Lecture 2: Boolean Algebra and Logic Gates

CS207: Digital Logic

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Acknowledgement

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M. M. Mano and M. Ciletti, *Digital design: with an introduction to the Verilog HDL*. Pearson, 2013

Recap: Last Week

- **Number systems**: decimal, binary, octal, hexadecimal systems.
- **Number-based conversions**:
  - Base-r system to decimal: for $(a_n a_{n-1} \ldots a_2 a_1 . b_1 b_2 \ldots b_m)_r$,

$$\begin{aligned} & a_n \cdot r^{n-1} + a_{n-1} \cdot r^{n-2} + a_{n-2} \cdot r^{n-3} + \cdots + a_2 \cdot r^1 + a_1 \cdot r^0 \\ & + b_1 r^{-1} + b_2 \cdot r^{-2} + \cdots + b_m \cdot r^{-m} \end{aligned}$$

  - Decimal to base-r system: division for integer part, multiplication for fraction
  - Conversion between binary and octal systems
- **Complements of numbers**: r's complement & $r-1$'s complement
  - For substraction
  - For representing signed binary numbers
- **Representation of data: code**

Exercise

1. Convert $(110.5)_{10}$ to base 3. For repeating decimals, write down four reptends (循环节).

Exercise

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1. Convert $(110.5)_{10}$ to base 3. For repeating decimals, write down four reptends (循环节).

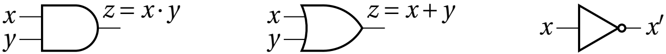
$$
\begin{aligned}
(110)_{10} &= 1*3^4+1*3^3+0*3^2+0*3^1+2*3^0=(11002)_3 \\
(0.5)_{10} &= 1*3^{-1}+1*3^{-2}+1*3^{-3}+1*3^{-4}+1*3^{-5}+1*3^{-6}+\cdots=(0.111111\ldots)_3 \\
(110.5)_{10} &= (11002.1111)_3 \qquad \textit{// write down four reptends}
\end{aligned}
$$

Exponent	Division	Quotient	Remainder
0	110/3	36	2
1	36/3	12	0
2	12/3	4	0
3	4/3	1	1
4	1/3	0	1

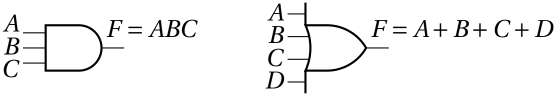
Exponent	Multiplication	Product
-1	$0.5*3$	1.5
-2	$0.5*3$	1.5
-3	$0.5*3$	1.5
-4	$0.5*3$	1.5
-5	$0.5*3$	1.5
-6	$0.5*3$	1.5
…	…	…

Logic Gate (逻辑门)

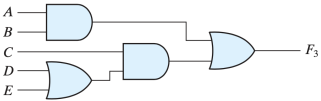
- **Logic gates** are electronic circuits that operates on one or more inputs signals to produce an output.



- It is fine to have more than two inputs for AND/OR.



- Any desired circuit can be realised through various combinations of logic gates.



Why Boolean Algebra (逻辑代数)?

- **Binary logic** is used in all of todays's digital computers and devices.
  - In a digital system, input is given with the help of switches: usually have two distinct discrete levels or values: `ON` and `OFF`.
  - Binary logic deals with variables that take on two discrete values (1 for true or switch `ON` and 0 for false or switch `OFF`) and with operations that assume logical meaning.

- The cost of the digital circuits is an important factor addressed by designers.
  Aim: Reduce cost.
  ← Finding simpler and cheaper, but equivalent, realizations of a circuit.
  ← Mathematical methods to simplify circuits.
  ← Key: Boolean algebra, a deductive mathematical system.

Outline of This Lecture

- Boolean Algebra
- Boolean Function
- Canonical Forms, Minterms, Maxterms
- Other Logic Operations
- Digital Logic Gates
- Summary of this Lecture

Boolean Algebra (逻辑代数)

- **Boolean algebra**, a deductive mathematical system developed by George Boole in 1854, deals with the rules by which logical operations are carried out.

