Assignment 6

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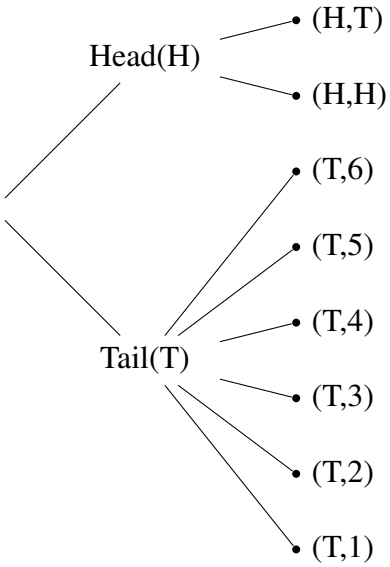
Question: Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

Solution:

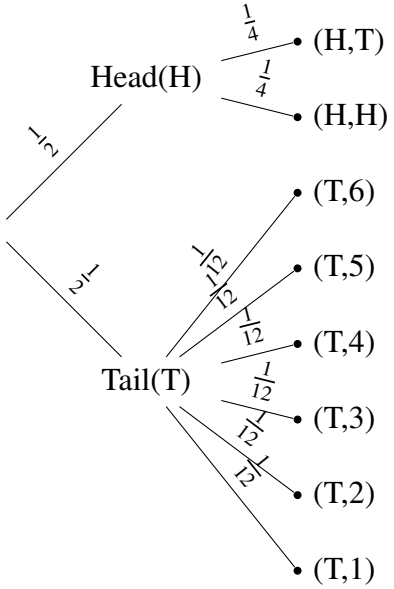
The outcomes of the experiment can be represented in following diagrammatic manner called the 'tree diagram'.



The sample space of the experiment may be described as

S = {(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

Thus, the probabilities assigned to the 8 elementary events (H, H), (H, T), (T, 1), (T, 2), (T, 3) (T, 4), (T, 5), (T, 6) are $\frac{1}{4},\frac{1}{4},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12}$ respectively which is clear from the below given tree diagram.



Let X be a randomn variable such that, X=0 be the event that 'there is at least one tail' and X=1 be the event 'the die shows a number greater than 4'. Then

$$P(X=0) = (H,T),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6) \tag{1}$$

$$P(X=1) = (T,5),(T,6) \text{ and } E \cap F = (T,5),(T,6) \tag{2}$$

Now

$$P(X=1) = \sum_{i=1}^{6} P((T,i)) \tag{3}$$

$$= \frac{1}{4}+\frac{1}{4}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12} = \frac{3}{4} \tag{4}$$

And

$$P(X=0 \cap X=1) = P((T,5)) + P((T,6)) \tag{5}$$

$$= \frac{1}{12}+\frac{1}{12} = \frac{1}{6} \tag{6}$$

hence

$$P(X = 0|X = 1) = \frac{P(X = 0 \cap X = 1)}{P(X = 1)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9} \quad (7)$$