Roll No. : 014

# MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

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**Sessional Examination – II : Odd Semester 2022-23**

**Course/Branch: B Tech – CSE/ME/ECE** **Semester: III**
**Subject Name: Engg. Mathematics IV** **Max. Marks: 60**
**Subject Code: KAS302** **Time: 120 min**

*CO-3: On completion of this course, the student will be able to apply descriptive Statistics.*
*CO-4: On completion of this course, the student will be able to apply the probability theory, distributions and their applications.*

## Section – A (CO3) # Attempt both the questions # 30 Marks

**Q.1:** Attempt any **SIX** questions (*Short Answer Type*). Each question is of two marks. (2 x 6 = 12 Marks)

(a) Classify the following partial differential equation

$$\left(\frac{\partial^2 u}{\partial x^2}\right) - 2\left(\frac{\partial^2 u}{\partial x \partial y}\right) + 3\left(\frac{\partial^2 u}{\partial y^2}\right) - 4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} - 6u = 0. \text{ (K2)}$$

(b) Explain the **Radio Equations**.(K2)

(c) Calculate the algebraic sum of the deviations of all the variates from their arithmetic mean. (K3)

(d) A frequency distribution gives the following results:
Coefficient of variation = 5, Standard deviation = 2.
Find the **arithmetic mean**. (K3)

(e) Write *Laplace's equation in two dimensions*. (K2)

(f) **The first four moments about mean** of a frequency distribution are 0, 3, 0, 26. Obtain the various characteristics of the distribution on the basis of the given information. Comment upon the nature of the distribution. (K3)

(g) The following data regarding the heights ($y$) and the weights ($x$) of 100 college students are given:
$\sum x = 15000$, $\sum x^2 = 2272500$, $\sum y = 6800$, $\sum y^2 = 463025$, $\sum xy = 1022250$.
Find the **correlation coefficient** between height and weight. (K3)

**Q.2:** Attempt any **THREE** questions (*Medium Answer Type*). Each question is of 6 marks. (3 x 6 = 18 Marks)

(a) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $u = u_0 sin^3\left(\frac{\pi x}{L}\right)$. If it is release from rest from the position, find the displacement $u(x, t)$. (K3)

(b) A rod of length $L$ with insulated sides is initially at a uniform temperature $u_0$. Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function $u(x, t)$. (K3)

(c) In a certain distribution, the first four moments about the point $x = 4$ are -1.5, 17, -30 and 308. Find the **moments about mean** and **about origin**. Also, calculate $\beta_1$ and $\beta_2$. (K3)

(d) Find the moment generating function (**mgf**) of the **discrete** binomial distribution given by

$$P(x) = {}^nC_x p^x q^{n-x} \quad (\text{where } q = 1 - p)$$

Also find its **mean** and **standard deviation**. (K3)

(e) Fit a **parabolic curve** of regression of $y$ **on** $x$ to the following data: (K3)

| $x$ | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|
| $y$ | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

## Section – B(CO4) # Attempt both the questions # 30 Marks

**Q.3:** Attempt any **SIX** questions (*Short Answer Type*). Each question is of two marks. (2 x 6 = 12 Marks)

(a) Write **normal equations** to fit a curve $y = ab^x$ by least squares method. (K2)

(b) Write the formula to find **skewness coefficient by Karl Pearson's method**. (K2)

(c) What is meant by **kurtosis**? (K2)

(d) If the lines of regress on of $y$ on $x$ and $x$ on $y$ are respectively $y = x + 5$ and $16x = 9y + 94$, then find the **correlation coefficients**. (K3)

(e) Write the **normal equations** of multiple linear regression of $z$ on $x$ and $y$. (K2)

(f) If the probability density function (p.d.f.) of the normal distribution is given by

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < x < \infty$$ where $\mu$ and $\sigma$ are mean and standard deviation respectively.

Then what will be the value of $\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$ and **why**? (K2)

(g) In a class of 10 students, 4 are boys and the rest are girls. Find the **probability** that a student selected will be a girl. (K3)

**Q.4:** Attempt any **THREE** questions (*Medium Answer Type*). Each question is of 6 marks. (3 x 6 = 18 Marks)

(a) Calculate the **standard deviation** and $\mu_3$ from the following data: (K3)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | 1 | 9 | 26 | 59 | 72 | 52 | 29 | 7 | 1 |

(b) Let the random variable X assume the value 'r' with the probability law

$$P(X = r) = pq^{r-1}, \quad r = 1, 2, 3, \ldots \quad (\text{where } q = 1 - p).$$

Find the moment generating function and hence find its mean and variance. (K3)

(c) Fit the curve $y = \frac{c_0}{x} + c_1\sqrt{x}$ to the following data by least squares method: (K3)

| $x$ | 0.1 | 0.2 | 0.4 | 0.5 | 1 | 2 |
|---|---|---|---|---|---|---|
| $y$ | 21 | 11 | 7 | 6 | 5 | 6 |

(d) In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible:

Standard deviation of $x = 3$,

Regression equations: $8x - 10y + 66 = 0$, $40x - 18y = 214$.

What were

- the mean values of $x$ and $y$
- the standard deviation of $y$ and the coefficient of correlation between $x$ and $y$ ? (K3)

(e) The contents of urns I, II and III are as follows:

1 white, 2 black and 3 red balls,
2 white, 1 black and 1 red balls, and
4 white, 5 black and 3 red balls.

One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urn I ? (K3)