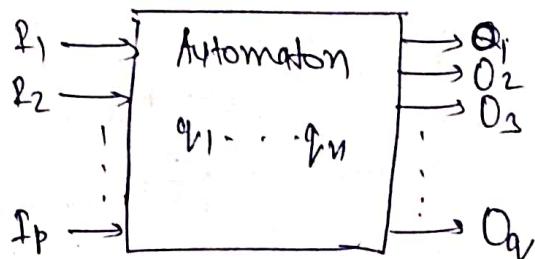


Definition of Automata:

Used to performing some function Without direct participation of man.

Ex:- Automatic machine tools , Automatic Packing machines & Automatic photo printing machine etc.



(i) 2IP :- At each of the discrete instants of time t_1, t_2, \dots, t_m the 2IP values f_1, f_2, \dots, f_p , each of which can take a finite no. of fixed values from the 2IP alphabet Σ .

(ii) OIP :- O_1, O_2, \dots, O_q are the oip of the model, each of which can take a finite no. of fixed values from an oip O

(iii) State :- At any state of time the automaton can be in one of the state q_1, q_2, \dots, q_n

(iv) State Relation :- The next state of the automaton at any ~~not~~ instant of time is determined by the present state and the present 2IP.

(v) OIP relation :- The oip is related to either state only or to both 2IP and the state

Theory of Computation (TOC) :

In Computation, any task that is perform by any Calculator or Computer.

Symbols : $a, b, c, 0, 1, 2, \dots, 9$

$|\Sigma|$ Alphabate : $\{a, b\}$ $\{0, 1\}$ $\{0, 1, \dots, 9\}$
 $\{a, b, c\}$

String : For $\Sigma = \{a, b\}$
1-bit = $\{a, b\} = 2^1 = 2$

2-bit = $\{aa, ab, ba, bb\} = 2^2 = 4$

3-bit = $\{aaa, aab, aba, abb, baa, bab, bba, bbb\} = 2^3 = 8$

No. of string $|\Sigma|^n$, where n size of string

language : An English language is collection of words, but the mlc language is collection of string.

for $\Sigma = \{a, b\}$

L_1 = Set of all string of length 2.
= $\{aa, ab, ba, bb\}$

L_2 = Set of all string of length 3
 $\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

L_3 = Set of all string, where each string start with a
 $\{a, aa, ab, aba, aab, abb, abba\}$

Power of Σ (sigma) :

For $\Sigma = \{a, b\}$

$\Sigma^0 = \boxed{\text{Set of all string over } \Sigma \text{ of length zero, } \{e\}}$

$\Sigma^1 = \text{Set of all string over } \Sigma \text{ of length 1, } \{a, b\}$

$\Sigma^2 = \text{for length 2 } \{aa, ab, ba, bb\} = \Sigma \cdot \Sigma = \{a, b\} \cdot \{a, b\}$

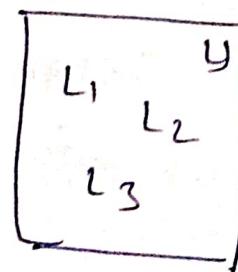
$\Sigma^3 = \text{for length 3}$

$\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

$\Sigma + \Sigma + \Sigma = \dots$

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$

= Universal set



Finite Automata

Finite Automata with output

Moore
Machine

Mealy Machine

Finite Automata without output

DFA

NFA

ε-NFA

Peterministic finite Automata

5-tuples $(Q, \Sigma, \delta, q_0, F)$

$Q \rightarrow$ is a finite non-empty set of state

$\Sigma \rightarrow$ is a finite non-empty set of input alphabate

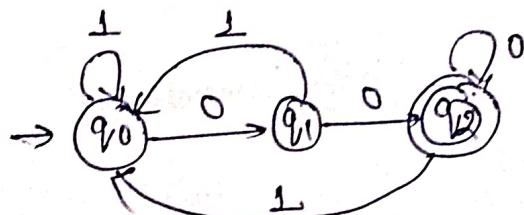
$\delta \rightarrow$ is a transition function which map $Q \times \Sigma$ into Q
and is usually called discrete transition function.

This is the function which describes the change of
state during the transition

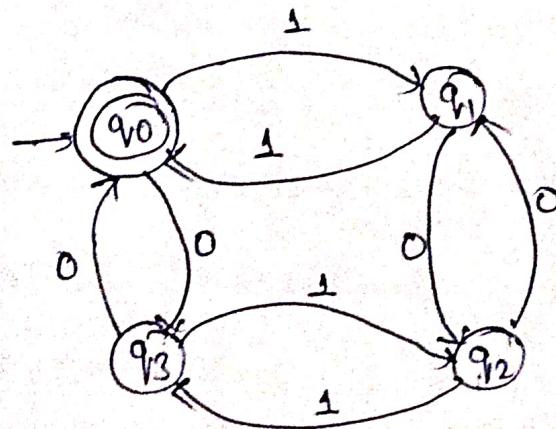
$q_0 \in Q$ is the initial state

$F \subseteq Q$ is the final state

Q: Design a DFA that accepts set of string such that every string ends
with 00, over alphabate $\Sigma = \{0, 1\}$

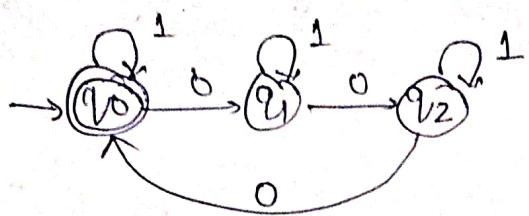


Q: Design a DFA that accept even no. of 0 and 1 over $\Sigma = \{0, 1\}$.

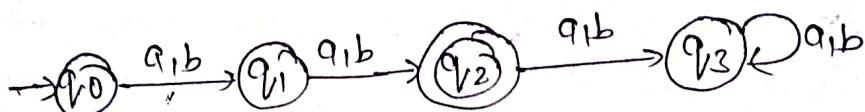


(3)

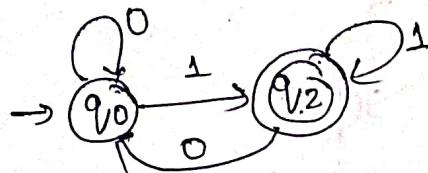
Construct A DFA that accept set of string Where the no. of 0's in every string is multiple of 3 over alphabet $\Sigma = \{0, 1\}$



Q. Construct a DFA Which accepts Set of all string over $\Sigma = \{a, b\}$ of length = 2.



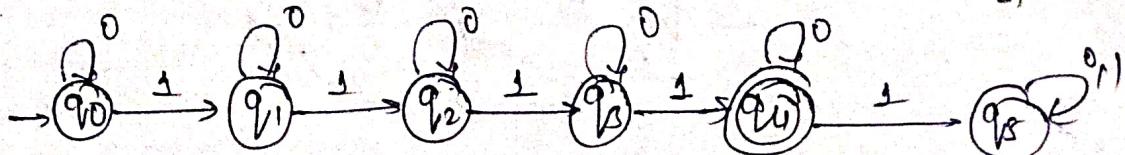
Q. Construct a DFA that accept set of strings such that every string ends with ~~01~~1, over Alphabet $\Sigma = \{0, 1\}$



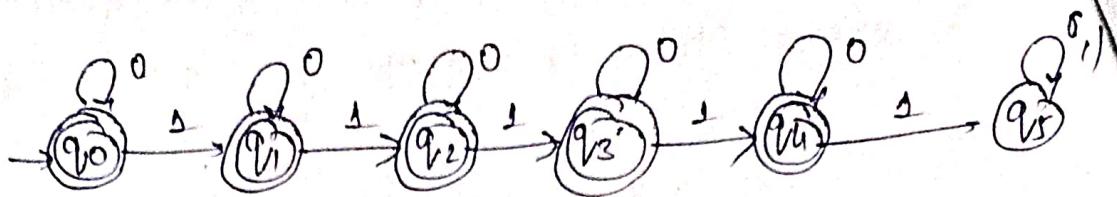
Q. Construct a DFA Which accept set of string Containing (i) exactly four 1's in every string over alphabet $\Sigma = \{0, 1\}$

