EE 324 Control Systems Lab

Problem Sheet 1 : Analysis in Laplace domain

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Question 1

$$G1(s) = \frac{10}{s^2 + 2s + 10}$$
$$G2(s) = \frac{5}{5 + s}$$

Part (a) : Cascade System
In Cascade System of two blocks $G1(s)$ and $G2(s)$, Transfer Function is :

$$TF1(s) = G1(s) * G2(s)$$

Using SciLab, we get :

$$\boxed{TF1(s) = \frac{50}{50 + 20s + 7s^2 + s^3}}$$

```
\\-----------------Q1------------------ \\
s= poly(0,'s');
G1 = syslin('c',10/(s^2+2*s+10));
G2 = syslin('c',5/(s+5));
\\---------------------------------------------
\\ a) Cascade System
TF1 = syslin('c', G1*G2);
disp(TF1, "Transfer Function TF1(s) is");
```

Part (b) : Parallel System
In Parallel System of two blocks $G1(s)$ and $G2(s)$, Transfer Function is :

$$TF2(s) = G1(s) + G2(s)$$

Using SciLab, we get :

$$\boxed{TF2(s) = \frac{100 + 20s + 5s^2}{50 + 20s + 7s^2 + s^3}}$$

```
\\-----------------Q1------------------ \\
s= poly(0,'s');
G1 = syslin('c',10/(s^2+2*s+10));
G2 = syslin('c',5/(s+5));
\\---------------------------------------------
\\ b) Parallel System
TF2 = syslin('c', G1+G2);
disp(TF2, "Transfer Function TF2(s) is");
```

Part (c) : Feedback System

In the given Feedback System of two blocks $G1(s)$ and $G2(s)$, Transfer Function is :

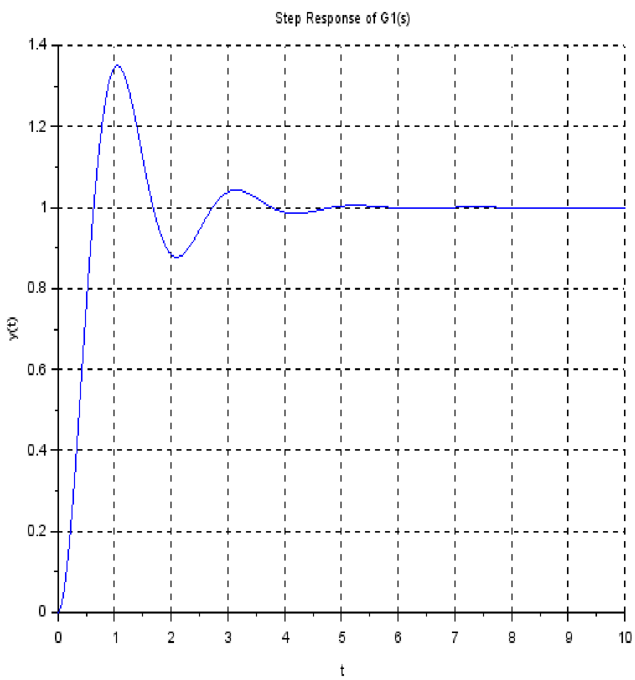
$$TF3(s) = \frac{G1(s)}{1 + G1(s) * G2(s)}$$

Using `SciLab`, we get :

$$\boxed{TF3(s) = \frac{50 + 10s}{50 + 20s + 7s^2 + s^3}}$$

```
\\--------------------Q1-------------------- \\
s= poly(0,'s');
G1 = syslin('c',10/(s^2+2*s+10));
G2 = syslin('c',5/(s+5));
\\-------------------------------------------------
\\ c) Feedback System
TF3 = syslin('c', G1/(1+G1*G2));
disp(TF3, "Transfer Function TF3(s) is");
```

Part (d) : Plot Unit Step Response of G1(s)




```
\\ Q1(d) Plot
t = 0:0.05:10;
plot2d(t, csim('step', t, G1), style=[color("blue")]);
\\to show the grids
xgrid(0);
\\ Title of Graph
title(["Step Response of G1(s)"]);
\\ X-axis label
xlabel("t");
\\ Y-axis label
```

```
ylabel("y(t)");
```

Question 2

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Poles and Zeros of the systems $TF1(s)$, $TF2(s)$ and $TF3(s)$

Part (a) : Cascade System

Poles :

