

- * Algorithm \Rightarrow finite set of steps to solve a problem is called Algorithms.
- * Analysis \Rightarrow It is a process of comparing two Algo.
- * Priority \rightarrow execution \neq part of Analysis. (approx. value)
- * Posterior \rightarrow execution \neq part of Analysis. (give exact value.)

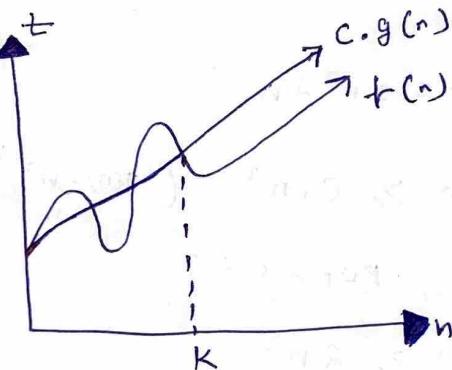
* Asymptotic Notation

\rightarrow It is the mathematical way to represent a time complexity.

① Big-Oh (O)

\rightarrow worst case

\rightarrow upper bound (at most)
least



$$f(n) = O(g(n))$$

$$f(n) \leq c \cdot g(n)$$

$$\left\{ \begin{array}{l} c > 0 \\ n \geq k \\ k \geq 0 \end{array} \right\}$$

$$\text{put } c = 3$$

~~$2n^2 + n \leq 3n^2$~~

$$n \leq 3n^2 - 2n^2$$

$$n \leq n^2$$

(divide by $\frac{1}{n^2}$)

$$1 \leq n^2, n \geq 1$$

For ex.

$$f(n) = 2n^2 + n$$

$$f(n) = O(n^2)$$

$$2n^2 + n \leq C \cdot g(n^2)$$

For all the value of n

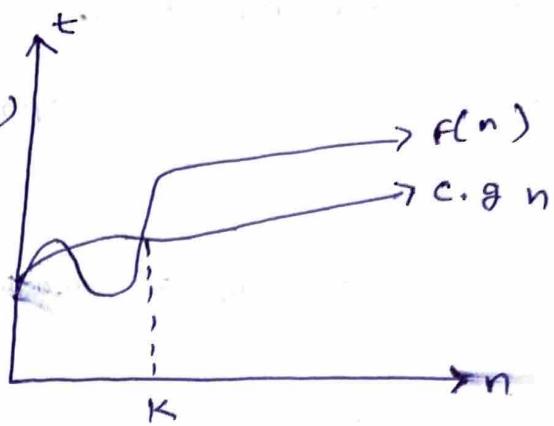
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2. Big - Omega (Ω)

→ Best case

→ Lower bound (at least)



$$f(n) = \Omega g(n)$$

$$f(n) \geq c \cdot g(n)$$

for ex.

$$f(n) = 2n^2 + n$$

$$2n^2 + n \geq c \cdot n^2 \quad (g(n) = n^2)$$

$$\text{Put } c = 2$$

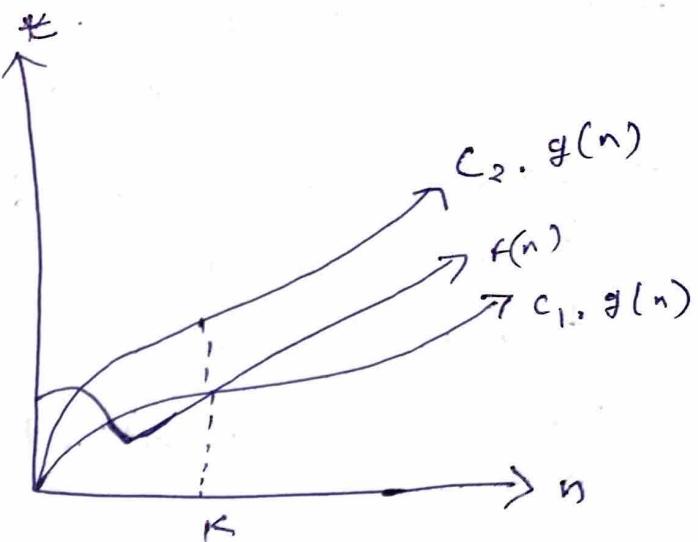
$$2n^2 + n \geq 2n^2$$

$$n \geq 2n^2 - 2n^2$$

$$n \geq 0$$

③ Theta (Θ)