- Boolean algebra is an algebraic structure defined by
  - a set of elements S: binary inputs;
  - a set of binary operators: rules;
  - and a number of unproved postulates.

Postulates (公理)

1. `Closure` (闭包): A set S is closed with respect to a binary operator if, for every pair of elements of S, the binary operator specifies a rule for obtaining a unique element of S.
2. `Associative law` (结合律): $A+(B+C)=(A+B)+C$ and $A\cdot(B\cdot C)=(A\cdot B)\cdot C$.
3. `Commutative law` (交换率): $A+B=B+A$ and $A\cdot B=B\cdot A$.
4. `Identity element` (单比特): A set S is to have an identity element with respect to a binary operation $*$ on S, if there exists an element $E\in S$ with the property $E*A=A*E=A$. Examples:
   - Element 0 is an identity element of + as $A+0=0+A=A$.
   - Element 1 is an identity element of · as $A\cdot 1=1\cdot A=A$.
5. `Distributive law` (分配律): $A\cdot(B+C)=A\cdot B+A\cdot C$ and $A+B\cdot C=(A+B)\cdot(A+C)$.
6. For every element $\in S$, there exists and element $x'\in S$ (called the `complement` of x) such that $x+x'=1$ and $x\cdot x'=0$.

Two-valued Boolean Algebra

- A **two-valued Boolean algebra** is defined on
  - a set of two elements: $\{0, 1\}$;
  - two binary operators $+$ and $\cdot$, and a complement operator $'$.
- It satisfied the aforementioned postulates.

AND

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

OR

x	y	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT

x	x'
0	1
1	0

In this course, we only deal with two-valued Boolean algebra. From now on, we use "Boolean algebra" to refer to "two-valued Boolean algebra" for short.

Properties of Boolean Algebra

Operator Precedence

- Operator precedence for evaluating Boolean expression:
  1. Parentheses $()$,
  2. NOT $'$;
  3. AND $\cdot$;
  4. OR $+$.

Properties of Boolean Algebra

Duality Property (对偶性)

- Every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged.
  - Change $+$ to $\cdot$ and vice versa.
  - Change 0 to 1 and vice versa.
- The **dual** of a Boolean expression is the expression one obtains by interchanging $+$ and $\cdot$ and interchanging 0s and 1s.
- Examples:
  - $A + A' = 1 \rightarrow A \cdot A' = 0$.
  - $A + B = B + A \rightarrow A \cdot B = B \cdot A$.
  - $A \cdot (B + C) = A \cdot B + A \cdot C \rightarrow A + B \cdot C = (A + B) \cdot (A + C)$.
  - $(A + B)' = A' \cdot B' \rightarrow (A \cdot B)' = A' + B'$.

Properties of Boolean Algebra

Basic Postulates and Theorems

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

Figure: Screenshot of Table 2.1 in [1].

Outline of This Lecture

Boolean Algebra

Boolean Function

Canonical Forms, Minterms, Maxterms

Other Logic Operations

Digital Logic Gates

Summary of this Lecture

Boolean Function

Algebraic Expression (代数表达式)

- A Boolean function is an expression formed with **binary variables**, the two **binary operators** `AND` and `OR`, one **unary operator** `NOT`, **parentheses** and **equal sign**.

- A Boolean function describes how to determine the binary output given binary variables as inputs and binary operators.

- The value of a function may be `0` or `1`, depending on the values of variables present in the Boolean function or expression.

- Example: $F = A \cdot B' \cdot C$.
  - $F = 1$ when $A = C = 1$ and $B = 0$,
  - otherwise $F = 0$.

- Boolean functions can also be represented by **truth tables**.
  - Tabular form of the values of a Boolean function according to the all possible values of its variables. n number of variables $\rightarrow 2^n$ combinations of 1's and 0'.s
  - Example: $F = A \cdot B + C$.

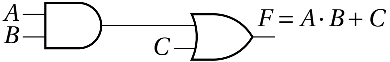
A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- Each Boolean function has one representation in truth table, but a variety of ways in algebraic form.

Boolean Function Simplification

Logic Diagram (逻辑图)

- A Boolean function from an algebraic expression can be realised to a **logic diagram** composed of logic gates.



