

4th UNIT PDA

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Check given Language is CFL OR NOT:-

- ① $L = \{a^h b^h c^m \mid n > m\}$
- ② $L = \{a^{m+n} b^n c^n \mid m, n \geq 1\} \rightarrow \text{DCFL, CFL}$
- ③ $L = \{a^m b^m c^n d^n \mid m, n \geq 1\} \rightarrow \text{DCFL, CFL}$
- ④ $L = \{a^m b^n c^m d^n \mid m, n \geq 1\} \rightarrow \text{NCFPL, CFL NOT CFL}$
- ⑤ $L = \{a^m b^l c^m d^j \mid m, l, j \geq 1\} \rightarrow \text{CFL, DCFL}$
- ⑥ $L = \{a^i b^j c^k \mid k = i+j\} \rightarrow \text{DCFL, CFL}$
- ⑦ $L = \{a^i b^j c^k d^l \mid i=k, j=l\} \rightarrow \text{not DCFL, not CFL}$
- ⑧ $L = \{a^n b^{2n} \mid n \geq 1\} \rightarrow \text{DCFL, CFL}$
- ⑨ $L = \{a^i b^j c^k d^l \mid i, j, k, l \geq 1\} \rightarrow \text{DCFL, CFL}$
- ⑩ $L = \{a^n b^n c^n \mid n \geq 1\} \rightarrow \text{not DCFL; not CFL}$

- ⑪ $L = a^m b^n c^n d^n \mid n \geq 1 \rightarrow \text{not DCFL, NOT CFL}$
- ⑫ $L = w c w^\dagger \mid w \in \{a, b\}^*$ $\rightarrow \text{DCFL, CFL}$
- ⑬ $L = w w^\dagger \mid w \in \{a, b\}^*$ $\rightarrow \text{DCFL, CFL only}$
- ⑭ $L = w w^\dagger \mid w \in \{a, b\}^*$ $\begin{matrix} w=abb \\ ww=abbabb \end{matrix} \rightarrow \text{DCFL, CFL only}$
- ⑮ $L = a^{n^2} \mid n \geq 1 \Rightarrow a, a^4, a^9, a^{16}, \dots \text{not in AP} \rightarrow \text{not DCFL}$
- ⑯ $L = a^{2^n} \mid n \geq 1 \Rightarrow a^2, a^4, a^8, \dots \text{not in AP} \rightarrow \text{not DCFL}$
- ⑰ $L = a^n b^{n^2} \mid n \geq 1 \rightarrow \text{not DCFL, not CFL}$
- ⑱ $L = a^n b^{2n} \mid n \geq 1 \rightarrow \text{not DCFL, not CFL}$
- ⑲ $L = a^n b^{2n} c^{3n} \mid n \geq 1 \rightarrow \text{not DCFL, not CFL}$
- ⑳ $L = h_a(w) = h_b(w) = h_c(w) \mid w \in \{a, b\}^* \rightarrow \text{not DCFL, not CFL}$

Ques:- Check give Language CFL or not $L = a^p \mid p \in \mathbb{N}$

Pumping Lemma [For Context Free Language]

* Pumping Lemma [for CFL] is used to prove that a language is NOT context free

Context Free Language:-

In formal language theory a context free language is a language generated by some context free grammar.

The set of all CFL is identical to the set of languages accepted by pushdown Automata

context free Grammar is identified by 4 tuples as

$$G = \{ V, \Sigma; S, P \} \text{ where}$$

V = Set of variables or Non-terminal

Σ = set of Terminal symbol

S = start symbol

P = Production Rule

context free grammar has production rule of the form.

$$A \rightarrow a$$

$$\text{where, } a = \{V \cup \Sigma\}^*$$

* If A is a context free language, then, A has a pumping length "p" such that any string ' s ' where $|s| \geq p$ may be divided into 5 pieces $s = uvxyz$ such that the following conditions

conditions must be true:

- (1) $uv^i x y^i z$ is in A for every $i \geq 0$
- (2) $|vy| > 0$
- (3) $|vxy| \leq p$

To prove that a language is Not context free using Pumping Lemma [For CFL] follow the steps given below:-

[We Prove using CONTRADICTION]

- ⇒ Assume that A is context free
- ⇒ It has to have a pumping length (say p)
- ⇒ All strings longer than p can be pumped $|S| \geq p$
- ⇒ Now find a string 's' in such that $|S| \geq p$
- ⇒ Divide s into 5 parts $uvxyz$
- ⇒ Show that $uv^i xy^i z \notin A$ for some i
- ⇒ They consider the ways that s can be divided into $uvxyz$.
- ⇒ Show that none of these can satisfy all the 3 pumping condition at the same time
- ⇒ S cannot be pumped = CONTRADICTION

Ques:- Show that $L = \{a^n b^n c^n | n \geq 0\}$ is context free.

take a string 's' such that $S = a^p b^p c^p$
Eg:- $p=4$

Case 1:- v & y each contains only one cube of

$$L = \{ \underbrace{aaaa}_{4} \underbrace{bbbb}_{x} \underbrace{cccc}_{yz} \}$$

(2) $|VY| > 0$, $|aac| = 3$ satisfy?

(3) $|VXY| \leq p$ $aaaaabbbbc$ $|V| \leq 4$ not satisfy

(i) $uv^ix^iy^iz$ $i=2$

$$\begin{aligned} & uv^2 x y^2 z \\ & a(aa)^2 abbbb c(c)^2 cc \\ & aaaaaabb b b cccccc \\ & a^6 b^4 c^5 \neq L \end{aligned}$$

Case 2: Either v or y has more than one kind of symbol.

$$L = \{ \underbrace{aaaa}_{u} \underbrace{abbb}_{v} \underbrace{cccc}_{xy} \underbrace{zz}_{z} \}$$

(i) $|aabbb| = 5$ $|VY| > 0$ ✓

(ii) $|aabbb| \leq 4$

$$\begin{aligned} (iii) \quad uv^i x y^i z &= aa (aabbb)^2 b cccc \\ &\Rightarrow aa aabbaabb b cccc \notin L \\ &\Rightarrow a^4 b^2 a^2 b^3 c^4 \text{ not CFG} \end{aligned}$$

Q4 Show that $L = \{ ww \mid w \in \{0, 1\}^* \}$ is Not Context free.

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- ⇒ Assume that L is context free
⇒ L must have a pumping length (say, p)
⇒ Now we take a string s such that
 $s = 0^p 1^p 0^p 1^p$
⇒ We divide s into parts $uvxyz$

Eg $p = 5$
 $s = 0^5 1^5 0^5 1^5$

CASE 1:- vxy does not straddle a boundary

00000 1111 "00000" 1111
u vxy z

$|vy| > 0$ $|v| > 0$ ✓
 $|vxy| \leq p$ $|v| + |y| \Rightarrow |z| \leq 15$, which p length

$uv^i xy^i z$ $i = 2$

$uv^2 xy^2 z$

000001[1]²1[1]²0000011111

00000111110000011111

0⁵ 1⁷ 0⁵ 1⁵

Here 1st half not equal to 2nd half.

∴

$0^5 1^7 0^5 1^5 \notin L$

CASE 2a: vxy straddles the first boundary

00000'11111'00000'11111
4 vxy z

$$|v^i y^i| > 0$$

$$|v^i x y^i| \leq p \quad |z| \leq 5$$

$$uv^i x y^i z \Rightarrow uv^2 x y^2 z$$

 $i=2$

000 [00] $\underbrace{[11]}_2^8$ $\underbrace{[11]}_2^2$ 1100000 [111]
0000000111111000000 $\underbrace{111}_3$
07 17 05 15

So 1st half not equal to 2nd Half.

$07 17 05 15 \notin L$

CASE 2b: vxy straddles the third boundary

00000'11111'00000'11111
u $\underbrace{vxy}_2 z$

$$uv^2 x y^2 z$$

00000111110000000111111
05 15 07 17

So 1st half not equal to 2nd Half.

$05 15 07 17 \notin L$

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CASE 3a :- vxy straddles the mid point.