Average case
exact time



$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

$$2n^2 \leq 2n^2 + n \leq 3n^2$$

* Dynamic Programming

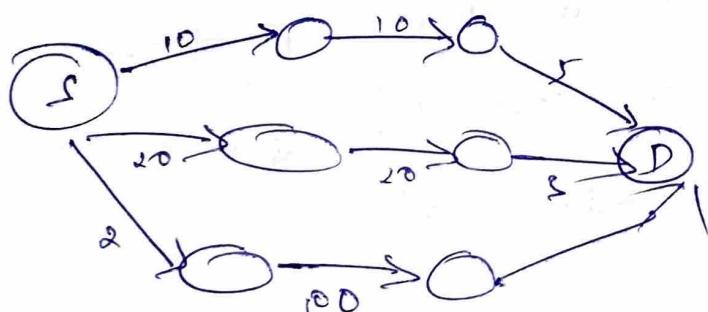
→ It is used to solve optimization problems

→ It divide a problem into series of overlapping subproblems.

Two features : ① optimal substructure

② overlapping subproblems

→ It always give the optimal answer



g Dynamic P

init check → init
~~path~~
solution open

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and prove with

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in Dynamic ~~arr~~ path check & if then ΘT
optimal solution will go.

→ Dynamic programming \rightarrow sub problems
to solution \rightarrow use table & store it &
it not in form same subproblem state not
start solution direct after it without solving.

★ Greedy Technique

- Follows local optimal choice of each stage with instead of finding global optimum.
- Min Cost
→ Max profit
→ Min Risk

→ Feasible solution
→ Optimal solution

Application:

- ① Knapsack problem
- ② Job sequencing
- ③ Minimum Spanning tree
- ④ Optimal Merge pattern
- ⑤ Huffman coding
- ⑥ Dijkstra algo.

→ Dynamic and greedy both work on optimization techniques. (min or max)

①

Greedy Method

① There is no principle of optimality.

② It is a deterministic approach to get a solution for a problem.

③ Greedy method never looks back or revisits previous choices.

④ More efficient compare to D.P.

⑤ Knapsack problem

Dynamic Prog.

① There is a principle of Optimality.

② It is not a deterministic approach to get a solution of a problem here we consider all the possible solution and select the best solution or optimal solution.

③ Dynamic prog look back on revisiting previous choices.

④ less efficient compare to G.M.

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Cook's Theorem

→ It states that the Satisfiability problem (SAT) is NP-complete.

What is (SAT)?

Cook's theorem → SAT is NP-complete.

SAT - A propositional logic formula (ϕ) is called a satisfiable if there is some assignment to its variable that makes it evaluate to true.

→ $P \wedge Q$ is satisfiable (if $P=1$ $Q=1$).

* Abstract decision problem = It can be viewed as a function that maps the instance set (I) to the solution set $\{0, 1\}$.

* P class problem

→ Problem which can be solved on a Polynomial time is known as P-class problem. (And also verified in ~~P~~ polynomial time.)

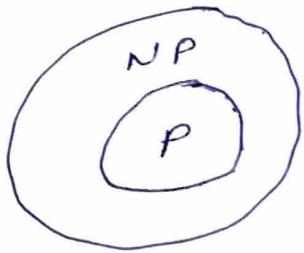
(ex.) All sorting and searching Algo.

* NP class problem

→ Problems which cannot be solved on a Polynomial time but is verified in polynomial time is known as Non-deterministic polynomial or NP-class problem.

(ex.) Sudoku, Prime factor, Scheduling, Travelling Salesman

(In this solving a problem is difficult but verifying is easy)

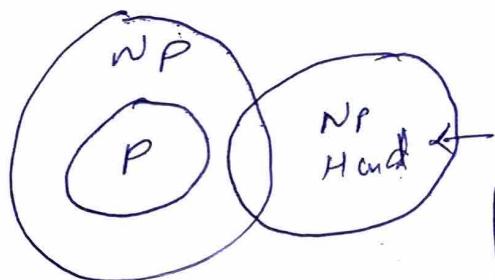


Those problems which come under P
are called Tractable problems

Those problems which come under
NP are called intractable problems

* NP Hard problem

→ A problem is NP Hard if every problem
in NP can be polynomial reduced to it,



intersection of NP Hard
and NP is NP
complete

* NP Complete problem

→ The problem is NP complete if it is
in NP and it is in NP Hard



NP complete problems are decision Problem

NP Hard problems are optimization problems.

