4-12-1985

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SPECTROPHOTOMETRIC COLOR FORMULATION BASED ON TWO-CONSTANT KUBELKA-MUNK THEORY

by

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A thesis submitted in partial fulfillment
of the requirements for the degree of
Bachelor of Science in the School of
Photographic Arts and Sciences in the
College of Graphic Arts and Photography
of the Rochester Institute of Technology

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ABSTRACT

A new approach to computer color formulation based on non-linear least-squares techniques has been developed to characterize colorants and predict their behavior in mixtures. This approach allows the optimization of absorption and scattering calculations to characterize these colorants. This same method has been used to directly match the spectral reflectance of a standard with a mixture of colorants that yields nearly the same spectral reflectance as the standard being matched. Several advantages have been gained over more traditional methods: Kubelka-Munk K and S has been determined without primary binary blends, the spectral reflectance of the standard and proposed formulation exhibit lower spectral difference, the use of spectrally similar colorants has been improved and, formulations can now be predicted for standards measured over wavelength regions other than the visible spectrum.

ACKNOWLEDGMENTS

The author wishes to express special gratitude to Mr. Cornelius McCarthy of Bausch & Lomb Company, Analytical Products Division and Dr. Roy Berns of The Rochester Institute of Technology, Munsell Color Science Laboratory, who have supplied equipment, ideas, time and inspiration toward the success of this thesis. Certainly, this research would not have been accomplished without their help.

A word of thanks also goes to Dominick A. Burlone of Badische, Anderson, South Carolina and Roland S. Connelly of Burlington Industries, Greensboro, North Carolina for their helpful discussions and provision of necessary data in a timely fashion.

Special thanks to Bausch & Lomb Company for funding this research.

Dedicated to Dr. Franc Grum

ROCHESTER INSTITUTE OF TECHNOLOGY

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Date: April 12, 1985

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II. INTRODUCTION

A. Background

For many years it has been the task of various industries to provide materials of a specific color requested by a customer. The customer provides a standard of the desired color and it becomes the job of the of the manufacturer to produce a product whose color matches that of the standard. For instance, in the textile industry, heather blends (fine tweeds) are made from fibers that, if each were viewed in bulk, appear to be of similar color. These fibers are blended in such a way as to yield a desired color. Similarly, a standard blend can be matched by choosing a field of colorants (the bulk fibers), which are combined in the proper proportions (concentrations) such that the mixture (blend) is of the same perceived color of the standard for specified viewing conditions. If successful, the match evokes the same perceptual response as that of the standard. One way of finding a match is the trial and error approach of mixing various colorants until the color of the mixture appears the same as that of the standard under specific viewing conditions. Alternatively, a spectrophotometer can be employed to measure reflectances, over the visible range of the spectrum. Spectrophotometric measurements of the standard and the prospective colorants can be analyzed to yield further information regarding the correct colorants and their relative proportions. Since many data points are gathered, this kind of matching operation is best performed using modern digital computers and is known as computer color formulation. The objective of computer color formulation is to find the proportions of each colorant in a set that, when mixed, have a reflectance, that appears to have the same color as that of the standard under specified viewing conditions. If a systematic prediction can be used to select the correct colorants and their relative proportions to accurately match the color of a standard, much savings in terms of time, money and, wasted material is gained over using a trial and error approach. This systematic procedure is based on the Kubelka-Munk theory and involves two steps. First, it is necessary to characterize the field of colorants (primaries) that will be used to formulate the proposed match in terms of absorption and scattering coefficients. Second, once the primaries have been characterized, the selection of the correct colorants and their relative proportions must be determined.

B. Kubelka-Munk Theory

Kubelka-Munk turbid media theory adequately explains the behavior of mixtures of colorants by quantitatively characterizing the absorption and scattering properties of the colorants and their mixtures over the visible region of the spectrum. A perfect black absorbs all of the radiation incident upon it and scatters back none of it, while a perfect white absorbs none and scatters all. Real colors lie between these extremes, but their absorption and scattering coefficients are expressed relative to black and white. For any wavelength, reflectance R and Kubelka-Munk absorption K and scattering S may be interchanged using the following transformations:

$$K/S = (1-R)^2 / 2R, \text{ and}$$

$$R = 1 + K/S - \sqrt{(K/S)^2 + 2(K/S)} \ .$$

Frequently, it is not necessary to know explicitly both the absorption and the scattering of the colorants, but rather the ratio K/S is enough to predict the spectral properties of mixtures. This is known as Kubelka-Munk single-constant theory [1,2]. Single-constant theory applies when the scattering of the mixture is not dependent on the colorant or its concentration or if the scattering of the colorants is negligible in comparison with that of the absorbing substrate, as in the case of dyes. Generally, scattering does depend upon colorant concentration and is not negligible in comparison with that of the substrate to which they are applied, as in the case of paints. Therefore, both K and S must be known explicitly and is known as Kubelka-Munk two-constant theory [1,2]. This research is confined to the case of industrial importance where two-constant theory accounts for the physical effects of turbid media. Two-constant theory is developed below as it applies to the characterization of colorants (K and S determination) and the calculation, of colorant concentrations (color matching).

1. K and S Determination

The conventional method for characterizing the absorption and scattering properties of colorants over wavelength has been through the preparation of special mixtures known as primary binary blends [3]. For every colorant of interest, a sample of each of the following is prepared: masstone (pure colorant), binary mixture of masstone with white, or a binary mixture of masstone with black, or both. The spectral reflectance of each sample is measured spectrophotometrically, and surface corrections, if any, are made [4]. The calculation of K and S for the colorant of interest actually requires only two samples which yield the two independent linear equations required to solve for K and S:

$$(K/S)m = (1-Rm)^2 / 2Rm$$

$$= CmKm / CmSm$$

$$(K/S)mix = (1-Rmix)^2 / 2Rmix$$

$$= (CmKm + CwKw) / (CmSm + CwSw)$$

Where (K/S)m (K/S of the masstone), Cm (concentration of the masstone), Rmix (reflectance of the binary mixture with, say, white), Cw (concentration of white), Kw (absorption coefficient of white), Sw (scattering of white) are all known. This yields two equations and two unknowns allowing the explicit solution for Km (absorption coefficient of the masstone) and Sm (scattering coefficient of the masstone).

An improvement can be made through the use of the third mixture which increases the reliability of the K and S determination [5]. This approach uses combinations of the three mixtures taken two at a time to compute a set of K and S values which are then used to predict the reflectance of the third mixture not used in the K and S determination. This is repeated for the other combinations and the K and S values predicted by each combination are compared. For each measured wavelength, the methods that predict the reflectance of the third mixture most accurately are averaged to yield the K and S values which are then stored for future use. This entire procedure is then repeated for each colorant of interest.

This procedure becomes quite time-consuming and expensive and does not always give accurate results. Frequently, the K and S values produced by the different combinations do not agree very well, and the accuracy of the K and S values determined in this fashion must seriously be questioned [6]. The validity of this method is also questionable since masstones tend to be mixed with each other when matching, rather than mixtures of masstone with black or white. If there is any effect on the absorption and scattering properties of a colorant by the presence of other colorants, as in the case of the blending of fibers, then the difference between the way colorants are characterized and the way they are used may be important. Furthermore, the discrepancies would be expected to grow when the nature of reflection, absorption, and scattering of the actual colorants is more complex than the simple two-flux assumption made under Kubelka-Munk theory [7].

2. Color Matching

Once the colorant characteristics have been determined, it is then desirable to know which ones are required, and in what proportions, to match the color of the standard. In practice only the ratio K/S of the standard is known, while the separate K and S values of the prospective colorants have been previously determined. K/S is calculated by dividing the sum of the products of colorant concentrations and absorption coefficients by the sum of the products of colorant concentrations and scattering coefficients for the i different colorants:

$$K/S \text{ predicted} = \Sigma_i CiKi \,/\, \Sigma_i CiSi.$$

The object is to find the concentrations Ci for each colorant such that K/S predicted becomes nearly equal to the actual K/S:

$$K/S \text{ actual} = (1\text{-Ractual})^2 / \ 2\text{Ractual}.$$

When this is satisfied, actual reflectance is nearly equal to the predicted reflectance. Typically, reflectance data is taken every ten nanometers from 400nm to 700nm to represent the standard and colorants, and four colorants are used to match the standard [8]. The K and S coefficients have previously been determined for each colorant at each wavelength and the reflectance of the standard has been converted to K/S. This yields thirty-one equations in the parameters K/S, C, K, and S, where all parameters are known except the concentrations of each colorant, C. The manner in which this overdetermined system of equations is solved for colorant concentration determines the type of match performed. If the system of equations is solved for the colorant concentrations yielding minimum tristimulus difference, a tristimulus match is performed. Alternatively, if the solution is made for the colorant concentrations that yield minimum spectral difference, a spectral match is performed. These methods are discussed in detail below. Both approaches have been used based on single-constant theory, while for the more complex two-constant theory, only tristimulus matching has been reported.

a. Tristimulus Matching

Tristimulus matching involves matching a standard's tristimulus values with a set of colorants that, when mixed, yield the same tristimulus values as that of the standard being matched [9-11]. Tristimulus values incorporate the reflectance data taken over all wavelengths by a relationship of the form,

$$TSVi = k\sum_{\lambda} R(\lambda) \times S(\lambda) \times O(\lambda)i,$$

where $R(\lambda)$ is the spectral reflectance of the standard over wavelength λ, $S(\lambda)$ is the spectral power distribution of the reference illuminant over wavelength, $O(\lambda)i$ are the three observer color matching functions i over wavelength, k is a normalizing factor, and TSVi is the resulting tristimulus value for each observer color matching function. A set of simultaneous equations with as many tristimulus values as colorants is created, which is then solved for colorant concentrations that yield the minimum tristimulus difference between standard and predicted match.

Given that different reflectance curves may integrate to the same tristimulus values, the predicted match may have a reflectance curve that is different from or metameric to that of the original sample even though the tristimulus values of the standard and the match are equal. Metamerism occurs when two samples have the same tristimulus values under a primary illuminant and different tristimulus values under secondary illuminants. The samples may look the same under one source but not the others if their spectral curves are not identical. Additionally, the color matching functions exist only over the visible spectrum, hence if spectral characteristics in a band outside the visible are of interest, tristimulus values yield no weight in the desired band. The military, for instance, is particularly interested in controlling the infrared reflectance of camouflage materials. Furthermore, tristimulus matching makes use of information of only a limited number of parameters, the tristimulus values of the standard, derived from all of the wavelength information available, and therefore may not yield the best values when the match is being made from colorants that are of quite similar color. Although changing the concentration of one of these colorants may change the predicted reflectance substantially, changing the concentrations of the other spectrally similar colorants in the set may have a nearly identical effect on the resulting tristimulus values. This poor sensitivity of the tristimulus values of the match to spectrally similar colorants frequently leads to unreasonable predicted concentrations and is typical of a class of problems where the parameters are highly correlated.

b. Spectral Matching

McGuiness proposed an algorithm that allows the reflectance spectrum of a standard to be compared directly to the reflectance produced by a set of colorants at concentration as determined using Kubelka-Munk single-constant theory [12]. Spectral matching eliminates illuminant and observer dependence in the calculations as well as the loss of sensitivity associated with tristimulus reductions. Matches generated in this manner will exhibit less spectral difference than tristimulus matches and can be performed over any spectral region.

Spectral matching, as it has been applied to the single-constant case, makes use of linear optimization to calculate the colorant concentrations that minimize the sum of squares difference between the K/S curves of the standard and predicted match at every wavelength of interest. Linear optimization is possible since by definition of Kubelka-Munk single-constant theory, K/S of the mixture is linear with the product of colorant concentration and K/S:

$$K/S = \sum_i Ci(K/S)i.$$

Little has been published explicitly on spectral matching for the non-linear two-constant case. A non-linear approach for two-constant theory seems necessary since by definition K/S of the mixture is proportional to the sum of the products of colorant concentrations and absorption coefficients but inversely proportional to the sum of the products of colorant concentrations and scattering coefficients [13].

C. Limitations of Computer Color Formulation

Computer color formulation, however helpful, is not perfect, namely because of the large number of parameters affecting successful implementation of formulated recipes [14-17]. Computer color formulation deals only with the manipulation of spectral data to yield better recipes. The production of the recipes has its associated problems, the solution of which are beyond the scope of this research. Briefly, confounding the manufacturing and industrial problems of mixing the formulae there are associated instrumental problems such as calibration as well as the problems implicit in sample preparation and presentation. Inherent complexities of turbid media theory are also a factor.

D. Outlook

Present methods of computer color formulation have their inherent limitations. K and S determination using primary binary blends may not always yield accurate matching results. This happens as the nature of the colorants depart from the assumptions of Kubelka-Munk theory. Tristimulus matching is limited to the visible spectrum and to well defined colorants and frequently results in a match whose spectral curve is quite different from that of the standard. Spectral matching eliminates these problems with tristimulus matching but has only been applied to single-constant theory. For these reasons, a more robust approach to K and

S determination and color matching is needed to alleviate some of the problems associated with traditional computer color formulation based on Kubelka-Munk two-constant theory.

III. EXPERIMENTAL

The technique described below extends the principles associated with spectral matching to two-constant theory. A single approach to both K and S determination and spectral matching based on two-constant theory is referred to here as spectrophotometric color formulation. This method uses a linear simplification followed by iterative improvement using non-linear optimization techniques resulting in a spectrophotometric color formulation algorithm that will handle the non-linear two-constant situation.

A. Theoretical Development

1. Background

As discussed above, spectral matching refers to the determination of the correct colorant concentrations that results in a spectral curve least different from that of the standard. To perform a spectral match, the reflectance of the standard over wavelength is converted to K/S, and the coefficients (concentrations) of K and S of the primaries being used to match the standard are selected such that the spectral difference between standard and match is a minimum. This same idea can be extended to K and S determination. If the reflectance at any wavelength of several mixtures of colorants and the proportions of each colorant are known, then the values of K and S that yield the closest values of the actual K/S of the mixtures can be determined.

It is possible to simplify the problem by assuming that reflectance can be accurately converted to K/S using:

$$K/S\ \text{actual} = (1-R\text{actual})^2 / 2R\text{actual}.$$

Ractual is the true reflectance value of a standard for a spectral match or a true reflectance value of a mixture for a K and S determination. Since,

$$K/S\ \text{predicted} = \sum_i CiKi / \sum_i CiSi,$$

and can be rewritten as,

$$-\sum_i CiKi + (K/S\ \text{actual}) \sum_i CiSi = 0,$$

then all terms are linear. The Ki and the Si are Kubelka-Munk absorption and scattering coefficients respectively for the i colorants at concentration Ci for a particular wavelength or mixture. K/S predicted is the quantity that must equal K/S actual.

It can be seen that if the Ki and the Si over wavelength are known then the Ci may be linearly determined to perform a spectral match. Alternatively, if the Ci are known for different mixtures then the Ki and the Si may be computed to perform a K and S determination.

At this point it is helpful to define several matrices. These matrices are depicted below.

When calculating the K and S values of a set of primaries, several mixtures of these primaries with each other are required at known concentration levels. A matrix [KSCOEFS] has as its row elements the coefficients of K and S, namely the concentrations of the primaries for that mixture. This matrix will have as many rows as mixtures. At least two mixtures are required for each pair of K and S parameters to satisfy the degrees of freedom of the problem. The solution vector [KANDS] will have in the first half of its elements the computed values of K for the primaries and in the second half it will have the computed values of S.

When performing a spectral match, the K and S values of the primaries are known over wavelength, and it is the concentrations that need to be determined. A matrix [CCOEFS] can be formed that has as its row elements the coefficients of concentration, namely the Ki and the Si. This matrix will have as many rows as wavelengths for which spectral data was obtained. At least as many wavelengths must be included as colorant concentrations to be determined to satisfy the degrees of freedom of the problem. The solution vector [C] will contain the computed colorant concentrations.

In both cases the vector [OBS] contains the right side of the equation, namely zeros except for a constraint. Unconstrained, these problems are indeterminate, since the observation vector would contain all zeros and there is an infinite number of solutions yielding a zero observation vector. One way of constraining the system of equations is to add a unit row to the coefficient matrices. In a K and S determination, this has the effect of forcing the sum of the absorption and scattering coefficients of a set of colorants to unity, or the sum of the concentrations to unity in the case of a spectral match.

The parameters of interest are then computed in one cycle using the well known matrix manipulation for solving an overdetermined system of linear equations [18]:

$$[B] = \{ [X]t[X] \}^{-1}[X]t[Y].$$

The solution vector [B] contains the parameters of interest, [X]t is the transpose of the coefficient matrix [X], and [Y] is the vector of observed values for the number of observations. In a K and S determination, [B] contains a K and S value for each primary at a particular wavelength, [X] contains colorant concentrations for the different mixtures. In the case of a spectral match, [B] contains colorant concentrations, [X] contains absorption and scattering coefficients over the wavelength range of interest.

