Submission for Computer Vision Exercise 1

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1 Pen and Paper task

1.1 Eigendecomposition of 2×2 matrix

First, let's find the eigenvalues of A.

$$
\begin{aligned}
0 &= \det\left(A - \lambda I\right) \\
0 &= \det\left(\begin{bmatrix} 9 & 1 \\ 8 & 7 \end{bmatrix} - \lambda I\right) \\
0 &= \det\left(\begin{bmatrix} 9-\lambda & 1 \\ 8 & 7-\lambda \end{bmatrix}\right) \\
0 &= (9-\lambda)\cdot(7-\lambda) - 8 \cdot 1 \\
0 &= \lambda^2 - 16\lambda + 55 \\
0 &= (\lambda - 11)\cdot(\lambda - 5)
\end{aligned}
$$

Hence, the eigenvalues are $\begin{bmatrix} 11 & 5 \end{bmatrix}$, now we find the corresponding eigenvectors:

$$
\begin{aligned}
0 &= (A - \lambda_1 I)\cdot \mathbf{v_1} \\
0 &= \begin{bmatrix} -2 & 1 \\ 8 & -4 \end{bmatrix} \cdot \mathbf{v_1}
\end{aligned}
$$

We solve for $\mathbf{v_1}$ and get $\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$. Similarly for $\mathbf{v_2}$ by substituting $\lambda = 5$, we get $\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}$.

$$
\begin{aligned}
A = \begin{bmatrix} 9 & 1 \\ 8 & 7 \end{bmatrix} &= Q\Lambda Q^{-1} \\
&= \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 11 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{5}}{4} & \frac{\sqrt{5}}{2} \\ \frac{\sqrt{5}}{4} & \frac{-\sqrt{5}}{2} \end{bmatrix}
\end{aligned}
$$

1.2 Computing A^{10} using eigendecomposition

$$
\begin{aligned}
A^{10} &= \left(Q\Lambda Q^{-1}\right)^{10} \\
&= \left(Q\Lambda Q^{-1}\right)\left(Q\Lambda Q^{-1}\right)\ldots \\
&= Q\Lambda^{10}Q^{-1} \\
&= \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 11^{10} & 0 \\ 0 & 5^{10} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{5}}{4} & \frac{\sqrt{5}}{2} \\ \frac{\sqrt{5}}{4} & \frac{-\sqrt{5}}{2} \end{bmatrix}
\end{aligned}
$$