(ii) At least four 1's
At Most four 1's

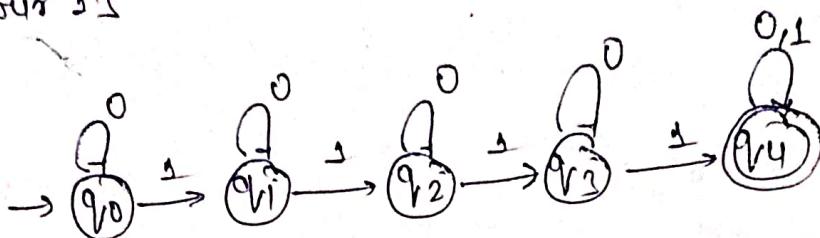
(iii) At least four 1's



(ii) At Most four 0's

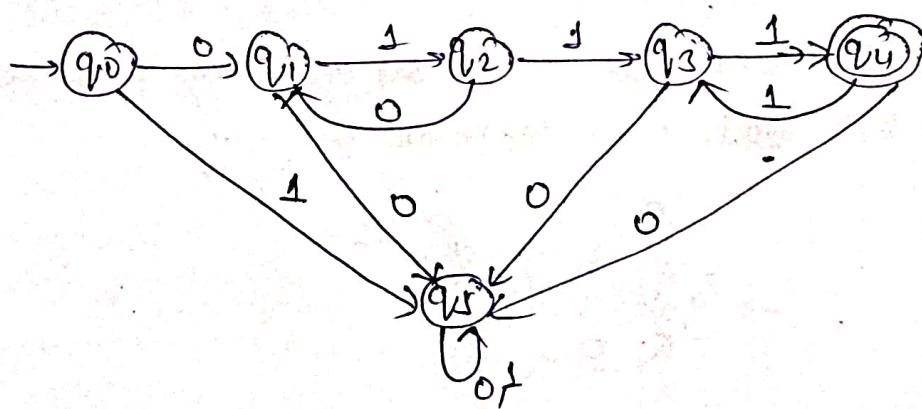


(iii) At least four 0's

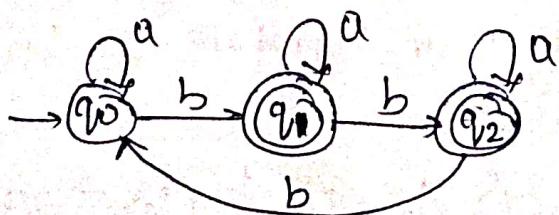


Q. Design a FA for the language.

$$L = \{ (01)^i 1^j \mid i \geq 1, j \geq 1 \} \text{ where } W \in \{0,1\}^*$$

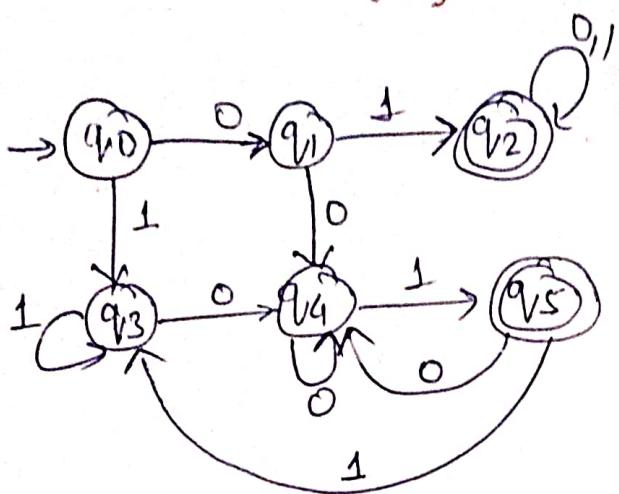


Q. Design a FA for the language $L = \{ w \in \{a,b\}^* \mid n_b(w) \bmod 3 > 2 \}$

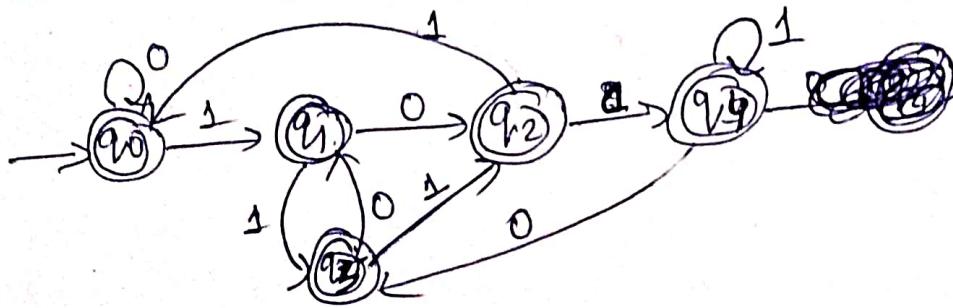


Draw a FA that accept a substring start with 01 or end with 01 for 2ip Alphabate {0,1} (4)

01 for 2ip Alphabate {0,1}



Q. Draw the DFA for $L = \{ 1^w 1^y 0^z \mid w \in (0,1)^*, y \in \{0,1\}^*, z \in \{0,1\}^* \}$



0000	-0
0001	-1
0010	-2
0011	-3
0100	-4
0101	-5
0110	-6
0111	-7
1000	-8
1001	-9
1010	-10
1011	-11
1100	-12
1101	-13
1110	-14
1111	-15

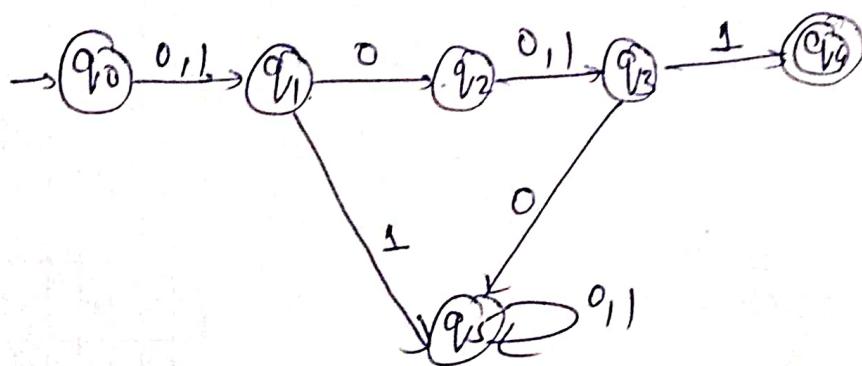
(5)

Automata

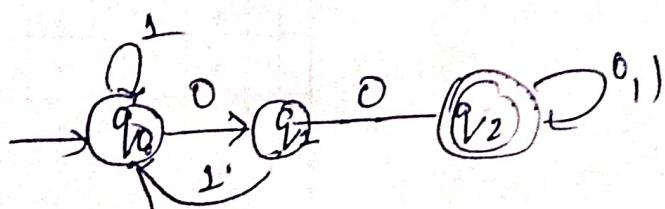
Q. Design a FA Which accepts the language.

$$L = \{ w \in \{0,1\}^* \mid \text{second symbol of } w \text{ is } 0 \text{ & fourth input is } 1 \}$$

Sol.



Q. for the given FA Write the language and also give the transition table.



$$L = \{ w \in \{0,1\}^* \mid \text{every string } w \text{ of the language containing } 00 \text{ as substring} \}$$

(6)

Minimization of DFA:

Minimization of DFA is required to obtain the minimal version of any DFA which consists of the minimum no. of states possible.

Two states 'A' and 'B' are said to be equivalent if

$$\begin{array}{ll} \delta(A, x) \rightarrow f & \delta(A, x) \xrightarrow{*} f \\ \text{OR} & \\ \delta(B, x) \rightarrow f & \delta(B, x) \xrightarrow{*} f \end{array}$$

Where x is any input string.

If $|x|=0$, then A & B are said to be 0-equivalent

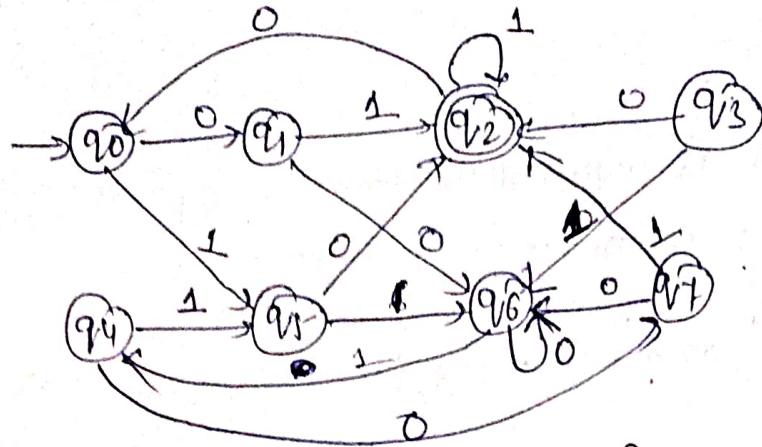
If $|x|=1$, then A & B 1-equivalent

If $|x|=2$, 2-equivalent

:

If $|x|=n$, then A & B are said to be n-equivalent

Expt.



First we remove q_3 which is not reachable state.

