CSCE 4603
Fundamentals of Computer Vision

Harris Corner Detector

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Fall **2025**

Text Book

- Computer Vision: Algorithms and Applications, Richard Szeliski, Springer, 2011.
- http://szeliski.org/Book/
- Section: 4.1.1

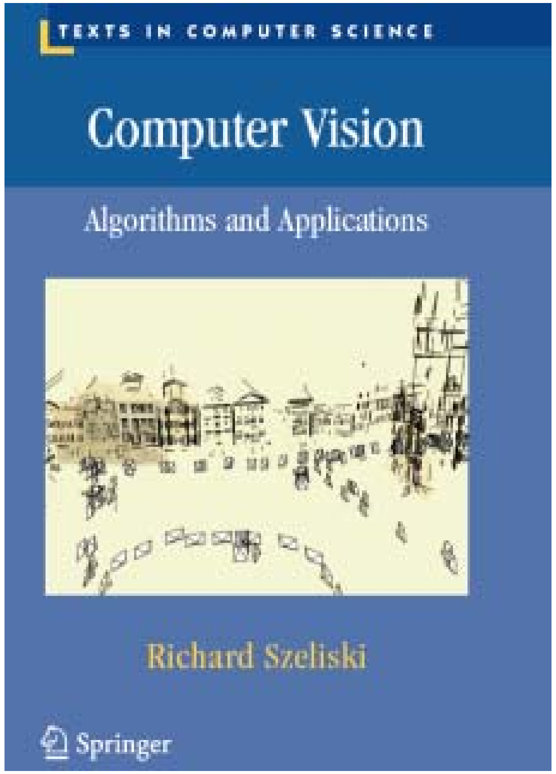


Image Matching

by Diva Sian