00000'11111'00000'11111
 \underbrace{\hspace{1cm}}_4 \underbrace{\hspace{1cm}}_{vxy} \underbrace{\hspace{1cm}}_z

uv^2xy^2z

00000111111000000011111

0⁵1⁷ 0⁷0⁵

1st half not equal to 2nd half

0⁵1⁷0⁷0⁵ $\notin L$

{ In 1211 4 cases our condition not satisfied
so our language is not context free }

★ PUSHDOWN AUTOMATA

A PDA is a way to implement a context free grammar in a similar way we design finite automata for regular grammar.

- * It is powerful than FSM
- * FSM has a very limited memory but PDA has more memory.
- * PDA = Finite State Machine + A stack.

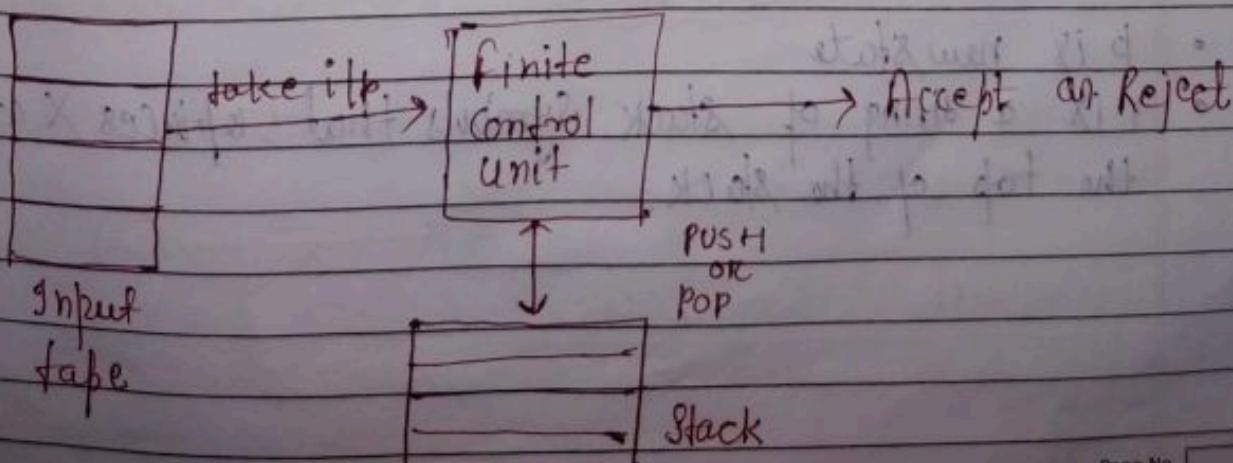
Stack:- It is a data structure, a stack is a way we arrange elements one on top of another.

A stack does two basic operations.

- **PUSH**:- A new element is added at the Top of the Stack
- **POP**:- The top element of the stack is read & removed.

Pushdown Automata has 3 components:-

- ① Input tape
- ② A finite control
- ③ A stack with infinite size.



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PDA :- A PDA is formally defined by 7 tuple as

shown.

$$P = (\mathcal{Q}, \Sigma, \Gamma, \delta; q_0, z_0; F)$$

Where,

$\mathcal{Q} \Rightarrow$ A finite set of symbols

$\Sigma \Rightarrow$ A finite set of input symbols

$\Gamma \Rightarrow$ A finite stack Alphabet

$\delta \Rightarrow$ The Transition Function

$q_0 \Rightarrow$ The start state

$z_0 \Rightarrow$ The start stack symbol

$F \Rightarrow$ The set of final/Accepting States

δ takes as argument a triple $\delta(q, a, X)$ where:

(i)

q is a state in \mathcal{Q}

(ii)

a is either an input symbol in Σ or $a = \epsilon$

(iii)

X is a stack symbol, that is a member of Γ

The output of δ is finite set of pairs (p, Y)
where:-

- p is new state

- Y is a string of stack symbols that replaces X at the top of the stack.

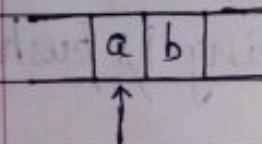
Eg: If $Y = \epsilon$ then the stack is popped.

If $Y = X$ then the stack is unchanged.

If $Y = YZ$ then X is replaced by Z and Y is pushed onto the stack.

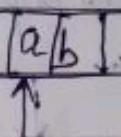
* Basic operation on stack:

(1) PUSH



$$(q_i, a, z_0) \xrightarrow{a, z_0 / 0z_0} (q_j)$$

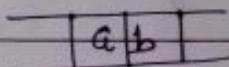
(2) POP



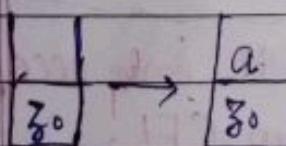
$$(q_i, a, c / \epsilon) \xrightarrow{a, c / \epsilon} (q_j)$$

(3) SKIP

(no push + no pop)

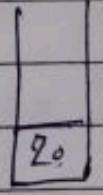
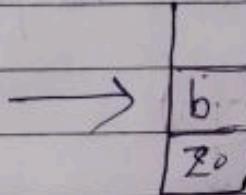