Transition Table:-

State	0	1	RIP
$\rightarrow q_0$	q_1	q_{15}	
q_1	q_6	q_2	
q_2	q_0	q_2	
q_3	q_2	q_6	X
q_4	q_7	q_5	
q_5	q_2	q_6	
q_6	q_6	q_4	
q_7	q_6	q_2	

0-equivalence:

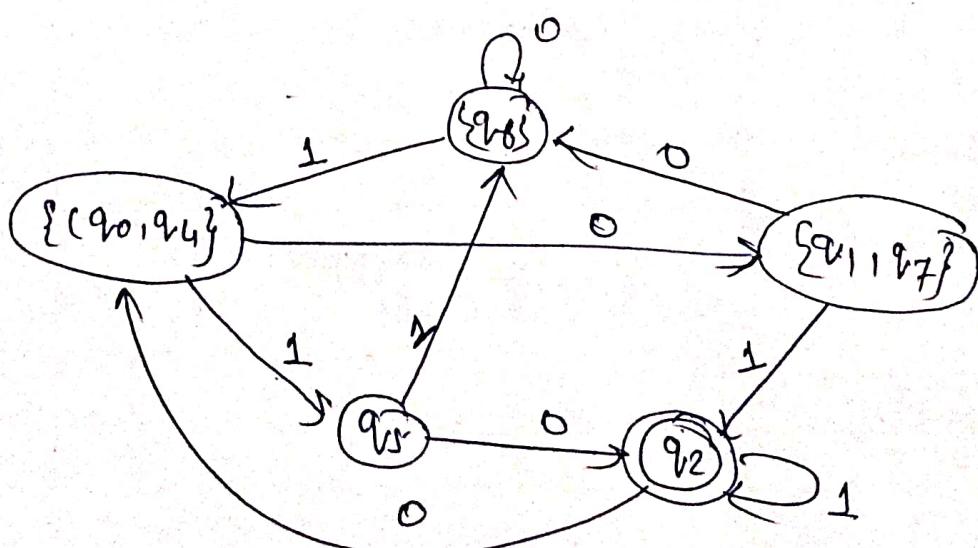
$$Q_0 = \{q_0, q_1, q_4, q_5, q_6, q_7\} \setminus \{q_3\}$$

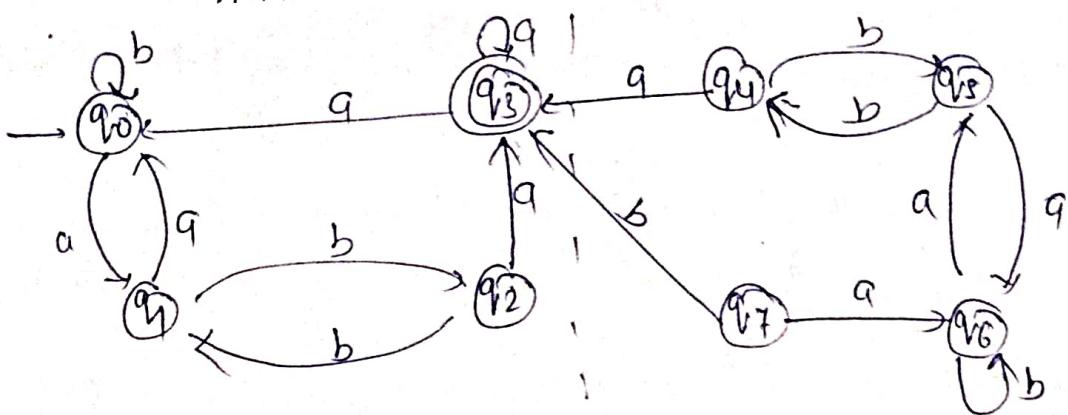
1-equivalence:

$$\Theta_1 = \{q_0, q_4, q_6\} \cup \{q_1, q_7\} \cup \{q_5\} \cup \{q_2\}$$

2-equivalence:

$$\Theta_2 = \{q_0, q_4\} \cup \{q_6\} \cup \{q_1, q_7\} \cup \{q_5\} \cup \{q_2\}$$





0-equivalence:

$$Q_0 = \{q_0, q_1, q_2\} \quad \{q_3\}$$

1-equivalence

$$\Theta_1 = \{q_0, q_1\} \quad \{q_2\} \quad \{q_3\}$$

2-equivalence

$$\Theta_2 = \{q_0\} \quad \{q_1, q_2\} \quad \{q_3\}$$

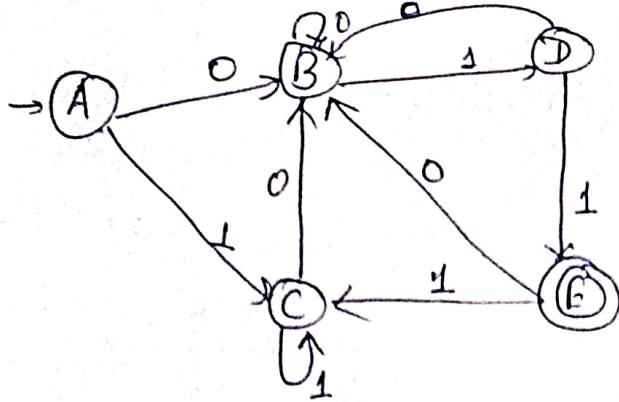
Transition table:

state	21P	
	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
q_{13}	q_3	q_0
q_3	q_5	q_5
q_4	q_5	q_6
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

Not Reachable state: $\{q_4, q_5, q_6, q_7\}$

Transition Table:

Step	RIP	
	0	1
$\rightarrow A$	B C	
B	B D	
C	B C	
D	B E	
E	B C	



0-equivalence

$$\pi_0 \rightarrow \{A, B, C, D\} \{E\}$$

1-equivalence

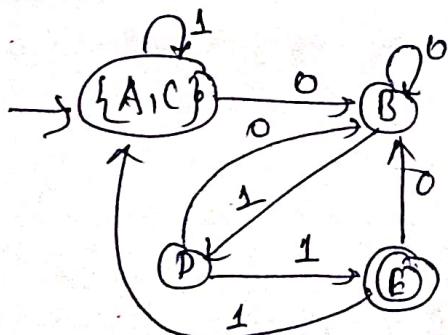
$$\pi_1 \rightarrow \{A, B, C\} \{D\} \{E\}$$

2-equivalence

$$\pi_2 \rightarrow \{A, C\} \{B\} \{D\} \{E\}$$

3-equivalence

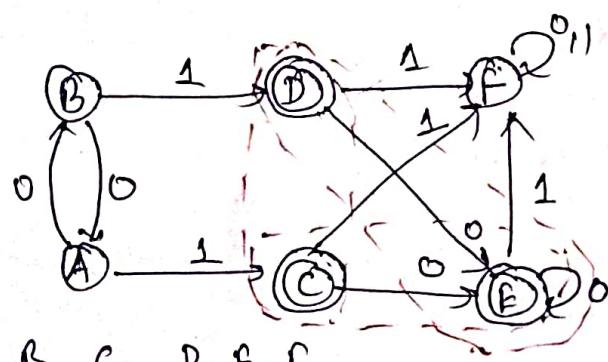
$$\pi_3 \rightarrow \{A, C\} \{B\} \{D\} \{E\}$$



(8)

Minimization of DFA - Table filling Method (Myhill-Nerode Theorem)

- Steps:
- ① Draw a table for all pairs of states (P, Q)
 - ② Mark all pairs where $P \in F$ and $Q \notin F$
 - ③ if there are any unmarked pairs (P, Q) such that $[\delta(P, x), \delta(Q, x)]$ is marked, then mark $[P, Q]$
Where x is an SLP symbol.
Repeat this until No more markings can be made
 - ④ Combine all the unmarked pairs and make them a single state in the minimized DFA



	A	B	C	D	E	F
A						
B						
C	✓	✓				
D	✓	✓	.	.		
E	✓	✓	.	.		
F	✓		✓	✓	✓	

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

$$\delta(B, A) = \delta(B, 0) \vdash A \quad \left. \begin{array}{l} \delta(B, 1) \vdash D \\ \delta(A, 0) \vdash B \\ \delta(A, 1) \vdash C \end{array} \right\} \text{unmarked}$$

$$\delta(D, C) = \delta(D, 0) \vdash E \quad \left. \begin{array}{l} \delta(D, 1) \vdash F \\ \delta(C, 0) \vdash E \\ \delta(C, 1) \vdash F \end{array} \right\} \text{unmarked}$$

$$\delta(E, C) = \delta(E, 0) \vdash E \quad \left. \begin{array}{l} \delta(E, 1) \vdash F \\ \delta(C, 0) \vdash E \\ \delta(C, 1) \vdash F \end{array} \right\} \text{unmarked}$$

$$(E,D) = \delta(E,0) \vdash F \left\{ \begin{array}{l} \delta(E,1) \vdash F \\ \delta(D,0) \vdash E \end{array} \right\} \text{Unmarked}$$

$$(F,A) = \delta(F,0) \vdash F \left\{ \begin{array}{l} \delta(F,1) \vdash F \\ \delta(A,0) \vdash B \end{array} \right\} \text{Marked}$$

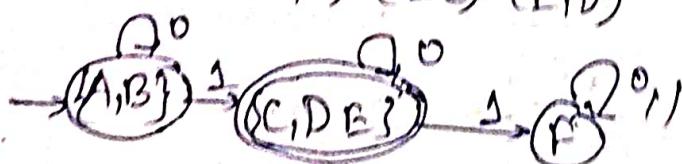
(F,C) is marked so (F,A) become marked.

$$(F,B) = \delta(F,0) \vdash F \left\{ \begin{array}{l} \delta(F,1) \vdash F \\ \delta(B,0) \vdash A \end{array} \right\} \text{Marked}$$

(F,A) is marked so (F,B) become marked.

Make a set of unmarked pair

$$(A,B) (D,C) (B,C) (E,D)$$



Transformation of NFA to DFA:

The DFA equivalent of NFA is to simulate the moves in parallel state of a DFA will be combination of one or more states of NFA, hence every state of a DFA will be represented by some subset of set of states of NFA. and therefore the transition of NFA to DFA is normally called a subset construction.

Equivalence of DFA and NFA:

We are going to prove following " if NDFA accepts a language 'L', then there exist a DFA that also accept 'L'.