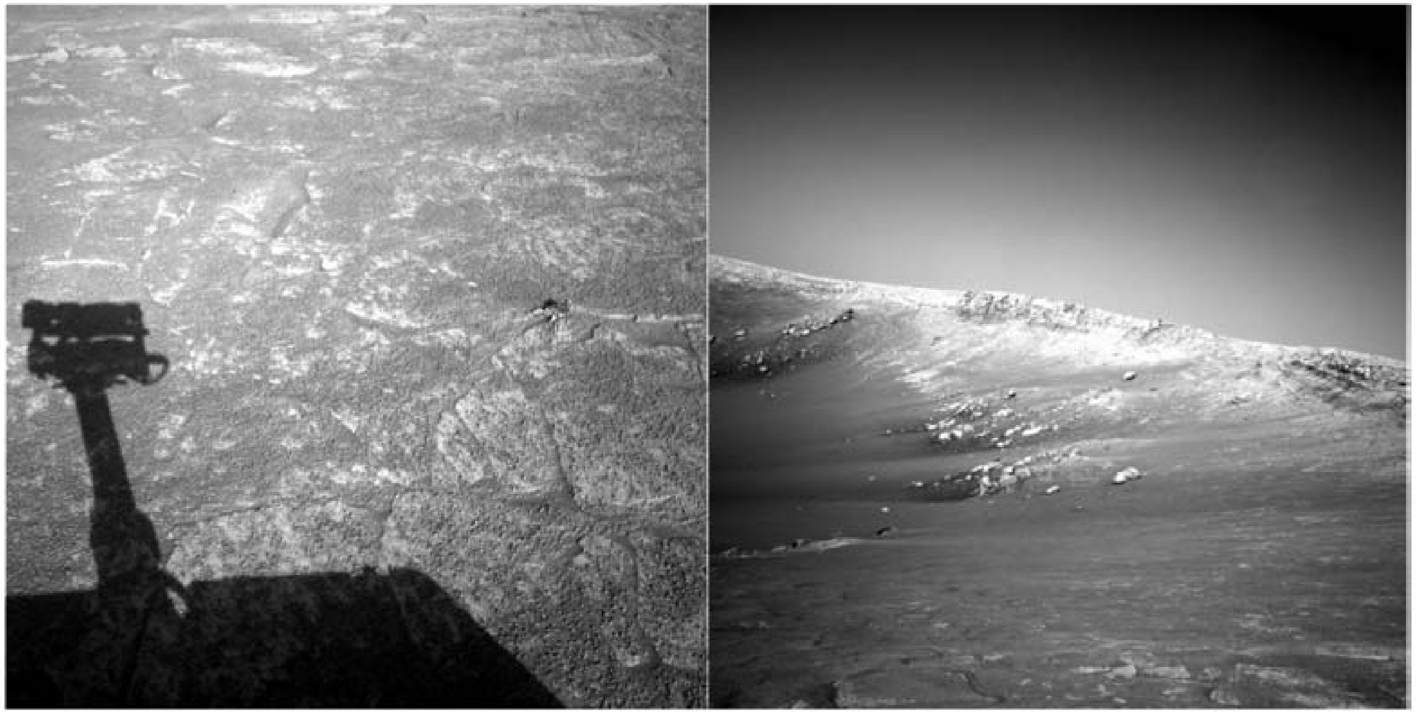
by swashford

Harder Case

by Diva Sian

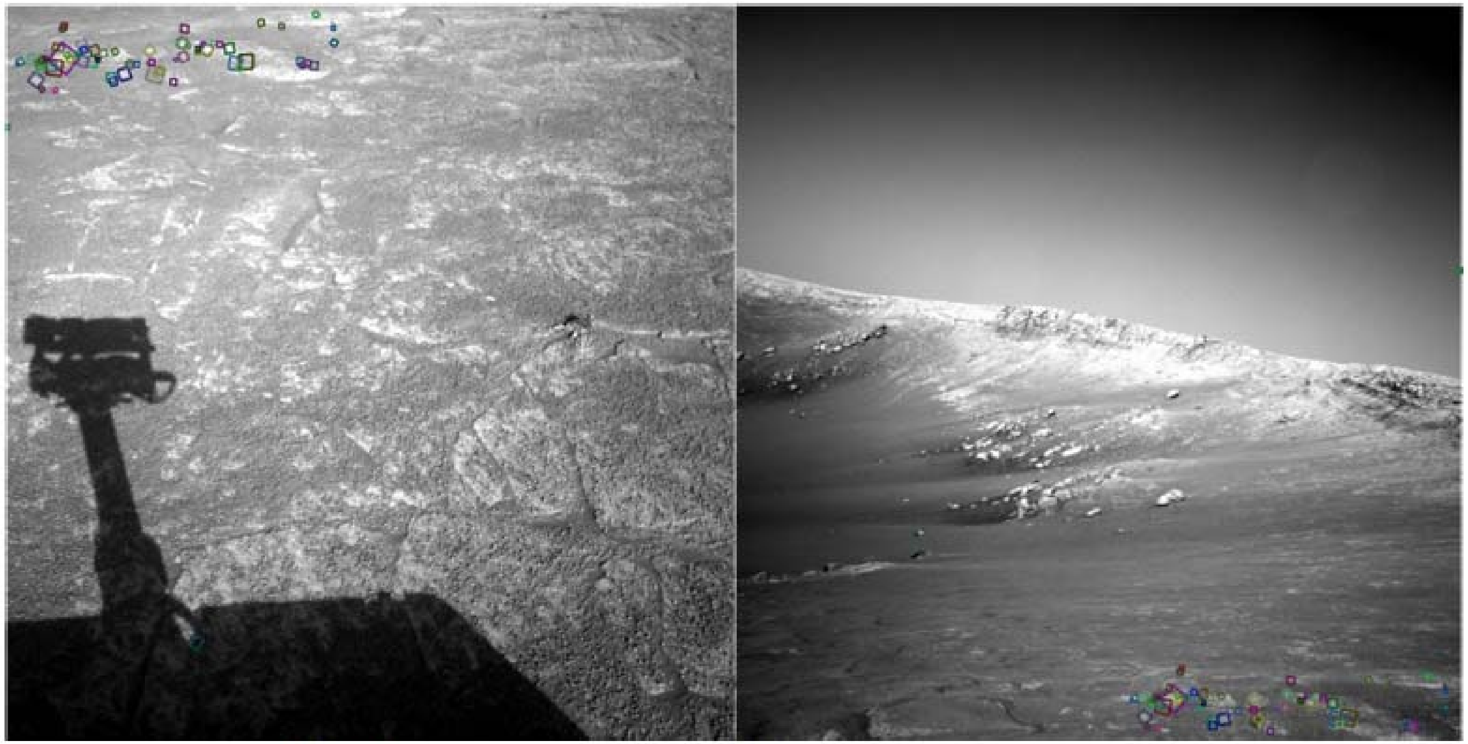
by scgbt

Still Harder



NASA Mars Rover images

Harder Case



NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

Image Stitching

Image Stitching

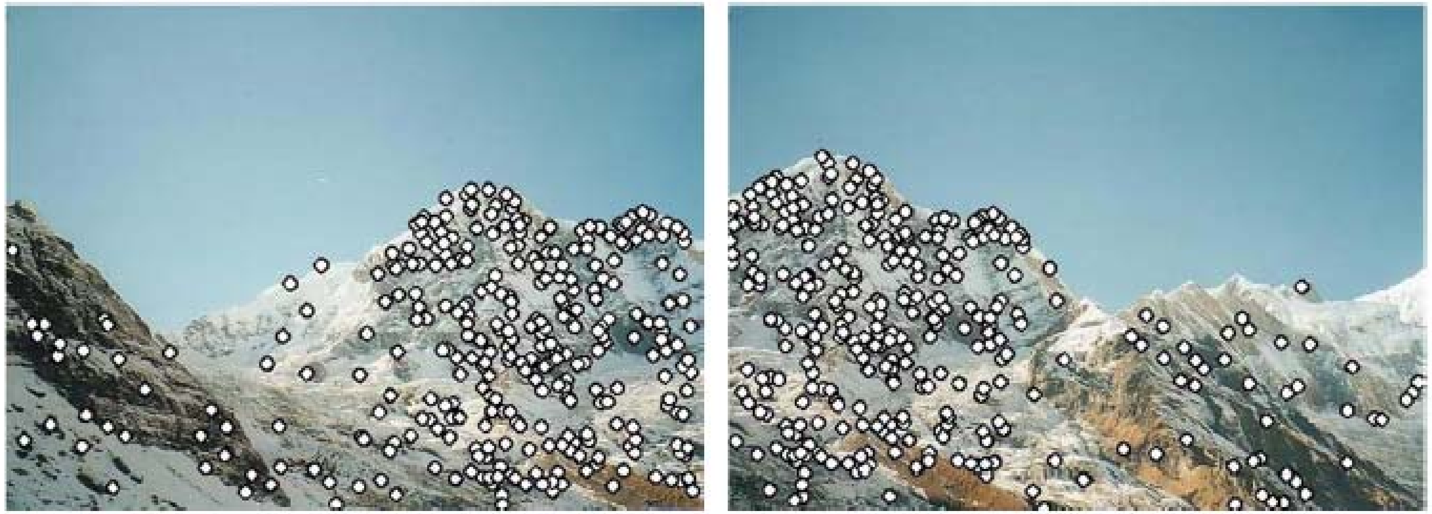


Image Stitching

