Indian Institute of Technology Bombay

EE338 : Digital Signal Processing

Filter Design Assignment

Mayur Ware

19D070070, Department of Electrical Engineering

Group 2

Filter Number : 74

Guided by :

Prof. V. M. Gadre

Assignment Report

March 31, 2022

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1. Reviews

1.1 Peer Review

Name of the reviewer : Siddhant Shingade
Roll Number of the reviewer : 19D070057
Group Number : 2

Review Comments :

The filter number assigned to Mayur is 74. He has implemented all four components of the mandatory part of the filter design assignment which are IIR Bandpass Filter, FIR Bandpass Filter, IIR Bandstop Filter and FIR Bandstop Filter by appropriate Buterworth and Chebyshev approximations. He has correctly implemented the Kaiser window specifications for the FIR filters.

Mayur has included all the plots in the report such as Magnitude Response, Frequency Response, Phase Response, Pole-Zero Map as well as Time Domain Response of the filters. He has made the Direct Form II realizations of the Discrete IIR filters. Finally, he has compared the FIR and IIR designs and also given the link to access his code.

I, Siddhant Shingade, hereby declare that Mayur Ware (19D070070) has completed all parts of the mandatory component of the filter design assignment.

1.2 Peer Review by me

Name of the peer : Rushikesh Metkar
Roll Number of the reviewer : 19D070034
Group Number : 2

Review Comments :

I will be reviewing the filter design assignment of Rushikesh Metkar, Roll Number 19D070034. The filter number assigned to him is 44. Following are my comments on his assignment :

1) Rushikesh has correctly used the specifications assigned to him for both the Bandpass and Bandstop filters.
2) He has correctly used the butterworth and chebyshev approximations to implement both IIR and FIR designs for each filter.
3) He has correctly used the Kaiser window specifications to implement the FIR designs for each filter.
4) He has included all the results and their plots in the report with proper explainations.

Hence, I ascertain that Rushikesh has completed every part of the mandatory component of the filter design assignment.

2. Bandpass Filter (Filter 1)

2.1 Un-normalized Discrete Time Filter Specifications

Filter Number : 74
q(m) = [0.1*m] = [7.4] = 7
r(m) = m - 10*q(m) = 74 - 10*7 = 4
B_L(m) = 10 + 5*q(m) + 13*r(m) = 10 + 5*7 + 13*4 = 97
B_H(m) = B_L(m) + 45 = 97 + 45 = 142
The first filter given is a **Bandpass** with the passband from B_L(m) kHz to B_H(m) kHz. So, the specifications of this filter are :

> Passband : 97 kHz to 142 kHz
> Transition Band : 3 kHz on either side of passband
> Stopband : 0-94 kHz and 145-270 kHz ($\because$ Sampling rate is 540 kHz)
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Monotonic
> Stopband Nature : Monotonic

2.2 Normalized Digital Filter Specifications

Our given Sampling Rate is 540 kHz. This rate corresponds to 2π on the normalized frequency axis. So, any frequency Ω upto 270 kHz (SamplingRate/2) can be represented on the normalised axis (ω) as

$$\omega = \frac{\Omega \times 2\pi}{\text{Sampling Rate}}$$

So, the specifications of this filter are :

> Passband : 0.359π to 0.525π
> Transition Band : 0.011π on either side of passband
> Stopband : 0 - 0.348π and 0.537π - π
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Monotonic
> Stopband Nature : Monotonic

2.3 Analog filter specifications for Bandpass analog filter using Bilinear Transformation

The bilinear transformation is given as :

$$\Omega = tan(\frac{\omega}{2})$$

By applying this transform at the bandedges, we get

ω	Ω
0	0
0.348π	0.608
0.359π	0.632
0.525π	1.081
0.537π	1.123
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are :

> Passband : 0.632(Ω_{P1}) to 1.081(Ω_{P2})
> Transition Band : 0.608 to 0.632 1.081 to 1.123
> Stopband : 0 to 0.608(Ω_{S1}) and 1.123(Ω_{S2}) to ∞
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Monotonic
> Stopband Nature : Monotonic

2.4 Frequency Transformation & Relevant Parameters

We transform the Bandpass analog filter to a Lowpass analog filter. We need two parameters in this case. We can make use of the Bandpass transformation which is given as :

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

The two parameters mention above are B and Ω_0. Considering the specifications defined for the analog filter, parameters can be calculated using the following equations :

$$\Omega_0 = \sqrt{\Omega_{P1} \times \Omega_{P2}} = \sqrt{0.632 \times 1.081} = 0.8265$$

$$B = \Omega_{P2} - \Omega_{P1} = 1.081 - 0.632 = 0.449$$

Ω	Ω_L
0^+	$-\infty$
0.608(Ω_{S1})	-1.1481($\Omega_{L_{S1}}$)
0.632(Ω_{P1})	-0.9996($\Omega_{L_{P1}}$)
0.8265(Ω_0)	0
1.081(Ω_{P2})	1($\Omega_{L_{P2}}$)
1.123(Ω_{S2})	1.1463($\Omega_{L_{S2}}$)
∞	∞

2.5 Frequency Transformed Lowpass Analog Filter Specifications

> Passband Edge : 1 (Ω_{L_P})
> Stopband Edge : min(-$\Omega_{L_{S1}}$, $\Omega_{L_{S2}}$) = min(1.1481, 1.1463) = 1.1463 (Ω_{L_S})
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Monotonic
> Stopband Nature : Monotonic

2.6 Lowpass Analog Filter Transfer Function

We need to design this filter using the Chebyshev approximation as we need an equiripple passband and a monotonic stopband. The Tolerance (δ) value for both the bands is 0.15. So, we define new quantities using the following equations :

$$D_1 = \frac{1}{(1-\delta^2)} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = 43.44$$

Now choosing the parameter ϵ of the Chebyshev filter to be $\sqrt{D_1}$, we get the minimum value of N (N_{min}) as:

$$N_{min} = \left\lceil \frac{log\sqrt{\frac{D_2}{D_1}}}{log\frac{\Omega_{LS}}{\Omega_{LP}}} \right\rceil$$

$$= \left\lceil \frac{log\sqrt{\frac{43.44}{0.3841}}}{log\frac{1.1463}{1}} \right\rceil = \left\lceil \frac{log(17.1593)}{0.0592} \right\rceil = \lceil 20.8530 \rceil = 20$$

Then,
The cut-off frequency (Ω_c) of the Analog Lowpass filter constraints are calculated using the following equation :

$$\frac{\Omega_p}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_s}{D_2^{\frac{1}{2N}}}$$

$$1.0242 \leq \Omega_c \leq 1.0431$$

Hence, we can safely assume the value of Ω_c as 1.03. So, the poles of the transfer function can be obtained using the following equation :

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s}{j1.03}\right)^{40} = 0$$

The roots of this equation are shown in Fig. 1 (calculated using `https://www.wolframalpha.com/`) :

