EE 324 Control Systems Lab

Problem Sheet 4

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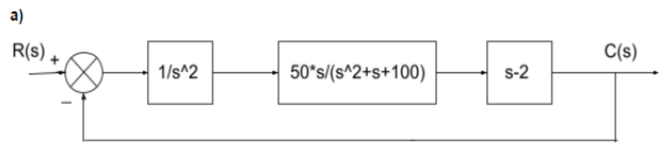
Question 1

Obtain input-output transfer functions for Complex Interconnected Systems.

Part (a) :

Given System is,



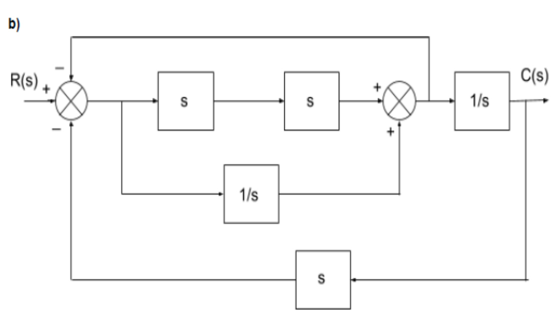
Transfer Function of the given function using `SciLab` is,

$$\mathrm{TF(s)} = \frac{50s - 100}{s^3 + s^2 + 150s - 100}$$

```
s = poly(0, 's');
G1 = 1/s^2;
G2 = (50*s)/(s^2+s+100);
G3 = s-2;
G4 = 1;
S1 = syslin('c', G1);
S2 = syslin('c', G2);
S3 = syslin('c', G3, 1);
S4 = syslin('c', G4, 1);
TF = (S1*S2*S3)/.S4;
```

Part (b) :

Given Transfer Function is,

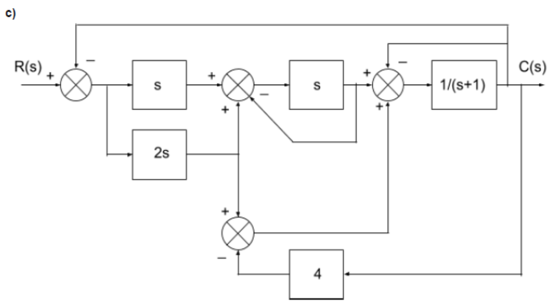


Transfer Function of the given function using `SciLab` is,

$$\mathrm{TF(s)} = \frac{s^3 + 1}{s(2s^3 + s + 2)}$$

```
G1 = s;
G2 = 1/s;
S1 = syslin('c', G1, 1);
S2 = S1*S1;
S3 = syslin('c', G2);
S4 = S2+S3;
S5 = S4/.syslin('c', 1, 1);
TF = (S5*syslin('c', G2))/.syslin('c', s, 1);
```

Part (c) :
Given Transfer Function is,



Transfer Function of the given function using `SciLab` is,

$$\text{TF(s)} = \frac{s(5s+2)}{3(2s^2+3s+2)}$$


```
G1 = s;
G2 = 1/s;
S1 = syslin('c', G1, 1);
S2 = S1*S1;
S3 = syslin('c', G2);
S4 = S2+S3;
S5 = S4/.syslin('c', 1, 1);
TF = (S5*syslin('c', G2))/.syslin('c', s, 1);
```


Question 2

Part (a) :
Given Transfer Function is,

$$G(s) = \frac{10}{s(s+2)(s+4)}$$

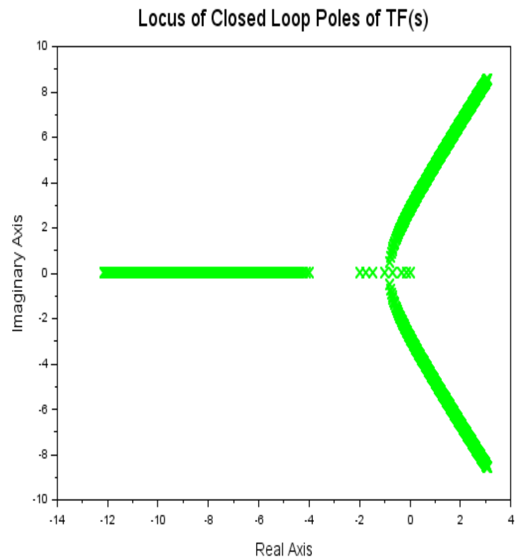
A proportionality Gain **K** has been put in the forward path in series with the plant and then the feedback loop has been closed with unity negative feedback.
Transfer Function of modified system is,

$$TF(s) = \frac{K}{1+KG(s)} = \frac{10K}{s^3+6s^2+8s+10K}$$

I took **K=7** for this part.

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
S = syslin('c', G);
K = 7;
TF = K*S/.syslin('c', 1, 1);
```

Part (b) :
Locus of Closed-Loop Poles of TF(s) as we vary K from 1 to 100 with a step of 0.1 is simulated using `SciLab`




```
K = 0:0.1:100;
for i=1:size(K, 2)
  k = K(i);
  S1 = k*S/.syslin('c', 1, 1);
  [zeros, poles, gain] = tf2zp(S1);
  plot(real(poles), imag(poles), 'gx');
end
xlabel("Real Axis");
ylabel("Imaginary Axis");
title("Locus of Closed Loop Poles of TF(s)");
```

Part (c) :
`'Estimate the critical value of K that takes the closed-loop system to the verge of instability.'`
This value of K can be found if any one poll falls in the Right-half plane.
I found the **critical value of K = 4.8**