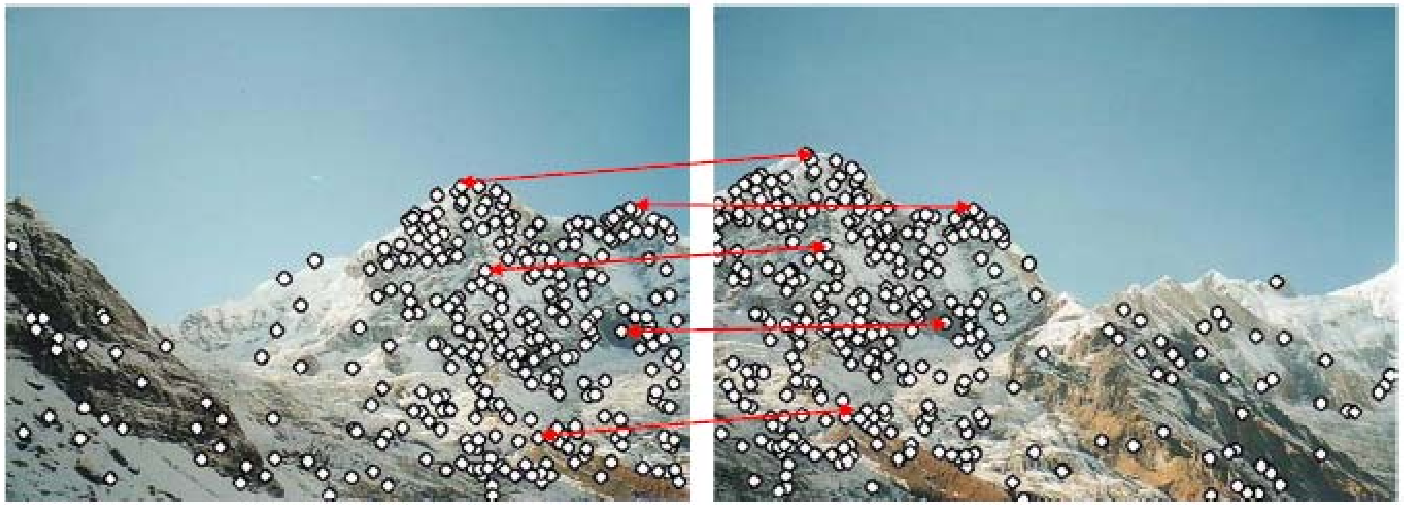


Image Stitching

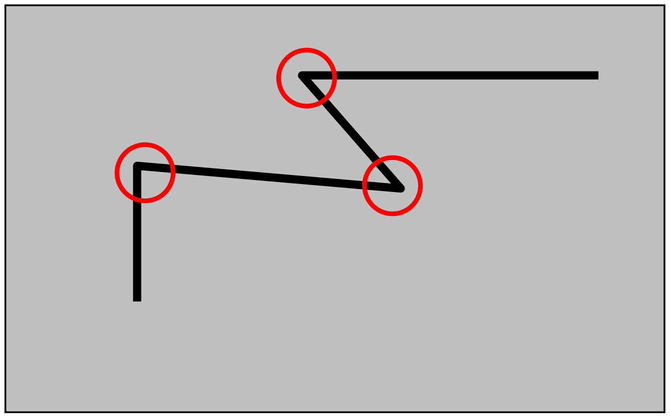
Building a Panorama

Applications

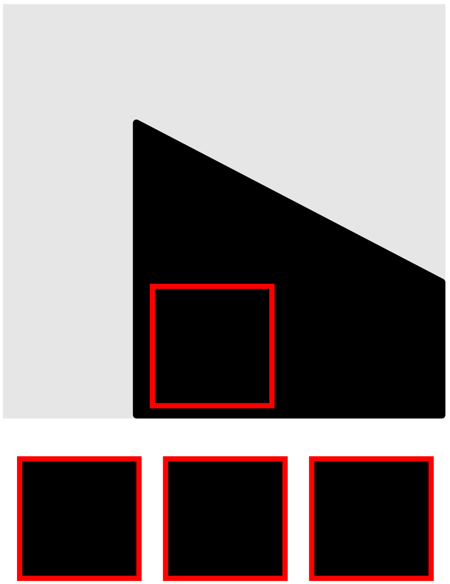
- Feature points are used for:
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Robot navigation
  - Indexing and database retrieval
  - Object recognition

Moravec corner detector (1980)

