

# Assignment 4

## Question 1 ✓

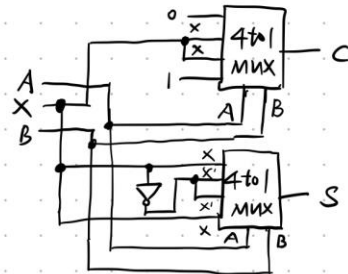
Input			Output	
A	B	X	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

当  $A=0, B=0$  时,  $C=0, S=X$

当  $A=0, B=1$  时,  $C=X, S=X'$

当  $A=1, B=0$  时,  $C=X, S=X'$

当  $A=1, B=1$  时,  $C=1, S=X$

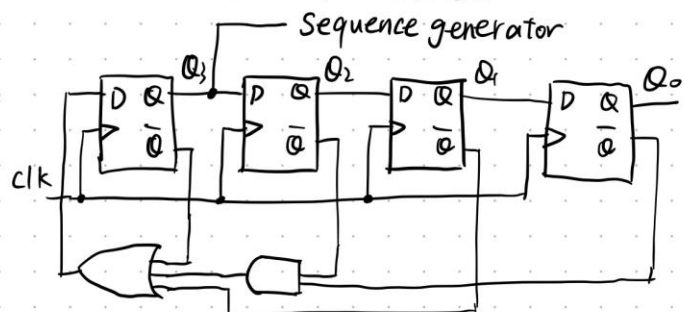


## Question 2 ✓

clk	$Q_3$	$Q_2$	$Q_1$	$Q_0$	Z
↑	1	1	0	1	1
↑	1	1	1	0	0
↑	0	1	1	1	1
↑	1	0	1	1	0
↑	0	1	0	1	1
↑	1	0	1	0	1
↑	1	1	0	1	1
↑	1	1	1	0	0
↑	0	1	1	1	1
↑	1	0	1	1	0

$Q_3 Q_2$ \ $Q_1 Q_0$	00	01	11	10
00	⊗	x	x	⊗
01	x	1	1	x
11	x	1	x	0
10	⊗	x	0	⓪

$$Z = Q_3' + Q_1' + Q_2' Q_0'$$



### Question 3

Input				Output			
A	B	C	D	X	Y	Z	R
0	0	0	0	1	0	0	1
0	0	0	1	1	0	0	0
0	0	1	0	0	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	0	0
0	1	1	0	0	0	1	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	X	X	X	X
10	0	0	X	X

$$X = A'B'C' \\ = (A+B+C)'$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	X	X	X	X
10	0	0	X	X

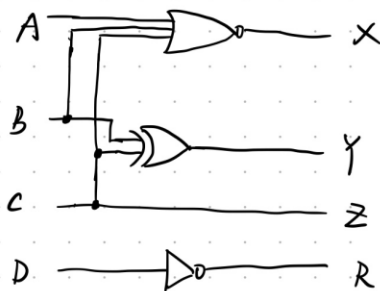
$$Y = BC' + B'C \\ = B \oplus C$$

AB \ CD	00	01	11	10
00			1	1
01			1	1
11	X	X	X	X
10			X	X

$$Z = C$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	X	X	X	X
10	1	0	X	X

$$R = D'$$



### Question 4

① 8个未使用状态

$Q_A$	$Q_B$	$Q_C$	$Q_D$
0	0	1	0
0	1	0	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	1

②

8个未使用状态及其接下来的状态

$Q_A$	$Q_B$	$Q_C$	$Q_D$	$Q_A'$	$Q_B'$	$Q_C'$	$Q_D'$
0	0	1	0	1	0	0	1
0	1	0	0	1	0	1	0
0	1	0	1	0	0	1	0
0	1	1	0	1	0	1	1
1	0	0	1	0	1	0	0
1	0	1	0	1	1	0	1
1	0	1	1	0	1	0	1
1	1	0	1	0	1	1	0