```
K_C = -1;
for i=1:size(K, 2)
    k = K(i);
    S2 = k*S/.syslin('c', 1, 1);
    [zeros, poles, gain] = tf2zp(S2);
    Real_Poles = real(poles);
    if Real_Poles(1) > 0 || Real_Poles(2) > 0 || Real_Poles(3) > 0
        K_C = k;
        break;
    end
end
```

Part (d) :
Verify the value of critical K using R-H Table.

$$\text{Denominator of TF(s)} = s^3 + 6s^2 + 8s + 10K$$

R-H Table of this function is

s^3	1	8
s^2	6	10K
s	$\frac{48-10K}{6}$	0
1	10K	0

If we take K = 4.8,

$$P(s) = 6s^2 + 48$$

$$P'(s) = 12$$

R-H Table of this modified system is,

s^3	1	8
s^2	6	10K
s	12	0
1	48	0

So, as we can notice, there is no sign change in the first column. So, we can conclude that K = 4.8 is the smallest value of K for which the system is stable.

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Question 3

Form the R-H table for the following polynomials.

Part (a) :

$$G1(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$$

s^5	1	5	1
s^4	3	4	3
s^3	3.6667	0	0
s^2	4	3	0
s	-2.75	0	0
1	3	0	0

```
s = poly(0, 's');
G1(s) = s^5+3*s^4+5*s^3+4*s^2+s+3;
disp(routh_t(G1(s)));
```

Part (b) :

$$G2(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

s^5	1	6	8
s^4	ϵ	5	20
s^3	$\frac{6\epsilon-5}{\epsilon}$	$\frac{8\epsilon-20}{\epsilon}$	0
s^2	$\frac{-8\epsilon^2+50\epsilon-25}{6\epsilon-5}$	20	0
s	$\frac{64\epsilon^2+160\epsilon}{8\epsilon^2-50\epsilon+25}$	0	0
1	20	0	0

```
s = poly(0, 's');
G2(s) = s^5+6*s^3+5*s^2+8*s+20;
disp(routh_t(G2(s)));
```

Part (c) :

$$G3(s) = s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4$$

s^5	1	3	2
s^4	-2	-6	-4
s^3	-8	-12	0
s^2	-3	-4	0
s	-1.333	0	0
1	-4	0	0

```
s = poly(0, 's');
G3(s) = s^5-2*s^4+3*s^3-6*s^2+2*s-4;
disp(routh_t(G3(s)));
```

Part (d) :

$$G4(s) = s^6 + s^5 - 6s^4 + s^2 + s - 6$$

s^6	1	-6	1	-6
s^5	1	0	1	0
s^4	-6	0	-6	0
s^3	-24	0	0	0
s^2	ϵ	-6	0	0
s	$\frac{-144}{\epsilon}$	0	0	0
1	-6	0	0	0

```
s = poly(0, 's');
G4(s) = s^6+s^5-6*s^4+s^2+s-6;
disp(routh_t(G4(s)));
```

Question 4

Part (a) :

For a 6 degree polynomial having entire row corresponding to s^3 equal to 0 in R-H Table, it's 4th degree R-H Table polynomial must be it's factor.

Let's assume,

$$G(s) = (3s^4 + 5)(2s^2 + 12s - 7) = 6s^6 + 36s^5 - 21s^4 + 10s^2 + 60s - 35$$

s^6	6	-21	10	-35
s^5	36	0	60	0
s^4	-21	0	-35	0
s^3	-84	0	0	0
s^2	ϵ	-35	0	0
s	$\frac{-2940}{\epsilon}$	0	0	0
1	$\frac{102900}{-2940}$	0	0	0

```
s = poly(0, 's');
G(s) = 6*s^6+36*s^5-21*s^4+10*s^2+60*s-35;
disp(routh_t(G(s)));
```

Part (b) :

For a 8 degree polynomial having entire row corresponding to s^3 equal to 0 in R-H Table, it's 4th degree R-H Table polynomial must be it's factor.

Let's assume,

$$G(s) = s^4(3s^4 + s^2 + 5) = 3s^8 + s^6 + 5s^4$$

s^8	3	1	5	0	0
s^7	24	6	20	0	0
s^6	0.25	2.5	0	0	0
s^5	-234	20	0	0	0
s^4	2.5213	0	0	0	0
s^3	20	0	0	0	0
s^2	60	0	0	0	0
s	120	0	0	0	0
1	0	0	0	0	0

```
s = poly(0, 's');
G(s) = 3*s^8+s^6+5*s^4;
disp(routh_t(G(s)));
```

Part (c) :
Let's assume,

$$G(s) = a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s^1 + a_0$$

For a 6 degree polynomial having it's first entry corresponding to s^3 in R-H Table equal to 0, it must satisfy the following equation.

$$(a_5a_2 - a_6a_1) = \frac{a_3}{a_5}(a_6a_3 - a_5a_4)$$

One Possible solution for the above condition is,
$a_6 = 1, a_5 = 0.5, a_2 = 1, a_3 = 1, a_0 = a_1 = a_4 = 0$

$$\implies G(s) = s^6 + 0.5s^5 + s^3 + s^2$$

s^6	1	0	1	0
s^5	0.5	1	0	0
s^4	4	0	0	0
s^3	ϵ	3	0	0
s^2	$\frac{12}{\epsilon}$	1	0	0
s	$\frac{36-\epsilon^2}{12\epsilon}$	0	0	0
1	1	0	0	0

```
s = poly(0, 's');
G(s) = s^6+0.5*s^5+s^3+s^2;
disp(routh_t(G(s)));
```

References

1) https://help.scilab.org/
2) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=English
3) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=Englishpage=2