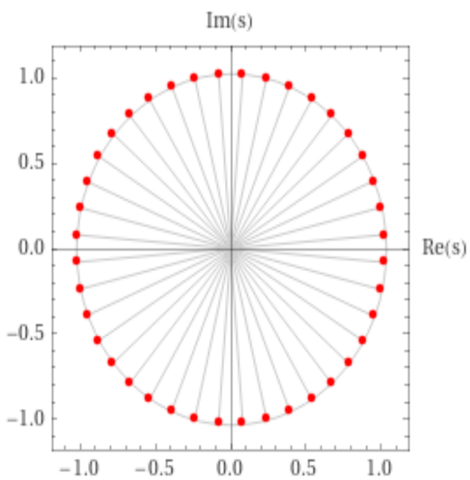

Fig. 1 : Poles of Transfer Function

Now, for the Analog Lowpass Filter to be stable, we must select the poles lying in the Left Half Plane (LHP) of the plot.
So, the selected poles are ,
p_1 = -1.02682 - 0.0808129i
p_2 = -1.02682 + 0.0808129i
p_3 = -1.00154 - 0.240449i
p_4 = -1.00154 + 0.240449i
p_5 = -0.951596 - 0.394164i
p_6 = -0.951596 + 0.394164i
p_7 = -0.878219 - 0.538174i
p_8 = -0.878219 + 0.538174i
p_9 = -0.783218 - 0.668931i
p_{10} = -0.783218 + 0.668931i
p_{11} = -0.668931 - 0.783218i
p_{12} = -0.668931 + 0.783218i
p_{13} = -0.538174 - 0.878219i
p_{14} = -0.538174 + 0.878219i
p_{15} = -0.394164 - 0.951596i
p_{16} = -0.394164 + 0.951596i
p_{17} = -0.240449 + 1.00154i
p_{18} = -0.240449 - 1.00154i
p_{19} = -0.0808129 - 1.02682i
p_{20} = -0.0808129 + 1.02682i
Using these LHP poles, we can write the Analog Lowpass Filter Transfer Function as :

$$H_{Analog,LPF}(s_L) =$$
$$\frac{(\Omega_c)^N}{(s_L-p_1)(s_L-p_2)(s_L-p_3)(s_L-p_4)(s_L-p_5)(s_L-p_6)(s_L-p_7)(s_L-p_8)(s_L-p_9)(s_L-p_{10})}$$
$$\times\frac{1}{(s_L-p_{11})(s_L-p_{12})(s_L-p_{13})(s_L-p_{14})(s_L-p_{15})(s_L-p_{16})(s_L-p_{17})(s_L-p_{18})(s_L-p_{19})(s_L-p_{20})}$$
$$=\frac{1.8061}{(s_L^2+2.05s_L+1.06)(s_L^2+2s_L+1.06)(s_L^2+1.9s_L+1.06)(s_L^2+1.756s_L+1.06)(s_L^2+1.566s_L+1.06)(s_L^2+1.336s_L+1.06)}$$
$$\times\frac{1}{(s_L^2+1.076s_L+1.06)(s_L^2+0.788s_L+1.06)(s_L^2+0.48s_L+1.06)(s_L^2+0.16s_L+1.06)}$$

Here, we have adjusted the numerator and denominator of the transfer function to have a DC gain 1.

2.7 Analog Bandpass Transfer Function

The Transfer Equation is given by :

$$s_L = \frac{s^2+\Omega_0^2}{Bs}$$

By substituting the values, B = 0.449 and Ω_0 = 0.8265, we get :

$$s_L = \frac{s^2+0.683}{0.449s}$$

Now, by substituting this value into $H_{Analog,LPF}(s_L)$, we get $H_{Analog,BPF}(s)$.
We can write $H_{Analog,BPF}(s)$ in the form of N(s)/D(s) to calculate the coefficients.
`Numerator :`

Degree	s^1
Coefficient	3.097e-06

`Denominator :`

Degree	s^{40}	s^{39}	s^{38}	s^{37}	s^{36}
Coefficient	1(a_{40})	6.75(a_{39})	36.52(a_{38})	139.11(a_{37})	455.76(a_{36})

Degree	s^{35}	s^{34}	s^{33}	s^{32}	s^{31}
Coefficient	1.24e+03(a_{35})	3.05e+03(a_{34})	6.62e+03(a_{33})	1.30e+04(a_{32})	2.34e+04(a_{31})

Degree	s^{30}	s^{29}	s^{28}	s^{27}	s^{26}
Coefficient	3.88e+04(a_{30})	5.93e+04(a_{29})	8.44e+04(a_{28})	1.11e+05(a_{27})	1.38e+05(a_{26})

Degree	s^{25}	s^{24}	s^{23}	s^{22}	s^{21}
Coefficient	1.60e+05(a_{25})	1.74e+05(a_{24})	1.78e+05(a_{23})	1.71e+05(a_{22})	1.54e+05(a_{21})

Degree	s^{20}	s^{19}	s^{18}	s^{17}	s^{16}
Coefficient	1.32e+05(a_{20})	1.05e+05(a_{19})	8.01e+04(a_{18})	5.69e+04(a_{17})	3.81e+04(a_{16})

Degree	s^{15}	s^{14}	s^{13}	s^{12}	s^{11}
Coefficient	2.39e+04(a_{15})	1.41e+04(a_{14})	7.82e+03(a_{13})	4.04e+03(a_{12})	1.94e+03(a_{11})

Degree	s^{10}	s^{9}	s^{8}	s^{7}	s^{6}
Coefficient	872.92(a_{10})	360.53(a_9)	137.47(a_8)	47.63(a_7)	15.04(a_6)

Degree	s^{5}	s^{4}	s^{3}	s^{2}	s^{1}	s^{0}
Coefficient	4.20(a_5)	1.04(a_4)	0.21(a_3)	0.03(a_2)	0.0050(a_1)	5.04e-04(a_0)

2.8 Discrete Time Bandpass Filter

We use the Bilinear Transformation given by the following equation to transform the analog domain bandpass filter transfer function into the discrete domain bandpass filter transfer function.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

By substituting this equation in $H_{Analog,BPF}(s)$, we get the discrete transfer function $H_{Discrete,BPF}(z)$. It can be written in the form N(z)/D(z) to calculate the coefficients :

`Numerator :`

Degree	z^{0}	z^{-1}	z^{-2}	z^{-3}	z^{-4}
Coefficient	1.7344e-12(b_0)	0(b_1)	-3.4687e-11(b_2)	0(b_3)	3.2953e-10(b_4)

Degree	z^{-5}	z^{-6}	z^{-7}	z^{-8}	z^{-9}
Coefficient	0(b_5)	-1.9772e-09(b_6)	0(b_7)	8.4030e-09(b_8)	0(b_9)

Degree	z^{-10}	z^{-11}	z^{-12}	z^{-13}	z^{-14}
Coefficient	3.88e+04(b_{10})	0(b_{11})	-2.6890e-08(b_{12})	0(b_{13})	6.7224e-08(b_{14})

Degree	z^{-15}	z^{-16}	z^{-17}	z^{-18}	z^{-19}
Coefficient	0(b_{15})	-1.3445e-07(b_{16})	0(b_{17})	2.1848e-07(b_{18})	0(b_{19})

Degree	z^{-20}	z^{-21}	z^{-22}	z^{-23}	z^{-24}
Coefficient	-2.9131e-07(b_{20})	0(b_{21})	3.2044e-07(b_{22})	0(b_{23})	-2.9131e-07(b_{24})

