# EE 324 Control Systems Lab

Problem Sheet 5

**Mayur Ware | 19D070070**

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## Question 1

Plot the root locus of the following systems

**Part (a) :**

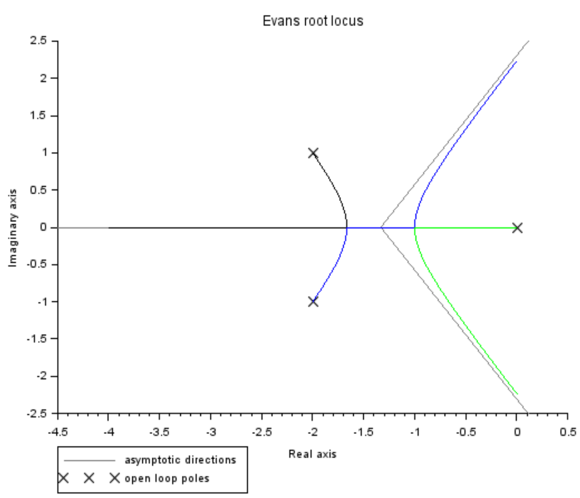
Given Closed Loop Transfer Function is,

$$S(s) = \frac{10}{(s^3 + 4s^2 + 5s + 10)} = \frac{KG(s)}{1 + KG(s)}$$

By solving this, we get the Open Loop Transfer Function G(s) as,

$$G(s) = \frac{1}{s(s^2 + 4s + 5)}$$

Locus of Closed Loop Poles using `SciLab` is,



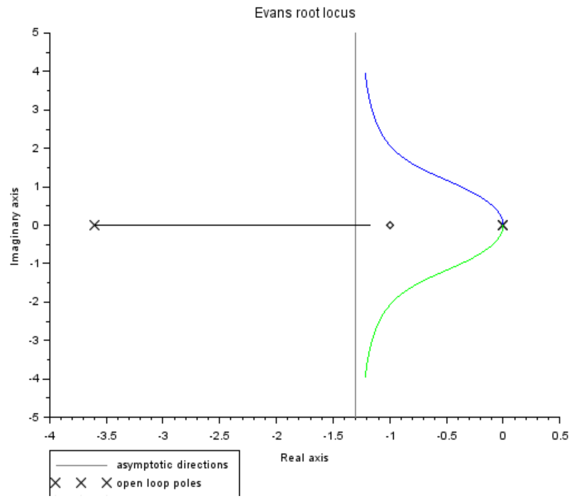

```
s = poly(0, 's');
G1 = 1/(s^3 + 4*s^2 + 5*s);
S1 = syslin('c', G1);
evans(S1, 20);
```

**Part (b) :**

Given Open Loop Transfer Function is,

$$G(s) = \frac{s+1}{s^2(s+3.6)}$$

Locus of Closed Loop Poles using `SciLab` is,

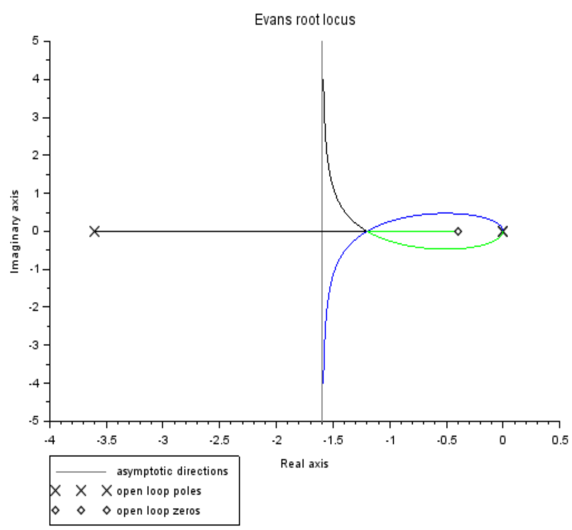



```
s = poly(0, 's');
G2 = (s+1)/(s^2*(s+3.6));
S2 = syslin('c', G2);
evans(S2, 20);
```

