EE 324 Control Systems Lab

Problem Sheet 3

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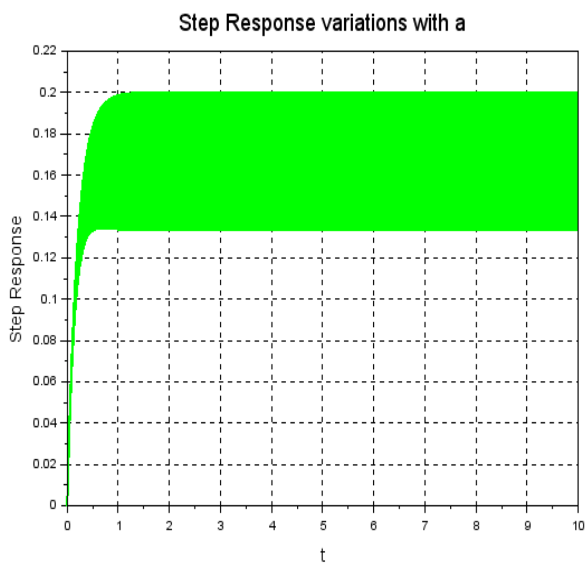
Question 1

Given Transfer Function is,

$$G(s) = \frac{s + 5 + a}{s^2 + 11s + 30}$$

Part (a) :

'a' is a real parameter to be varied from -1 to 1 in steps of 0.01.
Step Response of G(s) can be simulated in `SciLab` using `syslin` and `simp`

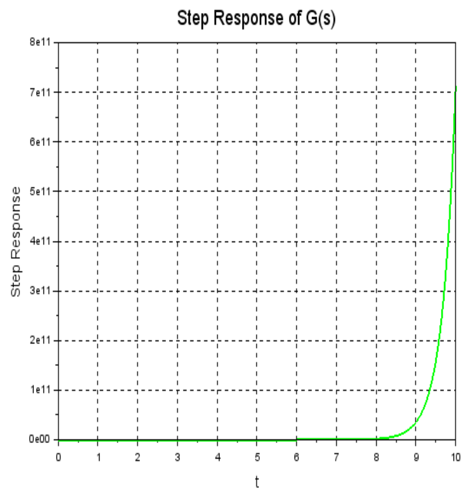



```
s = poly(0, 's');
P = -1:0.01:1;
for i=1:size(P, 2)
      a = P(i);
      [Num, Den] = simp(s + 5 + a, s^2 + 11*s + 30);
      Sys = syslin('c', Num, Den);
      t=0:0.05:10;
      plot(t, csim('step', t, Sys), 'g-');
      xgrid(0);
      xlabel("t");
      ylabel("Step Response");
      title("Step Response variations with a");
end
```

Part (b) :
Given Transfer Function is,

$$G(s) = \frac{1}{s^2 - s - 6}$$

Step Response of this system using `SciLab`, we get,

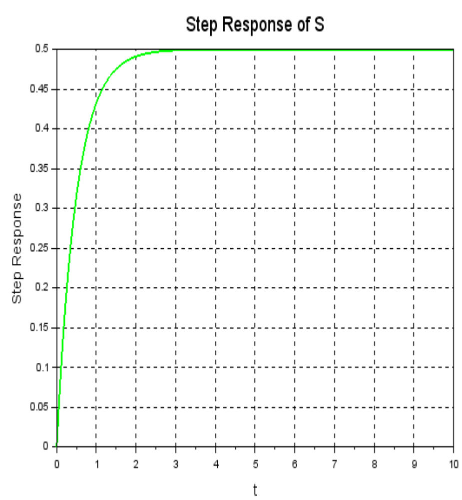



```
G = syslin('c', 1/(s^2-s-6));
t=0:0.05:10;
plot(t, csim('step', t, G), 'g-');
xgrid(0);
xlabel("t");
ylabel("Step Response");
title("Step Response of G(s)");
```