2. Explicit linear solution for K and S

In a K and S determination for, say, four colorants eight parameters need to be determined (the K and S values of each colorant at a particular wavelength). Therefore, at least eight mixtures are required to create an exactly determined system of eight equations in the eight unknowns. However, if the entire mixture history involving only these four colorants is available, then the additional mixtures can be used to define an overdetermined system of equations, thereby increasing the confidence in the calculated values of K and S. This method is outlined below:

1) K/S for a particular mixture at a particular wavelength is computed from reflectance and is known to contain the four colorants at known concentration, hence:

$$\left(\frac{K}{S}\right)_{mix} = \frac{C1K1 + C2K2\ C3K3 + C4K4}{C1S1 + C2S2\ C3S3 + C4S4}$$

2) Cross-multiplying and equating to zero yields:

$$-C1K1-C2K2-C3K3-C4K4 + (K/S)mix[\ C1S1+C2S2+C3S3+C4S4\ ] = 0$$

3) Since the concentrations of the colorants in the mixture and the K/S of the mixture are known constants, they are the terms of a row in the coefficient matrix. Making the substitutions: $X1 = -C1$, $X2 = -C2$, $X3 = -C3$, $X4 = -C4$,

X5 = (K/S)mixS1, X6 = (K/S)mixS2, X7 = (K/S)mixS3, X8 = (K/S)mixS4 and repeating for the j different mixtures at a particular wavelength yields:

$$[KSCOEFS] = \begin{array}{c} \begin{bmatrix} X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ & & & & & & & \\ X1 & X2 & X3 & X4 & X5 & X6 & X7 & X8 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} \text{mixture 1} \\ \text{mixture 2} \\ \text{mixture 3} \\ \text{mixture 4} \\ \text{mixture 5} \\ \text{mixture 6} \\ \text{mixture 7} \\ \text{mixture 8} \\ \quad : \\ \text{mixture j} \\ \text{constraint} \end{array}$$

4) The observation vector is formed from the right side of the j equations; zeros and a constraint:

$$[OBS] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \\ 0 \\ 1 \end{bmatrix} \begin{array}{l} \text{mixture 1} \\ \text{mixture 2} \\ \text{mixture 3} \\ \text{mixture 4} \\ \text{mixture 5} \\ \text{mixture 6} \\ \text{mixture 7} \\ \text{mixture 8} \\ \quad : \\ \text{mixture j} \\ \text{constraint} \end{array}$$

5) The parameters of interest, the K and S coefficients for the four colorants at a particular wavelength, can now be computed:

$$[KANDS] = \{ [KSCOEFS]t[KSCOEFS] \}^{-1} [KSCOEFS]t[OBS],$$

The solution vector [KANDS] has the following form:

$$
[KANDS] = \begin{bmatrix} K1 \\ K2 \\ K3 \\ K4 \\ S1 \\ S2 \\ S3 \\ S4 \end{bmatrix}
\begin{array}{l}
\text{absorption coefficient of colorant 1} \\
\text{absorption coefficient of colorant 2} \\
\text{absorption coefficient of colorant 3} \\
\text{absorption coefficient of colorant 4} \\
\text{scattering coefficient of colorant 1} \\
\text{scattering coefficient of colorant 2} \\
\text{scattering coefficient of colorant 3} \\
\text{scattering coefficient of colorant 4}
\end{array}
$$

6) Steps 1 - 5 are repeated for each wavelength for which spectrophotometric data is available. Normally, this is done from 400 to 700 nanometers in 10 nanometer increments yielding 31 points. This yields the four K and four S values for each colorant for each of the thirty-one wavelengths.

3. Explicit linear solution for C

In a spectral match from, say, four colorants four parameters need to be determined (the concentrations C of each colorant required to match the standard). Therefore, at least four wavelengths are required to create an exactly determined system of four equations in the four unknowns. However, if the thirty-one point reflectance data for the standard has been measured, as above, then the additional wavelengths can be used to define an overdetermined system of equations, thereby increasing the confidence in the calculated values of C. This method is detailed below:

1) K/S of the standard is computed from the reflectance of the standard at each wavelength. The colorants that will be used to match the standard as well as their K and S values have previously been determined, as above. The concentrations of each colorant required to match the K/S of the standard is implicit in:

$$\frac{K}{S}\text{ standard} = \frac{C1K1 + C2K2\ C3K3 + C4K4}{C1S1 + C2S2\ C3S3 + C4S4} \text{ or } (K/S) \text{ for brevity.}$$

2) Cross-multiplying, equating to zero, and rearranging the terms so they appear explicitly as coefficients of concentration yields:

$$C1[\ K1 - S1(K/S)\ ] + C2[\ K2 - S2(K/S)\ ] +$$
$$C3[\ K3 - S3(K/S)\ ] + C4[\ K4 - S4(K/S)\ ] = 0$$

3) Since the K and S values of the colorants, and the K/S of the standard are known constants, they are the terms of a row in the coefficient matrix. Making the substitution $X1 = C1[K1-S1(K/S)$, $X2 = C2[K2-S2(K/S)$, $X3 = C3[K3-S3(K/S)$, and $X4 = C4[K4-S4(K/S)$, and repeating for the 31 wavelengths yields:

$$[CCOEFS] = \begin{array}{l} \left|\begin{array}{cccc} X1 & X2 & X3 & X4 \\ X1 & X2 & X3 & X4 \\ X1 & X2 & X3 & X4 \\ X1 & X2 & X3 & X4 \\ & & & \\ X1 & X2 & X3 & X4 \\ 1 & 1 & 1 & 1 \end{array}\right| \end{array} \begin{array}{l} \text{wavelength} = 400 \text{ nm} \\ \text{wavelength} = 410 \text{ nm} \\ \text{wavelength} = 420 \text{ nm} \\ \text{wavelength} = 430 \text{ nm} \\ \qquad\qquad\quad \vdots \\ \text{wavelength} = 700 \text{ nm} \\ \text{constraint} \end{array}$$

4) The observation vector is formed from the right side of the 32 equations; zeros and a constraint:

$$[OBS] = \left|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \\ 0 \\ 1 \end{array}\right| \quad \begin{array}{l} \text{wavelength} = 400 \text{ nm} \\ \text{wavelength} = 410 \text{ nm} \\ \text{wavelength} = 420 \text{ nm} \\ \text{wavelength} = 430 \text{ nm} \\ \qquad\qquad\quad \vdots \\ \text{wavelength} = 700 \text{ nm} \\ \text{constraint} \end{array}$$

5) The parameters of interest, the colorant concentrations, can now be computed:

$$[C] = \{ [CCOEFS]t[CCOEFS] \}^{-1} [CCOEFS]t[OBS].$$

The solution vector [C] has the following form:

$$[C] = \begin{bmatrix} C1 \\ C2 \\ C3 \\ C4 \end{bmatrix} \begin{array}{l} \text{concentration for colorant 1} \\ \text{concentration for colorant 2} \\ \text{concentration for colorant 3} \\ \text{concentration for colorant 4} \end{array}$$

4. Non-linear iterative improvement

An iterative improvement can be obtained by then performing either type of calculation in reflectance space. Once [B] has been determined as above, predicted K/S and predicted reflectance can be calculated,

$$K/S \text{ predicted} = \Sigma_i CiKi \;/\; \Sigma_i CiSi, \text{ and}$$

$$R \text{ predicted} = 1 + (K/S)p - \sqrt{(K/S)p^2 + 2(K/S)p}\;.$$

Evaluation of the quality of the match can be done in the highly non-linear reflectance space using a non-linear optimization algorithm. Using these equations, the non-linear optimization algorithm adjusts [B] until R predicted nearly equals R actual.

The problem of determining [B] in situations where Kubelka-Munk two-constant approach is necessary, is essentially an exercise in non-linear optimization. The parameter to be minimized is the sum of squares difference between the actual reflectance curve and the predicted curve as computed from [B]. A non-linear approach is ultimately necessary if the predicted reflectance is going to be compared with the actual reflectance since the transformation to reflectance is not linear.

Several general optimization algorithms have appeared in the literature since 1963 and the interested reader should refer to Christian and Tucker [19-23] who discuss optimization, Bevington [24] and Marquardt [25] who discuss the Levenberg-Marquardt optimization algorithm that was implemented in this research for iterative improvements. Optimization algorithms based on several different strategies were implemented for this research. Gradient-following [24] and linearization [24] techniques similar to those found in computer statistical libraries were tried independently with only fair results. Finally, the Levenberg-Marquardt [25] strategy was used. This technique interpolates between the gradient and linearization approaches from the instantaneous position on the error hypersurface at any iteration to the predicted minimum. This error hypersurface is a map of residual error as a function of each n parameter and is hence n-dimensional. From the current position on the error hypersurface the gradient and linearization strategies tend to predict the position of the minimum , in the worst case, orthogonal to each other with the true minimum hopefully lying along some vector between them. The Levenberg-Marquardt algorithm attempts to compute this angle and control the descent to the true minimum by moving along this vector. Using this algorithm, convergence was rapid and reliable.

B. Verification of Theory

The research is twofold. The first is the application of optimization to K and S determination, while the second is the application of optimization to spectral matching. The testing followed this outline:

1. An optimization of Kubelka-Munk absorption coefficients K and scattering coefficients S was performed on a data set provided by Burlington Industries. From a set of twenty-five blends of blue, green, yellow and red fibers, as many as twenty four of these blends were used in a non-linear least-squares determination of K and S for the thirty-one point wavelength data. The reflectance data was measured on a Bausch & Lomb / Diano Match Scan spectrophotometer, specular component included with diffuse polychromatic illumination. This reflectance data is provided in Appendix I. The K and S coefficients determined in this fashion were used to predict the reflectance of the twenty-fifth blend, a twenty-five percent mixture of each fiber.

2. Tristimulus and spectral matches were performed for several types of matching situations. The primary K and S data had previously been determined using primary binary blends. Colorimetric differences and spectral differences were computed for both matching approaches.

a. A reflectance curve was matched that was previously synthesized from the primaries used to match it. This reflects the situation where a match is sought for a standard from the correct colorants with materials that follow Kubelka-Munk behavior closely and there is little measurement error.

b. Standard reflectance curves were matched that were synthesized with primaries other than those used to attempt to match the standard. This reflects the situation where the correct colorants are not known and a match is sought with an alternate set of colorants. These materials, however, are assumed to follow Kubelka-Munk behavior closely.

c. Fiber-blends provided by Badische Corporation were matched from the fibers used to create those blends. Colorant concentrations were computed with both spectral and tristimulus matching and compared to the concentrations actually used to prepare the original blends. This reflects the situation where there is substantial measurement error and it is known that the materials deviate significantly from Kubelka-Munk behavior [26]. The reflectances of the primary binary blends as well as the blend reflectance data and matching data is given in Appendix III.

By testing various aspects of computer color formulation, the advantages and limitations associated with spectrophotometric color formulation have been observed.

IV. RESULTS

Tables 1 and 2 pertain to the determination of K and S data for the four fibers. Although the optimized K and S data for each fiber may be found in Appendix II, Table 1 shows the difference between predicted and actual reflectance of the four-way blend. Table 2 has three items of interest. First is the color difference between predicted and actual reflectance of the four-way blend in CIELab units, two degree observer. Second is the percentages of each fiber required to match the actual four way blend as determined through the optimized K and S and using tristimulus matching. Third is the percentages of each fiber required to match the actual four way blend as determined through the optimized K and S and using spectral matching.

Tables 3, 4, and 5 pertain to color matching. Table 3 shows matches to a green standard from the primaries that were used to numerically synthesize that standard. The first match is a tristimulus match from these primaries and the related color differences while the second match is a spectral match and the related color differences. Table 4 summarizes tristimulus and spectral matches to gray standards from an alternate set of colorants. The actual matching data can be found in Appendix IV. Table 5 summarizes the tristimulus and spectral matches of the fiber blends from the fibers in the blends. The actual matching data can be found in Appendix V.

TABLE 1

PREDICTION OF FOUR WAY BLEND REFLECTANCE

WL	R predicted	R measured	Difference
400	3.11241	3.10000	0.01241
410	3.19328	3.18000	0.01328
420	3.32346	3.32000	0.00346
430	3.51839	3.51000	0.00839
440	3.75462	3.74000	0.01462
450	3.93744	3.93000	0.00743
460	4.07171	4.06000	0.01171
470	4.20132	4.20000	0.00132
480	4.40311	4.40000	0.00311
490	4.66537	4.66000	0.00537
500	4.97179	4.94000	0.03179
510	5.21765	5.15000	0.06765
520	5.30567	5.17000	0.13567
530	5.16815	4.98000	0.18815
540	4.95634	4.73000	0.22634
550	4.85363	4.59000	0.26363
560	4.81634	4.52000	0.29634
570	4.84285	4.51000	0.33285
580	5.14202	4.76000	0.38202
590	5.58195	5.36000	0.22195
600	6.64220	6.18000	0.46220
610	7.22156	6.77000	0.45156
620	7.46222	7.03000	0.43221
630	7.53312	7.11000	0.42312
640	7.72023	7.30000	0.42023
650	8.22978	7.81000	0.41978
660	9.27334	8.85000	0.42334
670	10.84142	10.41000	0.43142
680	13.12664	12.68000	0.44664
690	16.19110	15.69000	0.50110
700	20.75672	20.21000	0.54672

GOODNESS = 9.093E-06

NONLINEAR OPTIMIZATION OF K AND S USING TWENTY-FOUR BLENDS

TABLE 2 : Match of Four-way Blend

TRIAL IS -- 4 WAY MATCH NL24

STANDARD	TRIAL					
R25/G25/Y25/B25	4 WAY MATCH NL24					
ILL=D6 2 DE= 1.38	LD= 0.71	RG= 0.27	YB= 1.16	DC= 1.12	DH= 2.88	
ILL=A 2 DE= 1.55	LD= 0.80	RG= 0.35	YB= 1.28	DC= 1.14	DH= 3.62	
ILL=CW 2 DE= 1.54	LD= 0.80	RG= 0.15	YB= 1.30	DC= 1.23	DH= 3.50	

CIELAB GOODNESS = 0.9093E-05

12-APR-85 11:51:33 ITER= 1 COMBINATION 1

COLORANT	%
(15U)BLUE (NL/24)	== 24.8541
(16U)GREEN (NL/24)	== 27.1974
(17U)YELLOW (NL/24)	== 22.4374
(18U)RED (NL/24)	== 25.5111

COST= 1.000 MI = 0.04

ILL=D6 2 DE= 0.00	LD= 0.00	RG= 0.00	YB= 0.00	DC= 0.00	DH= 0.00
ILL=A 2 DE= 0.03	LD= 0.00	RG= -0.03	YB= 0.00	DC= -0.03	DH= 0.11
ILL=CW 2 DE= 0.03	LD= 0.01	RG= 0.01	YB= 0.02	DC= 0.03	DH= 0.02

CIELAB
GOODNESS = 1.3095500E-06

COLORANT PERCENTAGES

FIBER	1	BLUE (NL/24)	25.62%
FIBER	2	GREEN (NL/24)	26.19%
FIBER	3	YELLOW (NL/24)	22.33%
FIBER	4	RED (NL/24)	25.86%

12-APR-85 11:54:32 COMBINATION 1 AFTER 2 ITERATIONS FOR R25/G25/Y25/B25

ILL=D6 2 DE= 0.37	LD= 0.03	RG= 0.27	YB= -0.26	DC= -0.05	DH= -2.82
ILL=A 2 DE= 0.28	LD= 0.04	RG= 0.18	YB= -0.21	DC= 0.00	DH= -1.56
ILL=CW 2 DE= 0.33	LD= 0.03	RG= 0.20	YB= -0.26	DC= -0.12	DH= -2.49

CIELAB

MI = 0.10 GOODNESS = 0.2719E-06 AT A COST OF $1.0000

Table 3 : Match to Green Standard

```
TYPE REFLECTANCE FILE RECORD NUMBER  24,1
STANDARD           IS -- GREEN

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  6,1
RECORD CONTAINS YELLOW 142
COLORANT  2 ?  11,1
RECORD CONTAINS TIO2
COLORANT  3 ?  13,1
RECORD CONTAINS NEW BLUE
COLORANT  4 ?

NUMBER OF COLORANTS PER FORMULA?  3
10-APR-85 14:46:16 ITER=  1 COMBINATION     1
COLORANT                      %
(   6U)YELLOW 142       == 19.9985
(  11U)TIO2             == 60.0020
(  13U)NEW BLUE         == 19.9995

COST=   0.800      MI =  0.00
ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
ILL=A  2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
ILL=CW 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
CIELAB
GOODNESS =   9.5179609E-10




COLORANT PERCENTAGES

FIBER  1  YELLOW 142            20.00%
FIBER  2  TIO2                  60.00%
FIBER  3  NEW BLUE              20.00%

10-APR-85 14:47:09 COMBINATION  1 AFTER  1 ITERATIONS FOR GREEN

ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
ILL=A  2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
ILL=CW 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00

CIELAB

MI =   0.00 GOODNESS = 0.8919E-09 AT A COST OF $0.8000
```

TABLE 4

SUMMARY OF TRISTIMULUS AND SPECTRAL MATCHES
OF GRAY STANDARDS FROM ALTERNATE COLORANTS

STANDARD	Match Number	G tm	G sm	Delta E sm
Light Gray	1	4.70	3.44	2.91
	2	2.60	2.06	8.97
Medium Gray	3	1.55	.80	10.1
	4	1.93	1.27	7.70
Dark Gray	5	.31	.17	2.09
	6	23.4	1.17	4.55

Reference to a tristimulus match is indicated by tm, a spectral match by sm. G is goodness, the average squared spectral difference per wavelength. Delta E is the predicted color difference between standard and spectrally predicted match expressed in CIELab units, D65, two degree observer. By definition, tristimulus matches predict zero color difference under the primary illuminant.

TABLE 5

SUMMARY OF TRISTIMULUS AND SPECTRAL MATCHES
OF FIBER BLENDS FROM THE CORRECT FIBERS

BLEND	G tm	G sm	Delta E sm	Delta C tm	Delta C sm
1 H	5.64	3.11	.45	1.76	1.54
2 L	.78	.61	.19	2.25	2.28
4 L	1.81	1.28	.20	.84	.91
6 L	6.82	3.26	.27	1.09	.85
7 L	34.7	12.1	1.25	1.23	1.14
9 H	.53	.42	.09	3.62	3.65
10 L	.56	.37	.1	.77	.91
11 H	7.75	2.09	.57	4.27	4.16
12 L	47.7	29.8	1.38	4.36	1.99
15 L	48.2	4.96	1.39	10.77	4.10
17 H	.86	.68	.06	1.06	1.07
21 H	625.	.98	.41	11.02	1.10
22 L	2.48	.41	.39	2.23	1.02
25 L	2.17	.27	.13	2.40	1.91
27 L	8.61	6.83	.33	1.44	1.55
32 H	47.3	.71	.40	6.59	3.30
33 L	.30	.17	.40	4.54	4.53
34 L	5.36	4.98	.15	.93	1.34
Ave	47.0	4.06	.45	3.398	2.075

The blend number is that reported by Burlone [28]. An H or an L indicates that when the predicted match was actually prepared it had a high or low color difference from the standard. Reference to a tristimulus match is indicated by tm, a spectral match by sm. G is goodness, the average squared spectral difference per wavelength. Delta E is the predicted color difference between standard and spectrally predicted match expressed in CIELab units, D65, two degree observer. By definition, tristimulus matches predict zero color difference under the primary illuminant. Delta C is the average difference in concentration between the predicted and actual concentrations of fiber in the blend.

V. DISCUSSION

The research had two objectives. The first was the application of optimization to K and S determination, while the second was the application of optimization to spectral matching.

