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MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

Roll No. : ....

NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P.

Sessional Examination – I : Odd Semester 2022-23

Course/Branch: B Tech – CSE/ME/ECE
Subject Name: Engg. Mathematics IV
Subject Code: KAS302

Semester: III
Max. Marks: 60
Time: 120 min

CO-1: *On completion of this course, the student will be able to solve PDEs.*
CO-2: *On completion of this course, the student will be able to understand the importance of PDEs which are mathematical modeling of engineering problems.*

Section – A (CO - 1) # Attempt both the questions # 30 Marks

Q.1: Attempt any **SIX** questions (*Short Answer Type*). Each question is of two marks. (2 x 6 = 12 Marks)

(a) Find the order and degree of the following PDE

$$\left(\frac{\partial^2 z}{\partial x^2}\right)^{\frac{1}{3}} + \frac{\partial z}{\partial y} = z^2.$$

(b) Find the partial differential equation by eliminating arbitrary constants a and b from

$$z = axe^y + \frac{1}{2}a^2 e^{2y} + b.$$

(c) Classify the following first order partial differential equations
- $p + 3q = 5z + \tan(y - 3x)$
- $p = (z + qy)^2$
- $P(x,y)z\,p + Q(x,y)\,zq = R(x,y)$
- $pp + qq = 1.$

(d) Write Charpit's auxiliary equations for $f(x, y, z, p, q) = 0$.

(e) Find the complete solution (integral) of the equation

$$z = px + qy + \sqrt[3]{1 + p^2 + e^q}.$$

(f) Solve: $p + q = npq.$

(g) Solve: $(D^2 - \pi D')z = 0.$

Q.2: Attempt any **THREE** questions (*Medium Answer Type*). Each question is of 6 marks. (3 x 6 = 18 Marks)

(a) Find the general solution of the equation $(\beta z - \gamma y)\frac{\partial z}{\partial x} + (\gamma x - \alpha z)\frac{\partial z}{\partial y} = \alpha y - \beta x.$

(b) Solve: $(D^2 + DD' - 6D'^2)z = \cos(2x + y).$

(c) Solve the linear partial differential equation $(D^2 + DD' - 2D'^2)z = (y - 1)e^x.$

(d) Solve: $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y).$

(e) Solve the following linear partial differential equation

$$(x^2 D^2 + 2xyDD' + y^2 D'^2)z = x^m y^n \text{ where } (m + n) \notin \{0,1\}.$$

Section – B (CO - 2) # Attempt both the questions # 30 Marks

Q.3: Attempt any **SIX** questions (*Short Answer Type*). Each question is of two marks. (2 x 6 = 12 Marks)

(a) Classify the following partial differential equation

$$\left(\frac{\partial^2 u}{\partial x^2}\right) + 3\left(\frac{\partial^2 u}{\partial x \partial y}\right) + \left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$$

(b) Write limitations of method of separation of variables for PDEs.

(c) Find the *complementary function (C.F.)* of $D'(2D - 4D' + 5)^2 z = 0.$

(d) Find the *complementary function (C.F.)* of $(D^2 + D'^2)z = xe^{2x-7y}.$

(e) Find the *particular integral (P.I.)* of $(D - D')(D - 2D')z = e^{x+y}.$

(f) Classify the operator $D^2 + DD' + D'^2.$

(g) Find a partial differential equation by eliminating the function $\mathcal{F}$ from

$$z = \mathcal{F}\{(x^2 + y^2)^{1/2}\}.$$

Q.4: Attempt any **THREE** questions (*Medium Answer Type*). Each question is of 6 marks. (3 x 6 = 18 Marks)

(a) Use Cauchy's method of Characteristic to solve the following
$$u_x + yu_y = 2u, \qquad u(0,y) = y.$$

(b) Show that the equation $u_{xx} + 2xu_{xy} + (a^2 - y^2)u_{yy} = 0$ is elliptic for values of x and y in the region $x^2 + y^2 < a^2$, parabolic on the boundary and hyperbolic outside this region.

(c) Solve the following linear partial differential equation
$$(D + D' - 1)(D + 3D' - 2)z = xy.$$

(d) Use the method of separation of variables to solve the following
$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u, \quad \text{given that} \quad u(x,0) = 5e^{7x}.$$

(e) Use the method of separation of variables to solve the following equation
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c \neq 0 \text{ (only for negative value of constant after separation).}$$

Note:- This section is only for ECE students.

Section – B # Attempt both the questions # 30 Marks

Q.3: Attempt any **SIX** questions (*Short Answer Type*). Each question is of two marks.   (2 x 6 = 12 Marks)

(a) Find the median of 6,8,9,10,11,12,13.

(b) The mean of 200 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find out the correct mean.

(c) Find the *complementary function* (C.F.) of $D'(2D - 4D' + 5)^2 z = 0$.

(d) Find the *complementary function* (C.F.) of $(D^2 - D'^2)z = xe^{2x-7y}$.

(e) Find the *particular integral* (P.I.) of $(D - D')(D - 2D')z = e^{x+y}$.

(f) Solve: $(D^2 + DD' + D'^2)z = 0$.

(g) Find a partial differential equation by eliminating the function $\mathcal{F}$ from
$$z = \mathcal{F}\{(x^2 + y^2)^{1/2}\}.$$

Q.4: Attempt any **THREE** questions (*Medium Answer Type*). Each question is of 6 marks. (3 x 6 = 18 Marks)

(a) Use Cauchy's method of Characteristic to solve the following
$$u_x + yu_y = 2u, \qquad u(0,y) = y.$$

(b) Solve the following linear partial differential equation
$$(D + D' - 1)(D + 3D' - 2)z = xy.$$

(c) Compute the arithmetic mean for the following data:

Height (in cm)	219	216	213	210	207	204	201	198	195
No. of persons	2	4	6	10	11	7	5	4	1

(d) Find the mode of the following frequency distribution:

Size	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	3	8	15	23	35	40	32	28	20	45	14	6

(e) Calculate the mean and median from the following data:

Class interval	6.5-7.5	7.5-8.5	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Frequency	5	12	25	48	32	6	1