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Faculty of Engineering,
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MC 4010 : Assignment 2 Answers

30 minutes **30-10-2023**

Important instructions:

- **Answer all the questions (1-3).**
- **If it is determined that you have violated any policies during this exam, you will receive a score of zero for this assignment, without any exceptions or considerations.**

1. (a) Suppose that A is the set of fourth-semester students at the Faculty of Engineering, University of Jaffna, and B is the set of students in discrete mathematics at your faculty. Express each of these sets in terms of A and B.

   i. the set of fourth-semester students taking discrete mathematics at the Faculty of Engineering, University of Jaffna.

   ii. the set of fourth-semester students at the Faculty of Engineering, University of Jaffna, who are not taking discrete mathematics.

   iii. the set of students at the Faculty of Engineering, University of Jaffna, who either are fourth-semester students or are taking discrete mathematics.

   iv. the set of students at the Faculty of Engineering, University of Jaffna, who either are not fourth-semester students or are not taking discrete mathematics.

   (b) Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

   i. by showing each side is a subset of the other side.

   ii. using a membership table.

Solutions

(a) i. $A \cap B$

ii. $A \cap \bar{B}$ or $A - B$

iii. $A \cup B$

iv. $\bar{A} \cup \bar{B}$

(b) i. $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

first we want to show that $(\overline{A \cap B \cap C}) \subseteq \bar{A} \cup \bar{B} \cup \bar{C}$

$\Rightarrow \quad x \in \overline{A \cap B \cap C}$

$\Rightarrow \quad x \notin (A \cap B \cap C)$

$\Rightarrow \quad x \in> (A \cap B \cap C)$

$\Rightarrow \quad (x \in> A) \vee (x \in> B) \vee (x \in> C)$

$\Rightarrow \quad x \notin A \vee x \notin B \vee x \notin C$

$\Rightarrow \quad x \in \bar{A} \vee x \in \bar{B} \vee x \in \bar{C}$

$\Rightarrow \quad x \in (\bar{A} \cup \bar{B} \cup \bar{C})$

$$\therefore \quad \overline{A \cap B \cap C} \subseteq \bar{A} \cup \bar{B} \cup \bar{C} \longrightarrow (1)$$

Next we want to show that $\bar{A} \cup \bar{B} \cup \bar{C} \subseteq (\overline{A \cap B \cap C})$

$\Rightarrow \quad x \in (\bar{A} \cup \bar{B} \cup \bar{C})$

$\Rightarrow \quad x \in \bar{A} \vee x \in \bar{B} \vee x \in \bar{C}$

$\Rightarrow \quad x \notin A \vee x \notin B \vee x \notin C$

$\Rightarrow \quad (x \in> A) \vee (x \in> B) \vee (x \in> C)$

$\Rightarrow \quad x \in> (A \cap B \cap C)$

$\Rightarrow \quad x \notin (A \cap B \cap C)$

$\Rightarrow \quad x \in \overline{A \cap B \cap C}$

$$\therefore \quad \bar{A} \cup \bar{B} \cup \bar{C} \subseteq \overline{A \cap B \cap C} \longrightarrow (2)$$

(1) and (2)

$$\Rightarrow \quad \overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$$

ii.

A	B	C	$A \cap B \cap C$	$\overline{A \cap B \cap C}$
1	1	1	1	0
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	0	1

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{A} \cup \bar{B} \cup \bar{C}$
1	1	1	0	0	0	0
1	1	0	0	0	1	1
1	0	1	0	1	0	1
1	0	0	0	1	1	1
0	1	1	1	0	0	1
0	1	0	1	0	1	1
0	0	1	1	1	0	1
0	0	0	1	1	1	1

$$\therefore \quad \overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$$

2. (a) Show that the function $f(x) = |x|$ from the set of real numbers to the set of non negative real numbers is not invertible, but if the domain is restricted to the set of non negative real numbers, the resulting function is invertible.

   (b) Suppose that g is a function from A to B and f is a function from B to C.

      i. Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

      ii. Show that if both f and g are onto functions, then $f \circ g$ is also onto.

   (c) Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbb{R}$ to $\mathbb{R}$.

Solutions

(a) The function $f(x) = |x|$ is not one-to-one.
for example $f(2) = 2 = f(-2) \Rightarrow \quad 2 \neq -2$ two different elements in the domain map to the same element in the co-domain. $\therefore$ It is not invertible.
On the restricted domain, the function is the identity function from the set of non-negative real numbers to itself. $f(x) = x$, So it is one-to-one and onto.Therefore invertible.

