Department of Inter Disciplinary Studies,
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MC 4010 - Assignment 02

50 minutes **16-03-2023**

Answer all the questions(1-7), and a bonus question is given on the next page, giving you a chance to gain extra five marks.

1. Let p, q, and r be the propositions

$$p : \text{You get an A on the final exam.}$$
$$q : \text{You do every exercise in this class lecture slides.}$$
$$r : \text{You get an A in this assignment exam.}$$

Write these propositions using p, q, and r and logical connectives (including negations).

a) You get an A in this assignment exam, but you do not do every exercise in this class lecture slides.

b) You get an A on the final, you do every exercise in this class lecture slides, and you get an A in this assignment exam.

c) To get an A in this assignment exam, it is necessary for you to get an A on the final.

d) You get an A on the final, but you don't do every exercise in this class lecture slides; nevertheless, you get an A in this assignment exam.

e) Getting an A on the final and doing every exercise in this class lecture slides is sufficient for getting an A in this assignment exam.

f) You will get an A in this assignment exam if and only if you either do every exercise in this class lecture slides or you get an A on the final.

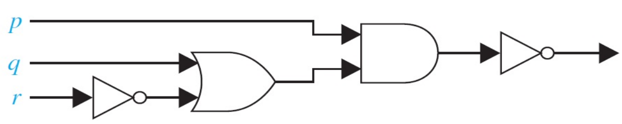
[18 Marks]

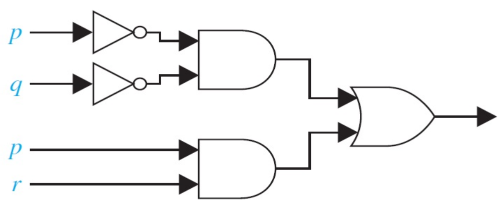
2. a) Build a digital circuit that produces the output $(\neg p \vee \neg r) \wedge (\neg p \vee (q \wedge \neg r))$ when given input bits p, q, and r.

b) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$ from input bits p, q, and r.

[12 Marks]

3. Find the output of each of these combinatorial circuits.

a) 

b) 

[10 Marks]

4. Show that following compound propositions are logically equivalent.

   a) $\neg(p \wedge q)$ and $\neg p \vee \neg q$
   b) $\neg(p \vee q)$ and $\neg p \wedge \neg q$
   c) $p \rightarrow q$ and $\neg p \vee q$

[15 Marks]

5. Let p, q, r be three statements. Simplify the following expressions,

   a) $\neg(p \vee q) \vee (\neg p \wedge q)$;
   b) $[(p \rightarrow q) \rightarrow p] \rightarrow p$;
   c) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$;

[15 Marks]

6. Construct a truth table for each of these compound propositions.

   a) $p \leftrightarrow \neg p$
   b) $(p \wedge q) \rightarrow (p \vee q)$
   c) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
   d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

[16 Marks]

7. a) Use a direct proof to show that (1) the sum of two odd integers is even. (2) the sum of two even integers is even.
   b) Give a proof by contradiction of the theorem "If $3n + 7$ is even, then $n + 2$ is odd."

[14 Marks]

***Bonus Question (5 Marks)**
Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = 0$ [Hint: Assume that $r = a/b$ is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]

****The maximum mark possible to obtain for this assignment is 100.***