All the NP complete problems are NP Hard,
But All the NP Hard are not NP complete



Linear Search

- ① In linear search technique the array of element is sequentially traversed to locate the given item.
- ② In linear search elements can store Randomly in Array list.
- ③ It is slower searching technique.

Binary Search

- ① In Binary search technique the given item is first compared with mid item of array, If given item is not equal to mid item than ~~searching~~ is performed once half of array. [above or below the mid item].
- ② In Binary search elements should be stored in Sorted array.
- ③ It is faster searching technique compared to linear search.

- ① Worst case time complexity of linear search is $O(n)$
- ② Best case time complexity of linear search is $O(1)$
- ③ It cannot be implemented on Array and linked list
- ④ Iterative in Nature
- ⑤ Recursive in nature and based on divide and conquer technique.
- ⑥ Tricky algorithm and elements should sorted in order
- ⑦ Less coding to write
- ⑧ More coding to write.
- ⑨ Worst case time complexity of binary search is $O(\log n)$.
- ⑩ Best case time complexity of Binary Search is $O(1)$

★ Topological sort

we get graph of the input for ex.

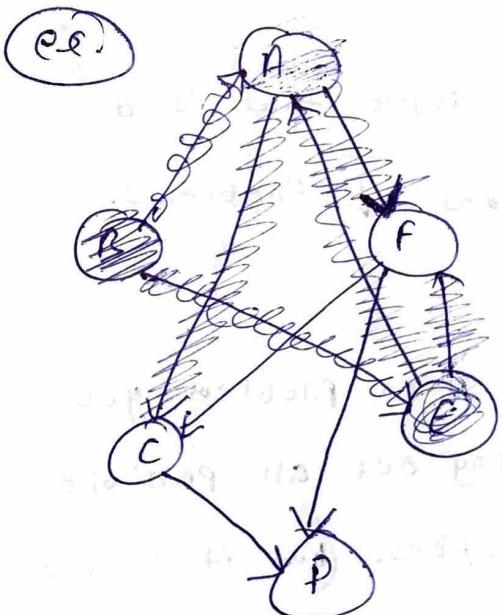
$\textcircled{v} \rightarrow \textcircled{w}$ And we get the latest order

write node in \textcircled{v} after \textcircled{w} in the next row

→ Applicable on DAG (Directed Acyclic graph)

→ Linear Running Time Complexity

[For ex. 2nd year st node will be first year complete
and "ed"]



Firstly

we get arrow with st node

of 3rd row like \textcircled{B} is st
Arrow get sorted by

like \textcircled{B}

like \textcircled{B}

we get Arrow or, st of
exit then same for
others node

B E A F C P

① State Space tree = It is a tree that Represent all of the possible states

of the problem from the root as an initial state to the leaf as a terminal state.

② Problem state = Each Node in the tree defines a Problem state.

③ Answer state = Answer state are those solution state (s) for which the path from the Root to (s)

' Defines a tuple that is a Member of the set of solutions of Problems.

④ Brute force Approach = for any given problem you should try out all possible solution and pickup desired solutions. (but it is not for Optimization problem)

⑤ Abstract problem = A function that maps the instance set I to the solution set $\emptyset \{0,1\}$

⑥ Search space = It is a set of all possible valid moves.

⑦ ~~Search~~

⑧ Abstract Problem $\Rightarrow Q$ is a binary relation \in set(I) of a problem instance and a set(s) of problem solution.

⑨ Dead node \Rightarrow If is not to be expanded.

⑩ Dynamic tree \Rightarrow It is a set of dynamically changing rooted tree.

⑪ Solution stat \Rightarrow present state leading to a tuple.