An optimization of absorption coefficients (K) and scattering coefficients (S) was performed on the data set provided by Burlington Industries. From a set of twenty-five blends of blue, green, yellow and red fibers, as many as twenty four of these blends were used in a non-linear least-squares determination of Kubelka-Munk absorption and scattering coefficients for the thirty-one point wavelength data. The K and S coefficients determined in this fashion were used to predict the reflectance of the twenty-fifth blend, a twenty-five percent mixture of each fiber. The agreement between predicted and actual reflectance of this four-way blend was quite good, and improved as the number of blends used to compute K and S was increased from eight to twenty-four. Eight blends provide the minimum number of independent equations to solve for the four unknown absorption and scattering coefficients. This is the same number that would be required to compute K and S using binary blends. From Table 1, the difference between predicted and actual reflectance of the four-way blend can be observed. Goodness is defined here as the average squared absolute error per wavelength and was calculated by summing the squared difference between the reflectance of the standard and that of the prediction and dividing by the number of data points (wavelengths). The calculated value of goodness for this prediction was 9.093E-06 or about an average of .3% difference in percent reflectance units. Although this seems quite small, from Table 2 this led to a predicted colorimetric difference of about 1.4 CIELab units for the listed conditions. Tristimulus and spectral matches of the standard four-way blend with these primaries resulted in predicted concentrations averaging 1.4% different from the nominal values. Although these kind of results would normally be considered unacceptable when dealing with homogeneous materials that follow Kubelka-Munk theory, conversations with several sources in the fiber industry indicate that these results are considered quite good in a fiber-blending situation. This stems from the fact that fiber blends appear quite heterogeneous to a human observer, and it is not known how this correlates with color differences as measured by a diffuse spectrophotometer which tends to make the sample appear homogeneous to the detector. Therefore, according to these sources, delta Es of up to two CIELab units and percentages of fiber of up to two percent difference usually yield quite acceptable matches. The advantage of this method is that no primary binary blends have to be prepared; the primary data can be computed from normally documented blend history. Further research is necessary to test the hypothesis that primaries optimized is this fashion yield better matching results than primaries determined using binary letdowns.

Tristimulus and spectral matches were performed for several types of matching situations, where the primary K and S data have been determined using primary binary blends. Colorimetric differences and spectral differences were computed for both matching approaches.

In the first situation, a reflectance curve was matched that was previously synthesized from the primaries used to match it. This represents the situation where a match is sought for a standard from the correct colorants with materials that follow Kubelka-Munk behavior closely and there is little measurement error. From Table 3, it can be seen that both tristimulus and spectral matching resulted in perfect spectral matches indicated by the near zero values of goodness and delta Es. Additionally, the predicted concentrations is both cases were the correct values.

In the second situation, reflectance curves were matched that were synthesized with primaries other than those used to attempt to match the standard. This is indicative of the situation where the correct colorants are not known and a match is sought with an alternate set of colorants. These materials are paints and follow Kubelka-Munk behavior closely. From Table 4 it can be seen that although spectral matching resulted in curves of much lower predicted spectral mismatch, it was at the expense of a colorimetric match. When matches are sought from colorants that are not in the standard, perfect spectral matches are not possible, hence any match will be metameric with that of the standard. Given the fact that some spectral mismatch must exist, tristimulus matching optimizes the mismatch to allow a colorimetric match. Alternatively, spectral matching optimizes the mismatch itself without regard to colorimetric difference. Unless it is necessary to specifically control spectral difference, tristimulus matching may be preferred for this type of situation. If many colorants are available, it is possible that spectral matching could produce matches of low spectral difference and acceptable colorimetric difference, although further research is necessary to test this hypothesis.

In the third situation, fiber-blends provided by Badische Corporation were matched from the fibers used to create those blends. Some of these fibers were of spectrally similar character. This reflects the situation where there is substantial measurement error, the colorants are similar, and it is known that the materials deviate from Kubelka-Munk behavior. Colorant concentrations were computed with both spectral and tristimulus matching and compared to the concentrations actually used to prepare the original blends. Significantly better results were gained with spectral matching over tristimulus matching. This improvement may be due to the increased sensitivity provided by the spectral matching approach. Where there is substantial deviation from optimal conditions associated with computer color formulation, it appears better to avoid the tristimulus reduction and rather try to minimize overall spectral error. From Table 5 it can be seen that spectral matching resulted in much better predicted spectral difference as well as providing significantly closer concentration values while providing acceptable colorimetric differences. In fact, average spectral difference was reduced an order of magnitude, average predicted concentration accuracy was increased by 1.3 percentage points while the average predicted colorimetric difference was less than .5 delta E CIELab units. These results indicate a real matching improvement.

VI. CONCLUSIONS

Spectrophotometric color formulation is a tool that could have widespread application in the colorant formulation industry. Since this approach produces better matches under certain conditions while potentially allowing the use of more colorants without unreasonably increasing the formulation time, savings in terms of money and time could be realized by reducing the number of wasted production batch formulations.

An application for this tool is in the fiber-blending industry. Here, spectrally similar fibers are woven together on a microscopic level to yield a macroscopically different color. Traditional approaches computer to color formulation as applied to fiber-blending have suffered from low accuracy. The measurement of these heterogeneous samples is difficult and it is been hypothesized that the turbid media effects associated with fiber depart from Kubelka-Munk theory. Furthermore, spectrally similar colorants are employed in matching fiber blends. It is in the area of fiber-blending that spectrophotometric color formulation could be most useful. Perhaps the combined approach of least-squares determination of K and S coupled with spectral matching would yield an even greater improvement in match prediction.

The value of using spectrally similar primaries transcends fiber-blending, however. Frequently, production batches that were intended to match a standard are slightly off-shade and are useless unless they can be worked into another formulation. The key is to use spectrally similar off-shade materials to produce an on-shade product. If formulations from these similar waste materials could be accurately produced, substantial savings would result. In the case where alternate formulations are being sought to match a standard with colorants other than those in the standard, the substitution of a colorant that is slightly different from one that is being considered may have certain advantages. It may be true that the new colorant, or combination of several colorants, will reduce metamerism or cost. It is apparent that the ability to distinguish between, and work with, mathematically similar colorants would be an improvement.

Another area spectrophotometric color formulation could be used is for matching standards in a wavelength region where there is no tristimulus information possible. If, for instance, the ultraviolet or infrared reflectance regions of a standard need to be matched, spectrophotometric color formulation could be used, just as in the visible region. The manufacture of camouflage materials is an example of the importance of controlling reflectance outside the visible region.

It is conceivable that spectrophotometric color formulation could replace many of the existing color formulation programs designed to handle various types of specific problems.

VII. REFERENCES

1) D.B. Judd, and G. Wyszecki, Color in Business, Science and Industry, Third edition, John Wiley and Sons, New York, 1975, pp. 438-461.

2) F.W. Billmeyer, and M. Saltzman, Principles of Color Technology, 2nd Edition, John Wiley and Sons, New York, 1981, pp. 156-157.

3) R.P. Best, "Computer Match Prediction in Laboratory and Factory", Instrumental Colour Systems Limited, Personal Communication to R.S. Berns, p. 144.

4) E. Allen, "Colorant Formulation and Shading" from F. Grum, and C.J. Bartleson, Optical Radiation Measurements, Volume 2: Color Measurement, Academic Press, New York, 1980, p. 310.

5) Bausch & Lomb Company, Match-Mate 4000 Match-Pak II Manual, Bausch & Lomb Company, Rochester, 1984, pp. 24-37.

6) ibid., p. 35

7) D.A. Burlone, "Theoretical and Practical Aspects of Selected Fiber-Blend Color Formulation Functions", Color Research and Application, 9, 213(1984).

8) E. Allen, "Colorant Formulation and Shading" from F. Grum, and C.J. Bartleson, Optical Radiation Measurements, Volume 2: Color Measurement, Academic Press, New York, 1980, p. 293.

9) ibid., pp. 290-334.

10) E. Allen, "Basic Equations Used in Computer Color Matching", Journal of the Optical Society of America, 56, 1256(1966).

11) E. Allen, "Basic Equations Used in Computer Color Matching, II. Tristimulus Match, Two-Constant Theory", Journal of the Optical Society of America, 64, 991(1974).

12) P.H. McGinnis, "Spectrophotometric Color Matching With the Least Squares Technique", Color Engineering, 5, 22(1967).

13) V. Gugerli, "Rezeptieren auf der Basis optischer Messungen", Textil-Rundschau, 18, 266(1963).

14) A. Nava, D. Bianchi, and D. Pitea, "Colors of Dope and Yarn of Spun-Dyed Cellulose Acetate", Color Research and Application, 8, 23(1983).

15) A. Brockes, "Computer Color Matching: A Review of its Limitations", Journal of the American Association of Textile Chemists and Colorists, 6, 98(1974).

16) H.R. Davidson, "Use and Misuse of Computers in Color Control", Journal of Coatings Technology, 54, 37(1982).

17) N. Ohta, "A Simplified Method for Formulating Pseudo-object Colors", 4th Congress of the Association Internationale de la Couleur, Berlin, Germany, Sept. 20-25, 1981.

18) B. Carnahan, H.A. Luther, and J.O. Wilkes, Applied Numerical Methods, John Wiley & Sons, Inc., New York, 1969, p 574.

19) S.D. Christian, and E.E. Tucker, "Least squares analysis with the microcomputer: Linear Relationship, Part One", American Laboratory, 8, 36(1982).

20) S.D. Christian, and E.E. Tucker, "Least squares analysis with the microcomputer: Part Two:Nonlinear Relationships", American Laboratory, 9, 31(1982).

21) S.D. Christian, and E.E. Tucker, "Part Three:Least squares analysis with the microcomputer: Solving Simultaneous Equations", American Laboratory, 5, 78(1983).

22) S.D. Christian, and E.E. Tucker, "Least squares analysis with the microcomputer: Part Four: Smoothing and Expanding Data Sets", American Laboratory, 9, 35(1983).

23) S.D. Christian, and E.E.Tucker, "Least squares analysis with the microcomputer: Part Five: General Least Squares with Variable Weighting", American Laboratory, 2, 30(1984).

24) P.R. Bevington, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill Book Company, New York, 1969, pp 204-246.

25) D.W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters", Journal of the Society of Industrial Applied Mathematics, 11, 431(1963).

26) D.A. Burlone, "Formulation of Blends of Precolored Nylon Fiber", Color Research and Application, 8, 114(1983).

VIII. Appendix I

Burlington blend reflectance data:

Reflectance data of fiber blends

Note: The next two pages are reflectances of the blends followed by their graphs.

GREEN 621 17-AUG-84 POL SIN -1.0000 1.00

1.97	1.93	1.93	1.99	2.13	2.37	2.74	3.28	4.06	4.94
5.60	5.64	5.14	4.40	3.57	2.98	2.59	2.33	2.11	1.95
1.87	1.84	1.85	1.85	1.84	1.84	1.91	2.13	2.73	4.08
7.04									

YELLOW 621 17-AUG-84 POL SIN -1.0000 1.00

2.24	2.18	2.14	2.13	2.12	2.13	2.21	2.41	2.89	3.86
5.79	8.73	12.84	17.53	22.93	28.28	33.79	38.48	42.45	45.18
47.11	48.11	48.69	48.93	49.08	49.32	49.75	50.41	51.39	52.47
54.17									

BLUE 621 17-AUG-84 POL SIN -1.0000 1.00

9.85	11.37	13.80	17.57	22.73	26.28	26.30	23.75	20.20	16.36
12.81	10.37	8.17	6.17	4.80	4.18	3.64	2.98	2.51	2.43
2.59	2.63	2.44	2.27	2.44	3.21	5.23	8.92	14.84	22.06
30.58									

RED 621 17-AUG-84 POL SIN -1.0000 1.00

2.31	2.27	2.17	2.05	1.94	1.86	1.79	1.74	1.70	1.69
1.68	1.67	1.68	1.70	1.73	1.79	1.97	2.50	4.01	7.11
12.05	17.21	21.64	24.48	26.39	27.50	28.26	28.80	29.30	29.82
30.63									

RED75,BLUE10,GREEN15/622 17-AUG-84 POL SIN -1.0000 1.00

2.44	2.44	2.42	2.41	2.41	2.42	2.42	2.43	2.45	2.46
2.45	2.40	2.31	2.19	2.09	2.03	2.08	2.34	3.20	4.90
7.35	9.54	11.08	11.92	12.51	13.06	13.89	15.02	16.61	18.74
21.83									

BLUE50,GREEN25,YELLOW25/622 17-AUG-84 POL SIN -1.0000 1.00

4.66	4.95	5.42	6.08	6.85	7.39	7.69	7.84	8.01	8.14
8.18	8.14	7.72	6.91	6.15	5.74	5.36	4.87	4.48	4.38
4.49	4.54	4.40	4.27	4.39	4.97	6.27	8.37	11.45	15.36
20.92									

RED25,GREEN25,YELLOW25,BLUE25/ 17-AUG-84 POL SIN -1.0000 1.00

3.10	3.18	3.32	3.51	3.74	3.93	4.06	4.20	4.40	4.66
4.94	5.15	5.17	4.98	4.73	4.59	4.52	4.51	4.76	5.36
6.18	6.77	7.03	7.11	7.30	7.81	8.85	10.41	12.68	15.69
20.21									

GREEN75,YELLOW25/624 17-AUG-84 POL SIN -1.0000 1.00

1.94	1.90	1.90	1.93	2.03	2.19	2.47	2.90	3.57	4.48
5.46	6.13	6.36	6.16	5.70	5.27	4.97	4.75	4.52	4.33
4.22	4.21	4.23	4.24	4.22	4.23	4.32	4.61	5.39	7.06
10.48									

BLUE25,YELLOW75/624 17-AUG-84 POL SIN -1.0000 1.00

3.15	3.22	3.35	3.53	3.73	3.85	3.94	4.08	4.43	5.15
6.55	8.56	10.90	12.86	14.57	16.08	17.12	17.29	17.19	17.45
18.03	18.24	17.87	17.53	17.88	19.36	22.50	27.11	32.96	38.83
44.83									

BLUE75,YELLOW25/624 17-AUG-84 POL SIN -1.0000 1.00

6.69	7.39	8.47	9.96	11.70	12.75	12.89	12.41	11.72	10.87
10.02	9.44	8.62	7.48	6.55	6.17	5.75	5.10	4.59	4.52
4.75	4.81	4.57	4.36	4.56	5.50	7.82	11.85	18.02	25.24
33.51									

BLUE25,GREEN75/624 17-AUG-84 POL SIN -1.0000 1.00

2.88	2.96	3.12	3.39	3.78	4.18	4.62	5.14	5.79	6.38
6.61	6.31	5.55	4.58	3.67	3.08	2.68	2.35	2.08	1.94
1.90	1.89	1.87	1.83	1.85	1.99	2.34	2.96	4.03	5.89
9.42									

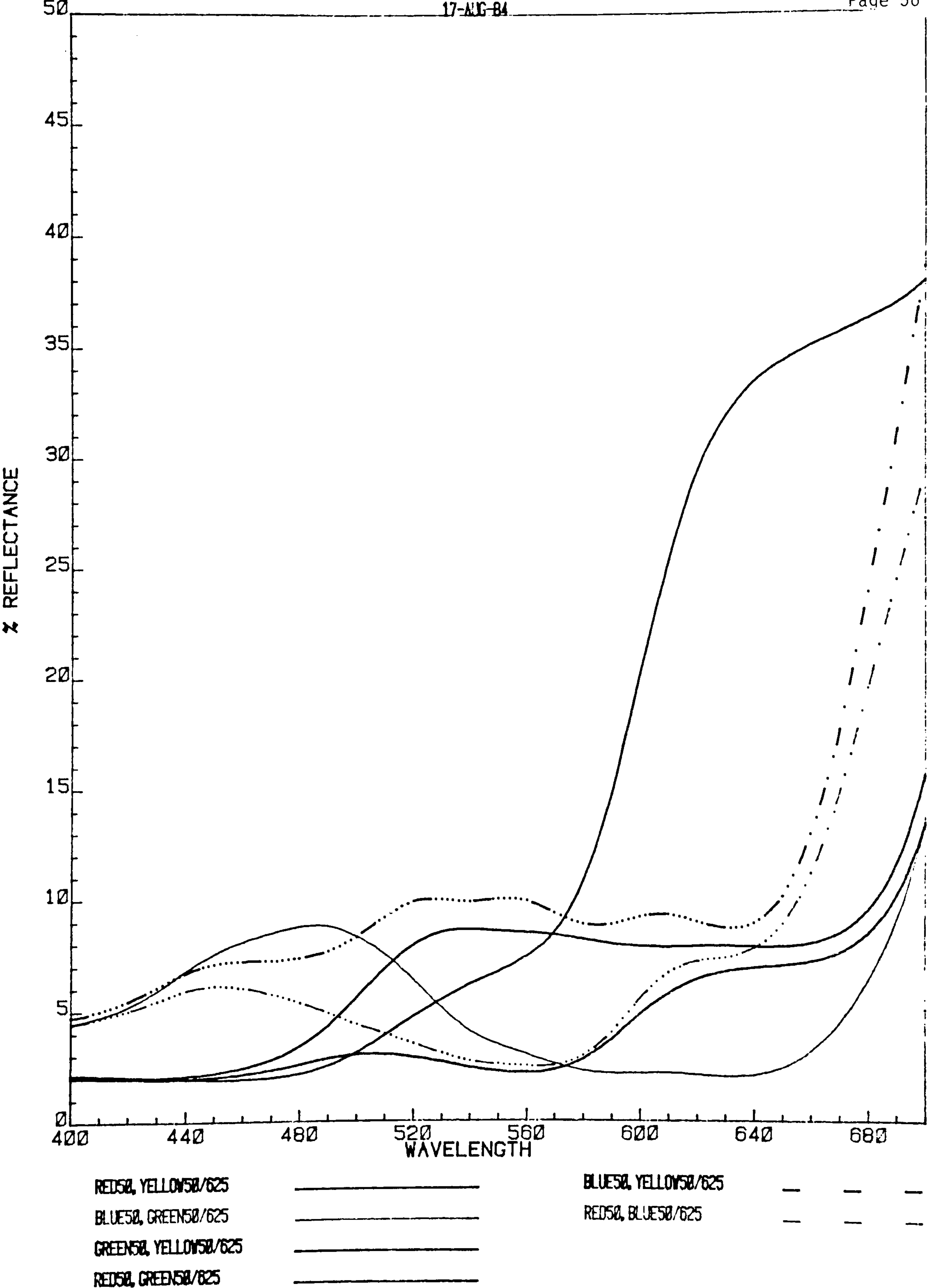
RED75,BLUE25/624 17-AUG-84 POL SIN -1.0000 1.00

