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Q1 let A be a set with n element. Justify that the power set P(A) has 2^n element.

Sol^n for a given set A with n element number of element in P(A) is 2^n as each element has two possibili-ties (present or absent)

Possible subset are $2 \times 2 \times 2 \cdots n$ time $= 2^n$
therefore power set P(A) contains 2^n element

Q2 let A, B, C be three non-empty sets Justify the following

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
Proof :-
let $(x, y) \in A \times (B \cup C)$
$x \in A$, $y \in B$ or $y \in C$
$x \in A$, $y \in B$ or $x \in A$, $y \in C$
$(x, y) \in A \times B$ or $(x, y) \in A \times C$
$(x, y) \in A \times B \cup (A \times C)$ — 1

let $(x, y) \in (A \times B) \cup (A \times C)$
$x \in A$, $y \in B$ or $x \in A$, $y \in C$

$x \in A,\ y \in B$ or $y \in C$

$(x, y) \in A \times (B \cup C)$ — (2)

from (1) and (2)

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Q. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proof :-

let $(x, y) \in A \times (B \cap C)$

$x \in A,\ y \in B \cap C$

$x \in A,\ y \in B$ AND $y \in C$

$x \in A,\ y \in B$ AND $x \in A,\ y \in C$

$(x, y) \in A \times B$ AND $(x, y) \in A \times C$

$(x, y) \in (A \times B) \cap (A \times C)$ — (1)

let $(x, y) \in (A \times B) \cap (A \times C)$

$x \in A,\ y \in B$ AND $x \in A,\ y \in C$

$x \in A,\ y \in B$, AND $y \in C$

$x \in A,\ y \in B \cap C$

$(x, y) \in A \times (B \cap C)$ — (2)

from (1) & (2)

$$\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$$

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3. $A-(B\cap C) = (A-B)\cup(A-C)$

Proof :-

$A-(B\cup C) = (A-B)\cup(A-C)$

we know $x-y = x\cap y^c$

so,

$A-(B\cap C) = A\cap(B\cap C)^c$

$= A\cap(B^c\cup C^c)$

[By demorgans law]

$(A\cap B^c)\cup(A\cap C^c)$

$(A-B)\cup(A-C)$

4. $(A^c\cup B^c)\cup(A^c\cup B^c) = A$

$(A\cap B)\cup(A\cap B^c)$

$A\cap(B\cup B^c)$

$A\cap U$

$A.$

Q3 find the union and intersection of multiset given by $m = [1,1,4,2,2,3]$ and $M = [1,2,2,6,3,3]$

soln

$m = [2\cdot 1, 2\cdot 2, 1\cdot 3, 1\cdot 4]$

$M = [1\cdot 1, 2\cdot 2, 2\cdot 3, 1\cdot 6]$

i) $m\cup M = [\max(2,1)\cdot 1]\ \max(2,2)\cdot 2$
$\max(1,2)\cdot 3, 1\cdot 4, 1\cdot 6]$

$= [2.1, 2.2, 2.3, 4.6]$

ii) $m \cap H = [\min(2,1), \min(2,2), 2$
$\min(7,2).3]$
$[1.1, 2.2, 1.3]$
$\{1, 2.2, 3\}$

Q4 How many reflexive and symmetric relations there are on a set A with n elements.

Ans On a set A with n element.

$\Rightarrow$ for reflexive relation $= 2^{(n^2)}$

$\Rightarrow$ for symmetric relation $= 2^n \times 2^{\frac{n(n-1)}{2}}$

Q5 let $A = \{1, 2, 3, 4\}$. Identify which of the following relations on A are reflexive, symmetric, transitive and antisymmetric

Ans $R_1 = \{(1,1), (1,2), (2,1), (2,3), (3,3), (4,1), (3,4)\}$

$\Rightarrow$ $(1,1), (3,3) \in R_1$ but $(2,2), (4,4) \notin R$

$\therefore R_1$ is not reflexive.