⇒ 电路一旦输入非法状态，无法通过电路形成合法状态。

③ 可以将所有非法状态都在 T-Y 时强制复位至初态 0000

Present				Next				DFF Input			
Q <sub>A</sub>	Q <sub>B</sub>	Q <sub>C</sub>	Q <sub>D</sub>	Q <sub>A</sub>	Q <sub>B</sub>	Q <sub>C</sub>	Q <sub>D</sub>	D <sub>A</sub>	D <sub>B</sub>	D <sub>C</sub>	D <sub>D</sub>
0	0	0	0	1	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0
1	0	0	0	1	1	0	0	1	1	0	0
1	0	0	1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0	0
1	1	0	0	1	1	1	0	1	1	1	0
1	1	0	1	0	0	0	0	0	0	0	0
1	1	1	0	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	0	1	1	1

**D<sub>A</sub>**

Q <sub>A</sub> Q <sub>B</sub>	Q <sub>C</sub> Q <sub>D</sub> 00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$D_A = Q_B'Q_C'Q_D' + Q_AQ_BQ_D'$

**D<sub>B</sub>**

Q <sub>A</sub> Q <sub>B</sub>	Q <sub>C</sub> Q <sub>D</sub> 00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$D_B = Q_AQ_C'Q_D' + Q_AQ_BQ_C$

**D<sub>C</sub>**

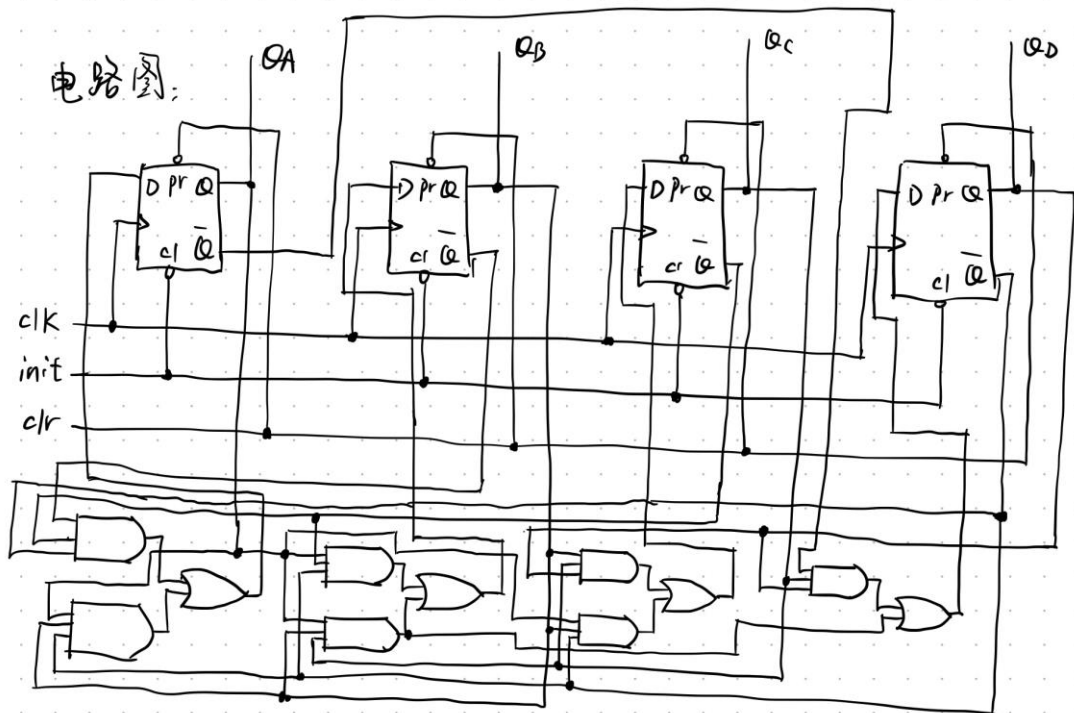
Q <sub>A</sub> Q <sub>B</sub>	Q <sub>C</sub> Q <sub>D</sub> 00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$D_C = Q_BQ_CQ_D + Q_AQ_BQ_D'$

**D<sub>D</sub>**

Q <sub>A</sub> Q <sub>B</sub>	Q <sub>C</sub> Q <sub>D</sub> 00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$D_D = Q_A'Q_CQ_D + Q_AQ_BQ_C$



