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MC 4010 - Tutorial 02

1. Find the cardinality in the following sets,

   (a) $A = \{x \in \mathbb{R} : x^2 + 2x + 2 = 0\}$

   (b) $B = \{a, b, c, \{a, b, c\}\}$.

   (c) $C = \{\Phi, \{\Phi\}, \{\Phi, \{\Phi\}\}\}$

2. Let $X = [0, 5)$, $Y = [2, 4]$, $Z = (1, 3]$ and $W = (3, 5)$ be intervals in $\mathbb{R}$. Find in each of the following sets:

   (a) $Y \cup Z$

   (b) $Z \cap W$

   (c) $X - (Z \cup W)$

   (d) $\overline{Z}$

3. Show that the following two sets are equal:

   A= $\{x \in \mathbb{Z} | x = 1 + 3q, \text{for some } q \in \mathbb{Z}\}$.

   B= $\{x \in \mathbb{Z} | x = -2 + 3q, \text{for some } q \in \mathbb{Z}\}$.

4. Simplify the set, $(A) \cap (\overline{\overline{A} \cap B})$.

5. Prove that $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

6. Prove that

   (a) $(A - B) \cup (A \cap B) \cup (B - A) = A \cup B$

   (b) $A \cap (B - C) = (A \cap B) - (A \cap C)$

   (c) $(A \times B) \cap (A \times C) = A \times (B \cap C)$

   (d) $(A \times B) \cup (A \times C) = A \times (B \cup C)$

   (e) If $A \cap B = A \cap C$, $A \cup B = A \cup C$ then $B = C$

7. For each of the following functions F determine whether or not F is (i) injective(one to one), (ii) surjective(onto). Justify your answers.

   (a) $F : \{a, b, c, d, e\} \to \{a, b, c, d, e, f\}$
   $a \to b$, $b \to e$, $c \to f$, $d \to c$, $e \to a$

   (b) $F : \{a, b, c, d, e, f, g\} \to \{a, b, c, d, e\}$
   $a \to e$, $b \to c$, $c \to d$, $d \to a$, $e \to d$, $f \to e$, $g \to a$

8. In a survey of 1400 households, 450 owned a home computer, 500 a mini-tab, 350 home theatre, and 450 households owned neither a home computer, nor a mini-tab, nor home theatre. Given that 200 households owned both a home computer and a mini-tab, 140 both a mini-tab and home theatre, and 110 both home theatre and a home computer, find the number of households surveyed which owned:

   (a) a home computer, a mini-taps and home theatre;

   (b) a mini-tab only;

   (c) home theatre and mini-tab but not a home computer;

   (d) a mini-tab and home computer but not home theatre.

9. Use a membership table to show that

   (a) $(A \cup B) - C = (A - C) \cup (B - C)$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   (c) $(A - B) \cup (A \cap B) \cup (B - A) = A \cup B$

10. Show that the function $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R} \setminus \{-1\}$ given that;

$$f(x) = \frac{x-3}{x+1}$$

is a bijective function.

11. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$ are function from $\mathbf{R}$ to $\mathbf{R}$.

12. Show that the function $f : \mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$ defined by

$$f(x) = \frac{x}{1+|x|}, \quad \forall x \in \mathbb{R}$$

is a bijective function.

13. Define the relation $\sim$ on $\mathbb{Z}$ by

$$x \sim y \text{ if and only if } \frac{x-y}{2} \in \mathbb{Z}$$

Show that $\sim$ is an equivalence relation. Describe the equivalence classes $[0], [1], \left[\frac{1}{2}\right]$.

14. Let S be a relation on the set R of all real numbers defined by

$$S = \left\{(a, b) \in \mathbb{R}^2 |\ a^2 + b^2 = 1\right\}.$$

Prove that S is not an equivalence relation on $\mathbb{R}$.

15. Let R be relation defined on the set of natural numbers $\mathbb{N}$ as follows

$$R = \{(x, y); x \in \mathbb{N}, y \in \mathbb{N},\ 2x + y = 41\}$$

Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

16. Let R and S be partial order on a set A. Determine whether the union relation $R \cup S$ is also partial order on A.

17. Draw a Venn diagram for the symmetric difference of the sets A and B
$A \oplus B = (A - B) \cup (B - A)$.

18. (a) Let $A = \{1, 2, 3, 4\}$ and let R be the relation on A defined by:

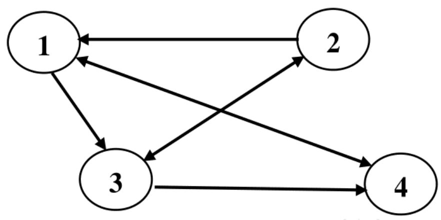
$$R = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 1), (4, 4)\}$$

i. Draw the directed graph of R.

ii. Write down the binary matrix of R

iii. Decide whether the relation is reflexive, symmetric or transitive and justify your answer.

(b) Faculty of Engineering, University of Jaffna arranged a quiz competition among four Departments. Each of them **1**- Civil, **2** - Mechanical, **3** - Computer, **4** - Electrical compete with every other departments. A relation R on the set $S = \{1, 2, 3, 4\}$ is defined by: xRy if and only if x beat y. The following diagram is the directed graph of R.



i. List the elements of R, and Prepare the points table (Total Quiz competitions, Win, Loss) for each department.

ii. Identify the Reflexive, Symmetry ,Anti-Symmetry and Transitive Properties of this Relation.

19. (a) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

(b) Let p be any odd number,Show that $A = \{x \mid x = p + 2m, \quad for\ some \quad m \in \mathbb{Z}\}$ is equal to the set of all odd numbers.

(c) The symmetric difference $X \oplus Y$ of sets X and Y is defined by:

$$X \oplus Y = (X - Y) \cup (Y - X)$$

Find a counter-example to disprove the proposition that, for all sets A,B and C.

$$A \cup (B \oplus C) = (A \cup B) \oplus (A \cup C)$$

20. (a) Let f be the function from the set $X = \{2, 3, 4, 5, 6, 7\}$ to set $Y = \{0, 1, 2, 3, 4\}$ defined by $f(x) = 2x \ (mod\ 5)$. Write f as a set of ordered pairs. Is f one-to-one or onto Y.

21. (a) Indicate whether the given relations are functions and justify your answer

i. $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 3)\}$

ii. $\{(x, |x|) : x \in \mathbb{R}\}$

(b) Consider $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 2x + 1$, Show that f is both injective and surjective.

22. The given directed graph indicate the R on a set A:

$$A = \{a, b, c, d, e\}$$

Check whether the above relation is satisfy the four properties and justify your answer.