Degree	z^{-25}	z^{-26}	z^{-27}	z^{-28}	z^{-29}
Coefficient	0(b_{25})	2.1848e-07(b_{26})	0(b_{27})	-1.3445e-07(b_{28})	0(b_{29})

Degree	z^{-30}	z^{-31}	z^{-32}	z^{-33}	z^{-34}
Coefficient	6.7224e-08(b_{30})	0(b_{31})	-2.6890e-08(b_{32})	0(b_{33})	8.4030e-09(b_{34})

Degree	z^{-35}	z^{-36}	z^{-37}	z^{-38}	z^{-39}	z^{-40}
Coefficient	0(b_{35})	-1.9772e-09(b_{36})	0(b_{37})	3.2953e-10(b_{38})	0(b_{39})	-3.4687e-11(b_{40})

`Denominator :`

Degree	z^{0}	z^{-1}	z^{-2}	z^{-3}	z^{-4}
Coefficient	1(a_0)	-6.0445(a_1)	29.6163(a_2)	-102.2560(a_3)	306.3950(a_4)

Degree	z^{-5}	z^{-6}	z^{-7}	z^{-8}	z^{-9}
Coefficient	-770.3209(a_5)	1.7409e+03(a_6)	-3.4952e+03(a_7)	8.4030e-09(a_8)	0(a_9)

Degree	z^{-10}	z^{-11}	z^{-12}	z^{-13}	z^{-14}
Coefficient	3.88e+04(a_{10})	-5.5634e+04(a_{11})	-2.6890e-08(a_{12})	6.4366e+03(a_{13})	-1.0815e+04(a_{14})

Degree	z^{-15}	z^{-16}	z^{-17}	z^{-18}	z^{-19}
Coefficient	1.6879e+04(a_{15})	-2.4394e+04(a_{16})	3.3016e+04(a_{17})	-4.1755e+04(a_{18})	4.9720e+04(a_{19})

Degree	z^{-20}	z^{-21}	z^{-22}	z^{-23}	z^{-24}
Coefficient	5.8822e+04(a_{20})	-5.8643e+04(a_{21})	5.5361e+04(a_{22})	-4.9372e+04(a_{23})	4.1733e+04(a_{24})

Degree	z^{-25}	z^{-26}	z^{-27}	z^{-28}	z^{-29}
Coefficient	-3.3342e+04(a_{25})	2.5247e+04(a_{26})	-1.8056e+04(a_{27})	1.2225e+04(a_{28})	-7.8020e+03(a_{29})

Degree	z^{-30}	z^{-31}	z^{-32}	z^{-33}	z^{-34}
Coefficient	4.7032e+03(a_{30})	-2.6626e+03(a_{31})	1.4187e+03(a_{32})	-705.7381(a_{33})	328.5949(a_{34})

Degree	z^{-35}	z^{-36}	z^{-37}	z^{-38}	z^{-39}	z^{-40}
Coefficient	-141.5331(a_{35})	56.5923(a_{36})	-20.6183(a_{37})	-2.0402(a_{38})	0.5434(a_{39})	3.5582e-04(a_{40})