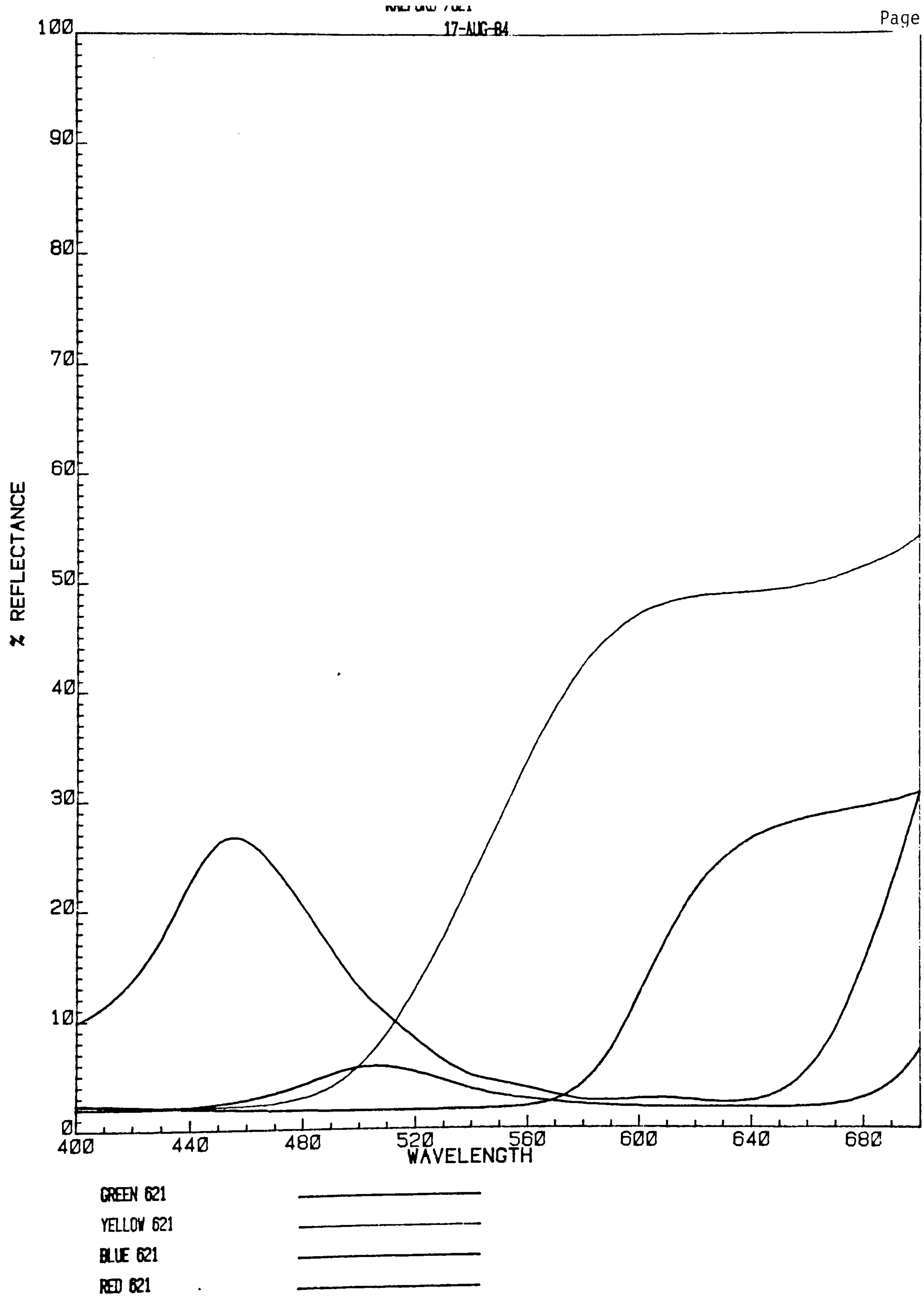
3.37	3.47	3.53	3.62	3.69	3.70	3.60	3.45	3.28	3.09
2.88	2.71	2.54	2.36	2.22	2.19	2.27	2.57	3.55	5.45
8.16	10.46	11.86	12.52	13.27	14.58	16.91	20.06	23.81	27.28
30.57									

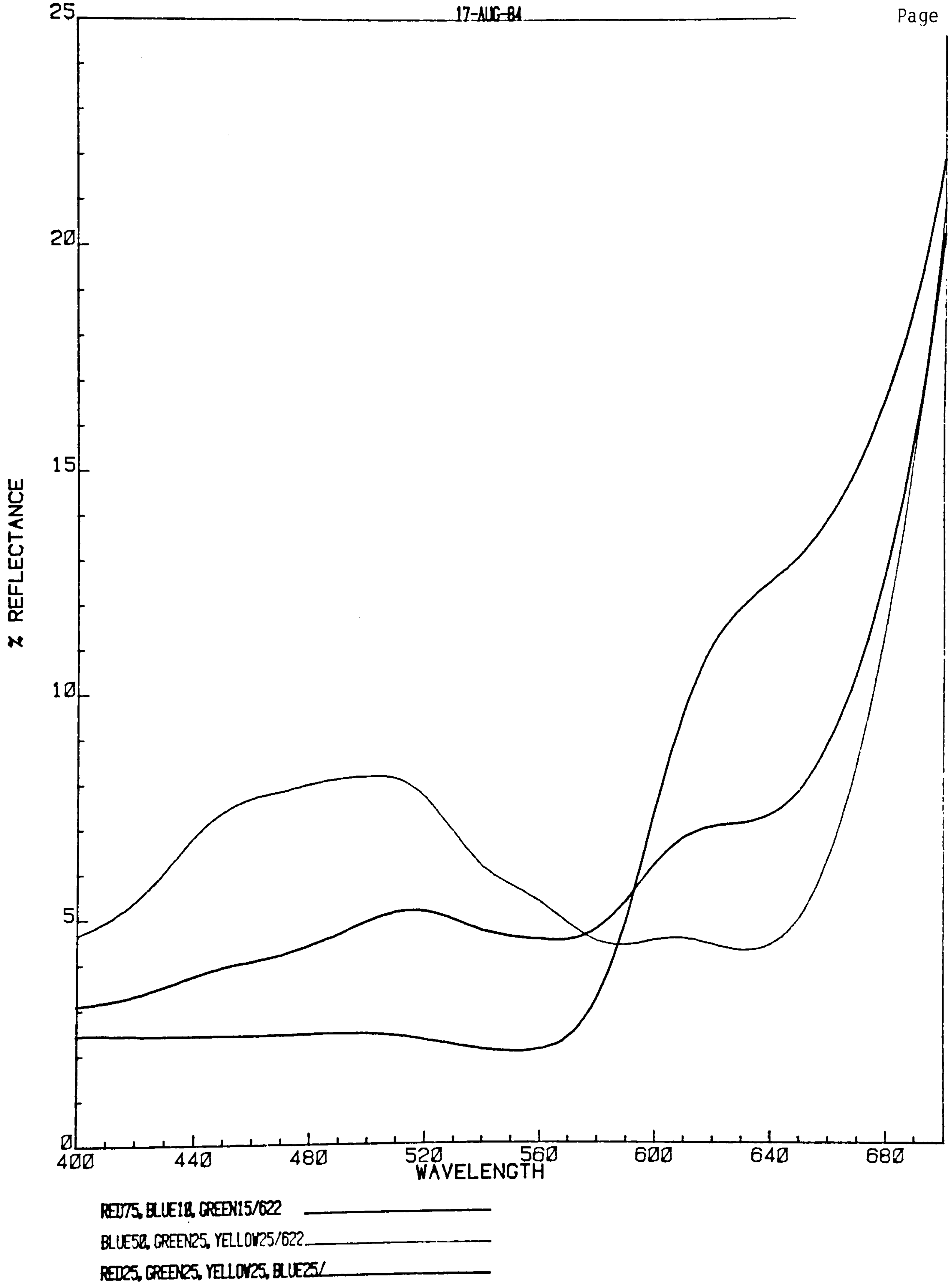
GREEN25,YELLOW75/624 17-AUG-84 POL SIN -1.0000 1.00

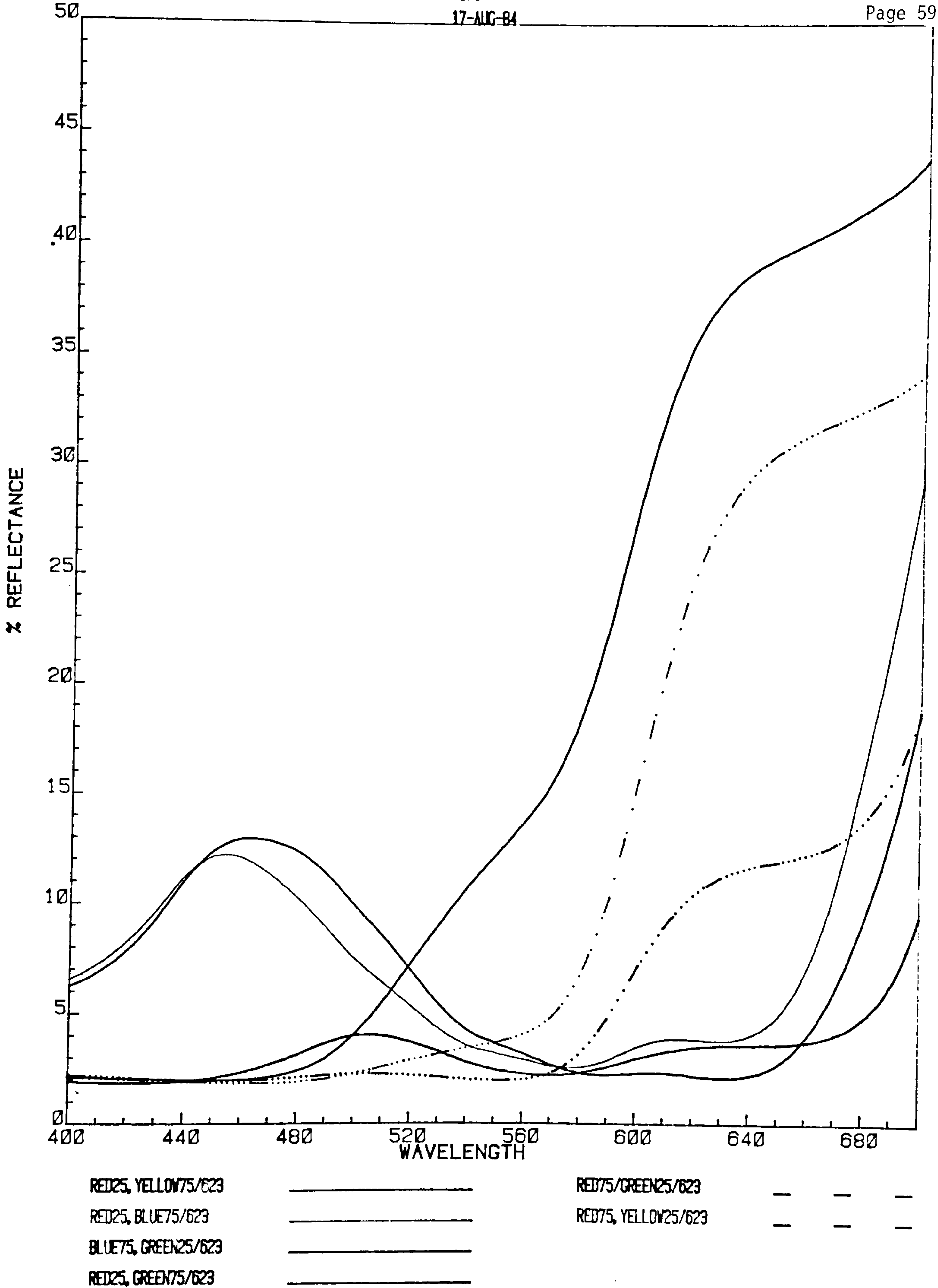
2.13	2.08	2.06	2.07	2.09	2.15	2.29	2.54	3.03	3.92
5.44	7.41	9.66	11.62	13.18	14.26	15.09	15.58	15.72	15.70
15.70	15.73	15.81	15.85	15.81	15.83	16.04	16.64	18.07	20.69

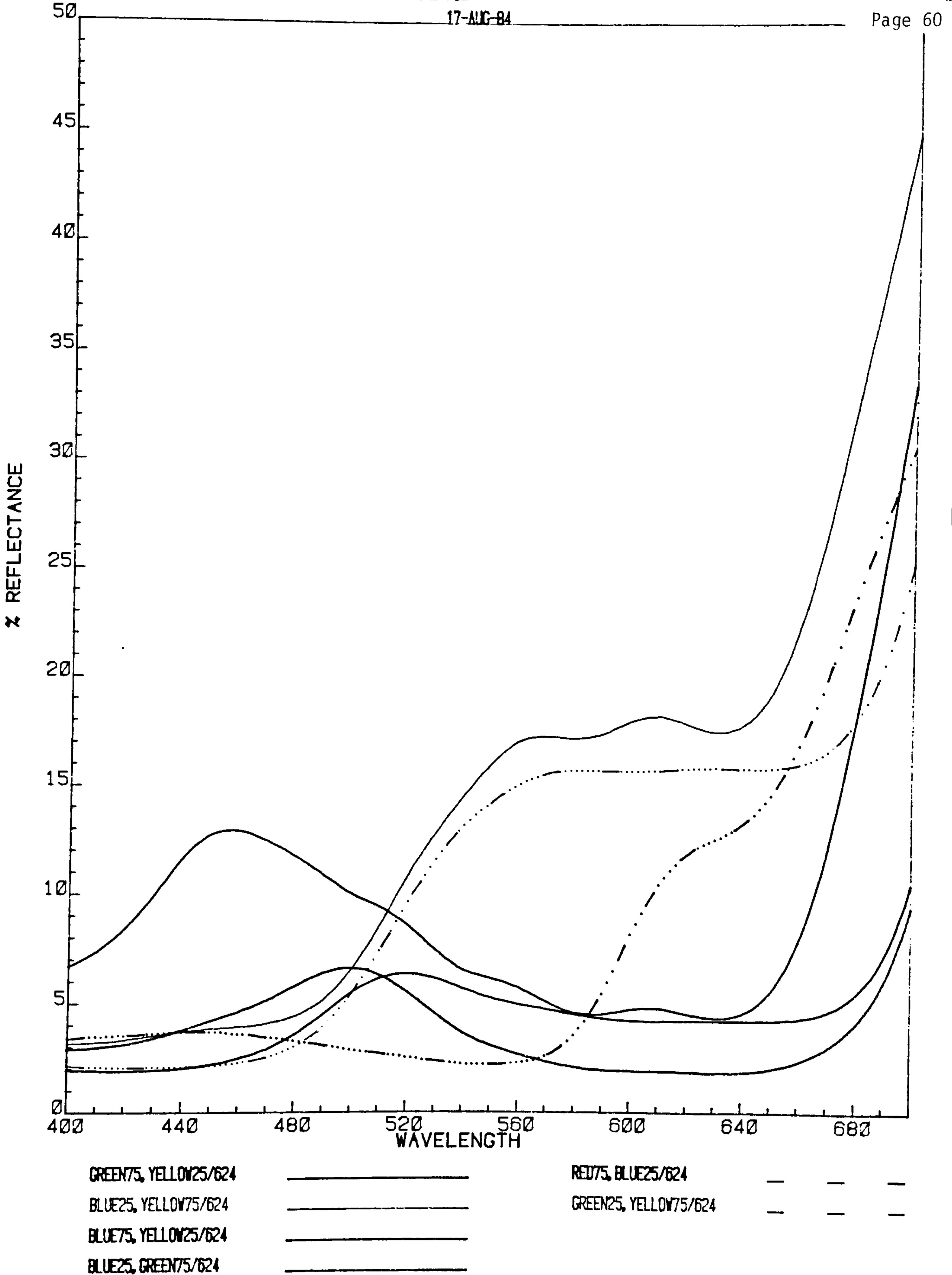
```
  2.15   2.09   2.04   2.01   1.97   1.96   2.00   2.11   2.38   2.9
  4.01   5.47   7.26   9.02  10.72  12.20  13.72  15.55  18.55  22.87
 28.20  32.71  35.98  37.86  38.98  39.67  40.30  40.94  41.73  42.59
 43.86
RED25,BLUE75/623                17-AUG-84   POL   SIN   -1.0000   1.00
  6.54   7.25   8.27   9.65  11.24  12.12  12.02  11.25  10.16   8.83
  7.45   6.39   5.34   4.32   3.56   3.21   2.93   2.62   2.55   2.87
  3.46   3.84   3.84   3.77   4.00   4.87   6.94  10.55  16.00  22.28
 29.25
BLUE75,GREEN25/623              17-AUG-84   POL   SIN   -1.0000   1.00
  6.22   6.86   7.83   9.26  11.05  12.37  12.91  12.80  12.30  11.29
  9.86   8.49   6.95   5.43   4.27   3.67   3.19   2.68   2.30   2.20
  2.28   2.31   2.18   2.07   2.18   2.68   3.88   5.93   9.06  13.11
 18.85
RED25,GREEN75/623               17-AUG-84   POL   SIN   -1.0000   1.00
  1.89   1.85   1.83   1.85   1.93   2.07   2.31   2.66   3.12   3.64
  4.00   4.00   3.72   3.29   2.81   2.45   2.24   2.18   2.28   2.56
  2.93   3.24   3.46   3.57   3.59   3.62   3.73   4.01   4.74   6.28
  9.42
RED75/GREEN25/623               17-AUG-84   POL   SIN   -1.0000   1.00
  2.08   2.05   1.99   1.93   1.90   1.89   1.93   1.99   2.09   2.19
  2.26   2.25   2.19   2.11   2.00   1.94   1.98   2.27   3.11   4.74
  6.97   8.94  10.38  11.19  11.65  11.92  12.23  12.72  13.73  15.54
 18.63
RED75,YELLOW25/623              17-AUG-84   POL   SIN   -1.0000   1.00
  2.21   2.17   2.09   2.01   1.93   1.87   1.83   1.83   1.89   2.03
  2.27   2.57   2.90   3.17   3.44   3.67   4.05   4.82   6.70  10.21
 15.47  20.75  25.12  27.86  29.66  30.72  31.49  32.08  32.67  33.32
 34.26
RED50,YELLOW50/625              17-AUG-84   POL   SIN   -1.0000   1.00
  2.14   2.10   2.04   1.98   1.93   1.90   1.90   1.97   2.15   2.49
  3.09   3.85   4.69   5.44   6.13   6.70   7.40   8.49  10.80  14.72
 20.19  25.36  29.45  31.92  33.49  34.42  35.13  35.71  36.31  36.97
 37.95
BLUE50,GREEN50/625              17-AUG-84   POL   SIN   -1.0000   1.00
  4.47   4.75   5.23   5.92   6.81   7.56   8.10   8.47   8.77   8.78
  8.29   7.48   6.33   5.07   4.03   3.44   2.99   2.57   2.25   2.13
  2.15   2.15   2.07   2.00   2.07   2.39   3.15   4.38   6.30   9.01
 13.43
GREEN50,YELLOW50/625            17-AUG-84   POL   SIN   -1.0000   1.00
  2.04   1.98   1.98   2.01   2.08   2.20   2.43   2.78   3.38   4.31
  5.58   6.87   7.93   8.49   8.61   8.54   8.48   8.36   8.16   7.95
  7.84   7.82   7.85   7.86   7.82   7.83   7.97   8.42   9.47  11.57
 15.57
RED50,GREEN50/625               17-AUG-84   POL   SIN   -1.0000   1.00
  2.01   1.99   1.94   1.92   1.94   2.00   2.13   2.31   2.55   2.82
  3.01   3.01   2.87   2.66   2.40   2.22   2.16   2.29   2.76   3.65
  4.79   5.71   6.35   6.69   6.87   6.97   7.14   7.52   8.40  10.11
 13.37
BLUE50,YELLOW50/625             17-AUG-84   POL   SIN   -1.0000   1.00
  4.72   5.00   5.46   6.07   6.71   7.09   7.21   7.22   7.34   7.65
  8.29   9.18   9.89   9.95   9.87  10.01   9.91   9.34   8.84   8.83
  9.16   9.27   8.95   8.65   8.89  10.12  12.94  17.49  23.92  30.95
 38.55
RED50,BLUE50/625                17-AUG-84   POL   SIN   -1.0000   1.00
  4.41   4.70   5.04   5.47   5.91   6.11   6.00   5.72   5.34   4.87
  4.36   3.93   3.50   3.05   2.69   2.55   2.47   2.49   2.93   3.98
  5.49   6.66   7.17   7.34   7.76   8.85  11.17  14.75  19.54  24.47
 29.47
```


