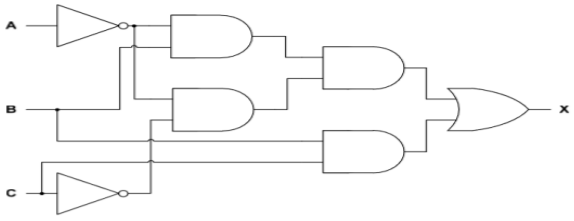
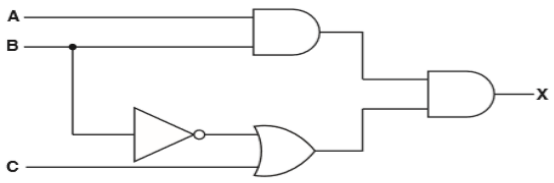
Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 4010 : Discrete Mathematics

Tutorial-01 **September 2023**

1. Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).
   (a) "The message is scanned for viruses whenever the message was sent from an unknown system."
   (b) "The message was sent from an unknown system but it was not scanned for viruses."
   (c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system."
   (d) "When a message is not sent from an unknown system it is not scanned for viruses."
2. Show that the following are tautologies
   (a) $[\neg p \wedge (p \vee q)] \rightarrow q$;
   (b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$;
   (c) $[p \wedge (p \rightarrow q)] \rightarrow q$;
   (d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$.
3. Construct a truth table for each of these compound propositions.
   (a) $\neg (p \rightarrow \neg q)$;
   (b) $p \oplus (p \vee q)$
   (c) $\neg (p \wedge q) \vee \neg (q \leftrightarrow p)$;
   (d) $(p \rightarrow q) \vee \neg (p \leftrightarrow \neg q)$;
   (e) $[p \rightarrow (\neg q \vee r)] \wedge \neg [q \vee (p \leftrightarrow \neg r)]$.
4. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the following outputs from input bits p, q, and r.
   (a) $(p \wedge \neg r) \vee (\neg q \wedge r)$
   (b) $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$
5. Write a logic statements that corresponds with the following logic circuits.





6. Which of the following pairs of propositional forms are logically equivalent,

   (a) $p \to (q \to r), (q \wedge \neg r) \to \neg p$;

   (b) $p \leftrightarrow (q \leftrightarrow r), (p \leftrightarrow q) \leftrightarrow r$;

   (c) $(p \to q) \vee (q \to r), p \wedge q \to r$;

7. Let p,q,r be three statements.Simplify the following expressions,

   (a) $\{(p \to q) \wedge \neg q\} \to \neg p$;

   (b) $[(p \to q) \to p] \to p$;

   (c) $\neg (p \wedge q) \to (p \vee \neg q)$.

8. Let $P(x), Q(x)$, and $R(x)$ be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and $P(x), Q(x)$, and $R(x)$.

   (a) All clear explanations are satisfactory.

   (b) Some excuses are unsatisfactory.

   (c) Some excuses are not clear explanations.

   (d) Does (c) follow from (a) and (b)?

9. Determine the validity of the following argument

   (a) If I enter the poodle den, then I will carry my electric poodle prod or my can of mace. I am carrying my electric poodle prod but not my can of mace. Therefore, I will enter the poodle den.

   (b) I will buy a new goat or a used Yugo. If I buy both a new goat and a used Yugo, I will need a loan. I bought a used Yugo and I don't need a loan. Therefore, I didn't buy a new goat.

   (c) If my car runs out of gas then I will not make it home. My car ran out of gas. Therefore, I did not make it home.

   (d) If my hair is too messy, then I will never get married. If I never get married, then I will begin to lose more weight. Therefore, If my hair is too messy, I will lose weight.

   (e) It is not sunny this afternoon and it is colder than yesterday. We will go swimming only if it is sunny. If we do not go swimming, then we will take a canoe trip. If we take a canoe trip, then we will be home by sunset. Therefore we will be home by sunset.

   (f) Wages will increase only if there is an inflation. If there is an inflation then the cost of living will increase. Wages will increase. Therefore the cost of living will increase.

(g) If the races are fixed or gambling houses are crooked then the tourist trade will decline and the town will suffer. If the tourist trade decreases then the police force will be happy. The police force is never happy. Therefore the races are not fixed.

10. Let $O(x)$ be the statement "x is in this exam hall", $W(x)$ be "x is a woman", let $M(x)$ be the statement "x is a man", and let $G(x)$ be the statement "x wears glasses". Express each of these statements in terms of $O(x), W(x), M(x), G(x)$, quantifiers, and logical connectives. Let the domain consist of all people.

    (a) Someone in this exam hall wears glasses.

    (b) Not everyone in this exam hall wears glasses.

    (c) No woman in this exam hall wears glasses.

    (d) All men in this exam hall wears glasses.

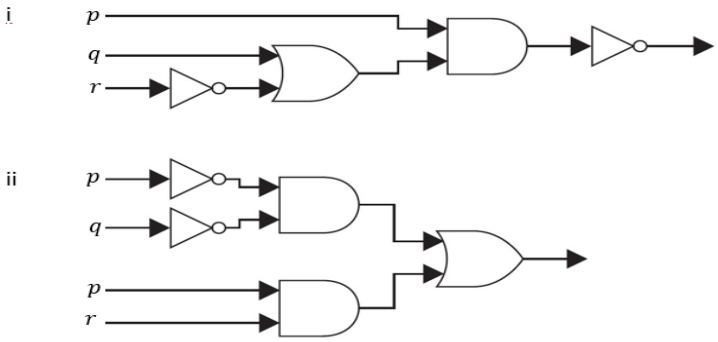
11. Using logical equivalences, show that each of the following compound propositions is a tautology.

    (a) $[p \wedge (p \rightarrow q)] \rightarrow q$

    (b) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

12. (a) Construct a combinatorial circuit using **inverters**, **OR** gates, and **AND** gates that produces the output $(p \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$ from input bits p, q, and r.

    (b) Find the output of each of these combinatorial circuits.



Note:
The tutorial discussions (Tutors: Miss S. Anushiya and Miss R. Meenadushanthani) are set for September 25th, September 27th, and October 2nd. Please make sure to prepare yourself ahead of the discussion sessions.