Proof:- Let M be any given NFA, Which accept L.

$$M = \{ Q, \Sigma, \delta, q_0, q_f \}$$

Now Let us Construct a DFA $M' = \{ Q', \Sigma, \delta', q'_0, q'_f \}$ where.

(i) $Q' = 2^Q$, i.e Q' contains the subset of Q (any state in Q' is denoted by $\{q_1, q_2, \dots, q_n\}$ where $q_1, q_2, \dots, q_n \in Q\}$

(ii) $q'_0 = \{q_0\}$

(iii) q'_f is the set of all subsets of Q containing an element of q_f

(iv) Transition function δ' is defined as follows

$$\begin{aligned} \delta'(\{q_1, q_2, \dots, q_n\}, a) &= \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_3, a) \dots \cup \delta(q_n, a) \\ &= P_1 \cup P_2 \cup \dots \cup P_r \end{aligned}$$

If and only if $\{\{q_1, q_2, \dots, q_n\}, a\} = \{P_1, P_2, \dots, P_r\}$

\Rightarrow Procedure for converting NFA to Equivalent DFA:

Let M be an NFA denoted by $\{\emptyset, \Sigma, \delta, q_0, q_f\}$ Which accepts L.

To obtain a equivalent DFA $M' = (\emptyset, \Sigma, \delta', q_0', q_f')$ Which accept the same language as given NFA $M = (\emptyset, \Sigma, \delta, q_0, q_f)$ over, we may proceed as follows:

Step 1 - Initially $Q' = \emptyset$

Step 2 - put $[q_0]$ into Q' . $[q_0]$ is the initial state of DFA M' .

Step 3. Add every new state q to Q'

Where $\delta'(q, a) = \bigcup_{p \in Q} \delta(p, a)$, δ on the right hand side is that of NFA 'M'.

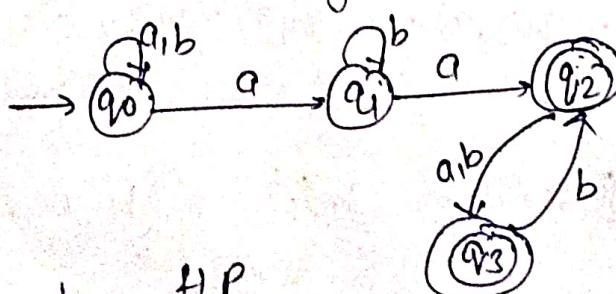
Step 4. Repeat step 3 till no new states are there to add in Q' ,

If there is no further new state found to add in Q'

The process terminated. All states in Q' that contain final state of M' are accepting state of M'

Note: The states which are not reached from the initial state should not be included in Q' . Thus the set of states (Q') is not necessarily equal to 2^Q .

Q. Convert the following NFA into DFA

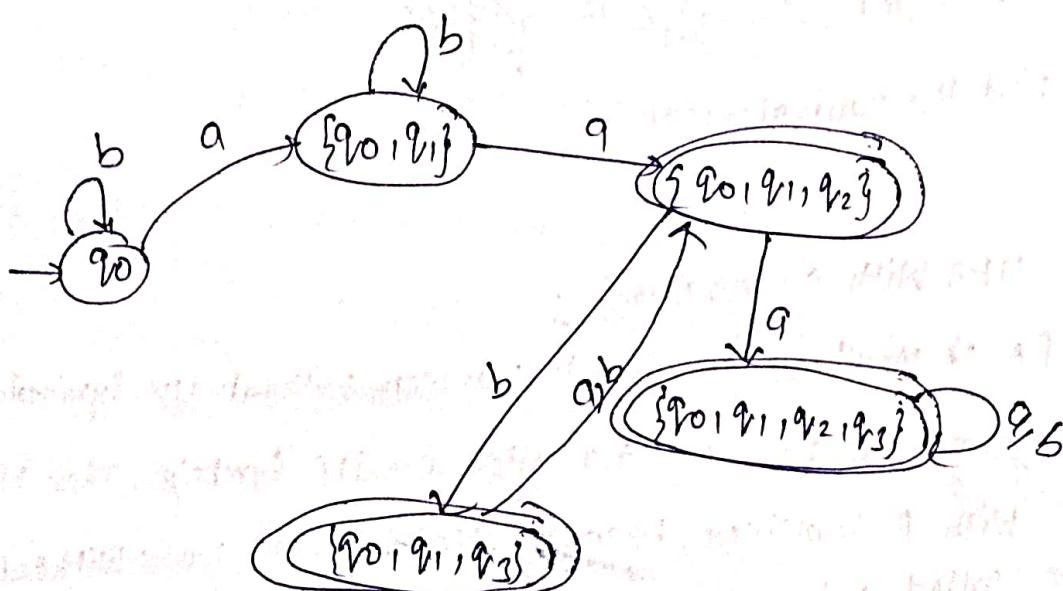


Ans:

State	HP	
	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3	ϵ	q_2

(4)

α	a	b	Tip
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$	
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$	
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	

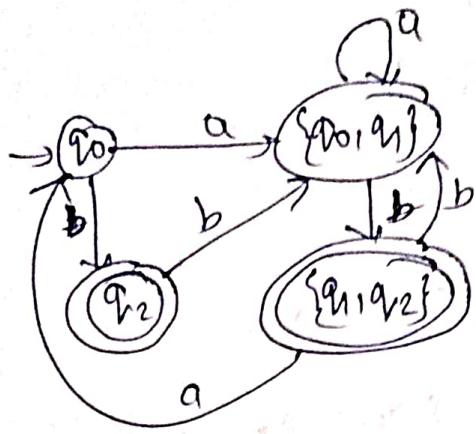


Q. Convert the following NFA into DFA

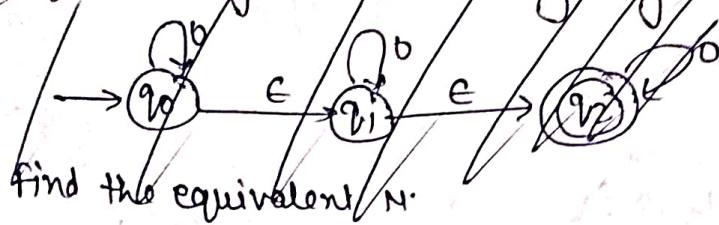
α	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_0\}$	$\{q_3\}$
(q_2)	ϕ	$\{q_0, q_1\}$

α	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
q_2	ϕ	$\{q_0, q_1\}$

α	a	b
$\{q_1, q_2\}$	$\{q_0\}$	$\{q_0, q_1\}$



Q/ Consider the NFA given by following diagram



find the equivalent N.

NFA-ε (NFA With ε-Transition) :

If a FA is modified to permit transition without IIP symbols, along with zero; one or more transition on IIP symbols, then we get a NFA with ϵ -transitions, because the transitions made without symbols are called ϵ -transition.

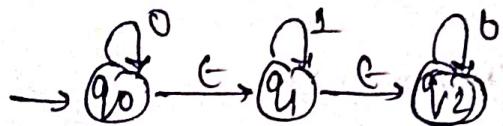


fig.1

fig.1 represent the NFA with ϵ -transition because it is possible to make transition from state q_0 to q_2 and q_1 to q_2 without consuming any of the IIP symbols.

tuple $(\Sigma, \delta, q_0, q_f)$

$$\delta \rightarrow Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$$

Note! for representing ϵ transition need to know ϵ -closure

ϵ -closure(q) = set of all those states which can be reached from q on the path labeled by ϵ

$$\boxed{\hat{\delta}(\epsilon\text{-closure}(q, qw)) \vdash \epsilon\text{-closure}(\delta(\hat{\delta}(\epsilon\text{-closure}q), q), w)}$$

$$\boxed{\hat{\delta}(q_1q) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1)\epsilon), q)}$$

$$\hat{\delta}(q_1\epsilon) \rightarrow \epsilon\text{-closure}(q_1)$$

A. Convert the given NFA With ϵ to NFA Without ϵ



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

$$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 0))$$

$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 0))$$

$$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), 0)$$

$$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$$

$$= \epsilon\text{-closure}(q_0 \cup \emptyset \cup \emptyset)$$

$$= \epsilon\text{-closure}(q_0)$$

$$= \{q_0, q_1, q_2\}$$

$$\hat{\delta}(q_0, 1) = \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 1))$$

$$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), 1)$$

$$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$$

$$= \epsilon\text{-closure}(\emptyset \cup q_1 \cup q_2)$$

$$= \epsilon\text{-closure}(q_1, q_2)$$

$$= \text{G-closure}(q_1) \cup \text{G-closure}(q_2)$$

$$= \{q_1, q_2\} \cup \{q_2\}$$

$$= \{q_1, q_2\}$$

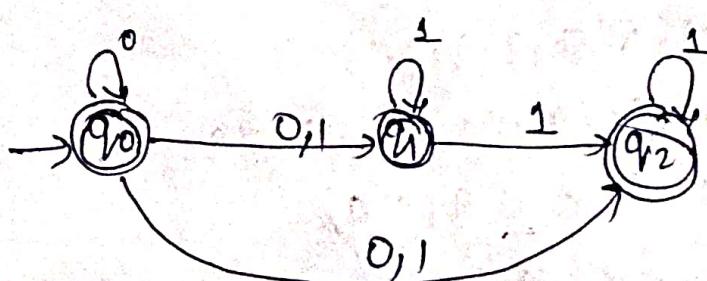
NFA (Non-Deterministic Finite Automaton)

$$\begin{aligned}\hat{\delta}(q_1, 0) &= \text{E-closure}(\delta(\delta(q_1, \epsilon), 0)) \\&= \text{E-closure}(\delta(q_1 q_2), 0) \\&= \text{E-closure}(\delta(q_1, 0) \cup \delta(q_2, 0)) \\&= \text{E-closure}(\emptyset \cup \emptyset) \\&= \text{E-closure}(\emptyset) = \emptyset\end{aligned}$$