- Minimisation of the number of literals and the number of terms leads to less complex circuits as well as lower number of gates.
- We first try use postulates and theorems of Boolean algebra to simplify.

$$
\begin{aligned}
F &= A\cdot B + B\cdot C + B'\cdot C \\
&= A\cdot B + C\cdot(B + B') \\
&= A\cdot B + C
\end{aligned}
$$

$$
\begin{aligned}
F &= A'\cdot B'\cdot C + A'\cdot B\cdot C + A\cdot B' \\
&= A'\cdot C\cdot(B' + B) + A\cdot B' \\
&= A'\cdot C + A\cdot B'
\end{aligned}
$$

Exercise

Exercise

1. Use postulates and theorems of Boolean algebra to simplify $F = A \cdot B \cdot C + A \cdot B' \cdot C + A \cdot B \cdot C'$

Exercise

Exercise

1. Use postulates and theorems of Boolean algebra to simplify $F = A{\cdot}B{\cdot}C + A{\cdot}B'{\cdot}C + A{\cdot}B{\cdot}C'$

$$
\begin{aligned}
F &= A{\cdot}B{\cdot}C + A{\cdot}B'{\cdot}C + A{\cdot}B{\cdot}C' \\
&= A{\cdot}B{\cdot}C + A{\cdot}B'{\cdot}C + A{\cdot}B{\cdot}C + A{\cdot}B{\cdot}C' \\
&= A{\cdot}C{\cdot}(B' + B) + A{\cdot}B{\cdot}(C + C') \\
&= A{\cdot}C + A{\cdot}B \\
&= A{\cdot}(C + B)
\end{aligned}
$$

Before simplifying: 3 terms, 9 literals
After simplifying: 2 terms, 3 literals

Algebraic Manipulation

- Reduce the total number of terms and literals.
- Usually not possible by hand for complex functions, use computer minimisation program.
- More advanced techniques in the next lectures.

Boolean Function Complement

- Complement a Boolean function from F to F'.
  - Change `0`'s to `1`'s and vice versa in the truth table.
  - Use DeMorgan's theorem for multiple variables.
    Recap: $(x+y)' = x' \cdot y'$, $(x \cdot y)' = x' + y'$

- Example: $F = x'yz' + x'y'z$.

Complement:

$$\begin{aligned} F' &= (x'yz' + x'y'z)' \\ &= (x'yz')'(x'y'z)' \\ &= (x+y'+z)(x+y+z') \end{aligned}$$

Dual:

$$F^* = (x'+y+z')(x'+y'+z)$$

Outline of This Lecture

Canonical Forms

- Logical functions are generally expressed in terms of different combinations of logical variables with their true forms as well as the complement forms: x and x'.

- An arbitrary logic function can be expressed in the following forms, called **canonical forms**:
  - **Sum of products** (SOP), and
  - **Product of sums** (POS).

- What are the products and sums?

Canonical Forms

- The logical product of several variables on which a function depends is considered to be a **product term**.
  - Called **minterms** when all variables are involved: For x and y, xy, $x'y$, xy', and $x'y'$ are all the minterms.
- The logical sum of several variables on which a function depends is considered to be a **sum term**.
  - Called **maxterms** when all variables are involved: For x and y, $x+y$, $x'+y$, $x+y'$, and $x'+y'$ are all the maxterms.
- `SOP`: The logical sum of two or more logical product terms is referred to as a **sum of products expression (SOP)**.
- `POS`: The logical product of two or more logical sum terms is referred to as a **product of sums expression (POS)**.

Minterms

- In the minterm, a variable will possess the value 1 if it is in true or uncomplemented form, whereas, it contains the value 0 if it is in complemented form.

A	B	C	Minterm
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

- It possesses the value of 1 for only one combination of n input variables. The rest of the $2^n - 1$ combinations have the logic value of 0.

Sum of Minterms

- **Canonical SOP** expression, or **sum of minterms**:
  A Boolean function expressed as the logical sum of all the minterms from the rows of a truth table with value 1.
- A compact form by listing the corresponding decimal-equivalent codes of the minterms:
  - Example: $F = AB + C = A'B'C + A'BC + AB'C + ABC' + ABC = \sum(1,3,5,6,7)$.