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity

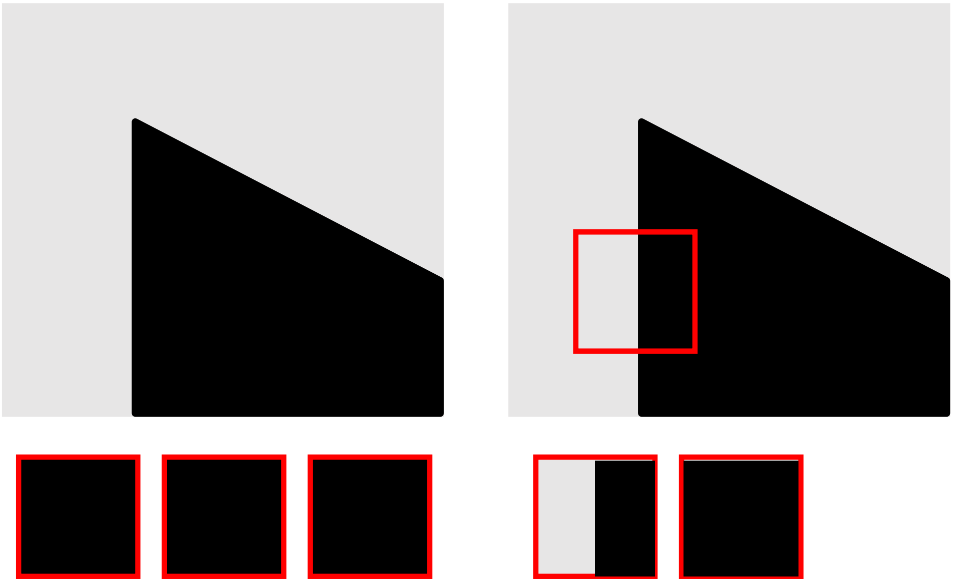


Moravec corner detector



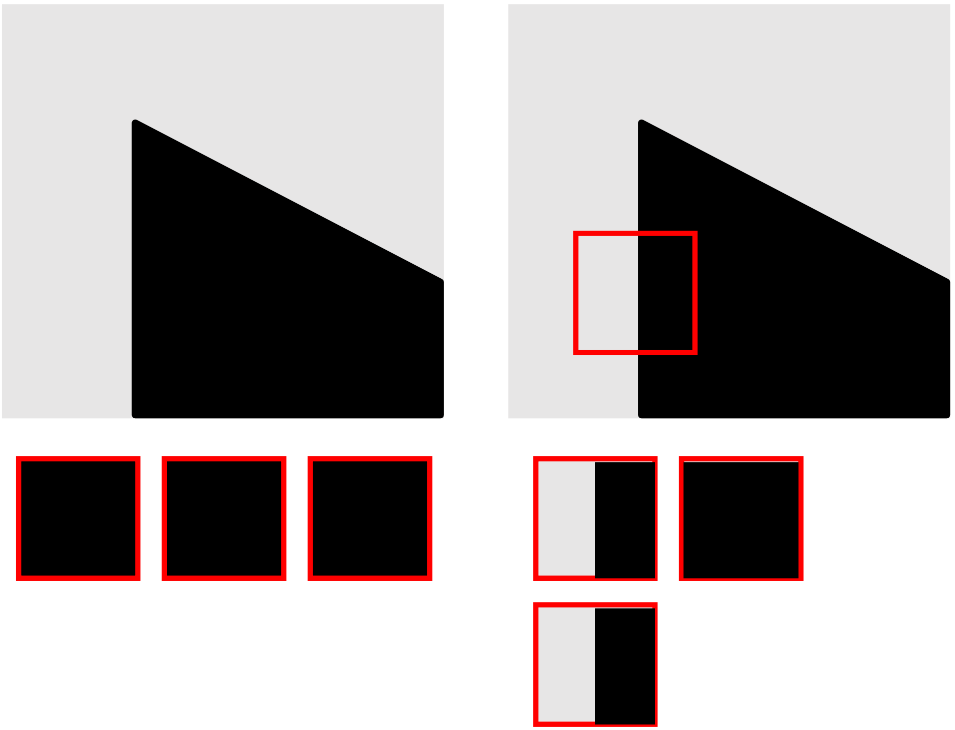
flat

Moravec corner detector



flat

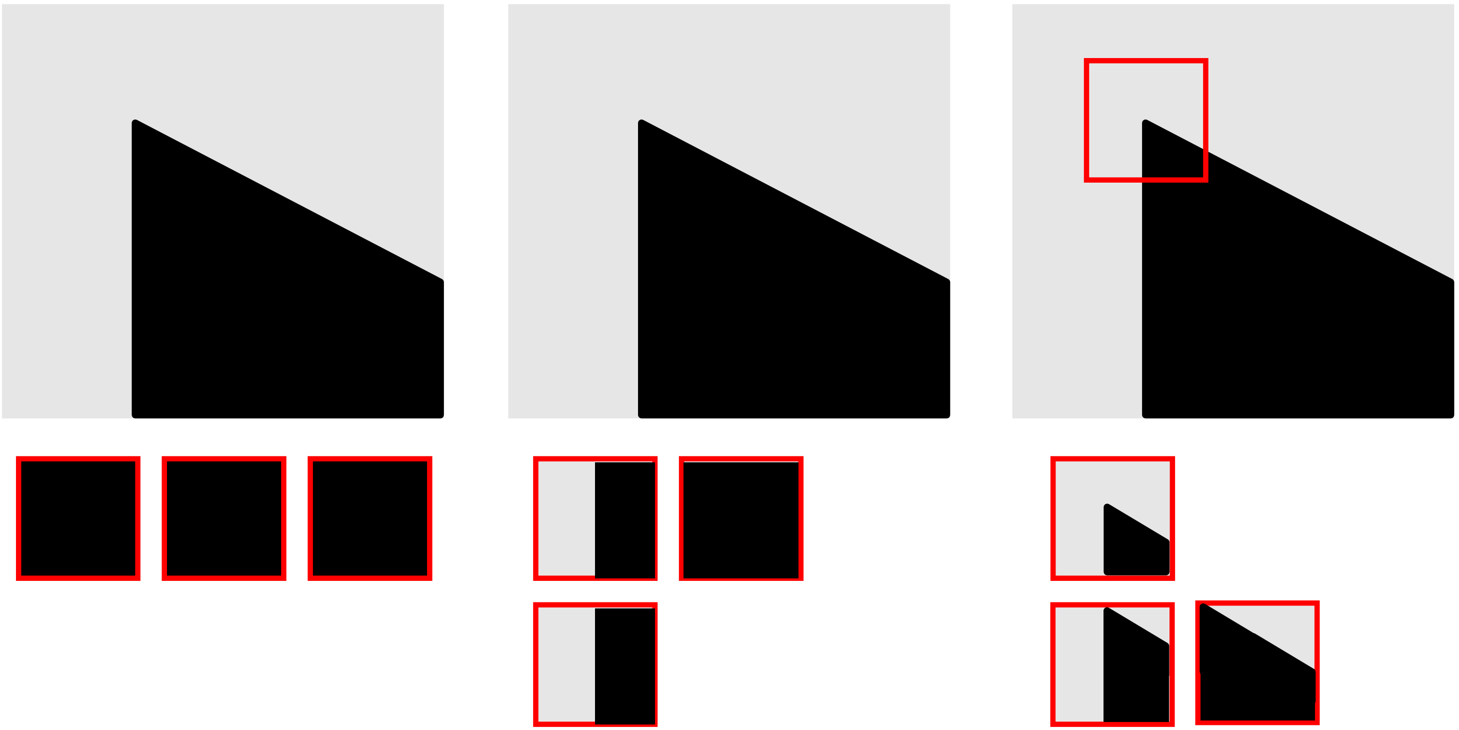
Moravec corner detector



flat

edge

Moravec corner detector



Moravec corner detector

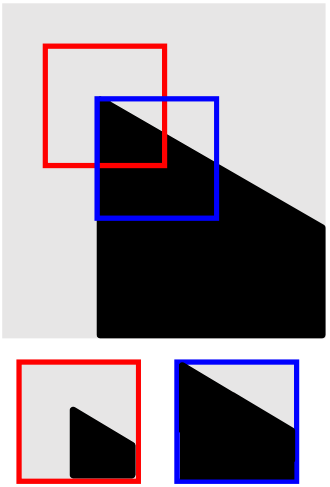
Change of intensity for the shift [*u,v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

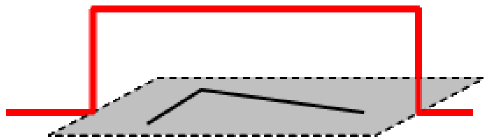
window function

shifted intensity

intensity



Window function $w(x,y)$ =



1 in window, 0 outside

Four shifts: (u,v) = (1,0), (1,1), (0,1), (-1, 1)

Look for local maxima in $min\{E\}$

Problems of Moravec detector

- Noisy response due to a binary window function
- Only a set of shifts at every 45 degree is considered
- Only minimum of E is taken into account

⇨Harris corner detector (1988) solves these problems.