Backtracking

Meaning = Go back the same way you come
First $\frac{2\pi}{n}$ then $\frac{\pi}{n}$ next $\frac{2\pi}{n}$ etc until n times

- In Backtracking we search depth first for solution.
 - In Backtracking technique we backtrack to the last valid path as soon as we hit a deadend.
 - Backtracking Reduce the search space since we no longer have to follow down any path we know as invalid.
 - Backtracking is an algorithmic technique that consider searching every possible combination in order to solve an optimization problem.
 - Backtracking is an approach to solving constraint satisfaction problem without trying all possibilities.
 - Backtracking technique is used for those problems which have many candidate solution but the majority of which do not satisfy the given constraints.
 - PFS used.
 - what if multiple solution exist & we want Backtracking to solve them &

- Backtracking is the depth first search with some bounding function
- In Backtracking method the solution can be expressed in a form of tuples.
 $(x_1, x_2, x_3, \dots, x_n)$



Branch and Bound

- The Branch and Bound Algo. is similar to Backtracking but is used for ~~multi~~ optimization problem.
- Branch bound is usually work on minimization.
- It performs a graph traversal on the state space tree by using BFS
- There are three B-Bound Techniques
 - FIFO Branch and Bound Technique (Queue)
 - LIFO Branch and Bound technique (Stack)
 - Least Cost Branch and Bound.

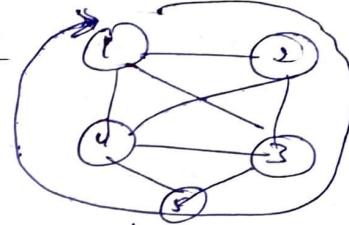
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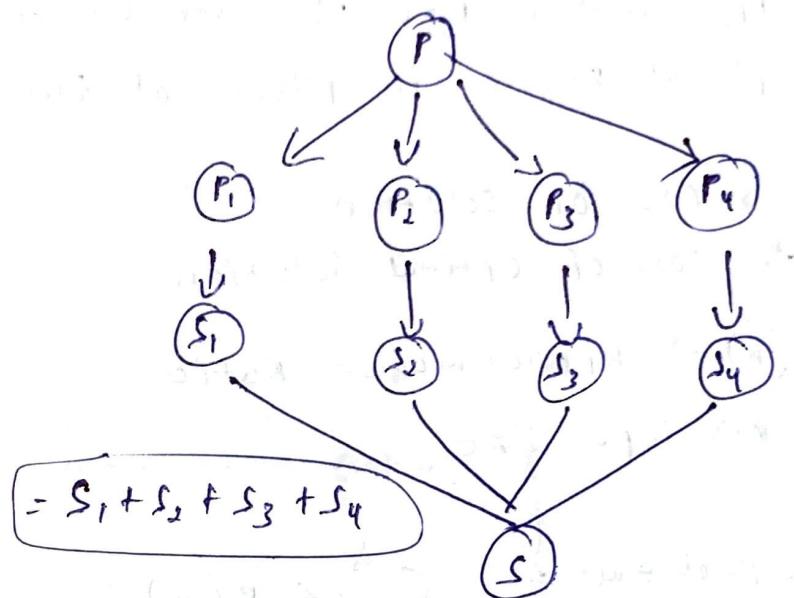
Hamiltonian Cycle



- It is the ~~part~~ of backtracking
- (1) Node \rightarrow start with \Rightarrow start end node
cover \rightarrow direct (1) Node \rightarrow end direct \Rightarrow)
- Hamiltonian ~~is~~ a cycle is a path in a given graph that visit each vertex exactly once and there is edge path b/w first and last vertex.
- There may be any number of Hamiltonian cycle exist in a graph

* Divide and Conquer

→ In this Approach we divide a big problem into subproblems then find a solutions of sub problems and then combine sub problems' solution.



- It is used to solve decision problem
- follows top-down Approach
- subproblems are interdependent
- solution of subproblems complete Recursively more than one.
- recursive in nature

★ Approximation Algo

- This Algorithm is used to deal NP-completeness for optimization problem.
- The Goal of Approximation Algo is come close as possible to optimal solution in polynomial time.