2.9 Realization using Direct Form II

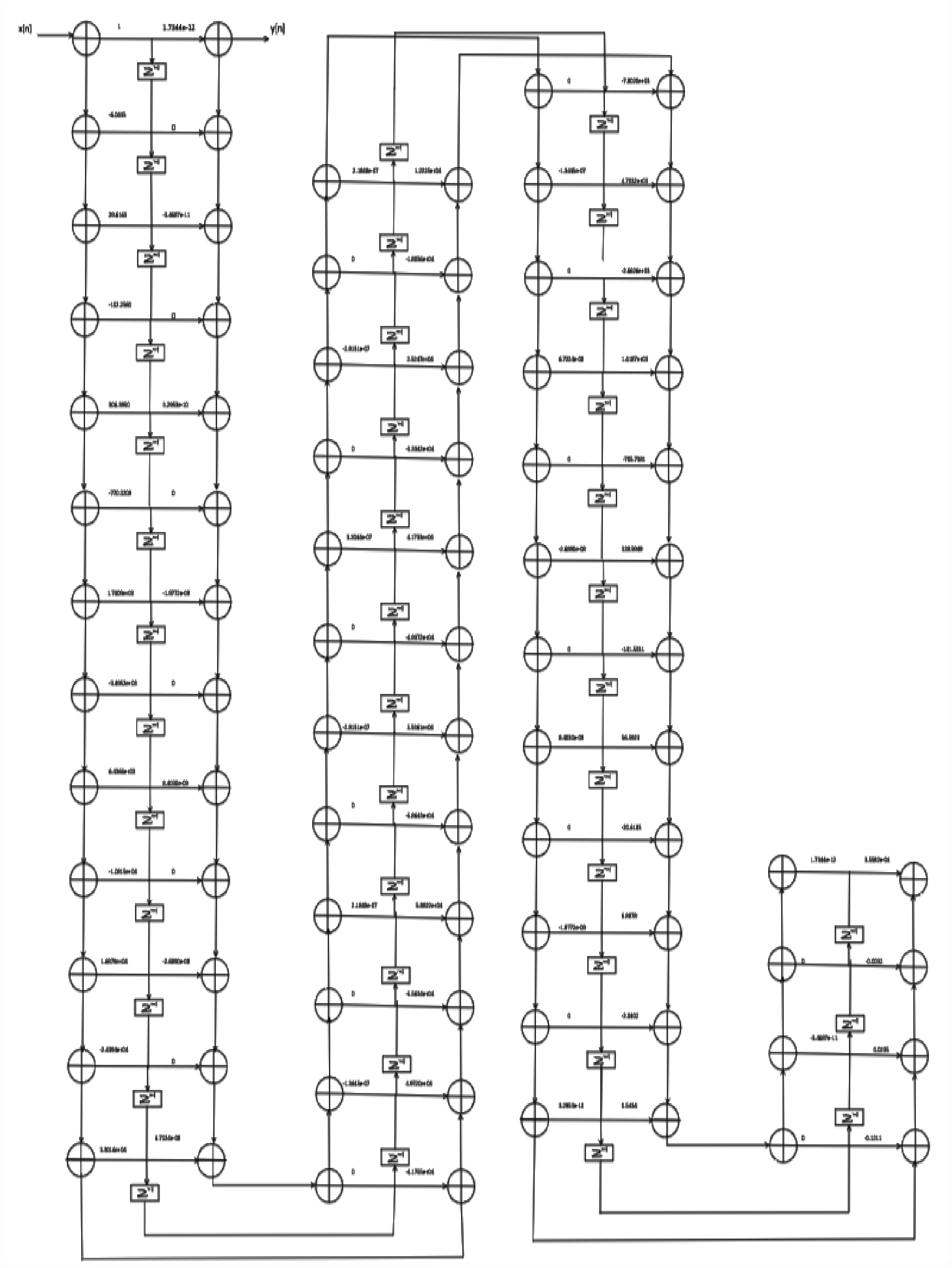

Fig. 2 : Realization of Bandpass Filter using Direct Form II

2.10 IIR Bandpass Filter Response

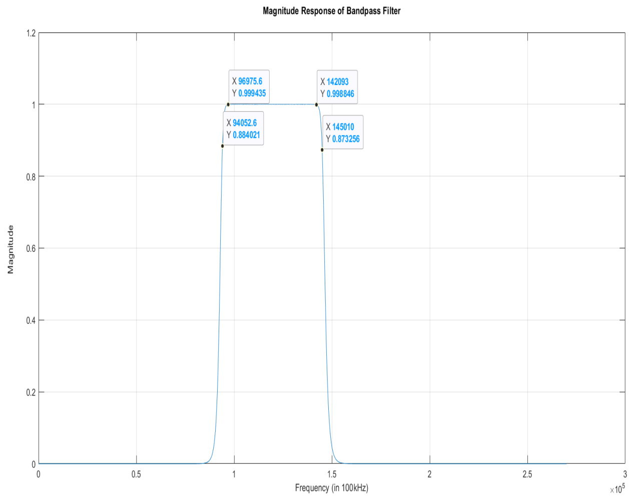

Fig. 3 : Magnitude Response of IIR Bandpass Filter

Fig. 3 shows the Magnitude Plot of IIR Bandpass Filter. The bandedge specifications are marked in the plot and they meet the required criteria in stopband and passband within the transition band of 3kHz.

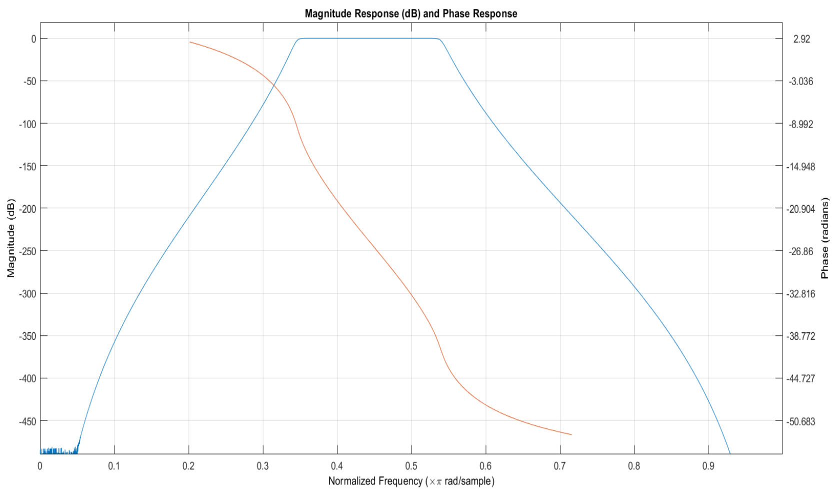

Fig. 4 : Magnitude and Phase Response of IIR Bandpass Filter (in dB)

Fig. 4 shows the Magnitude and Phase Response of IIR Bandpass Filter in dB scale. We can notice that the tolerance specifications have been met and also, the phase response in not exactly linear.

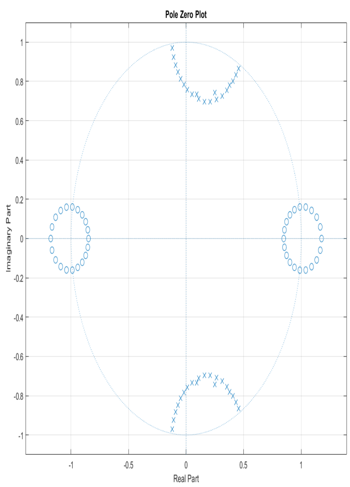

Fig. 5 : Pole-Zero Map of IIR Bandpass Filter

Fig. 5 shows the Pole-Zero Map of IIR Bandpass Filter. As all the poles are in the unit circle, the system is stable.

2.11 FIR Filter Transfer Function using Kaiser Window

We define minimum stopband attenuation in terms of stopband and passband tolerance value (δ) which is given as 0.15 to us.

$$A = -20log(\delta) = -20log(0.15) = 16.4782dB$$

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window. Now, to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here, $\Delta\omega_T$ is the minimum transition width which is same on the either sides of passband in our case.

$$\Delta\omega_T = \frac{3kHz * 2\pi}{540kHz} = 0.011\pi$$

$$\therefore N \geq 108$$

The first condition can be satisfied using N=108, but to satisfy the other conditions, on successive trials using MATLAB, it was found that a window length of **135** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.
For designing the bandpass filter according to specifications, we first require an ideal bandpass filter. This is obtained by subtracting two lowpass filters of appropriate frequencies. We take the midpoint of the transition band in each case as the cutoff frequencies for the ideal lowpass filter.
By generating the ideal impulse response samples for the same length as that of the window, we obtained the time domain coefficients. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. A separate function is made to generate the impulse response of an ideal Lowpass filter. The bandpass impulse response samples were generated as the difference between three Lowpass filters :

```
Columns 1 through 14

 -0.0006    0.0035    0.0045   -0.0049   -0.0088    0.0026    0.0100    0.0010   -0.0070   -0.0022    0.0016   -0.0008    0.0022    0.0067

Columns 15 through 28

 -0.0018   -0.0115   -0.0021    0.0117    0.0059   -0.0073   -0.0054    0.0017   -0.0003    0.0005    0.0081    0.0027   -0.0129   -0.0088

Columns 29 through 42

  0.0115    0.0126   -0.0056   -0.0099    0.0004    0.0010   -0.0012    0.0085    0.0089   -0.0124   -0.0182    0.0082    0.0220   -0.0001

Columns 43 through 56

 -0.0159   -0.0037    0.0031   -0.0028    0.0078    0.0183   -0.0084   -0.0335   -0.0021    0.0374    0.0149   -0.0259   -0.0160    0.0061

Columns 57 through 70

 -0.0039    0.0053    0.0417    0.0075   -0.0806   -0.0490    0.0987    0.1069   -0.0816   -0.1575    0.0316    0.1775    0.0316   -0.1575

Columns 71 through 84

 -0.0816    0.1069    0.0987   -0.0490   -0.0806    0.0075    0.0417    0.0053   -0.0039    0.0061   -0.0160   -0.0259    0.0149    0.0374

Columns 85 through 98

 -0.0021   -0.0335   -0.0084    0.0183    0.0078   -0.0028    0.0031   -0.0037   -0.0159   -0.0001    0.0220    0.0082   -0.0182   -0.0124

Columns 99 through 112

  0.0089    0.0085   -0.0012    0.0010    0.0004   -0.0099   -0.0056    0.0126    0.0115   -0.0088   -0.0129    0.0027    0.0081    0.0005

Columns 113 through 126

 -0.0003    0.0017   -0.0054   -0.0073    0.0059    0.0117   -0.0021   -0.0115   -0.0018    0.0067    0.0022   -0.0008    0.0016   -0.0022

Columns 127 through 135

 -0.0070    0.0010    0.0100    0.0026   -0.0088   -0.0049    0.0045    0.0035   -0.0006
```

