MC4010

Tutorial - 01 Answers

1) p - The message is scanned for viruses.
q - The message was sent from an unknown system.

a) $q \longrightarrow p$

b) $q \wedge \neg p$

c) $q \longrightarrow p$

d) $\neg q \longrightarrow \neg p$

2) a) $[\neg p \wedge (p \vee q)] \longrightarrow q$.

last output

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \longrightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

P∨r

b) $[\underbrace{(p \rightarrow q)}_{(\neg p \vee q)} \wedge \underbrace{(q \rightarrow r)}_{(\neg q \vee r)}] \longrightarrow \underbrace{(p \rightarrow r)}_{(\neg p \vee r)}. \equiv P$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	P.
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c) $[p \wedge (p \rightarrow q)] \rightarrow q \equiv P$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	P
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d) $[(p \vee q) \wedge (p \overset{\overset{Q}{\equiv}}{\rightarrow} r) \wedge (q \rightarrow r)] \rightarrow r \equiv P$

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r)$	Q	P
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

3) a) $\neg (p \rightarrow \neg q)$

p	q	$\neg q$	$p \rightarrow \neg q$	$\neg (p \rightarrow \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	T	F
F	F	T	T	F

TT	F
TF	T
FT	T
FF	F

$p \oplus q$.

3] b) $p \oplus (p \vee q)$ Xor

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F.

TT T
FF T

c) $\neg(p \wedge q) \vee \neg(q \leftrightarrow p) \equiv p$.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$q \leftrightarrow p$	$\neg(q \leftrightarrow p)$	p
T	T	T	F	T	F	F
T	F	F	T	F	T	T
F	T	F	T	F	T	T
F	F	F	T	T	F	T

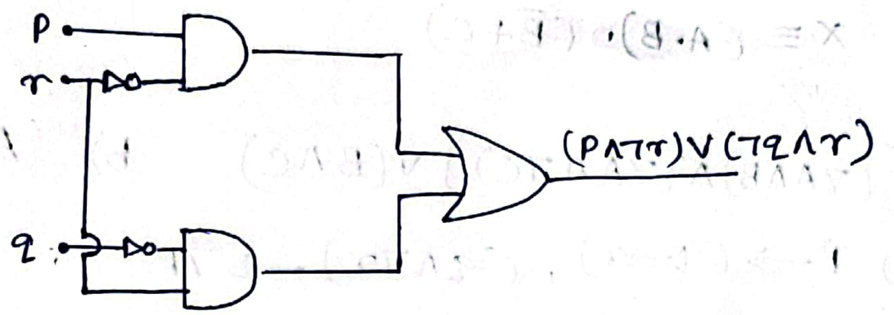
d) $(p \rightarrow q) \vee \neg(p \leftrightarrow \neg q) \equiv p$

p	q	$p \rightarrow q$	$\neg q$	$p \leftrightarrow \neg q$	$\neg(p \leftrightarrow \neg q)$	p
T	T	T	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	T
F	F	T	T	F	T	T.

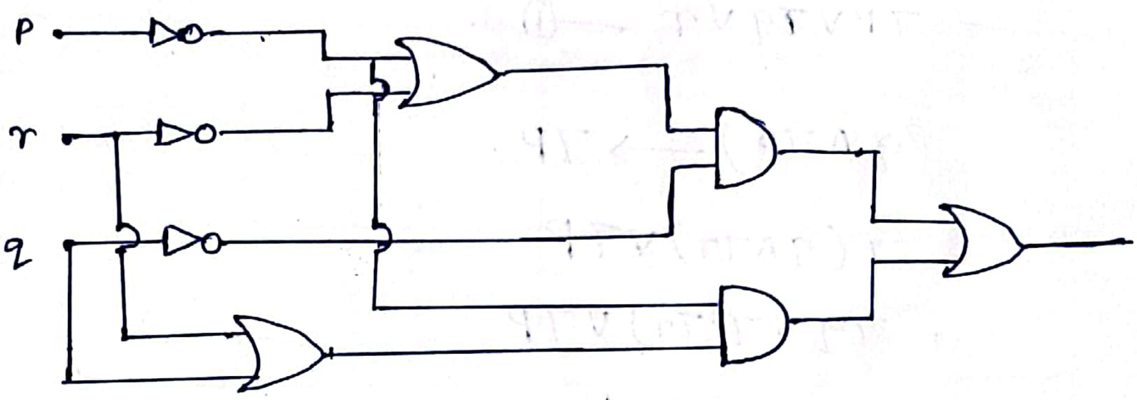
e) $[p \rightarrow (\neg q \vee r)] \wedge \neg [q \vee (p \leftrightarrow \neg r)] \equiv p$