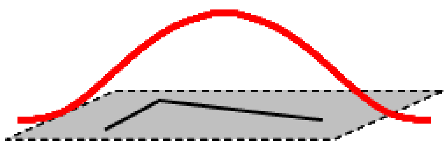
C.Harris, M.Stephens. “A Combined Corner and Edge Detector”. 1988

Harris corner detector

Noisy response due to a binary window function

➢Use a Gaussian function

$$w(x,y) = \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

Window function $w(x,y) =$ 

Gaussian

Taylor Series for 2D Functions

$$f(x+u,y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y) +$$

First partial derivatives

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uv f_{xy} x,y + v^2 f_{yy}(x,y)\right] +$$

Second partial derivatives

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third partial derivatives

$+ \ \ldots$ (Higher order terms)

First order approx

$$f(x+u,y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$$

Harris corner detector

Only a set of shifts at every 45 degree is considered

- Consider all small shifts by Taylor’s expansion

Harris corner detector

Only a set of shifts at every 45 degree is considered

➢Consider all small shifts by Taylor's expansion

$$E(u,v) \approx \sum_{x,y} w(x,y)[I(x,y) + uI_x + vI_y - I(x,y)]^2$$

$$= \sum_{x,y} w(x,y)[uI_x + vI_y]^2$$

$$= \sum_{x,y} w(x,y)(u \quad v)\begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

Harris corner detector

Equivalently, for small shifts [u,v] we have a *bilinear* approximation:

$$E(u,v) \cong \begin{bmatrix} u & v \end{bmatrix} \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

, where M is a 2×2 matrix computed from image derivatives:

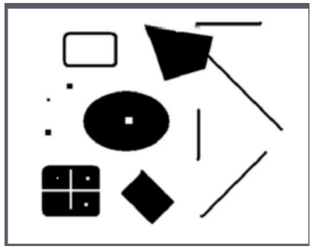
$$\mathbf{M} = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

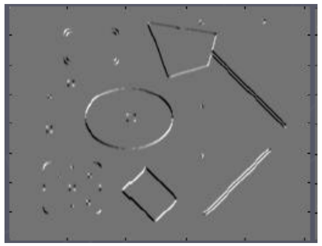
M is also called “structure tensor”

Harris corner detector

M is a 2×2 matrix computed from image derivatives:

$$\mathbf{M} = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$





Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $\quad I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $\quad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

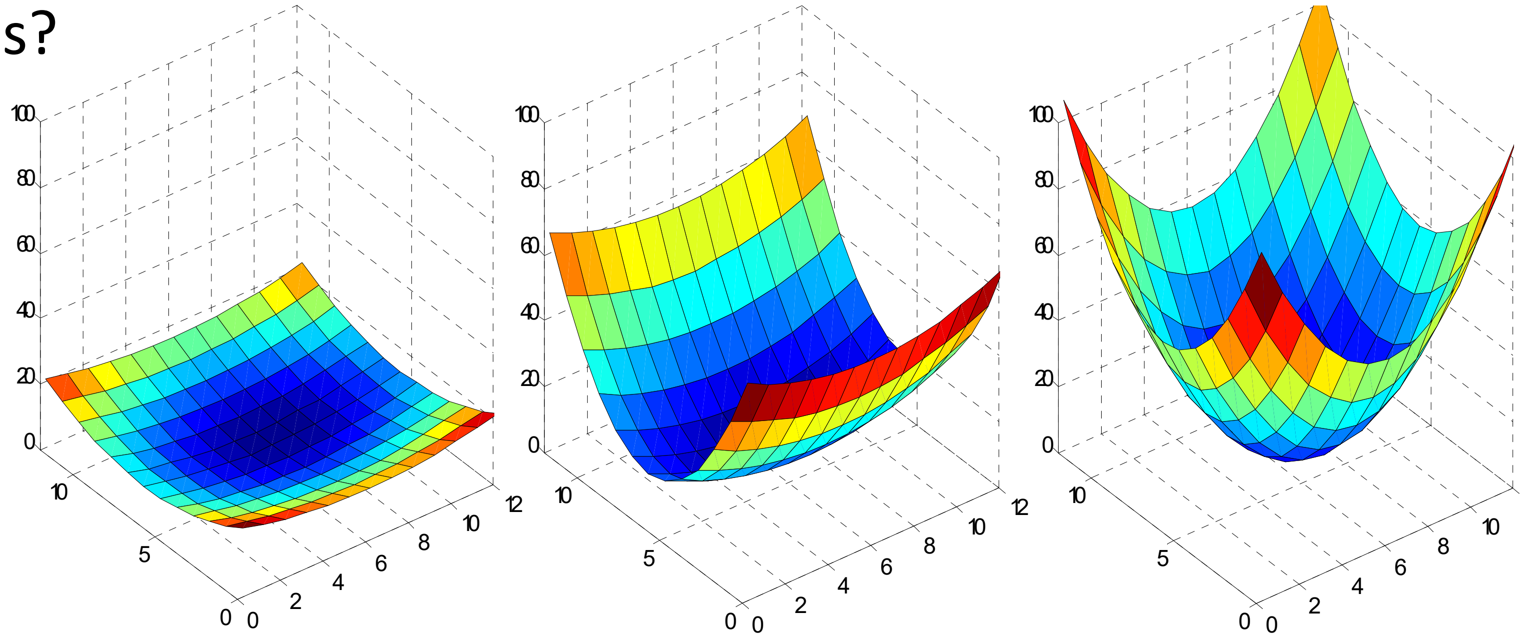
Harris corner detector

Only minimum of E is taken into account

➢A new corner measurement by investigating the shape of the error function

Harris corner detector

High-level idea: what shape of the error function will we prefer for features?