1) -5 + 0i

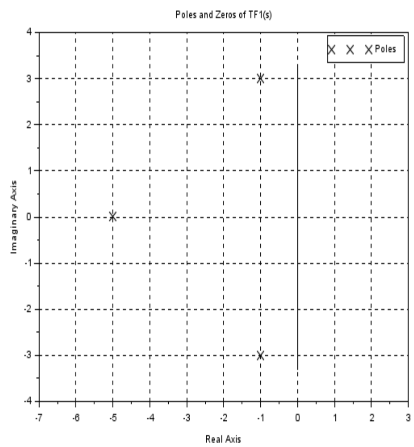
2) -1 + 3i

3) -1 - 3i

Zeros :

None

Pole-Zero Plot :




```
\\------------------------Q2------------------------ \\
\\ a) Cascade System
[zeros_a, poles_a, gain_a] = tf2zp(TF1);
disp(zeros_a,"Zeros of TF1(s)");
disp(poles_a,"Poles of TF1(s)");
plzr(TF1);
\\to show the grids
xgrid(0);
\\ Title of Graph
title("Poles and Zeros of TF1(s)");
\\X-axis label
xlabel("Real Axis");
\\Y-axis label
ylabel("Imaginary Axis");
```

Part (b) : Parallel System

Poles :

1) -5 + 0i

2) -1 + 3i

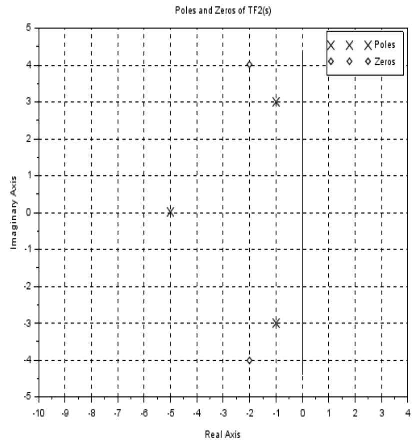
3) -1 - 3i

Zeros :

1) -2 + 4i

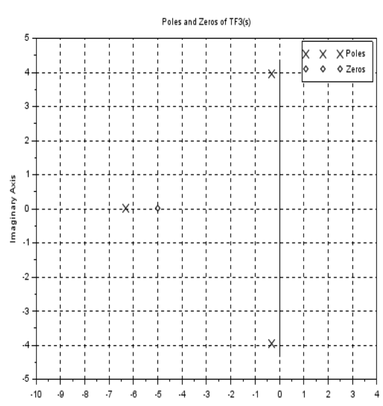
2) -2 - 4i

Pole-Zero Plot :




```
\\----------------------Q2---------------------- \\
\\ b) Parallel System
[zeros_b, poles_b, gain_b] = tf2zp(TF2);
disp(zeros_b,"Zeros of TF2(s)");
disp(poles_b,"Poles of TF2(s)");
plzr(TF2);
\\to show the grids
xgrid(0);
\\ Title of Graph
title("Poles and Zeros of TF2(s)");
\\X-axis label
xlabel("Real Axis");
\\Y-axis label
ylabel("Imaginary Axis");
```

Part (c) : Feedback System
Poles :
1) -6.3347665 + 0i
2) -0.3326167 + 3.9592004i
3) -0.3326167 - 3.9592004i
Zeros :
1) -5 + 0i
Pole-Zero Plot :



```
\\---------------------Q2---------------------- \\
\\ c) Feedback System
[zeros_c, poles_c, gain_c] = tf2zp(TF3);
disp(zeros_c,"Zeros of TF3(s)");
disp(poles_c,"Poles of TF3(s)");
plzr(TF3);
\\to show the grids
xgrid(0);
\\ Title of Graph
title("Poles and Zeros of TF3(s)");
\\X-axis label
xlabel("Real Axis");
\\Y-axis label
ylabel("Imaginary Axis");
```

Question 3

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Matrix Calculations Practice

Let matrices A and B be :

$$A = \begin{bmatrix} 3s^2+5s+4 & 7s & 47 \\ 34 & 1/s & 29s \\ 2s^3+5 & 5 & 3/s \end{bmatrix}$$

$$B = \begin{bmatrix} 4s^3 & 4/(3s+5) & 23 \\ 41s+6 & 3s^2+5s-14 & 57 \\ s^2-6 & 3s & 78 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 4+5s+3s^2+4s^3 & \frac{4+35s+21s^2}{5+3s} & 70 \\ 40+41s & \frac{1-14s+5s^2+3s^3}{s} & 57+29s \\ -1+s^2+2s^3 & 5+3s & \frac{3+78s}{s} \end{bmatrix}$$