As we can notice, poles of the system are **3** and **-2**. To cancel the Right-half pole 3, we will add a zero at 3. So, the new system will be,

$$G(s) = \frac{1}{s + 2}$$

Step Response of this system using `SciLab`, we get,

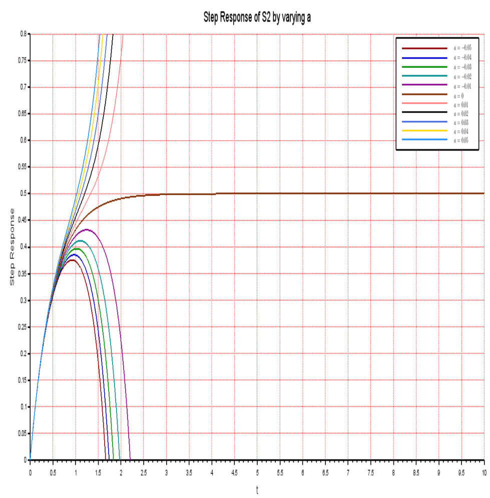


```
S = syslin('c', 1/(s + 2));
plot(t, csim('step', t, S), 'g-');
xgrid(0);
xlabel("t", 'fontsize', 3);
ylabel("Step Response");
title("Step Response of S");
```

Now, instead of directly cancelling the pole, we will vary the Right-half zero.

$$G(s) = \frac{s - 3 + a}{s^2 - s - 6}$$

The variations is Step Response using `MatLab` are,




```
Q = -0.05:0.01:0.05;
colors = ["red", "blue", "green", "cyan", "magenta", "brown", "pink", "black", "royalblue",
"gold", "violet"];
plotcolors = ["scilabred4", "scilab blue2", "scilab green4", "scilab cyan4",...
"scilabmagenta4", "scilabbrown4", "scilabpink4", "black", "royalblue","gold",...
"dodgerblue1"];
for i=1:size(Q, 2)
   a = Q(i);
   [Num, Den] = simp(s-3+a, s^2-s-6);
   S2 = syslin('c', Num, Den);
   t=0:0.01:10;
   plot2d(t, csim('step', t, S2), style=[color(plotcolors(i))]);
   xgrid(5,0.5,7);
   xlabel("t", 'fontsize', 3);
   ylabel("Step Response");
   title("Step Response of S2 by varying a");
end
set(gca(),"data_bounds",[0,0;10,0.8]);
legend(["a =  0  .05","a =  0  .04","a =  0  .03","a =  0  .02","a =  0  .01","a = 0",
"a = 0.01","a = 0.02","a = 0.03","a = 0.04","a = 0.05"]);
```

`An unstable plant cannot be rendered stable by canceling unstable poles by adding zeros attempting to cancel the unstable pole.`

As, we can notice in the above graph, response of the plant changes drastically on very low variations in the compensating zero.

So, if we don't know the exact location of Right-half pole, the response of the system can go unstable.

Also, sometimes, plant can be already stable with initial conditions, so adding another zero to the system might make it unstable.

Question 2

Part a :

Given Transfer Function is,

$$G(s) = \frac{85}{s^3 + 7s^2 + 27s + 85}$$

We will find poles of the system using `SciLab`

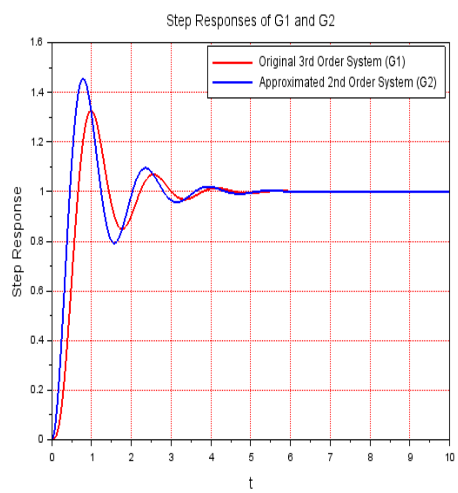
Poles of the system are -5, 1+4j and 1-4j

Here, 1+4j and 1-4j are the dominant poles for the second order approximation.

