

# Probability and Random Variables

## Assignment 2

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**Question:** Solve the following system of linear equations using matrix method :

$$\begin{aligned}\frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= 9 \\ \frac{2}{x} + \frac{5}{y} + \frac{7}{z} &= 52 \\ \frac{2}{x} + \frac{1}{y} - \frac{1}{z} &= 0\end{aligned}$$

**Solution:** Let us substitute  $a$  with  $\frac{1}{x}$ ,  $b$  with  $\frac{1}{y}$  and  $c$  with  $\frac{1}{z}$  in the given system of linear equations. we get,

$$a + b + c = 9 \quad (1)$$

$$2a + 5b + 7c = 52 \quad (2)$$

$$2a + b - c = 0 \quad (3)$$

In order to represent the above system of equation in the form of matrices, we can write that,

$$\mathbf{AX} = \mathbf{B} \quad (4)$$

where,

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix} \quad (5)$$

Forming the augmented matrix and pivoting,

$$\left( \begin{array}{ccc|ccc} \textcircled{1} & 1 & 1 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right) \quad (6)$$

$$\begin{array}{l} \xleftarrow{R_3 \leftarrow R_3 - 2R_1} \\ \xleftarrow{R_2 \leftarrow R_2 - 2R_1} \end{array} \left( \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & \textcircled{3} & 5 & -2 & 1 & 0 \\ 0 & -1 & -3 & -2 & 0 & 1 \end{array} \right) \quad (7)$$

$$\begin{array}{l} \xleftarrow{R_3 \leftarrow -3R_3 + R_2} \\ \xleftarrow{R_1 \leftarrow -3R_1 - R_2} \end{array} \left( \begin{array}{ccc|ccc} 3 & 0 & -2 & 5 & -1 & 0 \\ 0 & 3 & 5 & -2 & 1 & 0 \\ 0 & 0 & \textcircled{-4} & -8 & 1 & 3 \end{array} \right) \quad (8)$$

$$\begin{array}{l} \xleftarrow{R_2 \leftarrow -4R_2 + 5R_3} \\ \xleftarrow{R_1 \leftarrow 2R_1 - R_3} \end{array} \left( \begin{array}{ccc|ccc} 6 & 0 & 0 & 18 & -3 & -3 \\ 0 & 12 & 0 & -48 & 9 & 15 \\ 0 & 0 & -4 & -8 & 1 & 3 \end{array} \right) \quad (9)$$

$$\begin{array}{l} \xleftarrow{R_3 \leftarrow -3R_3} \\ \xleftarrow{R_1 \leftarrow 2R_1} \end{array} \left( \begin{array}{ccc|ccc} 12 & 0 & 0 & 36 & -6 & -6 \\ 0 & 12 & 0 & -48 & 9 & 15 \\ 0 & 0 & 12 & 24 & -3 & -9 \end{array} \right) \quad (10)$$

After pivoting the formed augmented matrix, we get:

$$\mathbf{A}^{-1} = \frac{1}{12} \begin{pmatrix} 36 & -6 & -6 \\ -48 & 9 & 15 \\ 24 & -3 & -9 \end{pmatrix} \quad (11)$$

Thus, letting

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} \quad (12)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 36 & -6 & -6 \\ -48 & 9 & 15 \\ 24 & -3 & -9 \end{pmatrix} \begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix} \quad (13)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \quad (14)$$

From the above equation, we get,  $a = 1$ ,  $b = 3$ ,  $c = 5$ .

Now, lets re-substitute  $a$ ,  $b$ , and  $c$  as:

$$a \leftrightarrow \frac{1}{x}, b \leftrightarrow \frac{1}{y}, c \leftrightarrow \frac{1}{z} \quad (15)$$

thus, we get the following as our final answer:

$$x = 1, y = \frac{1}{3}, z = \frac{1}{5} \quad (16)$$