IMPORTANT FORMULAS AND DEFINITIONS

UNIT-1

- Cases to find C.F

1. Both M_1 and M_2 real and Distinct

$$y(x) = C_1 e^{M_1 x} + C_2 e^{M_2 x}$$

2. If M_1 and M_2 real and equal

$$y(x) = (C_1 + xC_2) e^{M_1 x} \text{ or } = (C_1 + xC_2 + x^2 C_3) e^{Mx}$$

3. If $M_1 = \alpha \pm i\beta$

$$y(x) = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$$

4. If $M_1 = M_2 = \alpha + i\beta$ and $M_3 = M_4 = \alpha - i\beta$

$$y(x) = e^{\alpha x}[(C_1 + xC_2)\cos\beta x + (C_3 + xC_4)\sin\beta x]$$

5. If $M_1 = \alpha + \sqrt{\beta}$, $M_2 = \alpha - \sqrt{\beta}$

$$y(x) = e^{\alpha x}(C_1 \cos\sqrt{\beta} x + C_2 \sin \beta x)$$

- Rules to find P.I / cases:

i. $\phi = e^{ax}$ ie

$$P.I = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$$

$\therefore$ Failure

$$\frac{1}{f(D)}\phi = x \cdot \frac{1}{f'(a)} \cdot \phi.$$

Case : 2

If $Q = \sin(ax+b)$ or $(\cos(ax+b))$

$$\frac{1}{f(D)^2}\sin(ax+b) = \frac{1}{f(-a^2)}\sin(ax+b)$$

Fails:

$$\frac{1}{f(D)^2}\sin(ax+b) = x\frac{1}{f'(D^2)}\sin(ax+b)$$

3.) $Q = x^M$

$$P.I = \frac{1}{f(D)} x^M.$$

∴ Expand through binomial:

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$$

4. $Q = e^{ax} V$

$$P.I = \frac{1}{f(D)}(e^{ax}V) = e^{ax}\frac{1}{f(D+a)}V$$

5. $$\frac{1}{f(D)}x^n(\cos ax + i\sin ax) = \frac{1}{f(D)}x^n e^{iax}$$

∴ $e^{iax} = \cos ax + i\sin ax.$

Case 6 :

(i) $\frac{1}{D-\alpha} Q = e^{\alpha x} \int e^{-\alpha x} Q \, dx$

(ii) $\frac{1}{D+\alpha} Q = e^{-\alpha x} \int e^{\alpha x} Q \, dx$

• Steps to solve simultaneous Diff eqn :

(i) Linear :

→ Make perfect eqn using given eqns.

→ Use elimination method to find eqn and find CF and PI for x or y.

→ After find x and y find data of one eqn ie, $\frac{d}{dx}$ or $\frac{d^2}{dx^2}$ then put in eqn to find another x and y.

→ Question completed but if conditions were given then find them. By putting.

(ii) Order two.

- Make eqns.
- Use elimination method to eliminate x or y.
- Find C.F and P.I of the new eqn. then Put x or y = C.F + P.I.
- Find eqn data i.e, $\frac{d}{dt}$ or $\frac{d^2}{dt^2}$. Put in eqn.
- Use the given data if needed.

- Cauchy - Eulers equations.

Steps to find :

① Put $x = e^z$, $z = \log x$

② Replace $x\frac{d}{dx}$ by D

$x^2\frac{d^2}{dx^2}$ by $D(D-1)$

$x^3\frac{d^3}{dx^3}$ by $D(D-1)(D-2)$

- then find Auxiliary eqn, C.F, P.I
  then Put in
  eqn

$$y = C.F + P.I.$$

- Change of Independent Variable

Gen eqn: $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ — ①

Variables changes from x to z.
then eqn:

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1 \quad \text{— ②}$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}, \quad Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}, \quad R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$$

- Steps:

1. Make 1'st coeff 1
2. compare given eqn with ①
3. Find P, Q, R
4. choose z such that $\left(\frac{dz}{dx}\right)^2 = Q$.

- find $\frac{dz}{dx}$, z and $\frac{d^2z}{dx^2}$

5. Then find P_1, Q_1, and R_1

6. Put everything in eqn ② only P, Q, R
   → Find C.F and P.I.

7. Now again put z as x.

• Method of Variation of Parameters.

eqn: $$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{—①}$$

steps.