p	q	r	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$	$\neg r$	$p \leftrightarrow \neg r$	$q \vee (p \leftrightarrow \neg r)$	$\neg [q \vee (p \leftrightarrow \neg r)]$	p
T	T	T	F	T	T	F	F	T	F	F
T	T	F	F	F	F	T	T	T	F	F
T	F	T	T	T	T	F	F	F	T	T
T	F	F	T	T	T	T	T	T	F	F
F	T	T	F	T	T	F	T	T	F	F
F	T	F	F	F	T	T	F	T	F	F
F	F	T	T	T	T	F	T	T	F	F
F	F	F	T	T	T	T	F	F	T	T

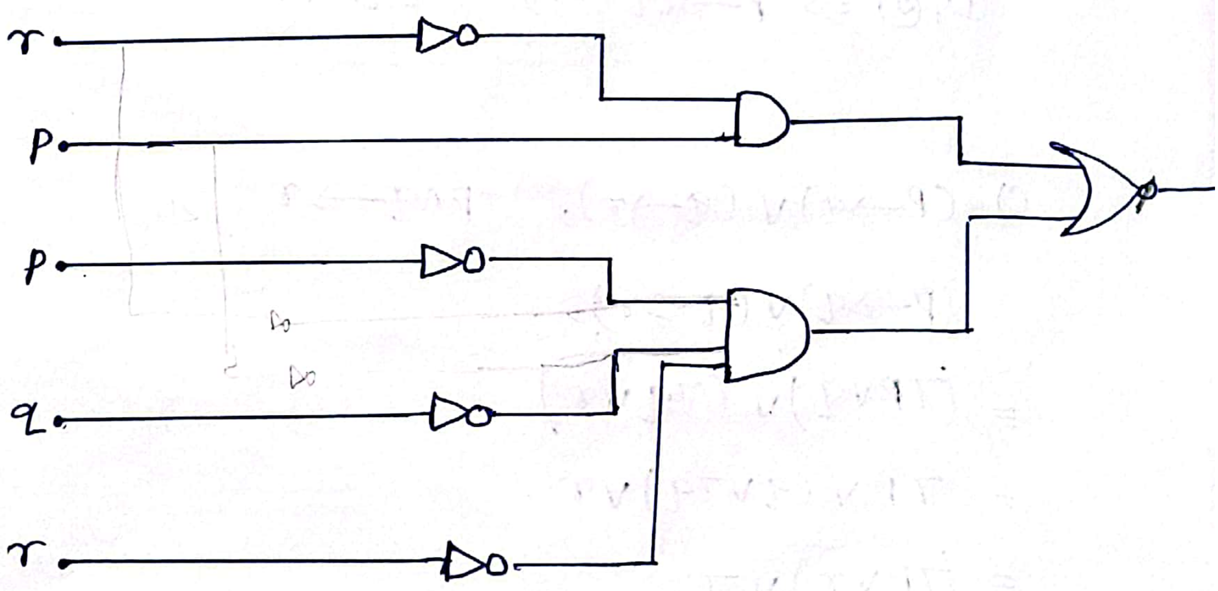
4) a) $(p \wedge \neg r) \vee (\neg q \wedge r)$.



b) $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$.



12) ~~a)~~ $(p \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$.



5]

a) $[(\neg A \wedge B) \wedge (\neg A \wedge \neg C)] \vee (B \wedge C)$ b) $(A \wedge B) \wedge (\neg B \vee C)$

6] a) $p \rightarrow (q \rightarrow r)$, $(q \wedge \neg r) \rightarrow \neg p$

$p \rightarrow (q \rightarrow r)$

$\equiv p \rightarrow (\neg q \vee r)$

$\equiv \neg p \vee (\neg q \vee r)$

$\equiv \neg p \vee \neg q \vee r$ — ①

$(q \wedge \neg r) \rightarrow \neg p$.

$\equiv \neg (q \wedge \neg r) \vee \neg p$.

$\equiv \neg q \vee \neg(\neg r) \vee \neg p$

$\equiv \neg q \vee r \vee \neg p$

$\equiv \neg p \vee \neg q \vee r$ — ②

①, ② $\Rightarrow p \rightarrow (q \rightarrow r) \equiv (q \wedge \neg r) \rightarrow \neg p$.

c) $(p \rightarrow q) \vee (q \rightarrow r)$, $p \wedge q \rightarrow r$

$(p \rightarrow q) \vee (q \rightarrow r)$

$\equiv (\neg p \vee q) \vee (\neg q \vee r)$

$\equiv \neg p \vee (q \vee \neg q) \vee r$

$\equiv (\neg p \vee r) \vee T$

$\equiv \neg p \vee r$ — ①

$p \wedge q \longrightarrow r$.