Fig. 6 : TIme Domain Sequence values for FIR Bandpass Filter

2.12 FIR Bandpass Filter Response

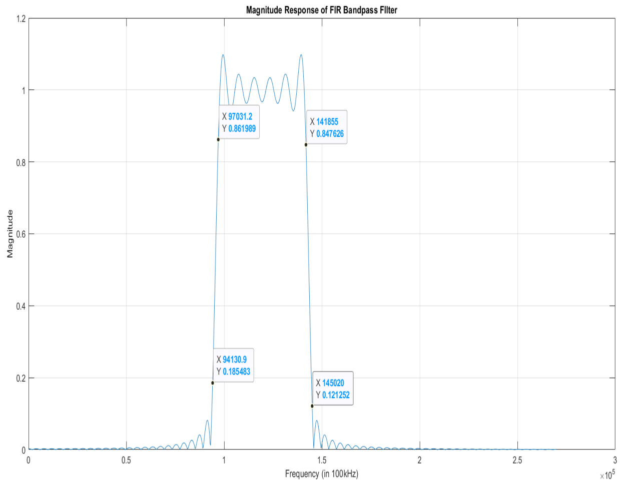

Fig. 7 : Magnitude Response of FIR Bandpass Filter

Fig. 7 shows the Magnitude Plot of FIR Bandpass Filter. The bandedge specifications are marked in the plot and they meet the required criteria in stopband and passband within the transition band of 3kHz.

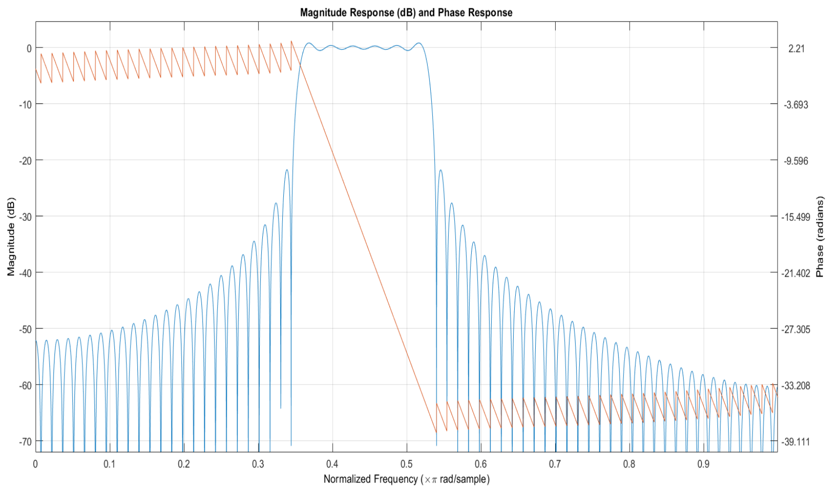

Fig. 8 : Magnitude and Phase Response of FIR Bandpass Filter (in dB)

Fig. 8 shows the Magnitude and Phase Response of FIR Bandpass Filter in dB scale. We can notice that the tolerance and attenuation specifications have been met. The phase response decreases linearly in the passband.

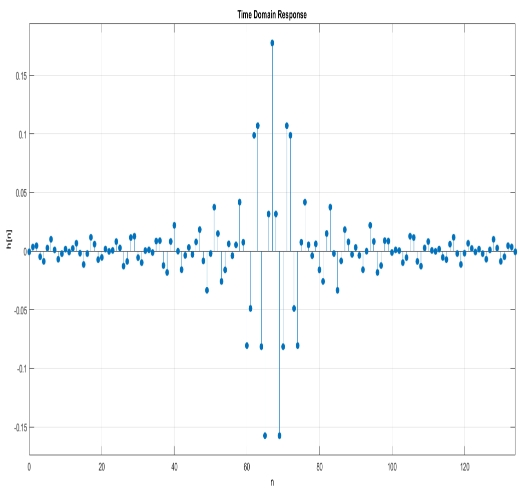

Fig. 9 : Time Domain Response of FIR Bandpass Filter

Fig. 9 shows the Time Domain Response of FIR Bandpass Filter.

3. Bandstop Filter (Filter 2)

3.1 Un-normalized Discrete Time Filter Specifications

Filter Number : 74
q(m) = [0.1*m] = [7.4] = 7
r(m) = m - 10*q(m) = 74 - 10*7 = 4
B_L(m) = 5 + 3*q(m) + 11*r(m) = 5 + 3*7 + 11*4 = 70
B_H(m) = B_L(m) + 45 = 70 + 25 = 95
The second filter given is a **Bandstop** with the stopband from B_L(m) kHz to B_H(m) kHz. So, the specifications of this filter are :
> Stopband : 70 kHz to 95 kHz
> Transition Band : 3 kHz on either side of passband
> Passband : 0-67 kHz and 98-200 kHz ($\because$ Sampling rate is 400 kHz)
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Equiripple
> Stopband Nature : Monotonic

3.2 Normalized Digital Filter Specifications

Our given Sampling Rate is 400 kHz. This rate corresponds to 2π on the normalized frequency axis. So, any frequency Ω upto 200 kHz (SamplingRate/2) can be represented on the normalised axis (ω) as

$$\omega = \frac{\Omega \times 2\pi}{\text{Sampling Rate}}$$

So, the specifications of this filter are :
> Stopband : 0.35π to 0.475π
> Transition Band : 0.015π on either side of stopband
> Passband : 0 - 0.335π and 0.49π - π
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Equiripple
> Stopband Nature : Monotonic

3.3 Analog filter specifications for Bandstop analog filter using Bilinear Transformation

The bilinear transformation is given as :

$$\Omega = tan(\frac{\omega}{2})$$

By applying this transform at the bandedges, we get

ω	Ω
0	0
0.335π	0.580
0.35π	0.612
0.475π	0.924
0.49π	0.969
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are :
> Stopband : 0.612(Ω_{S1}) to 0.924(Ω_{S2})
> Transition Band : 0.580 to 0.612 1.081 to 1.123
> Passband : 0 to 0.580(Ω_{P1}) and 0.969(Ω_{P2}) to ∞
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Equiripple
> Stopband Nature : Monotonic

3.4 Frequency Transformation & Relevant Parameters

We transform the Bandstop analog filter to a Lowpass analog filter. We need two parameters in this case. We can make use of the Bandstop transformation which is given as :

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

The two parameters mention above are B and Ω_0. Considering the specifications defined for the analog filter, parameters can be calculated using the following equations :

$$\Omega_0 = \sqrt{\Omega_{P1} \times \Omega_{P2}} = \sqrt{0.580 \times 0.969} = 0.7496$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.969 - 0.580 = 0.389$$

Ω	Ω_L
0^+	$-\infty$
$0.580(\Omega_{P1})$	$1(\Omega_{L_{P1}})$
$0.612(\Omega_{S1})$	$1.2706(\Omega_{L_{S1}})$
$0.745(\Omega_0^-)$	∞
$0.745(\Omega_0^+)$	$-\infty$
$0.924(\Omega_{S2})$	$-1.2314(\Omega_{L_{S2}})$
$0.969(\Omega_{P2})$	$-1(\Omega_{L_{P2}})$
∞	0^-

3.5 Frequency Transformed Lowpass Analog Filter Specifications

> Passband Edge : 1 (Ω_{L_P})
> Stopband Edge : min($\Omega_{L_{S1}}$, -$\Omega_{L_{S2}}$) = min(1.2706, 1.2314) = 1.2314 (Ω_{L_S})
> Tolerance : 0.15 in magnitude for both Passband and Stopband
> Passband Nature : Equiripple
> Stopband Nature : Monotonic