$$A*B = \begin{bmatrix} -282+42s+334s^2+16s^3+20s^4+12s^5 & \frac{16+235s+316s^2+210s^3+63s^4}{5+3s} & 3758+514s+69s^2 \\ \frac{6+41s-174s^2+165s^4}{s} & \frac{-70+119s+30s^2+444s^3+261s^4}{5s+3s^3} & \frac{57+782s+2262s^2}{s} \\ \frac{-18+30s+208s^2+20s^4+8s^7}{s} & \frac{-285-58s+150s^2+53s^3}{5+3s} & \frac{234+400s+46s^4}{s} \end{bmatrix}$$

$$B^{-1} = \frac{\begin{bmatrix} -5460-2181s+1827s^2+702s^3 & -312+345s+207s^2 & -847+741s-621s^4 \\ -4050-18420s-9309s^2+171s^3 & 690+414s-115s^2+1491s^3+936s^4 & 528s-175s^3+451s^5+669s^7 \\ -420-12s+919s^2+440s^3-30s^4-9s^5 & -24+4s^2-60s^4-36s^5 & 2744+475s^3-603s^6-376s^7 \end{bmatrix}}{-12900-13068s+21365s^2-11720s^3-9414s^4+7101s^5+2808s^6}$$

$$det(B) = \begin{bmatrix} \frac{-12900-13068s+21365s^2-11720s^3-9414s^4+7101s^5+2808s^6}{5+3s} \end{bmatrix}$$

```
\\---------------------Q3------------------ \\
\\ Matrix Calculations Practice
s = poly(0,'s');
\\ 3*3 Matrix A
A = [ 3*s^2+5*s+4, 7*s, 47; 34, 1/s, 29*s; 2*s^3+5, 5, 3/s];
\\ 3*3 Matrix B
B = [4*s^3, 4/(3*s+5), 23; 41*s+6, 3*s^2+5*s-14, 57; s^2-6, 3*s, 78];
\\ Addition
disp(A+B, "A+B is :");
\\ Multiplication
disp(A*B, "A*B is :");
\\ Inverse
```

```
disp(inv(B), "Determinant of B is :");
\\ Determinant
disp(det(B), "Determinant of B is :");
```

Question 4

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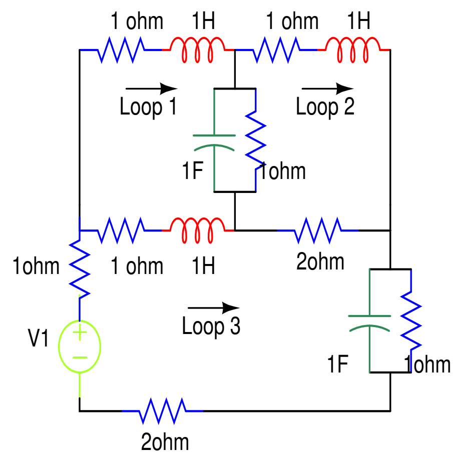
Mesh Analysis
Circuit :



Considering I_1 flowing through Loop 1, I_2 flowing through Loop 2 and I_3 flowing through Loop 3, using KVL, we make the following Impedance Matrix Z(s) :

$$Z(s) = \begin{bmatrix} 1+2\,(s+1)^2 & -1 & -(s+1)^2 \\ -1 & (s+2)^2 & -2(s+1) \\ -(s+1)^2 & -2(s+1) & s^2+7s+7 \end{bmatrix}$$

and

$$V(s) = \begin{bmatrix} 0 \\ 0 \\ V_1(s+1) \end{bmatrix}$$

On solving the above matrix equation for $I(s)$ using `Scilab`, we get

$$I(s) = Z^{-1}(s) * V(s)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = V_1 * \begin{bmatrix} \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \\ \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \\ \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix}$$

Hence, the Transfer Functions are as follows:

$$\boxed{\frac{I_1(s)}{V_1(s)} = \frac{6+14s+13s^2+6s^3+s^4}{57+144s+147s^2+74s^3+17s^4+s^5}}$$

$$\boxed{\frac{I_2(s)}{V_1(s)} = \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5}}$$

$$\boxed{\frac{I_3(s)}{V_1(s)} = \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5}}$$

```
\\------------------Q4------------------ \\
s = poly(0,'s');
\\ Impedance Matrix
Z=[2+2s+1/(1+s),  1 /(1+s), -s-1;  1 /(1+s), 3+s+1/(1+s),  2 ;  s -1,  2 , 6+s+1/(1+s)];
\\ Voltage Matrix
V = [0; 0; 1];
\\ I = V*Z^-1
inv(Z)*V;
```

References

1) https://help.scilab.org/
2) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=English
3) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=Englishpage=2