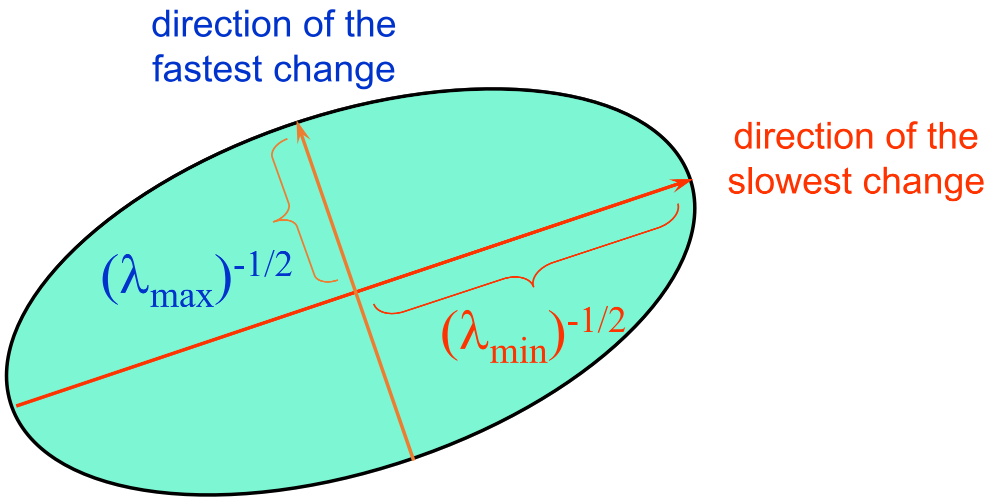


flat edge corner

General Case

Since M is symmetric, we have $M = X^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X$

We can visualize *M* as an ellipse with axis lengths determined by the eigenvalues and orientation determined by *X*



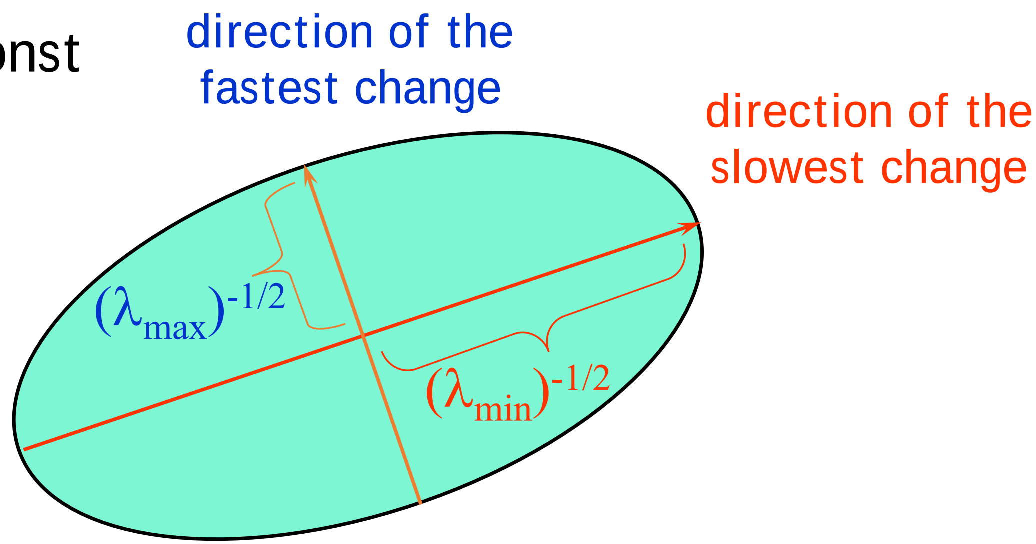
Harris corner detector

Intensity change in shifting window: eigenvalue analysis

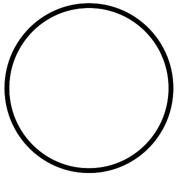
$$E(u,v) \cong [u,v]\,\mathbf{M}\begin{bmatrix} u \\ v \end{bmatrix}$$

λ_1, λ_2 – eigenvalues of M

Ellipse *E(u,v)* = const

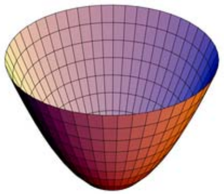


Visualizing quadratics



Equation of a circle

$$1 = x^2 + y^2$$



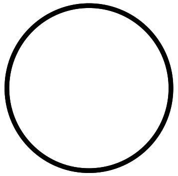
Equation of a 'bowl' (paraboloid)

$$f(x, y) = x^2 + y^2$$

If you slice the bowl at

$$f(x, y) = 1$$

what do you get?



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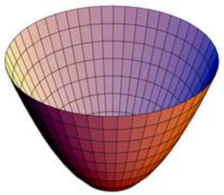

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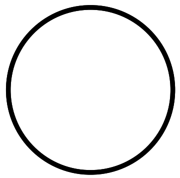


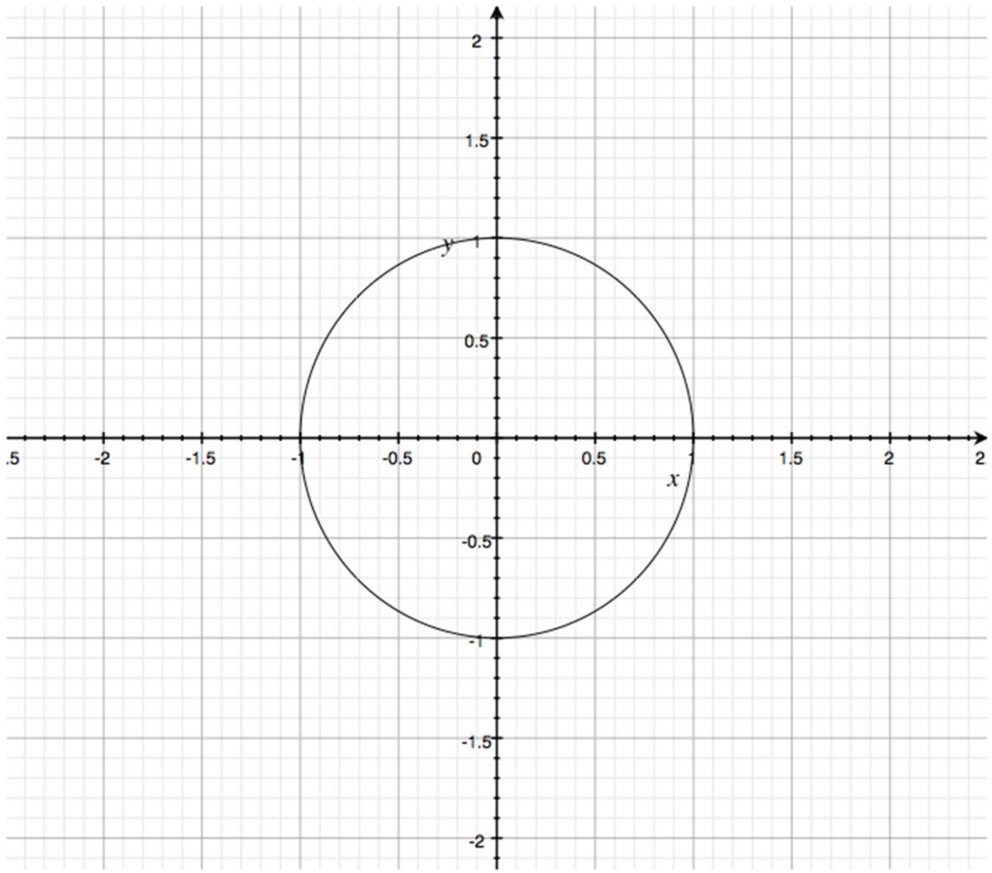
$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