$$(q_i, a, z_0) \xrightarrow{a, z_0 / 0z_0} (q_j)$$

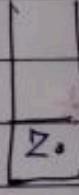


Stack

$$S(q_i, a, z_0) \rightarrow (q_j, a z_0)$$



\rightarrow

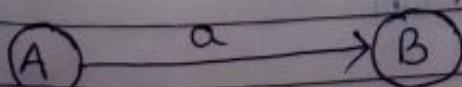


$$S(q_i, a, z_0) \rightarrow (q_j, z_0)$$

$$S(q_i, a, c) \rightarrow (q_j, \epsilon)$$

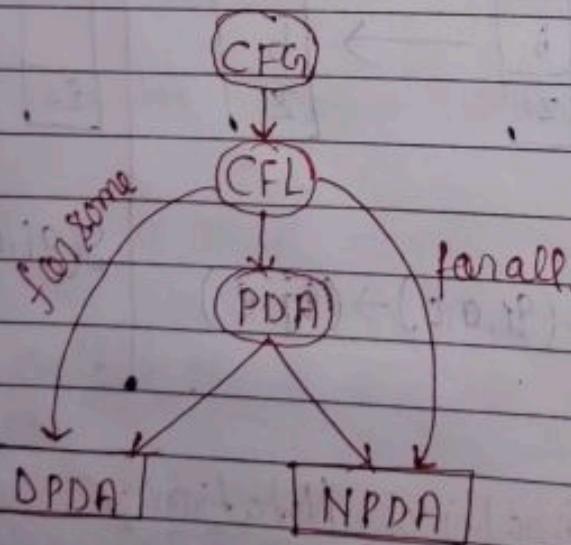
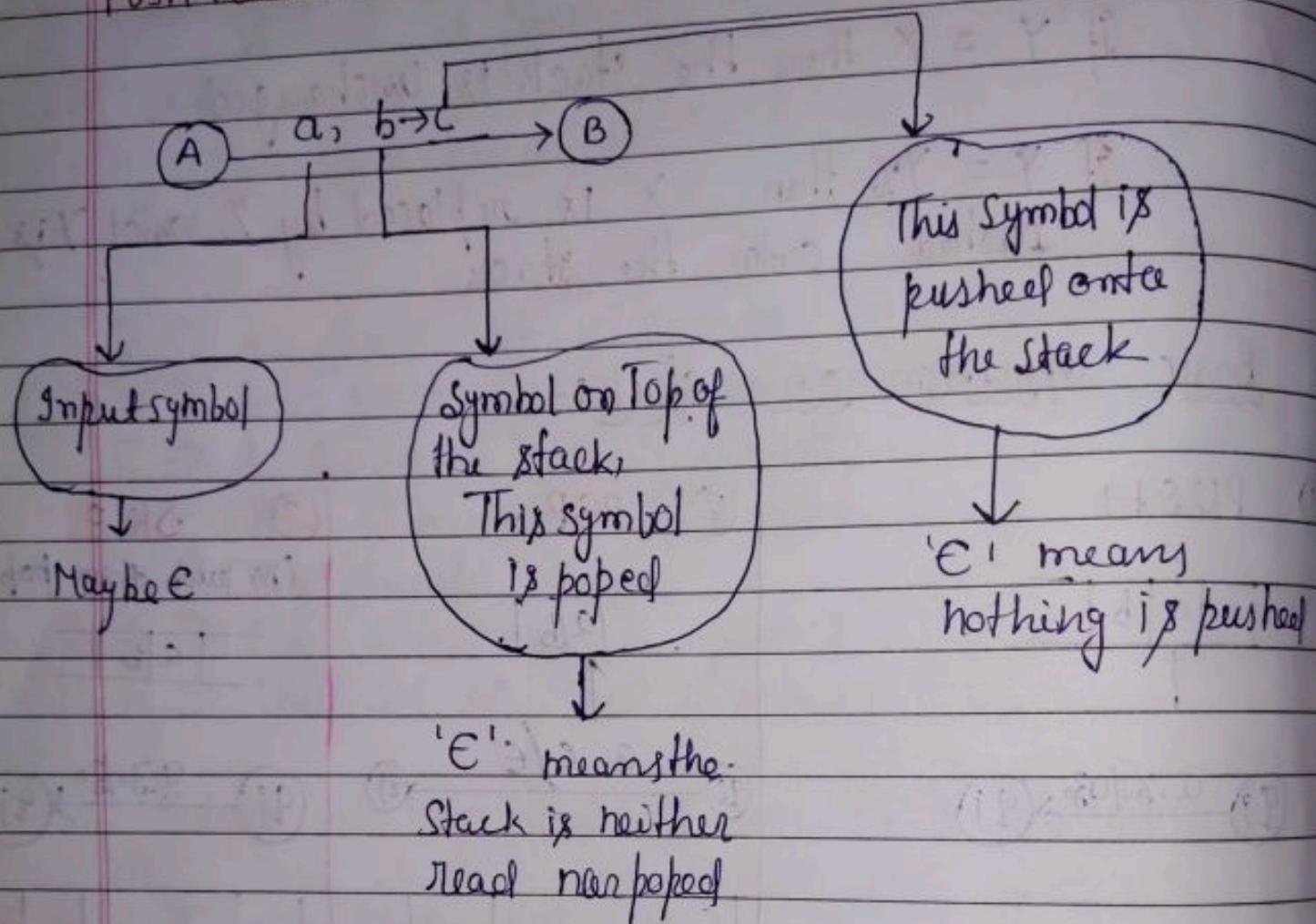
PUSHDOWN Automata {Graphical Notation}

Finite state Machine



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PUSH down Automata



DPDA only accept some CFL, which is known as Deterministic CFL, [DCFL]

NPDA accepts all CFL

DPDA \rightarrow Deterministic Pushdown Automaton,
NPDA \rightarrow Non-deterministic PDA.

$$P(\text{NPDA}) > P(\text{DPDA})$$

NPDA is more powerful than DPDA

Transition function of DPDA & NPDA.

DPDA:-

$$\{\delta : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*\}$$

NPDA:-

$$\{\delta : Q \times \Sigma \times \Gamma \rightarrow (Q \times \Gamma^*)\}$$

Eg:- Let PDA ($A = \{Q, \Sigma, T, \delta, q_0, z_0, F\}$)

$$Q = \{q_0, q_1, q_f\} \quad \Sigma = \{a, b\} \quad T = \{a, z_0\} \quad F = \{q_f\}$$

$$\delta(q_0, a, z_0) = \{(q_0, az_0)\}$$

$$\delta(q_0, a, a) = \{(q_0, aa)\}$$

$$\delta(q_0, b, a) = \{(q_1, \epsilon)\}$$

$$\delta(q_0, b, a) = \{(\epsilon, \epsilon)\}$$

$$\delta(q_1, \epsilon, z_0) = \{(q_0, \epsilon)\}$$

input string
 $w = aabb$

Qn: $\delta(q_0, aabb, z_0)$ $(q_0, \overset{a}{abb}, az_0)$

$$(q_0, \overset{b}{bb}, aaz_0)$$

\hookrightarrow (Here 'a' will be popped)

$$(q_1, \overset{b}{b}, aaz_0)$$

\hookrightarrow (Here 'a' is popped)

$$(q_1, \epsilon, az_0)$$

$$(q_0, \epsilon, \epsilon)$$

String accepted

\Rightarrow string accepted case:-

Eg:- If stack is empty

In this example we are on initial state but stack is empty.

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Eg: Let input string $aacabb$.

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, A)$$

$$\delta(q_1, b, a) = (q_1, A)$$

$$\delta(q_1, A, z_0) = (q_f, z_0)$$

$$\rightarrow (q_0, aacabb, z_0) \xrightarrow{\quad} (q_0, aacabb, az_0)$$

$$\xrightarrow{\quad} (q_0, acabb, aaaz_0)$$

$$\xrightarrow{\quad} (q_0, bbb, aaaaz_0)$$

$$\xrightarrow{\quad} (q_1, bb, aaaz_0)$$

$$\xrightarrow{\quad} (q_1, b, aaaz_0)$$

$$\xrightarrow{\quad} (q_1, A, z_0)$$

$$\xrightarrow{\quad} (q_f, A, z_0)$$

In this example we are on final state but stack is non empty.

PDA Acceptance string:

?) String is accepted by the PDA when we get to the acceptance state and see that the stack is empty.

) PDA can accept by accepting both empty state & empty stack.

\Rightarrow Graphical representation of PDA

Eg:-

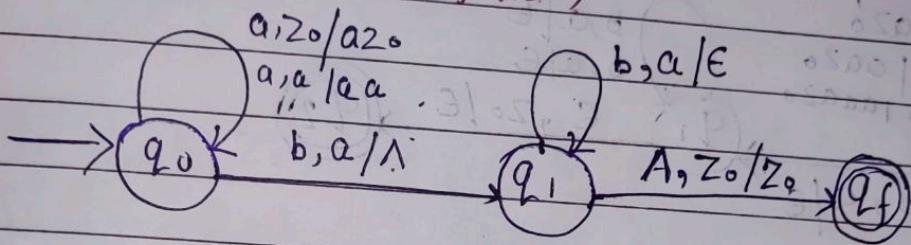
$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \lambda)$$

$$\delta(q_1, b, a) = (q_1, \lambda)$$

$$\delta(q_1, \lambda, z_0) = (q_f, z_0)$$



Ques Construct the PDA for the language $L = \{a^n b^n \mid n \geq 1\}$

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$$

Step 1:-

Initially push all 'a's onto the stack

Step 2:-

Whenever "b" occurs change the state of pop "a" from the stack.

Step 3:-

Repeat Step 2 until stack is empty.

$$L = \{ab, aabb, aaabb - \dots\}$$

a	a	a	b	b	b	ε
---	---	---	---	---	---	---

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa z_0)$$

$$\delta(q_0, a, a) = (q_0, aaa z_0)$$

$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

a

a

a

z0

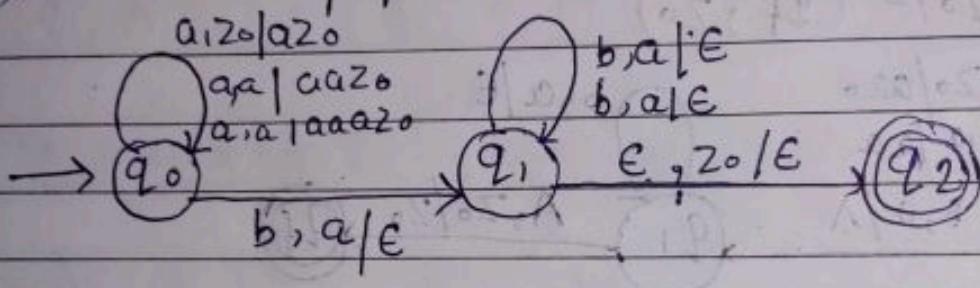
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$$S(q_1, \epsilon, z_0) = (q_2, \epsilon)$$



↓
Here input string completely encoded, so we will change the state.