3.6 Lowpass Analog Filter Transfer Function

We need to design this filter using the Butterworth approximation as it has both a monotonic passband as well as a monotonic stopband. The Tolerance (δ) value for both the bands is 0.15. So, we define new quantities using the following equations :

$$D_1 = \frac{1}{(1-\delta^2)} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = 43.44$$

The inequality for calculating the order N of the butterworth filter is given by :

$$N_{min} = \left\lceil \frac{cosh^{-1}\sqrt{\frac{D_2}{D_1}}}{cosh^{-1}\frac{\Omega_s}{\Omega_p}} \right\rceil$$

$$= \left\lceil \frac{cosh^{-1}\sqrt{\frac{43.44}{0.3841}}}{cosh^{-1}\frac{1.2314}{1}} \right\rceil = \left\lceil \frac{3.5348}{0.6678} \right\rceil = \lceil 5.2932 \rceil = 5$$

Now, the poles of the transfer function can be obtained using the following equation :

$$1 + D_1 cosh^2(N_{min} cosh^{-1}(\frac{s}{j})) = 0$$

$$\implies 1 + 0.3841 cosh^2(5 cosh^{-1}(\frac{s}{j})) = 0$$

The roots of this equation are shown in Fig. 10 (calculated using `https://www.wolframalpha.com/`) :

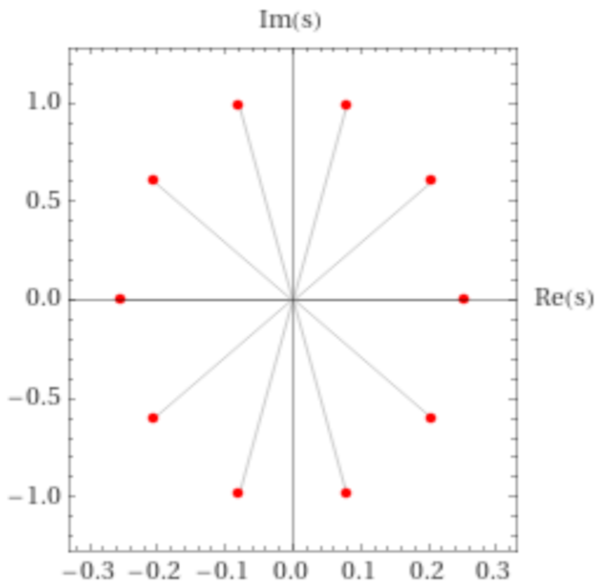

Fig. 10 : Poles of Transfer Function

Now, for the Analog Lowpass Filter to be stable, we must select the poles lying in the Left Half Plane (LHP) of the plot.
So, the selected poles are :
p_1 = -0.078453 + 0.98123i
p_2 = -0.078453 - 0.98123i
p_3 = -0.20539 + 0.60643i
p_4 = -0.20539 - 0.60643i
p_5 = -0.25388
As the number of poles is even and all the poles lie in the Left Half Plane (LHP), we write the Analog Transfer Function as :

$$H_{Analog,LPF}(s_L) = \frac{(-1)^5 p_1 p_2 p_3 p_4 p_5}{\sqrt{1+D_1}(s_L-p_1)(s_L-p_2)(s_L-p_3)(s_L-p_4)(s_L-p_5)}$$

3.7 Analog Bandstop Transfer Function

The Transfer Equation is given by :

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

By substituting the values, B = 0.389 and Ω_0 = 0.7496, we get :

$$s_L = \frac{0.389s}{s^2 + 0.562}$$

Now, by substituting this value into $H_{Analog,LPF}(s_L)$, we get $H_{Analog,BSF}(s)$.
We can write $H_{Analog,BSF}(s)$ in the form of N(s)/D(s) to calculate the coefficients.
`Numerator :`

Degree	s^{10}	s^9	s^8	s^7	s^6
Coefficient	1(a_{10})	0(a_9)	2.8144(a_8)	0(a_7)	3.1683(a_6)

Degree	s^5	s^4	s^3	s^2	s^1	s^0
Coefficient	0(a_5)	1.7834(a_4)	0(a_3)	0.5019(a_2)	0(a_1)	0.0565(a_0)

`Denominator :`

Degree	s^{10}	s^9	s^8	s^7	s^6
Coefficient	1(a_{10})	1.9810(a_9)	4.0530(a_8)	5.3814(a_7)	5.4450(a_6)

Degree	s^5	s^4	s^3	s^2	s^1	s^0
Coefficient	4.8903(a_5)	3.0649(a_4)	1.7050(a_3)	0.7228(a_2)	0.1989(a_1)	0.0565(a_0)

3.8 Discrete Time Bandstop Filter

We use the Bilinear Transformation given by the following equation to transform the analog domain bandpass filter transfer function into the discrete domain bandpass filter transfer function.

$$s = \frac{1-z^{-1}}{1+z^{-1}}$$

By substituting this equation in $H_{Analog,BSF}(s)$, we get the discrete transfer function $H_{Discrete,BSF}(z)$. It can be written in the form N(z)/D(z) to calculate the coefficients :
`Numerator :`

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}
Coefficient	0.3272(b_0)	-0.9151(b_1)	2.6597(b_2)	-4.2332(b_3)	6.5035(b_4)

Degree	z^{-5}	z^{-6}	z^{-7}	z^{-8}	z^{-9}	z^{-10}
Coefficient	-6.6540(b_5)	6.5035(b_6)	-4.2332(b_7)	2.6597(b_8)	-0.9151(b_9)	0.3272(b_{10})

`Denominator :`

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}
Coefficient	1(a_0)	-2.2155(a_1)	4.9042(a_2)	-6.2092(a_3)	7.4836(a_4)

Degree	z^{-5}	z^{-6}	z^{-7}	z^{-8}	z^{-9}	z^{-10}
Coefficient	-6.0731(a_5)	4.5888(a_6)	-2.2698(a_7)	0.9977(a_8)	-0.1829(a_9)	0.0065(a_{10})