‘sliced at 1’



*What happens if you **increase** coefficient on **x**?*

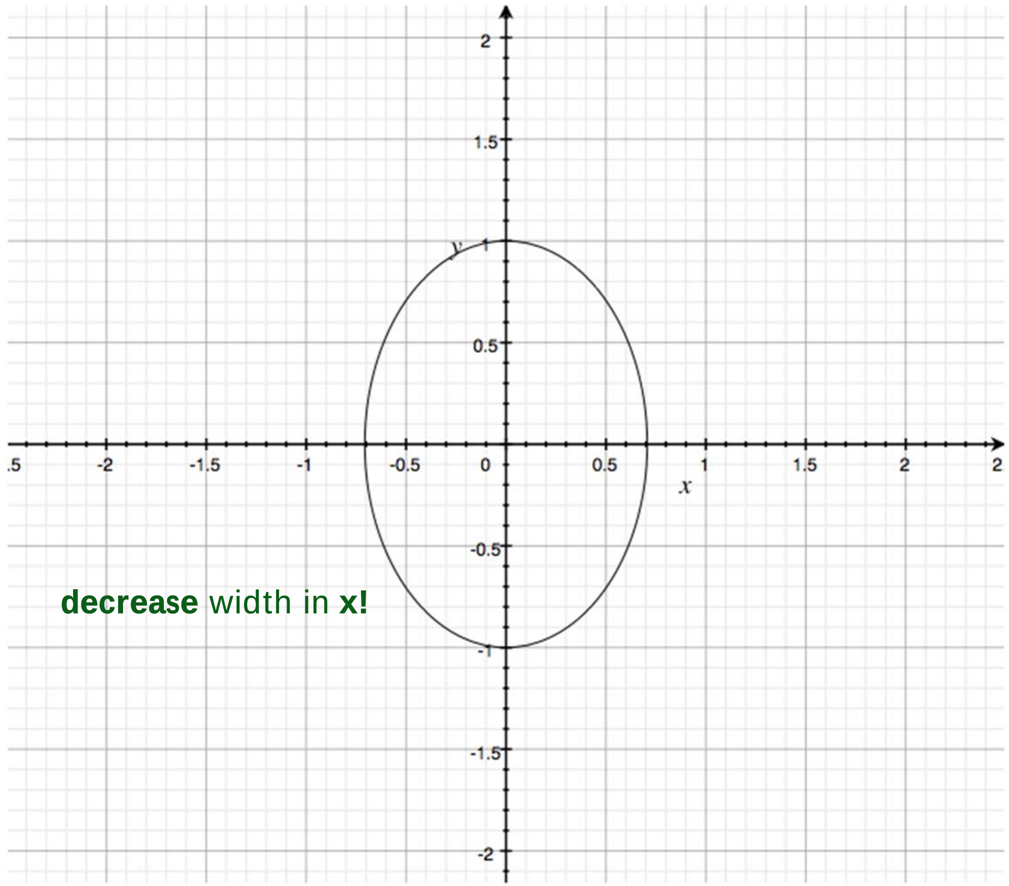
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **x**?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



*What happens if you **increase** coefficient on **y**?*

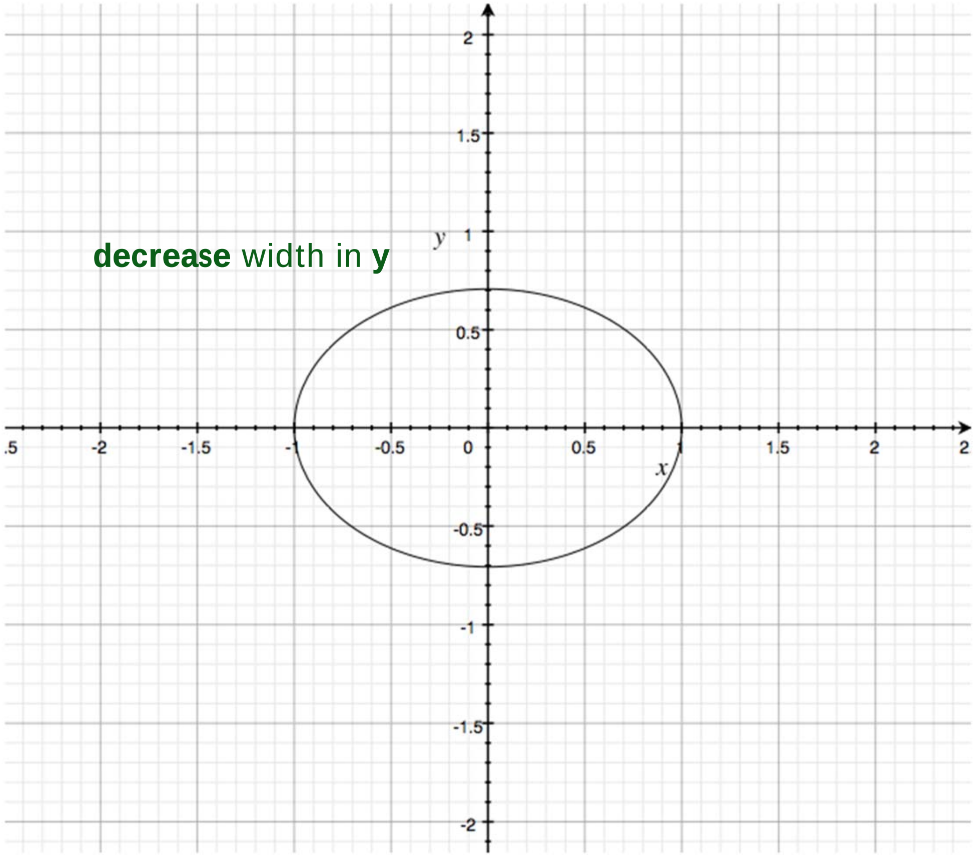
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **y**?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



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can be written in matrix form like this...