% also pop, now stack becomes empty.



Now check whether this string accepts or not
• w2: aabb

$$S(q_0, aabbz_0) \rightarrow (q_0, abb, az_0)$$

$$\rightarrow (q_0, bb, aaaz_0)$$

$$\rightarrow (q_1, b, az_0)$$

$$\rightarrow (q_1, \epsilon, z_0)$$

$$\rightarrow (q_2, \epsilon)$$

↓ ↓
final state & stack is empty.

Design PDA for

$$(i) L = a^n b^n \mid n \geq 0$$

$$(ii) L = a^n b^{2n} \mid n \geq 0$$

$$(iii) L = a^n b^n c^m \mid n, m \geq 1, m \neq n$$

$$(iv) L = a^n b^n c^n \mid n, m \geq 1, m \neq n$$

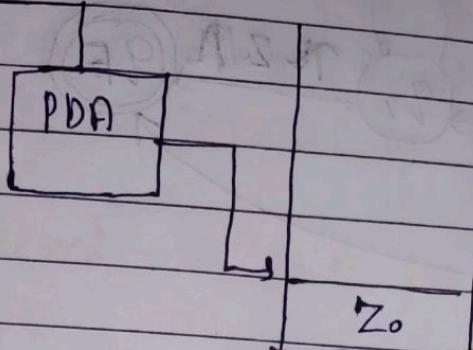
$$(v) L = a^n b^n \mid n \geq 0$$

$$L = \{ \text{aa, ab, aabb, aaabb} \}$$

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'a a a b b b' \in PDA



$$\delta(q_0, \lambda, z_0) \rightarrow (q_f, \lambda)$$

$$\delta(q_0, a, z_0) \rightarrow (q_1, az_0)$$

$$\delta(q_1, a, a) \rightarrow (q_1, aa z_0)$$

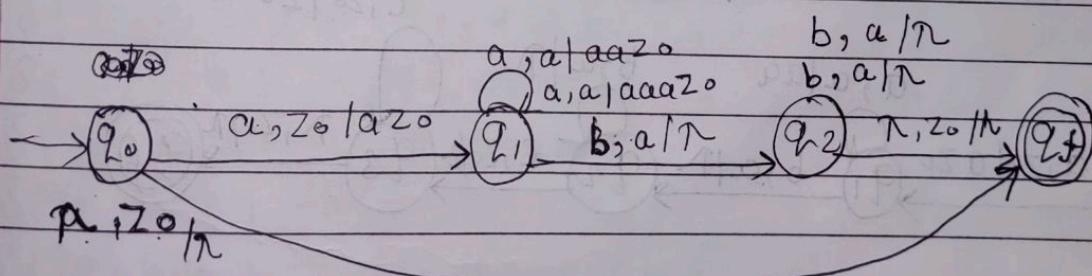
$$\delta(q_1, a, a) \rightarrow (q_1, a a a z_0)$$

$$\delta(q_1, b, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, b, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, b, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, \lambda, z_0) \rightarrow (q_f, \lambda)$$



$$(9i) L = a^n b^{2n} \mid n \geq 0 \Rightarrow L = \{ \lambda, abb, aa bbbb, aaa bbbb bbb, \dots \}$$

$$\delta(q_0, \lambda, z_0) \rightarrow (q_f, \lambda)$$

$$\delta(q_0, a, z_0) \rightarrow (q_1, a a z_0)$$

$$\delta(q_1, a, a) \rightarrow (q_1, a a a a z_0)$$

$$\delta(q_1, b, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, b, a) \rightarrow (q_2, \lambda)$$

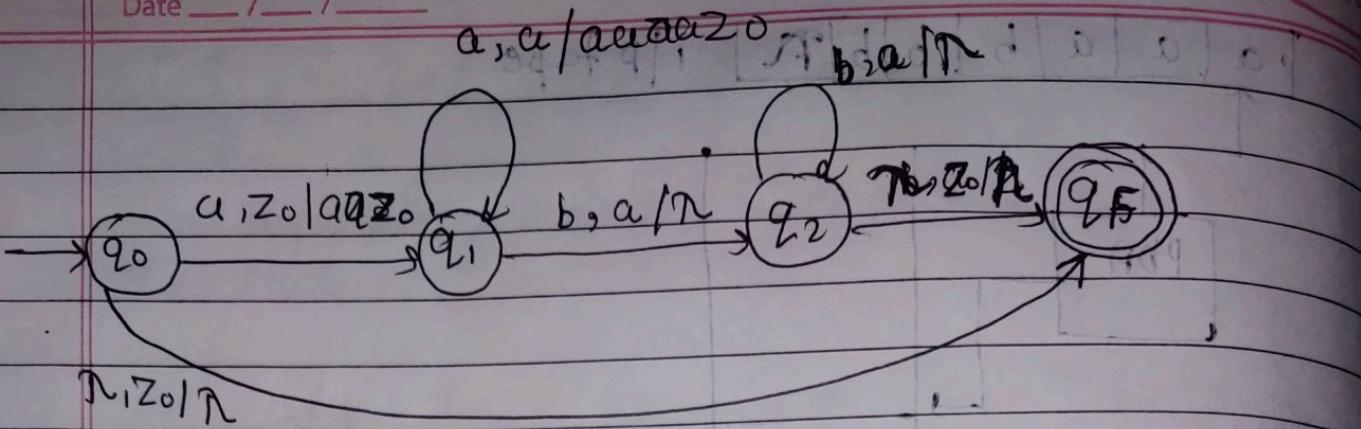
$$\delta(q_2, \lambda, z_0) \rightarrow (q_f, \lambda)$$

faisanya

we insert two
a's into
stack

a
a
a
a
z0

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$$(iii) \quad t = a^n b^m c^m \mid n, m \geq 1, \quad (m \neq n)$$

$$L = \{ abcc, aabbccc \dots \}$$

$(\tau, \eta p) \leftarrow (\circ\delta, \tau, \circ p)$

$$s(q_0, a, z_0) \rightarrow (q_1, q_{z0})$$

$$s(q_1, q, a) \rightarrow (q_1, qa) \xrightarrow{S_0(\mu)} (s_0, s_1(\mu))$$

$$s(q_1, b, a) \rightarrow (q_2, \pi) \quad \text{with } (\pi) \leftarrow (a \mid 0 \cup p)$$

$\delta(q_2, b, a) \rightarrow (q_2, \pi)(r, cp) \leftarrow (a, d, 18)$

$$S(q_2, c) \otimes_0 \rightarrow (q_3, \otimes_0) \cap S(p) \leftarrow (q_1, d) \otimes_0 S$$

$$\delta(q_3, c, z_0) \rightarrow (q_3, z_0) \leftarrow (p_3, s_3)$$

$$\delta(q_3, \gamma, z_0) \rightarrow (q_f, \gamma)$$

$$(iv) \quad L = a^m b^n c^n \mid n, m \geq 1, m \neq n$$

$$L = \{ abbcc, aabbccc, aaaa bbbb cccc \dots \}$$

$$\delta(q_0, a, z_0) \rightarrow (q_1, z_1)$$

$$s(q_1, a, z_0) \rightarrow (q_1, z_0)$$

$$\delta(q_1, b, z_0) \rightarrow (q_2, bz_0)$$

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$$\delta(q_2, b, b) \rightarrow (q_2, bb)$$

