Roll No.: 16/1/23

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MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P.

Sessional Examination -II (SET-B) :Odd Semester 2022-23

Course/Branch: B Tech –I Years **Semester: I**

Subject Name :Engg. Mathematics-I **Max. Marks :60**

Subject Code : BAS-103

Sections : (OP4, OP6, OP8, OP10, OP12, OC4, OC6, OC8, OC10) **Time: 120 min**

CO-3 *: On completion of this course, the student will learn to deal with functions of several variables that is essential in optimizing the results of real life problems*

CO-4 *: On completion of this course, the student will be able to apply integral calculus in various field of engineering and have a basic understanding of Beta and Gamma functions and application of Dirichlet's integral*

Section – A (CO - 3) # Attempt both the questions # 30 Marks

Q.1: Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	A balloon in the form of right circular cylinder of radius 1.5m and length 4m is surmounted by hemispherical ends. If the radius is increased by 0.01m find the percentage change in the volume of the balloon.	(BKL:K2 Level)
b)	If $f(x) = x^3 + 6x^2 + 9$ then find the value of $f\left(\frac{11}{10}\right)$ by Taylor's Theorem 17.321 17.26	(BKL:K2 Level)
c)	Find the stationary point of: $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	(BKL:K2 Level)
d)	Calculate $\frac{\partial(u,v)}{\partial(x,y)}$ for , $x = e^u \cos v$ *and* $y = e^u \sin v$.	(BKL:K2 Level)
e)	Are the functions: $u = xy + yz + zx,\ v = x^2 + y^2 + z^2,\ w = x + y + z$ functionally dependent?	(BKL:K2 Level)
f)	If $RI = E$ and possible error in E and I are 20% and 10% respectively, then find the error in R.	(BKL:K2 Level)
g)	State the Maclaurin's Theorem for two variables.	(BKL:K1 Level)

Q.2:Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	Expand $f(x,y) = x^y$ about $(1,1)$ upto second degree terms and hence evaluate $(1.1)^{1.02}$.	(BKL:K3 Level)
b)	Find approximate value of: $f(x,y) = x^2 y^{\frac{1}{10}}$ when $x = 1.99$ *and* $y = 3.01$	(BKL:K3 Level)
c)	Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$, if $x = \sqrt{vw},\ y = \sqrt{uw},\ z = \sqrt{uv}$ *and* $u = r\sin\theta\cos\phi,\ v = r\sin\theta\sin\phi,\quad w = r\cos\theta$.	(BKL:K3 Level)
d)	Find the dimension of rectangular box of maximum capacity whose surface area is given when Box is closed.	(BKL:K3 Level)
e)	Find the minimum distance from point (1, 2,0) to the cone $x^2 + y^2 = z^2$.	(BKL:K3 Level)

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Section – B (CO - 4) # Attempt both the questions # 30 Marks

Q.3: Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	Calculate the volume of the solid bounded by the surface $x = 0, y = 0, z = 0$ and $x + y + z = 1$	(BKL:K2 Level)
b)	Evaluate $\int_0^1 \sqrt{\frac{1-x}{x}}\, dx$	(BKL:K2 Level)
c)	Change the order of integration $\int_0^2 \int_{\frac{x^2}{4}}^{3-x} f(x,y)dydx$	(BKL:K2 Level)
d)	Evaluate $\Gamma(3/4)\Gamma(1/4)$	(BKL:K2 Level)
e)	Find area bounded by $r = 2sin\theta$ and $r = 4\, sin\theta$.	(BKL:K2 Level)
f)	Evaluate $\int_0^\infty \frac{1}{1+x^4}\, dx$	(BKL:K2 Level)
g)	Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dxdy$	(BKL:K2 Level)

Q.4: Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing into polar coordinates. Hence evaluate $\int_0^\infty e^{-x^2} dx$	(BKL:K3 Level)
b)	Evaluate $\iiint x^2yz\, dxdydz$ through out the volume bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	(BKL:K3 Level)
c)	Show that $\iiint \frac{dxdydz}{(x+y+z+1)^3} = \frac{1}{2} log\, 2 - \frac{5}{16}$ the integral being taken throughout the volume bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$	(BKL:K3 Level)
d)	Using transformation $x + y = u, y = uv$, Evaluate $\int_0^1 \int_0^{1-x} e^{y/(x+y)}\, dy\, dx$	(BKL:K3 Level)
e)	Evaluate the following integral by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$	(BKL:K3 Level)

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NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P.