## Question 5

Present				Next				JKFF Input							
$A_3$	$A_2$	$A_1$	$A_0$	$A_3$	$A_2$	$A_1$	$A_0$	$J_3$	$K_3$	$J_2$	$K_2$	$J_1$	$K_1$	$J_0$	$K_0$
0	0	0	0	0	0	1	0	0	x	0	x	1	x	0	x
0	0	0	1	0	0	1	0	0	x	0	x	1	x	x	1
0	0	1	0	0	1	1	0	0	x	1	x	x	0	0	x
0	0	1	1	0	0	1	0	0	x	0	x	x	0	x	1
0	1	0	0	0	1	0	0	0	x	x	1	1	x	0	x
0	1	0	1	0	0	1	0	0	x	x	1	1	x	x	1
0	1	1	0	1	0	0	1	1	x	x	1	x	1	1	x
1	0	0	0	1	1	0	0	x	0	1	x	0	0	x	1
1	0	0	1	1	0	0	0	x	1	0	x	0	0	x	1
1	0	1	0	0	0	1	0	x	1	0	x	x	0	x	1
1	0	1	1	0	0	1	1	x	0	x	0	0	1	x	1
1	1	0	0	0	0	1	0	x	1	x	1	1	x	x	1
1	1	0	1	0	0	1	0	x	1	x	1	1	x	x	1
1	1	1	0	0	0	1	0	x	1	x	1	x	0	0	x
1	1	1	1	0	0	1	0	x	1	x	1	x	0	x	1

$J_3$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	0	0	0	0	0	0
01	0	0	0	0	0	1
11	x	x	x	x	x	x
10	x	x	x	x	x	x

$$J_3 = A_2 A_1 A_0'$$

$K_3$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	x	x	x	x	x	x
01	x	x	x	x	x	x
11	0	1	1	1	1	1
10	0	0	1	1	1	1

$$K_3 = A_2 A_0 + A_1$$

$J_2$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	0	0	0	0	1	1
01	0	x	x	x	x	x
11	x	x	x	x	x	x
10	1	0	0	0	0	0

$$J_2 = A_3 A_1 A_0' + A_3' A_1 A_0'$$

$K_2$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	x	x	x	x	x	x
01	1	1	1	1	1	1
11	0	1	1	1	1	1
10	x	x	x	x	x	x

$$K_2 = A_3' + A_0 + A_1$$

$J_1$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	1	1	x	x	x	x
01	1	1	x	x	x	x
11	0	1	x	x	x	x
10	0	0	x	x	x	x

$$J_1 = A_3' + A_2 A_0$$

$K_1$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	x	x	0	0	0	0
01	x	x	0	0	0	0
11	x	x	0	0	0	0
10	x	x	0	0	0	0

$$K_1 = A_3' A_2 A_0'$$

$J_0$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	0	x	x	x	x	x
01	0	x	x	x	x	x
11	1	x	x	x	x	x
10	0	x	x	x	x	x