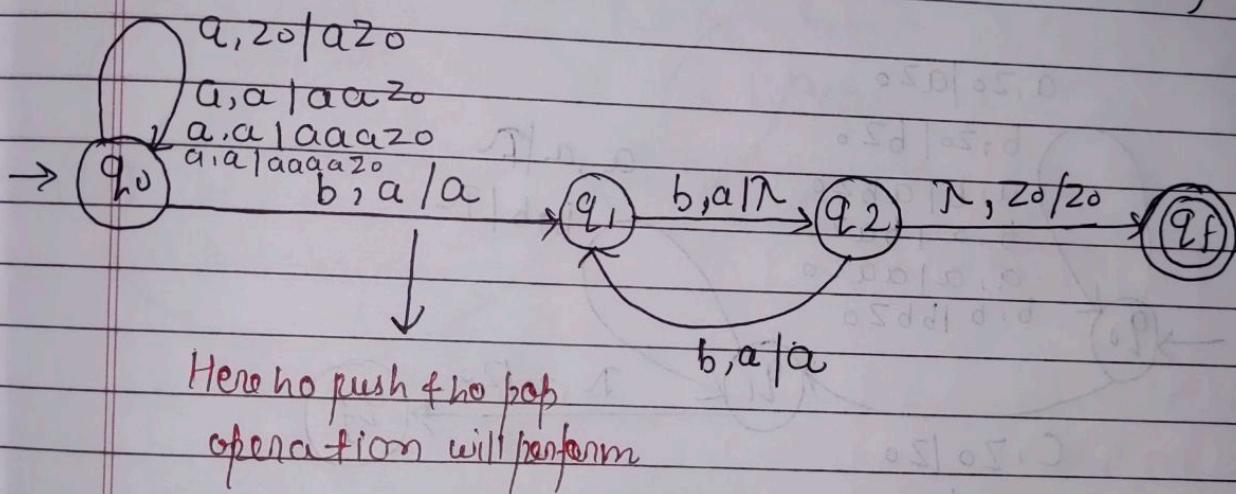
$$\delta(q_3, c, b) \rightarrow (q_4, \gamma)$$

$$\delta(q_4, c, b) \rightarrow (q_4, \gamma)$$

$$\delta(q_4, \gamma, z_0) \rightarrow (q_f, \gamma)$$

Ques construct a PDA for $L = a^n b^{2n} / n \geq 1$

$$L = \{abb, aabbff, aaeebbbbb\}$$



$$\delta(q_0, a, z_0) \rightarrow (q_0, aza_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aaaza_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aaaaza_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aaaaaza_0)$$

$$\delta(q_0, b, a) \rightarrow (q_1, aaaaza_0)$$

$$\delta(q_1, b, a) \rightarrow (q_2, \gamma)$$

$$\delta(q_2, b, a) \rightarrow (q_1, aaaza_0)$$

$$\delta(q_2, \gamma, z_0) \rightarrow (q_f, z_0) \quad \text{accepted}$$

Ques construct PDA for the given $L = (wCw^* | w \in (a,b)^*)$

$$w = ab \quad w^* = ba \quad wCw^* = abcba$$

$$w = baba \quad w^* = babab \quad wCw^* = babaCbabab$$

$$w = a \quad w^* = a \quad wCw^* = aca$$

$$w = \epsilon \quad w^* = \epsilon \quad wCw^* = \epsilon C \epsilon = \epsilon$$

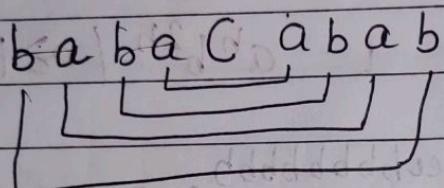
Here C will be decision making

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Language will be

$$L = \{c, aca, bcb, (abcba, babaCabab) \}$$



a, z₀ | a₂z₀

b, z₀ | b₂z₀

a, b | ab₂z₀

b, a | ba₂z₀

a, a | aa₂z₀

b, b | bb₂z₀

a, a | F

b, b | N

N, z₀ | Z₀

$\rightarrow q_f$

C, z₀ | Z₀

C, a | a

C, b | b

b
a
z ₀

a'
a
z ₀

b
b
z ₀

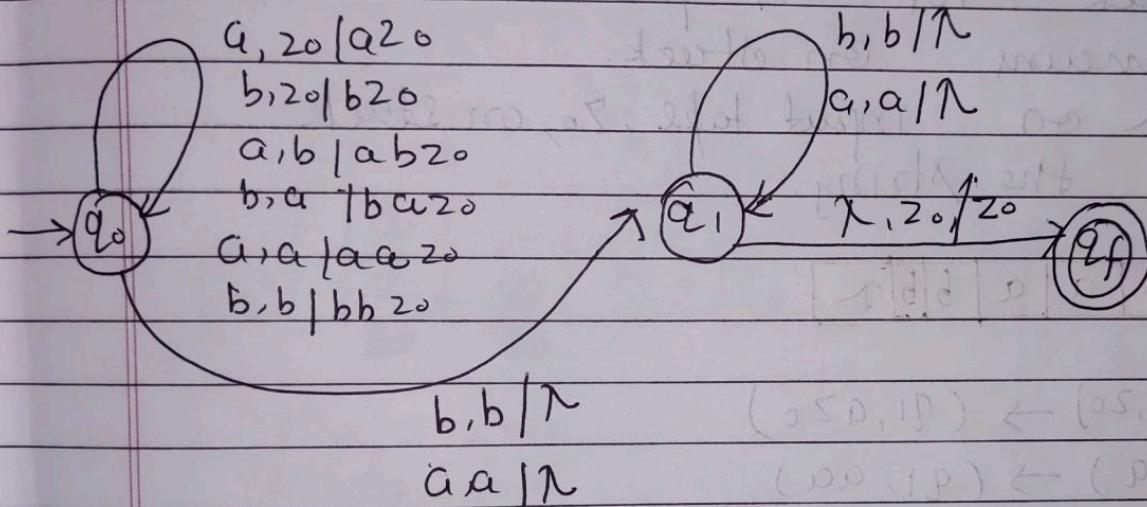
Ques Construct PDA for $L = \{ww^R \mid w \in (a,b)^*\}$

$$w = a \quad w^R = a \quad ww^R = a a$$

$$w = ab \quad w^R = ba \quad ww^R = abba$$

$$w = abab \quad w^R = baba \quad ww^R = ababbaba$$

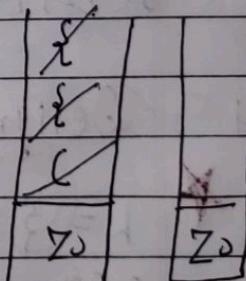
$$L = \{aa, abba, ababbaba, \dots\}$$



Ques Design PDA for Balanced parenthesis

({{}})

- ⇒ 1st push C into stack
- ⇒ Now push all { into stack
- ⇒ When } comes, pop each {
- ⇒ When) comes, pop each (



$$\delta(q_0, C, z_0) \rightarrow (q_1, (z_0))$$

$$\delta(q_1, \{, C) \rightarrow (q_2, \{C)$$

$$\delta(q_2, \{\{) \rightarrow (q_2, \{\{C)$$

$$\delta(q_2, \}, \{) \rightarrow (q_3, R)$$

$$\delta(q_2, \}, \{) \rightarrow (q_3, R)$$

$$\delta(q_3,), () \rightarrow (q_4, R)$$

$$\delta(q_3, R, z_0) \rightarrow (q_f, R)$$

Date _____

Design PDA for $L = \{a^m b^n \mid m > n \geq 1\}$

$$L = \{aab, aaab, aabb, \dots\}$$

- \Rightarrow Push all a's into stack
- \Rightarrow When b's comes, pop a's from stack
- \Rightarrow If empty string λ comes, and a's on stack, pop a's from stack till λ remains on stack.
- \Rightarrow If λ on input tape, λ , on stack accept the string.