Decimal	A	B	C	F	Minterms
0	0	0	0	0	$A'B'C'$
1	0	0	1	1	$A'B'C$
2	0	1	0	0	$A'BC'$
3	0	1	1	1	$A'BC$
4	1	0	0	0	$AB'C'$
5	1	0	1	1	$AB'C$
6	1	1	0	1	ABC'
7	1	1	1	1	ABC

Procedure of Determining SOP

The canonical sum of products form of a logic function can be obtained by using the following procedure.

1. Check each term in the given logic function.
   - If it is a minterm,
     continue to examine the next term in the same manner.
   - Otherwise,
     - examine for the variables that are missing in each product which is not a minterm.
     - if the missing variable in the minterm is X, multiply that minterm with $(X + X')$.
       Example: $A + B \rightarrow A \cdot (B + B') + (A + A') \cdot B$
2. Combine all the products and discard the redundant terms.

Procedure of Determining SOP

Example

$$
\begin{aligned}
F(A,B,C,D) &= AB + ACD \\
&= AB(C + C')(D + D') + ACD(B + B') \\
&= (ABC + ABC')(D + D') + ABCD + AB'CD \\
&= ABCD + ABCD' + ABC'D + ABC'D' + ABCD + AB'CD \\
&= ABCD + ABCD' + ABC'D + ABC'D' + AB'CD
\end{aligned}
$$

Recap:

1. Check each term in the given logic function.
   - If it is a minterm,
     continue to examine the next term in the same manner.
   - Otherwise,
     - examine for the variables that are missing in each product which is not a minterm.
     - if the missing variable in the minterm is X, multiply that minterm with $(X + X')$.
       Example: $A + B \rightarrow A \cdot (B + B') + (A + A') \cdot B$
2. Combine all the products and discard the redundant terms.

Maxterms

- In the maxterm, a variable will possess the value 0, if it is in true or uncomplemented form, whereas, it contains the value 1, if it is in complemented form.

A	B	C	Maxterm
0	0	0	$A+B+C$
0	0	1	$A+B+C'$
0	1	0	$A+B'+C$
0	1	1	$A+B'+C'$
1	0	0	$A'+B+C$
1	0	1	$A'+B+C'$
1	1	0	$A'+B'+C$
1	1	1	$A'+B'+C'$

- It possesses the value of 0 for only one combination of n input variables. The rest of the $2^n - 1$ combinations have the logic value of 1.

Product of Maxterms

- **Canonical POS** expression, or **product of maxterms**:
  A Boolean function expressed as the logical product of all the maxterms from the rows of a truth table with value 0.

- A compact form by listing the corresponding decimal-equivalent codes of the maxterms:
  - Example: $F = (A+B+C)(A+B'+C)(A'+B+C') = \prod(0,2,5)$.

Decimal	A	B	C	Maxterm
0	0	0	0	$A+B+C$
1	0	0	1	$A+B+C'$
2	0	1	0	$A+B'+C$
3	0	1	1	$A+B'+C'$
4	1	0	0	$A'+B+C$
5	1	0	1	$A'+B+C'$
6	1	1	0	$A'+B'+C$
7	1	1	1	$A'+B'+C'$

- Example: $F(A,B,C) = A + B'C$.

$$\begin{aligned}
F(A,B,C) &= A + B'C \\
&= AA + B'C \\
&= (A+B')(A+C) \text{ using the distributive property: } X+YZ=(X+Y)(X+Z) \\
&= (A+B'+CC')(A+C+BB') \\
&= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C) \text{ using the distributive property: } X+YZ=(X+Y)(X+Z) \\
&= (A+B'+C)(A+B'+C')(A+B+C)
\end{aligned}$$

Derive from A Truth Table

Decimal	A	B	C	F	Minterm	Maxterm
0	0	0	0	0		$A+B+C$
1	0	0	1	0		$A+B+C'$
2	0	1	0	1	$A'BC'$	
3	0	1	1	0		$A+B'+C'$
4	1	0	0	1	$AB'C'$	
5	1	0	1	1	$AB'C$	
6	1	1	0	1	ABC'	
7	1	1	1	0		$A'+B'+C'$

- The final canonical SOP for the output F is derived by summing or performing an OR operation of the four product terms as shown below:
  - $F = A'BC' + AB'C' + AB'C + ABC' = \sum(2,4,5,6)$.
- The final canonical POS for the output F is derived by summing or performing an AND operation of the four sum terms as shown below:
  - $F = (A+B+C)(A+B+C')(A+B'+C')(A'+B'+C') = \prod(0,1,3,7)$.