17-AUG-84
% REFLECTANCE
WAVELENGTH
RED50, YELLOW50/625
BLUE50, GREEN50/625
GREEN50, YELLOW50/625
RED50, GREEN50/625
BLUE50, YELLOW50/625
RED50, BLUE50/625


17-AUG-84
100
90
80
70
60
50
40
30
20
10
0
% REFLECTANCE
400
440
480
520
560
600
640
680
WAVELENGTH
GREEN 621
YELLOW 621
BLUE 621
RED 621


17-AUG-84
% REFLECTANCE
25
20
15
10
5
0
400
440
480
520
560
600
640
680
WAVELENGTH
RED75, BLUE10, GREEN15/622
BLUE50, GREEN25, YELLOW25/622
RED25, GREEN25, YELLOW25, BLUE25/


50
45
40
35
30
25
20
15
10
5
0
% REFLECTANCE
400
440
480
520
560
600
640
680
WAVELENGTH
RED25, YELLOW75/623
RED25, BLUE75/623
BLUE75, GREEN25/623
RED25, GREEN75/623
RED75/GREEN25/623
RED75, YELLOW25/623


17-AUG-84
% REFLECTANCE
WAVELENGTH
GREEN75, YELLOW25/624
BLUE25, YELLOW75/624
BLUE75, YELLOW25/624
BLUE25, GREEN75/624
RED75, BLUE25/624
GREEN25, YELLOW75/624

IX. Appendix II

K and S data for Burlington blends:

Optimized K and S coefficients for the four fibers

JOB CODE? 21
RECALL K&S DATA FROM FILE
TYPE K&S FILE RECORD NUMBER 15,1

K & S DATA RECORD
BLUE (NL/24)
PRICE= 1.000
STRFAC= 1.000

K & S DATA RECORD
@

WL	K	S
400.0	6.91171	1.66654
410.0	6.07239	1.75082
420.0	5.01133	1.85656
430.0	3.81107	1.96812
440.0	2.71423	2.06539
450.0	2.17408	2.10180
460.0	2.13745	2.06884
470.0	2.42394	1.97812
480.0	2.90031	1.83613
490.0	3.51273	1.63673
500.0	4.19516	1.40335
510.0	4.92864	1.24102
520.0	6.39597	1.18170
530.0	7.70761	1.03017
540.0	8.81245	0.88350
550.0	9.37626	0.80105
560.0	9.98352	0.72500
570.0	10.85088	0.62508
580.0	11.24832	0.52900
590.0	11.32959	0.56992
600.0	9.51084	0.45045
610.0	8.98888	0.43188
620.0	9.11162	0.40078
630.0	9.32487	0.37848
640.0	8.98200	0.39507
650.0	7.80947	0.47235
660.0	5.91502	0.62583
670.0	4.02957	0.80549
680.0	2.50274	0.96620
690.0	1.51734	1.04823
700.0	0.87489	1.06599

JOB CODE? 21
RECALL K&S DATA FROM FILE
TYPE K&S FILE RECORD NUMBER 16,1

K & S DATA RECORD
GREEN (NL/24)
PRICE= 1.000
STRFAC= 1.000

K & S DATA RECORD
@

WL	K	S
400.0	18.47473	0.72972
410.0	18.33967	0.71137
420.0	17.91206	0.70066
430.0	16.99678	0.69016
440.0	15.80589	0.69581
450.0	14.65801	0.72082
460.0	13.49940	0.77451
470.0	12.27406	0.85003
480.0	10.82498	0.93664
490.0	9.46746	1.01352
500.0	8.45785	1.03728
510.0	8.10466	1.00057
520.0	8.44530	0.94256
530.0	9.29608	0.87555
540.0	10.46715	0.79062
550.0	11.44854	0.71273
560.0	12.18456	0.65386
570.0	12.71350	0.60876
580.0	12.86120	0.54965
590.0	13.27128	0.59216
600.0	11.83783	0.44558
610.0	11.43332	0.43511
620.0	11.19638	0.43582
630.0	11.09442	0.43528
640.0	11.01524	0.43085
650.0	10.75069	0.42308
660.0	10.09666	0.41336
670.0	9.05872	0.41498
680.0	7.58384	0.44580
690.0	5.89010	0.52892
700.0	4.05349	0.66108

JOB CODE? 21
RECALL K&S DATA FROM FILE
TYPE K&S FILE RECORD NUMBER 17,1

K & S DATA RECORD
YELLOW (NL/24)
PRICE= 1.000
STRFAC= 1.000

K & S DATA RECORD
@

WL	K	S
400.0	16.43665	0.74714
410.0	16.26427	0.71683
420.0	15.80248	0.68372
430.0	15.21797	0.65306
440.0	14.58635	0.61933
450.0	14.14320	0.60130
460.0	13.69165	0.60424
470.0	13.06378	0.63097
480.0	11.95068	0.69781
490.0	10.38368	0.82990
500.0	8.43085	1.04943
510.0	6.21535	1.24044
520.0	4.13531	1.37031
530.0	3.10691	1.59195
540.0	2.34422	1.80666
550.0	1.77621	1.95278
560.0	1.33288	2.05539
570.0	1.03933	2.11414
580.0	0.81145	2.07968
590.0	0.69082	2.06932
600.0	0.54007	1.81095
610.0	0.48716	1.72449
620.0	0.46265	1.68606
630.0	0.45419	1.67356
640.0	0.44601	1.65330
650.0	0.42980	1.61288
660.0	0.39992	1.53860
670.0	0.36180	1.44793
680.0	0.31846	1.35258
690.0	0.27993	1.27063
700.0	0.23591	1.19090

```
JOB CODE?  21
RECALL K&S DATA FROM FILE
TYPE K&S FILE RECORD NUMBER  18,1

K & S DATA RECORD
RED (NL/24)
PRICE=    1.000
STRFAC=   1.000


K & S DATA RECORD
@
```

WL	K	S
400.0	18.49532	0.85660
410.0	18.01753	0.82098
420.0	17.51757	0.75906
430.0	16.88891	0.68866
440.0	16.23574	0.61948
450.0	15.89828	0.57608
460.0	15.87148	0.55240
470.0	15.92647	0.54088
480.0	15.83452	0.52941
490.0	15.59868	0.51985
500.0	15.24245	0.50993
510.0	15.18728	0.51797
520.0	14.82503	0.50543
530.0	14.69114	0.50233
540.0	14.82776	0.51922
550.0	14.70250	0.53344
560.0	14.12050	0.56575
570.0	12.79089	0.65202
580.0	10.07712	0.84167
590.0	6.64996	0.76859
600.0	4.35464	1.29303
610.0	2.92986	1.40852
620.0	2.18019	1.47735
630.0	1.82658	1.51268
640.0	1.61702	1.52078
650.0	1.47663	1.49169
660.0	1.34112	1.42222
670.0	1.21448	1.33161
680.0	1.09369	1.23542
690.0	0.98891	1.15222
700.0	0.88628	1.08204

X. Appendix III

Alternate matches:

Tristimulus and spectral matches of gray standards

Note: The next three pages are tristimulus matches of the gray standards followed by three pages of spectral matches to the same gray standards.

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  13,1
STANDARD           IS -- LIGHT GRAY

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  10,1
RECORD CONTAINS TIO2
COLORANT  2 ?  12,1
RECORD CONTAINS YELLOW PY74
COLORANT  3 ?  17,1
RECORD CONTAINS GREEN PG17
COLORANT  4 ?  28,1
RECORD CONTAINS PURPLE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 14:06:39 ITER=  3 COMBINATION     1
COLORANT                        %
(  10U)TIO2             == 37.6942
(  12U)YELLOW PY74      ==  4.1773
(  17U)GREEN PG17       == 43.1394
(  28U)PURPLE           == 14.9891

COST=   1.000      MI =  1.12
ILL=D6 2 DE=  0.01   LD=  0.00   RG=  0.01   YB= -0.01   DC=  0.00   DH=  0.24
ILL=A  2 DE=  1.12   LD=  0.01   RG=  1.07   YB=  0.34   DC= -0.59   DH= 28.31
ILL=CW 2 DE=  1.24   LD= -0.36   RG= -0.38   YB= -1.13   DC=  1.18   DH= -3.61
CIELAB
GOODNESS =   4.6967864E-03

JOB CODE?  3

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  10,1
RECORD CONTAINS TIO2
COLORANT  2 ?  12,1
RECORD CONTAINS YELLOW PY74
COLORANT  3 ?  29,1
RECORD CONTAINS BLUE PB60
COLORANT  4 ?  28,1
RECORD CONTAINS PURPLE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 14:07:25 ITER=  3 COMBINATION     1
COLORANT                        %
(  10U)TIO2             ==  4.5487
(  12U)YELLOW PY74      ==  1.9353
(  29U)BLUE PB60        == 91.7684
(  28U)PURPLE           ==  1.7475

COST=   1.000      MI =  1.60
ILL=D6 2 DE=  0.02   LD=  0.01   RG= -0.01   YB=  0.01   DC= -0.01   DH= -0.39
ILL=A  2 DE=  1.59   LD= -0.02   RG=  1.58   YB= -0.22   DC=  0.03   DH= 40.64
ILL=CW 2 DE=  1.24   LD= -0.41   RG=  0.86   YB= -0.79   DC=  0.82   DH= 19.42
CIELAB
GOODN
```

```
ENTER
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  14,1
STANDARD           IS -- MEDIUM GRAY

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  10,1
RECORD CONTAINS TIO2
COLORANT  2 ?  12,1
RECORD CONTAINS YELLOW PY74
COLORANT  3 ?  29,1
RECORD CONTAINS BLUE PB60
COLORANT  4 ?  28,1
RECORD CONTAINS PURPLE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 14:12:30 ITER=  3 COMBINATION     1
COLORANT                         %
(  10U)TIO2             ==  1.8310
(  12U)YELLOW PY74      ==  1.8817
(  29U)BLUE PB60        == 94.4009
(  28U)PURPLE           ==  1.8864

COST=  , 1.000      MI =  1.62
ILL=D6 2 DE=  0.02   LD=  0.01   RG= -0.01   YB=  0.01   DC= -0.01   DH= -0.36
ILL=A  2 DE=  1.60   LD= -0.01   RG=  1.58   YB= -0.27   DC=  0.05   DH= 33.06
ILL=CW 2 DE=  1.15   LD= -0.40   RG=  0.82   YB= -0.71   DC=  0.75   DH= 14.97
CIELAB
GOODNESS =   1.5534506E-03

JOB CODE?  3

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  10,1
RECORD CONTAINS TIO2
COLORANT  2 ?  15,1
RECORD CONTAINS RED PV19
COLORANT  3 ?  17,1
RECORD CONTAINS GREEN PG17
COLORANT  4 ?  29,1
RECORD CONTAINS BLUE PB60
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 14:13:16 ITER=  3 COMBINATION     1
COLORANT                         %
(  10U)TIO2             ==  1.1843
(  15U)RED PV19         ==  1.4189
(  17U)GREEN PG17       ==  4.7083
(  29U)BLUE PB60        == 92.6885

COST=   1.000      MI =  0.74
ILL=D6 2 DE=  0.03   LD=  0.01   RG=  0.01   YB= -0.02   DC=  0.02   DH=  0.26
ILL=A  2 DE=  0.72   LD=  0.00   RG=  0.39   YB=  0.61   DC= -0.71   DH=  2.97
ILL=CW 2 DE=  1.25   LD= -0.25   RG= -0.23   YB= -1.20   DC=  1.22   DH= -1.75
CIELAB
GOODNESS =   1.9339784E-03
```

```
JOB        _

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  15,1
STANDARD          IS -- DARK GRAY

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  10,1
RECORD CONTAINS TIO2
COLORANT  2 ?  12,1
RECORD CONTAINS YELLOW PY74
COLORANT  3 ?  28,1
RECORD CONTAINS PURPLE
COLORANT  4 ?  30,1
RECORD CONTAINS BLUEGREEN
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 14:14:30 ITER=  3 COMBINATION     1
COLORANT                     %
(  10U)TIO2             ==  6.7194
(  12U)YELLOW PY74      == 26.8751
(  28U)PURPLE           == 51.0937
(  30U)BLUEGREEN        == 15.3118

COST=   1.000       MI =  0.99
ILL=D6 2 DE=  0.03   LD=  0.01   RG= -0.02   YB=  0.02   DC= -0.02   DH= -0.34
ILL=A  2 DE=  0.97   LD=  0.00   RG=  0.92   YB= -0.32   DC=  0.14   DH= 17.60
ILL=CW 2 DE=  0.47   LD= -0.15   RG=  0.44   YB= -0.08   DC=  0.09   DH=  8.02
CIELAB
GOODNESS =   3.0586662E-04

JOB CODE?  3

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  11,1
RECORD CONTAINS MARS BLACK
COLORANT  2 ?  17,1
RECORD CONTAINS GREEN PG17
COLORANT  3 ?  29,1
RECORD CONTAINS BLUE PB60
COLORANT  4 ?  28,1
RECORD CONTAINS PURPLE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 14:15:09 ITER=  4 COMBINATION     1
COLORANT                     %
(  11U)MARS BLACK       ==  0.7608
(  17U)GREEN PG17       ==  1.8579
(  29U)BLUE PB60        == 96.2599
(  28U)PURPLE           ==  1.1214