1. Make coeff of 1st as 1
2. Compare eqn with ① to find P, Q, R.
3. Find out C.F of the eqn:
4. Take one part of C.F as U and another as V.
5. Then find W as $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$
6. complet sol of Ques is

$$y = Au + Bv$$

7. Find $A = -\int \frac{Rv}{W}$, $B = \int \frac{Ru}{W}$

8. Put in Ques.

UNIT-2 : Laplace Transformation

Def : Let $F(t)$ be a function of t defined for all $t \geq 0$. Then the laplace transformation of $F(t)$, denoted by $L\{F(t)\}$ is defined by

$$L\{F(t)\} = f(P) = \int_0^\infty e^{-pt} F(t)\, dt.$$

* $L^{-1}\{F(P)\} = F(t)$

$F(T)$	$L\{F(t)\}$	$L^{-1}(F(t) \rightarrow$ soln
$1 \rightarrow$	$1/p$	$L^{-1}[1/p] = 1$
$t \rightarrow$	$1/p^2$	$L^{-1}\left[\frac{1}{p-a}\right] = e^{at}$
$t^n \rightarrow$	$n!/p^{n+1}$	$L^{-1}\left[\frac{1}{p^n}\right] = \frac{t^{n-1}}{(n-1)!}$
t^n	$\sqrt{n+1}/p^{n+1}$	
$e^{at} \rightarrow$	$1/p-a$	$L^{-1}\left[\frac{1}{(p-a)^n}\right] = e^{at}\frac{t^{n-1}}{(n-1)!}$
$e^{-at} \rightarrow$	$1/p+a$	
$\sin at \rightarrow$	a/p^2+a^2	$L^{-1}\left[\frac{1}{p^2 \pm a^2}\right] = \frac{1}{a}\sin at$, $\frac{1}{a}\sinh at$.
$\cos at \rightarrow$	p/p^2+a^2	$L^{-1}\left\{\frac{p}{p^2 \pm a^2}\right\} = \cos at$, $\cosh t$
$\sinh at \rightarrow$	$\frac{a}{p^2-a^2}$	
$\cosh at \rightarrow$	$\frac{p}{p^2-a^2}$	

• First shifting Property

$$L\{e^{at}F(t)\} = \int_0^\infty e^{-pt}e^{at}F(t)\,dt$$

$$= \int_0^\infty e^{-(p-a)t}F(t)\,dt = f(p-a)$$

* $L^{-1}\{f(p-a\} = e^{at}F(t)$

• Second Translation Property: $G(t) = \begin{cases} F(t-a) & t>a \\ 0 & t<a \end{cases}$

$$L\{G(t)\} = e^{-ap}f(p)$$

* $L^{-1}\{e^{-ap}f(p) = G(t)$

• Change of scale

$L\{F(t)\} = f(P)$
then

$\therefore L^{-1}f(ap) = \frac{1}{a}F\left(\frac{t}{a}\right)$

$$L\{F(at)\} = \frac{1}{a}f\left(\frac{P}{a}\right).$$

• Transformation of Integrals.

$L\{\int_0^t F(t)dt = \frac{1}{p}f(P)$; Put $p=1$ in last for Any

• Multiplication by t^n

$L\{t^nF(t) = (-1)^n\frac{d^n}{dp^n}f(P)$ $\therefore L^{-1}\{p f(P) = F'(t).$

• Division by t

$L\{F(t)\} = f(P)$ then

$L\{\frac{1}{t}F(t)\} = \int_P^\infty f(P)\,dP$

$\therefore \frac{L^{-1}f(p)}{p^n} = \int_0^t \int_0^t \cdots \int_0^t F(u)\,du$

• Unit step function:

$L\{u(t-a)\} = \int_0^\infty e^{-pt} u(t-a)\,dt.$

$= \frac{1}{p} e^{-ap}$

and

$L(u(t)) = \frac{1}{p}$

• Second shifting theo

$L\{F(t)\} = f(P)$ then

$L\{F(t-a) \cdot u(t-a)\} = e^{-ap} f(P)$

• To solve Ques.

