EE 324 Control Systems Lab

Problem Sheet 10

Mayur Ware | 19D070070

October 25, 2021

Question 1

I've chosen A, B, C and D matrices are as follows :

$$A = \begin{bmatrix} 5 & 9 & 3 \\ 10 & 2 & 5 \\ 8 & 2 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 \\ 9 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 6 & 10 \end{bmatrix} \qquad D = 1$$

Calculating the Transfer Function $G(s) = D + C(sI - A)^{-1}B$

$$(sI - A) = \begin{bmatrix} s-5 & -9 & -3 \\ -10 & s-2 & -5 \\ -8 & -2 & s-4 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^3 - 11s^2 - 86s - 2} \begin{bmatrix} s^2 - 6s - 2 & 9s - 30 & 3s + 39 \\ 10s & s^2 - 9s - 4 & 5s + 5 \\ 8s + 4 & 2s + 62 & s^2 - 7s - 80 \end{bmatrix}$$

$$G(s) = D + C(sI - A)^{-1}B$$

$$= \frac{s^3 + 63s^2 + 376s + 4310}{s^3 - 11s^2 - 86s - 2}$$

```
s = poly(0, 's');
T = [1, 2, 8;
     4, 5, 6;
     7, 8, 9];
A = [5, 9, 3;
     10, 2, 5;
     8, 2, 4];
B = [5; 9; 1];
C = [2, 6, 10];
D = 1;
I = eye(3, 3);
G = syslin('c', A, B, C, D);
G_Tf = ss2tf(G);
```

I choose the matrix T as

$$T = \begin{bmatrix} 1 & 2 & 8 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

On applying the given transformations we will find the new Transfer Function

$$A' = T^{-1}AT = \begin{bmatrix} 1 & 2 & 8 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 5 & 9 & 3 \\ 10 & 2 & 5 \\ 8 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 8 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \frac{1}{15} \begin{bmatrix} -1020 & -1359 & -2803 \\ 375 & 1308 & 2801 \\ 0 & -9 & -123 \end{bmatrix}$$

$$B' = T^{-1}B = \begin{bmatrix} 1 & 2 & 8 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{15} \begin{bmatrix} -371 \\ 367 \\ -36 \end{bmatrix}$$

$$C' = CT = \begin{bmatrix} 2 & 6 & 10 \end{bmatrix} \begin{bmatrix} 1 & 2 & 8 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 96 & 114 & 142 \end{bmatrix}$$

$$G_{New} = D' + C'(sI - A')^{-1}B'$$

$$= \frac{s^3 + 63s^2 + 3765 + 4310}{s^3 - 116^2 - 865 - 2}$$

```
A_New = pinv(T)*A* T;
B_New = pinv(T)*B;
C_New = C*T;
GT = syslin('c', A_New, B_New, C_New, D);
G_New_Tf = ss2tf(GT);
```

We can see that Transfer Functions in both cases are same.
Eigenvalues of A are the poles of G(s) are,
16.2876
-5.2643
-0.0233
Now,
The Proper Transfer Function G_Proper is,

$$G_Proper(s) = \frac{s^2 + 5s + 7}{s^2 + 9s + 1}$$

The State Space Respresentation of G_Proper is,
A = $\begin{bmatrix} -9 & -1 \\ 1 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, C = $\begin{bmatrix} -4 & 6 \end{bmatrix}$ D=1
The Strictly Proper Transfer Function G_S_Proper is,

$$G_SProper(s) = \frac{s + 3}{s^2 + 4s + 9}$$

The State Space Respresentation of G_S_Proper is,
A = $\begin{bmatrix} -4 & -9 \\ 1 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 3 \end{bmatrix}$ D=0

```
GT = syslin('c', A_New, B_New, C_New, D);
G_New_Tf = ss2tf(GT);
eig_val = spec(A);
[z, p, k] = ss2zp(G);
p
```

```
G_Proper = syslin('c', (s^2+5*s+7)/(s^2+9*s+1));
X = tf2ss(G_Proper);
X
G_S_Proper = syslin('c', (s+3)/(s^2+4*s+9));
Y = tf2ss(G_S_Proper);
Y
```

