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TECHNICAL MEMORANDUM 3-81

DESIGN CURVES FOR OCEANOGRAPHIC  
PRESSURE-RESISTANT HOUSINGS

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August 1981

Office of Naval Research Contract No.  
N00014-73-C-0097; NR 265-107



## Abstract

Curves have been prepared to serve as a guide in designing externally pressurized cylindrical and spherical housings and flat end closures. The plots show wall thickness-to-diameter ratio as a function of collapse depth over the range 0-10,000 meters (0-32,800 feet). Curves are plotted for the commonly used stainless steels and aluminum and titanium alloys. Also, there are three dimensionless curves that can be used for any structural material. Included is a brief review of the equations used in plotting the curves.

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## I. Introduction

The following pages present curves for use in the design of externally pressurized cylindrical and spherical oceanographic housings. Design curves also are included for flat circular plates, which commonly are used as end closures for cylindrical vessels.

Curves have been plotted for the commonly used stainless steel types 302, 304, 316 and 17-14 PH; the aluminum alloys 6061-T6 and 7075-T6; and for unalloyed titanium ASTM Grade 2, and titanium alloy 6Al-4V. In each case the thickness/diameter ratio is shown as a function of collapse depth to a maximum depth of 10,000 meters (32,800 feet).

For thin-walled vessels, collapse can occur at wall stress levels below the elastic limit. This is called elastic buckling or instability failure. For relatively thicker shells that fail at stress levels above the elastic limit, the failure is due to plastic yielding of the material. Most of the present curves clearly show these two types of failure. In plotting the sphere curves for aluminum alloy 6061-T6, the stainless steel 300 series, and unalloyed titanium, it was found that the governing elastic portion covers a very small range of depths (to less than 500 meters)

and so it was omitted. However, in two of the cylinder curves (those for stainless steel 17-4 PH and titanium alloy 6Al-4V) the elastic failure mode governs over practically the entire range of ocean depths considered.

The elastic curves are based on standard equations found in elastic buckling theory. For spheres, the equation used is a modification of the classical buckling equation derived for the ideal case. The inelastic portions of the curves are based on the thick-wall theory of Lamé. Specifically, the Lamé equations used were those for maximum normal stress. All of the equations are discussed in another section of this report.

For materials not covered by the main body of curves, three additional non-dimensional plots have been included. Since the parameters plotted are dimensionless ratios, any system of units may be used. Also, because the material yield strength  $s_y$  and the modulus of elasticity  $E$  are contained in the ratios, pressure vessels of any engineering material can be designed. A minor disadvantage of these curves is that two separate determinations of the required  $t/D$  ratio must be made, and the larger of the two must be used. In order to allow the use of dimensionless ratios the

collapse pressure was substituted for collapse depth. The conversion to depth can readily be made by the user.

## II. Equations

### Cylinders

The curves for cylinders are based on the following equations:

#### A. Elastic Buckling Collapse Pressure

$$p_e = \frac{2E}{1-\mu^2} \left(\frac{t}{D_o}\right)^3 \quad (\text{Ref. 1})$$

where  $p_e$  = elastic buckling pressure

$E$  = modulus of elasticity

$\mu$  = Poisson's ratio

$t$  = wall thickness of tube

$D_o$  = outside diameter of tube

#### B. Tube Length (Elastic Failure)

The elastic curves for cylindrical vessels are plotted for so-called "long tubes" where the length must be greater than

$$L = 4.90 r (r/t)^{1/2} \quad (\text{Ref. 1})$$

where  $r$  = tube radius

$t$  = wall thickness of tube

For shorter tubes the collapse pressure would be somewhat greater than that predicted by the equation of

part A. above. This would be the result of reinforcement provided by the end closures.

#### C. Inelastic Failure

$$(s)_{\max.} = \frac{2b^2}{b^2-a^2} (p) \quad (\text{Ref. 1})$$

where  $(s)_{\max.}$  = maximum stress at tube inner surface

p = external pressure

b = outer radius of tube

a = inner radius of tube

Failure is considered to occur at that value of pressure where  $(s)_{\max.}$  is equal to the yield strength of the material.

#### D. Flat End Caps

Cylindrical pressure cases commonly use flat circular plates as end closures. These plates usually contain some type of seal, such as an o-ring, and some means of attachment to the cylinder. For practical design purposes the equation for a flat circular plate, simply supported around its edge, can be used. The diameter used in the equation is the diameter of the unsupported portion of the plate in the actual case. For greatest accuracy the plate thickness should be no more than about one-quarter of the effective diameter. The flat

plate equation is

$$(s)_{\max.} = \frac{3(3m+1)a^2}{8mt^2} (p) \quad (\text{Ref. 1})$$

where  $(s)_{\max.}$  = maximum stress at center of plate

$p$  = external pressure (pressure uniformly distributed over plate surface)

$a$  = radius of unsupported plate

$t$  = thickness of unsupported plate

$m$  = reciprocal of Poisson's ratio

Failure is considered to occur at that value of pressure where  $(s)_{\max.}$  is equal to the yield strength of the material.

### Spheres

The following equations served as the basis for plotting the curves for spherical pressure vessels.

#### A. Elastic Buckling Collapse Pressure

The classical elastic buckling pressure for an ideal sphere is given by

$$p_e = \frac{2Et^2}{\sqrt{3(1-\mu^2)r^2}} \quad (\text{Ref. 1})$$

where  $p_e$  = elastic buckling pressure

$E$  = modulus of elasticity

$t$  = spherical shell thickness

$r$  = radius to shell midsurface

$\mu$  = Poisson's ratio

This reduces to

$$p_e = 1.21 E(t/r)^2$$

where Poisson's ratio is taken as 0.3.

Early experimental work showed wide disagreement with the above equation, probably due to imperfections in geometry and material. Extensive recent testing at the David Taylor Naval Ship Research and Development Center (DTNSRDC) has indicated that the coefficient 1.21 can be attained in an ideal spherical shell, but that in most cases, even when imperfections are almost unmeasurably small, the coefficient more properly should be about 70 percent of the classical value. DTNSRDC has recommended that the classical equation be modified for near-perfect spherical shells to be

$$p_e = 0.84 E(t/r_o)^2 \quad (\text{Ref. 2})$$

where Poisson's ratio is equal to 0.3, and  $r_o$  is the radius to the shell outside surface. The elastic portions of the spherical shell curves of this report are based on the above modified equation.

## B. Inelastic Failure

$$(s)_{\max.} = \frac{3b^3}{2(b^3-a^3)} (p) \quad (\text{Ref. 1})$$

where  $(s)_{\max.}$  = maximum stress at sphere inner surface

p = external pressure

b = outer radius of sphere

a = inner radius of sphere

As in the case of the cylinders, failure is considered to occur when  $(s)_{\max.}$  is equal to the yield strength of the material.

### Thin Wall Stress Equations

Where the wall thickness t of a vessel is less than about one tenth the radius r the stresses can be considered uniform throughout the thickness of the wall and the thin-wall, or membrane stress equations

$$(s)_{\max.} = \frac{pr}{t} \quad (\text{cylinder}) \quad (\text{Ref. 1})$$

$$\text{and} \quad (s)_{\max.} = \frac{pr}{2t} \quad (\text{sphere})$$

can be used to predict inelastic behavior. The present curves did not make use of these equations, however. The so-called Lamé equations, discussed above, were used throughout the inelastic range of t/D values. The membrane

or thin-wall curves would practically coincide with the Lameé curves at the lower end of the range. It should be noted, however, that it is in that region of  $t/D$  values that elastic or instability failure might govern. For the higher values of  $t/D$  the thin-wall equations could be in error by as much as 20 percent.

#### Dimensionless Curves

The three dimensionless curves (Figs. 19, 20, and 21) are based on the same equations that were used for the other curves, but their arrangement is somewhat different. On the horizontal axes are plotted dimensionless ratios that involve the collapse pressure, and either the modulus of elasticity  $E$  or the yield strength  $s_y$ . The vertical scales express as before the ratio of thickness  $t$  to outside diameter  $D_o$ . Thus, for any collapse depth and any material,  $t/D_o$  values can be found for elastic and inelastic failure modes. Of the two  $t/D_o$  ratios determined, the greater value must be used. The left hand vertical scale and the bottom horizontal scale are used with the inelastic curve, while the right hand vertical and upper horizontal scales go with the elastic buckling curve.

### Pressure-Depth Conversion

In performing the calculations required for plotting the curves the following approximate pressure-depth relationship was used:

$$p = 0.01h$$

where  $p$  = pressure in megapascals (MPa)

$h$  = depth in meters

or  $p = \frac{64h}{144} = 0.444h$

where  $p$  = pressure in pounds per square inch

$h$  = depth in feet

A more exact, non-linear determination of the pressure-depth equation would take into account the effect of the compressibility of sea water, but this added precision was not considered necessary here. The approximate linear equation shown above is in error by about two percent at a depth of 10,000 meters.

### Poisson's Ratio

In all cases where Poisson's ratio was required in the calculations, a value of 0.3 was used.

### III. Design Curves for Cylindrical Housings

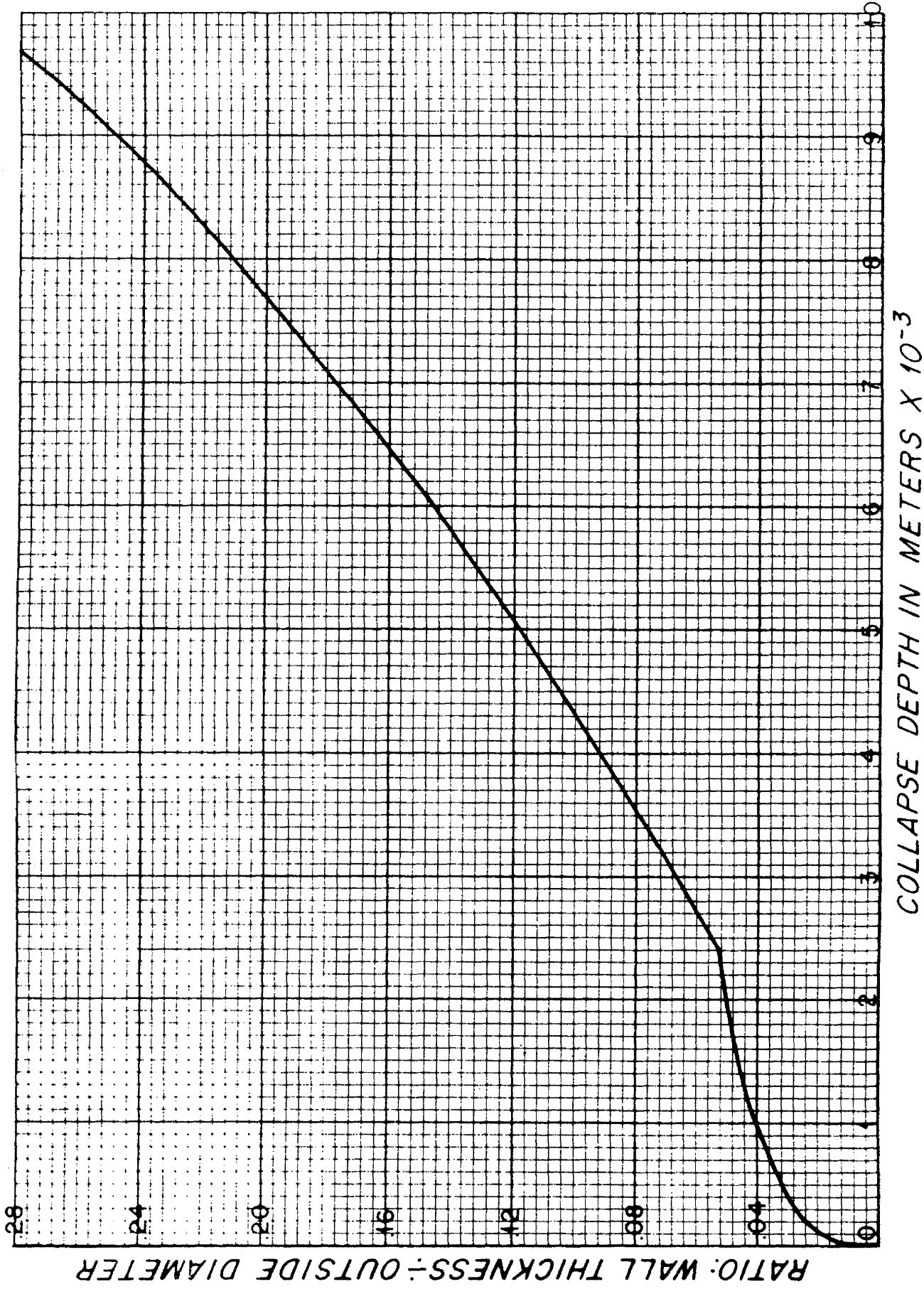


Figure 1 Collapse Depths of Cylinders; Aluminum Alloy 6061-T6; Yield Strength 241 MPa (35,000 psi)