a	a	a	a	b	b	λ
---	---	---	---	---	---	-----------

$$\delta(q_0, a, z_0) \rightarrow (q_1, a z_0)$$

$$\delta(q_1, a, a) \rightarrow (q_1, a a)$$

$$\delta(q_1, b, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, b, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, \lambda, a) \rightarrow (q_2, \lambda)$$

$$\delta(q_2, \lambda, z_0) \rightarrow (q_f, \lambda)$$

Design PDA $L = \{h_a(w) = h_b(w) \mid w \in \{a, b\}^*\}$

$$L = \{\lambda, a, b, ab, ba, aabb, bbba\}$$

$$\delta(q_0, \lambda, z_0) \rightarrow (q_f, \lambda)$$

$$\delta(q_0, a, z_0) \rightarrow (q_0, q_2)$$

$$\delta(q_0, b, z_0) \rightarrow (q_0, b z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, a a)$$

$$\delta(q_0, b, b) \rightarrow (q_0, b b)$$

$$\delta(q_0, a, b) \rightarrow (q_0, \lambda)$$

$$\delta(q_0, b, a) \rightarrow (q_0, \lambda)$$

a	a	b	a	b	b	a	λ
---	---	---	---	---	---	---	-----------

a	a	b		
z ₀				

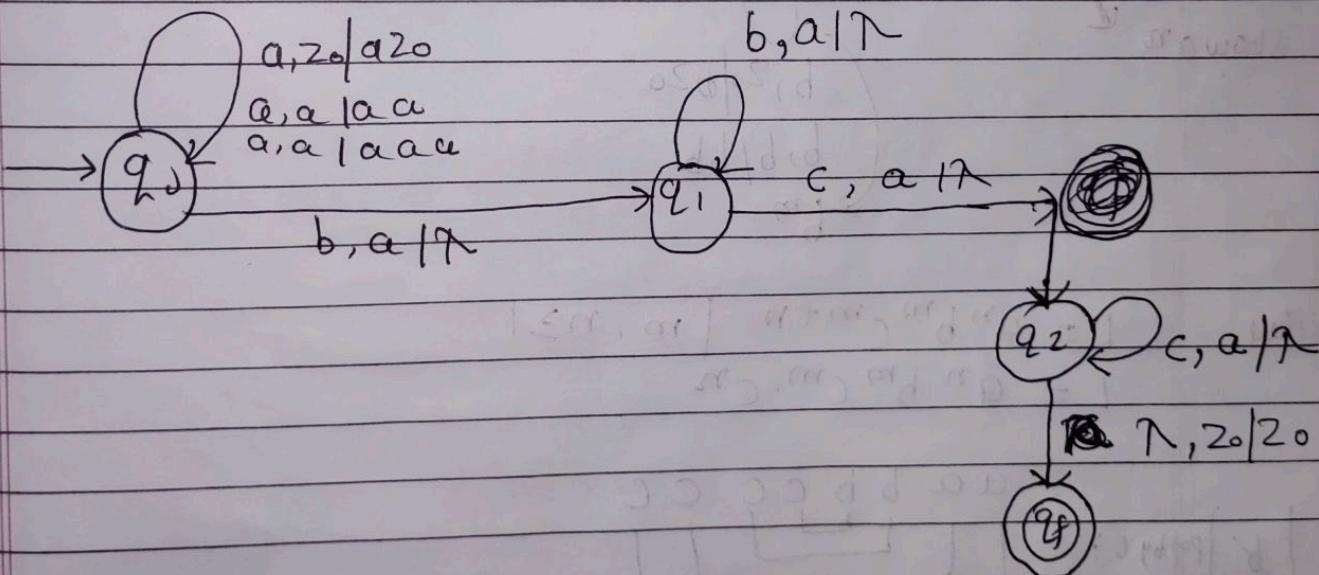
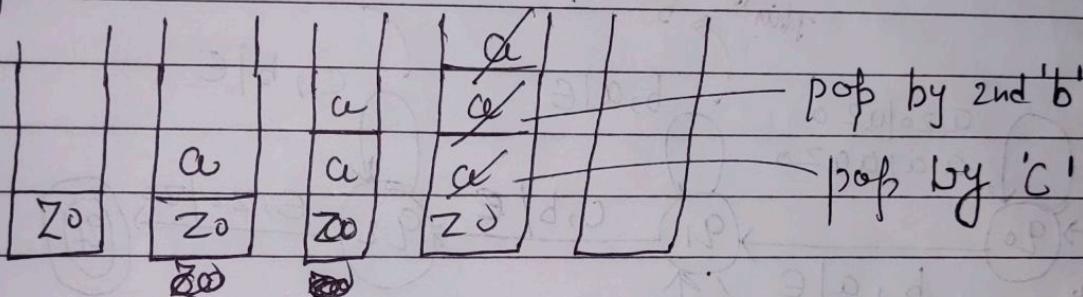
Q11

Construct PDA $L = a^{m+n} b^m c^n \mid m, n \geq 1$

$\{ m=1, n=1 \} \cup \{ m=2, n=1 \}$
 $\{ t = aabc, aaabbc, \dots \}$

- * In this all 'a's will be PUSH into stack & 'a' will be pop when insert 'b', & 'a' also pop when insert 'c'

Eg: a a a b b c , pop by 1st 'b'



Q12

$$L = a^n b^{m+n} c^m \mid m, n \geq 1$$

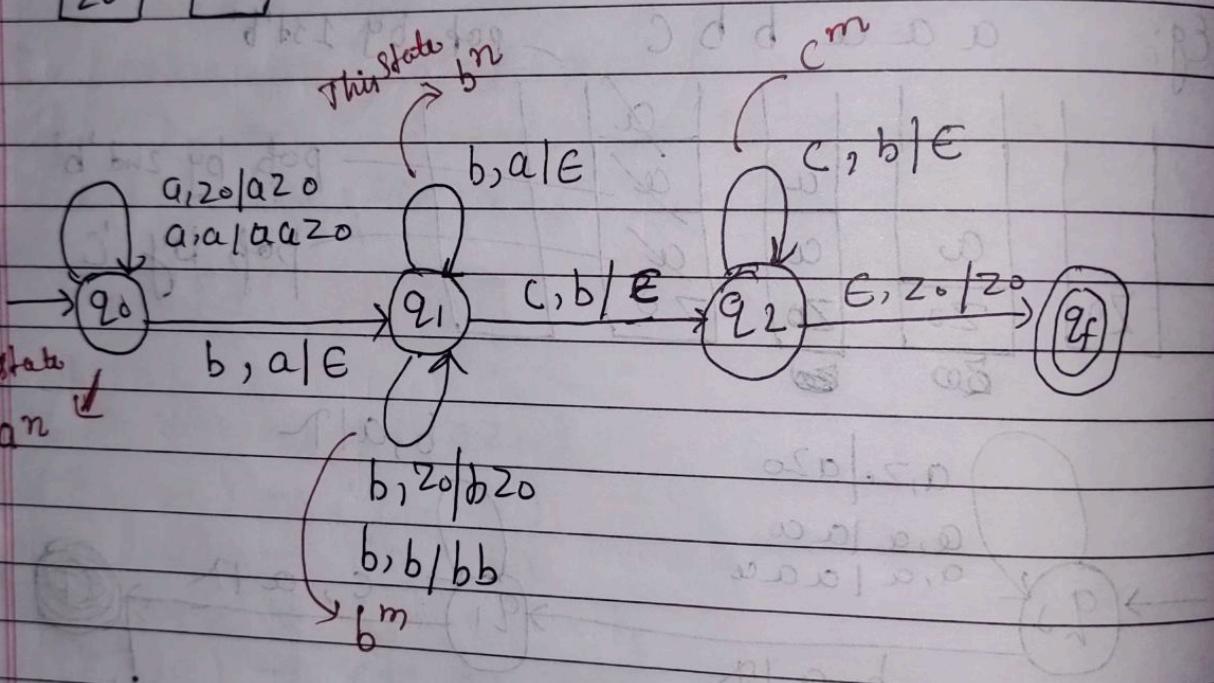
$$= a^n b^n b^m c^m$$

- Q0 no 'a's PUSH into stack & Pop by all no of 'b's (b^n)
- then all no of 'b's (b^m) pop by all no of 'c's (c^m)