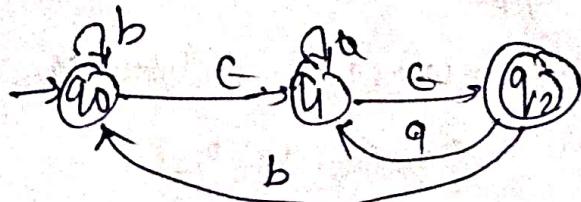
$$\begin{aligned}\hat{\delta}(q_1, 1) &= \text{E-closure}(\delta(\delta(q_1, \epsilon), 1)) \\&= \text{E-closure}(\delta(q_1 q_2), 1) \\&= \text{E-closure}(\delta(q_1, 1) \cup \delta(q_2, 1)) \\&= \text{E-closure}(q_1, q_2) \\&= \text{E-closure}\{q_1\} \cup \text{E-closure}(q_2) \\&= \{q_1, q_2\} \cup \{q_2\} \\&= \{q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\hat{\delta}(q_2, 0) &= \text{E-closure}(\delta(\delta(q_2, \epsilon), 0)) \\&= \text{E-closure}(\delta(q_2), 0) \\&= \text{E-closure}(\emptyset) = \emptyset\end{aligned}$$

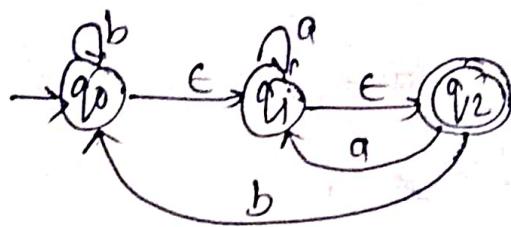
$$\begin{aligned}\hat{\delta}(q_2, 1) &= \text{E-closure}(\delta(\delta(q_2, \epsilon), 1)) \\&= \text{E-closure}(\delta(q_2), 1) \\&= \text{E-closure}(q_2) \\&= q_2\end{aligned}$$



Q.



(6)



Convert the given NFA with ϵ to NFA without ϵ .

$$\epsilon\text{-closure}(q_0) \rightarrow \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) \rightarrow \{q_1, q_2\}$$

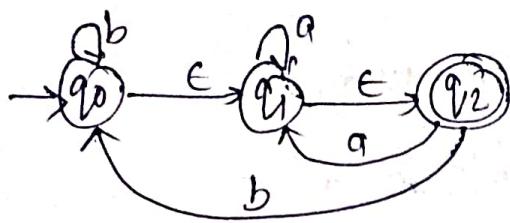
$$\epsilon\text{-closure}(q_2) \rightarrow \{q_2\}$$

$$\begin{aligned}\hat{\delta}(q_0, a) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), a)) \\&= \epsilon\text{-closure}(\delta(\{\hat{\delta}(q_0, q_1, q_2), q\}, a)) \\&= \epsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a)) \\&= \epsilon\text{-closure}(\epsilon \cup q_1 \cup q_2) \\&= \epsilon\text{-closure}\{q_1\} \\&= \{q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\hat{\delta}(q_0, b) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), b)) \\&= \epsilon\text{-closure}(\delta(\{\hat{\delta}(q_0, q_1, q_2), b\}, b)) \\&\rightarrow \epsilon\text{-closure}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b)) \\&= \epsilon\text{-closure}(\{q_0\} \cup \{\epsilon\} \cup \{q_0\}) \\&= \epsilon\text{-closure}(q_0) \\&= \{q_0, q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\hat{\delta}(q_1, a) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), a)) \\&= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, a)) \\&= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a)) \\&= \epsilon\text{-closure}(\phi \cup q_1) \\&= \{q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\hat{\delta}(q_1, b) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), b)) \\&= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, b)) \\&= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b)) \\&= \epsilon\text{-closure}(\epsilon \cup q_0) = \{q_0, q_1, q_2\}\end{aligned}$$



(6)

Convert the given NFA with
ε to NFA without G.

$$\epsilon\text{-closure}(q_0) \rightarrow \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) \rightarrow \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) \rightarrow \{q_2\}$$

$$\begin{aligned}
 \hat{\delta}(q_0, a) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), a)) \\
 &= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, a)) \\
 &= \epsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a)) \\
 &= \epsilon\text{-closure}(\epsilon \cup q_1 \cup q_2) \\
 &= \epsilon\text{-closure}(q_1) \\
 &= \{q_1, q_2\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{\delta}(q_0, b) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), b)) \\
 &= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, b)) \\
 &= \epsilon\text{-closure}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b)) \\
 &= \epsilon\text{-closure}(\{q_0\} \cup \{\epsilon\} \cup \{q_0\}) \\
 &= \epsilon\text{-closure}(q_0) \\
 &= \{q_0, q_1, q_2\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{\delta}(q_1, a) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), a)) \\
 &= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, a)) \\
 &= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a)) \\
 &= \epsilon\text{-closure}(\phi \cup q_1) \\
 &= \{q_1, q_2\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{\delta}(q_1, b) &= \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), b)) \\
 &\Rightarrow \epsilon\text{-closure}(\delta(\{q_1, q_2\}, b)) \\
 &= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b)) \\
 &= \epsilon\text{-closure}(\epsilon \cup q_0) = \{q_0, q_1, q_2\}
 \end{aligned}$$

$\hat{\delta}(q_2, a) = \text{E-closure}(\delta(q_2, a))$

$$\delta(q_2, a) = \text{E-closure}(\delta(\delta(q_2, \epsilon), a))$$

$$= \text{E-closure}(\delta(q_2, a))$$

$$= \text{E-closure}(q_1)$$

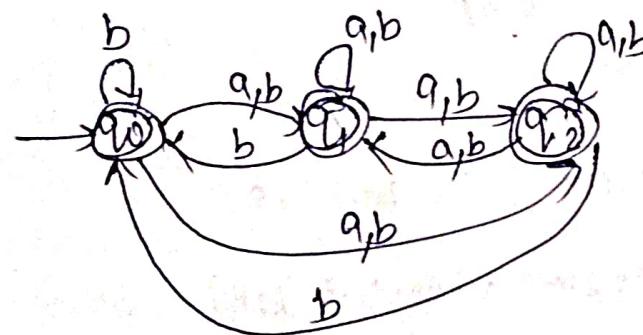
$$= \{q_1, q_2\}$$

$$\hat{\delta}(q_2, b) = \text{E-closure}(\delta(\delta(q_2, \epsilon), b))$$

$$= \text{E-closure}(\delta(q_2, b))$$

$$= \text{E-closure}(q_0)$$

$$= \{q_0, q_1, q_2\}$$



NFA - With Out E

NFA (Non-Deterministic finite Automata)

5-tuple $(Q, \Sigma, \delta, q_0, F)$

$Q \rightarrow$ Non-Empty set of states

$\Sigma \rightarrow$ Input Alphabet

$\delta \rightarrow$ Transition function shown the mapping b/w $Q \times (\Sigma \cup \{\epsilon\}) \rightarrow \mathcal{P}(Q)$

$q_0 \in Q$ is the initial state

$F \subseteq Q$ is the final state

Note!:- NFA make difference with DFA, In NFA it's contain all possible combination some may contain true or may not be.

* it's used for backtracking

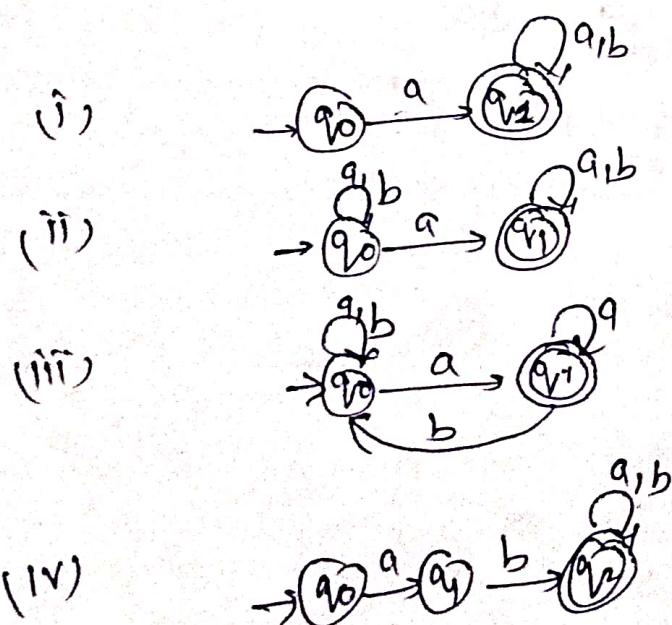
Q. Draw the NFA for $\Sigma = \{a, b\}$

(i) $L_1 = \{\text{start with } a\}$

(ii) $L_2 = \{\text{containing } a\}$

(iii) $L_3 = \{\text{end with } a\}$

(iv) $L_4 = \{\text{start with } ab\}$

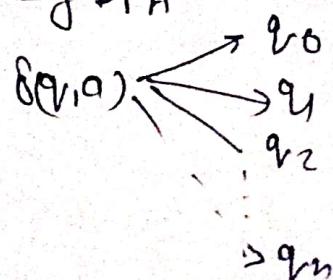


Note!:- In Non-deterministic accepter, the range of δ is the power set of 2^Q , so it's value is not a single element of Q , but a subset of it.