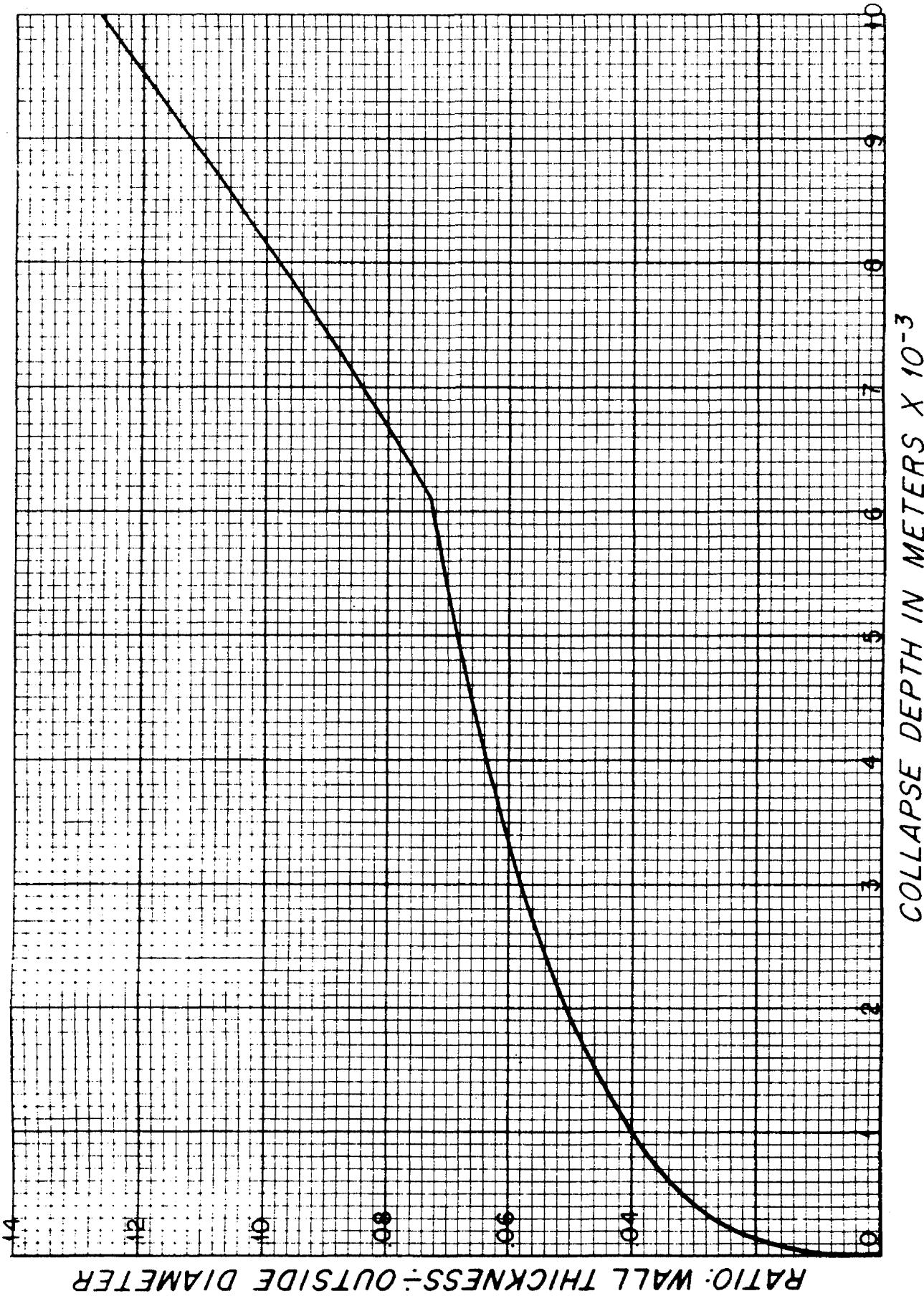


Figure 2 Collapse Depths of Cylinders; Aluminum Alloy 7075-T6; Yield Strength 455 MPa (66,000 psi)

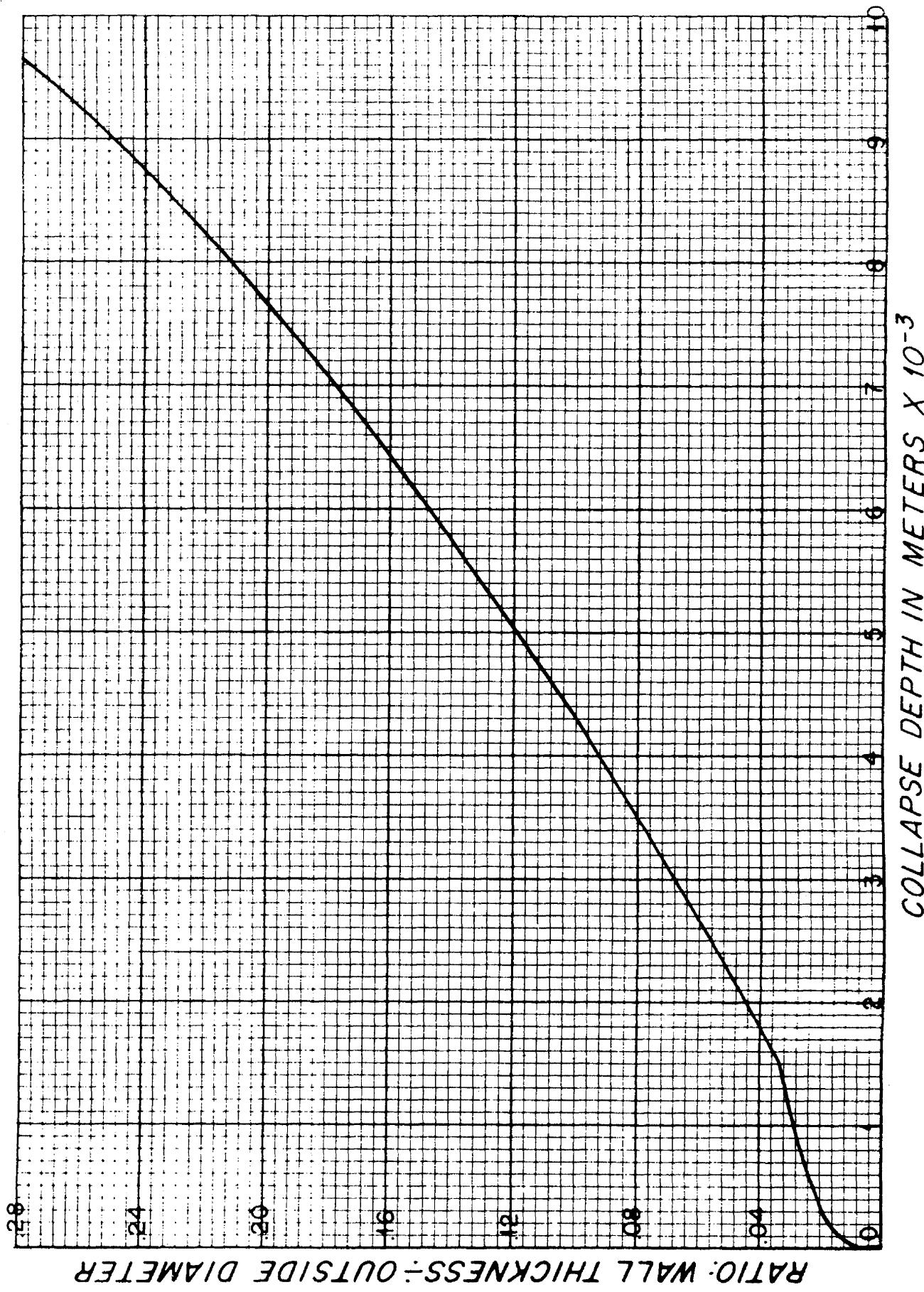


Figure 3      Collapse Depths of Cylinders; Stainless Steel  
 Types 302, 304, 316; Yield Strength 241 MPa  
 (35,000 psi)

