

Probability and Random Variables

Assignment 1

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Question:

The vertices of a triangle ABC are A(3,8), B(-1,2) and C (6,-6). Find:

- Slope of BC
- Equation of a line perpendicular to BC and passing through A

Solution:

- Let \mathbf{A} , \mathbf{B} , \mathbf{C} be the points vectors.

$$\mathbf{A} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \quad (1)$$

\therefore The direction vector of BC is,

$$\mathbf{m} = \mathbf{C} - \mathbf{B} \quad (2)$$

$$\mathbf{m} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (3)$$

$$\mathbf{m} = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \quad (4)$$

We know that, if the direction vector of a line is represented by a matrix $\mathbf{m} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ then the slope for the same can be represented by $(\frac{d_2}{d_1})$.

Therefore in this case the slope of line BC can be given as:

$$slope = \frac{-8}{7} \quad (5)$$

- Now, let n_{BC} be the normal vector of the line BC, hence we know that;

$$\mathbf{m}^\top \mathbf{n}_{BC} = 0 \quad (6)$$

$$(7 \ -8) \mathbf{n}_{BC} = 0 \quad (7)$$

$$\mathbf{n}_{BC} = \begin{pmatrix} 8 \\ 7 \end{pmatrix} \quad (8)$$

$$\mathbf{n}_{BC}^\top = (8 \ 7) \quad (9)$$

Let L be the line that passes through A and is perpendicular to BC , and let n_L be the normal vector of the line L , in such a case we can say that;

$$\mathbf{n}_{BC}^\top \mathbf{n}_L = 0 \quad (10)$$

$$(8 \ 7) \mathbf{n}_L = 0 \quad (11)$$

$$\mathbf{n}_L = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \quad (12)$$

$$\mathbf{n}_L^\top = (7 \ -8) \quad (13)$$

The normal equation of the line L is given by,

$$\mathbf{n}_L^\top (\mathbf{X} - \mathbf{A}) = 0 \quad (14)$$

$$(7 \ -8) \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 3 \\ 8 \end{pmatrix} \right) = 0 \quad (15)$$

$$(7 \ -8) \begin{pmatrix} x-3 \\ y-8 \end{pmatrix} = 0 \quad (16)$$

$$\text{Thus, line } L \equiv (7 \ -8) \begin{pmatrix} x-3 \\ y-8 \end{pmatrix} = 0$$