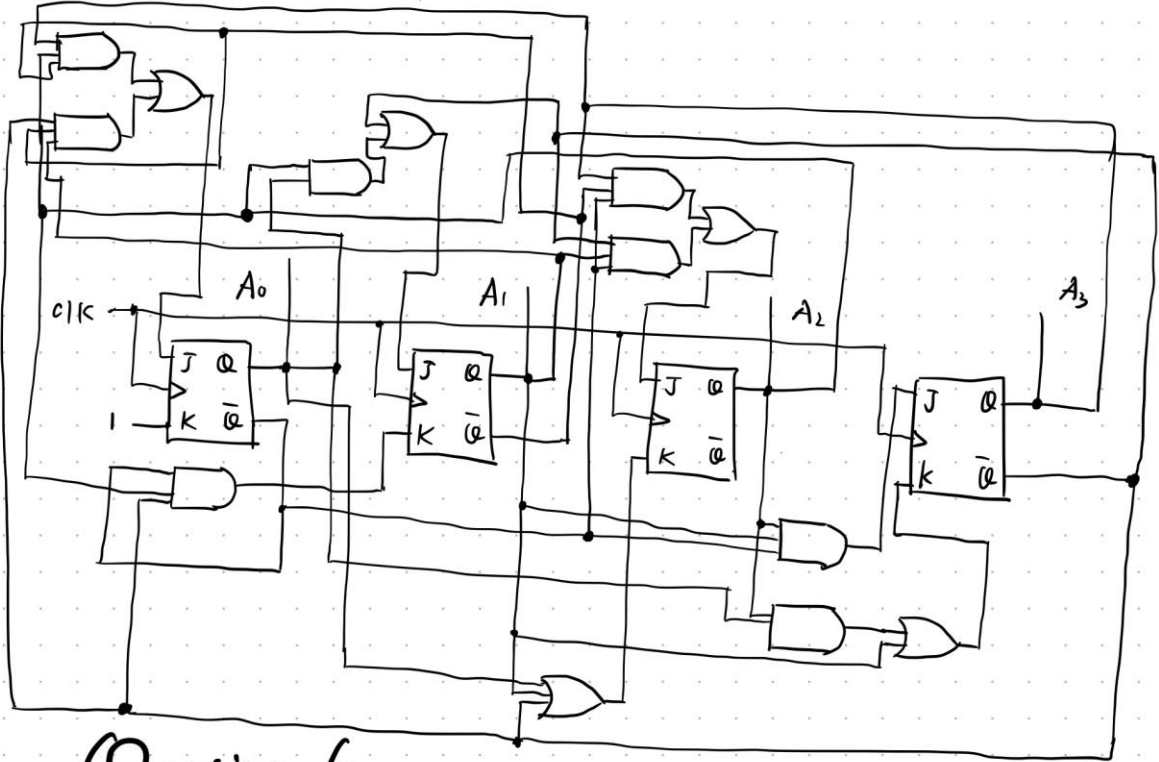
$$J_0 = A_3 A_2 A_1' + A_3' A_2 A_1$$

$K_0$	$A_3 A_2$	$A_1 A_0$	00	01	11	10
00	x	1	1	x	x	x
01	x	1	1	x	x	x
11	x	1	1	x	x	x
10	x	1	1	x	x	x

$$K_0 = 1$$

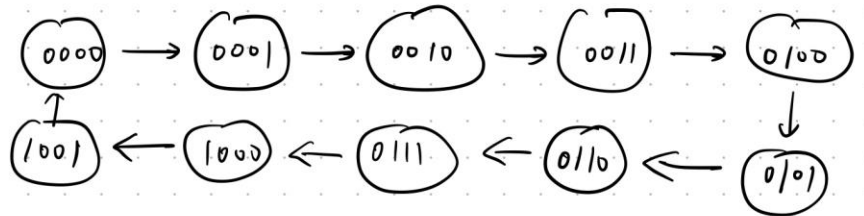
$$\begin{cases} J_3 = A_2 A_1 A_0' \\ K_3 = A_2 A_0 + A_1 \end{cases} \begin{cases} J_2 = A_3 A_1' A_0' + A_3' A_1 A_0' \\ K_2 = A_3' + A_0 + A_1 \end{cases}$$

$$\begin{cases} J_1 = A_3' + A_2 A_0 \\ K_1 = A_3' A_2 A_0' \end{cases} \begin{cases} J_0 = A_3 A_2 A_1' + A_3' A_2 A_1 \\ K_0 = 1 \end{cases}$$

