Numerical Analysis Project 1

- **Pseudo Code**
  - ***Bisection***

```
prev_xr = 0
for i = 1 : max_iter
        xr = (xl+xu)/2
        if f(xr)==0
                root = xr
                break
        ea = (xr – prev_xr)/xr
        if ea<=es
                root = xr
                break
        if f(xr)*f(xl)<0
                xu = xr
        else
                xl = xr
```

  - ***False Position***

```
prev_xr = 0
for i = 1 : max_iter
        xr = (xl*f(xu)-xu*f(xl))/(f(xu)-f(xl))
        if f(xr)==0
                root = xr
                break
        ea = (xr – prev_xr)/xr
        if ea<=es
                root = xr
                break
        if f(xr)*f(xl)<0
                xu = xr
        else
                xl = xr
```

  - ***Fixed Point***

```
for i = 1 : max_iter
        x_new = g(x_old)
        ea = (x_new – x_old)/x_new
        if ea<=es
                root = xr
                break
```

```
x_old = x_new
```

- ***Newton-Raphson***

```
for i = 1 : max_iter
	xr_new = xr_old-f(xr_old)/diff_f(xr_old)
	if f(xr_new)==0
		root = xr_new
		break
	ea = (xr_new – xr_old)/xr_new
	if ea<=es
		root = xr_new
		break
	xr_old = xr_new
```

- ***Secant***

```
for i = 1 : max_iter
	x3 = x2 – (f(x2)*(x2-x1))/(f(x2)-f(x1))
	if f(x3)==0
		root = x3
		break
	ea = (x3 – x2)/x3
	if ea<=es
		root = x3
		break
	x1,x2 <- x2,x3
```

- **DS used**
  - ***Output Matrix from each root finding method***
    Each row represents an iteration in the algorithm. Each column represents a certain variable value. It was very efficient because it could be added in the table object used in our GUI.
- **Analysis**
  - ***Example 1: y = cos(x)-exp(x)+x^3***
    Bisection: finds root after 15 iterations
    False Position: finds root after 11 iterations
    Fixed Point: diverges in given g(x)
    Newton-Raphson: finds root after 7 iterations
    Secant: finds root after 7 iterations
  - ***Example 2: y = x^4-2x^3-2x^2+4x+4***
    Bisection: finds root after 15 iterations
    False Position: finds root after 10 iterations
    Fixed Point: diverges in given g(x)
    Newton-Raphson: finds root after 5 iterations