Question 2

The system given to us is,

$$G(s) = \frac{s+3}{s^2+5s+4}$$

State Space Representation of G(s) using SciLab is,

```
A =

 -1.5384615   0.3076923
  4.3076923  -3.4615385

B =

 -1.1094004
  1.6641006

C =

 -0.9013878   0.

D =

  0.

E =

  1.   0.
  0.   1.
```

Now, we modify the system by shifting the zero from -3 to -1

$$G(s) = \frac{s+1}{s^2+5s+4}$$

State Space Representation of G(s) using SciLab is,

```
A =

  -4.   0.
   0.  -4.

B =

   1.
   0.

C =

   1.   0.

D =

   0.

E =

   1.   0.
   0.   1.
```

```
s = poly(0, 's');
G = syslin('c', (s+3)/(s^2+5*s+4));
M = tf2des(G);
M
G1 = syslin('c', (s+1)/(s^2+5*s+4));
M1 = tf2des(G1);
M1
```

Question 3

I've chosen A, B, C and D matrices are as follows :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix} \qquad D = 5$$

The poles and zeros of the following system are 1, 2, 3 and -6.2036, 1.084, 2.319 respectively.
When we make one entry of the column vector B zero, lets say the second entry then B becomes [1, 0, 6]. In this case the poles and zero of the system are as shown below. As we can see there is a zero at 2 and a pole at 2. Thus a pole-zero cancellation occurs, and the corresponding entry which is zero in B that pole is cancelled.
Z (3x1) :
1.1749
-3.5749
2.0000
P (3x1) :
1
2
3
When we make one entry of the column vector C zero, lets say the second entry then B becomes [2, 3, 0]. In this case the poles and zero of the system are as shown below. As we can see there is a zero at 3 and a pole at 3. Thus a pole-zero cancellation occurs, and the corresponding entry which is zero in C that pole is cancelled.
Z (3x1) :
-1.0000
1.2000
3.0000
P (3x1) :
1
2
3

```
A = [1,0,0;
     0,2,0;
     0,0,3];
B = [1;4;6];
C = [2,3,5];
D = 5;
[z1,p1,k1] = ss2tf(A,B,C,D)
A2 = [1,0,0;0,2,0;0,0,3];
```

```
B2 = [1;0;6];
C2 = [2,3,5];
D2 = 5;
[z2,p2,k2] = ss2tf(A2,B2,C2,D2)
A3 = [1,0,0;
      0,2,0;
      0,0,3];
B3 = [1;4;6];
C3 = [2,3,0];
D3 = 5;
[z3,p3,k3] = ss2tf(A3,B3,C3,D3)
```

Question 4

I've chosen an Upper Half Matrix A, B, C and D matrices are as follows :

$$A = \begin{bmatrix} 9 & 6 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 3 & 9 \end{bmatrix} \qquad D = 4$$

Using SciLab,
Eigenvalues of A are :
9
4
2
And, Poles of G(s) are :
9
4
2
Now, I made the first two elements of diagonal same as 4.

$$A\_New = \begin{bmatrix} 4 & 6 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

Using SciLab,
Eigenvalues of A are :
4
4
2
And, Poles of G(s) are :
3.999999
4.000001
2

```
s = poly(0, 's');
A = [9, 6, 0;
     0, 4, 3;
     0, 0, 2];
B = [2; 3; 7];
C = [4, 3, 9];
D = 4;
```

```
eig_val = spec(A);
eig_val
G = syslin('c', A, B, C, D);
G = ss2tf(G);
[z, p, k] = tf2zp(G);
p
A_New = [4, 6, 0;
         0, 4, 3;
         0, 0, 2];
eig_val = spec(A_New);
eig_val
G = syslin('c', A_New, B, C, D);
G = ss2tf(G);
[z, p, k] = tf2zp(G);
p
```

References

1) https://help.scilab.org/
2) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=English
3) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=Englishpage=2