COST=   1.000       MI =  0.56
ILL=D6 2 DE=  0.04   LD=  0.00   RG=  0.02   YB= -0.04   DC=  0.04   DH=  0.35
ILL=A  2 DE=  0.53   LD=  0.00   RG=  0.12   YB=  0.51   DC= -0.52   DH= -1.38
ILL=CW 2 DE=  0.51   LD= -0.11   RG= -0.33   YB= -0.37   DC=  0.39   DH= -5.27
CIELAB
GOODNESS =   2.3436528E-02
```

COLORANT PERCENTAGES

FIBER	1	TIO2	32.09%
FIBER	2	YELLOW PY74	2.17%
FIBER	3	GREEN PG17	53.14%
FIBER	4	PURPLE	12.60%

10-APR-85 14:26:55 COMBINATION 1 AFTER 1 ITERATIONS FOR LIGHT GRAY

ILL=D6 2	DE= 2.91	LD= -0.71	RG= -2.41	YB= 1.47	DC= 0.97	DH=-69.39
ILL=A 2	DE= 2.06	LD= -0.85	RG= -1.08	YB= 1.53	DC= -0.09	DH=-49.83
ILL=CW 2	DE= 2.72	LD= -1.09	RG= -2.47	YB= 0.26	DC= 1.23	DH=-48.51

CIELAB

MI = 1.33 GOODNESS = 0.3440E-02 AT A COST OF $1.0000

COLORANT PERCENTAGES

FIBER	1	TIO2	2.18%
FIBER	2	YELLOW PY74	1.00%
FIBER	3	BLUE PB60	96.43%
FIBER	4	PURPLE	0.39%

10-APR-85 14:27:40 COMBINATION 1 AFTER 1 ITERATIONS FOR LIGHT GRAY

ILL=D6 2	DE= 8.97	LD= 0.55	RG= -8.86	YB= 1.26	DC= 7.42	DH=-73.20
ILL=A 2	DE= 7.08	LD= -0.30	RG= -7.04	YB= -0.68	DC= 6.24	DH=-44.29
ILL=CW 2	DE= 5.91	LD= -0.26	RG= -5.90	YB= -0.15	DC= 4.50	DH=-61.88

CIELAB

MI = 2.79 GOODNESS = 0.2059E-02 AT A COST OF $1.0000

COLORANT PERCENTAGES

FIBER	1	TIO2	0.60%
FIBER	2	YELLOW PY74	0.84%
FIBER	3	BLUE PB60	98.29%
FIBER	4	PURPLE	0.27%

10-APR-85 14:32:58 COMBINATION 1 AFTER 26 ITERATIONS FOR MEDIUM GRAY

ILL=D6 2	DE= 10.05	LD= -0.21	RG= -9.90	YB= 1.69	DC= 7.88	DH=-77.70
ILL=A 2	DE= 7.90	LD= -1.14	RG= -7.80	YB= -0.57	DC= 6.68	DH=-46.61
ILL=CW 2	DE= 6.68	LD= -1.11	RG= -6.58	YB= 0.18	DC= 4.60	DH=-64.62

CIELAB

MI = 3.22 GOODNESS = 0.7997E-03 AT A COST OF $1.0000

COLORANT PERCENTAGES

FIBER	1	TIO2	0.27%
FIBER	2	RED PV19	0.14%
FIBER	3	GREEN PG17	1.36%
FIBER	4	BLUE PB60	98.23%

10-APR-85 14:34:50 COMBINATION 1 AFTER 26 ITERATIONS FOR MEDIUM GRAY

ILL=D6 2	DE= 7.70	LD= -2.05	RG= -6.18	YB= -4.13	DC= 6.81	DH=-36.66
ILL=A 2	DE= 9.33	LD= -2.98	RG= -7.13	YB= -5.23	DC= 8.56	DH=-22.59
ILL=CW 2	DE= 8.46	LD= -2.87	RG= -4.96	YB= -6.22	DC= 7.63	DH=-24.91

CIELAB

MI = 1.73 GOODNESS = 0.1268E-02 AT A COST OF $1.0000

COLORANT PERCENTAGES

FIBER	1	TIO2	5.36%
FIBER	2	YELLOW PY74	21.19%
FIBER	3	PURPLE	56.18%
FIBER	4	BLUEGREEN	17.28%

10-APR-85 14:37:18 COMBINATION 1 AFTER 26 ITERATIONS FOR DARK GRAY

ILL=D6 2	DE= 2.09	LD= -1.34	RG= 0.84	YB= -1.37	DC= 1.43	DH= 12.43
ILL=A 2	DE= 2.36	LD= -1.34	RG= 1.14	YB= -1.57	DC= 1.38	DH= 21.23
ILL=CW 2	DE= 2.25	LD= -1.47	RG= 0.90	YB= -1.44	DC= 1.51	DH= 11.83

CIELAB

MI = 0.36 GOODNESS = 0.1731E-03 AT A COST OF $1.0000

COLORANT PERCENTAGES

FIBER	1	MARS BLACK	0.26%
FIBER	2	GREEN PG17	0.51%
FIBER	3	BLUE PB60	98.67%
FIBER	4	PURPLE	0.55%

10-APR-85 14:39:40 COMBINATION 1 AFTER 26 ITERATIONS FOR DARK GRAY

ILL=D6 2	DE= 4.55	LD= -0.77	RG= 1.37	YB= -4.27	DC= 4.37	DH= 12.64
ILL=A 2	DE= 4.17	LD= -0.92	RG= 0.17	YB= -4.06	DC= 3.93	DH= 12.83
ILL=CW 2	DE= 5.21	LD= -0.99	RG= 0.88	YB= -5.04	DC= 5.07	DH= 7.28

CIELAB

MI = 1.23 GOODNESS = 0.1171E-02 AT A COST OF $1.0000

XI. Appendix IV

Badische blend reflectance data:

Reflectance data of fiber blends

Badische Corporation

Badische

August 9, 1984

Cornelius McCarthy

Dear Mickey:

Enclosed is the data you requested. Page 1 contains data on 9 solids. Pages 1 to 3 contain data on the 42 blends discussed in the Color Research and Application paper. The blend information starts at line 19. There are 8 numbers on line 190. The first four are the numbers of the colors in blend #1 and the last four are their respective concentrations in blend #1. Lines 195 and 200 are the 16 reflectances (400 to 700 nanometers, 20 nanometer intervals) for blend #1. The rest of the blends follow the same format.

Pages 4 and 5 contain information on the "Primaries". These curves were synthesized from the K and S data stored in the data file.

I am very interested in the outcome of your study. I am especially interested in your non-linear optimization routine particularly as it pertains to calculation of K and S data from blends other than the conventional binary blends with white and black. If the program is successful in accomplishing this and could be made available by itself, we would be interested in talking about a purchase.

When you are in this area, please come by to talk about our common interests. In the meantime, please let me know if there is any other information you need.

Dom Burlone

D. A. Burlone
R & D Department

/ct

Enclosure

Telephone: , TWX:

Member of the BASF Group

BASF

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100   73.92  75.84  77.34  78.83  79.97  80.78  81.62  81.69  WHITE #1
105   82.08  82.01  81.98  82.11  82.51  83.11  83.65  84.14
110    2.84   2.77   2.91   3.60   4.58   5.96   7.56   9.28  GINGER #2
115   12.21  15.10  17.33  18.49  19.44  20.12  20.70  21.56
120    5.25   3.80   2.98   3.00   3.98   7.16  12.18  16.42  ORANGE #3
125   22.08  32.80  46.18  53.26  57.42  60.38  62.94  65.52
130    3.32   3.10   3.56   5.64  12.15  19.75  21.61  17.28  GREEN #4
135   12.86   9.96   8.40   8.20   8.22  10.30  17.87  33.20
140    3.70   3.43   2.82   2.43   1.95   1.83   1.78   2.09  RED #5
145    2.42   5.09  12.91  18.41  20.22  22.81  28.27  34.63
150   14.23  10.70   8.53   8.65  12.36  21.52  35.74  45.24  YELLOW #6
155   53.95  59.43  63.58  66.34  68.76  71.34  74.18  76.86
160    1.88   1.84   1.77   1.87   1.81   1.80   1.65   1.75  BLACK #7
165    1.57   1.53   1.56   1.58   1.74   2.28   5.43  14.13
170    2.94   2.69   2.46   2.46   2.45   2.69   3.01   3.51  BROWN #8
175    3.79   4.24   4.89   4.80   6.56  14.61  29.39  41.23
180    5.63   6.59   7.50   7.24   5.56   4.66   3.74   3.45  BLUE #9
185    2.91   2.66   2.52   2.51   2.57   3.20   6.55  16.43
190   3 9 4 1 20.0  5.0 15.0 60.0
195   20.36  19.37  19.15  20.25  23.45  28.08  31.82  32.35
200   32.34  32.78  33.27  33.62  33.98  35.96  42.28  53.33
205   8 2 9 1 14.3 33.3  9.5 42.9
210   10.55  10.43  10.51  11.13  11.78  12.86  14.07  15.31
215   17.11  18.81  20.23  20.68  22.00  25.51  30.09  34.17
220   2 9 4 1 15.0 15.0 15.0 55.0
225   14.64  14.74  15.34  16.68  18.66  20.60  21.36  21.14
230   21.01  20.86  20.75  20.93  21.23  22.70  27.35  35.64
235   8 6 4 1 30.0 30.0  5.0 35.0
240   13.37  12.07  11.16  11.39  12.91  15.68  18.46  19.79
245   20.58  21.20  22.03  21.94  23.89  31.99  45.15  55.80
250   7 2 5 8  5.0 10.0 30.0 55.0
255    2.95   2.73   2.44   2.42   2.36   2.57   2.83   3.31
260    3.72   4.89   7.14   7.99   9.53  15.35  25.08  33.41
265   8 2 5 1  5.6 11.1  5.6 77.8
270   27.74  27.29  27.01  27.38  27.78  28.68  29.75  30.86
275   32.64  35.43  39.23  40.83  42.48  46.20  50.49  53.23
280   8 3 5 1 20.0 30.0  5.0 45.0
285   13.57  12.30  11.35  11.28  11.97  14.08  16.76  18.78
290   20.96  24.63  29.03  30.50  32.93  40.54  51.38  59.00
295   6 2 5 1 35.0 20.0 10.0 35.0
300   15.01  13.53  12.51  12.84  14.73  18.40  22.52  25.10
305   28.13  32.09  37.25  39.95  41.31  42.63  44.39  46.18
310   3 5 4 1 20.0 30.0 10.0 40.0
315   13.35  12.53  11.86  12.03  12.92  14.51  15.84  16.50
320   17.27  20.46  26.90  30.61  31.86  34.35  40.19  47.92
325   8 2 4 1 25.0 20.0 15.0 40.0
330   10.58  10.29  10.36  11.23  12.84  14.56  15.73  16.18
335   16.64  17.10  17.69  17.75  19.10  24.32  32.66  40.09
340   8 5 9 3 15.0 10.0 20.0 55.0
345    5.02   4.36   3.86   3.79   4.06   5.37   6.97   8.27
```

350	9.54	11.88	14.57	15.54	16.67	20.16	27.08	37.43
355	8 2 5 9	20.0	30.0	20.0	30.0			
360	3.63	3.66	3.66	3.75	3.59	3.78	4.00	4.51
365	5.07	6.29	8.16	9.00	9.82	12.27	17.09	24.25
370	8 3 4 6	10.0	10.0	30.0	50.0			
375	7.70	6.30	5.59	6.37	9.80	15.50	20.79	21.89
380	21.75	21.47	21.47	21.57	22.46	26.75	36.12	49.41
385	3 9 4 2	30.0	20.0	10.0	40.0			
390	4.17	3.84	3.77	4.20	5.13	6.89	8.64	9.88
395	11.39	13.25	14.72	15.38	15.89	17.10	20.57	26.99
400	8 2 4 6	14.3	33.3	9.5	42.9			
405	6.61	5.55	4.94	5.40	7.25	10.67	14.58	16.99
410	19.59	21.66	23.30	23.97	25.47	29.25	34.03	37.67
415	3 2 4 1	5.0	5.0	20.0	70.0			
420	23.76	23.31	23.78	26.12	31.18	36.79	39.68	38.78
425	37.51	36.49	35.94	36.10	36.35	38.77	45.63	55.72
430	3 6 4 1	15.0	25.0	10.0	50.0			
435	18.89	16.91	15.75	16.43	19.75	26.16	33.46	36.65
440	39.05	41.56	43.77	44.95	45.70	48.19	54.62	63.98
445	1 7 4 5	15.0	35.0	5.0	45.0			
450	5.21	5.07	4.67	4.58	4.36	4.41	4.31	4.45
455	4.50	5.71	8.47	9.91	10.56	12.01	16.67	25.60
460	7 6 2 1	10.0	20.0	30.0	40.0			
465	13.42	12.73	12.34	13.03	14.55	17.12	19.65	21.32
470	23.43	25.18	26.50	27.20	28.02	29.27	32.05	36.36
475	2 5 9 1	20.0	15.0	25.0	40.0			
480	11.77	12.00	12.08	12.18	11.69	11.74	11.75	12.10
485	12.58	13.89	16.10	17.25	17.72	18.90	22.66	29.93
490	7 8 5 3	5.0	20.0	20.0	55.0			
495	4.55	3.67	2.99	2.96	3.44	5.02	7.03	8.66
500	10.45	13.98	18.93	21.01	22.94	28.28	36.72	44.76
505	1 7 8 4	10.0	30.0	20.0	40.0			
510	3.92	3.77	3.82	4.58	6.34	8.07	8.43	7.79
515	6.80	6.13	5.88	5.80	6.29	8.75	15.37	27.10
520	7 8 3 1	5.0	30.0	15.0	50.0			
525	13.27	12.58	11.96	11.99	12.27	13.44	14.80	15.92
530	16.98	18.38	19.90	20.04	21.81	28.67	40.06	50.33
535	7 8 3 1	10.0	10.0	10.0	70.0			
540	21.94	21.22	20.52	20.58	21.03	22.40	23.67	24.52
545	25.33	26.40	27.54	27.72	29.06	33.44	41.27	50.73
550	7 8 9 6	10.0	15.0	10.0	65.0			
555	9.24	7.61	6.44	6.53	8.19	11.78	15.58	17.41
560	18.42	19.02	19.62	19.74	20.79	24.58	31.86	42.03
565	7 8 9 1	5.0	5.0	5.0	85.0			
570	33.04	33.40	33.51	33.56	32.82	32.70	32.29	32.06
575	31.80	31.65	31.78	31.73	32.54	35.41	41.68	51.40
580	7 6 4 1	10.0	20.0	5.0	65.0			
585	26.61	25.12	24.08	24.77	27.53	31.99	35.47	36.16
590	36.08	35.70	35.57	35.71	36.28	38.17	43.52	52.61
595	1 7 9 2	20.0	20.0	10.0	50.0			

600	5.72	5.75	5.89	6.43	6.87	7.61	8.26	9.03
605	10.09	11.08	11.83	12.22	12.71	13.66	16.54	22.11
610	1	7	5	3	5.3	15.8	26.3	52.6
615	5.11	4.34	3.66	3.60	3.95	5.27	6.83	8.10
620	9.46	12.62	17.54	19.97	21.14	23.11	28.22	36.79
625	7	5	4	1	5.0	15.0	5.0	75.0
630	24.41	24.21	23.76	24.00	24.42	25.20	25.22	24.99
635	24.87	26.58	30.74	33.01	33.94	36.17	42.03	50.66
640	7	8	3	1	15.0	15.0	20.0	50.0
645	14.89	14.19	13.53	13.58	14.02	15.38	16.75	17.71
650	18.54	19.78	21.04	21.31	22.49	26.17	33.33	43.16
655	7	8	9	3	5.3	15.8	15.8	63.2
660	4.96	4.17	3.60	3.61	4.13	5.82	7.85	9.37
665	10.82	12.98	14.93	15.49	16.58	19.82	26.33	36.57
670	7	6	4	9	20.0	10.0	10.0	60.0
675	4.53	4.78	5.01	5.10	4.87	5.07	4.84	4.66
680	4.15	3.86	3.73	3.72	3.82	4.57	8.18	18.03
685	7	6	2	1	5.0	15.0	20.0	60.0
690	20.87	19.97	19.41	20.15	21.92	25.03	28.10	30.00
695	32.43	34.34	35.82	36.58	37.49	38.68	41.15	44.71
700	7	8	9	5	5.0	10.0	25.0	60.0
705	3.69	3.68	3.38	3.11	2.52	2.34	2.19	2.40
710	2.51	3.86	7.25	9.14	10.00	12.16	17.44	26.36
715	1	2	5	7	25.0	5.0	10.0	60.0
720	5.96	5.90	5.72	5.82	5.70	5.73	5.60	5.71
725	5.64	5.91	6.60	6.95	7.34	8.36	12.32	21.20
730	7	2	9	6	15.0	10.0	5.0	70.0
735	9.22	7.53	6.37	6.57	8.59	12.82	17.50	19.78
740	21.31	22.21	22.89	23.29	23.98	25.55	30.04	38.22
745	2	9	4	7	25.0	15.0	25.0	35.0
750	2.69	2.70	2.86	3.47	4.47	5.52	5.84	5.81
755	5.62	5.55	5.57	5.68	5.91	6.92	10.83	19.33
760	7	9	4	3	5.0	20.0	5.0	70.0
765	5.34	4.42	3.81	3.92	4.82	7.23	10.07	11.95
770	13.86	16.85	19.46	20.43	21.09	22.70	27.67	37.71
775	7	6	5	1	10.0	5.0	5.0	80.0
780	29.98	29.50	28.73	28.71	28.79	29.61	30.02	30.19
785	30.36	31.42	33.61	34.68	35.59	37.48	42.50	50.68
790	7	9	4	8	20.0	5.0	15.0	60.0
795	2.68	2.52	2.42	2.63	2.96	3.44	3.66	3.84
800	3.73	3.80	4.08	4.01	4.95	9.22	17.90	29.29
805	7	8	3	1	25.0	25.0	25.0	25.0
810	7.29	6.73	6.22	6.29	6.60	7.60	8.64	9.49
815	10.16	11.22	12.28	12.49	13.59	17.31	24.62	34.75

WL	WHITE STD.	WHITE MATCH
400	73.92	73.92
420	75.84	75.84
440	77.34	77.34
460	78.83	78.83
480	79.97	79.97
500	80.78	80.78
520	81.62	81.62
540	81.69	81.69
560	82.08	82.08
580	82.01	82.01
600	81.98	81.98
620	82.11	82.11
640	82.51	82.51
660	83.11	83.11
680	83.65	83.65
700	84.14	84.14

WL	BLUE 100% STD.	25% BLUE 75% WHITE MATCH
400	5.63	29.72
420	6.59	30.86
440	7.50	31.84
460	7.24	31.59
480	5.56	29.68
500	4.66	28.61
520	3.74	27.54
540	3.45	27.02
560	2.91	26.30
580	2.66	25.88
600	2.52	25.59
620	2.51	25.52
640	2.57	25.64
660	3.20	26.78
680	6.55	31.36
700	16.43	41.55

WL	100% BLACK STD.	25% BLACK 75% WHITE MATCH
400	1.88	21.91
420	1.84	21.86
440	1.77	21.59
460	1.87	21.77
480	1.81	21.66
500	1.80	21.61
520	1.65	21.24
540	1.75	20.99
560	1.57	20.55
580	1.53	20.30
600	1.56	20.43
620	1.58	20.51
640	1.74	21.17
660	2.28	22.70
680	5.43	27.85
700	14.13	37.95

WL	100% RED STD.	25% RED 75% WHITE MATCH
400	3.70	25.41
420	3.43	25.10
440	2.82	24.26
460	2.43	23.72
480	1.95	22.81
500	1.83	22.56
520	1.78	22.50
540	2.09	22.86
560	2.42	23.55
580	5.09	27.08
600	12.91	35.21
620	18.41	40.10
640	20.22	41.73
660	22.81	44.04
680	28.27	48.59
700	34.63	53.77

WL	100% BROWN STD.	25% BROWN 75% WHITE MATCH
400	2.94	25.01
420	2.69	24.67
440	2.46	24.27
460	2.46	24.29
480	2.45	24.24
500	2.69	24.71
520	3.01	25.41
540	3.51	26.13
560	3.79	26.74
580	4.24	27.44
600	4.89	28.36
620	4.80	28.22
640	6.56	30.52
660	14.61	39.51
680	29.39	52.79
700	41.23	61.64

WL	100% ORANGE STD.	25% ORANGE 75% WHITE MATCH
400	5.81	27.13
420	4.32	25.29
440	3.41	24.04
460	3.43	24.11
480	4.63	25.56
500	8.04	29.72
520	13.15	35.62
540	17.31	39.99
560	22.75	45.44
580	32.49	54.02
600	44.47	63.11
620	50.91	67.47
640	54.84	70.08
660	57.79	72.01
680	60.36	73.66
700	63.07	75.20

WL	100% GREEN STD.	25% GREEN 75% WHITE MATCH
400	3.32	25.65
420	3.10	25.36
440	3.56	26.09
460	5.64	28.89
480	12.15	36.20
500	19.75	43.98
520	21.61	46.35
540	17.28	42.31
560	12.86	37.99
580	9.96	34.78
600	8.40	32.91
620	8.20	32.66
640	8.22	32.75
660	10.30	35.18
680	17.87	42.95
700	33.20	56.20