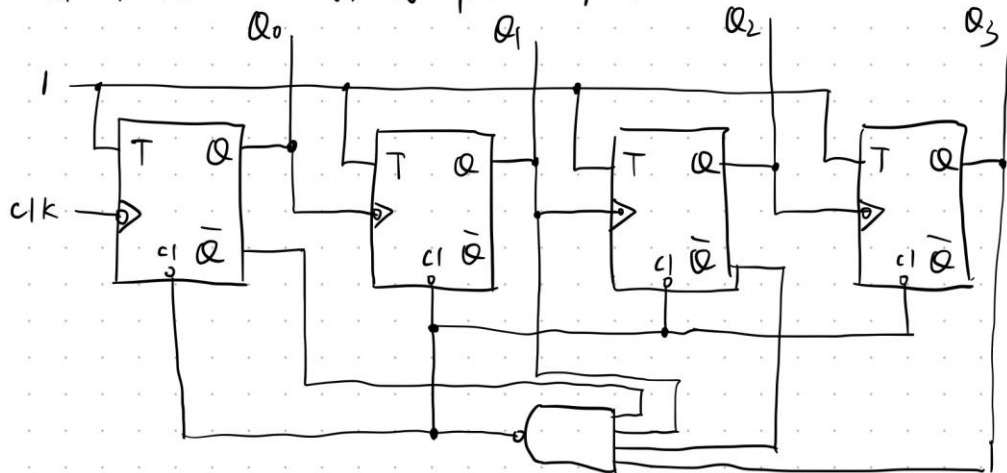


## Question 6

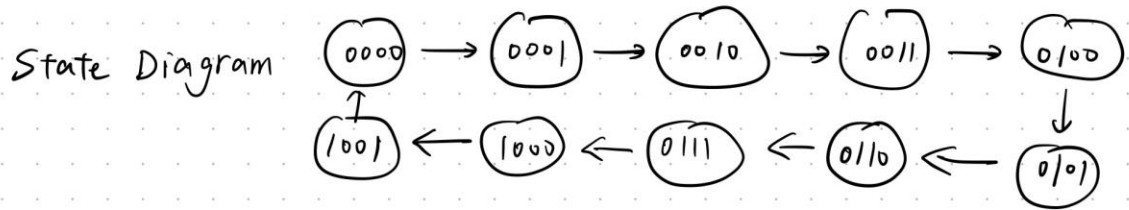
State Diagram



当状态为 1010 时, 将利用 cl 端将 counter 置为 0000 状态



## Question 7



① JK flip-flops

JK flip-flops Input

Present				Next				JK flip-flops Input							
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$J_3$	$K_3$	$J_2$	$K_2$	$J_1$	$K_1$	$J_0$	$K_0$
0	0	0	0	0	0	0	1	0	x	0	x	0	x	1	x
0	0	0	1	0	0	1	0	0	x	0	x	1	x	x	1
0	0	1	0	0	0	1	1	0	x	0	x	x	0	1	x
0	0	1	1	0	1	0	0	0	x	1	x	x	1	x	1
0	1	0	0	0	1	0	1	0	x	x	0	0	x	1	x
0	1	0	1	0	1	1	0	0	x	x	0	1	x	x	1
0	1	1	0	0	1	1	1	0	x	x	0	x	0	1	x
0	1	1	1	1	0	0	0	1	x	x	1	x	1	x	1
1	0	0	0	1	0	0	1	x	0	0	x	0	x	1	x
1	0	0	1	0	0	0	0	x	1	0	x	0	x	x	1

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	0	0	0	0
00	01	0	0	1	0
00	11	x	x	x	x
00	10	x	x	x	x

$$J_3 = Q_2 Q_1 Q_0$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	x	x	x	x
00	01	x	x	x	x
00	11	x	x	x	x
00	10	x	x	x	x

$$K_3 = Q_0$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	0	1	x	x
00	01	0	1	x	x
00	11	x	x	x	x
00	10	x	x	x	x

$$J_1 = Q_3' Q_0$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	1	x	x	1
00	01	1	x	x	1
00	11	x	x	x	x
00	10	1	x	x	x

$$J_0 = 1$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	0	0	1	0
00	01	x	x	x	x
00	11	x	x	x	x
00	10	0	0	x	x

$$J_2 = Q_1 Q_0$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	x	x	x	x
00	01	0	0	1	0
00	11	x	x	x	x
00	10	x	x	x	x

$$K_2 = Q_1 Q_0$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	x	x	1	0
00	01	x	x	x	x
00	11	x	x	x	x
00	10	x	x	x	x

$$K_1 = Q_0$$

$Q_3 Q_2$	$Q_1 Q_0$	00	01	11	10
00	00	x	1	1	x
00	01	x	x	1	x
00	11	x	x	x	x
00	10	x	1	x	x

$$K_0 = 1$$

所以  $J_3 = Q_2 Q_1 Q_0$   $J_2 = Q_1 Q_0$   $J_1 = Q_3' Q_0$   $J_0 = 1$   
 $K_3 = Q_0$   $K_2 = Q_1 Q_0$   $K_1 = Q_0$   $K_0 = 1$