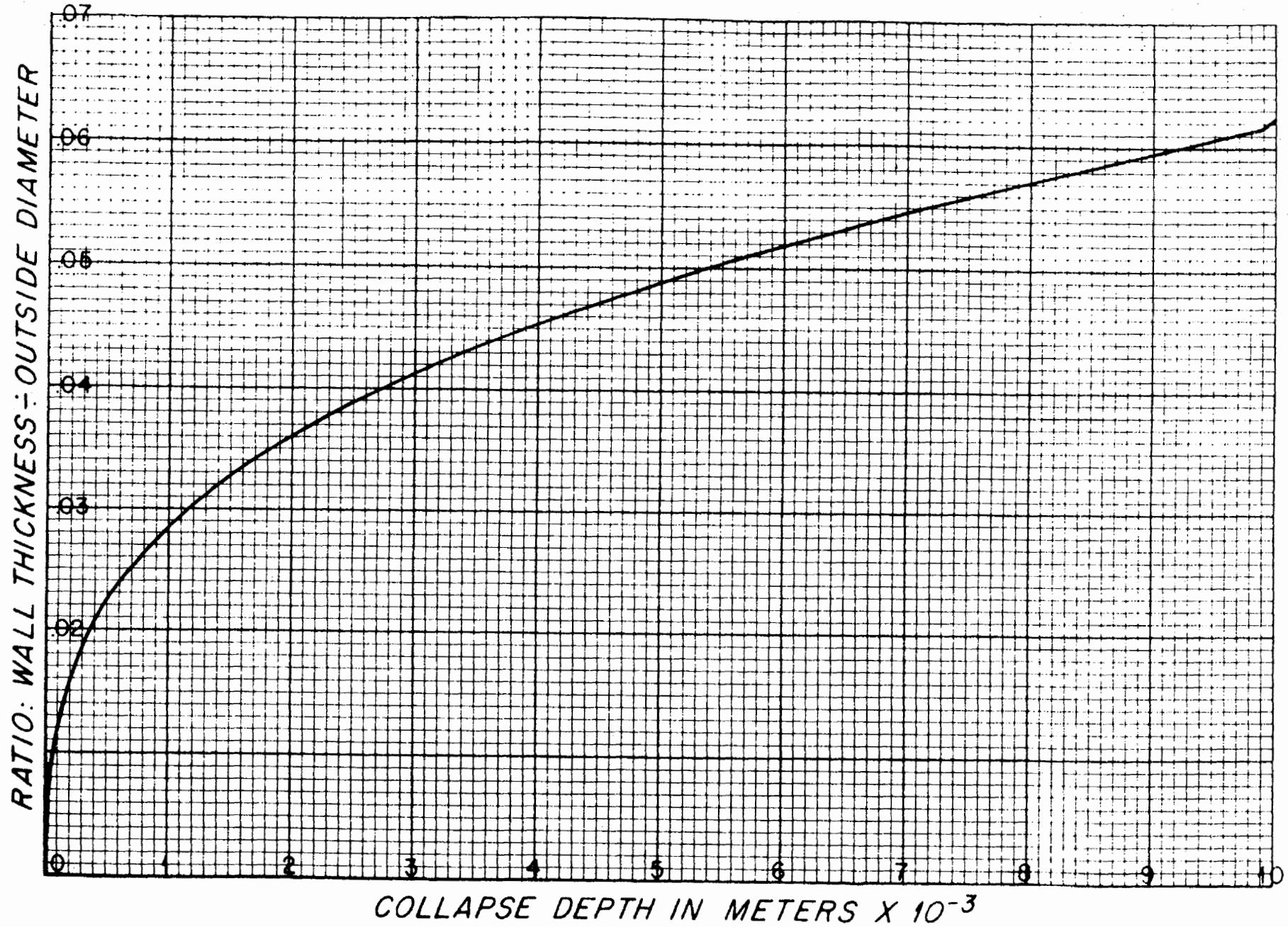


Figure 4      Collapse Depths of Cylinders: Stainless Steel  
Type 17-4 PH; Yield Strength 862 MPa (125,000  
psi)

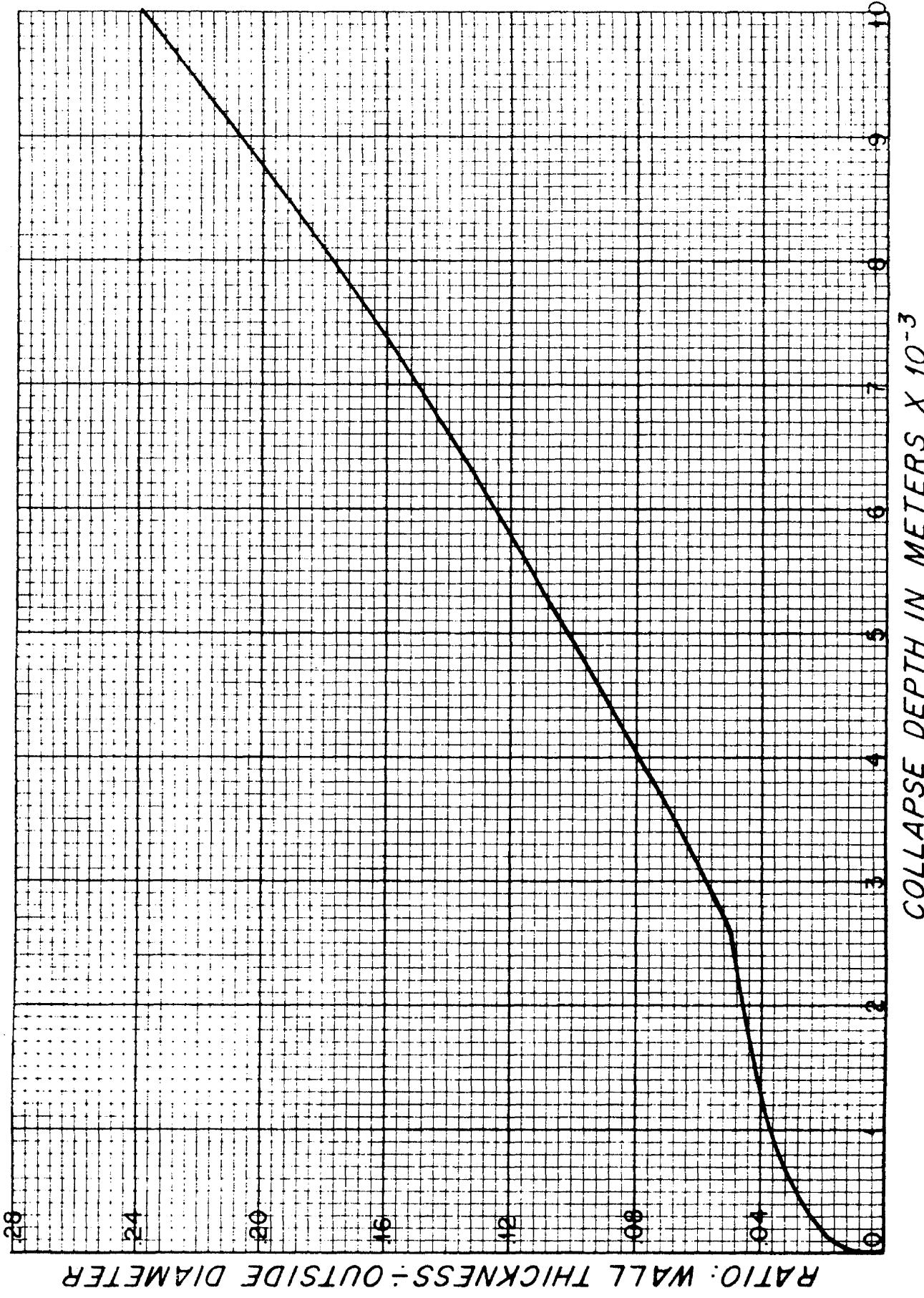


Figure 5      Collapse Depths of Cylinders; Unalloyed  
                  Titanium ASTM Grade 2; Yield Strength 276  
                  MPa (40,000 psi)

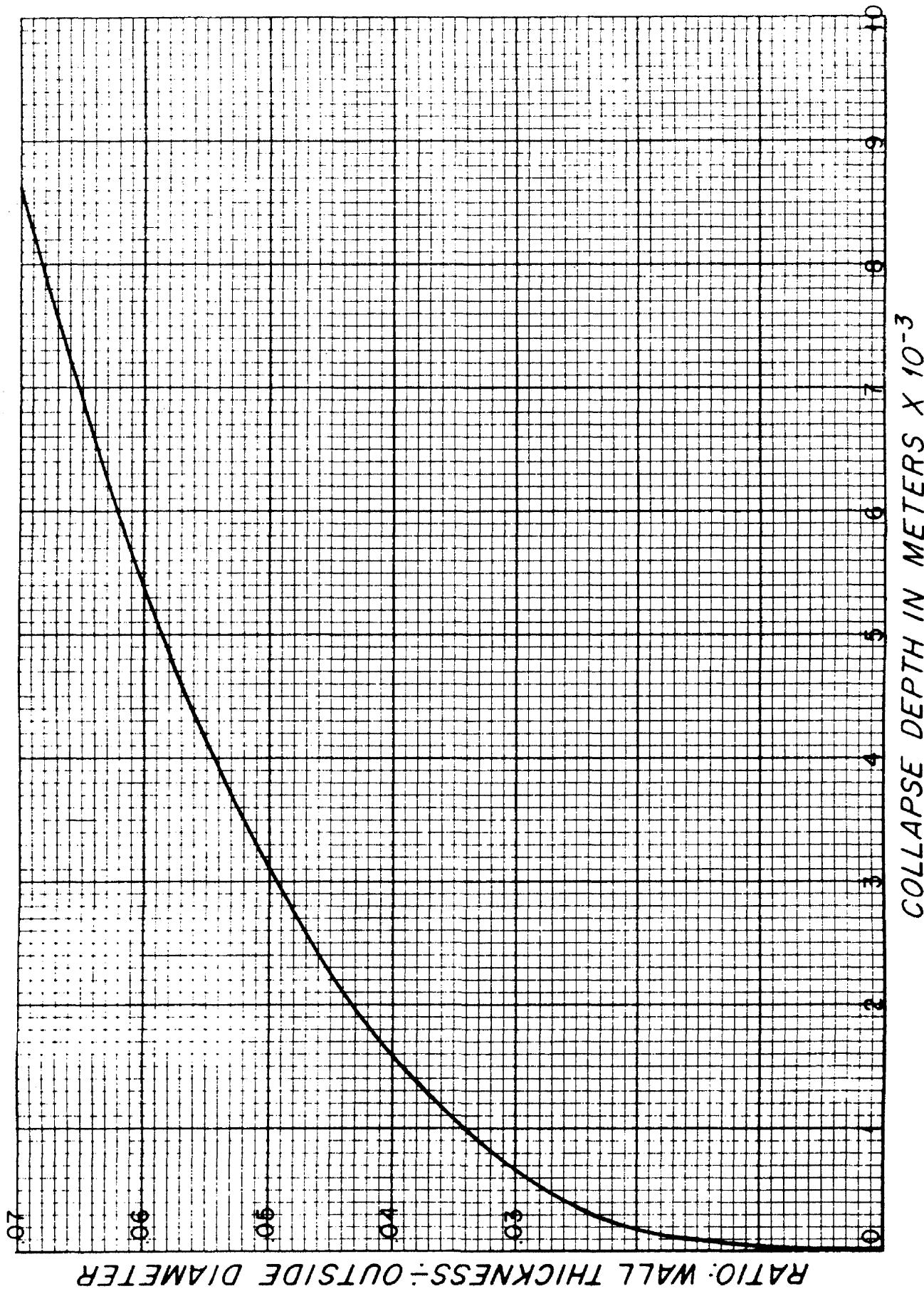


Figure 6      Collapse Depths of Cylinders; Titanium Alloy  
6Al-4V; Yield Strength 828 MPa (120,000 psi)

IV. Design Curves For Flat End Caps

RATIO: THICKNESS : UNSUPPORTED DIAMETER



Figure 7

(psi)

)

psf)