Date / /

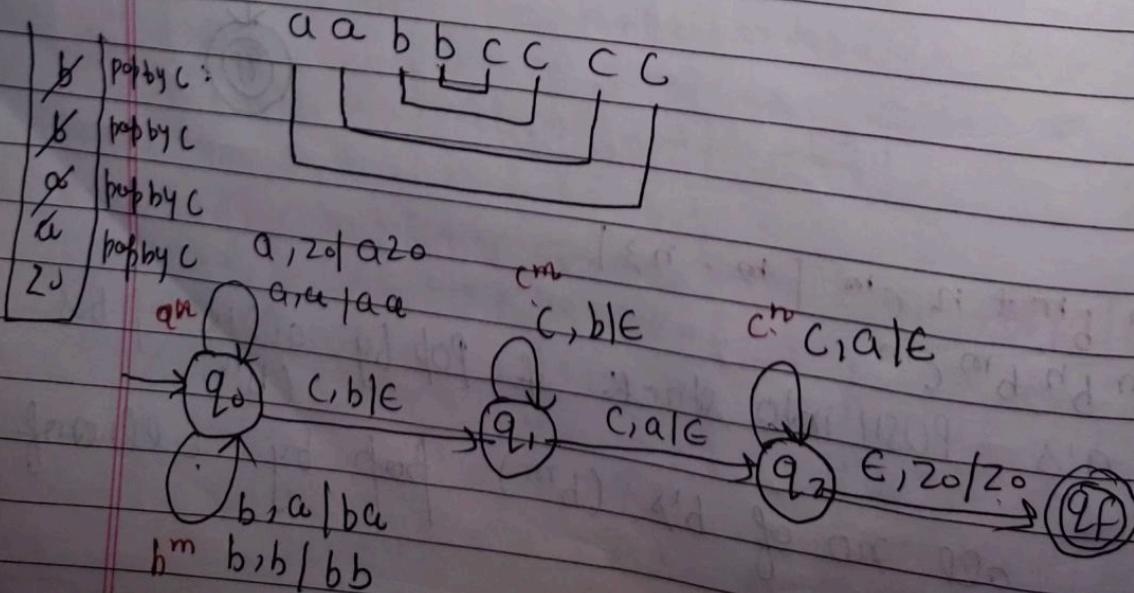
eg $aabbcc$

all 'a's	α	b	all 'b's pop
pop by	α	b	by c 's
'b'	z_0	z_0	



$$L = a^n b^m c^{m+n} \quad | \quad m, n \geq 1$$

$$L = a^n b^m c^m \cdot c^n$$

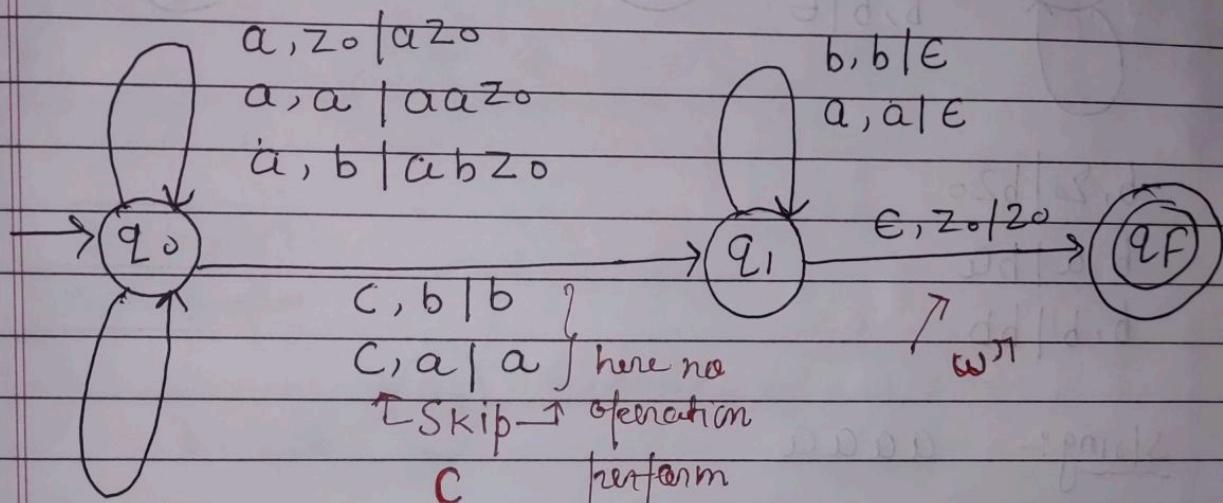
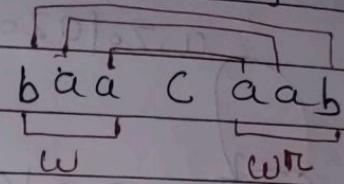
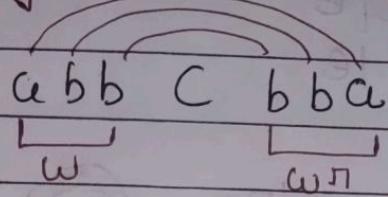


Date _____ / _____ / _____



Saathi

Design PDA $L = \{w c w^{\pi} \mid w \in \{a, b\}^*\}$



$b, z_0 | b z_0$

$b, b | b b z_0$

$b, a | b a z_0$

all these represent w

On state q_0 , when i/p a
then top of stack all are different
symbols, $\{z_0, a, b\}$

q_0 , when i/p b, then top
of stack $\{z_1, b, a\}$
so this is DPDA

Check string: $b a a c a a b$
 $\uparrow \uparrow \uparrow$ for 'c' skip operation

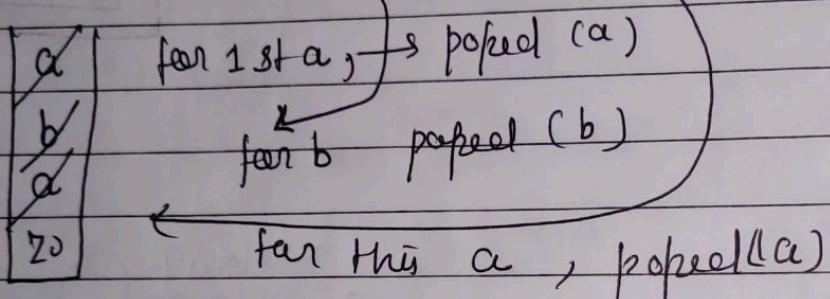
q_0
q_1
b
z_0

Date / /

Ques Design PDA for the given Language $L = \{ww^R\}$ [we will check whether it is DPDA + NPDA.]

$w = aba$ $w^R = aba$ $ww^R = abaaba$
 $w = aa$ $w^R = aa$ $ww^R = aaaa$

⇒ String abaaba



⇒ String aaaaa

for this 'a' we have two choice

a		
a	a	
z0	z0	

'a' will be PUSH,
'a' will be POP

*] means when top of stack 'a' & input 'a'
then we will pop.

* on other side if 'a' & top of stack a
then we will PUSH 'a'

At a time same input symbol 'a'
we will perform two transition on 2 State.
(operation PUSH & POP)
which is the case of NPDA

Saat

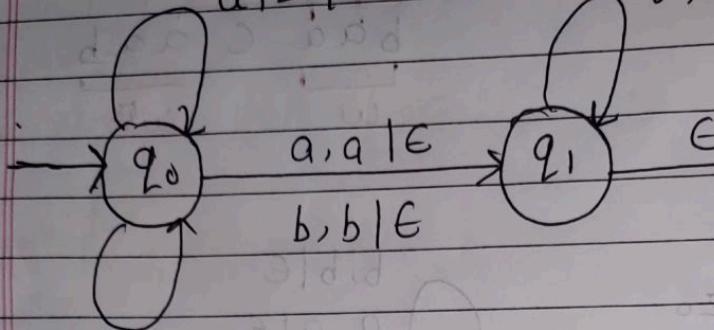
Date _____ / a, a | aa

a, b | ab

a, z₀ | a z₀

a, a | ε

b, b | ε



b, z₀ | b z₀

b, a | ba

b, b | bb

String:- aaaa

(q₀, aaaa, z₀)

(q₀, aaa, a z₀)

(q₀,

(q₀, aa, a a z₀)

(PUSH)

(POP)

Here this
transition
is not define
so Stop

(q₀, a, a a a z₀) (q₁, a, a z₀)

(q₀, ε, a a a a z₀)

Stop

(q₁, ε, a a z₀)

Stop

(q₁, ε, z₀)

q_f

find final
state.

