2019-12-31

$\boxed{\text{Prop 1.}} \longrightarrow \forall\, m, k$ $\boxed{\text{indep't}}$

$\theta = \mathrm{vec}(\xi_j)_{j=1}^{m}$ $\qquad \theta(t=0) =: \theta_0$

$$g^{\infty}(\theta_0) = \theta(t=\infty)$$

$$\|$$

$$\mathrm{vec}(\xi_j(t=\infty))_{j=1}^{m}$$

$$\frac{1}{m}\sum_{j=1}^{m} \delta_{\xi_j(t=\infty)} \in \mathbb{R}^{d+2}$$

$\boxed{\text{Thm 1}} \Longrightarrow \downarrow$

$$\mu$$

$$\frac{1}{m}\sum_{j=1}^{m} \delta^{k}_{\xi_j(t=\infty)} \in \mathbb{R}^{d+2}$$

$$\downarrow$$

$$\mu^{k}$$

$\boxed{\text{Thm 2}}$

$\lim_{k \to \infty} \mu^{k} \to \mu^{\infty}$

$$\nabla_\theta \left[\sum_{i=1}^{n} \frac{1}{2} (f_\theta - f)^2 \right]$$

Part 2 Let $\boxed{k = m^\gamma}$

$$\frac{1}{m} \sum_{j=1}^{m} \delta_{\tilde{\varepsilon}_j (t \to \infty)} \subset S^d$$

$\boxed{\text{Thm } 3} \Longrightarrow \Big\downarrow (m \to \infty)$

μ

$\boxed{\text{Thm } 4}$ If $\gamma > 1$

let μ be the distribution in Thm 3

? $\text{supp}(\mu) \neq \cancel{\mathbb{R}^{d+2}}\ S^d$

$$\mathbb{E}_{\theta(0)} \, Q_j(m)$$

$$= \mathbb{E}_{\theta(0)} \int_0^\infty \frac{1}{m}\sum_{j=1}^{m} \left\| \frac{d}{dt} q_j(t) \right\| dt$$

$$\mathbb{E} \, \Theta_j(m)$$

$$= \mathbb{E}_{\theta(0)} \int_0^\infty \frac{1}{\sqrt{m}} \left\| \frac{d}{dt} \theta(t) \right\| dt$$

FNL dyn. $\longleftrightarrow$ NKT dyn

(GD & DNN) (GD & linDNN)

$K = K(0)$

$\| \theta(t) - \theta_{NTK}(t) \| \ll$

$$Q_\gamma(m)$$

$$= \int_0^\infty \frac{1}{m} \sum_{j=1}^{m} \left\| \frac{d}{dt} \xi_j \right\| dt$$

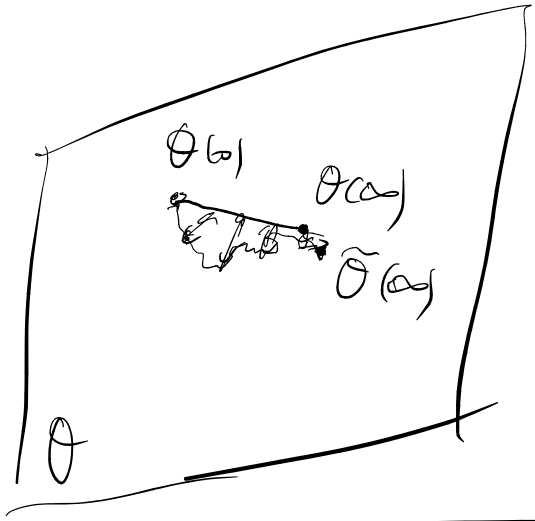
$$Q_\gamma^{lin}(m)$$

$$= \int_0^\infty \frac{1}{m} \sum_{j=1}^{\infty} \left\| \frac{d}{dt} \xi_j^{lin} \right\| dt$$

$$\| \xi_j^{lin}(t) \quad \cancel{e^{-\lambda t} \xi_j^{lin}(0)}$$

$$- \xi_j^{lin}(\infty) \| \le e^{-\lambda t} \| \xi_j^{lin}(0)$$

$$- \xi_j^{lin}(\infty) \|$$



$$\sup \| \theta(t) - c_2 \theta^{lin}(c, t) \|_2$$

$t \in [0, T]$

$\boxed{\gamma > 1}$

$$\lim_{m\to\infty} \int_0^\infty \frac{1}{m}\sum_{j=1}^{m} \left\| \frac{d}{dt} \xi_j \right\| dt$$

$$\boxed{\gamma \geq 1} \geq \lim_{m\to\infty} \frac{1}{m}\sum_{j=1}^{m} \left\| \xi_j(t=\infty) - \xi_j(t=0) \right\|$$

$$= +\infty$$

$\boxed{\gamma < 1}$

$$\lim \int_0^\infty \frac{1}{m}\sum_{j=1}^{m} \left\| \frac{d}{dt} \xi_j^{(t)} \right\| dt$$

$$\leq \lim \int_0^\infty \frac{1}{m}\sum_{j=1}^{m} \left\| \frac{d}{dt} \xi_j^{lin\,(ct)} \right\| dt$$

$$+ \lim \int_0^\infty \frac{1}{m}\sum_{j=1}^{m} \left\| \frac{d}{dt} \left(\xi_j^{lin\,(ct)} - \xi_j(t) \right) \right\| dt$$

$$\leq \underset{?}{\underline{\underline{0}}} + 0$$

$f(x, \theta(\infty))$ $\qquad \theta(\infty) = (a_j, w_j, c_j)_{j=1}^m$

$= \frac{1}{m^\gamma} \sum_j a_j \sigma(w_j x + c_j)$

若 $\gamma > 1$ $\qquad \theta(\infty)_{jk} < O(1)$

$|f(x, \theta(\infty)) - f^*| > O(1)$ $\quad \exists\, w_2 > c_0$

$\downarrow$

$o(1)$

$m \to \infty$

$f^* \sim O(1)$

Ass $\quad |a_j|\, \|w\|\, |c| < c_0$

$\|f_{\theta\gamma}\|_2^2 \leq \frac{1}{m^{2\gamma}} \sum \|a_j\|_2^2 \|\sigma(wx + c)\|_2^2$

$m\, O(1)$

$0 \leq \frac{1}{m^\gamma} m\, O(1)$

$\frac{1}{m^{\gamma-1}} O(1)$ $\quad c_0$

0

$$\sup_{\theta \in B(\theta_0, r)} \| f_\theta(x) \|_2 \leq g_1 r,$$

$$r = \frac{1}{m}\sum_{j=1}^{m} \| \theta_j - \theta_0 \|_2 \int_0^\infty \quad dt$$

记得 : $\boxed{W_0 \quad \delta \quad \boxed{f(x_i, \theta_\infty) = f(x_i)}}$

$$\theta(s) \int_0^\infty$$

$\Delta \sim \cancel{I}\infty$