So, the modified system is,

$$G(s) = \frac{17}{s^2 + 2s + 17}$$

Step Response of the modified system using `SciLab` is,




```
s = poly(0, 's');
G1 = syslin('c', 85/(s^3+7*s^2+27*s+85));
[Zeros,Poles,Gain] = tf2zp(Gl);
disp(Poles);
G2 = syslin('c',17/(s^2+2*s+17));
t = 0:0.01:10;
plot(t, csim('step', t, G1), 'r--');
plot(t, csim('step', t, G2), 'b--');
xgrid(5,0.5,7);
xlabel("t");
ylabel("Step Response");
title("Step Responses of G1 and G2")
legend(["Original 3rd Order System", "Approximated 2nd Order System"]);
```

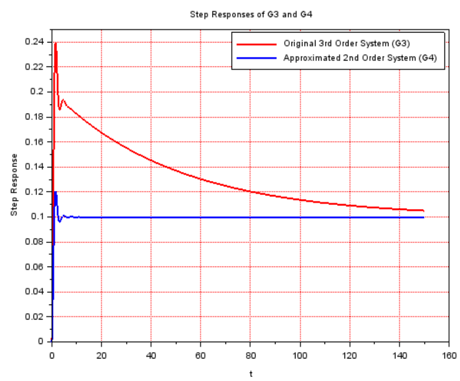
Part (b) :
Given Transfer Function is,

$$G(s) = \frac{s+0.01}{s^3+(101/50)s^2+(126/25)s+0.1}$$

We will find poles of the system using `SciLab`
Poles of the system are -0.02, -1+2j and -1-2j while -0.01 is a zero.
Here, -1+2j and -1-2j are the dominant poles for the second order approximation.
So, the modified system is,

$$G(s) = \frac{0.5}{s^2+2s+5}$$

Step Response of the modified system using `SciLab` is,




```
s = poly(0, 's');
G3 = syslin('c', (s+0.01)/(s^3+(101/50)*s^2+(126/25)*s+0.1));
[Zeros,Poles,Gain] = tf2zp(G3);
disp(Poles);
G4 = syslin('c', 0.5/(s^2+2*s+5));
t = 0:0.01:150;
plot(t, csim('step', t, G3), 'r–');
plot(t, csim('step', t, G4), 'b–');
xgrid(5,0.5,7);
xlabel("t");
ylabel("Step Response");
title("Step Responses of G3 and G4");
set(gca(),"data_bounds",[0,0;150,0.25]);
legend(["Original 3rd Order System (G3)", "Approximated 2nd Order System (G4)"]);
```

Question 3

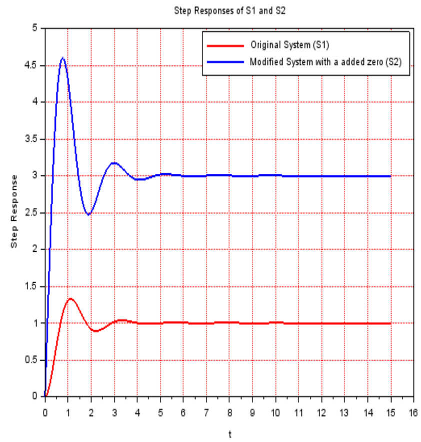
Part a :
Given Transfer Function is,

$$G(s) = \frac{9}{s^2+2s+9}$$

I added a zero at -3. Then, the system becomes,

$$G(s) = \frac{9(s+3)}{s^2+2s+9}$$

Step Response of the modified system using `SciLab` is,




```
s = poly(0, 's');
//S1 = syslin('c', 9/(s^2+2*s+9));
//trfmod(S1);
S2 = syslin('c', 9*(s+3)/(s^2+2*s+9));
t = 0:0.01:15;
plot(t, csim('step', t, S1), 'r-');
plot(t, csim('step', t, S2), 'b-');
xgrid(5,0.5,7);
xlabel("t");
ylabel("Step Response");
title("Step Responses of S1 and S2");
legend(["Original System (S1)", "Modified System with a added zero (S2)"]);
```