WL	100% GINGER STD.	25% GINGER 75% WHITE MATCH
400	2.84	25.70
420	2.77	25.54
440	2.91	25.83
460	3.60	27.11
480	4.58	28.63
500	5.96	30.64
520	7.56	32.78
540	9.28	34.71
560	12.21	37.96
580	15.10	40.84
600	17.33	43.00
620	18.49	44.13
640	19.44	45.11
660	20.12	45.90
680	20.70	46.58
700	21.56	47.39

WL	100% Yellow STD.	25% ~~Brown~~ Yellow 75% WHITE MATCH
400	14.50	39.08
420	11.14	35.93
440	8.99	33.67
460	9.11	33.97
480	12.87	38.27
500	21.85	47.09
520	35.60	58.18
540	44.53	64.27
560	52.42	69.20
580	57.25	71.88
600	60.92	73.82
620	63.65	75.25
640	66.28	76.71
660	69.06	78.28
680	72.29	79.95
700	75.18	81.36

WL	100% BLUE STD.	90% BLUE 10% BLACK MATCH
400	5.63	5.09
420	6.59	5.89
440	7.50	6.62
460	7.24	6.43
480	5.56	5.02
500	4.66	4.26
520	3.74	3.45
540	3.45	3.22
560	2.91	2.73
580	2.66	2.50
600	2.52	2.39
620	2.51	2.39
640	2.57	2.46
660	3.20	3.08
680	6.55	6.42
700	16.43	16.16

WL	100% RED STD.	90% RED 10% BLACK MATCH
400	3.70	3.49
420	3.43	3.25
440	2.82	2.70
460	2.43	2.37
480	1.95	1.93
500	1.83	1.83
520	1.78	1.77
540	2.09	2.05
560	2.42	2.32
580	5.09	4.61
600	12.91	11.05
620	18.41	15.37
640	20.22	16.88
660	22.81	19.18
680	28.27	24.59
700	34.63	31.87

WL	100% BROWN STD.	90% BROWN 10% BLACK MATCH
400	2.94	2.82
420	2.69	2.59
440	2.46	2.38
460	2.46	2.39
480	2.45	2.38
500	2.69	2.58
520	3.01	2.84
540	3.51	3.28
560	3.79	3.49
580	4.24	3.85
600	4.89	4.40
620	4.80	4.34
640	6.56	5.84
660	14.61	12.40
680	29.39	24.41
700	41.23	35.94

WL	100% ORANGE STD.	90% ORANGE 10% BLACK MATCH
400	5.81	5.34
420	4.32	4.04
440	3.41	3.23
460	3.43	3.26
480	4.63	4.31
500	8.04	7.22
520	13.15	11.32
540	17.31	14.46
560	22.75	18.16
580	32.49	24.03
600	44.47	29.94
620	50.91	32.44
640	54.84	34.20
660	57.79	36.39
680	60.36	41.12
700	63.07	48.96

WL	100% GREEN STD.	90% GREEN 10% BLACK MATCH
400	3.32	3.15
420	3.10	2.95
440	3.56	3.34
460	5.64	5.13
480	12.15	10.41
500	19.75	16.00
520	21.61	17.02
540	17.28	13.99
560	12.86	10.68
580	9.96	8.45
600	8.40	7.25
620	8.20	7.10
640	8.22	7.18
660	10.30	9.01
680	17.87	15.84
700	33.20	29.90

WL	100% GINGER STD.	90% GINGER 10% BLACK MATCH
400	2.84	2.72
420	2.77	2.66
440	2.91	2.77
460	3.60	3.37
480	4.58	4.19
500	5.96	5.33
520	7.56	6.59
540	9.28	7.95
560	12.21	10.11
580	15.10	12.18
600	17.33	13.77
620	18.49	14.58
640	19.44	15.38
660	20.12	16.21
680	20.70	17.86
700	21.56	20.51

WL	100% YELLOW STD.	90% YELLOW 10% BLACK MATCH
400	14.50	12.13
420	11.14	9.54
440	8.99	7.81
460	9.11	7.93
480	12.87	10.81
500	21.85	17.12
520	35.60	25.03
540	44.53	29.05
560	52.42	31.67
580	57.25	32.91
600	60.92	33.89
620	63.65	34.50
640	66.28	35.52
660	69.06	37.45
680	72.29	42.55
700	75.18	51.33

XII. Appendix V

Matches of Badische blends:

Tristimulus and spectral matches of fiber blends

Note: The first match on each page is a tristimulus match to the indicated standard while the second match is a spectral match to the same standard.

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  27,1
STANDARD           IS -- 190 BURLONE 1

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  2 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 13:59:19 ITER=  1 COMBINATION     1
COLORANT                        %
(   3U)ORANGE FIBER    == 15.4258
(   9U)BLUE FIBER      ==  4.1816
(   4U)GREEN FIBER     == 16.6643
(   1U)WHITE BASE      == 63.7283
COST=   1.000       MI =  0.04
ILL=D6 2 DE=  0.01   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH= -0.01
ILL=A  2 DE=  0.04   LD=  0.00   RG=  0.04   YB= -0.02   DC= -0.02   DH= -0.12
ILL=CW 2 DE=  0.02   LD=  0.00   RG=  0.01   YB=  0.01   DC=  0.01   DH= -0.03
CIELAB
GOODNESS =   5.6396066E-06



COLORANT PERCENTAGES

FIBER  1  ORANGE FIBER          15.27%
FIBER  2  BLUE FIBER             4.65%
FIBER  3  GREEN FIBER           16.14%
FIBER  4  WHITE BASE            63.94%

11-APR-85 14:19:15 COMBINATION  1 AFTER 26 ITERATIONS FOR 190 BURLONE 1

ILL=D6 2 DE=  0.45   LD= -0.10   RG=  0.17   YB= -0.40   DC= -0.43   DH= -0.27
ILL=A  2 DE=  0.40   LD= -0.10   RG=  0.10   YB= -0.38   DC= -0.37   DH= -0.41
ILL=CW 2 DE=  0.44   LD= -0.10   RG=  0.14   YB= -0.41   DC= -0.42   DH= -0.21

CIELAB

MI =   0.08 GOODNESS = 0.3119E-05 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  64,1
STANDARD          IS -- 220 BURLONE 2

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  2,1
RECORD CONTAINS GINGER FIBER
COLORANT  2 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:00:07 ITER=  1 COMBINATION     1
COLORANT                    %
(   2U)GINGER FIBER    == 19.4877
(   9U)BLUE FIBER      == 14.8736
(   4U)GREEN FIBER     == 14.7086
(   1U)WHITE BASE      == 50.9301
COST=   1.000      MI =  0.02
ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.01
ILL=A  2 DE=  0.02   LD=  0.00   RG= -0.01   YB=  0.01   DC=  0.01   DH=  0.10
ILL=CW 2 DE=  0.02   LD=  0.00   RG=  0.02   YB=  0.00   DC= -0.01   DH= -0.10
CIELAB
GOODNESS =   7.8108275E-07



COLORANT PERCENTAGES

FIBER  1  GINGER FIBER         19.56%
FIBER  2  BLUE FIBER           14.62%
FIBER  3  GREEN FIBER          15.00%
FIBER  4  WHITE BASE           50.82%

11-APR-85 14:20:11 COMBINATION  1 AFTER  1 ITERATIONS FOR 220 BURLONE 2

ILL=D6 2 DE=  0.19   LD=  0.03   RG= -0.10   YB=  0.16   DC=  0.18   DH=  0.21
ILL=A  2 DE=  0.16   LD=  0.03   RG= -0.06   YB=  0.14   DC=  0.15   DH=  0.40
ILL=CW 2 DE=  0.18   LD=  0.04   RG= -0.06   YB=  0.16   DC=  0.17   DH=  0.04

CIELAB

MI =   0.04 GOODNESS = 0.6061E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  62,1
STANDARD           IS -- 235 BURLONE 4

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  2 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:01:25 ITER=  1 COMBINATION     1
COLORANT                        %
(   8U)BROWN FIBER     == 31.2510
(   6U)YELLOW FIBER    == 30.4340
(   4U)GREEN FIBER     ==  4.2230
(   1U)WHITE BASE      == 34.0921
COST=   1.000      MI =  0.00
ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.01
ILL=A  2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.01
ILL=CW 2 DE=  0.01   LD=  0.00   RG=  0.01   YB=  0.01   DC=  0.01   DH= -0.03
CIELAB
GOODNESS =   1.8058562E-06



COLORANT PERCENTAGES

FIBER  1  BROWN FIBER            31.50%
FIBER  2  YELLOW FIBER           30.31%
FIBER  3  GREEN FIBER             3.84%
FIBER  4  WHITE BASE             34.35%

11-APR-85 14:21:12 COMBINATION  1 AFTER  2 ITERATIONS FOR 235 BURLONE 4

ILL=D6 2 DE=  0.20   LD= -0.01   RG=  0.16   YB= -0.11   DC= -0.11   DH= -0.52
ILL=A  2 DE=  0.14   LD=  0.00   RG=  0.12   YB= -0.07   DC= -0.03   DH= -0.39
ILL=CW 2 DE=  0.17   LD=  0.00   RG=  0.12   YB= -0.11   DC= -0.12   DH= -0.35

CIELAB

MI =   0.06 GOODNESS = 0.1277E-05 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD           DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  66,1
STANDARD            IS -- 265 BURLONE 6

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  2 ?  2,1
RECORD CONTAINS GINGER FIBER
COLORANT  3 ?  5,1
RECORD CONTAINS RED FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:02:53 ITER=  1 COMBINATION      1
COLORANT                     %
(   8U)BROWN FIBER      ==  3.8214
(   2U)GINGER FIBER     == 12.0393
(   5U)RED FIBER        ==  6.6154
(   1U)WHITE BASE       == 77.5239
COST=   1.000       MI =  0.09
ILL=D6 2 DE=  0.04   LD=  0.00   RG=  0.04   YB=  0.00   DC=  0.03   DH= -0.15
ILL=A  2 DE=  0.13   LD=  0.00   RG=  0.12   YB= -0.03   DC=  0.06   DH= -0.43
ILL=CW 2 DE=  0.08   LD= -0.02   RG=  0.07   YB= -0.02   DC=  0.02   DH= -0.36
CIELAB
GOODNESS =   6.8169243E-06



COLORANT PERCENTAGES

FIBER  1  BROWN FIBER            4.30%
FIBER  2  GINGER FIBER          11.41%
FIBER  3  RED FIBER              6.51%
FIBER  4  WHITE BASE            77.78%

11-APR-85 14:22:40 COMBINATION  1 AFTER 26 ITERATIONS FOR 265 BURLONE 6

ILL=D6 2 DE=  0.27   LD= -0.01   RG= -0.07   YB= -0.26   DC= -0.25   DH= -0.56
ILL=A  2 DE=  0.33   LD= -0.03   RG= -0.04   YB= -0.32   DC= -0.26   DH= -0.76
ILL=CW 2 DE=  0.32   LD= -0.05   RG= -0.03   YB= -0.32   DC= -0.29   DH= -0.64

CIELAB

MI =   0.07 GOODNESS = 0.3259E-05 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  68,1
STANDARD          IS -- 280 BURLONE 7

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  2 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  3 ?  5,1
RECORD CONTAINS RED FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:04:03 ITER=  1 COMBINATION     1
COLORANT                     %
(   8U)BROWN FIBER      == 17.5113
(   3U)ORANGE FIBER     == 29.9799
(   5U)RED FIBER        ==  6.4078
(   1U)WHITE BASE       == 46.1011
COST=   1.000      MI =  0.05
ILL=D6 2 DE=  0.01   LD=  0.00   RG= -0.01   YB=  0.00   DC=  0.00   DH=  0.02
ILL=A  2 DE=  0.06   LD=  0.00   RG= -0.06   YB=  0.00   DC= -0.03   DH=  0.09
ILL=CW 2 DE=  0.02   LD=  0.01   RG=  0.02   YB=  0.00   DC=  0.01   DH= -0.04
CIELAB
GOODNESS =   3.4686833E-05



COLORANT PERCENTAGES

FIBER  1  BROWN FIBER          19.28%
FIBER  2  ORANGE FIBER         31.59%
FIBER  3  RED FIBER             3.45%
FIBER  4  WHITE BASE           45.68%

11-APR-85 14:23:44 COMBINATION  1 AFTER 26 ITERATIONS FOR 280 BURLONE 7

ILL=D6 2 DE=  1.25   LD=  0.36   RG= -0.88   YB=  0.81   DC=  0.34   DH=  2.63
ILL=A  2 DE=  1.10   LD=  0.31   RG= -0.83   YB=  0.65   DC=  0.12   DH=  2.00
ILL=CW 2 DE=  1.23   LD=  0.41   RG= -0.74   YB=  0.90   DC=  0.64   DH=  2.17

CIELAB

MI =   0.17 GOODNESS = 0.1213E-04 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  70,1
STANDARD          IS -- 310 BURLONE 9

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  2 ?  5,1
RECORD CONTAINS RED FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:04:43 ITER=  1 COMBINATION     1
COLORANT                    %
(   3U)ORANGE FIBER     == 15.5397
(   5U)RED FIBER        == 27.4856
(   4U)GREEN FIBER      ==  9.7287
(   1U)WHITE BASE       == 47.2460
COST=   1.000      MI =  0.03
ILL=D6 2 DE=  0.01   LD=  0.00   RG= -0.01   YB=  0.01   DC=  0.00   DH=  0.03
ILL=A  2 DE=  0.04   LD=  0.00   RG= -0.01   YB=  0.04   DC=  0.02   DH=  0.07
ILL=CW 2 DE=  0.07   LD=  0.00   RG= -0.05   YB= -0.05   DC= -0.07   DH=  0.06
CIELAB
GOODNESS =   5.2538650E-07



COLORANT PERCENTAGES

FIBER  1  ORANGE FIBER          15.40%
FIBER  2  RED FIBER             27.48%
FIBER  3  GREEN FIBER            9.83%
FIBER  4  WHITE BASE            47.29%

11-APR-85 14:24:42 COMBINATION  1 AFTER  2 ITERATIONS FOR 310 BURLONE 9

ILL=D6 2 DE=  0.09   LD= -0.01   RG= -0.07   YB= -0.05   DC= -0.09   DH=  0.06
ILL=A  2 DE=  0.09   LD= -0.01   RG= -0.08   YB= -0.05   DC= -0.09   DH=  0.06
ILL=CW 2 DE=  0.16   LD= -0.01   RG= -0.10   YB= -0.12   DC= -0.15   DH=  0.07

CIELAB

MI =   0.01 GOODNESS = 0.4177E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  72,1
STANDARD             IS -- 325 BURLONE 10

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  2 ?  2,1
RECORD CONTAINS GINGER FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:05:17 ITER=  1 COMBINATION      1
COLORANT                         %
(   8U)BROWN FIBER      == 24.4210
(   2U)GINGER FIBER     == 20.7237
(   4U)GREEN FIBER      == 14.0196
(   1U)WHITE BASE       == 40.8357
COST=   1.000       MI =  0.01
ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.01
ILL=A  2 DE=  0.01   LD=  0.00   RG=  0.01   YB=  0.00   DC=  0.01   DH= -0.04
ILL=CW 2 DE=  0.01   LD=  0.00   RG=  0.00   YB=  0.01   DC=  0.01   DH= -0.01
CIELAB
GOODNESS =   5.5642658E-07



COLORANT PERCENTAGES

FIBER  1  BROWN FIBER            24.24%
FIBER  2  GINGER FIBER           21.11%
FIBER  3  GREEN FIBER            13.94%
FIBER  4  WHITE BASE             40.72%

11-APR-85 14:25:48 COMBINATION  1 AFTER  2 ITERATIONS FOR 325 BURLONE 10

ILL=D6 2 DE=  0.10   LD=  0.00   RG=  0.05   YB=  0.08   DC=  0.08   DH= -0.21
ILL=A  2 DE=  0.12   LD=  0.01   RG=  0.06   YB=  0.10   DC=  0.11   DH= -0.13
ILL=CW 2 DE=  0.11   LD=  0.01   RG=  0.04   YB=  0.10   DC=  0.10   DH= -0.19

CIELAB

MI =   0.02 GOODNESS = 0.3742E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  74,1
STANDARD          IS -- 340 BURLONE 11

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  2 ?  5,1
RECORD CONTAINS RED FIBER
COLORANT  3 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT  4 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:05:49 ITER=  1 COMBINATION     1
COLORANT                         %
(   8U)BROWN FIBER      == 17.8965
(   5U)RED FIBER        == 12.1552
(   9U)BLUE FIBER       == 23.4975
(   3U)ORANGE FIBER     == 46.4508
COST=   1.000      MI =  0.06
ILL=D6 2 DE=  0.01   LD=  0.00   RG=  0.01   YB=  0.00   DC=  0.01   DH= -0.03
ILL=A  2 DE=  0.07   LD=  0.00   RG=  0.07   YB=  0.00   DC=  0.04   DH= -0.12
ILL=CW 2 DE=  0.09   LD= -0.02   RG=  0.03   YB= -0.08   DC= -0.06   DH= -0.11
CIELAB
GOODNESS =   7.7493642E-06



COLORANT PERCENTAGES

FIBER  1  BROWN FIBER           21.07%
FIBER  2  RED FIBER             10.10%
FIBER  3  BLUE FIBER            22.15%
FIBER  4  ORANGE FIBER          46.68%

11-APR-85 14:27:33 COMBINATION  1 AFTER  6 ITERATIONS FOR 340 BURLONE 11

ILL=D6 2 DE=  0.57   LD=  0.06   RG= -0.46   YB=  0.34   DC=  0.12   DH=  1.19
ILL=A  2 DE=  0.40   LD=  0.02   RG= -0.32   YB=  0.24   DC=  0.05   DH=  0.71
ILL=CW 2 DE=  0.50   LD=  0.03   RG= -0.40   YB=  0.29   DC=  0.16   DH=  0.98

CIELAB

MI =   0.18 GOODNESS = 0.2093E-05 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  76,1
STANDARD           IS -- 355 BURLONE 12

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  2 ?  2,1
RECORD CONTAINS GINGER FIBER
COLORANT  3 ?  5,1
RECORD CONTAINS RED FIBER
COLORANT  4 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 15:46:38 ITER=  1 COMBINATION     1
COLORANT                     %
(   8U)BROWN FIBER      == 28.0516
(   2U)GINGER FIBER     == 30.6591
(   5U)RED FIBER        == 16.5835
(   9U)BLUE FIBER       == 24.7058