Alloy 6061-T6; Yield Strength 241 MPa (35,000

COLLAPSE DEPTHS OF FLAT END CAPS; ALUMINUM

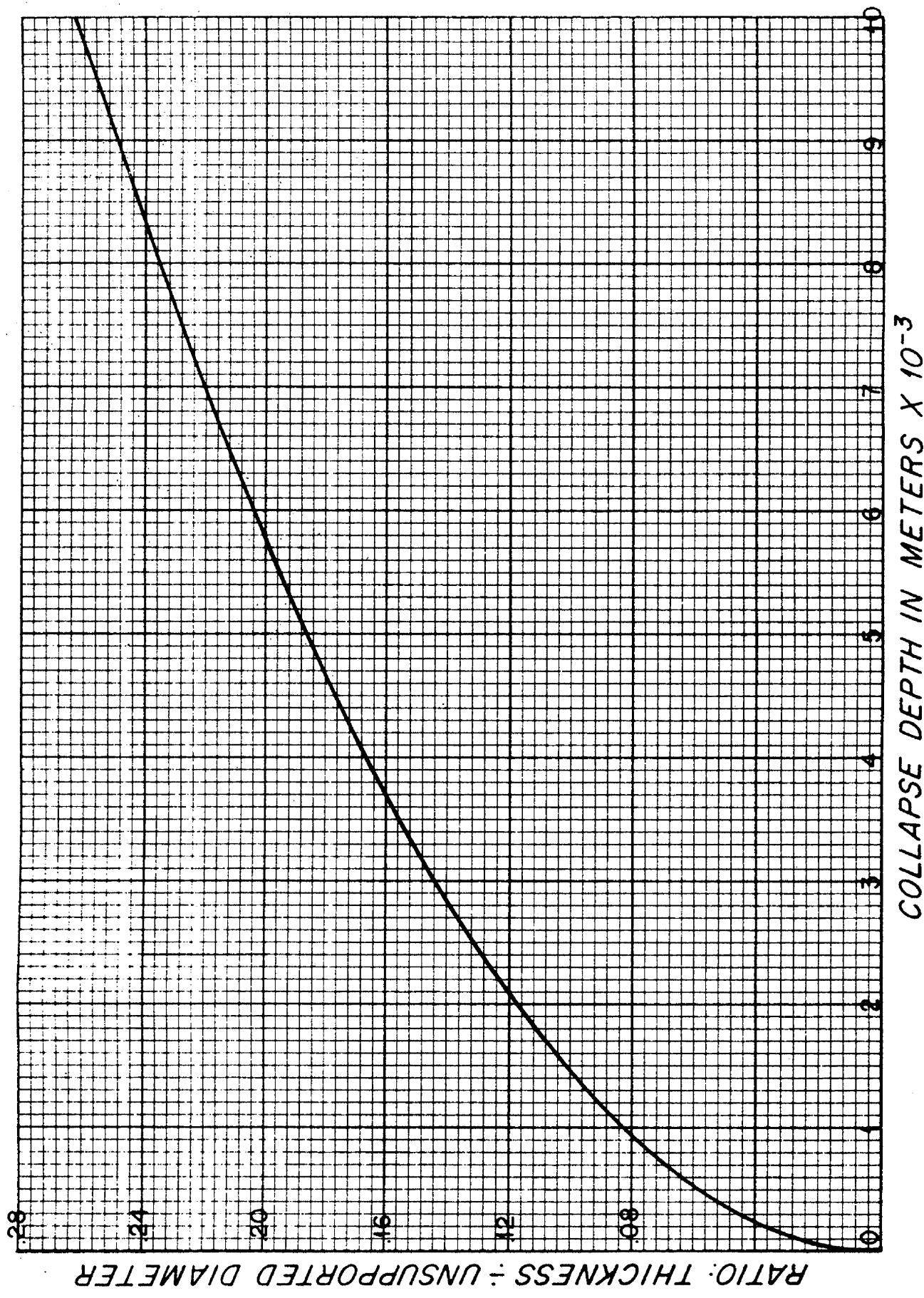


Figure 8 Collapse Depths of Flat End Caps; Aluminum Alloy 7075-T6; Yield Strength 455 MPa (66,000 psi)

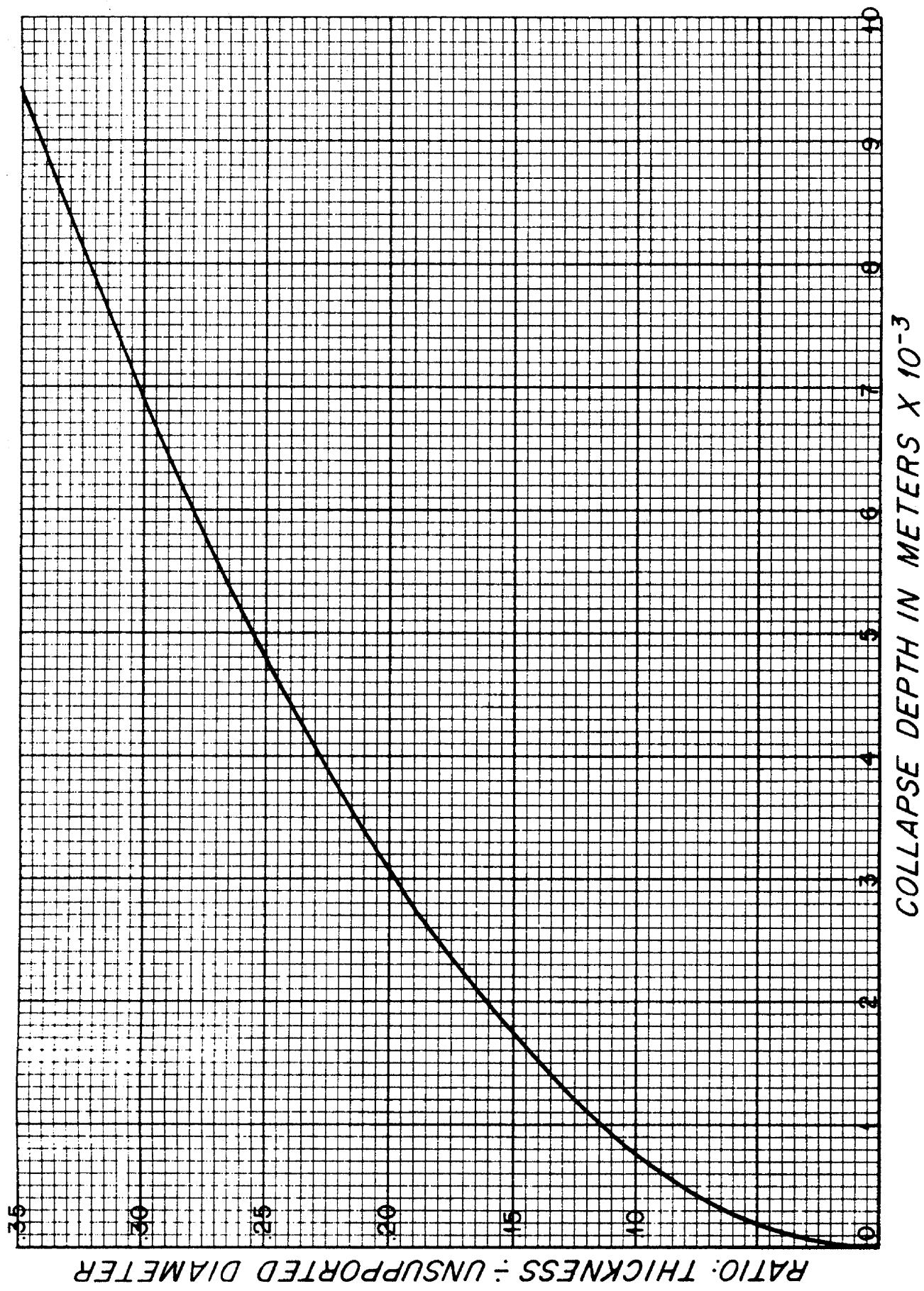


Figure 9      Collapse Depths of Flat End Caps; Stainless Steel Types 302, 316; Yield strength 241 MPa (35,000 psi)

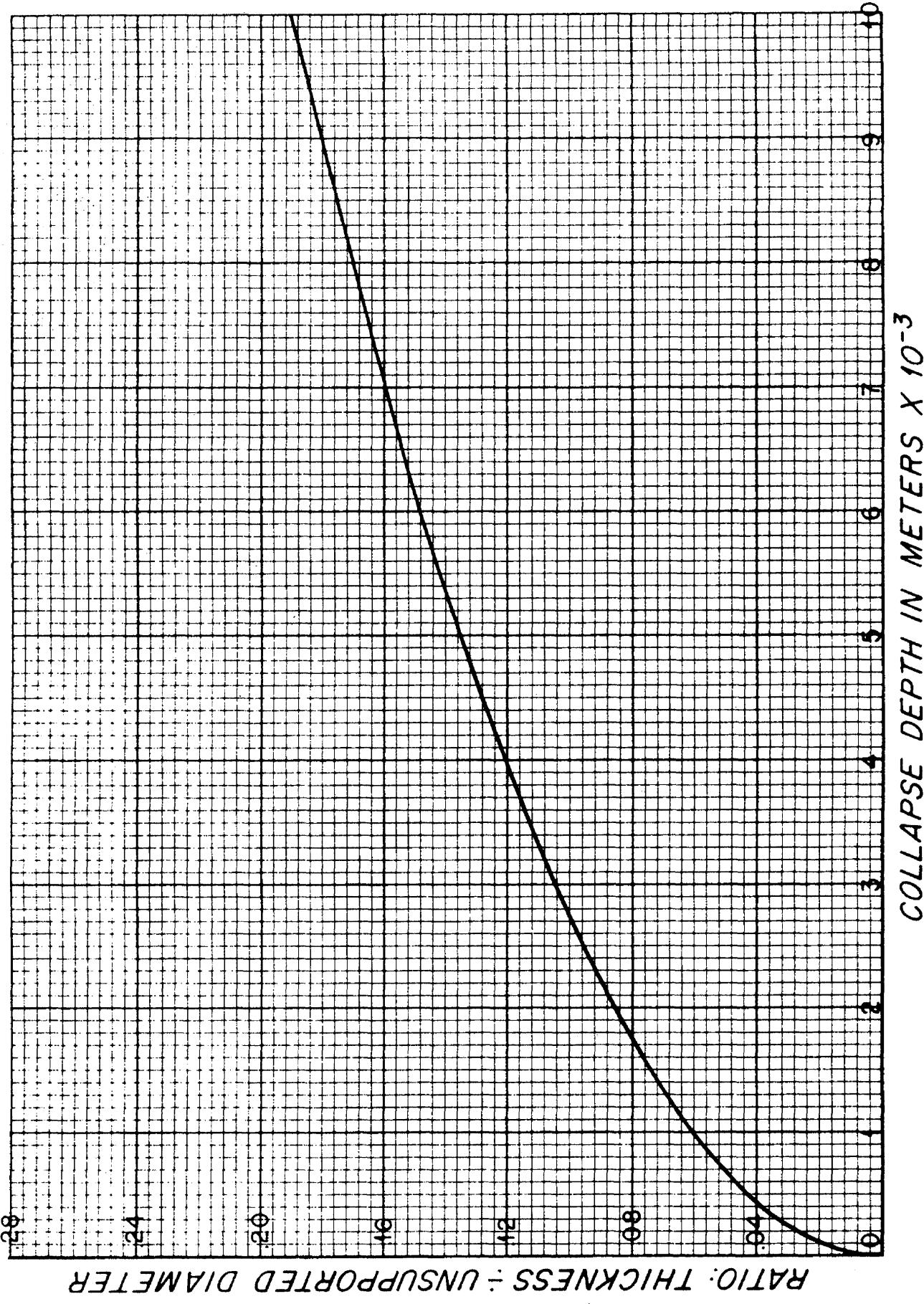


Figure 10 Collapse Depths of Flat End Caps; Stainless Steel Type 17-4 PH; Yield Strength 862 MPa (125,000 psi)

