Probability and Random Variables Assignment 2

Mayuri Chourasia
BT21BTECH11001

Question: Solve the following system of linear equations:

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$$
$$\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$$
$$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

Solution: Let us substitute a with $\frac{1}{x}$, b with $\frac{1}{y}$ and c with $\frac{1}{z}$ in the given system of linear equations.
we get,

$$a+b+c=9 \tag{1}$$
$$2a+5b+7c=52 \tag{2}$$
$$2a+b-c=0 \tag{3}$$

In order to represent the above system of equation in the form of matrices,
we can write that,

$$\mathbf{A}\mathbf{X}=\mathbf{B} \tag{4}$$

where,

$$\mathbf{A}=\begin{pmatrix}1&1&1\\2&5&7\\2&1&-1\end{pmatrix},\mathbf{X}=\begin{pmatrix}a\\b\\c\end{pmatrix},\mathbf{B}=\begin{pmatrix}9\\52\\0\end{pmatrix} \tag{5}$$

Forming the augmented matrix and pivoting,

$$\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\2&5&7&0&1&0\\2&1&-1&0&0&1\end{array}\right) \tag{6}$$

$$\xleftrightarrow[R_2\leftarrow R_2-2R_1]{R_3\leftarrow R_3-2R_1}\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&3&5&-2&1&0\\0&-1&-3&-2&0&1\end{array}\right) \tag{7}$$

$$\xleftrightarrow[R_1\leftarrow 3R_1-R_2]{R_3\leftarrow 3R_3+R_2}\left(\begin{array}{ccc|ccc}3&0&-2&5&-1&0\\0&3&5&-2&1&0\\0&0&-4&-8&1&3\end{array}\right) \tag{8}$$

$$\xleftrightarrow[R_1\leftarrow 2R_1-R_3]{R_2\leftarrow 4R_2+5R_3}\left(\begin{array}{ccc|ccc}6&0&0&18&-3&-3\\0&12&0&-48&9&15\\0&0&-4&-8&1&3\end{array}\right) \tag{9}$$

$$\xleftrightarrow[R_1\leftarrow 2R_1]{R_3\leftarrow -3R_3}\left(\begin{array}{ccc|ccc}12&0&0&36&-6&-6\\0&12&0&-48&9&15\\0&0&12&24&-3&-9\end{array}\right) \tag{10}$$

After pivoting the formed augmented matrix, we get:

$$\mathbf{A}^{-1}=\frac{1}{12}\begin{pmatrix}36&-6&-6\\-48&9&15\\24&-3&-9\end{pmatrix} \tag{11}$$

Thus, letting

$$\mathbf{X}=\mathbf{A}^{-1}\mathbf{B} \tag{12}$$

$$\begin{pmatrix}a\\b\\c\end{pmatrix}=\frac{1}{12}\begin{pmatrix}36&-6&-6\\-48&9&15\\24&-3&-9\end{pmatrix}\begin{pmatrix}9\\52\\0\end{pmatrix} \tag{13}$$

$$\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1\\3\\5\end{pmatrix} \tag{14}$$

From the above equation, we get, $a=1$, $b=3$, $c=5$.

Now, lets re-substitute a , b, and c as:

$$a\leftrightarrow\frac{1}{x},b\leftrightarrow\frac{1}{y},c\leftrightarrow\frac{1}{z} \tag{15}$$

thus, we get the following as our final answer:

$$x=1,y=\frac{1}{3},z=\frac{1}{5} \tag{16}$$