System	Rise Time (Tr)	Percentage Overshoot (%OS)
Original System	0.46s	32.92%
System with added zero at -3	0.26s	53.29%

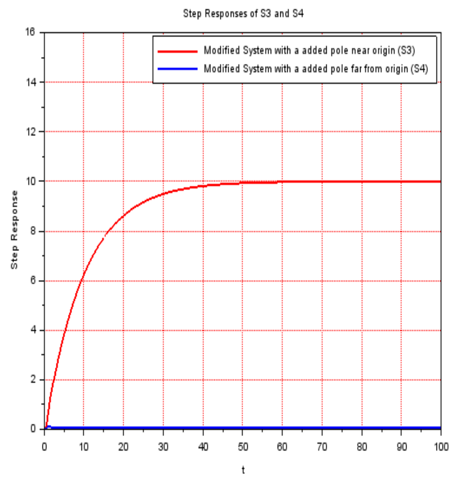
```
Sys = csim('step',t,S2);
steady_value = Sys(size(Sys,2));
peak_value = max(Sys);
OS = 100*(peak_value-steady_value)/steady_value;
disp(OS);
time_rise_h = 0;
time_rise_l = 0;
for i=1:size(Sys, 2)
  if(Sys(i)-(0.1*steady_value)<= 1e-5);
  time_rise_l = t(i);
  break;
  end
end
for i=1:size(Sys, 2)
   if(Sys(i)-(0.9*steady_value)<= 1e-5);
   time_rise_h = t(i);
   break;
   end
end
time_rise = (time_rise_h-time_rise_l);
disp(time_rise);
```

Part b :
I added poles at -0.1 and -12 respectively. The corresponding systems are,

$$G1(s) = \frac{9}{(s+0.1)(s^2+2s+9)}$$

$$G2(s) = \frac{9}{(s+12)(s^2+2s+9)}$$

Step Response of the modified systems using `SciLab` is,




```
s = poly(0, 's');
S3 = syslin('c', 9/((s+0.1)*(s^2+2*s+9)));
S4 = syslin('c', 9/((s+12)*(s^2+2*s+9)));
t = 0:0.01:100;
plot(t, csim('step', t, S3), 'r--');
plot(t, csim('step', t, S4), 'b--');
xgrid(5,0.5,7);
xlabel("t");
ylabel("Step Response");
title("Step Responses of S3 and S4");
set(gca(),"data_bounds",[0,0;150,15]);
```

System	Rise Time (Tr)	Percentage Overshoot (%OS)
Added pole at -0.01	10.99s	0%
Added pole at -12	0.48s	31.83%

I calculated Rise Time and Percentage Overshoot using the same code used for Part a.

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Part c :
`Observations on the effect of additional poles and zeros on the system :`

1) Adding zeros effects the amplitude of the response but response of the system remains the same.

2) Poles which are closer to the zero are dominant poles and have dominant effect on the response of the approximated 2 pole system.

3) Effect of additional poles on the overall response would be negligible.

4) For a complex conjugate three pole system (2 complex+ 1 real), the real pole should be atleast 5 times the real part of complex poles for the complex conjugates poles to be dominant.

