Probability and Random Variables Assignment 1

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Question:

The vertices of a triangle ABC are A(3,8), B(-1,2) and C (6,-6). Find:

a) Slope of BC
b) Equation of a line perpendicular to BC and passing through A

Solution:

1) Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be the points vectors.

$$\mathbf{A} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \tag{1}$$

$\therefore$ The direction vector of BC is,

$$\mathbf{d} = \mathbf{C} - \mathbf{B} \tag{2}$$

$$\mathbf{d} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{3}$$

$$\mathbf{d} = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \tag{4}$$

We know that, if the direction vector of a line is represented by a matrix d=$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ then the slope for the same can be represented by $(\frac{d_2}{d_1})$.

Therefore in this case the slope can be given as:

$$m = \frac{-8}{7} \tag{5}$$

The slope of BC is $\frac{-8}{7}$

2) Now, let **z** be the normal vector of the line BC, hence we know that;

$$\mathbf{d}^\top \mathbf{z} = 0 \tag{6}$$

$$\begin{pmatrix} 7 & -8 \end{pmatrix} \mathbf{z} = 0 \tag{7}$$

$$\mathbf{z} = \begin{pmatrix} 8 \\ 7 \end{pmatrix} \tag{8}$$

$$\mathbf{z}^\top = \begin{pmatrix} 8 & 7 \end{pmatrix} \tag{9}$$

Let L be the line that passes through A and is perpendicular to BC, and let **n** be the normal vector of the line L, in such a case we can say that;

$$\mathbf{z}^\top \mathbf{n} = 0 \tag{10}$$

$$\begin{pmatrix} 8 & 7 \end{pmatrix} \mathbf{n} = 0 \tag{11}$$

$$\mathbf{n} = \begin{pmatrix} 7 \\ -8 \end{pmatrix} \tag{12}$$

$$\mathbf{n}^\top = \begin{pmatrix} 7 & -8 \end{pmatrix} \tag{13}$$

The normal equation of the line L is given by,

$$\mathbf{n}^\top (\mathbf{X} - \mathbf{A}) = 0 \tag{14}$$

$$\begin{pmatrix} 7 & -8 \end{pmatrix} (\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 3 \\ 8 \end{pmatrix}) = 0 \tag{15}$$

$$\begin{pmatrix} 7 & -8 \end{pmatrix} \begin{pmatrix} x-3 \\ y-8 \end{pmatrix} = 0 \tag{16}$$

Thus, line L $\equiv \begin{pmatrix} 7 & -8 \end{pmatrix} \begin{pmatrix} x-3 \\ y-8 \end{pmatrix} = 0$