**Department of Inter Disciplinary Studies,**
**Faculty of Engineering,**
**University of Jaffna, Sri Lanka**
**MC 4010 : Assignment 1 Answers**

**40 minutes** **04-10-2023**

**Important instructions:**

- **Answer all the questions (1-7).**
- **If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, without any exceptions or considerations.**

1. Let $p$, $q$, and $r$ represent the following propositions in the context of computer engineering:

$p$ : The CPU executes instructions correctly.
$q$ : The memory is error-free.
$r$ : The program passes all test cases.

(a) You passed all test cases, but the memory is error-prone.
(b) The CPU executes instructions correctly, the memory is error-free, and the program passes all test cases.
(c) To pass all test cases, it is necessary for the CPU to execute instructions correctly.
(d) The CPU executes instructions correctly, but you didn't pass all test cases; nonetheless, the program passes all test cases.
(e) Executing instructions correctly in the CPU and having error-free memory is sufficient for the program to pass all test cases.
(f) You will pass all test cases if and only if you either have error-free memory or the CPU executes instructions correctly.

**Solutions**

(a) $r \wedge \neg q$
(b) $p \wedge q \wedge r$
(c) $r \rightarrow p$
(d) $p \wedge \neg r \wedge r$
(e) $(p \wedge q) \rightarrow r$
(f) $r \leftrightarrow (q \vee p)$

2. Let $N(x)$ be the statement "$x$ has visited Colombo Port City," where the domain consists of the students in your batch (E21). Express each of these quantifications in English.

(a) $\exists x N(x)$
(b) $\forall x N(x)$
(c) $\exists x \neg N(x)$

(d) $\neg \exists x N(x)$

**Solutions**

(a) $\exists x N(x)$ - There exists a student who has visited Colombo Port City.

(b) $\forall x N(x)$ - Every student has visited Colombo Port City.

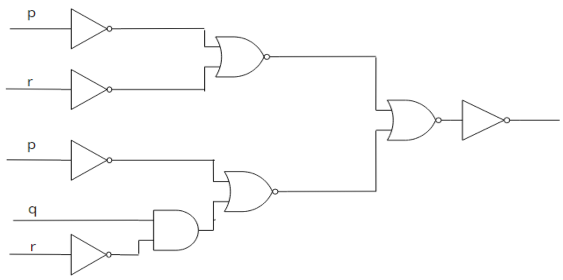
(c) $\exists x \neg N(x)$ - There exists a student who has not visited Colombo Port City.

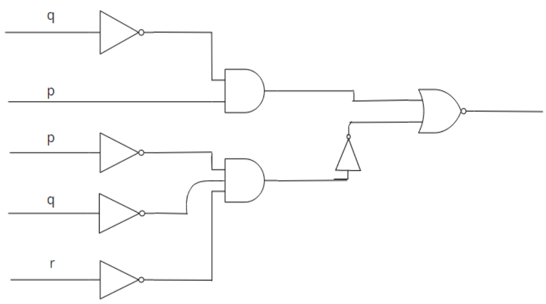
(d) $\neg \exists x N(x)$ - There does not exist a student who has visited Colombo Port city.

3. (a) Design a digital circuit that produces the output $(\neg p \vee \neg r \vee s) \wedge ((\neg p \wedge s) \vee (q \wedge \neg r))$ when given input bits $p, q, r$ and $s$.

   (b) Construct a combinatorial circuit using inverters, **OR** gates, and **AND** gates that produces the output $(\neg p \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge q)$ from input bits $p, q$ and $r$.

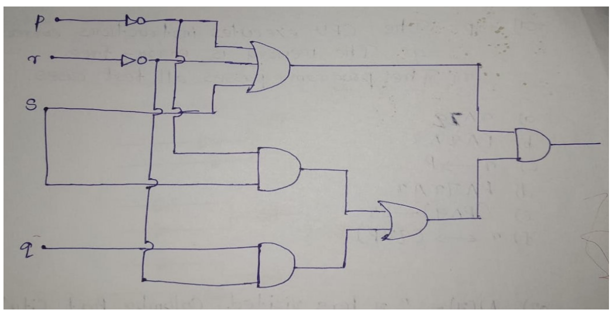
   (c) Find the output of each of these combinatorial circuits based on the provided diagrams:



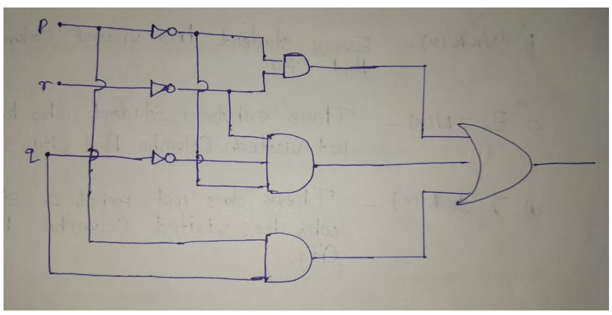




**Solutions**

(a)




(b)



(c) i. $\neg[(\neg q \wedge p) \vee \neg(\neg p \wedge \neg q \wedge \neg r)]$

ii. $\neg[\neg(\neg(\neg p \vee \neg r) \vee \neg(\neg p \vee (q \wedge \neg r)))]$

4. Prove that the following compound propositions are logically equivalent:

(a) $\neg(p \wedge q)$ and $\neg p \vee \neg q$

(b) $p \vee q$ and $\neg p \to q$

(c) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$

**Solutions**

(a)

| $p$ | $q$ | $p \wedge q$ | $\neg(p \wedge q)$ |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

| $p$ | $q$ | $\neg p$ | $\neg q$ | $\neg p \vee \neg q$ |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

$$\therefore \quad \neg(p \wedge q) \equiv \neg p \vee \neg q$$

(b)