## ② D flip-flops

D flip-flops Input

Present				Next				D flip-flops Input			
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$D_3$	$D_2$	$D_1$	$D_0$
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	0
0	0	1	0	0	0	1	1	0	0	1	1
0	0	1	1	0	1	0	0	0	1	0	0
0	1	0	0	0	1	0	1	0	1	0	1
0	1	0	1	0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	1	0	1	1	1
0	1	1	1	1	0	0	0	1	0	0	0
1	0	0	0	1	0	0	1	1	0	0	1
1	0	0	1	0	0	0	0	0	0	0	0

$D_3$

$Q_3 Q_2 \backslash Q_1 Q_0$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	x	x	x	x
10	1	0	x	x

$$D_3 = Q_2 Q_1 Q_0 + Q_3 Q_1' Q_0'$$

$D_2$

$Q_3 Q_2 \backslash Q_1 Q_0$	00	01	11	10
00	0	0	1	0
01	1	1	0	1
11	x	x	x	x
10	0	0	x	x

$$D_2 = Q_2 Q_1' + Q_2' Q_1 Q_0 + Q_2 Q_1 Q_0'$$

$D_1$

$Q_3 Q_2 \backslash Q_1 Q_0$	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	x	x	x	x
10	0	0	x	x

$$D_1 = Q_3' Q_1' Q_0 + Q_1 Q_0'$$

$D_0$

$Q_3 Q_2 \backslash Q_1 Q_0$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	x	x	x	x
10	1	0	x	x

$$D_0 = Q_0'$$

ΣF<sub>ms</sub> {

$$\begin{aligned} D_3 &= Q_2 Q_1 Q_0 + Q_3 Q_1' Q_0' \\ D_2 &= Q_2 Q_1' + Q_2' Q_1 Q_0 + Q_2 Q_1 Q_0' \\ D_1 &= Q_3' Q_1' Q_0 + Q_1 Q_0' \\ D_0 &= Q_0' \end{aligned}$$



若将上述两个设计与课件中只使用 TFF 的 4 bit 同步的 counter,  
我认为课件中只使用 TFF 的 4 bit 同步的 Counter 效率更高。

$$\begin{aligned} \text{在课件中} \quad \begin{cases} TQ_0 = 1 \\ TQ_1 = Q_1' Q_0 \\ TQ_2 = Q_1 Q_0 \\ TQ_3 = Q_1 Q_0 + Q_2 Q_1 Q_0 \end{cases} \quad \begin{aligned} \text{AND gates: } & 4 \\ \text{OR gates: } & 1 \\ \text{Total gates: } & \underline{5} \end{aligned} \end{aligned}$$

在使用 JKFF 设计中

$$\begin{aligned} JQ_3 &= Q_2 Q_1 Q_0 & JQ_2 &= Q_1 Q_0 & JQ_1 &= Q_3' Q_0 & JQ_0 &= 1 \\ KQ_3 &= Q_0 & KQ_2 &= Q_1 Q_0 & KQ_1 &= Q_0 & KQ_0 &= 1 \end{aligned}$$

$$\begin{aligned} \left( \begin{array}{l} \text{AND gates: } 4 \\ \text{OR gates: } 0 \end{array} \right) \rightarrow \text{Total gates: } \underline{4} \end{aligned}$$

在使用 DFF 设计中

$$\begin{cases} D_3 = Q_2 Q_1 Q_0 + Q_3 Q_1' Q_0' \\ D_2 = Q_2 Q_1' + Q_2' Q_1 Q_0 + Q_2 Q_1 Q_0' = Q_2 \oplus Q_1 Q_0 \\ D_1 = Q_3' Q_1' Q_0 + Q_1 Q_0' \\ D_0 = Q_0' \end{cases}$$

$$\begin{aligned} \text{AND gates: } & 5 \quad \rightarrow \quad \text{Total gates: } \underline{8} \\ \text{OR gates: } & 2 \\ \text{XOR gates: } & 1 \end{aligned}$$

所以, 使用 JK Flip-flops 的设计最有效率, 因为它需要最少

的门电路