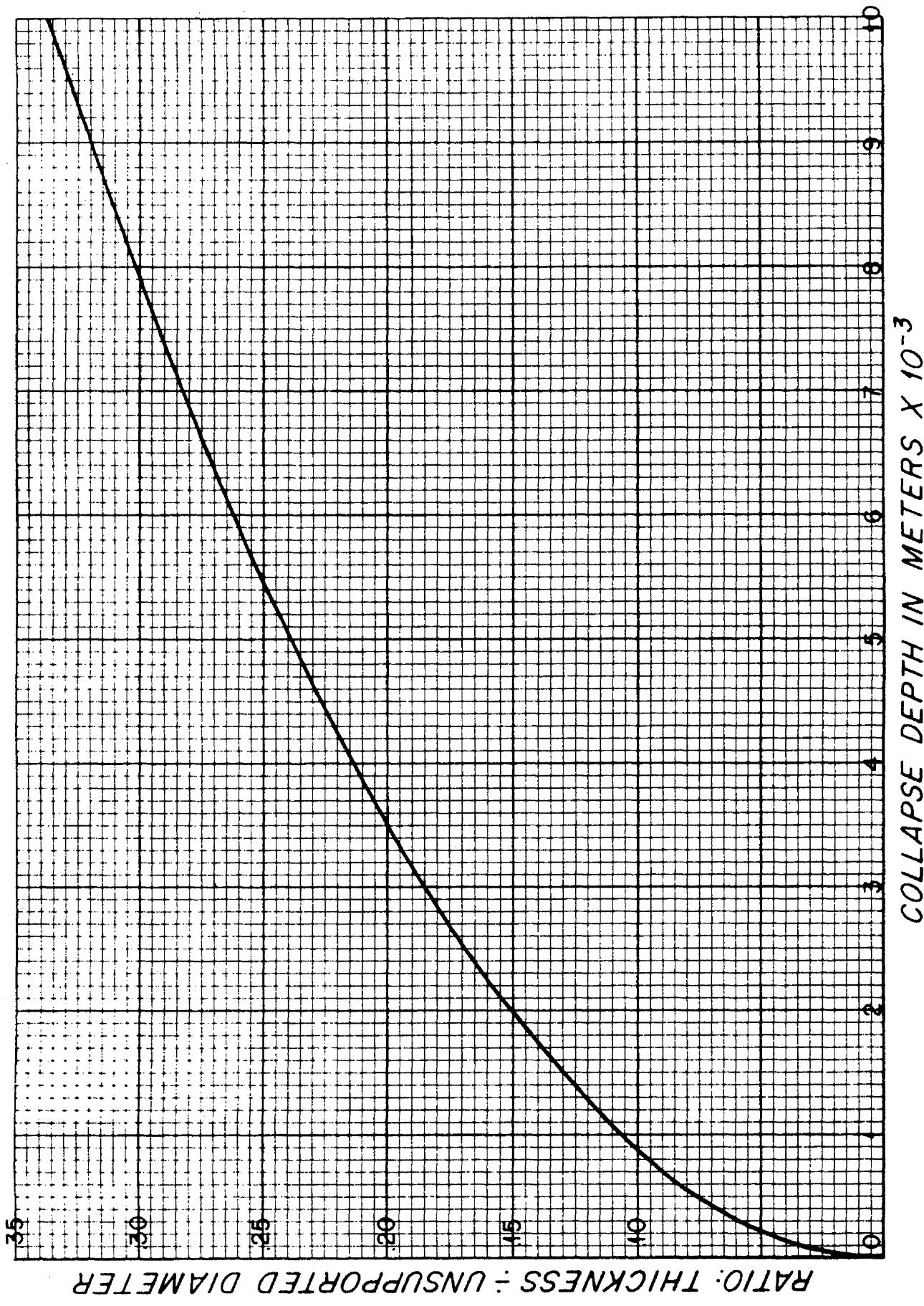


Figure 11 Collapse Depths of Flat End Caps; Unalloyed Titanium ASTM Grade 2; Yield Strength 276 MPa (40,000 psi)

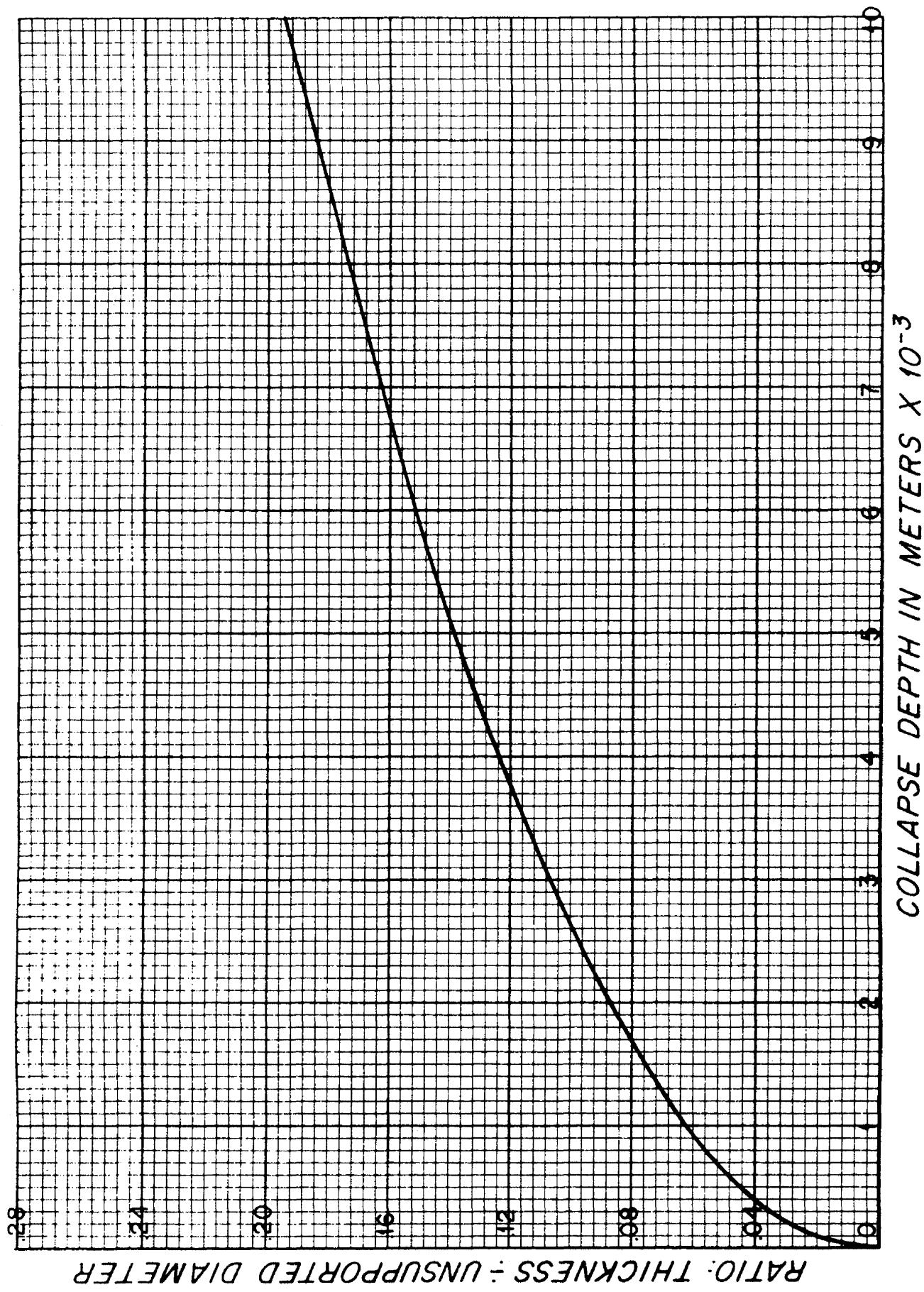


Figure 12 Collapse Depths of Flat End Caps; Titanium Alloy 6Al-4V; Yield Strength 828 MPa (120,000 psi)

V. Design Curves For Spherical Housings

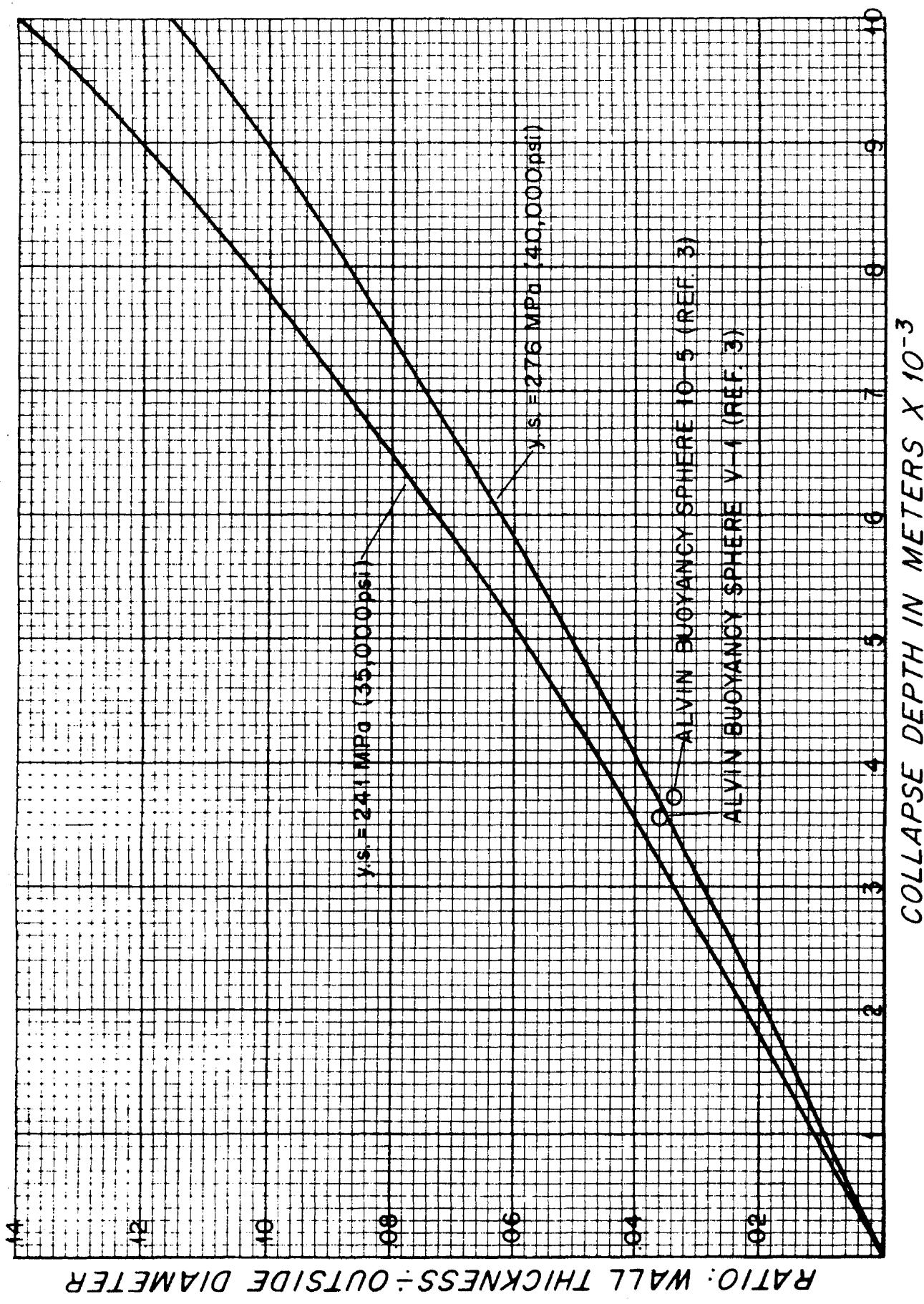


Figure 13 Collapse Depths of Spheres; Aluminum Alloy 6061-T6; Yield Strength 241 MPa (35,000 psi)

