Practical - 5

Topic ÷ Integration

1] Solve the following integration

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

2) $\int (4e^{3x}+1)\,dx$

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

5] $\int E^{t} \sin(2t+4)\,dt$

6] $\int \sqrt{x}\,(x^2-1)\,dx$

7] $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

8] $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\,dx$

9] $\int e^{\cos^2 x} \sin 2x\,dx$

10] $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

→1] $\int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$

Put $x+1=t$

$dx = \frac{1}{t}\,dt$ where $t=1$ $\quad t=x+1$

$\int \frac{1}{\sqrt{t^2-4}}\,dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

→2] $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= 4\frac{e^{3x}}{3} + x$ $\quad \because \int e^{ax}\,dx = \frac{1}{a} \times e^{ax}$

$= 4\frac{e^{3x}}{3} + x + C$

→3] $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3+10x\sqrt{x}}{3} + 3\cos x + C$

→4] $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\, dx + \int 3x^{1/2}\, dx + \int \frac{4}{x^{1/2}}\, dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

→8] $\int t^7 \times \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, du$

$= \int \frac{t^4 \sin(2t^4)}{8}\, du$

Substitute t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \ \sin(u/2)}{8}\, du$

$= \int \frac{u \times \sin(u)}{2} \Big| 8 \quad du$

$= \int \frac{u \times \sin(u)}{16}\, du$

$= \frac{1}{16} \int u \times \sin(u)\, du$

$\int u\, dv = uv - \int v\, du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\, du \quad v = -\cos(u)$

$= \frac{1}{16} (u \times (-\cos(u)) - \int -\cos(u)\, du$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\, du$

$= \frac{1}{16} \times (u \times (-\cos u) + \sin u)$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4) \quad \because u = 2t^4$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

→6] $\int \sqrt{x}(x^2-1)\, dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

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$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$$

8] $\int \frac{\cos x}{\sqrt[3]{\sin(x)}}\,dx$

$$= \int \frac{\cos x}{\sin x^{1/3}}\,dx$$

Let $t = \sin(x)$

$dt = \cos(x)$

$$= \int \frac{\cos(x)}{\sin(x)^{1/3}} \times \frac{1}{\cos(x)}\,dt$$

$$= \frac{1}{\sin x^{1/3}}\,dt$$

$$= \frac{1}{t^{1/3}}\,dt$$

$$I = \int \frac{1}{t^{1/3}}\,dt = \frac{-1}{(1/3 - 1)\,t^{1/3 - 1}}$$

$$= \frac{-1}{-2/3\; t^{1/3-1}} = \frac{1}{2/3\; t^{-2/3}} = \frac{t^{2/3}}{2/3} = 3t^{2/3}$$

$$= 3\sqrt[3]{t}$$

$\because t = \sin(x)$

$$= 3\sqrt[3]{\sin x} + C$$

$$\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\,dx$$

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Let $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\,dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\,dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\,dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3}\,dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)}\,dt = \int \frac{1}{3t}\,dt$$

$$= \frac{1}{3} \int \frac{1}{t}\,dt$$

$$= \frac{1}{3} \times \ln(t) + C$$

$$= \frac{1}{3} \times \ln(|x^3 - 3x^2 + 1|) + C$$

9] $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$$

Let $\frac{1}{x^2} = t$

$x^2 = t$

$\frac{-2}{x^3} \, dx = dt$

$I = \frac{-1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= \frac{-1}{2} \int \sin t$

$= \frac{-1}{2} (-\cos t) + c$

$= \frac{1}{2} \cos t + c$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$

2] $\int e^{\cos^2 x} \sin 2x \, dx$

→ Let $\cos^2 x = t$

$-2 \cos x \cdot \sin x \, dx = dt$

$-2 \sin 2x \, dx = dt$

$I = \int - \sin 2x \, e^{\cos^2 x} \, dx$

$= - \int e^t \, dt$

$= e^t + C$

Resubstituting $t = \cos^2 x$

$= e^{\cos^2 x} + C$

Practical - 6

Topic :- Application of Integration & Numerical integration

Q1] Find the length of integration curve:

1] $x = t \sin t \quad y = 1 - \cos t \quad [0, 2\pi]$

for t belong to $[0, 2\pi]$

2] $y = \sqrt{4 - x^2} \quad x \in [-2, 2]$

3] $y = x^{3/2}$ in $[0, 4]$

4] $x = 3 \sin t, \; y = 3 \cos t \quad t \in [0, 2\pi]$

5] $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q2] Using Simpson's Rule solve the following

1] $\int_0^2 e^{x^2} \, dx$ with $n = 4$

2] $\int_0^4 x^2 \, dx$ with $n = 4$

3] $\int_0^{\pi/3} \sqrt{\sin x} \, dx$ with $n = 6$

1] $y = \sqrt{4 - x^2}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4 - x^3}}\, dx$$

$$= 2 \int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$$

$$= 2 \left[\sin^{-1}(x/2)\right]^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right]$$

$$= 2\pi$$

1] $y = x^{3/2}$ $x \in [0, 4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$I = \int_{0}^{4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{0}^{4} \sqrt{1 + \frac{9}{4} x}\, dx$$

$$= \frac{1}{2} \int_{0}^{4} \sqrt{4 + 9x}\, dx$$

$$= \frac{1}{2} \left[\frac{[4 + 9x]^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27} \left[(4 + 9x)^{3/2}\right]$$

$$= \frac{-1}{27} \left[(4 + 0)^{3/2} - (4 + 31)^{3/5}\right]$$

$$= \frac{1}{27} (40^{3/2} - 8) \text{ units}$$

2a] $x = 3\sin t$ $y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$$

$$I = \int_{0}^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_{0}^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_{0}^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_{0}^{2\pi} 3\, dt$$

$$= 3 \int_{0}^{2\pi} dt$$

$= 3\,[x]_0^{2\pi}$

$= 3\,[2\pi - 0]$

$I = 6\pi$ units

5] $x = \frac{1}{6}y^3 + \frac{1}{2y}$

$\therefore \frac{dx}{dy} = \frac{y^3}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$I = \int_0^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

$= \int_0^2 \sqrt{1+\frac{(y^4-1)^2}{4y^4}}\,dy$

$= \int_0^2 \sqrt{\frac{(y^4-1)-4\times y^4\times 1}{4y^4}}\,dy$

$= \int_0^2 \sqrt{\frac{(y^4+1)^2}{2(y^2)^2}}\,dy$

$= \int_0^2 \frac{y^4+1}{2y^2}\,dy$

$= \frac{1}{2}\int_0^2 y^2\,dy + \frac{1}{2}\int_1^2 y^{-2}\,dy$

$= \frac{1}{2}\left[\frac{y^3}{3} \quad y^{-1}\right]^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$= \frac{17}{12}$ units

Q2] i] $\int_0^2 e^{x^2}\,dx$ with $n=4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$\int_0^2 e^{x^2}\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$

$= \frac{0.5}{3}\left[(1+54.5982) + 4(1.284+9.4877) + 2(2.7183)\right]$

$= \frac{0.5}{3}\left[55.5982 + 43.0866 + 5.4366\right]$

$= \int_0^2 e^{x^2}\,dx = 17.3535$

2] $\int_0^4 x^2\,dx \qquad n=4$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$

$$= \frac{1}{3}\left[0+16+4(1+9)+2\times 4\right]$$

$$= \frac{1}{3}\left[16+4(10)+8\right]$$

$$= \frac{64}{3}$$

$$= \int_0^4 x^2\,dx = 21.3333$$

→ 3] $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ $\quad n = 6$

$$h = \frac{\frac{\pi}{3} - 0}{6} = \pi/18$$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752	0.930
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x}\,dx = \frac{h}{3}\left[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167+0.936+4(0.4167+0.7071+0.9152)+2(0.5848+0.8017)\right]$$

$$= \frac{\pi}{54}\left[1.3473+4(1.999)+2(1.3865)\right]$$

$$= \frac{\pi}{54}\left[1.3473+7.996+2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x\,dx} = 0.7049$$