Conversion between Minterms and Maxterms

- Minterms are the complement of corresponding maxterms: $m_i = M_i'$.
- Example: $A' + B' + C' = (ABC)'$.

$$
\begin{aligned}
F(A,B,C) &= \sum(2,4,5,6) = m_2 + m_4 + m_5 + m_6 \\
&= A'BC' + AB'C' + AB'C + ABC' \\
F'(A,B,C) &= \sum(0,1,3,7) = m_0 + m_1 + m_3 + m_7 \\
F(A,B,C) &= \left(F'(A,B,C)\right)' = (m_0 + m_1 + m_3 + m_7)' \\
&= m_0' m_1' m_3' m_7' \\
&= M_0 M_1 M_3 M_7 \\
&= \prod(0,1,3,7) \\
&= (A+B+C)(A+B+C')(A+B'+C')(A'+B'+C').
\end{aligned}
$$

Outline of This Lecture

Boolean Algebra

Boolean Function

Canonical Forms, Minterms, Maxterms

Other Logic Operations

Digital Logic Gates

Summary of this Lecture

Other Logic Operations

- When the three operators AND, OR, and NOT are applied on two variables x and y, they form 16 Boolean functions:

Boolean Expressions for the 16 Functions of Two Variables

Boolean Functions	Operator Symbol	Name	Comments
$F_0 = 0$		Null	Binary constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_3 = x$		Transfer	x
$F_4 = x'y$	y/x	Inhibition	y, but not x
$F_5 = y$		Transfer	y
$F_6 = xy' + x'y$	$x \oplus y$	Exclusive-OR	x or y, but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x + y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$(x \oplus y)'$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y, then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x, then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary constant 1

Figure: Screenshot of Table 2.8 in [1].

Outline of This Lecture

Boolean Algebra

Boolean Function

Canonical Forms, Minterms, Maxterms

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Digital Logic Gates

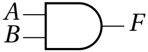
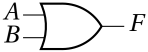
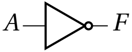
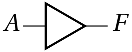
Summary of this Lecture

Construct Logic Gates

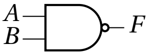
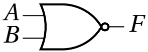
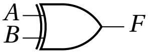
- As Boolean functions are expressed in terms of `AND`, `OR`, and `NOT` operations, it is easier to implement the Boolean functions with these basic types of gates.
  - It is possible to construct other types of logic gates.

- The following factors are to be considered for construction of other types of gates.
  - The `feasibility` and economy of producing the gate with physical parameters.
  - The possibility of `extending` to more than two inputs.
  - The basic properties of the binary operator such as `commutability` and `associability`.
  - The ability of the gate to `implement Boolean functions` alone or in conjunction with other gates.

Digital Logic Gates (1/2)

Name	Graphic symbol	Algebraic function	A	B	F
AND		$F = AB$	0	0	0
			0	1	0
			1	0	0
			1	1	1
OR		$F = A + B$	0	0	0
			0	1	1
			1	0	1
			1	1	1
NOT		$F = A'$	0	-	1
			1	-	0
Buffer		$F = A$	0	-	0
			1	-	1

Digital Logic Gates (2/2)

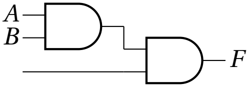
Name	Graphic symbol	Algebraic function	A	B	F
NAND		$F = (AB)'$	0	0	1
			0	1	1
			1	0	1
			1	1	0
NOR		$F = (A + B)'$	0	0	1
			0	1	0
			1	0	0
			1	1	0
XOR		$F = AB' + A'B$ $= A \oplus B$	0	0	0
			0	1	1
			1	0	1
			1	1	0

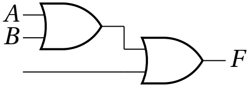
Multiple Input Logic Gates

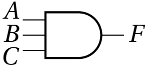
AND and OR

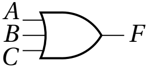
- A gate can be extended to have multiple inputs if its binary operation is commutative and associative.