3.9 Realization using Direct Form II

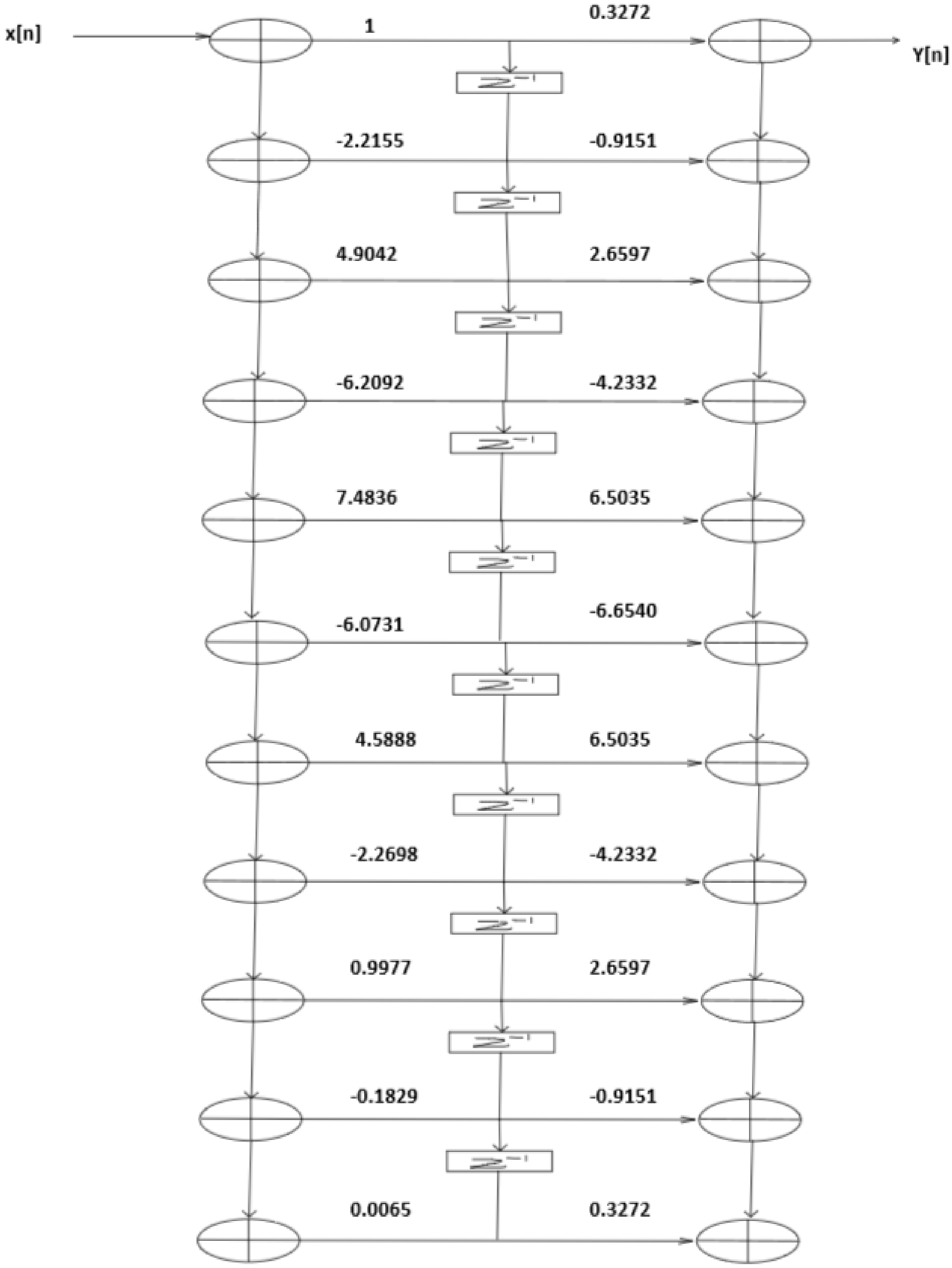

Fig. 11 : Realization of Bandstop Filter using Direct Form II

3.10 IIR Bandstop Filter Response

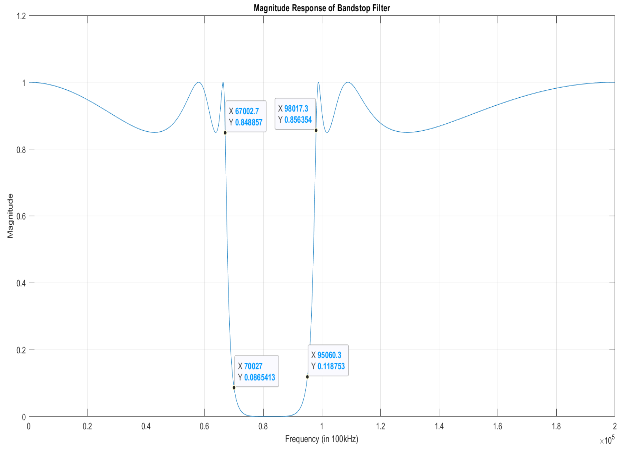

Fig. 12 : Magnitude Response of IIR Bandstop Filter

Fig. 12 shows the Magnitude Plot of IIR Bandstop Filter. The bandedge specifications are marked in the plot and they meet the required criteria in stopband and passband within the transition band of 3kHz.

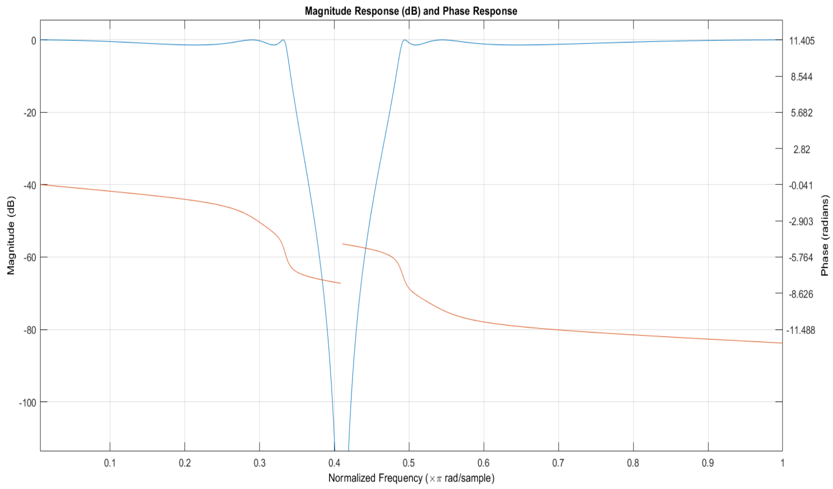

Fig. 13 : Magnitude and Phase Response of IIR Bandstop Filter (in dB)

Fig. 13 shows the Magnitude and Phase Response of IIR Bandstop Filter in dB scale. We can notice that the tolerance specifications have been met and also, the phase response in not exactly linear and it has a discontinuity in between.

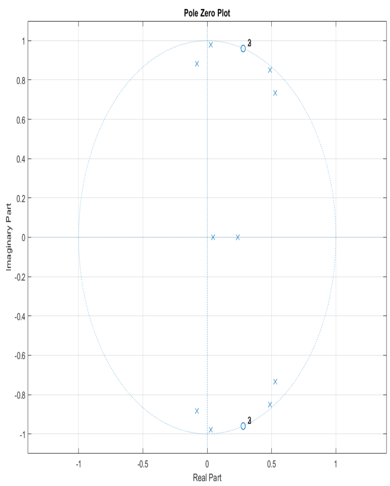

Fig. 14 : Pole-Zero Map of IIR Bandstop Filter

Fig. 14 shows the Pole-Zero Map of IIR Bandstop Filter. As all the poles are in the unit circle, the system is stable.

3.11 FIR Filter Transfer Function using Kaiser Window

We define minimum stopband attenuation in terms of stopband and passband tolerance value (δ) which is given as 0.15 to us.

$$A = -20log(\delta) = -20log(0.15) = 16.4782dB$$

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window. Now, to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here, $\Delta\omega_T$ is the minimum transition width which is same on the either sides of passband in our case.

$$\Delta\omega_T = \frac{3kHz * 2\pi}{400kHz} = 0.047\pi$$

$$\therefore N \geq 80$$

The first condition can be satisfied using N=80, but to satisfy the other conditions,On successive trials using MATLAB, it was found that a window length of **105** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

For designing the bandstop filter according to specifications, we first require an ideal bandstop filter. This is obtained by adding two lowpass filters of appropriate frequencies. We take the midpoint of the transition band in each case as the cutoff frequencies for the ideal lowpass filter.