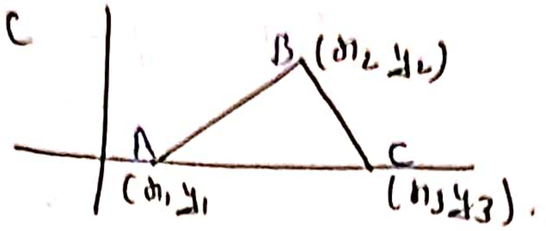
1. Compare with eqn $F(t-a)\,u(t-a)$

$a =$ ___ , $f(t) =$ ___

2. Then $L\{F(t)\} = f(P)$.

• If Ques is given in Graph

i.e



→ Find AB eqn by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Then write

$$f(t) = \begin{cases} \text{AB eqn } (t-a), & x < t < y \\ \text{BC eqn } (a-t), & x_1 < t < y_1 \end{cases}$$

then

$$\therefore F(t) = (t-a)\{u(t-x) - u(t-y)\} + (a-t)\{u(t-x_1) - u(t-y_1)\}$$

→ Then find Laplace of terms.

• Periodic function

$L\{f(t+T)\} = f(t)$ then

$$L\{f(t)\} = \frac{1}{1-e^{-pT}} \int_0^T e^{-pt} f(t)\,dt.$$

• Convolution Theorem:

$L^{-1}\{f(P)\} = F(t)$ and $L^{-1}\{g(P)\} = G(t)$ then

$$L^{-1}\{f(P)\,g(P)\} = F * G = \int_0^t F(u)\,G(t-u)\,du$$

ex: $L^{-1}\left\{\frac{p}{(p^2+1)(p^2+4)}\right\}$

If $L^{-1}\{f(P)\} = f(t)$ & $L^{-1}\{g(P)\} = G(t)$ then

$$L^{-1}\{f(P)\cdot g(P)\} = \int_0^t F(u)\cdot G(t-u)\,du$$

Let $f(P) = \frac{1}{p^2+4}$ and $g(P) = \frac{p}{p^2+1}$

$L^{-1}(F(t)) = \frac{1}{2}\sin 2t$ $\qquad L^{-1}\,G(P)$ or $G(t) = \cos t$

$\therefore F(u) = \frac{1}{2}\sin 2u$ $\qquad G(t-u) = \cos(t-u)$

$\therefore L^{-1}\{\text{function}\} = \int_0^t \frac{1}{2}\sin 2u \cos(t-u)\,du$

$$= \frac{1}{4}\left[-\frac{4}{3}\cos 2t + \frac{4}{3}\cos t\right]$$ ✓

- Normal Trigno formula:

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

- Application of Differential eqn:

Steps to solve:

Put $\frac{d^2y}{dt^2} = y''$, $\frac{dy}{dt} = y'$, $y = y$.

1. Take laplace transformation of both sides of the given differential eqn.

solve as

$$L\left(\frac{d^2y}{dt^2}\right) = \left(p^2\bar{y} - py(0) - y'(0)\right)$$

2. solve the algebraic eqn to get $\bar{y}$ in terms of p.
3. Take inverse laplace transform of both sides. This Gives y as a function of t which is the desired solⁿ.

ex:

$y'' + 9y = 6\cos t$:- laplace taking

$$[p^2\bar{y} - py(0) - y'(0)] + 9\bar{y} = \frac{6}{p^2+9} \Rightarrow (p^2+9)\bar{y} - 2p = \frac{6p}{p^2+9}$$

$$\bar{y} = \frac{6p}{(p^2+9)^2} + \frac{2p}{p^2+9}$$ (Inverse) $\rightarrow t\sin t + 2\cos 3t$.

• simultaneous diff eqn:

ex: $\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin t$ given $x(0) = 1, y(0) = 0$

sol:

Laplace Transform

$\cancel{[p\bar{y} - y(0)] - \bar{y}}$

$[p\bar{x} - x(0)] - \bar{y} = \frac{1}{p-1}$

$p\bar{x} - 1 - \bar{y} = \frac{1}{p-1}$

$p\bar{x} - \bar{y} = \frac{p}{p-1}$ —(1)

$[p\bar{y} - y(0) + \bar{x} = \frac{1}{p^2+1}$

$p\bar{y} + \bar{x} = \frac{1}{p^2+1}$ —(2)

solving (1) & (2) we get

$$\bar{x} = \frac{p^2}{(p-1)(p^2+1)} + \frac{1}{(p^2+1)^2}$$

and

$$\bar{y} = \frac{p}{(p^2+1)^2} - \frac{1}{2}\left[\frac{1}{p-1} - \frac{p}{p^2+1} + \frac{1}{p^2+1}\right]$$

Now Inverse laplace then

$x = \frac{1}{2}[e^t + \cos t + 2\sin t - t\sin t]$, $y = \frac{1}{2}[t\sin t - e^t + \cos t - \sin t]$

• UNIT - 3

1. Sequences: A sequence is a function $f: N \to S$, whose domain is the set N of all natural no whereas the range may be any set S.