$C \rightarrow$ Cost of solution

$C^* \rightarrow$ Cost of optimal solution

$P(n) \rightarrow$ Approximation Ratio

$n \rightarrow$ input size

Maximization problem = $\frac{C^*}{C} \leq P(n)$

Minimization problem = $\frac{C}{C^*} \leq P(n)$

$P(n) \leq 1$

★ Vertex Cover Problem

→ Vertex Cover Problem is the example of (App. Algo)

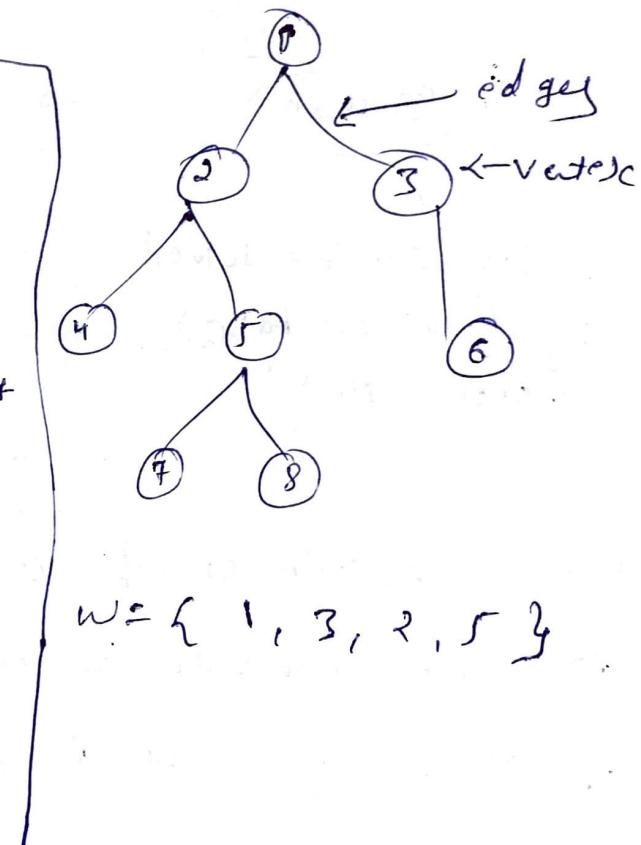
→ ~~Find~~ ^(अमीर जै नहीं) ~~Best~~ Vertex find that $\frac{v}{e}$ in what edges in cover are in ~~in~~ ~~of~~ graph ~~in~~

→ A vertex cover of a graph is a subset of vertices which cover every edges. -

→ Vertex cover problem is the minimum size vertex cover.

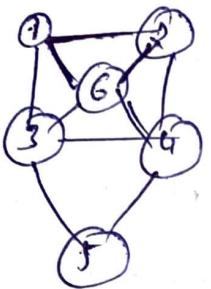
APPROX-VERTEX-COVER (G_1)

1. $C \leftarrow \emptyset$
2. $E' \leftarrow E[G_1]$
3. while $E' \neq \emptyset$
4. do let (u, v) an arbitrary edge in E'
5. $C \leftarrow C \cup \{u, v\}$
Remove from E' every edge incident on either u or v
6. return C



★ Clique Problem

→ Complete subgraph of a graph is clique.



1-2-3 is a complete subgraph
It is a Clique of size 3

→ A clique problem has two types



If ~~is~~ can be solved
by (True or False)
(Yes or No)

It can be solved
by maximum clique.
size of graph and
minimum.

→ NP completeness of clique problem:

- ① Clique problem is in NP
- ② Clique problem is NP-Hard

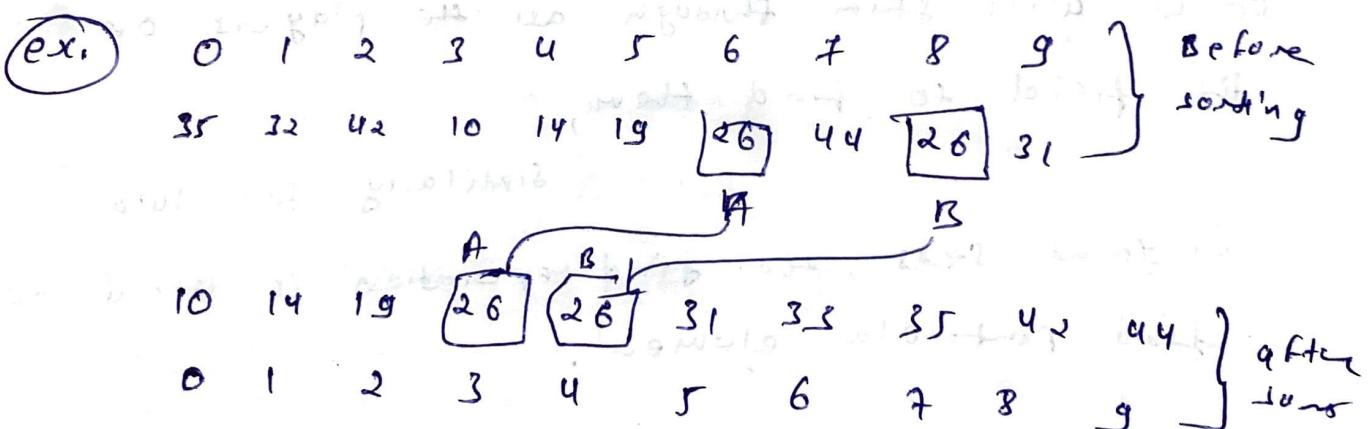
* Stable Sorting

Consider an integer array that contains some integers.

We are sorting the given elements of array by using a sorting algorithm,

If the sorting algorithm does not change the sequence of similar elements in which they appear,

then that type of sorting is called stable sorting.



Knapsack Problem

→ Greedy about both
weight
Profit

* Union Find

⇒ T.C = $O(\log N)$, where (n) is the no. of nodes

S.C = $O(n)$, where (n) is the no. of nodes.

* Find: Suppose player ① in the orange team
is our friend, and we want to
find him on the field during the match,

so we will skim through all the players on
the field to find them.

Similarly for two
disjoint sets, the find operation is used to
find particular element.

* Union: Let assume that the purple and orange
teams are the best polo teams from
our state.

Now for a national competition
we need to send eight players to represent the
state,

since we have purple and orange teams

are best

' we will combine them and send them
as a team

This describe the union operation.

* In Degree out degree

Time complexity $\in O(V+E)$

$V \rightarrow$ vertices

$E \rightarrow$ edges

* Transitive closure (warshall algo used)

T.C = $O(M)$

* TC of quick sort

Best case $O(n \log n)$

Worst case $\Theta(n^2)$

Average case $O(n \log n)$

* Kruskal Algo

T.C = $O(E \lg V)$

SC

* Selection sort T.C

worst and best as same $O(N^2) \approx O(N^2) \approx O(N^2)$

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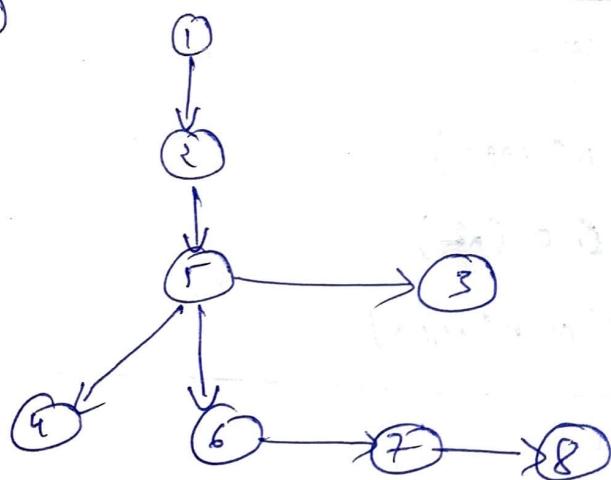
Amark

① Forward edge \Rightarrow If it is a non-tree edge (u,v) such that there exist a path b/w (u) to (v) in the tree.

② Cross edge \Rightarrow It is a non-tree edge (u,v) which connect two nodes such that neither of these is ancestor of the other in the tree.

③ Back edge \Rightarrow It is a non-tree edge (u,v) such that (v) is ancestor of node (u) in the tree.

e.g.



Forward edge $\Rightarrow (1 \rightarrow 3), (2 \rightarrow 4)$

Cross edge $\Rightarrow (4 \rightarrow 6), (4 \rightarrow 8), (3 \rightarrow 7), (3 \rightarrow 8)$

Back edge $\Rightarrow (4 \rightarrow 5)$