



Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 4010 : Discrete Mathematics

Tutorial-04

November 2023

1. Let n be a positive integer. Show that

$$\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+2}{n+1} / 2$$

2. Suppose that a password for a computer system in computer lab of Faculty of Engineering at Ariviyal Nagar, Kilinochchi must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the eight special characters $\star, >, <, !, +, @, \%$ and $=$.

- (a) How many different passwords are available for this computer system?
- (b) How many of these passwords contain at least one occurrence of at least one of the eight special characters?
- (c) Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes two nanoseconds (one nanosecond is one-billionth (10^{-9}) of a second) for a hacker to check each possible password.
3. Use proof by mathematical induction method (provide an appropriate induction steps to get full marks) to show that

- (a) for any positive integer n , $13 \times 7^{n-1} + 5^{2n-1}$ is divisible by 18.

(b) for any positive integer n ,
$$\sum_{k=1}^n \frac{k+4}{k(k+1)(k+2)} = \frac{n(3n+7)}{2(n+1)(n+2)}$$

4. (a) How many bit strings of length 8 are there?
- (b) How many bit strings of length 8 or less are there? (Count the empty string of length zero also.)
- (c) How many strings of 6 lower case English letters are there that have the letter x in them somewhere? Here strings may use the same letter more than once. (Hint: It might be easier to first count the strings that don't have an x in them.)
- (d) A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in 2 sizes for each sex. How many different types of this shirt are made?

5. How many strings of four decimal digits

- (a) Do not contain the same digit twice
- (b) End with an odd digit?
- (c) Have exactly three digits that are 8

6. How many license plates can be made using either four digits followed by two letters, or two letters followed by four digits?

7. From a group of 15 men, 7 women, 5 boys and 4 girls,
 - (a) How many ways can a man, a woman, a boy and a girl be selected?
 - (b) How many ways can a male and a female be selected?
 - (c) How many ways can a person be selected?
8.
 - (a) What is the maximum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade of there are five possible grade A,B,C,D and F?
 - (b) Prove that in any group of six people there must be at least three mutual friends or three mutual enemies.
 - (c) Show that if seven colours are used to paint 50 bicycles, at least 8 bicycles will be the same colour.
9. The Engineering student union (ESU) has Seven members: 4 women and 3 men. Three are selected to attend a workshop seminar related ragging awareness program, which is held in Colombo.
 - a) How many ways are there that all three selected will be women?
 - b) How many ways are there that all three selected will be men?
 - c) How many ways will two men and one woman be selected?
 - d) How many ways will one man and two women be selected?
10. In the Internet, which is made up of interconnected physical networks of computers, each computer (or more precisely, each network connection of a computer) is assigned an Internet address. In Version 4 of the Internet Protocol (IPv4), now in use an address is a string of 32 bits. It begins with a network number (netid). The netid is followed by a host number (hostid), which identifies a computer as a member of a particular network. Class C addresses, used for the smallest networks, consist of 110, followed by a 21-bit netid and an 8-bit hostid.

Class C :

1	1	0	netid	hostid
			21 bit	8 bit

- (a) What is the total IPv4 addresses are available in class C type?
- (b) How many IPv4 addresses are not included in class C type?
- (c) How many different hostids of one particular netid contain following properties
 - i. exactly four 1s?
 - ii. at most four 1s?
 - iii. both begin and end with a 1?
 - iv. an equal number of 0s and 1s?
- (d) What is the minimum amount of different IP addresses consist at least three host ids are same

11. A certain club has 5 male and 7 female members.
 - (a) How many ways can any 7 member committee be selected from the membership?
 - (b) How many ways are there to form a 7 member committee consisting of 3 men and 4 women?
 - (c) How many ways are there to form a committee of 6 people if 2 women refuse to serve together?
12. A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?
13. How many strings of six lowercase letters from the English alphabet contain
 - (a) the letter a ?
 - (b) the letter a and b ?
 - (c) the letters a and b in consecutive positions with a preceding b , with all the letters distinct?
 - (d) the letters a and b , where a is somewhere to the left of b in the string, with all the letters distinct?
14. How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?
15. In how many ways can a photographer at a wedding arrange 6 people in a row from a group of people, where the bride and the groom are among these 10 people, if
 - (a) the bride must be in the picture?
 - (b) both the bride and groom must be in the picture?
 - (c) exactly one of the bride and the groom is in the picture?
16. Thirteen women and seven men are on the ESU committee formed for the E-week program in the Faculty of Engineering at the University of Jaffna.
 - (a) How many ways are there to select a sub committee of five members of the ESU committee if at least one woman must be on the committee?
 - (b) How many ways are there to select a sub committee of five members of the ESU committee if at least one woman and at least one man must be on the committee?
17. Let n be a non-negative even integer. Then prove that

$$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} = \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n}$$

(*hint*: you may use binomial theorem with suitable substitutions)