• Convergent: A sequence $\{a_n\}$ is said to be convergent if $\lim_{n\to\infty} a_n$ is finite

ex: $\langle \frac{1}{2^n} \rangle = \{ \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3} \cdots \frac{1}{2^n} \rangle$

• Divergent: A sequence $\{a_n\}$ is said to be divergent if $\lim_{n\to\infty} a_n$ is not finite

i.e $\lim_{n\to\infty} a_n = \infty$ or $-\infty$

• Oscillatory sequence: If a sequence $\{a_n\}$ neither converges to finite number nor diverges to ∞ or $-\infty$ it. is called Oscillatory sequence.

③ **Bounded sequence:** A sequence is said to be bounded if $\exists$ two real no k and K $(k \le K)$ such that

$k \le a_n \le K$, $\forall n \in N$

ex: $\langle \frac{1}{n} \rangle = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdots\}$

- **Unbounded sequence:** If $\exists$ no real no M such that $|a_n| \le M$, $\forall n \in N$, then the seq $\langle a_n \rangle$ is said to be unbounded.

ex: $\langle 2^{n-1} \rangle = \{1, 2, 2^2, 2^3 \cdots\}$

④ **Monotonic sequence:-** A sequence is said to be Monotonic if it is either Monotonically increasing or Monotonically decreasing sequence.

Now

A seq $\{a_n\}$ is said to be Monotonically increasing if $a_{n+1} \ge a_n$

• p-series Test:

$$\sum \frac{1}{n^p} \Rightarrow \begin{array}{l} \text{convergent if } p>1 \\ \text{divergent if } p\leq 1 \end{array}$$

• Comparision Test:

$$\lim_{n\to\infty}\left(\frac{u_n}{v_n}\right) = l \text{ (fixed, finite, non-zero)}$$

$\Rightarrow$ Test is applicable

• D'Alembert Ratio Test:

$$\lim \frac{u_{n+1}}{u_n} = k$$

$\therefore$ convergent if $k<1$

Dgt $k>1$

Test fails $=1$

• Raabe's Test

$$\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right) = k.$$

Cgt >1

$k<1$

$k=1$

• Fourier Series:

Fourier series for the function $f(x)$ in the interval $c<x<c+2\pi$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx\, dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx\, dx$$

- **Dirichlet's conditions:**

Any function $f(x)$ can be expressed as Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, where a_0, a_n, b_n are constant, provided.

(i) $f(x)$ is periodic, single valued function.

(ii) $f(x)$ has a finite no of finite discontinuities in any one period.

(iii) $f(x)$ has a finite no of maxima and minima.

- Even and odd functio

A function is said to be even if

$$f(-x) = f(x)$$

ex: $x^2, \cos x, \sin^2 x$

A function is said to be odd if

$$f(-x) = -f(x)$$

ex: $x, x^3, \sin x, \tan x$

$\int_{-\pi}^{\pi} f(x)\,dx$ even $2\int_0^{\pi} f(x)\,dx$, If $f(x)$ is even

odd 0 If $f(x)$ is odd functio

- $\cos n\pi = (-1)^n$

- Change of Interval:

fourier series $f(x)$ in the interval

$$c < x < c+2l$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx.$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

- Half range series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

- Half range sine

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

• UNIT-4

• complex Variable ( ) : $x+iy$ is a complex Variable & it is denoted by z, so $z = x+iy$, where $i = \sqrt{-1}$

1) $z = x+iy \longrightarrow$ cartesian form.
2) $z = r(\cos\theta + i\sin\theta) \longrightarrow$ Polar form.
   where $r = \sqrt{x^2+y^2}$ & $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.
3) $z = re^{i\theta} \longrightarrow$ (Exponential form)
   $e^{i\theta} = \cos\theta + i\sin\theta$
   & $e^{-i\theta} = \cos\theta - i\sin\theta$

• continuity : If $f(z)$ is a function of complex Variable z.

Then $f(z)$ is said to be continuous at $z = z_0$ if

$$\lim_{z \to z_0} f(z) = f(z_0)$$

• Differentiablity : Let $f(z)$ be a single valued function

$$f'(z) = \frac{dw}{dz} = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$$

• **Analytic function :** A function $f(z)$ is said to be analytic at a point z_0, if $f(z)$ differentiable not only at z_0 but every point of some neighbourhood of z_0.