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=) $(1,2) \in R_1$ so $(2,1) \in R_1$
$(2,3) \in R_1$ but $(3,2) \notin R_1$

so R_1 is not symmetric

=) $(1,2) \in R_1$ and $(2,1) \in R_1$ so $(1,1) \in R_1$
$(2,3) \in R_1$ and $(3,3) \in R_1$ but
so R_1 is transitive

$(1,2) \in R_2$ and $(2,1) \in R_1$ but
$1 \neq 2$
so R_1 is not Antisymmetric

(b) $B = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$

=) $(1,1), (2,2), (3,3), (4,4) \in R_2$
so R_2 is reflexive

=) $(1,2) \in R_2$ so $(2,1) \in R_2$
$(1,4) \in R_2$ so $(4,1) \in R_2$
so R_2 is symmetric

=) $(1,2) \in R_2$ and $(2,1) \in R_2$
so $(1,1) \in R_2$

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$(1,4) \in R_2$ and $(4,1) \in R_2$

so $(1,1) \in R_2$

so R_2 is transitive

$(1,2) \in R_2$, $(2,1) \in R_2$

but $1 \neq 2$

so R_2 is not Antisymmetric

Q5. let $A = \{a,b,c\}$ and consider the relation R on A given by the matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

. find the matrix of the relations R^{-1}, R^2 and R^c. Also draw diagraphs for these three relations.

soln let $A = \{a,b,c\}$

given R on A $= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$R = \{(a,b), (a,c), (b,a), (b,c), (c,b)\}$

$R^{-1} = \{(b,a), (c,a), (a,b), (c,b), (b,c$

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$R^2 = R \circ R$

$\{(a,a), (a,b), (b,b), (a,c), (c,c), (b,a), (b,c), (c,b)\}$

$R^c = (A \times A) - R$

$= \{(a,b), (a,a), (a,c), (b,a), (b,c), (b,b), (c,a), (c,b), (c,c)\} - R$

$= \{(a,a), (b,b), (c,c), (c,a)\}$

matrices for relations

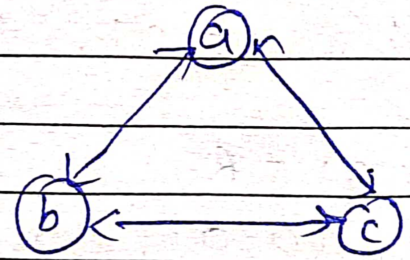
(i) $R^{-1} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} (a) & (b) & (c) \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$

(ii) $R^2 = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$

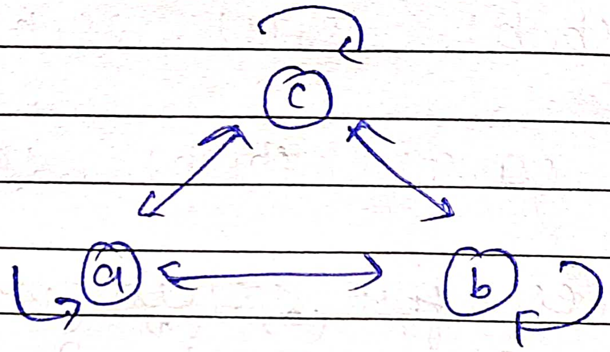
(iii) $R^c = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$

Diagraphs for relations

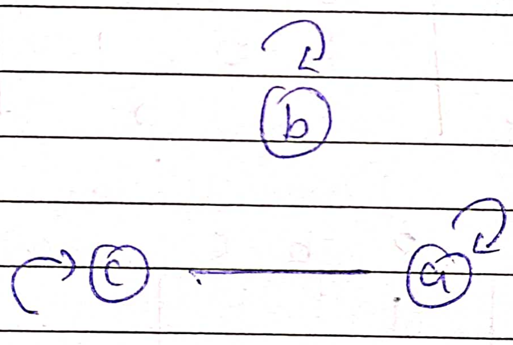
(i) $R^{-1} \Rightarrow$



(ii) $R^2 \Rightarrow$



(iii) R^c



Q7 Let $A = \{1, 2, 3, 4, 5, 6\}$ and consider the relations R on A given by R $\{(1,2), (1,3), (2,4), (5,6)\}$.

Find the reflexive, symmetric and the transitive closure of R.