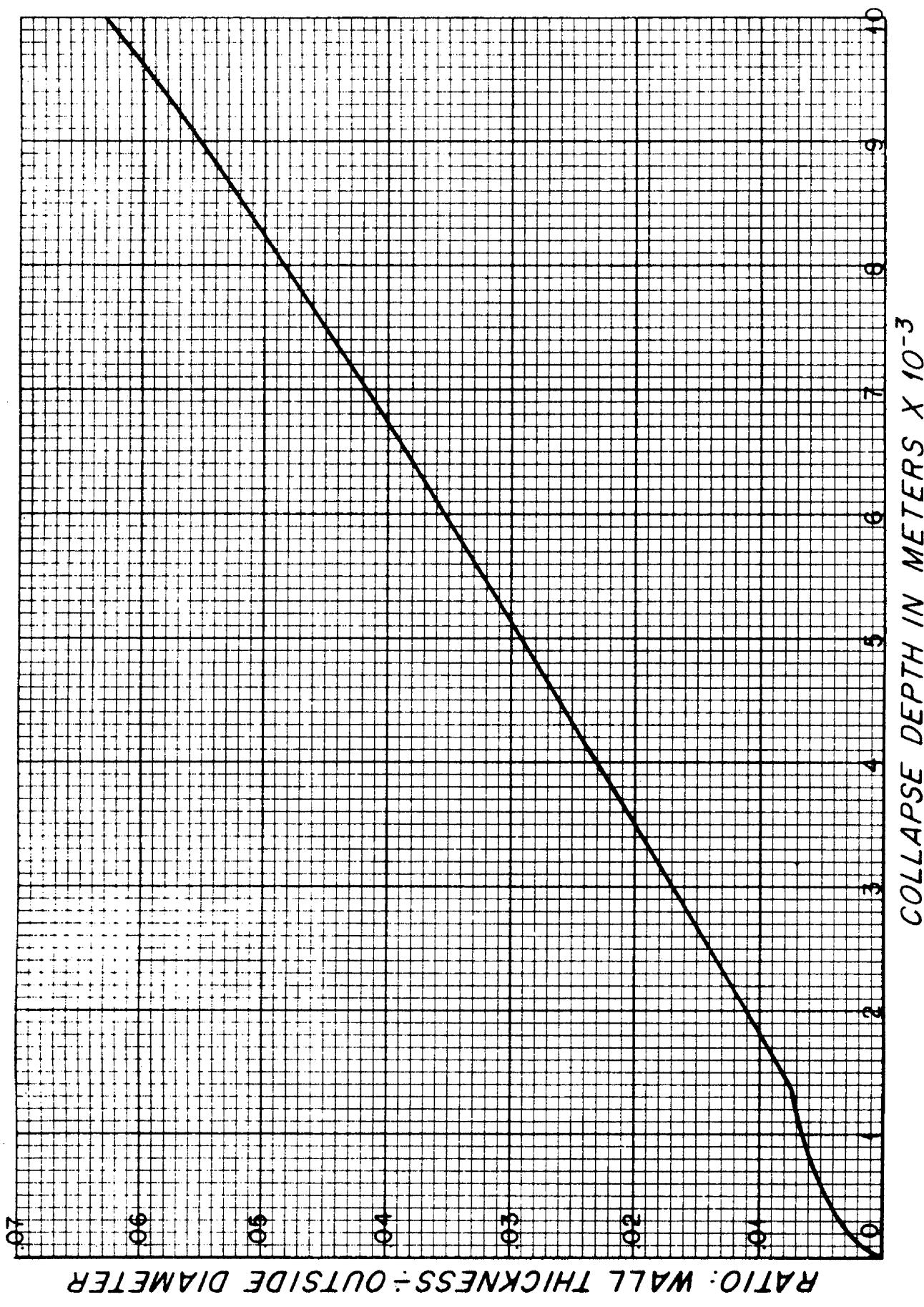


Figure 14 Collapse Depths of Spheres; Aluminum Alloy  
7075-T6; Yield Strength 455 MPa (66,000  
psi)

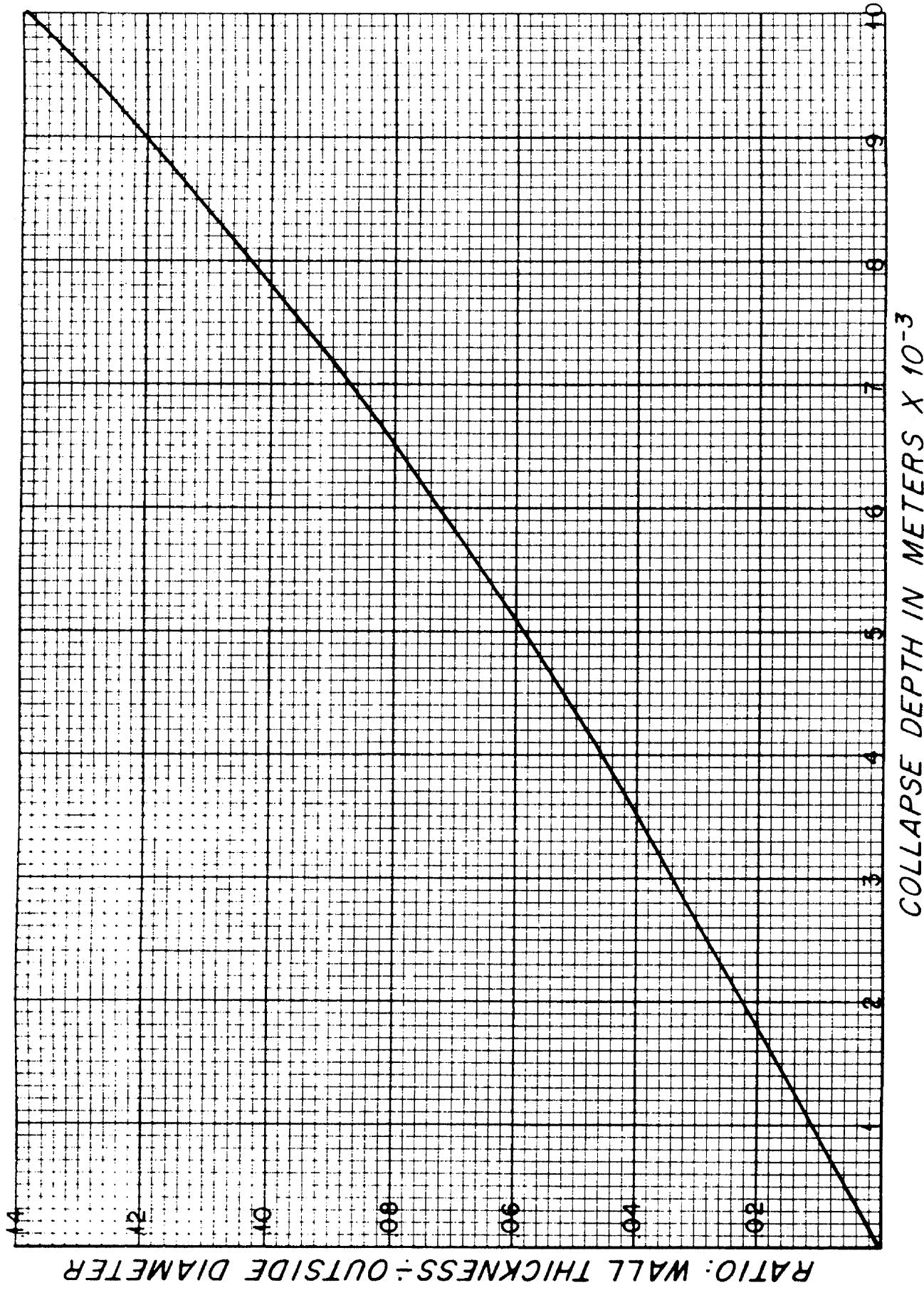


Figure 15 Collapse Depths of Spheres; Stainless Steel  
Types 302, 304, 116; Yield Strength 241 MPa  
(35,000 psi)

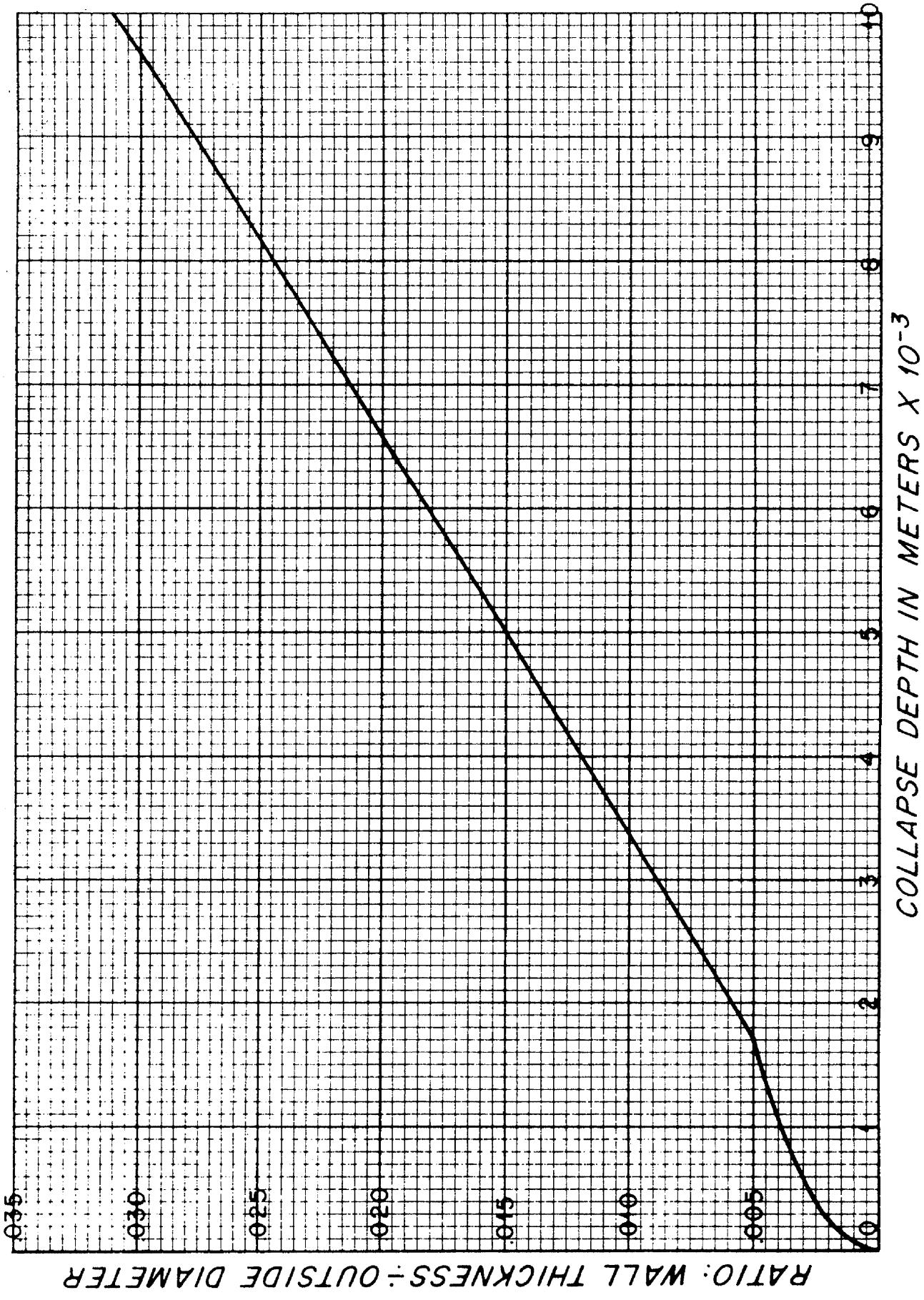


Figure 16 Collapse Depths of Spheres; Stainless Steel  
Type 17-4 PH; Yield Strength 862 MPa (125,000  
psi)

