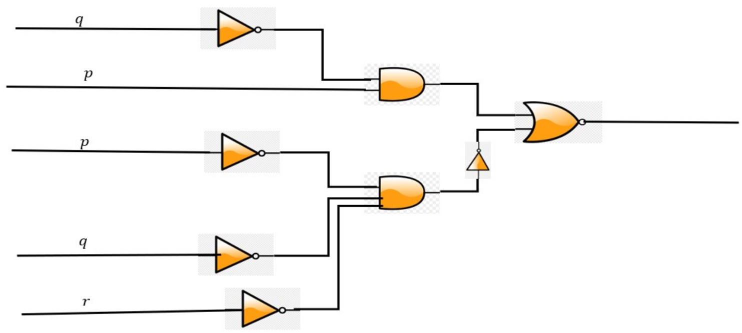
(c) $\exists x \neg N(x)$

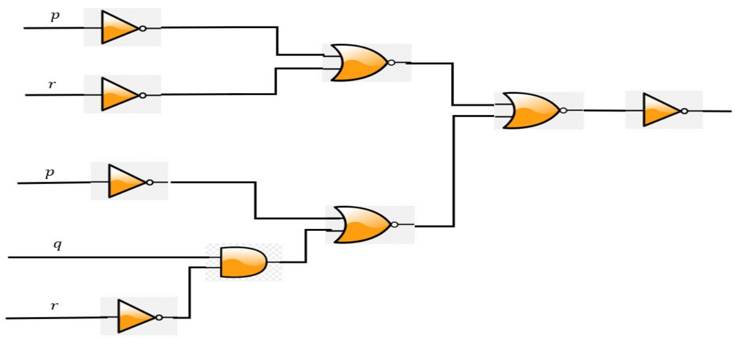
(d) $\neg \exists x N(x)$

3. a) Design a digital circuit that produces the output $(\neg p \vee \neg r \vee s) \wedge ((\neg p \wedge s) \vee (q \wedge \neg r))$ when given input bits p, q, r and s.

b) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(\neg p \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge q)$ from input bits p, q, and r.

(a) Find the output of each of these combinatorial circuits based on the provided diagrams:





4. Prove that the following compound propositions are logically equivalent:

    a) $\neg(p \wedge q)$ and $\neg p \vee \neg q$.
    b) $p \vee q$ and $\neg p \rightarrow q$.
    c) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$.

5. For statements p, q, r, simplify the following expressions:

    a) $\neg(p \vee q) \vee (\neg p \wedge q)$.
    b) $[(p \rightarrow q) \rightarrow p] \rightarrow p$.
    c) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$.

6. Construct a truth table for each of these compound propositions:

    a) $p \leftrightarrow \neg p$.
    b) $(p \wedge q) \rightarrow (p \vee q)$.
    c) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$.
    d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$.

7. a) Use a direct proof to show that (1) the sum of two odd integers is even. (2) the sum of two even integers is even.

    b) Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = 0$ [Hint: Assume that $r = a/b$ is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]