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sol^n let $A = \{1, 2, 3, 4, 5, 6\}$ given that R on A

$R = \{(1,2), (1,3), (2,4), (5,6)\}$

(i) for reflexive closure
let $\Delta R = A \times A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Reflexive closure

$S_{ref} = R \cup \Delta R$

$S_{ref} = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (1,3), (2,4), (5,6)\}$

(ii) for symmetric closure
let $R^{-1} = \{(2,1), (3,1), (4,2), (6,5)\}$

symmetric $= R \cup R^{-1}$

$S_{sy} = \{(1,2), (1,3), (2,4), (5,6), (2,1), (3,1), (4,2), (6,5)\}$

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(iii) for transitive closure.

now $R^2 = R \circ R$.

$\{(1,2), (1,3), (2,4), (5,6), (6,4)\}$

$R^* = R \cup R^2 = \{(1,2), (1,3), (2,4), (5,6), 6,5)\}$

Q8 let $R = \{Z\}$ the set of the integ-erated for $a, b \in A$ define $a \sim b \Leftrightarrow$ 5 divides $b-a$. Justify that $\sim$ is an equivalent relation. Also, write all elements of the set $Z/6$

Soln given that $A = Z/$ the set of integer

$(a,b) \in A$

$a \sim b \Leftrightarrow (b-a)/6$

now relation $(\sim) = \{(a,b) \in A \times A$ $(b-a)$ is divisible by 6 $\}$

① for reflexive relation:
let $x \in A$, now we check $(x,x) \in R$.

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$x - x = \frac{0}{6} = 0$

$R(x)$ is reflexive.

(ii) for symmetric
let $(x,y) \in R$, check $(y,x) \in R$.

$(x-y) = 6k_1$

$y - x = -6k_1$

$\therefore \{ k_1 \in A, -k_1 \in A \}$
as $A = Z$

$y - x$ is divisible by 6

$(y,x) \in R$.

so R is symmetric

(iii) for transitive

let $(x,y) \in R$, $(y,z) \in R$
check $(x,z) \in R$

$(x-y)$ is divisible by 6

$x - y = 6k_1$ — (1)

$(y,z) \in R$

$y - z = 6k_2$ — (2)

eq^n (1) + (2)

$(x-y) + (y-z) = 6k_1 + 6k_2$

$x - z = 6(k_1 + k_2)$

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$x - 2 = 6k_3$ let $k_1 + k_2 = k_3$.

$\sim$ is a equivalence relation

Q9 Let $A = \{1, 2, 3, 4\}$. Use Warshall algorithm to compute the transitive closure of the relation R on A given by R $\{(1,1), (1,3), (2,1), (2,2), (3,3), (3,4)\}$

Soln from relation R, using Warshall algorithm

$$W_0 = M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step-1.

computing W_1

In W_0, row index of C_1 is 1, 2

column index of R_1 is 1, 3.

make entries '1' at (1,1), (1,3), (2,1), (2,3)

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$$w_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2 :-

computing w_2
In w_1, Row index of C_2 : 2
column index of R_2 : 1, 2, 3.
make entries '1' at (2,1), (2,2),
(2,3)

$$w_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 3 :-

computing w_3 :
In w_2, Row index of C_3 : 1, 2, 3.
column index of R_3 - 3, 4.
make entries at '1' at
(1,3), (1,4), (2,3), (2,4), (3,3),
(3,4).

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$$w_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

<u>Step 4 :-</u>

computing w_4 :-
In w_3, row index of C_4 : 1, 2, 3
column index of R_4 : 0

So $\boxed{w_3 = w_4}$

Now, transitive element like of R.

$R^* = \{(1,1), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,3), (3,4)\}$

$\Delta R = \{(1,4), (2,3), (2,4)\}$

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Q10 Let P consist of eight sets S0, S1, S2, S3, S4, S5, S6, S7 given by

$S_0 = \{a, b, c, d, e, f\}$

$S_1 = \{a, b, c, d, e\}$

$S_2 = \{a, b, c, d, e, f\}$

$S_3 = \{a, b, c, e\}$

$S_4 = \{a, b, c\}$

$S_5 = \{a, b\}$

$S_6 = \{a, c\}$

$S_7 = \{a\}$

Draw the Hasse diagram of poset $(P, \subseteq)$.

Soln $P(S) = \{\phi, S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$