The subset defines the set of possible states that can be reached by transition.

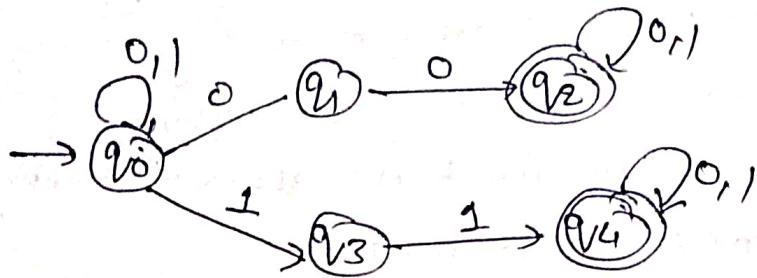
Let q be an state of Q and $a \in \Sigma$ then.

We can ~~transit~~ transit from a state q , on some input a to different states q_0, q_1, \dots, q_n in Q . This not possible in the case of DFA

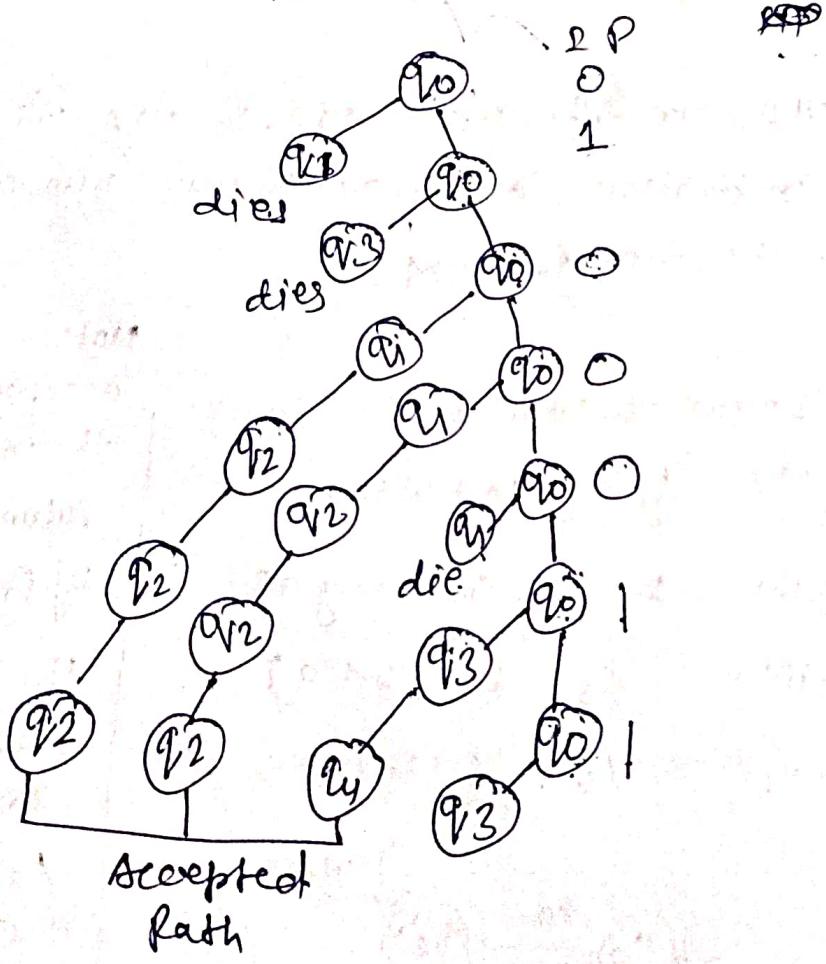


Q. Design a NFA for a language $L = \text{all strings over } \{0, 1\}^*$ that has at least two consecutive 0's or 1's

三

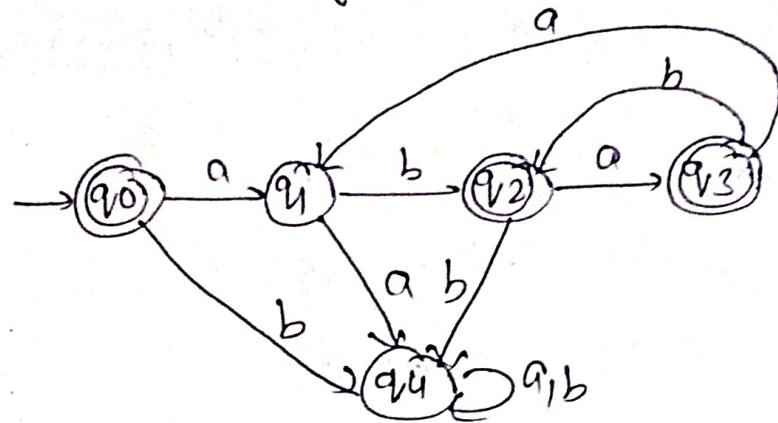


The tree of states this NFA is in for the SIP 0100011

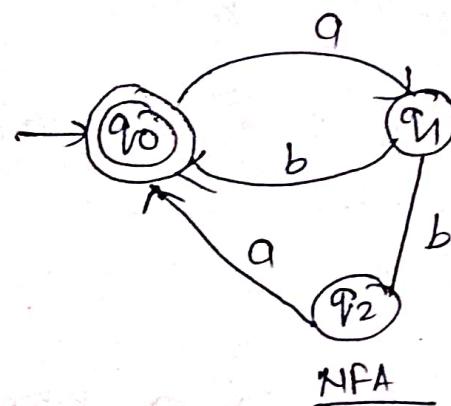


Ex Design a NFA for the language $L = (ab \cup aba)^*$.

Ans



DFA



NFA

Q. Draw the state diagram for NFA accepting string.

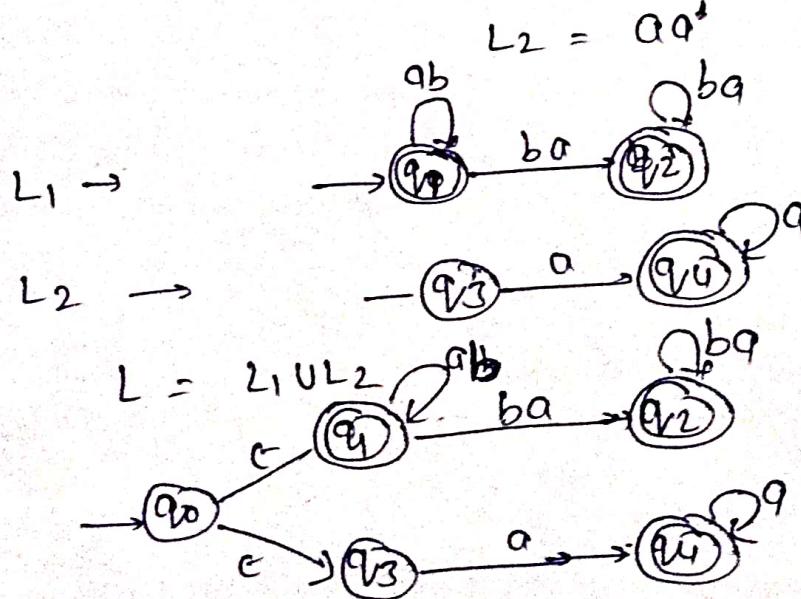
$$L = (ab)^* (ba)^* \cup aa^*$$

Ans: We construct NFA for the language L in two paths i.e

$$L = L_1 \cup L_2$$

$$L_1 = (ab)^* (ba)^*$$

$$L_2 = aa^*$$



Q. find NFA with four state for the language.

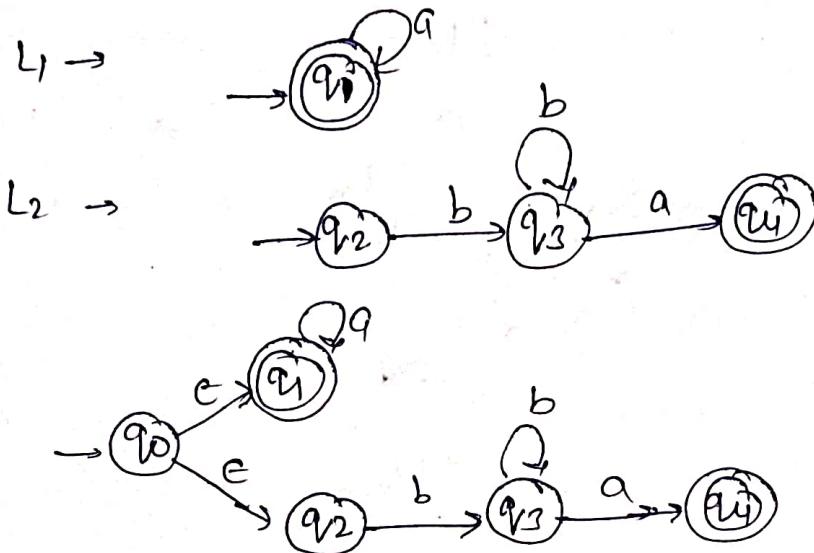
$$L = \{(a^n : n \geq 0) \cup (b^n a : n \geq 1)\}$$