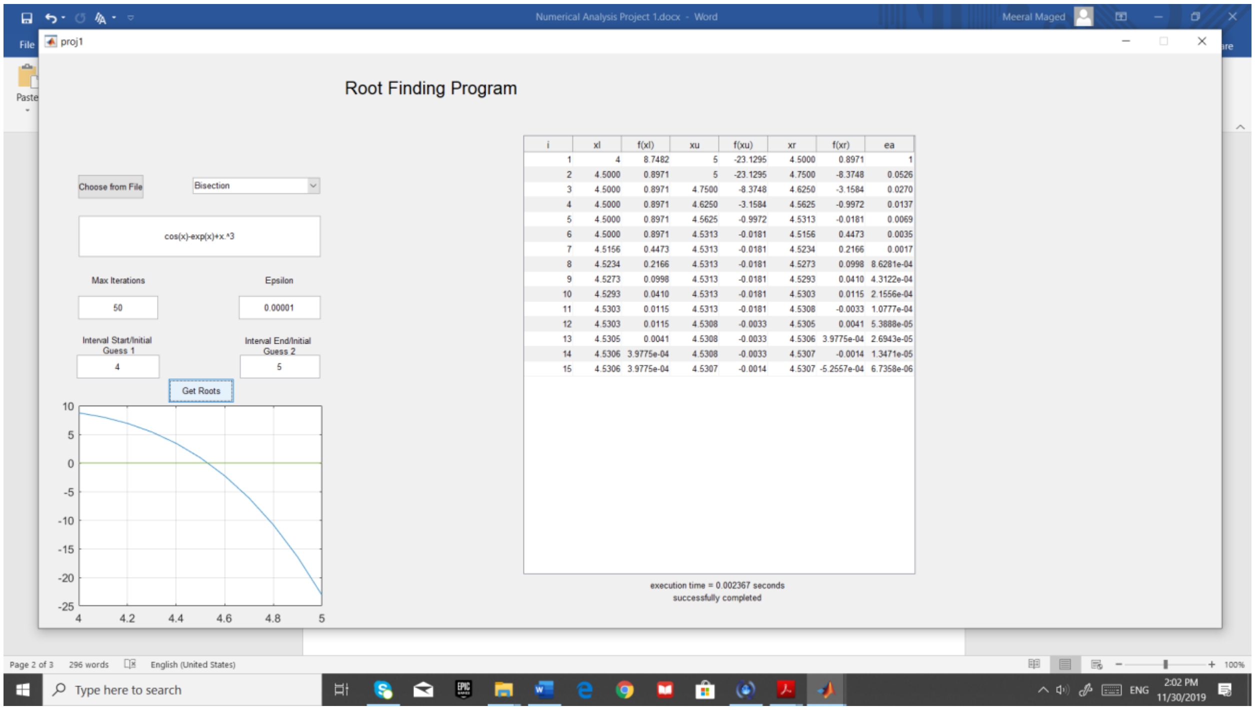
Secant: finds root after 6 iterations

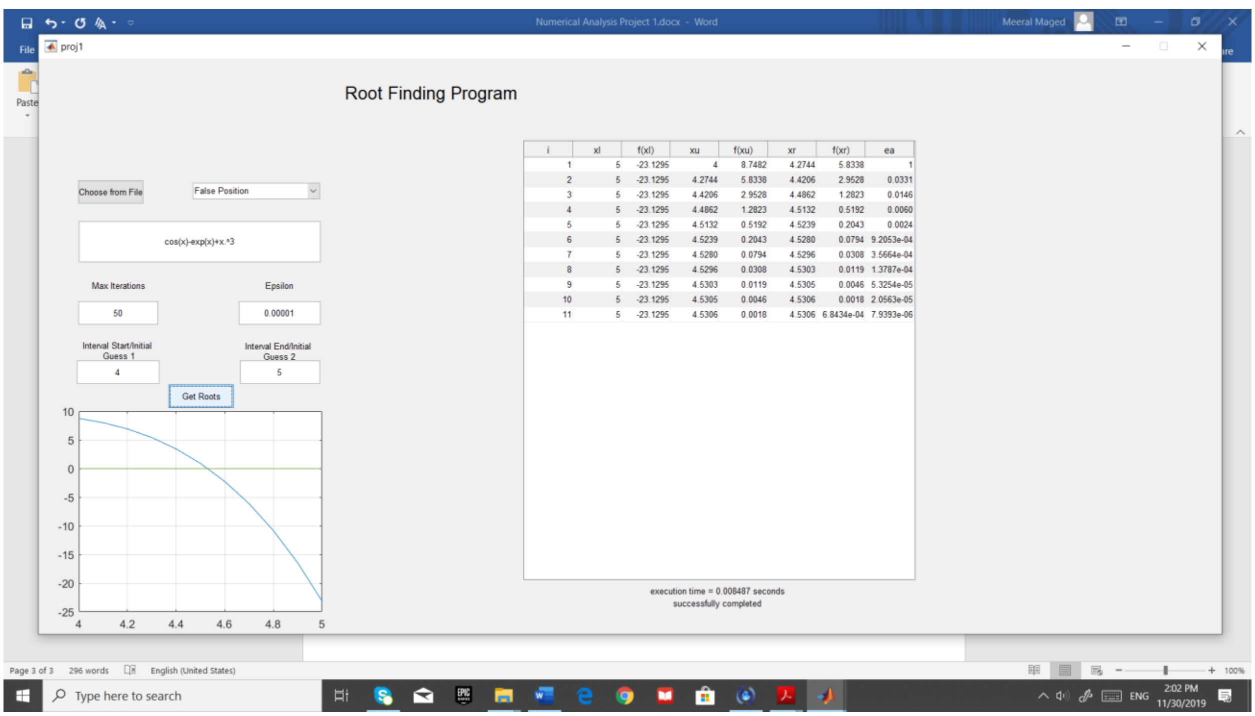
- ***Example 3: y = x^3-25***

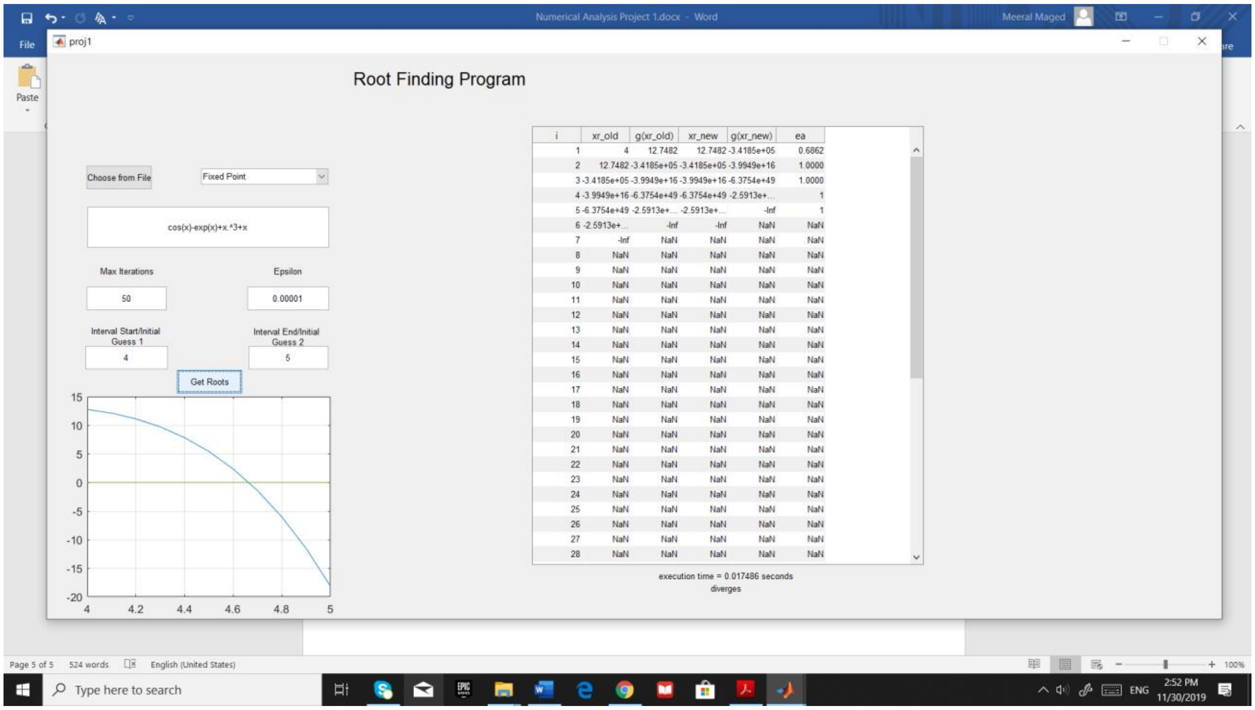
Bisection: finds root after 15 iterations
False Position: finds root after 14 iterations
Fixed Point: diverges in given g(x)
Newton-Raphson: finds root after 13 iterations
Secant: finds root after 11 iterations

- **<u>Problematic functions</u>**
  - ***Functions with imaginary parts with Bisection and False Position.***
    An example which illustrates the issue is the square root function. The root is the leftmost real value in the function. If any range was provided to find the root there would be found imaginary values which do not benefit the search procedure. Therefore, in functions like this, other methods should be used.
  - ***First order functions with Newton-Raphson and Fixed-Point methods***
    Finding the derivative for first order functions doesn't work as it is zero so other methods can be used.
  - ***Reaching xr = 0***
    In calculating ea, when xr = 0 the relative error is infinity even though it doesn't describe the actual relative error as the root could actually be zero or close to it. Our solution was to first check if f(xr) == 0 before calculating ea to handle the case where the root is at the origin. As for roots close to the origin, we divide ea by epsilon (the built-in value in MATLAB) rather than zero. We found another solution – which we didn't implement – where we calculate (in this case only) ea = x-xold instead of ea = x-xold/x.