(b) i. Assume that both f and g are one-to-one.
$x_1 = x_2$
$g(x_1) = g(x_2)$ [$\because$ g is one-to-one]
$f(g(x_1)) = f(g(x_2))$ [$\because$ f is one-to-one]

$(f \circ g)(x) = f(g(x)) \Rightarrow f(g(x_1)) = f(g(x_2))$
$f \circ g$ is one to one function.

ii. Assume that both f and g are onto.
z is any element of c then there is some element $x \in A$ such that $f(g(x)) = z$
f is onto then $y \in B$ such that $f(y) = z$
g is onto & $y \in B$, then there is an element $x \in A$ such that $g(x) = y$.

$$z = f(y)$$
$$= f(g(x))$$
$$\therefore \quad f(g(x)) = z$$

$\therefore$ $f \circ g$ is onto.

(c)
$$(f \circ g)(x) = f(g(x)) = f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 5$$
$$(g \circ f)(x) = g(f(x)) = g(x^2+1) = x^2 + 1 + 2 = x^2 + 3$$

3. (a) Determine whether the relation R on the set of all people is reflexive, symmetric, anti-symmetric, and/or transitive, where $(a, b) \in R$ and justify your answers if and only if

i. a is taller than b.
ii. a and b were born on the same day.
iii. a has the same first name as b.
iv. a and b have a common grandparent.

(b) Suppose we have a set of workstation computers, denoted as C(= Workstation 1, Workstation 2, Workstation 3, Workstation 4), in an engineering lab. The relation R on C is defined as follows:
$R =$ (Workstation 1, Workstation 1), (Workstation 1, Workstation 3), (Workstation 2, Workstation 2), (Workstation 3, Workstation 1), (Workstation 3, Workstation 3), (Workstation 4, Workstation 1), (Workstation 4, Workstation 4)

i. Create a directed graph that represents the relationship R among the workstation computers in the engineering lab.
ii. Construct a binary matrix that corresponds to relation R for these workstation computers.
iii. Decide whether the relation is reflexive, symmetric, or transitive and justify your answer.

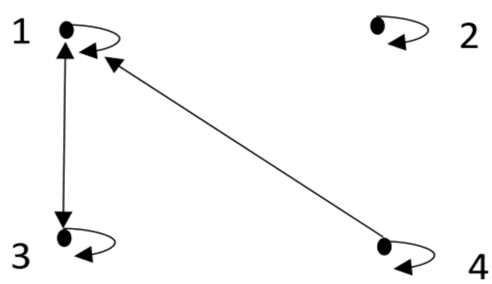
Solutions

(a) i. **reflexive** : this is not reflexive.because not every person taller then themselves.
symmetric : this is not symmetric.because a is taller than b it does not necessarily imply that b is taller than a.
anti-symmetric : this is anti-symmetric.because $(a, b) \in R$ & $(b, a) \notin R$.
transitive : this is transitive. because a is taller than b and b is taller than c. so a is taller than c.

ii. **reflexive** : this is reflexive. because every person shares their birth date with themselves.
symmetric : this is symmetric. because a and b were born on the same day implies that b and a were born on same day.
anti-symmetric : this is not anti-symmetric.
transitive : this is transitive. because (a, b) born on same day $\wedge$ (b, c) born same day than (a, c) born same day.

iii. **reflexive** : this is reflexive. because every person shares their first name with themselves.
symmetric : this is symmetric. because a has the same first name as b it implies that b has the same first name as a.
anti-symmetric : this is not anti-symmetric.
transitive : this is transitive. because (a, b) as the same first name and (b, c) as the same first name than (a, c) as the same first name.

iv. **reflexive** : this is reflexive. because every person has themselves as a common grandparent.
symmetric : this is symmetric. because (a, b) have a common grandparent. it implies that (b, a) have a common grandparent.
anti-symmetric : this is not anti-symmetric. (Ex):my cousin and I have a common grandparent , and I and my cousin have a common grandparent, but I'm not equal to my cousin.
transitive : this is not transitive. (Ex):I and my cousin have a common grandparent ,my cousin and her cousin on other side of family have a common grandparent. I and My cousin's cousin don't have a common grandparent.

(b) i. 

ii. $$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

iii. $(a, a) \in R$ for all $a \in A$.
we have $(1,1),(2,2),(3,3)$, and $(4,4) \in R$,
$\therefore$ R is reflexive.

$(a, b) \in R$ and $(b, a) \in R$.
we have $(4,1) \in R$ but $(1,4) \notin R$
$\therefore$ R is not symmetric.

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \;\; \forall a, b, c \in A$.
$(4,1),(1,3) \in R$ but $(4,3) \notin R$,
Therefore, R is not transitive.