Probability and Random Variables Assignment 1

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Question:

The vertices of a triangle ABC are A(3,8), B(-1,2) and C (6,-6). Find:

a) Slope of BC
b) Equation of a line perpendicular to BC and passing through A

Solution:

1) Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be the points vectors.

$$\mathbf{A} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \tag{1}$$

$\therefore$ The direction vector of BC is,

$$\mathbf{m} = \mathbf{C} - \mathbf{B} \tag{2}$$

$$\implies \mathbf{m} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{3}$$

$$\implies \mathbf{m} = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \tag{4}$$

We know that, if the direction vector of a line is represented by a matrix m=$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ then the slope for the same can be represented by $\left(\frac{d_2}{d_1}\right)$.

Therefore in this case the slope of line BC can be given as:

$$slope = \frac{-8}{7} \tag{5}$$

2) Now, let n_{BC} be the normal vector of the line BC, hence we know that;

$$\mathbf{m}^\top \mathbf{n_{BC}} = 0 \tag{6}$$

$$\implies \begin{pmatrix} 7 & -8 \end{pmatrix} \mathbf{n_{BC}} = 0 \tag{7}$$

$$\implies \mathbf{n_{BC}} = \begin{pmatrix} 8 \\ 7 \end{pmatrix} \tag{8}$$

$$\implies \mathbf{n}_{\mathbf{BC}}^\top = \begin{pmatrix} 8 & 7 \end{pmatrix} \tag{9}$$

Let L be the line that passes through A and is perpendicular to BC, and let n_L be the normal vector of the line L, in such a case we can say that;

$$\mathbf{n}_{\mathbf{BC}}^\top \mathbf{n_L} = 0 \tag{10}$$

$$\implies \begin{pmatrix} 8 & 7 \end{pmatrix} \mathbf{n_L} = 0 \tag{11}$$

$$\implies \mathbf{n_L} = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \tag{12}$$

$$\implies \mathbf{n_L}^\top = \begin{pmatrix} 7 & -8 \end{pmatrix} \tag{13}$$

The normal equation of the line L is given by,

$$\mathbf{n_L}^\top (\mathbf{X} - \mathbf{A}) = 0 \tag{14}$$

$$\implies \begin{pmatrix} 7 & -8 \end{pmatrix} \left(\mathbf{X} - \begin{pmatrix} 3 \\ 8 \end{pmatrix} \right) = 0 \tag{15}$$

$$\implies \begin{pmatrix} 7 & -8 \end{pmatrix} \mathbf{X} - \begin{pmatrix} 7 & -8 \end{pmatrix} \begin{pmatrix} 3 \\ 8 \end{pmatrix} = 0 \tag{16}$$

$$\implies \begin{pmatrix} 7 & -8 \end{pmatrix} \mathbf{X} = (-43) \tag{17}$$

Thus, line L $\equiv \begin{pmatrix} 7 & -8 \end{pmatrix} \mathbf{X} = (-43)$