- **<u>Sample runs</u>**






Numerical Analysis Project 1.docx - Word
Meeral Maged
proj1
Root Finding Program
Choose from File
Fixed Point
cos(x)-exp(x)+x.^3+x
Max Iterations
Epsilon
50
0.00001
Interval Start/Initial Guess 1
Interval End/Initial Guess 2
4
5
Get Roots
i xr_old g(xr_old) xr_new g(xr_new) ea
1 4 12.7482 12.7482 -3.4185e+05 0.6862
2 12.7482 -3.4185e+05 -3.4185e+05 -3.9949e+16 1.0000
3 -3.4185e+05 -3.9949e+16 -3.9949e+16 -6.3754e+49 1.0000
4 -3.9949e+16 -6.3754e+49 -6.3754e+49 -2.5913e+... 1
5 -6.3754e+49 -2.5913e+... -2.5913e+... -Inf 1
6 -2.5913e+... -Inf -Inf NaN NaN
7 -Inf NaN NaN NaN NaN
8 NaN NaN NaN NaN NaN
9 NaN NaN NaN NaN NaN
10 NaN NaN NaN NaN NaN
11 NaN NaN NaN NaN NaN
12 NaN NaN NaN NaN NaN
13 NaN NaN NaN NaN NaN
14 NaN NaN NaN NaN NaN
15 NaN NaN NaN NaN NaN
16 NaN NaN NaN NaN NaN
17 NaN NaN NaN NaN NaN
18 NaN NaN NaN NaN NaN
19 NaN NaN NaN NaN NaN
20 NaN NaN NaN NaN NaN
21 NaN NaN NaN NaN NaN
22 NaN NaN NaN NaN NaN
23 NaN NaN NaN NaN NaN
24 NaN NaN NaN NaN NaN
25 NaN NaN NaN NaN NaN
26 NaN NaN NaN NaN NaN
27 NaN NaN NaN NaN NaN
28 NaN NaN NaN NaN NaN
execution time = 0.017486 seconds
diverges
15
10
5
0
-5
-10
-15
-20
4
4.2
4.4
4.6
4.8
5
Page 5 of 5
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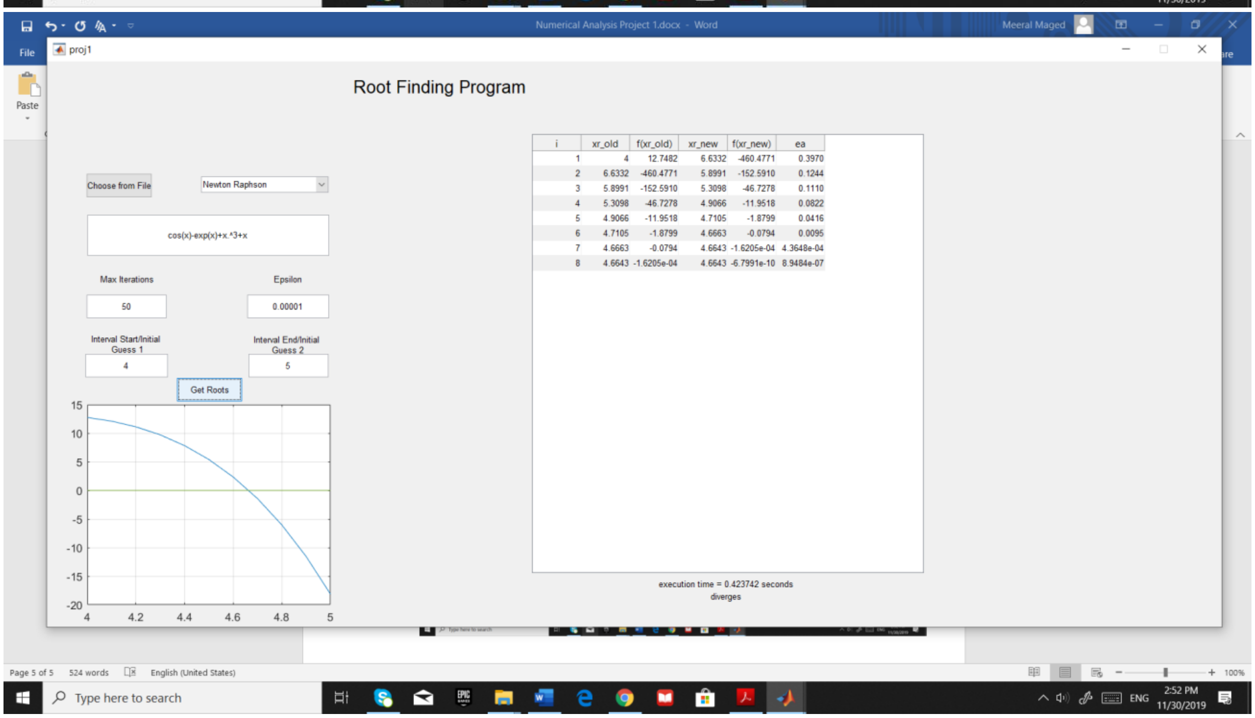
Numerical Analysis Project 1.docx - Word
Meeral Maged
proj1
Root Finding Program
Choose from File
Newton Raphson
cos(x)-exp(x)+x.^3+x
Max Iterations
Epsilon
50
0.00001
Interval Start/Initial Guess 1