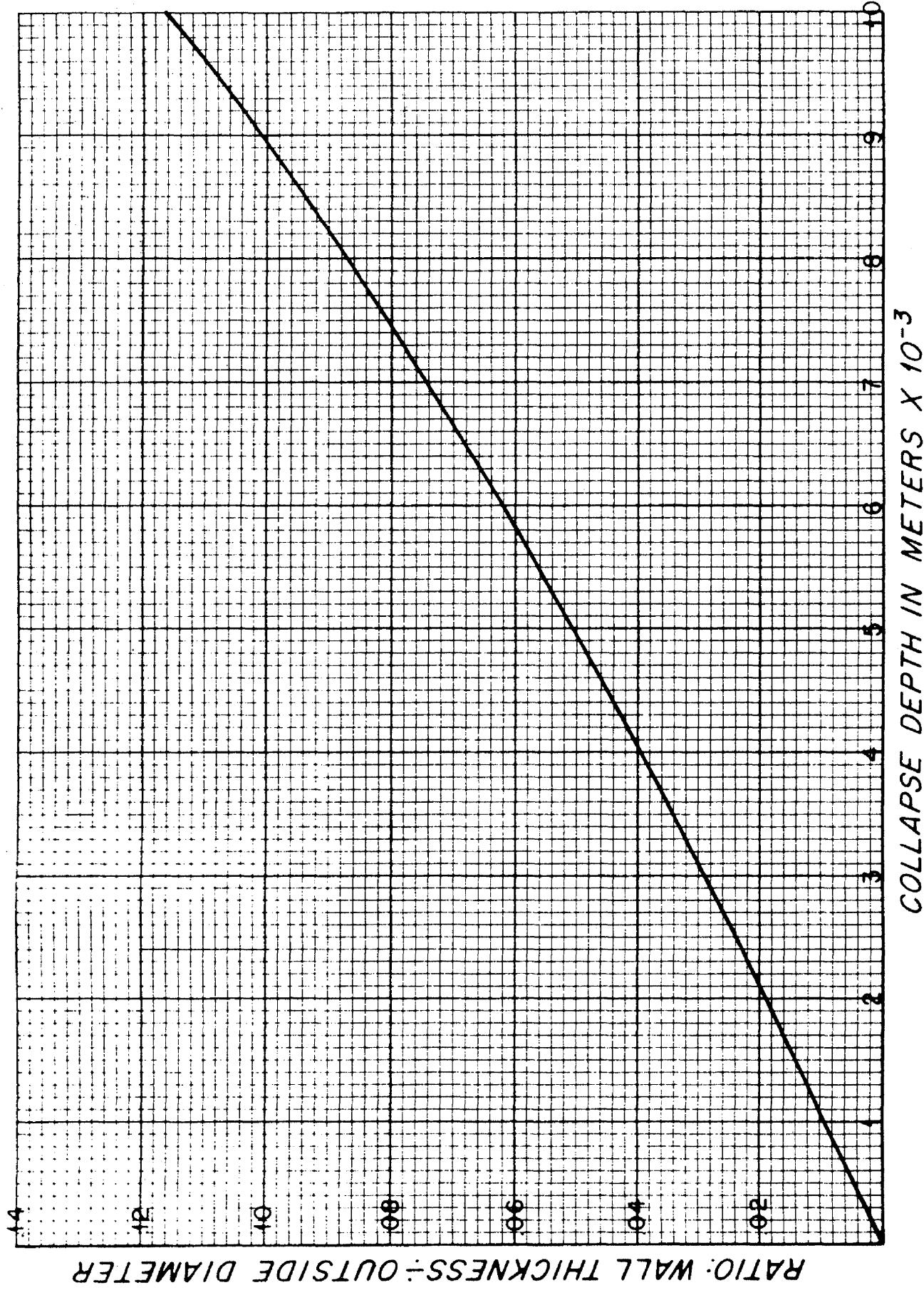


Figure 17 Collapse Depths of Spheres; Unalloyed Titanium  
ASTM Grade 2; Yield Strength 276 MPa (40,000  
psi)

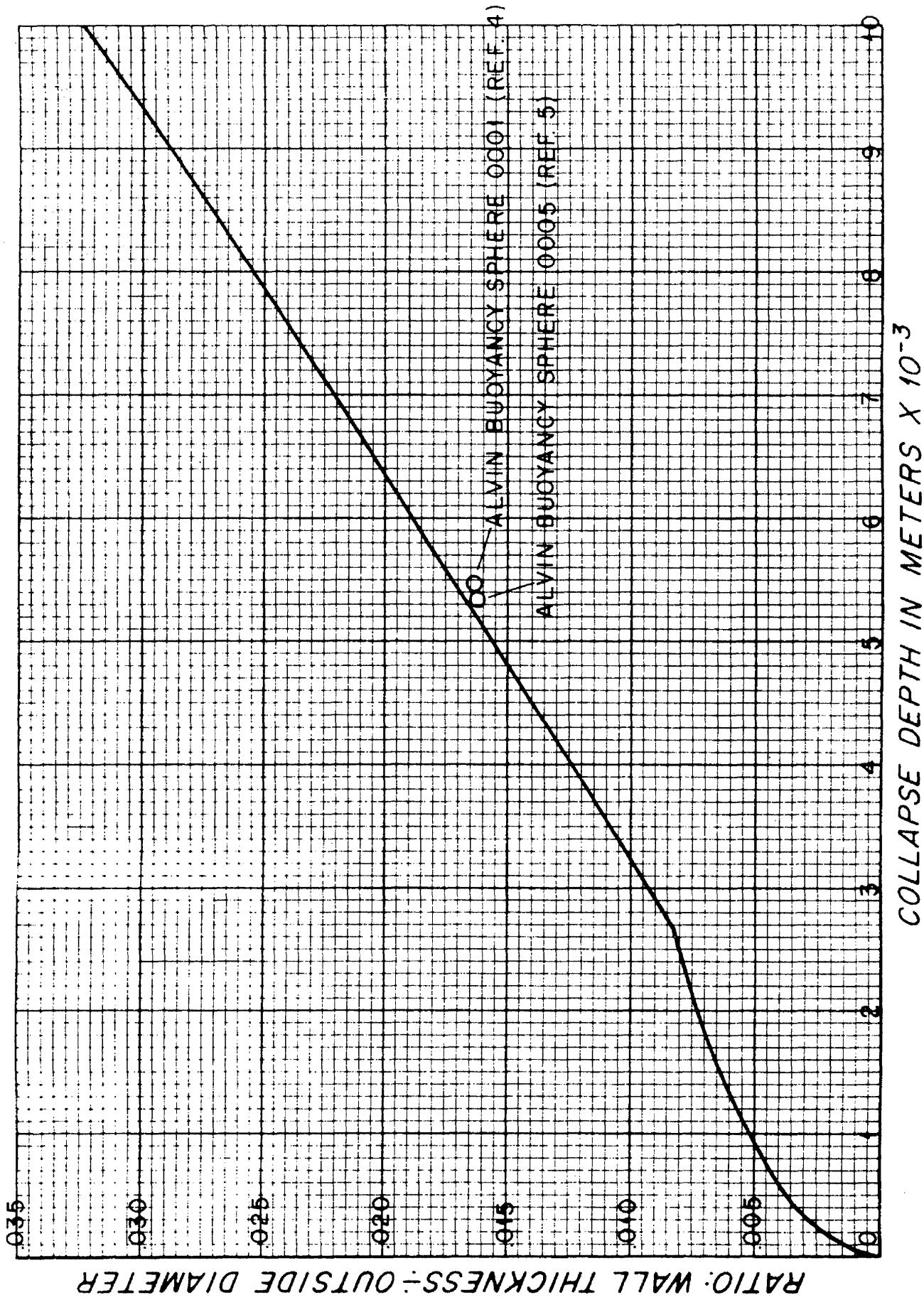


Figure 18 Collapse Depths of Spheres; Titanium Alloy  
6Al-4V; Yield strength 828 MPa (120,000 psi)