Sessional Examination -I (SET-B) :Odd Semester 2022-23 6/12/22

Course/Branch: B Tech –I Years **Semester: ISubject Name**

:Engg. Mathematics-I **Max. Marks :60**

Subject Code : BAS-103(OP4,OP6,OP8,[redacted],OP10,[redacted],OP12,OC4,OC6,OC8,OC10,[redacted])

Time : 120 min OC2

CO-1 *: On completion of this course, the student will learn the essential tools of matrices, Eigen values and its application in a comprehensive manner.*

CO-2 *: On completion of this course, the student will be able to apply the knowledge of differential calculus in the field of engineering*

Section – A (CO - 1) # Attempt both the questions # 30 Marks

Q.1: Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	Show that the matrix $\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2 = 1$.	(BKL:K2 Level)
b)	The Eigen values of a matrix A are 2,3,1 then find the Eigen values of $A^{-1}+A^2$.	(BKL:K2 Level)
c)	State Rank-Nullity theorem.	(BKL:K1 Level)
d)	Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ in to the normal form and find its rank.	(BKL:K2 Level)
e)	Find the value of 'b' so that the rank of A = $\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{bmatrix}$ is 2.	(BKL:K2 Level)
f)	Let A be a 3×3 matrix with real entries such that det(A) = 6 and the trace of A is 0. If $\lvert A+I\rvert = 0$ where I denotes the 3×3 identity matrix then find the eigen values of A.	(BKL:K2 Level)
g)	If the Eigen values of the matrix A are 1, 1, 1 then find the Eigen values of $A^2+2A+3I$.	(BKL:K2 Level)

Q.2:Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	For what values of λ and μ, the system of linear equations: $x+y+z=6$, $x+2y+5z=10$ and $2x+3y+\lambda z=\mu$, has: (i) a unique solution (ii) no solution and (iii) Infinite solution. Also find the solution for λ = 2 and μ = 8.	(BKL:K3 Level)
b)	Find non-singular matrices P and Q such that PAQ is in Normal form of the matrix and hence find the rank of matrix $A=\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$.	(BKL:K3 Level)
c)	Find the eigen values and eigen vectors of matrix $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$	(BKL:K3 Level)

d)	Compute the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by employing elementary row transformations.	(BKL:K3 Level)
e)	If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find A^{-1} and A^4 using Cayley-Hamilton's theorem.Also show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$.	(BKL:K3 Level)

Section – B (CO - 2) # Attempt both the questions # 30 Marks

Q.3: Attempt any SIX questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	If $I_n = \frac{d^n}{dx^n}(x^n \log x)$ then show that $I_n = n\, I_{n-1} + (n-1)!$	(BKL:K2 Level)
b)	If $x^2 = au + bv, y^2 = au - bv$ Evaluate $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v$.	(BKL:K2 Level)
c)	Find $\frac{du}{dt}$ if $u = x^3 + y^3, x = a\cos t, y = b\sin t$.	(BKL:K2 Level)
d)	Find the n^{th} derivative of $y = x^2 \sin x$.	(BKL:K2 Level)
e)	If $u = \left(x^{\frac{1}{4}} + y^{\frac{1}{4}}\right)\left(x^{\frac{1}{5}} + y^{\frac{1}{5}}\right)$,then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$	(BKL:K2 Level)
f)	If $V = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $6\frac{\partial V}{\partial x} + 4\frac{\partial V}{\partial y} + 3\frac{\partial V}{\partial z} = 0.$	(BKL:K2 Level)
g)	Find n^{th} derivative of $y = \sin 2x \sin 3x$.	(BKL:K2 Level)

Q.4:Attempt any THREE questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	If $y = \left(\frac{1+x}{1-x}\right)^{1/2}$ then prove that $(1-x^2)y_n - [2(n-1)x+1]y_{n-1} - (n-1)(n-2)y_{n-2} = 0$.	(BKL:K3 Level)
b)	If $y = \sin(nx) + \cos(nx)$ prove that $y_r = n^r[1 + (-1)^r \sin 2nx]^{1/2}$, where y_r is the r^{th} differential coefficient of y with respect to x.	(BKL:K3 Level)
c)	If $w = \sqrt{x^2 + y^2 + z^2}$ & $x = u\cos v, y = u\sin v, z = uv$,then prove that $\left[u\frac{\partial w}{\partial u} - v\frac{\partial w}{\partial v}\right] = \frac{u}{\sqrt{1+v^2}}$.	(BKL:K3 Level)
d)	If $u = \operatorname{cosec}^{-1}\left(\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}\right)^{1/2}$, Find $\left(x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial u}{\partial y}\right)$.	(BKL:K3 Level)
e)	If $y = \log\left(x + \sqrt{(x^2 + a^2)}\right)$, Prove that $(a^2 + x^2)y_2 + xy_1 = 0$. Differentiate this differential equation n times and prove that $\lim_{x\to 0}\frac{y_{n+2}}{y_n} = -\frac{n^2}{a^2}$.	(BKL:K3 Level)

$\cos 3x = 4\cos^3 x - 3\cos x$