Date _____

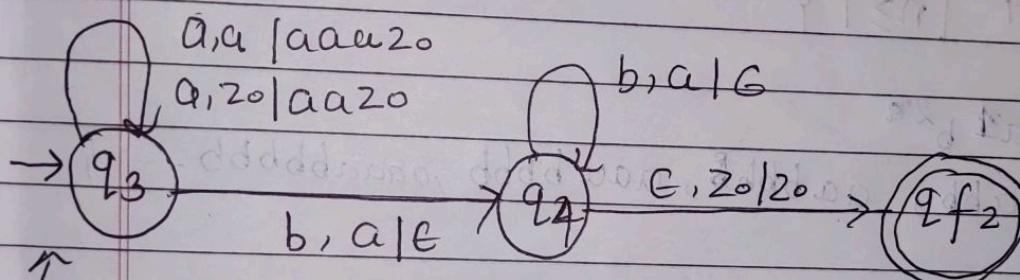
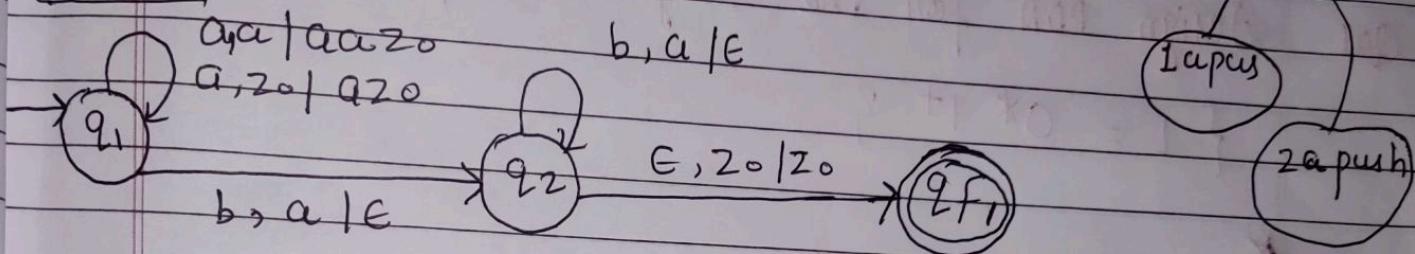
Ques

Design PDA for $\{ L = a^n b^n \text{ UNION } a^n b^{2n} \mid n \geq 1 \}$

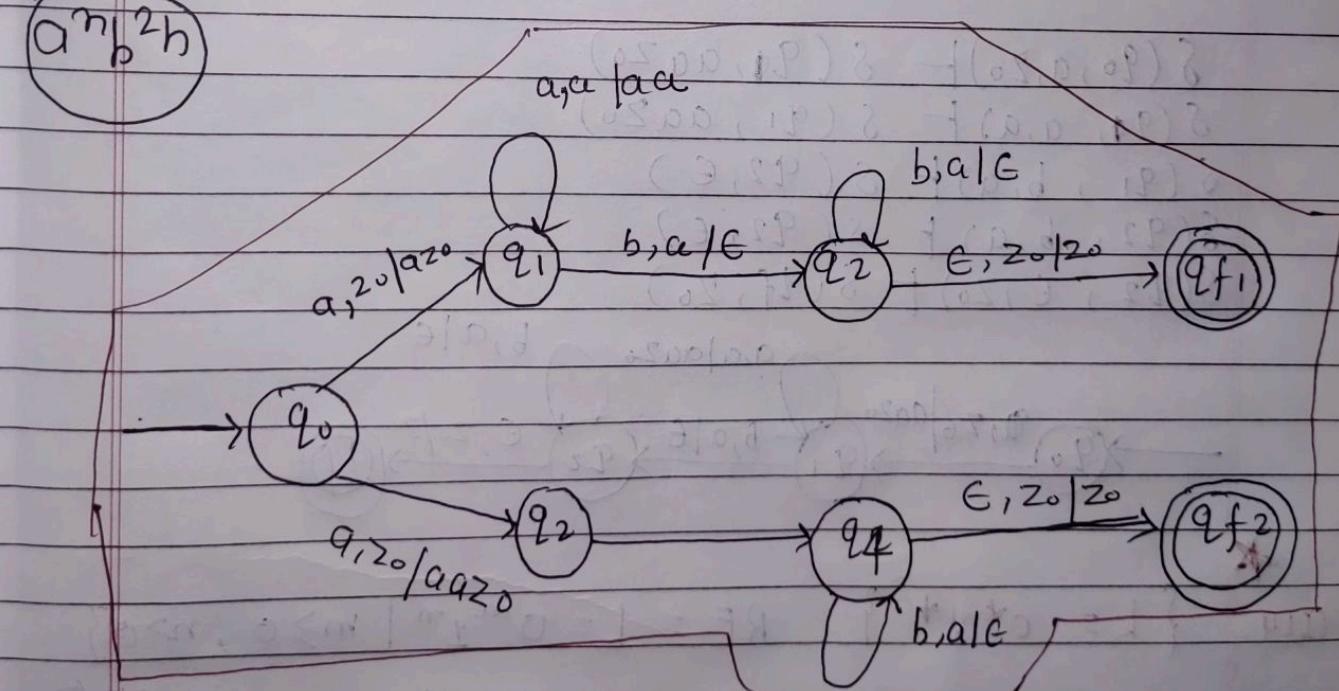
Saathi

$$L = \{ a^n b^n \} \cup \{ a^n b^{2n} \} \mid n \geq 1 \}$$

$a^n b^n$



$a^n b^{2n}$



This is also ND PDA

Date _____

Ques

Design a PDA for the language
 $\{L = a^n b^{n+1} \mid n \geq 1\}$

Ques 2) Design PDA for the regular Expression

$\{L = 0^* 1^*\}$

Ques: $\{L = a^n b^{n+1} \mid n \geq 1\}$

$L = a^1 b^2$

$\{L = abb, aabb, aaabb, aaaaabbb, \dots\}$

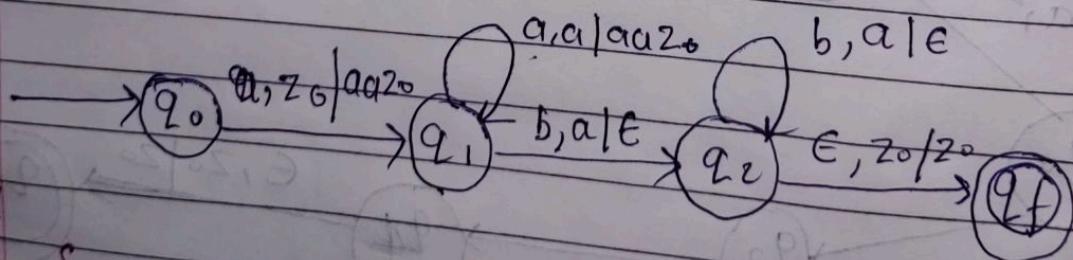
$\delta(q_0, a, z_0) \vdash \delta(q_1, a a z_0)$

$\delta(q_1, a, a) \vdash \delta(q_1, a a z_0)$

$\delta(q_1, b, a) \vdash \delta(q_2, \epsilon)$

$\delta(q_2, b, a) \vdash \delta(q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) \vdash \delta(q_f, z_0)$



Ques

$\{L = 0^* 1^*\}$

RE = $L = 0^m 1^n \mid m \geq 0, n \geq 0$

$\delta(q_0, 0, z_0) \vdash (q_0, \bullet z_0)$

$\delta(q_0, 1, z_0) \vdash (q_f, z_0)$

$\delta(q_1, 1, z_0) \vdash (q_f, z_0)$

