

# Probability and Random Variables

## Assignment 1

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### Question:

The vertices of a  $\triangle ABC$  are  $A(3,8)$ ,  $B(-1,2)$  and  $C(6,-6)$ . Find:

- Slope of BC
- Equation of a line perpendicular to BC and passing through A

### Solution:

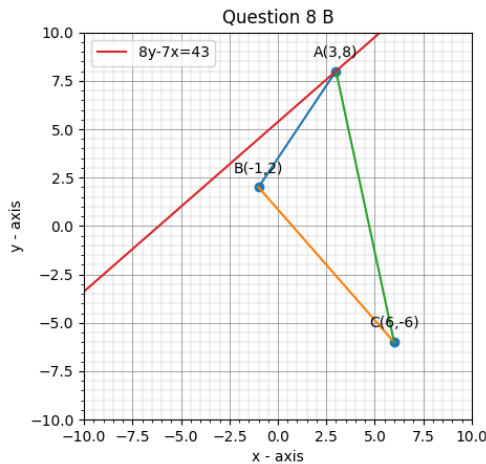


Fig. 1. Plot showing  $\triangle ABC$ , line BC and line perpendicular to BC.

- Let  $A, B, C$  be the points vectors.

$$A = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \quad (1)$$

$\therefore$  The direction vector of  $BC$  is,

$$\mathbf{m} = \mathbf{C} - \mathbf{B} \quad (2)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (3)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \quad (4)$$

We know that, if the direction vector of a line is represented by a matrix  $\mathbf{m} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$  then the

slope for the same can be represented by  $\left(\frac{d_2}{d_1}\right)$ .

Therefore in this case the slope of line BC can be given as:

$$\text{slope} = \frac{-8}{7} \quad (5)$$

- Now, let  $n_{BC}$  be the normal vector of the line BC, hence we know that;

$$\mathbf{m}^\top \mathbf{n}_{BC} = 0 \quad (6)$$

$$\Rightarrow (7 \ -8) \mathbf{n}_{BC} = 0 \quad (7)$$

$$\Rightarrow \mathbf{n}_{BC} = \begin{pmatrix} 8 \\ 7 \end{pmatrix} \quad (8)$$

$$\Rightarrow \mathbf{n}_{BC}^\top = (8 \ 7) \quad (9)$$

Let  $L$  be the line that passes through  $A$  and is perpendicular to  $BC$ , and let  $n_L$  be the normal vector of the line  $L$ , in such a case we can say that;

$$\mathbf{n}_{BC}^\top \mathbf{n}_L = 0 \quad (10)$$

$$\Rightarrow (8 \ 7) \mathbf{n}_L = 0 \quad (11)$$

$$\Rightarrow \mathbf{n}_L = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \quad (12)$$

$$\Rightarrow \mathbf{n}_L^\top = (7 \ -8) \quad (13)$$

The normal equation of the line  $L$  is given by,

$$\mathbf{n}_L^\top (\mathbf{X} - \mathbf{A}) = 0 \quad (14)$$

$$\Rightarrow (7 \ -8) \left( \mathbf{X} - \begin{pmatrix} 3 \\ 8 \end{pmatrix} \right) = 0 \quad (15)$$

$$\Rightarrow (7 \ -8) \mathbf{X} - (7 \ -8) \begin{pmatrix} 3 \\ 8 \end{pmatrix} = 0 \quad (16)$$

$$\Rightarrow (7 \ -8) \mathbf{X} = (-43) \quad (17)$$

Thus, line  $L \equiv (7 \ -8) \mathbf{X} = (-43)$