$\equiv \neg(p \wedge q) \vee r$.

$\equiv \neg p \vee \neg q \vee r$ —— ②

①, ② $\Rightarrow$ $(p \rightarrow q) \vee (q \rightarrow r) \not\equiv p \wedge q \longrightarrow r$

b) $p \leftrightarrow (q \leftrightarrow r)$, $(p \leftrightarrow q) \leftrightarrow r$

p	q	r	$p \leftrightarrow q$	$q \leftrightarrow r$	$p \leftrightarrow (q \leftrightarrow r)$	$(p \leftrightarrow q) \leftrightarrow r$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	T	T	T
F	T	T	F	T	F	F
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	F	T	T	F	F

$\therefore p \leftrightarrow (q \leftrightarrow r) \equiv (p \leftrightarrow q) \leftrightarrow r$.

9

7) a) $\{(p \rightarrow q) \wedge \neg q\} \longrightarrow \neg p$

$\equiv \neg[(p \rightarrow q) \wedge \neg q] \vee \neg p$

$\equiv \neg[(\neg p \vee q) \wedge \neg q] \vee \neg p$

$\equiv [\neg(\neg p \vee q) \vee q] \vee \neg p$ (De-morgans law)

$\equiv [(p \wedge \neg q) \vee q] \vee \neg p$ (De-morgans law)

$\equiv [(p \vee q) \wedge (\neg q \vee q)] \vee \neg p$ (Distributive law)

$\equiv [(p \vee q) \wedge T] \vee \neg p$ (Negation law)

$\equiv (p \vee q) \vee \neg p$ (Identity law)

$\equiv (p \vee \neg p) \vee q$ (Associative law)

$\equiv T \vee q$ (Negation law)

$\equiv T$ (Domination law).

b) $[(p \rightarrow q) \rightarrow p] \longrightarrow p$

$\equiv \neg[(p \rightarrow q) \rightarrow p] \vee p$

$\equiv \neg[\neg(p \rightarrow q) \vee p] \vee p$

$\equiv \neg[\neg(\neg p \vee q) \vee p] \vee p$

$\equiv [(\neg p \vee q) \wedge \neg p] \vee p$ (De-morgans law)

$\equiv [(\neg p \wedge \neg p) \vee (q \wedge \neg p)] \vee p$ (Distributive law)

$\equiv [\neg p \vee (q \wedge \neg p)] \vee p$ (Idempotent law).

$\equiv (\neg p \vee p) \vee (q \wedge \neg p)$ (Associative law)

$\equiv T \vee (q \wedge \neg p)$ (Negation law)

$\equiv T$ (Domination law).

c) $\neg (p \wedge q) \rightarrow (p \vee \neg q)$

$\equiv \neg [\neg (p \wedge q)] \vee (p \vee \neg q)$

$\equiv (p \wedge q) \vee (p \vee \neg q)$

$\equiv p \wedge (q \vee \neg q)$

$\equiv p \wedge T$

$\equiv p$

8) ~~a)~~ ~~Assuming~~ P(x) – x is a clear explanation

Q(x) – x is satisfactory

R(x) – x is an excuse.

a) $\forall x \, (P(x) \rightarrow Q(x))$

b) $\exists x \, (R(x) \wedge \neg Q(x))$

c) $\exists x \, (R(x) \wedge \neg P(x))$

d) Assume that (a) and (b) are true.

i.e) $\exists x \, (R(x) \wedge \neg Q(x))$ is true.

and

$\forall x, (P(x) \rightarrow Q(x))$ is true.

If we take the same x that satisfies $(R(x) \wedge \neg Q(x))$, we can conclude that $\neg P(x)$ must be true.

Because if it were not, then $(P(x) \rightarrow Q(x))$ would require $Q(x)$ to be true, contradicting $\neg Q(x)$ from (b).

So (c) is also true because we found an x that satisfies $(R(x) \wedge \neg P(x))$.

$\therefore$ (c) follows from (a) and (b).

9) a) p – I enter the Poodle den.
q – I will carry my electric poodle prod.
r – My can of mace.

$p \rightarrow (q \vee r), \; q \wedge \neg r \;\vdash p$.

Assume

$p \rightarrow q \vee r \equiv T$ — (1)

$q \wedge \neg r \equiv T$ — (2)

(2) $\Rightarrow q \wedge \neg r \equiv T$

$\therefore \neg r \equiv T \;\wedge\; q \equiv T$

$\Rightarrow r \equiv F$.