- AND and OR gates are both commutative and associative.
  - $F = ABC = (AB)C$.
  - $F = A + B + C = (A + B) + C$.







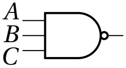


Multiple Input Logic Gates

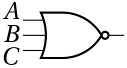
NAND and NOR

- The NAND and NOR functions are the complements of AND and OR functions respectively.
  - They are commutative, but not associative.
  - $((AB)'C)' \neq (A(BC)')'$: does not follow associativity.
  - $((A+B)'+C)' \neq (A+(B+C)')'$: does not follow associativity.

- We modify the definition of multi-input NAND and NOR:



$F = (ABC)' = A' + B' + C'$

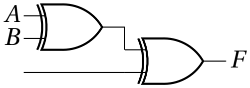


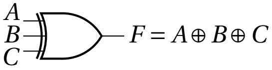
$F = (A+B+C)' = A'B'C'$

Multiple Input Logic Gates

XOR

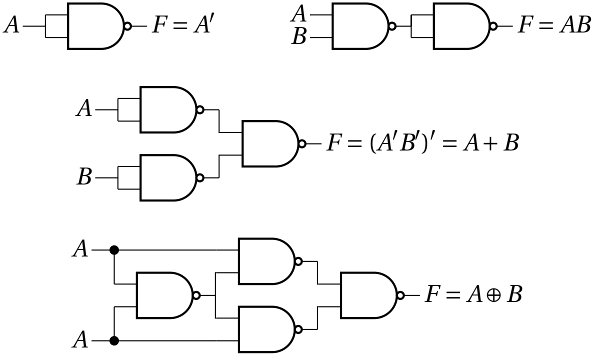
- The XOR gates and equivalence gates both possess commutative and associative properties.
  - Gate output is low when even numbers of 1's are applied to the inputs, and when the number of 1's is odd the output is logic 0.
  - Multiple-input exclusive-OR and equivalence gates are uncommon in practice.





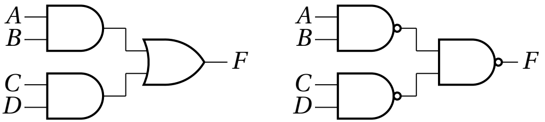
Universal Gates (1/2)

- NAND gates and NOR gates are called **universal gates** or **universal building blocks**.
  - Any type of gates or logic functions can be implemented by these gates.



Universal Gates (2/2)

- Universal gates are easier to fabricate with electronic components.
- Also reduce the number of varieties of gates.
- Example: $F = AB + CD$ requires
  - two AND and one OR gates,
  - or three NAND gates: $F = AB + CD = ((AB + CD)')' = ((AB)'(CD)')'$



Outline of This Lecture

Boolean Algebra

Boolean Function

Canonical Forms, Minterms, Maxterms

Other Logic Operations

Digital Logic Gates

Summary of this Lecture

Summary

- Boolean algebra help reduce the complexity and size of digital circuit (reduce the total number of terms and literals).
  - Postulates of Boolean algebra are key to it.
- Boolean functions can be expressed by algebraic expressions, true tables or logic diagrams. (We will see other representations in the future.)
- An arbitrary logic function can be expressed in canonical forms, thus sum of products (SOP) and product of sums (POS), determined by minterms or maxterms.
- It is possible to construct other types of logic gates (and of multiple inouts) with `AND`, `OR`, and `NOT` operations.

Next week: Gate-Level Minimisation (逻辑化简) using Karnaugh map.

Essential Reading

- Essential reading for this lecture: pages 38-68 of the textbook.
- Essential reading for next lecture: pages 73-118 of the textbook.

[1] M. M. Mano and M. Ciletti, *Digital design: with an introduction to the Verilog HDL*. Pearson, 2013