**Part (c) :**
Given Open Loop Transfer Function is,

$$G(s) = \frac{s + 0.4}{s^2(s + 3.6)}$$

Locus of Closed Loop Poles using `SciLab` is,

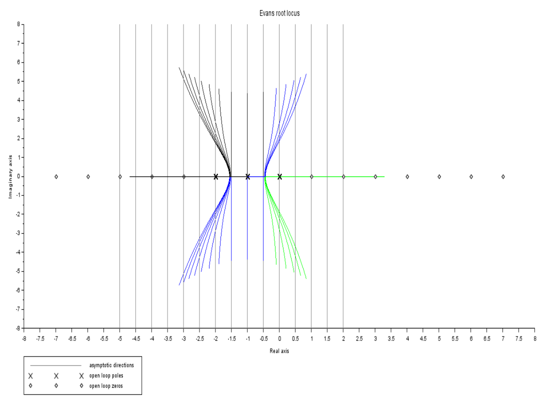



```
s = poly(0,'s');
G3 = (s+0.4)/(s^2*(s + 3.6));
S3 = syslin('c', G3);
evans(S3, 20);
```

**Part (d) :**
Given Open Loop Transfer Function is,

$$G(s) = \frac{s+p}{s(s+1)(s+2)}$$

Locus of Closed Loop Poles by varying p using `SciLab` is,




For $p < 0$, the system will be unstable. If there is pole-zero cancellation, then the system will be stable for all values of proportional gain. For $p = 3$, the root locus has an asymptote on j axis. If $p > 3$, the system will again be unstable for some values of K.

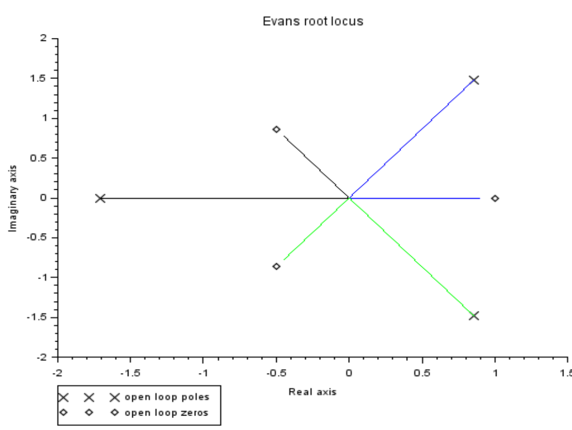
```
p = -7:1:7;
for i=1:size(p, 2)
  P = p(i);
  G4 = (s+P)/(s*(s+1)*(s+2));
  S4 = syslin('c', G4);
  evans(S4, 20);
end
```

# Question 2

Design following Transfer Functions

**Part (a) :**
The break-away and break-in points coincide.

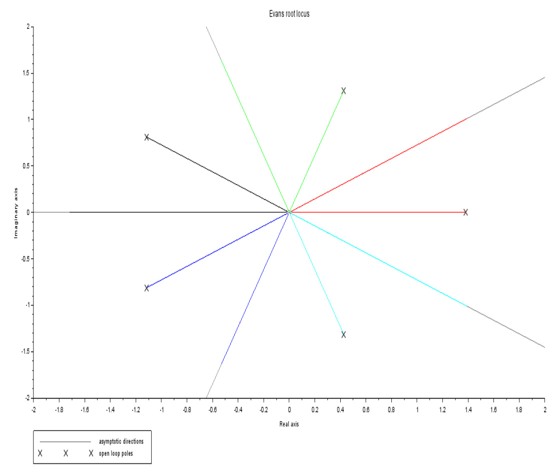
$$G(s) = \frac{s^3 - 1}{s^3 + 5}$$



```
s = poly(0, 's');
G1 = (s^3-1)/(s^3+5);
S1 = syslin('c', G1);
```

**Part (b) :**
The number of branches at the breakaway or breakin point is more than 4.

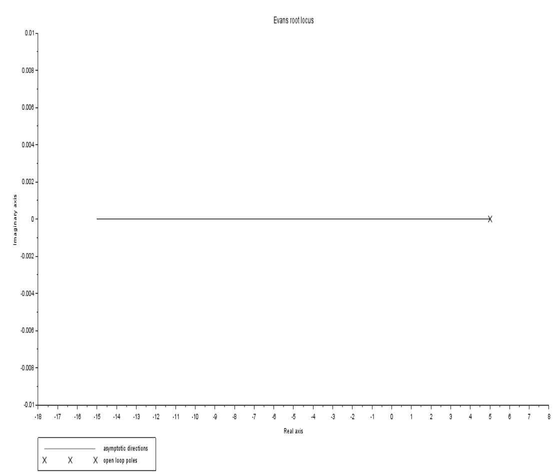
$$G(s) = \frac{1}{s^5 - 5}$$




```
s = poly(0,'s');
G2 = 1/(s^5-5);
S2 = syslin('c', G2);
evans(S2, 20);
```

