

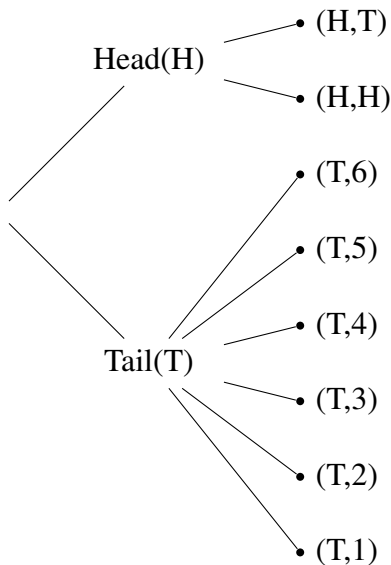
Assignment 4

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Question: Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

Solution:

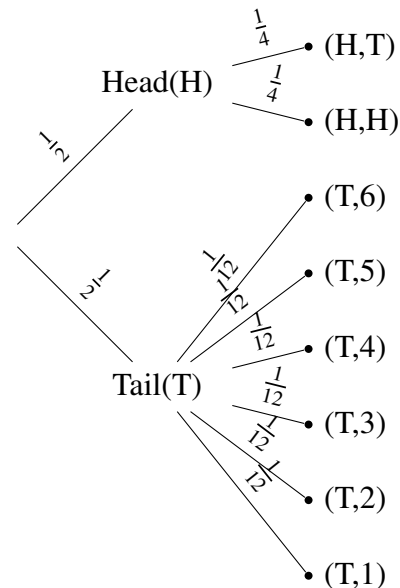
The outcomes of the experiment can be represented in following diagrammatic manner called the 'tree diagram'.



The sample space of the experiment may be described as

$S = \{(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

Thus, the probabilities assigned to the 8 elementary events $(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)$ are $\frac{1}{4}, \frac{1}{4}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}$ respectively which is clear from the below given tree diagram.



Let F be the event that 'there is at least one tail' and E be the event 'the die shows a number greater than 4'.

Then

$$F = (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6) \quad (1)$$

$$E = (T, 5), (T, 6) \text{ and } E \cap F = (T, 5), (T, 6) \quad (2)$$

Now

$$P(F) = \sum_{i=1}^6 P((T, i)) \quad (3)$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{4} \quad (4)$$

And

$$P(E \cap F) = P((T, 5)) + P((T, 6)) \quad (5)$$

$$= \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \quad (6)$$

hence

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9} \quad (7)$$