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

| $p$ | $q$ | $\neg p$ | $\neg p \to q$ |
|---|---|---|---|
| T | T | F | T |
| T | F | F | T |
| F | T | T | T |
| F | F | T | F |

$$\therefore \quad p \vee q \equiv \neg p \to q$$

(c)

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

| $p$ | $q$ | $(p \to q)$ | $(q \to p)$ | $(p \to q) \wedge (q \to p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

$$\therefore \quad p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

5. For statements $p, q, r, s$ Simplify the following expressions.

(a) $\neg(p \vee q) \vee (\neg p \wedge \neg q)$

(b) $[(p \to q) \to p] \to p$

(c) $[(p \to q) \wedge \neg q] \to \neg p$

**Solutions**

(a) $\neg(p \vee q) \vee (\neg p \wedge \neg q)$
$\equiv \quad (\neg p \wedge \neg q) \vee (\neg p \wedge q)$
$\equiv \quad \neg p \wedge (\neg q \vee q)$
$\equiv \quad \neg p \wedge T$
$\equiv \quad \neg p$

(b) $[(p \to q) \to p] \to p$
$\equiv \quad \neg[\neg(\neg p \vee q) \vee p] \vee p$
$\equiv \quad [(\neg p \vee q) \wedge \neg p] \vee p$
$\equiv \quad [(\neg p \wedge \neg p) \vee (q \wedge \neg p)] \vee p$
$\equiv \quad (\neg p \vee p) \vee (q \wedge \neg p)$

$\equiv \quad T \vee (q \wedge \neg p)$
$\equiv \quad T$

(c) $[(p \to q) \wedge \neg q] \to \neg p$
$\equiv \quad \neg[(\neg p \vee q) \wedge \neg q] \vee \neg p$
$\equiv \quad [\neg(\neg p \vee q) \vee q] \vee \neg p$
$\equiv \quad [(p \wedge \neg q) \vee q] \vee \neg p$
$\equiv \quad [(p \vee q) \wedge (\neg q \vee q)] \vee \neg p$
$\equiv \quad [(p \vee q) \wedge T] \vee \neg p$
$\equiv \quad (p \vee q) \vee \neg p$
$\equiv \quad (p \vee \neg p) \vee q$
$\equiv \quad T \vee q$
$\equiv \quad T$

6. Construct a truth table for each of these compound propositions:

(a) $p \leftrightarrow \neg p$
(b) $(p \wedge q) \to (p \vee q)$
(c) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

**Solutions**

(a)

| $p$ | $\neg p$ | $p \leftrightarrow \neg p$ |
|---|---|---|
| T | F | F |
| F | T | F |

(b)

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $(p \wedge q) \to (p \vee q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | F | T |

(c)

| $p$ | $q$ | $\neg p$ | $q \to \neg p$ | $p \leftrightarrow q$ | $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | F | T | F | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

(d)

| $p$ | $q$ | $\neg q$ | $p \to q$ | $p \to \neg q$ | $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| F | T | F | F | T | T |
| F | F | T | T | F | T |

7. (a) Use a direct proof to show that, (1) the sum of two odd integers is even. (2) the sum of two even integers is even.

(b) ) Use a proof by contradiction to show that there is no rational number $r$ for which $r^3 + r + 1 = 0$ [Hint: Assume that $r = a/b$ is a root, where $a$ and $b$ are integers and $a/b$ is in lowest terms. Obtain an equation involving integers by multiplying by $b^3$. Then look at whether $a$ and $b$ are each odd or even.]

**Solutions**

(a) i. Let $x, y$ an odd integers.
$\exists n_1 \in \mathbb{N}$ such that $x = 2n_1 + 1$
$\exists n_2 \in \mathbb{N}$ such that $y = 2n_2 + 1$
consider $x + y = (2n_1 + 1) + (2n_2 + 1) = 2(n_1 + n_2 + 1)$
since $n_1 + n_2 + 1$ is also integer.
$\therefore \quad x + y = 2k$ where $k = n_1 + n_2 + 1$
Therefore sum of two odd integers is even.

ii. Let $a, b$ an even integers.
$\exists n \in \mathbb{N}$ such that $a = 2n$
$\exists m \in \mathbb{N}$ such that $b = 2m$
consider $a + b = 2n + 2m = 2(n + m)$
since $n + m$ is also integer.
$\therefore \quad a + b = 2k$ where $k = n + m$
Therefore sum of two even integers is even.

(b) given equation $r^3 + r + 1 = 0$
Let's assume a rational number $r = a/b$
where $a, b$ are integers and $b \neq 0$
substitute $r = a/b$ in the given equation.
$r^3 + r + 1 = 0 \Rightarrow (a/b)^3 + (a/b) + 1 = 0 = a^3 + ab^2 + b^3 = 0$
There are 3 cases there for $a, b$.

Case 1:-
If $a$ is an even and $b$ is an odd then $a^3$ is an even term, $ab^2$ is an even and $b^3$ is an odd term. So there by sum of $a^3, ab^2, b^3$ gives an odd term but zero is an even number. So there is a contradiction here.

Case 2:-
If $a$ is an odd and $b$ is an even then $a^3$ is an odd term, $ab^2$ is an even and $b^3$ is an even term. So there by sum of $a^3, ab^2, b^3$ gives an odd term but zero is an even number. So there is a contradiction here.

Case 3:-
If both $a, b$ are odd numbers with no common factors. Then $a^3$ is an odd term, $ab^2$ is an odd and $b^3$ is also an odd term. So there by sum of $a^3, ab^2, b^3$ gives an odd term but zero is an even number. So there is a contradiction here.

All our assumption leads to contradiction. So there is no rational number $r$ for $r^3 + r + 1 = 0$