**Part (c) :**
The branches of the root locus coincide with their asymptotes.

$$G(s) = \frac{1}{s - 5}$$



```
s = poly(0,'s');
G3 = 1/(s-5);
S3 = syslin('c', G2);
evans(S3, 20);
```

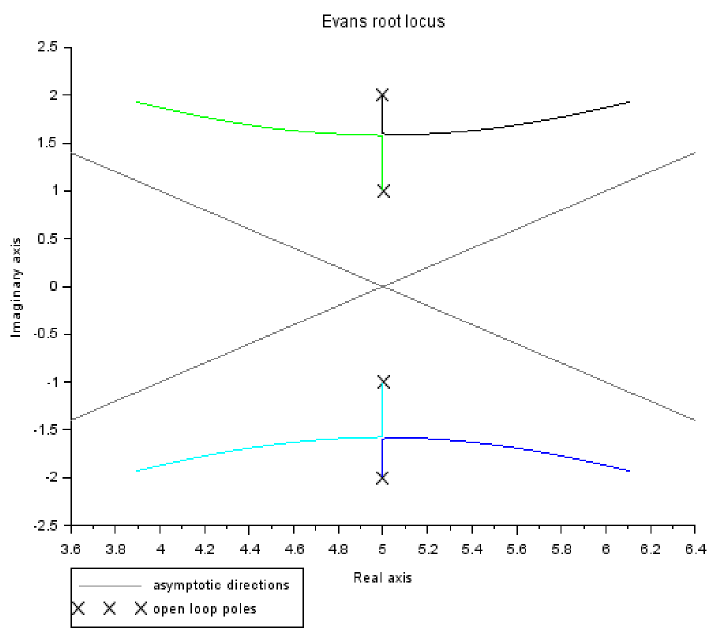
**Part (d) :**

The breakaway or break in points are complex numbers by following the steps given below.
i) Consider a transfer function with no zeros and with poles as real and symmetric about the jw axis.
ii) Now substitute $s^2$ with -$s^2$ (write the higher powers such as $s^4$, $s^6$ in terms of $s^2$) in the transfer function you have designed in part (i), and plot the root locus.
iii) Now substitute s with s-k where k being a positive integer of your choice, in the transfer function you have designed in part (ii) and plot the root locus.

$$G1(s) = \frac{1}{s^4 - 5s^2 + 4}$$

$$G2(s) = \frac{1}{s^4 + 5s^2 + 4}$$

$$G3(s) = \frac{1}{(s-5)^4 + 5(s-5)^2 + 4}$$



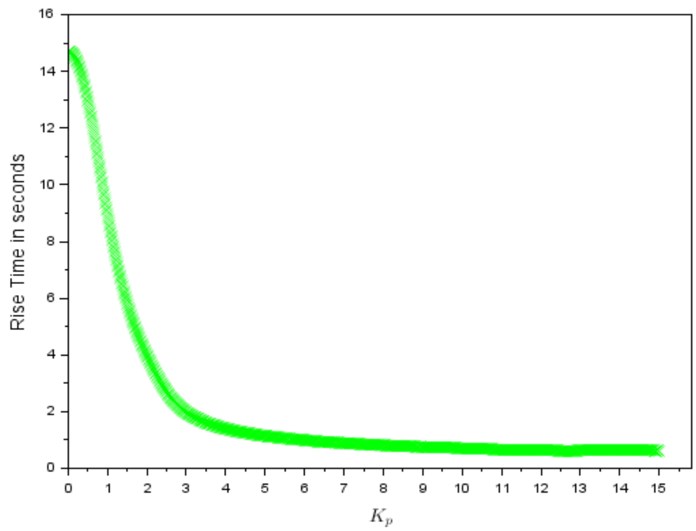

```
s = poly(0,'s')
G4a =  1/(s^4-5*s^2+4);
//Replace s^2 to -s^2
G4b = 1/(s^4+5*s^2+4);
//Substitute s as (s-5)
G4 = 1/((s-5)^4+5*(s-5)^2+4);
S4 = syslin('c',G4);
evans(S4,20);
```

