NOTES. The lowest grade among all twelve homework will be dropped, so **NO** late submission will be accepted. All homework assignment must be written on standard 8.5 by 11 paper. Computer generated output without detailed explanations and remarks will not receive any credit. You may type out your answers, but make sure to use different fonts to distinguish your own words from the computer output. Please submit the hard copy of your homework solution in class. For the simulation and data analysis problems, keep the code you develop as you may be asked to present your work later.

1 (25 pts). Find three univarite time series of you interest. Plot the series, and comment on the pattern you observe. [Note. Please find data of different types. For example, you should NOT use the return series of three stocks, or of three time periods. Find three time series of different nature.]

2 (25 pts). Let U_1 and U_2 be independent random variables with zero means and $\text{Var}(U_1) = \text{Var}(U_2) = \sigma^2$. Consider the time series
$$x_t = U_1 \sin(2\pi\omega t) + U_2 \cos(2\pi\omega t),$$
where $\omega \in [0, 1)$ is a fixed constant. Show that the autocovariance function of this process is given by
$$\gamma_{t,t+h} := \text{Cov}(x_t, x_{t+h}) = \sigma^2 \cos(2\pi\omega h).$$
Find the autocorrelation function as well.

3 (25 pts). Let $\{w_t, \ t \in \mathbb{Z}\}$ be iid $N(0, 1)$, and consider the time series
$$x_t = w_t w_{t-1}, \quad y_t = x_t^2.$$
  (a) Find the mean, autocovariance, and autocorrelation functions of x_t.
  (b) Simulate x_t of length $T = 500$. Give the time series plot. Comment.
  (c) Find the mean, autocovariance, and autocorrelation functions of y_t.
  (b) Simulate y_t of length $T = 500$. Give the time series plot. Comment.

4 (25 pts). For each of the three series you found for Homework 01, Problem 1, do the following. [Note. If your series $\{x_t\}$ does not look stationary, try to do the following for the process $\{y_t\}$, where $y_t := x_t - x_{t-1}$.]
  (a) Plot the sample autocorrelation functions.
  (b) Test whether ρ_1 is zero, using 95% level.
  (c) Perform the Ljung-Box test, using a suitable number (your own choice) of lags.