Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 4010 : Discrete Mathematics

Assignment-04 Solutions

1. A - Event that an email is detected as spam
   B - Event that an email is spam
   C - Event that an email is not spam.

   $P(B) = P(B^c) = 0.5$
   $P(A|B) = 0.99$
   $P(A|B^c) = 0.05$

$$\begin{aligned} P(B^c|A) &= \frac{P(A|B^c)\,P(B^c)}{P(A|B)\,P(B) + P(A|B^c)\,P(B^c)} \\ &= \frac{0.05 * 0.5}{0.05 * 0.5 + 0.99 * 0.5} \\ &= \frac{5}{104} \\ &= 0.0481 \end{aligned}$$

2. Let X be the no of engineering students completed internship at top technology companies.

   $X \sim Bin(30, 0.2)$

   (a)

$$\begin{aligned} P(X = 0) &= \binom{30}{0}(0.2)^0(0.8)^{30} \\ &= 0.0012 \end{aligned}$$

   (b)

$$\begin{aligned} P(X = 10) &= \binom{30}{10}(0.2)^{10}(0.8)^{20} \\ &= 0.0354 \end{aligned}$$

   (c)

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 0.8773 \end{aligned}$$

3. Let X be the no of teams successfully completed their projects.

   (a) $X \sim poi\,(0.1/4h)$
   Poisson distribution
   average rate is 0.1 for 4 hours

   (b)

$$\begin{aligned} E(X) &= \lambda \\ &= 0.1 \end{aligned}$$

   (c)

$$\begin{aligned} P\,(X=2) &= \frac{e^{-\lambda} * \lambda^x}{x!} \\ &= \frac{e^{-0.1} * 0.1^2}{2!} \\ &= 0.0045 \end{aligned}$$

   (d) for $4h \rightarrow 0.1$
   $30min \rightarrow \dfrac{0.1}{60*4} * 30$
   $= 0.0125$

$$\begin{aligned} P\,(X \geq 1) &= 1 - P\,(X=0) \\ &= 1 - \frac{e^{-0.0125} * 0.0125}{0!} \\ &= 0.0124 \end{aligned}$$

4. No. of students 60
   Let N be no of satistactory students out of 60.

Satisfactory	Unsatisfactory
N	60-N

$\rightarrow$

Sat	Unsat
3	2

   Let X be the no.of students with satisfactory attendance $P\,(X=3) = \dfrac{\binom{N}{3}\binom{60-N}{2}}{\binom{60}{5}}$

5. (a)

$$\begin{aligned} Var &= (X+Y) = E\,(X+Y)^2 - (E\,(X+Y))^2 \\ &= E\left(X^2 + 2XY + Y^2\right) - (E\,(X) + E\,(Y))^2 \\ &= E(X^2) + 2E(X)E(Y) + E(Y^2) - \left[(E(X))^2 + (E(Y))^2 + 2 * E(X)E(Y)\right] \\ &= E(X^2 - (E(X))^2 + E(Y^2) - (E(Y))^2) \\ &= Var(X) + Var(Y) \end{aligned}$$

(b) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
Let X be the random variable that assigns to an outcome the no of heads in the outcome.
$X = \{0, 1, 2, 3\}$

$P(X = 0) = \frac{1}{8}$

$P(X = 1) = \frac{3}{8}$

$P(X = 2) = \frac{3}{8}$

$P(X = 3) = \frac{1}{8}$

$$\begin{aligned} E(X) &= \sum_{x=0}^{3} x * P(X = x) \\ &= 0 * \frac{1}{8} + 1 * \frac{3}{8} + 2 * \frac{3}{8} + 3 * \frac{1}{8} \\ &= \frac{12}{8} \\ &= 1.5 \end{aligned}$$

(c)

$$\begin{aligned} Var(X) &= E\left(X^2\right) - \left(E\left(X\right)\right)^2 \\ Var(Y) &= E\left(Y^2\right) - \left(E\left(Y\right)\right)^2 \end{aligned}$$

$$\begin{aligned} Var(XY) &= E\left((XY)^2\right) - \left(E\left(XY\right)\right)^2 \\ &= E\left(X^2\right) E\left(Y^2\right) - \left(E\left(X\right)\right)^2 \left(E\left(Y\right)\right)^2 \\ &= \left(Var\left(X\right) + \left(E(X)\right)^2\right) \left(Var\left(Y\right) + \left(E(Y)\right)^2\right) - \left(E\left(X\right)\right)^2 \left(E\left(Y\right)\right)^2 \\ &= Var(X)Var(Y) + \left(E\left(Y\right)\right)^2 Var(X) + \left(E\left(X\right)\right)^2 Var(Y) + \left(E\left(X\right)\right)^2 \left(E\left(Y\right)\right)^2 \\ &- \left(E\left(X\right)\right)^2 \left(E\left(Y\right)\right)^2 \\ &= \left(E\left(X\right)\right)^2 Var(Y) + \left(E\left(Y\right)\right)^2 Var(X) + Var(X)Var(Y) \end{aligned}$$