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MC 4010 : Discrete Mathematics

Tutorial-05 **November 2023**

1. A space probe orbiting around Neptune uses bit strings (sequence of bits) to communicate with Earth. The probe sends a 1 in one-third of its transmissions and a 0 in two-thirds of its transmissions. When the probe sends a 0, there is a 90% chance that it will be received correctly, and a 10% chance that it will be received incorrectly as a 1. Similarly, when the probe sends a 1, there is an 80% chance that it will be received correctly, and a 20% chance that it will be received incorrectly as a 0.
   (a) Find the probability that a 0 is received.
   (b) Use Bayes' theorem to find the probability that a 0 was transmitted, given that a 0 was received.

2. A company produces machine components which pass through an automatic testing machine. 5% of the components entering the testing machine are defective. However, the machine is not entirely reliable. If a component is defective , there is 4% probability that it will not be rejected. If a component is not defective, there is 7% probability that it will be rejected.
   (a) What fraction of all the components are rejected.
   (b) What fraction of the components rejected are actually not defective.
   (c) What fraction of those not rejected, are defective.

3. An octahedral die has 8 faces that are numbered 1 through 8.
   (a) What is the expected value of the number that comes up when a fair octahedral die is rolled?
   (b) What is the variance of the number that comes up when a fair octahedral die is rolled

4. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that
   (a) at least 10 survive,
   (b) from 3 to 8 survive,
   (c) exactly 5 survive

5. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.
   (a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?
   (b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

6. From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that
   (a) all 4 will fire?
   (b) at most 2 will not fire?
7. What is the probability that a waitress will refuse to serve alcoholic beverages to only 2 minors if she randomly checks the IDs of 5 among 9 students, 4 of whom are minors?
8. An inventory study determines that, on average, demands for a particular item at a warehouse are made 5 times per day. What is the probability that on a given day this item is requested
   (a) more than 5 times?
   (b) not at all?
9. On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection
   (a) exactly 5 accidents will occur?
   (b) fewer than 3 accidents will occur?
   (c) at least 2 accidents will occur?
10. Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations L1, L2, L3, and L4 will be operated $40\%, 30\%, 20\%$ and 30% of the time. If a person who is speeding on her way to work has probabilities of $0.2, 0.1, 0.5$ and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?
11. (a) Show that if X and Y are independent random variables, then
    $$Var(XY) = E(X)^2Var(Y) + E(Y)^2Var(X) + Var(X)Var(Y)$$
    (b) Show that $Var(X) = np(1-p)$ if random variable X follows binomial distribution with parameters (n, p). Also show that $Var(X) = \lambda$ if random variable X follows Poisson distribution with parameter λ.
    (c) The final exam of a discrete mathematics course consists of 50 true/false questions, each worth two points, and 25 multiple-choice questions, each worth four points. The probability that Linda answers a true/false question correctly is 0.9, and the probability that she answers a multiple-choice question correctly is 0.8. What is her expected score on the final?
12. In a support system in the U.S. space program, a single crucial component works only 90% of the time. In order to enhance the reliability of the system, it is decided that 3 components will be installed in parallel such that the system fails only if they all fail. Assume the components act independently and that they are equivalent in the sense that all 3 of them have an 90% success rate. Consider the random variable X as the number of components out of 3 that fail.
    (a) Write out a probability function for the random variable X.
    (b) What is $E(X)$ (i.e., the mean number of components out of 3 that fail)?
    (c) What is $Var(X)$?
    (d) What is the probability that the entire system is successful?
    (e) What is the probability that the system fails?