Ans:

$$L = L_1 \cup L_2$$

$$L_1 = a^n : n \geq 0$$

$$L_2 = b^n a : n \geq 1$$

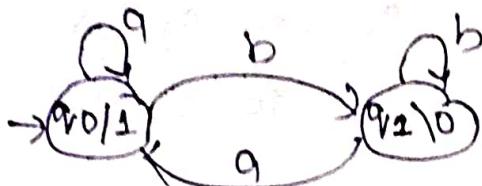


FA with Output

\downarrow
Moore machine

$$\lambda: Q \rightarrow \Delta$$

$$x(t) = \lambda q(t)$$



6-tuple $(Q, \Sigma, \delta, \lambda, q_0, q_f)$

$Q \rightarrow$ finite set of state

$\Sigma \rightarrow$ non-empty set of IIP alphabet

$\delta \rightarrow$ transition function $Q \times \Sigma \rightarrow Q$

$q_0 \in Q$ initial state

~~$q_f \in Q$ final state~~

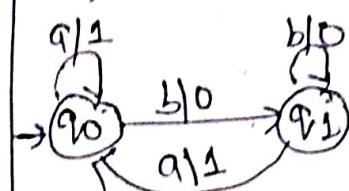
$\Delta \rightarrow$ output alphabet

$\lambda \rightarrow$ output function

Mealy machine

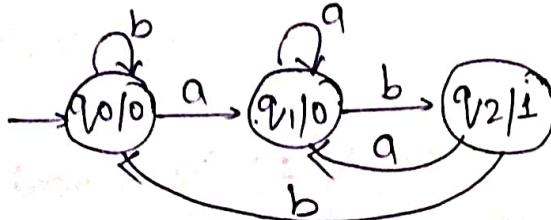
$$\lambda: Q \times \Sigma \rightarrow \Delta$$

$$x(t) = \lambda(q(t), \alpha(t))$$



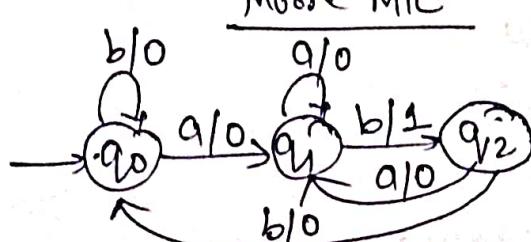
Q: Construct a Moore machine that takes set of all string over $\Sigma = \{a, b\}$ as IIP and print '1' as op for every occurrence of 'ab' as a substring

Moore.



PS	Next state		out put Δ
	a	b	
q_0	q_1	q_0	0
q_1	q_1	q_2	0
q_2	q_1	q_0	1

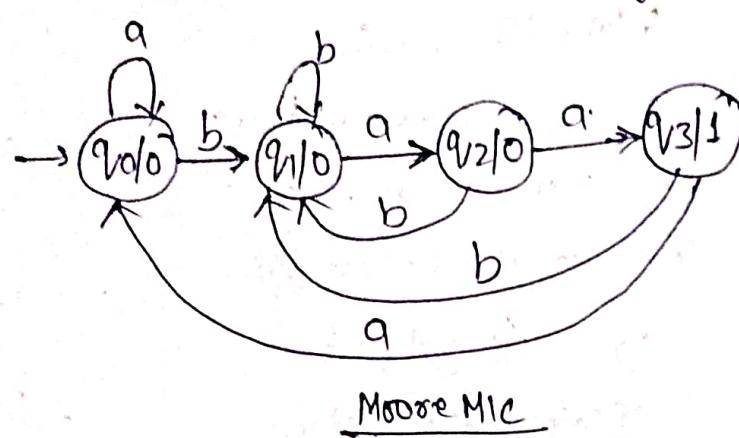
Moore MC



Mealy MC

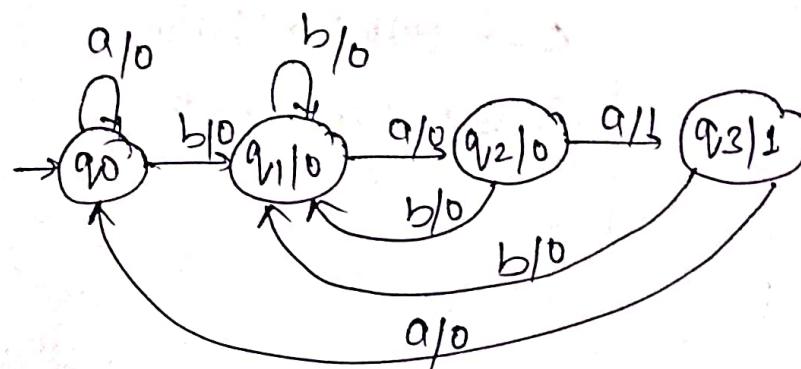
P.F.O

Q. Construct a Moore Mlc that takes set of all string over $\{a, b\}$ and counts no. of occurrence of substring 'baa'



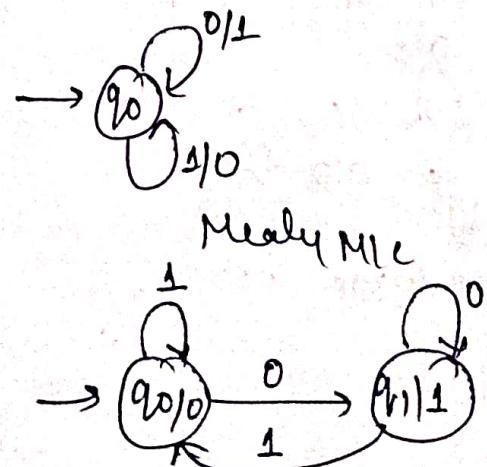
T.R. $p_{f,a}$	Next State		0/1
	a	b	
q_0	q_0	q_1	0
q_1	$q_{1/0}$	q_1	0
q_2	$q_{1/0}$	q_1	0
q_3	$q_{1/0}$	q_1	1

Q.