COST=   1.000       MI =  0.22
ILL=D6 2 DE=  0.03   LD=  0.00   RG=  0.03   YB=  0.01   DC=  0.03   DH= -0.06
ILL=A  2 DE=  0.25   LD=  0.02   RG=  0.25   YB=  0.02   DC=  0.20   DH= -0.44
ILL=CW 2 DE=  0.19   LD= -0.06   RG= -0.15   YB= -0.10   DC= -0.17   DH=  0.25
CIELAB
GOODNESS =   4.7703379E-05



COLORANT PERCENTAGES

FIBER  1  BROWN FIBER          18.52%
FIBER  2  GINGER FIBER         32.57%
FIBER  3  RED FIBER            17.50%
FIBER  4  BLUE FIBER           31.40%

10-APR-85 15:55:10 COMBINATION  1 AFTER  6 ITERATIONS FOR 355 BURLONE 12

ILL=D6 2 DE=  1.38   LD=  0.17   RG= -0.16   YB= -1.36   DC= -0.98   DH= -3.61
ILL=A  2 DE=  1.48   LD=  0.10   RG= -0.28   YB= -1.45   DC= -1.16   DH= -2.74
ILL=CW 2 DE=  1.54   LD=  0.10   RG= -0.12   YB= -1.53   DC= -1.26   DH= -3.77

CIELAB

MI =   0.16 GOODNESS = 0.2976E-05 AT A COST OF $1.0000
```

```
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  80,1
STANDARD            IS -- 400 BURLONE 15

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT   1 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT   2 ?  2,1
RECORD CONTAINS GINGER FIBER
COLORANT   3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT   4 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT   5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 15:47:31 ITER=  2 COMBINATION     1
COLORANT                          %
(    8U)BROWN FIBER     ==  0.8215
(    2U)GINGER FIBER    == 54.5400
(    4U)GREEN FIBER     ==  7.5223
(    6U)YELLOW FIBER    == 37.1162

COST=   1.000       MI =  0.28
ILL=D6 2 DE=  0.01   LD=  0.00   RG=  0.00   YB= -0.01   DC= -0.01   DH=  0.00
ILL=A  2 DE=  0.29   LD= -0.02   RG= -0.19   YB= -0.22   DC= -0.26   DH=  0.18
ILL=CW 2 DE=  0.41   LD=  0.06   RG=  0.33   YB=  0.24   DC=  0.26   DH= -0.46
CIELAB
GOODNESS =   4.8211752E-04




COLORANT PERCENTAGES

FIBER  1  BROWN FIBER          13.60%
FIBER  2  GINGER FIBER         36.11%
FIBER  3  GREEN FIBER           2.20%
FIBER  4  YELLOW FIBER         48.10%

10-APR-85 15:57:53 COMBINATION  1 AFTER  5 ITERATIONS FOR 400 BURLONE 15

ILL=D6 2 DE=  1.39   LD= -0.03   RG=  0.25   YB= -1.37   DC= -1.33   DH= -0.66
ILL=A  2 DE=  1.25   LD= -0.05   RG=  0.07   YB= -1.24   DC= -1.18   DH= -0.63
ILL=CW 2 DE=  1.46   LD= -0.05   RG=  0.20   YB= -1.45   DC= -1.43   DH= -0.43

CIELAB

MI =   0.21 GOODNESS = 0.4959E-05 AT A COST OF $1.0000
```

```
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  84,1
STANDARD            IS -- 430 BURLONE 17

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  2 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:07:21 ITER=  1 COMBINATION      1
COLORANT                         %
(   3U)ORANGE FIBER     == 16.5451
(   6U)YELLOW FIBER     == 23.9363
(   4U)GREEN FIBER      == 10.5822
(   1U)WHITE BASE       == 48.9364
COST=   1.000       MI =  0.02
ILL=D6 2 DE=  0.01   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
ILL=A  2 DE=  0.02   LD=  0.00   RG= -0.01   YB= -0.02   DC= -0.02   DH=  0.01
ILL=CW 2 DE=  0.03   LD=  0.01   RG=  0.02   YB=  0.01   DC=  0.01   DH= -0.04
CIELAB
GOODNESS =   8.5976313E-07



COLORANT PERCENTAGES

FIBER  1  ORANGE FIBER          16.60%
FIBER  2  YELLOW FIBER          23.88%
FIBER  3  GREEN FIBER           10.53%
FIBER  4  WHITE BASE            48.99%

11-APR-85 14:30:51 COMBINATION  1 AFTER 14 ITERATIONS FOR 430 BURLONE 17

ILL=D6 2 DE=  0.06   LD=  0.02   RG=  0.06   YB=  0.01   DC=  0.01   DH= -0.11
ILL=A  2 DE=  0.05   LD=  0.02   RG=  0.05   YB=  0.01   DC=  0.02   DH= -0.07
ILL=CW 2 DE=  0.08   LD=  0.03   RG=  0.07   YB=  0.02   DC=  0.02   DH= -0.11

CIELAB

MI =   0.02 GOODNESS = 0.6807E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  82,1
STANDARD          IS -- 490 BURLONE 21

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT  2 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  3 ?  5,1
RECORD CONTAINS RED FIBER
COLORANT  4 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:07:55 ITER=  1 COMBINATION     1
COLORANT                     %
(   7U)BLACK BASE      == 15.3451
(   8U)BROWN FIBER     == -2.0423
(   5U)RED FIBER       == 27.1909
(   3U)ORANGE FIBER    == 59.5063
COST=   1.000      MI =  0.15
ILL=D6 2 DE=  0.03   LD=  0.01   RG=  0.01   YB=  0.03   DC=  0.03   DH=  0.01
ILL=A  2 DE=  0.13   LD= -0.02   RG= -0.12   YB= -0.05   DC= -0.10   DH=  0.10
ILL=CW 2 DE=  0.42   LD=  0.01   RG=  0.42   YB=  0.00   DC=  0.14   DH= -0.59
CIELAB
GOODNESS =   6.2492618E-04




COLORANT PERCENTAGES

FIBER  1  BLACK BASE            3.50%
FIBER  2  BROWN FIBER          21.42%
FIBER  3  RED FIBER            20.78%
FIBER  4  ORANGE FIBER         54.30%

11-APR-85 14:32:18 COMBINATION  1 AFTER 26 ITERATIONS FOR 490 BURLONE 21

ILL=D6 2 DE=  0.41   LD=  0.04   RG= -0.39   YB= -0.12   DC= -0.29   DH=  0.44
ILL=A  2 DE=  0.33   LD=  0.00   RG= -0.25   YB= -0.21   DC= -0.31   DH=  0.13
ILL=CW 2 DE=  0.41   LD= -0.01   RG= -0.33   YB= -0.24   DC= -0.34   DH=  0.35

CIELAB

MI =   0.18 GOODNESS = 0.9836E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  86,1
STANDARD            IS -- 505 BURLONE 22

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  2 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT  3 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  4 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 15:51:03 ITER=  1 COMBINATION     1
COLORANT                      %
(   1U)WHITE BASE       ==  8.4071
(   7U)BLACK BASE       == 27.1426
(   8U)BROWN FIBER      == 23.3644
(   4U)GREEN FIBER      == 41.0859

COST=   1.000       MI =  0.17
ILL=D6 2 DE=  0.04   LD=  0.00   RG= -0.02   YB=  0.03   DC=  0.04   DH= -0.05
ILL=A  2 DE=  0.15   LD=  0.00   RG=  0.14   YB= -0.05   DC= -0.11   DH= -0.61
ILL=CW 2 DE=  0.09   LD= -0.02   RG= -0.03   YB=  0.09   DC=  0.09   DH= -0.12
CIELAB
GOODNESS =   2.4840026E-06



COLORANT PERCENTAGES

FIBER  1  WHITE BASE            9.13%
FIBER  2  BLACK BASE           28.84%
FIBER  3  BROWN FIBER          21.89%
FIBER  4  GREEN FIBER          40.14%

10-APR-85 16:03:42 COMBINATION  1 AFTER  7 ITERATIONS FOR 505 BURLONE 22

ILL=D6 2 DE=  0.39   LD= -0.08   RG=  0.15   YB= -0.35   DC= -0.36   DH=  0.57
ILL=A  2 DE=  0.43   LD= -0.08   RG=  0.15   YB= -0.39   DC= -0.41   DH=  0.41
ILL=CW 2 DE=  0.35   LD= -0.09   RG=  0.11   YB= -0.32   DC= -0.32   DH=  0.49

CIELAB

MI =   0.04 GOODNESS = 0.4141E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD           DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  88,1
STANDARD           IS -- 550 BURLONE 25

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT   1 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT   2 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT   3 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT   4 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT   5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:09:17 ITER=  1 COMBINATION     1
COLORANT                       %
(   7U)BLACK BASE      == 11.4922
(   8U)BROWN FIBER     == 14.5036
(   9U)BLUE FIBER      ==  5.6965
(   6U)YELLOW FIBER    == 68.3076
COST=   1.000       MI =  0.03
ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.01
ILL=A  2 DE=  0.03   LD=  0.00   RG= -0.03   YB=  0.00   DC= -0.01   DH=  0.06
ILL=CW 2 DE=  0.01   LD=  0.01   RG=  0.01   YB=  0.01   DC=  0.01   DH= -0.02
CIELAB
GOODNESS =   2.1747298E-06



COLORANT PERCENTAGES

FIBER  1  BLACK BASE          10.47%
FIBER  2  BROWN FIBER         15.48%
FIBER  3  BLUE FIBER           6.19%
FIBER  4  YELLOW FIBER        67.86%

11-APR-85 14:33:37 COMBINATION  1 AFTER  4 ITERATIONS FOR 550 BURLONE 25

ILL=D6 2 DE=  0.13   LD= -0.02   RG=  0.11   YB= -0.06   DC= -0.07   DH= -0.23
ILL=A  2 DE=  0.09   LD= -0.01   RG=  0.08   YB= -0.03   DC= -0.02   DH= -0.17
ILL=CW 2 DE=  0.10   LD= -0.01   RG=  0.08   YB= -0.06   DC= -0.07   DH= -0.15

CIELAB

MI =   0.04 GOODNESS = 0.2715E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  92,1
STANDARD           IS -- 580 BURLONE 27

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT  2 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT  3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:13:01 ITER=  1 COMBINATION     1
COLORANT                     %
(   7U)BLACK BASE      ==  7.9211
(   6U)YELLOW FIBER    == 19.4315
(   4U)GREEN FIBER     ==  4.7713
(   1U)WHITE BASE      == 67.8761
COST=   1.000      MI =  0.05
ILL=D6 2 DE=  0.00   LD=  0.00   RG=  0.00   YB=  0.00   DC=  0.00   DH=  0.00
ILL=A  2 DE=  0.05   LD=  0.00   RG=  0.05   YB= -0.01   DC= -0.01   DH= -0.21
ILL=CW 2 DE=  0.03   LD= -0.01   RG=  0.00   YB= -0.03   DC= -0.03   DH=  0.04
CIELAB
GOODNESS =   8.6077580E-06



COLORANT PERCENTAGES

FIBER  1  BLACK BASE           8.13%
FIBER  2  YELLOW FIBER        19.26%
FIBER  3  GREEN FIBER          4.51%
FIBER  4  WHITE BASE          68.10%

11-APR-85 14:35:26 COMBINATION  1 AFTER 26 ITERATIONS FOR 580 BURLONE 27

ILL=D6 2 DE=  0.33   LD= -0.12   RG=  0.15   YB= -0.26   DC= -0.30   DH= -0.27
ILL=A  2 DE=  0.30   LD= -0.12   RG=  0.13   YB= -0.24   DC= -0.24   DH= -0.54
ILL=CW 2 DE=  0.35   LD= -0.13   RG=  0.11   YB= -0.31   DC= -0.33   DH= -0.14

CIELAB

MI =   0.03 GOODNESS = 0.6826E-05 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]  F
TYPE REFLECTANCE FILE RECORD NUMBER  94,1
STANDARD          IS -- 655 BURLONE 32

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT  2 ?  8,1
RECORD CONTAINS BROWN FIBER
COLORANT  3 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT  4 ?  3,1
RECORD CONTAINS ORANGE FIBER
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
10-APR-85 15:51:46 ITER=  1 COMBINATION     1
COLORANT                       %
(   7U)BLACK BASE      ==  3.5402
(   8U)BROWN FIBER     == 25.0229
(   9U)BLUE FIBER      == 18.1472
(   3U)ORANGE FIBER    == 53.2898

COST=   1.000      MI =  0.07
ILL=D6 2 DE=  0.05   LD=  0.00   RG=  0.05   YB= -0.01   DC=  0.00   DH= -0.10
ILL=A  2 DE=  0.10   LD=  0.02   RG=  0.08   YB=  0.05   DC=  0.08   DH= -0.10
ILL=CW 2 DE=  0.13   LD=  0.00   RG= -0.10   YB= -0.08   DC= -0.10   DH=  0.17
CIELAB
GOODNESS =   4.7329424E-05



COLORANT PERCENTAGES

FIBER  1  BLACK BASE             7.76%
FIBER  2  BROWN FIBER           18.46%
FIBER  3  BLUE FIBER            18.15%
FIBER  4  ORANGE FIBER          55.64%

10-APR-85 16:04:51 COMBINATION  1 AFTER 20 ITERATIONS FOR 655 BURLONE 32

ILL=D6 2 DE=  0.40   LD=  0.10   RG= -0.32   YB=  0.20   DC=  0.10   DH=  0.74
ILL=A  2 DE=  0.38   LD=  0.08   RG= -0.34   YB=  0.16   DC=  0.01   DH=  0.66
ILL=CW 2 DE=  0.36   LD=  0.12   RG= -0.28   YB=  0.19   DC=  0.13   DH=  0.59

CIELAB

MI =   0.05 GOODNESS = 0.7064E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  96,1
STANDARD            IS -- 670 BURLONE 33

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT   1 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT   2 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT   3 ?  4,1
RECORD CONTAINS GREEN FIBER
COLORANT   4 ?  9,1
RECORD CONTAINS BLUE FIBER
COLORANT   5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:13:38 ITER=  1 COMBINATION      1
COLORANT                     %
(    7U)BLACK BASE      == 29.0686
(    6U)YELLOW FIBER    ==  9.2246
(    4U)GREEN FIBER     ==  8.8616
(    9U)BLUE FIBER      == 52.8451
COST=    1.000      MI =  0.08
ILL=D6 2 DE=  0.04   LD=  0.00   RG=  0.03   YB= -0.03   DC=  0.01   DH=  0.60
ILL=A  2 DE=  0.12   LD=  0.00   RG=  0.10   YB= -0.07   DC=  0.02   DH=  1.48
ILL=CW 2 DE=  0.03   LD= -0.01   RG=  0.03   YB=  0.01   DC= -0.02   DH=  0.22
CIELAB
GOODNESS =   3.0129013E-07




COLORANT PERCENTAGES

FIBER  1  BLACK BASE            29.06%
FIBER  2  YELLOW FIBER           9.42%
FIBER  3  GREEN FIBER            8.03%
FIBER  4  BLUE FIBER            53.49%

11-APR-85 14:39:55 COMBINATION  1 AFTER 26 ITERATIONS FOR 670 BURLONE 33

ILL=D6 2 DE=  0.40   LD= -0.10   RG=  0.30   YB= -0.25   DC=  0.08   DH=  5.54
ILL=A  2 DE=  0.35   LD= -0.09   RG=  0.25   YB= -0.23   DC=  0.11   DH=  4.15
ILL=CW 2 DE=  0.33   LD= -0.10   RG=  0.22   YB= -0.22   DC=  0.11   DH=  3.56

CIELAB

MI =   0.05 GOODNESS = 0.1746E-06 AT A COST OF $1.0000
```

```
JOB CODE?  1

ENTER STANDARD          DATA
USE MATCH SCAN? [T/F]   F
TYPE REFLECTANCE FILE RECORD NUMBER  98,1
STANDARD          IS -- 685 BURLONE 34

OUTPUT SIZE ?

ENTER RECORD NUMBER FROM K&S FILE FOR EACH COLORANT
COLORANT  1 ?  7,1
RECORD CONTAINS BLACK BASE
COLORANT  2 ?  6,1
RECORD CONTAINS YELLOW FIBER
COLORANT  3 ?  2,1
RECORD CONTAINS GINGER FIBER
COLORANT  4 ?  1,1
RECORD CONTAINS WHITE BASE
COLORANT  5 ?

NUMBER OF COLORANTS PER FORMULA?  4
11-APR-85 14:14:25 ITER=  1 COMBINATION     1
COLORANT                      %
(   7U)BLACK BASE      ==  4.2252
(   6U)YELLOW FIBER    == 13.9194
(   2U)GINGER FIBER    == 21.1464
(   1U)WHITE BASE      == 60.7091
COST=   1.000      MI =  0.03
ILL=D6 2 DE=  0.01   LD=  0.00   RG=  0.01   YB=  0.00   DC=  0.00   DH= -0.02
ILL=A  2 DE=  0.04   LD=  0.00   RG=  0.01   YB= -0.03   DC= -0.03   DH= -0.05
ILL=CW 2 DE=  0.04   LD=  0.00   RG=  0.03   YB=  0.03   DC=  0.03   DH= -0.08
CIELAB
GOODNESS =   5.3586587E-06




COLORANT PERCENTAGES

FIBER  1  BLACK BASE            4.02%
FIBER  2  YELLOW FIBER         13.27%
FIBER  3  GINGER FIBER         21.74%
FIBER  4  WHITE BASE           60.97%

11-APR-85 14:40:56 COMBINATION  1 AFTER 26 ITERATIONS FOR 685 BURLONE 34

ILL=D6 2 DE=  0.15   LD=  0.02   RG=  0.14   YB= -0.06   DC= -0.04   DH= -0.47
ILL=A  2 DE=  0.14   LD=  0.03   RG=  0.12   YB= -0.06   DC= -0.02   DH= -0.39
ILL=CW 2 DE=  0.14   LD=  0.03   RG=  0.14   YB= -0.01   DC=  0.00   DH= -0.40

CIELAB

MI =   0.02 GOODNESS = 0.4978E-05 AT A COST OF $1.0000
```

XIII. VITA

Eric Walowit received the B.S. degree in Imaging and Photographic Science from the Rochester Institute of Technology in May, 1985.

He has been employed by Bausch and Lomb Company, Analytical Products Division, Rochester, NY working on color software development since March, 1984. As of June 3, 1985 he is employed by Burlington Industries, Corporate Research and Development, Greensboro, North Carolina, as a Color Scientist. His research interests are least-squares solutions of non-linear problems associated with computer color formulation.