# Question 3

We have been given a plant with Open Loop Transfer Function,

$$G(s) = \frac{1}{s(s^2 + 3s + 2)}$$

Again, we design a Proportional Controller (with gain $K_p$)
Variations in Rise Time ($T_r$) with $K_p$ are plotted using `SciLab`




The value of $K_p$ for a rise time of 1.5s is 3.748. Whereas, the minimum possible rise time for the given system for stability 0.57s.

```
function [time_rise] = trise(t, S, flag)
   out_step = csim('step', t, S);
   steady_val = mean(out_step(size(out_step, 2)-200:size(out_step, 2)));
   if flag then
     steady_val = 1;
   end
   peak_val = max(out_step);
   time_rise_l = 0;
   time_rise_h = 0;
   for i=1:size(out_step, 2)
     if(out_step(i)-(0.1 * steady_val)>= 1e-4)
        time_rise_l = t(i);
        break;
     end
   end
   for i=1:size(out_step, 2)
     if(out_step(i)-0.9 * steady_val>= 1e-4)
     time_rise_h = t(i);
     break;
     end
   end
   time_rise = time_rise_h - time_rise_l;
endfunction
s = poly(0, 's');
```

```
G = 1/(s*(s^2 + 3*s + 5));
K = 0.01:0.01:kpure(G);
t = 0:0.01:20;
candidates = [];
for i=1:size(K, 2)
  k = K(i);
  T = syslin('c', k*G);
  T = T/.syslin('c', 1, 1);
  tr = trise(t, T, %f);
  if i == size(K, 2)
   tr = trise(t, T, %t);
  end
  plot(k, tr, 'gx', 'LineWidth', 0.25);
  if tr == 1.5
   candidates = [candidates, k];
  end
end
xlabel("$K_p$");
ylabel("Rise Time in seconds");
title(["Rise time vs ", "$K_p$");
disp(candidates(1));
k_crit = kpure(G);
T = syslin('c', k_crit*G);
T = T/.syslin('c', 1, 1);
trise_min = trise(t, T, %t);
disp(trise_min);
```

# Question 4

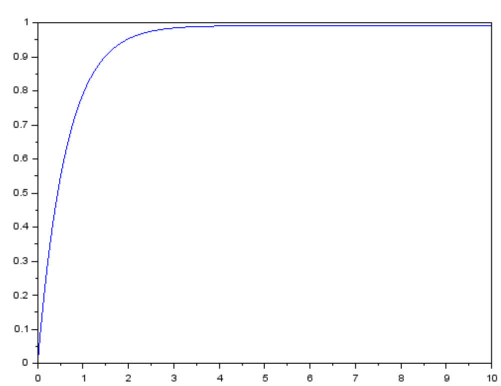
Given Open Loop Transfer Function is,

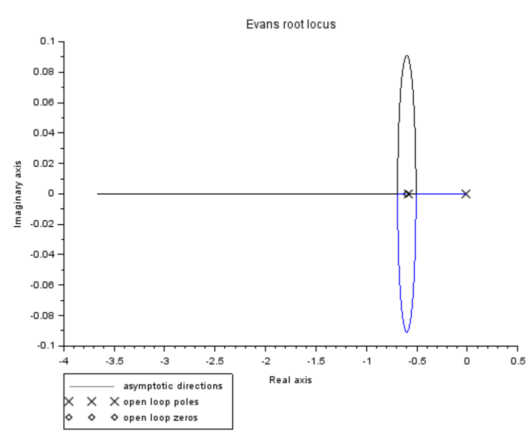
$$G(s) = \frac{0.11(s + 0.6)}{6s^2 + 3.6127s + 0.0572}$$

Steady state error of it's feedback system is 1% and can be formulated as,

$$e(\infty) = \frac{1}{1 + K_p G(0)} = 0.01$$

By solving this, we will get Kp = 85.8.
Whereas, the value of $K_p$ for the system to be marginally stable is -0.87.
The Unit Step Response and Root Locus of the system using `Scilab` is,