By generating the ideal impulse response samples for the same length as that of the window, we obtained the time domain coefficients. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. A separate function is made to generate the impulse response of an ideal Lowpass filter. The bandstop impulse response samples were generated as the difference between three Lowpass filters :

```
FIR_BandStop =

  Columns 1 through 14

   -0.0017   -0.0121   -0.0049    0.0100    0.0097   -0.0037   -0.0088   -0.0013    0.0032    0.0003    0.0015    0.0059   -0.0000   -0.0118

  Columns 15 through 28

   -0.0077    0.0112    0.0157   -0.0035   -0.0174   -0.0055    0.0110    0.0082   -0.0025   -0.0021    0.0004   -0.0073   -0.0085    0.0100

  Columns 29 through 42

    0.0213   -0.0010   -0.0279   -0.0147    0.0214    0.0252   -0.0060   -0.0210   -0.0045    0.0055   -0.0022    0.0057    0.0252    0.0045

  Columns 43 through 56

   -0.0476   -0.0402    0.0459    0.0853   -0.0081   -0.1113   -0.0557    0.0955    0.1156   -0.0377    0.8600   -0.0377    0.1156    0.0955

  Columns 57 through 70

   -0.0557   -0.1113   -0.0081    0.0853    0.0459   -0.0402   -0.0476    0.0045    0.0252    0.0057   -0.0022    0.0055   -0.0045   -0.0210

  Columns 71 through 84

   -0.0060    0.0252    0.0214   -0.0147   -0.0279   -0.0010    0.0213    0.0100   -0.0085   -0.0073    0.0004   -0.0021   -0.0025    0.0082

  Columns 85 through 98

    0.0110   -0.0055   -0.0174   -0.0035    0.0157    0.0112   -0.0077   -0.0118   -0.0000    0.0059    0.0015    0.0003    0.0032   -0.0013

  Columns 99 through 105

   -0.0088   -0.0037    0.0097    0.0100   -0.0049   -0.0121   -0.0017
```

Fig. 15 : TIme Domain Sequence values for FIR Bandstop Filter

3.12 FIR Bandstop Filter Response

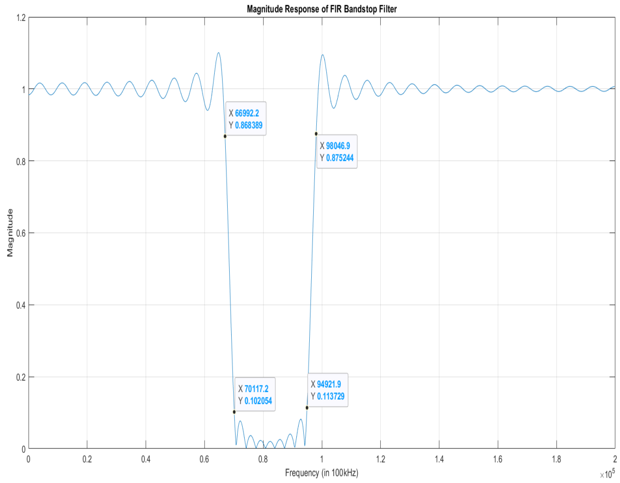

Fig. 16 : Magnitude Response of FIR Bandstop Filter

Fig. 16 shows the Magnitude Plot of FIR Bandstop Filter. The bandedge specifications are marked in the plot and they meet the required criteria in stopband and passband within the transition band of 3kHz.

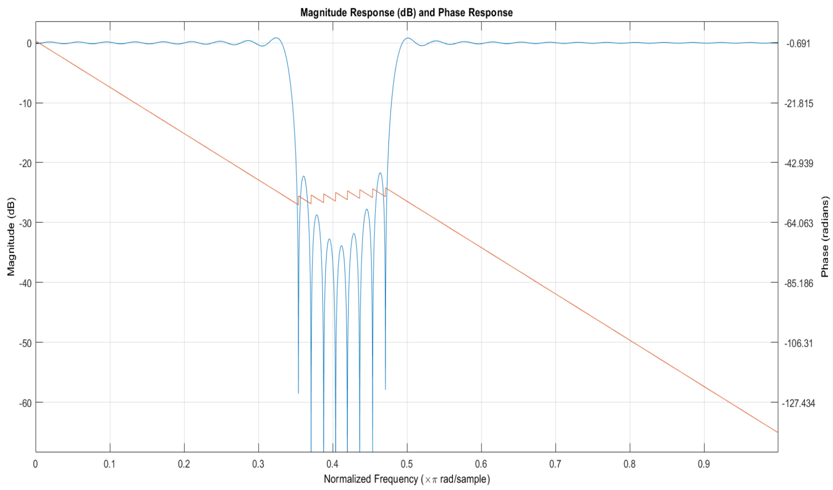

Fig. 17 : Magnitude and Phase Response of FIR Bandstop Filter (in dB)

Fig. 17 shows the Magnitude and Phase Response of FIR Bandstop Filter in dB scale. We can notice that the tolerance and attenuation specifications have been met. The phase response decreases linearly in the passband and fluctuates in the stopband.

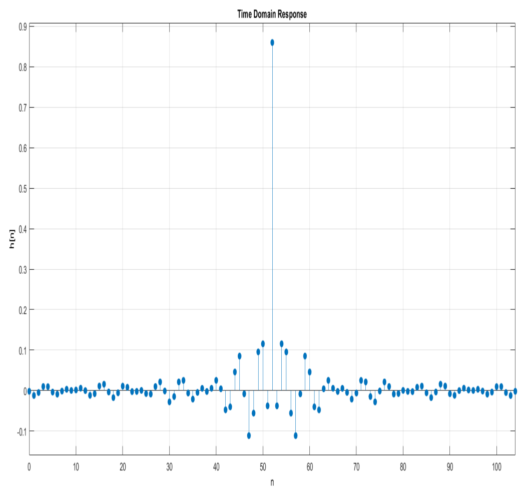

Fig. 18 : Time Domain Response of FIR Bandstop Filter

Fig. 18 shows the Time Domain Response of FIR Bandstop Filter.

4. Comparison between IIR and FIR designs

Following are the comparisons made between the FIR and the IIR designs :
1) FIR filter is easier to implement as compared to the IIR filter due to less order. Less order means less use of resources.
2) The phase response is discontinuous in the IIR Bandstop filter and not very smooth. Whereas, the phase response of FIR filter is both smooth as well as linear.
3) Because of higher order, the IIR design has a very sharp transition band as compared to the FIR design.
4) The magnitude response of FIR filter in dB scale has some fluctuations as compared to the magnitude response of IIR filter in dB scale.

5. MATLAB Code

All filter plots and code files used by me to generate the results throughout the assignment are there in the GitHub repository link given below :
`https://github.com/MayurWare/EE338-FilterDesign`

■