Probability and Random Variables Assignment 2

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Question: Solve the following system of linear equations using matrix method :

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$$
$$\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$$
$$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

Solution: Let us substitute a with $\frac{1}{x}$, b with $\frac{1}{y}$ and c with $\frac{1}{z}$ in the given system of linear equations.
we get,

$$a+b+c=9 \tag{1}$$
$$2a+5b+7c=52 \tag{2}$$
$$2a+b-c=0 \tag{3}$$

In order to represent the above system of equation in the form of matrices,
we can write that,

$$\mathbf{AX}=\mathbf{B} \tag{4}$$

where,

$$\mathbf{A}=\begin{pmatrix}1 & 1 & 1\\ 2 & 5 & 7\\ 2 & 1 & -1\end{pmatrix}, \mathbf{X}=\begin{pmatrix}a\\ b\\ c\end{pmatrix}, \mathbf{B}=\begin{pmatrix}9\\ 52\\ 0\end{pmatrix} \tag{5}$$

Forming the augmented matrix and pivoting,

$$\left(\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 2 & 5 & 7 & 0 & 1 & 0\\ 2 & 1 & -1 & 0 & 0 & 1\end{array}\right) \tag{6}$$

$$\xleftrightarrow[R_2\leftarrow R_2-2R_1]{R_3\leftarrow R_3-2R_1}\left(\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 3 & 5 & -2 & 1 & 0\\ 0 & -1 & -3 & -2 & 0 & 1\end{array}\right) \tag{7}$$

$$\xleftrightarrow[R_1\leftarrow 3R_1-R_2]{R_3\leftarrow 3R_3+R_2}\left(\begin{array}{ccc|ccc}3 & 0 & -2 & 5 & -1 & 0\\ 0 & 3 & 5 & -2 & 1 & 0\\ 0 & 0 & -4 & -8 & 1 & 3\end{array}\right) \tag{8}$$

$$\xleftrightarrow[R_1\leftarrow 2R_1-R_3]{R_2\leftarrow 4R_2+5R_3}\left(\begin{array}{ccc|ccc}6 & 0 & 0 & 18 & -3 & -3\\ 0 & 12 & 0 & -48 & 9 & 15\\ 0 & 0 & -4 & -8 & 1 & 3\end{array}\right) \tag{9}$$

$$\xleftrightarrow[R_1\leftarrow 2R_1]{R_3\leftarrow -3R_3}\left(\begin{array}{ccc|ccc}12 & 0 & 0 & 36 & -6 & -6\\ 0 & 12 & 0 & -48 & 9 & 15\\ 0 & 0 & 12 & 24 & -3 & -9\end{array}\right) \tag{10}$$

After pivoting the formed augmented matrix, we get:

$$\mathbf{A}^{-1}=\frac{1}{12}\begin{pmatrix}36 & -6 & -6\\ -48 & 9 & 15\\ 24 & -3 & -9\end{pmatrix} \tag{11}$$

Thus, letting

$$\mathbf{X}=\mathbf{A}^{-1}\mathbf{B} \tag{12}$$

$$\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\frac{1}{12}\begin{pmatrix}36 & -6 & -6\\ -48 & 9 & 15\\ 24 & -3 & -9\end{pmatrix}\begin{pmatrix}9\\ 52\\ 0\end{pmatrix} \tag{13}$$

$$\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}1\\ 3\\ 5\end{pmatrix} \tag{14}$$

From the above equation, we get, $a=1$, $b=3$, $c=5$.

Now, lets re-substitute a , b, and c as:

$$a\leftrightarrow\frac{1}{x}, b\leftrightarrow\frac{1}{y}, c\leftrightarrow\frac{1}{z} \tag{15}$$

after the re-substitution of a, b and c in (14), we get;

$$\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{pmatrix}=\begin{pmatrix}1\\ 3\\ 5\end{pmatrix} \tag{16}$$

thus, we get the following as our final answer:

$$x=1, y=\frac{1}{3}, z=\frac{1}{5} \tag{17}$$