NOTES. The lowest grade among all twelve homework will be dropped, so **NO** late submission will be accepted. All homework assignment must be written on standard 8.5 by 11 paper. Computer generated output without detailed explanations and remarks will not receive any credit. You may type out your answers, but make sure to use different fonts to distinguish your own words from the computer output. Please submit the hard copy of your homework solution in class. For the simulation and data analysis problems, keep the code you develop as you may be asked to present your work later.

1 (10 pts)**.** Rewrite the following models in the original form **without** using the back-shift operator B.

(a) $(1-.3B)(1-.5B)(1-.6B+.4B^2)(r_t-.5)=a_t$.
(b) $(1-.3B)(1-.5B)(1-.6B+.4B^2)(r_t-.5)=(1-.3B)(1+.8B)a_t$.

2 (10 pts)**.** Suppose that the daily log return of a security follows the model

$$r_t = 0.01 + 0.2r_{t-2} + a_t,$$

where $\{a_t\}$ is a Gaussian white noise series with mean zero and variance 0.02. What are the mean and variance of the return series r_t? Calculte all the autocorrelations of r_t.

3 (35 pts)**.** Suppose that the daily log return of a security follows the model

$$r_t = 0.01 + 0.6r_{t-1} - 0.4r_{t-2} + a_t,$$

where $\{a_t\}$ is a white noise series with mean zero and variance 0.02.

(a) What is the mean of the return series r_t?
(b) Calculate the lag-1, lag-2 and lag-3 autocorrelations of r_t.
(c) Calculate the variance, lag-1 and lag-2 autocovariances of r_t. [Do not use the computer for this problem, unless you need to use it to solve the linear system of equations. Please show details of your solutions.]
(d) Use what you found in part (c) to calculate the autocovariannces of lags 3, 4, 5, 6, 7. Plot the autocovariances of lags 0 to 7 in the same graph.
(e) Simulate a time series of length $T = 2000$ from this model. Creat a time series plot, and a sample autocorrelation plot. Compute the lag-1, lag-2 and lag-3 sample autocorrelations, and sample autocovariances.

4 (45 pts)**.** Consider the following models.

(i) AR(3): $r_t = 0.3 + 0.8r_{t-1} - .5r_{t-2} - .2r_{t-3} + a_t$.
(ii) MA(3): $r_t = 0.3 + a_t + 0.8a_{t-1} - .5a_{t-2} - .2a_{t-3}$.
(iii) ARMA(3,2): $r_t = 0.3 + 0.8r_{t-1} - .5r_{t-2} - .2r_{t-3} + a_t + 0.5a_{t-1} + 0.3a_{t-2}$.

Assume all a_t are i.i.d $N(0,4)$. For each of the three preceding model, do the following:

(1) Simulate a series of length $T = 600$, give the time series plot.
(2) Use the function `memory()` to calculate the memory functions of lag 0 to lag 10, and plot them.
(3) Use the function `auto.cov()` to calculate the autocorrelation functions of lag 0 to lag 10, and plot them.
(4) Compare the true auto correlations plot with the sample auto correlations plot.