Question 4

With the Natural Frequency $(\omega_n) = 1$,

Undamped System :

$$G1(s) = \frac{1}{s^2 + 1}$$

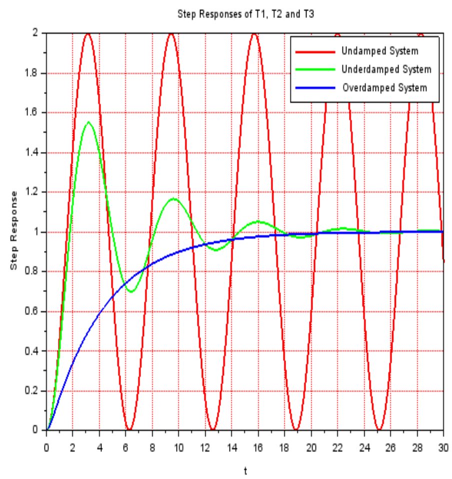
Underdamped System :

$$G2(s) = \frac{1}{s^2 + (3/8)s + 1}$$

Overdamped System :

$$G3(s) = \frac{1}{s^2 + (9/2) + 1}$$

Unit Step Response using `SciLab` is as follows :



System	%OS	Peak Time (Tp)	Delay Time (Td)	Rise Time (Tr)	2% Settling Time
Undamped	NaN	3.14s	0.96s	0.91s	-
Underdamped	24.59%	3.27s	1.13s	1.19s	10.66s
Overdamped	0%	∞	2.38s	8.36s	16.53s

```
s = poly(0,'s');
z1 = 3/8;
z2 = 9/2;
T1 = syslin('c', 1/(s^2+1));
T2 = syslin('c', 1/(s^2+z1*s+1));
T3 = syslin('c', 1/(s^2+z2*s+1));
t=0:0.01:30;
plot(t, csim('step', t, T1), 'r--','Linewidth',2);
plot(t, csim('step', t, T2), 'g--','Linewidth',2);
plot(t, csim('step', t, T3), 'b--','Linewidth',2);
xgrid(5,0.5,7);
xlabel("t");
ylabel("Step Response");
title("Step Responses of T1, T2 and T3");
legend(["Undamped System", "Underdamped System", "Overdamped System"]);
```

Observations :

1) Percentage Overshoot (%OS) : %OS decreases with increase in ζ. Undamped and Overdamped System have the highest and lowest %OS respectively.

2) Peak Time (Tp) : Tp increases with increase in ζ. Overdamped System has Peak Time ∞.

3) Delay Time (Td) : Td increases with increase in ζ.

4) Rise Time (Tr) : Tr increases with increase in ζ. Undamped System has Tr below 1s whereas Overdamped System has Tr greater than 8.

5) 2% Settling Time (Ts) : Undamped System never settles. For Underdamped and Overdamped Systems, Ts increases with increase in ζ.

```
time_peak1 = 3.14/sqrt(1-(z1)^2);
time_settle1= 4/z1
Sys = csim('step',t,T1);
steady_value = Sys(size(Sys,2));
peak_value = max(Sys);
time_peak=0;
for i=1:size(Sys, 2)
   if(Sys(i)-(peak_value)<= 1e-5);
   time_peak = t(i);
   break;
   end
end
OS = 100*(peak_value-steady_value)/steady_value;
disp(OS);
time_delay = 0
for i=1:size(Sys, 2)
   if(Sys(i)-(0.5*steady_value)<= 1e-5)
   time_delay = t(i);
   break;
   end
end
disp(time_delay);
time_rise_h = 0;
time_rise_l = 0;
for i=1:size(Sys, 2)
  if(Sys(i)-(0.1*steady_value)<= 1e-5);
  time_rise_l = t(i);
  break;
  end
end
for i=1:size(Sys, 2)
   if(Sys(i)-(0.9*steady_value)<= 1e-5);
   time_rise_h = t(i);
   break;
   end
end
time_rise = (time_rise_h-time_rise_l);
disp(time_rise);
time_s=();
for i=1:size(Sys, 2)
   if(Sys(i)-(0.98*steady_value)<= 1e-5);
   time_s($+1) = t(i);
   break;
   end
```

```
end
time_settle = time_s(size(time_s));
disp(time_settle);
```

References

1) EE302 : Control Systems lectures by Prof. Harish Pillai
2) https://help.scilab.org/
3) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=English
4) https://spoken-tutorial.org/tutorial-search/?search_foss=Scilabsearch_language=Englishpage=2