- A function $f(z)$ is analytic in a domain if it is analytic at every point of the domain.

• Necessary and sufficient condition for $f(z)$ to be Analytic:

(i) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$

(ii) $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

are known as cauchy-Riemann equation in Cartesian form provided $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial x}$ & $\dfrac{\partial v}{\partial y}$ exist.

• $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$

$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

$\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

→ $\sinh(x+iy)$

$\sinh x \cos y + i\cosh x \sin y$.

- To check Analyticity at origin:

$$\frac{\partial u}{\partial x} = \lim_{h \to 0} \frac{u(h,0) - u(0,0)}{h}$$

$$\frac{\partial u}{\partial y} = \lim_{k \to 0} \frac{u(0,k) - u(0,0)}{k}$$

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z} \quad \text{— (1)}$$

- Cauchy-Riemann equations in Polar form.

We know that

$$f(z) = f(x+iy) = f[r(\cos\theta + i\sin\theta)]$$
$$= f[re^{i\theta}] = u+iv$$

then C-R equation in Polar is

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad \& \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

$$x = r\cos\theta, \quad y = r\sin\theta$$

• Harmonic function: A function which satisfies Laplace eqn:

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 0.$$

To find conjugate: ① Prove Harmon function.

② find $\frac{\partial v, u}{\partial x}$, $\frac{\partial}{\partial y}$ then put in Milne-thomson method.

③ Take $f(z) \to$ outcome function put $\underline{z = (x+iy)}$ and $\boxed{e^{iy} = x\cos y + y\sin y}$

• Milne thomson - Method.

1. When u is given

$$f(z) = \int [\phi_1(z,0) - i\,\phi_2(z,0)]dz + c$$

where

$$\phi_1(x,y) = \frac{du}{dx}, \quad \& \ \phi_2(x,y) = \frac{du}{dy}$$

2. When img part is given

$$f(z) = \int [\psi_1(z,0) + \psi_2(z,0)]dz + c$$

$$\psi_1(x,y) = \frac{\partial v}{\partial y}, \quad \psi_2(x,y) = \frac{\partial v}{\partial x}$$

Case III when $u-v$ is given. —(1)

then

$$if(z) = iv - v \quad \text{—(2)}$$

Adding (1) & (2)

$$(1+i)f(z) = (u-v) + i(u+v)$$

$$F(z) = U + iV$$

$$\boxed{U = u-v, \quad V = u+v}$$

$$\boxed{F(z) = (1+i)f(z)}$$

- Conformal Mapping: A transformation which preserves angles both in magnitude & and senses b/w every pair of curves through a point is said to be conformal at the point.

- Bilinear Transformation:

$$\frac{(w_1-w_2)(w_3-w_4)}{(w_2-w_3)(w_4-w_1)} = \frac{(z_1-z_2)(z_3-z_4)}{(z_2-z_3)(z_4-z_1)}$$

UNIT - 5.

- **Cauchy Integral thm**: If $f(z)$ is analytic fuction and $f'(z)$ is continuous at each point within and on a simple closed curve C, then $\oint_C f(z)\,dz = 0$

- **Cauchy Integral formula**: If $f(z)$ is analytic within and on a closed curve C and a is any point within C, then

$$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}\,dz$$

Or

$$\oint \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$$

- **Cauchy Integral formula for Derivative**:

$$\int_C \frac{f(z)}{(z-a)^{n+1}}\,dz = \frac{2\pi i}{n!} f^{n}(a)$$

Or

$$\int_C \frac{f(z)}{(z-a)^{n}}\,dz = \frac{2\pi i}{(n-1)!} f^{n-1}(a).$$

• Taylor's series

$$f(z) = f(a) + (z-a)f'(a) + \frac{(z-a)^2}{2} f''(a) + \frac{(z-a)^n}{n!} f^n(a)$$

• Laurent expansion:

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} b_n (z-a)^{-n}$$

• Residue of pol

① simple pol

$$R = \lim_{z \to a} (z-a) f(z)$$

② of order m

$$R = \frac{1}{(m-1)!} \lim_{z-a} \frac{d^{m-1}}{dz^{m-1}} \left[(z-a)^m f(z) \right]$$