VI. Three Dimensionless Curve Sheets

$(P_c/E)10^3$  — ELASTIC BUCKLING

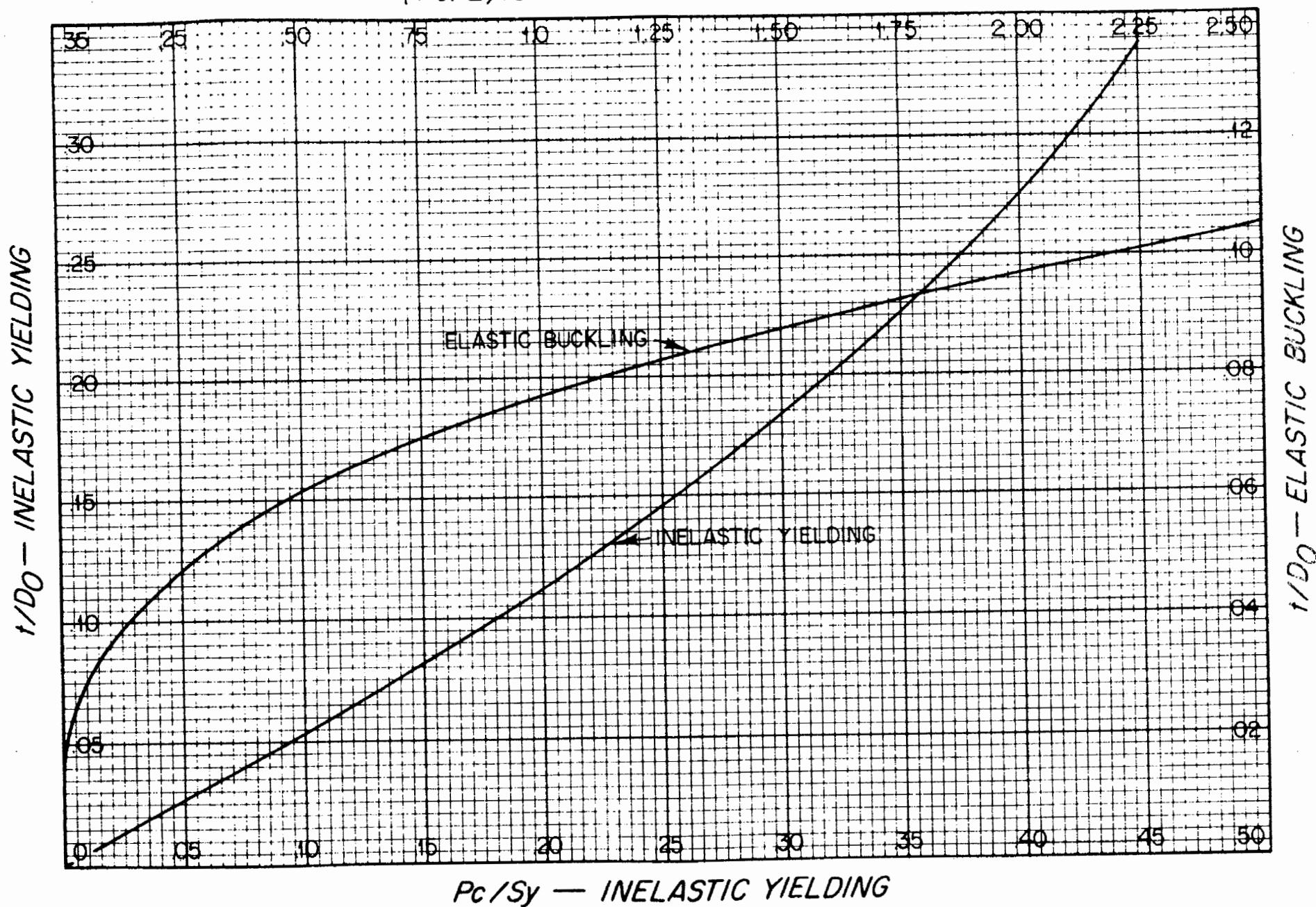


Figure 19 Collapse Pressure Ratios for Cylinders;  
Dimensionless Curves

$t/D_0 - \text{ELASTIC BUCKLING}$

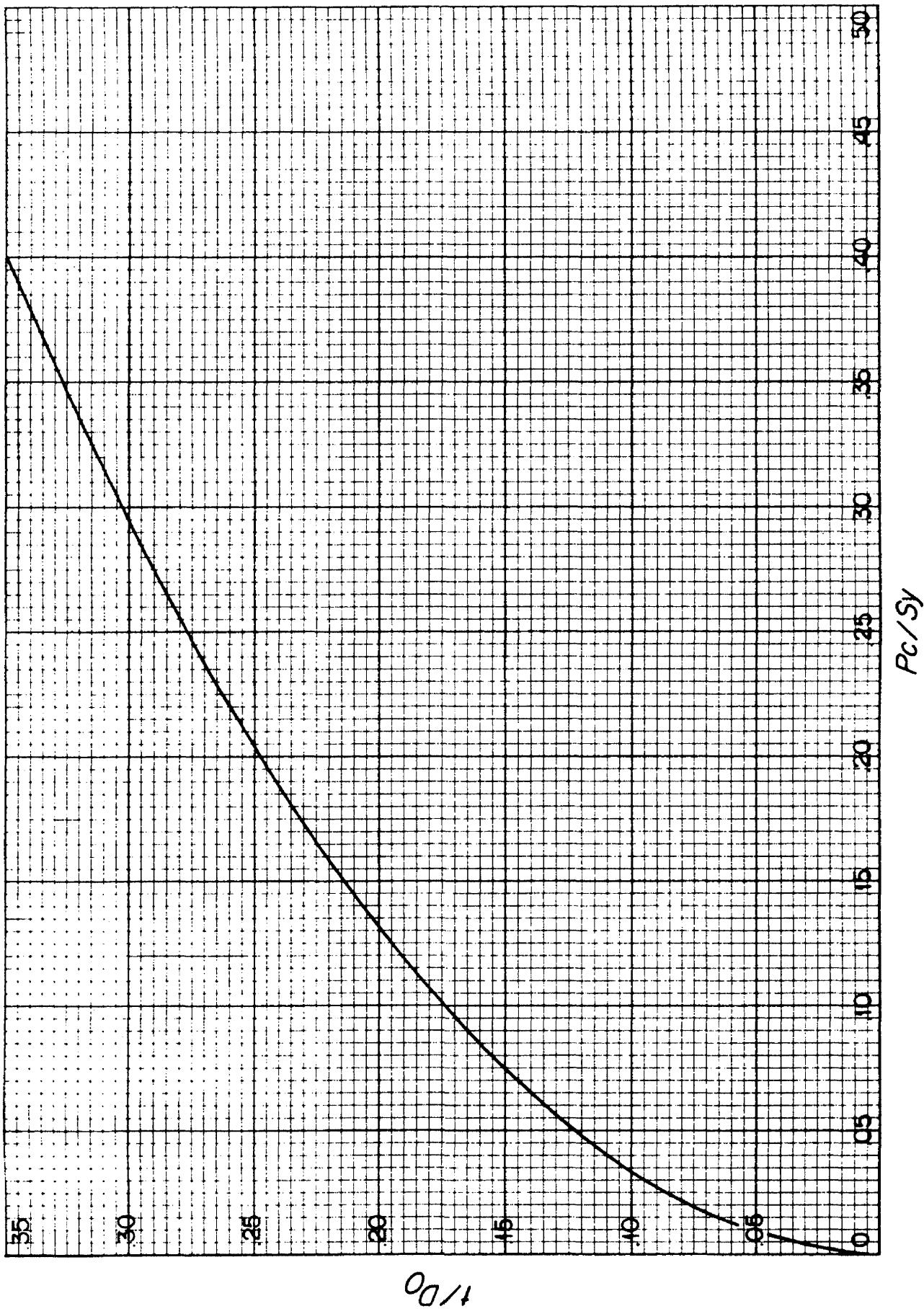
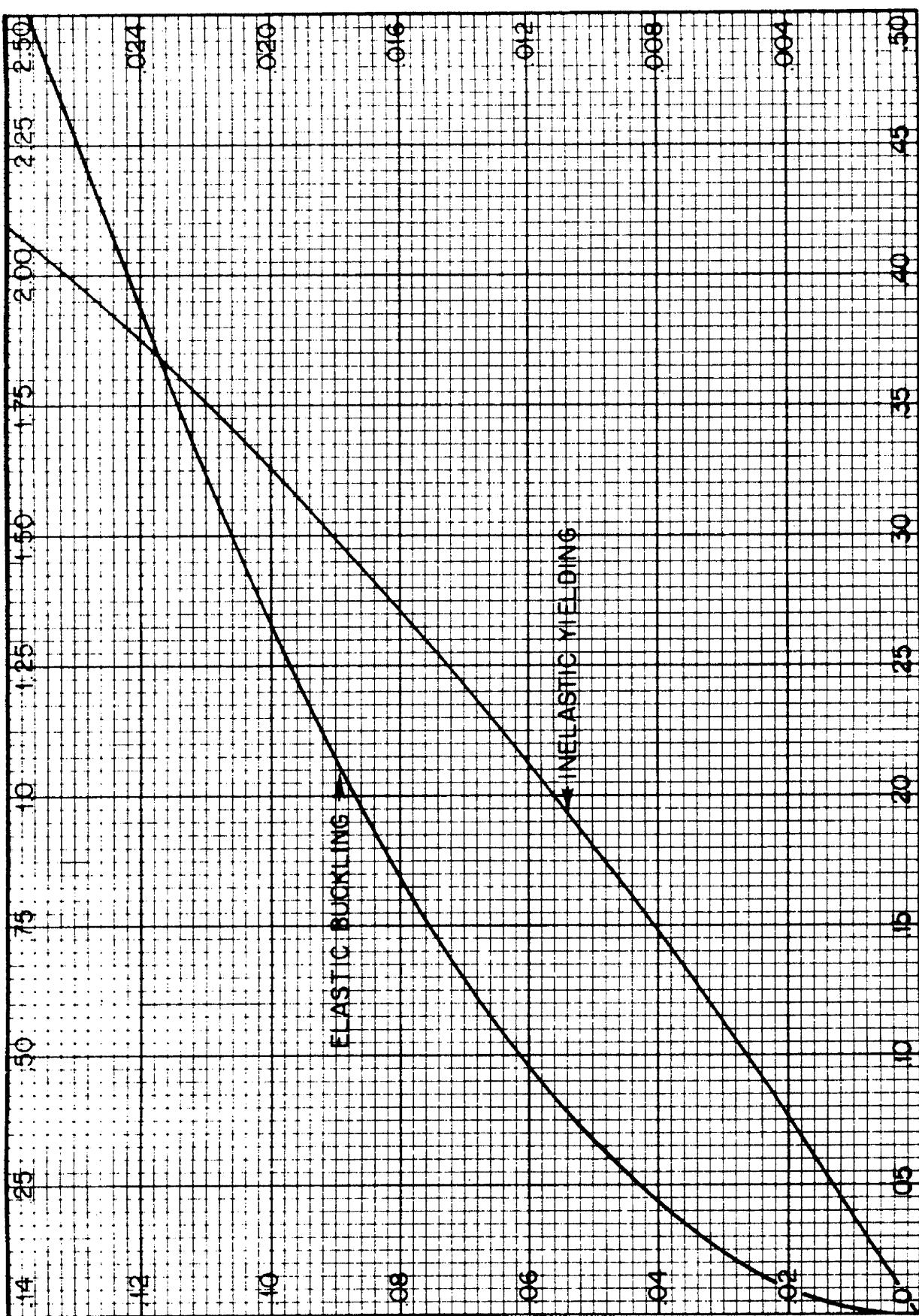


Figure 20 Collapse Pressure Ratios for Flat End Caps;  
Dimensionless Curve

$(P_c/E) \cdot 10^3 - ELASTIC BUCKLING$



$1/D_0 - INELASTIC YIELDING$

Figure 21 Collapse Pressure Ratios for Spheres;  
Dimensionless Curves

## VII. Discussion

It is common to design vessels to have a factor of safety suitable to the intended application, i.e., to have a collapse depth greater than the operating depth by some known factor. In some cases proof test procedures are required that dictate a larger safety factor (and greater weight penalty) than otherwise might have been used. For example, in the case of ALVIN, a U.S. Navy certified deep-submersible, all implodable instrument housings are proof-tested to a pressure that is 1.5 times the operating pressure. This means that the design collapse pressure must exceed the test pressure by a safe margin.

The present curves are intended to serve as a guide in designing or in predicting the collapse depth of cylindrical or spherical pressure cases. The actual collapse depth of a tube or sphere depends upon several factors not considered here, such as out-of-roundness, variations in wall thickness, flat spots, and flaws or lack of homogeneity in material. These defects are usually small in magnitude and may be expected to decrease collapse strengths slightly from those values given by the curves. On the other hand, the curves of this report are based on the minimum mechanical

properties of the materials in question. Average values of yield strength, however, are often found to be 10 to 15 percent greater than the minimum. This could result in a tendency toward conservative design when using the present curves (see Figure 13).

The curves for flat end caps are those for the basic solid flat circular plate with a simply supported edge. Any holes or penetrations in such a plate would cause stress concentrations to be present near the hole boundaries. These regions of higher stress are quite localized, but an increase in plate thickness may be necessary to prevent local yielding of the end cap material. Standard handbooks on the use of stress concentration factors, such as that of Peterson (Ref. 6), are available. With their use the designer can determine peak values of stress around penetrator holes, and select a suitable thickness/diameter ratio accordingly.

On two of the curves (Figures 13 and 18) documented results of collapse tests known to the author have been plotted for comparison with the curves. These were spheres designed for the variable ballast and fixed buoyancy systems of the deep-submersible ALVIN. The aluminum alloy spheres were two of those originally installed in ALVIN, while the titanium alloy samples

were intended for destructive testing, part of the group of new spheres that replaced the older aluminum ones.

For critical applications (for example, in manned deep submersible hulls) deviations from ideal geometry can be measured and their effects on the ultimate strength of the vessel determined. Researchers at the David Taylor Naval Ship Research and Development Center have developed a method for spherical shells that makes use of local geometry i.e., local thickness and radius of curvature over a predetermined critical arc length. Krenzke (Ref. 7) reported on this procedure that predicts closely the collapse pressures of near-perfect and initially-imperfect spheres. This technique in most cases would predict collapse at a somewhat lower pressure than do the curves of this report. However, the method requires many accurate measurements of radius and thickness and is therefore expensive. In less critical uses, the curves of this report, combined with an adequate factor of safety and a proof-test procedure, should provide a practical approach to pressure vessel design.

## VIII. References

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