Interval End/Initial Guess 2
4
5
Get Roots
i xr_old f(xr_old) xr_new f(xr_new) ea
1 4 12.7482 6.6332 -460.4771 0.3970
2 6.6332 -460.4771 5.8991 -152.5910 0.1244
3 5.8991 -152.5910 5.3098 -46.7278 0.1110
4 5.3098 -46.7278 4.9066 -11.9518 0.0822
5 4.9066 -11.9518 4.7105 -1.8799 0.0416
6 4.7105 -1.8799 4.6663 -0.0794 0.0095
7 4.6663 -0.0794 4.6643 -1.6205e-04 4.3648e-04
8 4.6643 -1.6205e-04 4.6643 -6.7991e-10 8.9484e-07
execution time = 0.423742 seconds
diverges
15
10
5
0
-5
-10
-15
-20
4
4.2
4.4
4.6
4.8
5
Page 5 of 5
524 words
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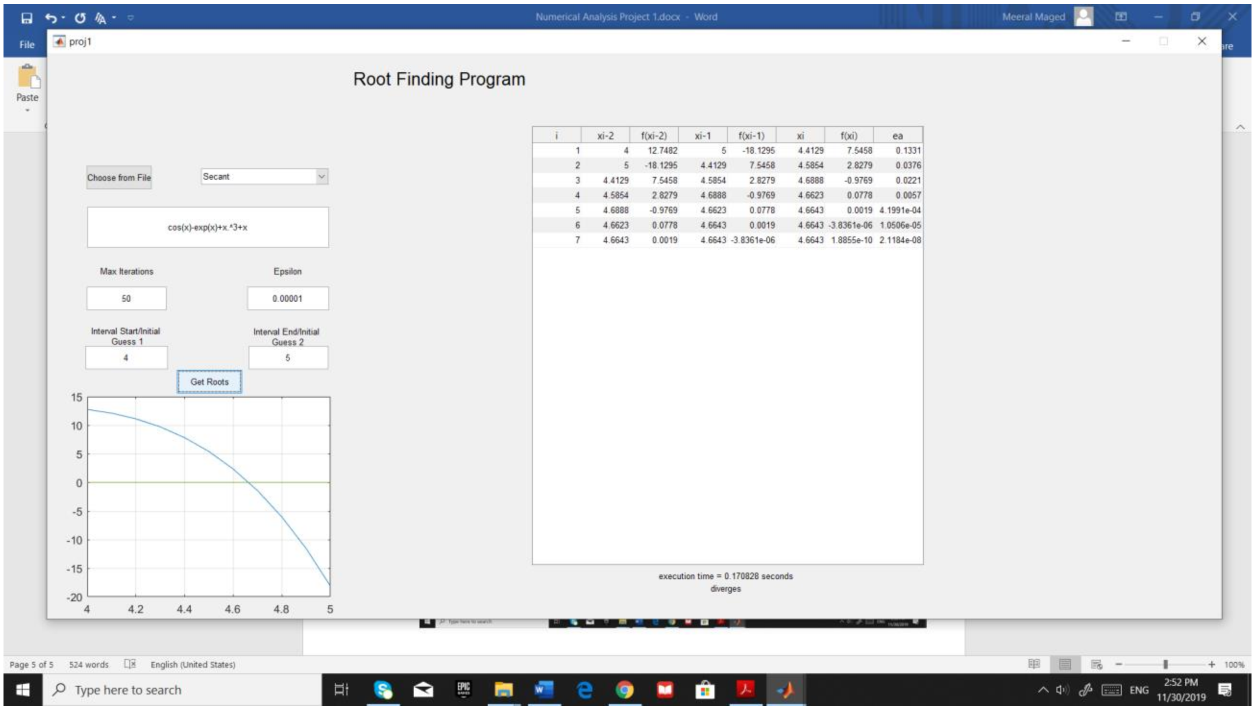
Numerical Analysis Project 1.docx - Word
Meeral Maged
proj1
Root Finding Program
Choose from File
Secant
cos(x)-exp(x)+x.^3+x
Max Iterations
Epsilon
50
0.00001
Interval Start/Initial Guess 1
Interval End/Initial Guess 2
4
5
Get Roots
i | xi-2 | f(xi-2) | xi-1 | f(xi-1) | xi | f(xi) | ea
1 | 4 | 12.7482 | 5 | -18.1295 | 4.4129 | 7.5458 | 0.1331
2 | 5 | -18.1295 | 4.4129 | 7.5458 | 4.5854 | 2.8279 | 0.0376
3 | 4.4129 | 7.5458 | 4.5854 | 2.8279 | 4.6888 | -0.9769 | 0.0221
4 | 4.5854 | 2.8279 | 4.6888 | -0.9769 | 4.6623 | 0.0778 | 0.0057
5 | 4.6888 | -0.9769 | 4.6623 | 0.0778 | 4.6643 | 0.0019 | 4.1991e-04
6 | 4.6623 | 0.0778 | 4.6643 | 0.0019 | 4.6643 | -3.8361e-06 | 1.0506e-05
7 | 4.6643 | 0.0019 | 4.6643 | -3.8361e-06 | 4.6643 | 1.8855e-10 | 2.1184e-08
execution time = 0.170828 seconds
diverges
Page 5 of 5
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