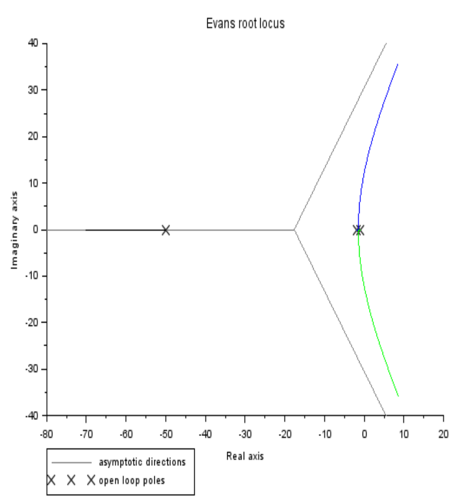
```
s = poly(0, 's');
G = 0.11*(s+0.6)/(6*s^2 + 3.6127*s +0.0572);
S = syslin('c', G);
evans(G, 200);
t = 0:0.01:10;
k = 85.8;
scf();
plot(t, csim('step', t, (k*G)/.syslin('c', 1, 1)));
K = -1:0.01:-0.1;
for i=1:size(K, 2)
   k = K(i);
   A = G*k;
   G = syslin('c', A);
   T = G/.syslin('c', 1, 1);
   [zeros, poles, gain] = tf2zp(T);
   x = real(poles);
   if abs(x(1)) <= 1e-4 || abs(x(2)) <= 1e-4
   disp(k);
   end
end
```

# Question 5

I assumed this Open Loop Transfer Function,

$$G(s) = \frac{75}{(s+1)(s+2)(s+25)}$$

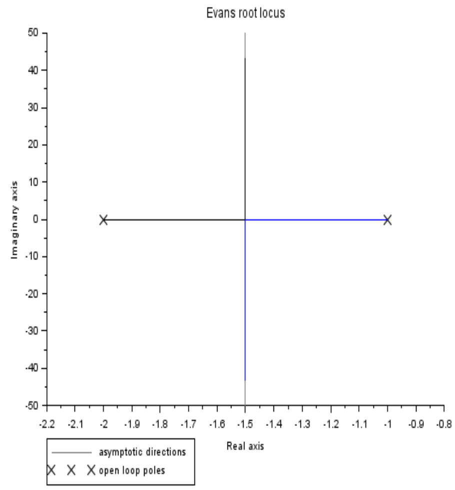
Root Locus of the system using `SciLab` is,



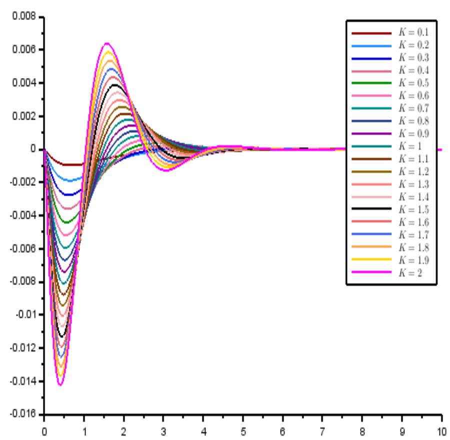

Now, system modified using the second order approximation will be,

$$T(s) = \frac{3}{(s+1)(s+2)}$$

Root Locus of the modified system using `SciLab` is,




If we plot the difference between unit negative feedback for G(s) and T(s), we will get the following graph from which we can notice that as K rises, difference increases



```
s = poly(0, 's');
G1 = 75/((s+1)*(s+2)*(s+25));
S1 = syslin('c', G1);
G2 = 3/((s+1)*(s+2));
S2 = syslin('c', G2);
scf();
evans(S1);
scf();
evans(S2);
K = 0.1:0.1:2;
t = 0:0.01:10;
scf();
for i=1:size(K, 2)
    k = K(i);
    S3 = syslin('c', k*G1);
    S4 = syslin('c', k*G2);
    T1 = S3/.syslin('c', 1, 1);
```

```
    T2 = S4/.syslin('c', 1, 1);
    xset("thickness",2);
    plot2d(t, csim('step', t, T1-T2));
end
title(["Difference in unity gain negative feedback");
xlabel("$t$");
ylabel("Step response");
```

## References


1) https://help.scilab.org/
2) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=English
3) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=Englishpage=2