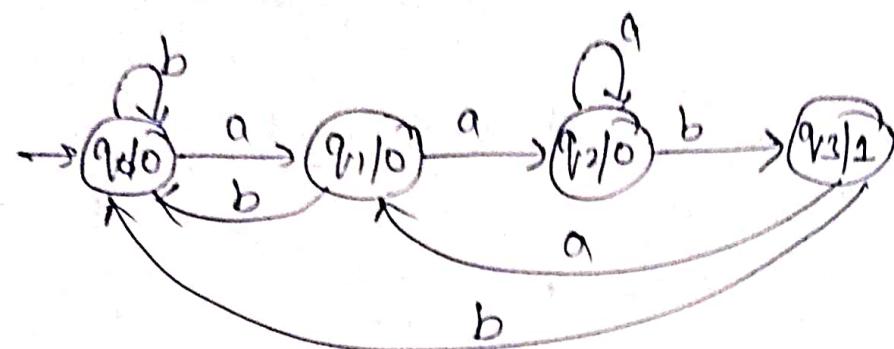
Mealy MLC

Q. Design a Mealy Mlc Which prints the Complement of input string over alphabet $\Sigma = \{0, 1\}$

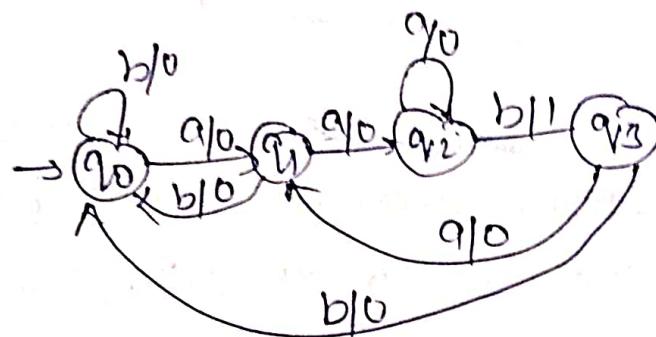


Moore MLC

Q. Design a Moore M/C Which Count the occurrence of substring aab in LIP string (2)

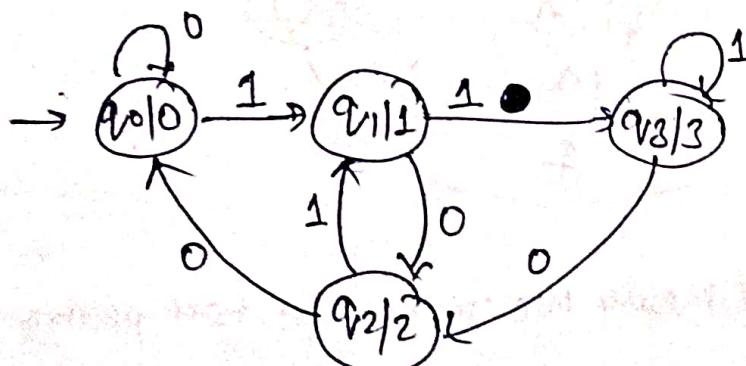


Moore M/C



Mealy M/C

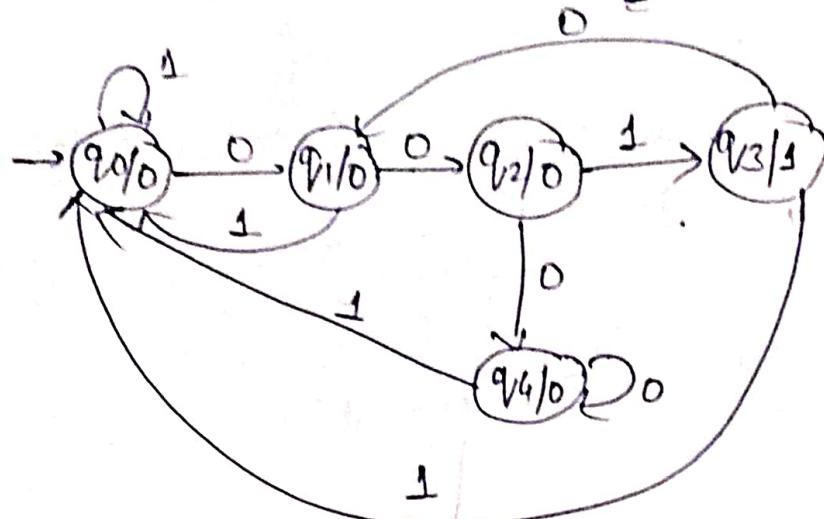
Q. Construct a Moore M/C Which calculate residue mode 4 for each binary string treated as binary integer.



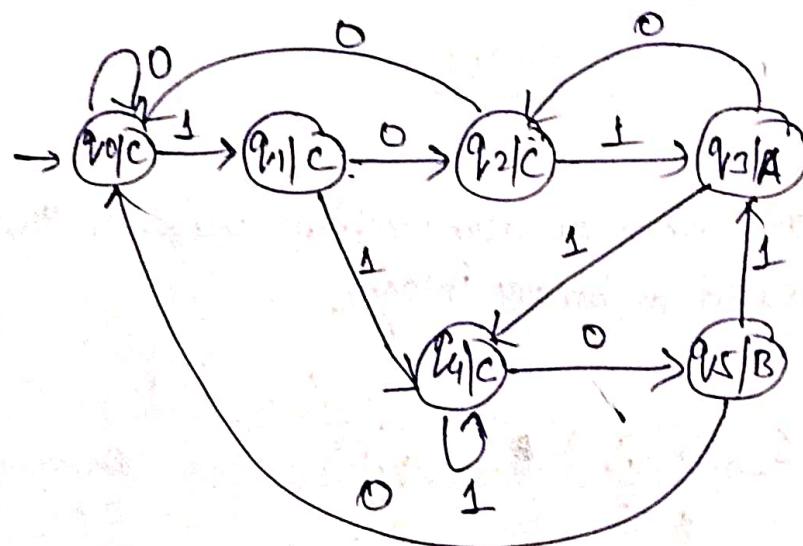
000	-6
001	-1
010	-2
011	-3
100	-4
101	-5
110	-6
111	-7

P.G.D

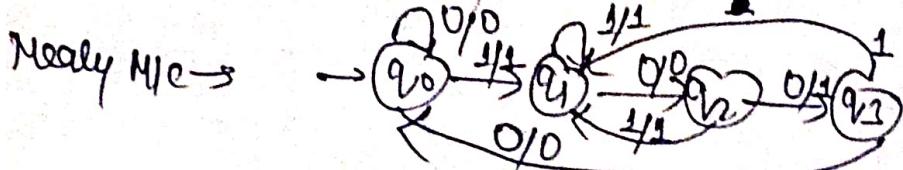
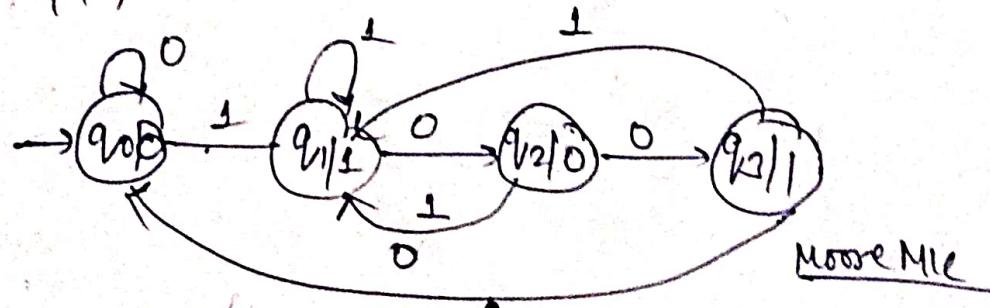
Q. Design a Moore M/c. that gives an O/P 1 if ~~the~~ Input of binary sequence a '3' is preceded by exactly two zeros



Q. Design a Moore M/c for a binary I/p sequence if it ends in 101 O/p is A, if it ends in 110 O/p is B.

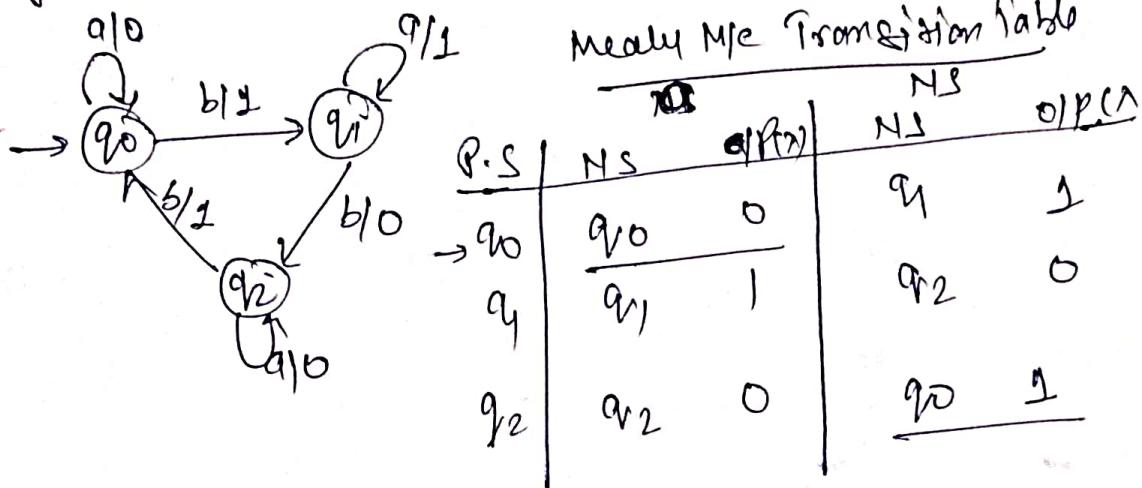


Q. Design a Moore & Mealy M/c to Convert each occurrence of substring 100 by 101



Conversion of Mealy to Moore

a. Convert the given Mealy Mle into Moore Mle



Note!

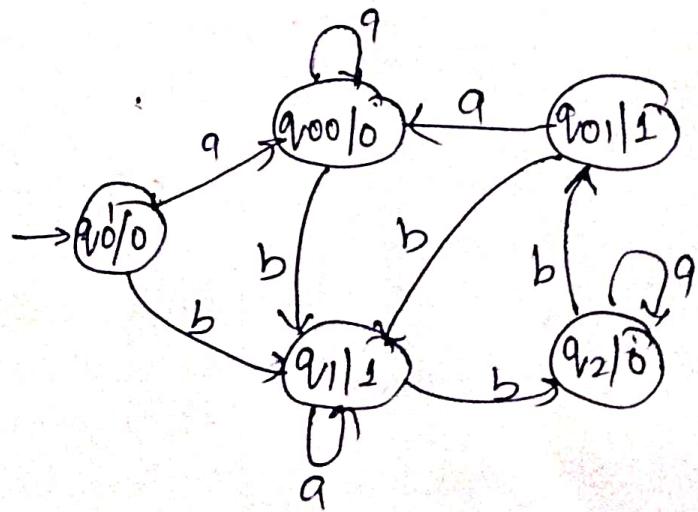
In the transition table of Mealy mle we found q_0 generated two different types of OIP. Now we reconstruct Mealy Mle Transition table.

Updated Mealy Mle Transition Table:

Present state	a		b		
	NS	OIP	NS	OIP	
$\rightarrow q_{00}$	q_{00}	0	q_1	1	
q_{01}	q_{00}	0	q_1	1	
q_1	q_1	1	q_2	0	
q_2	q_2	0	q_{01}	1	

Moore Mle Transition Table:

P.S.	a		b		
	NS		NS		
$\rightarrow q_{00}$	q_{00}	q_1	q_1	0	
q_{00}	q_{00}	q_1	q_1	0	
q_{01}	q_{00}	q_1	q_1	1	
q_1	q_1	q_2	q_2	1	
q_2	q_2	q_{01}	q_{01}	0	



Moore Mle