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What's the shape?
What are the eigenvectors?
What are the eigenvalues?

$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x,y) = \left[\begin{array}{cc} x & y \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \left[\begin{array}{c} x \\ y \end{array}\right]$$

Result of Singular Value Decomposition (SVD)

eigenvectors

eigenvalues along diagonal

$$\left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]^\top$$

axis of the ‘ellipse slice’

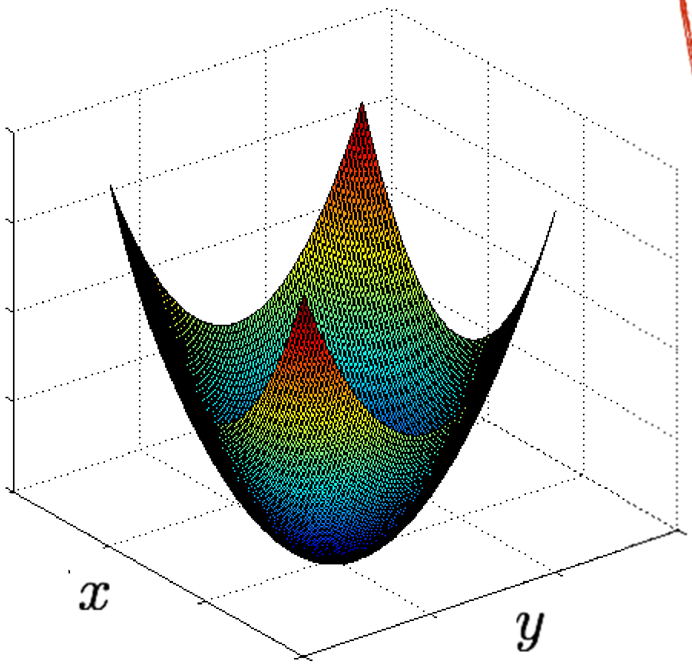
Inverse sqr of length of the quadratic along the axis

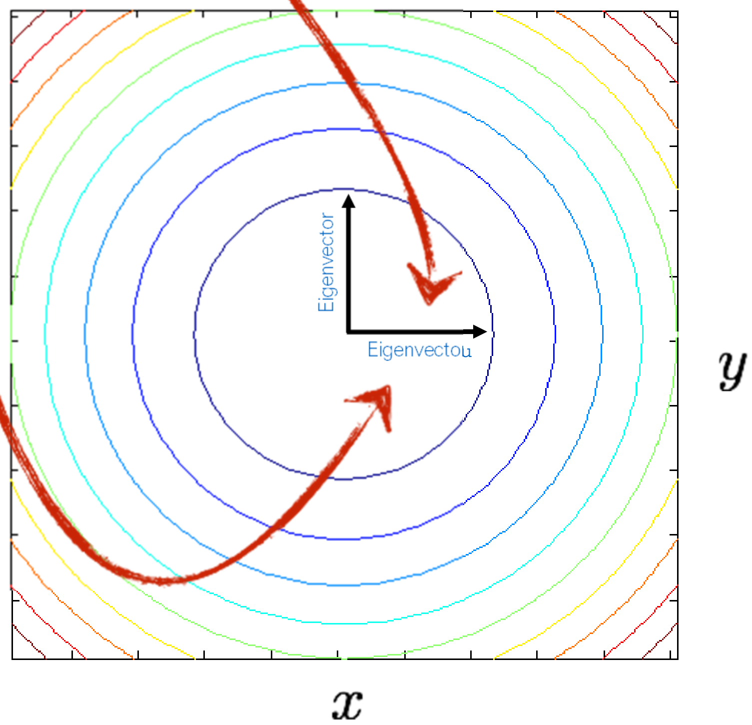
Eigenvectors Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

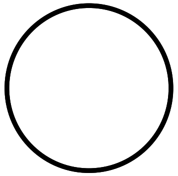
Eigenvectors

Inverse sqr of the size of the axis



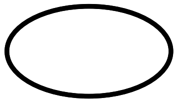


Recall:



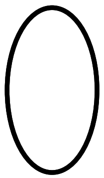
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **y** direction



$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **x** direction



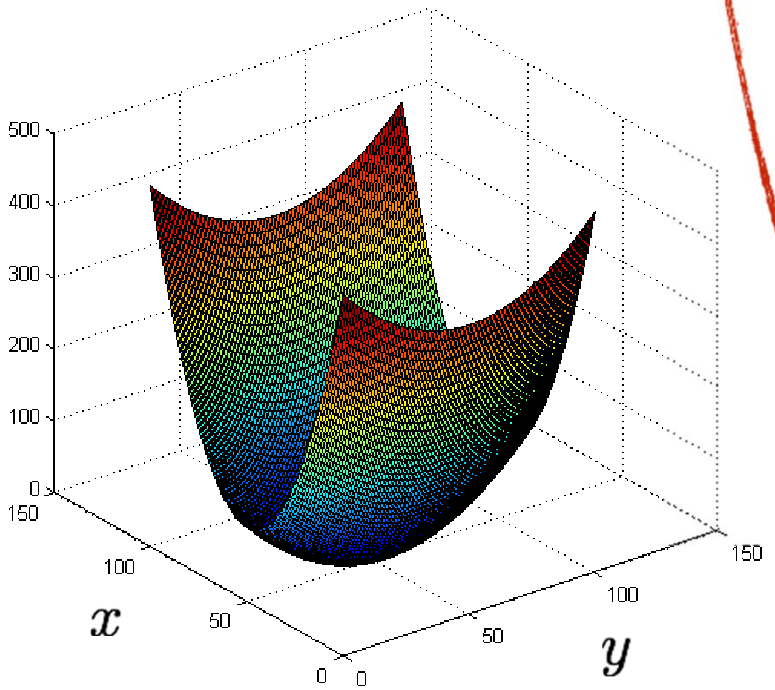
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

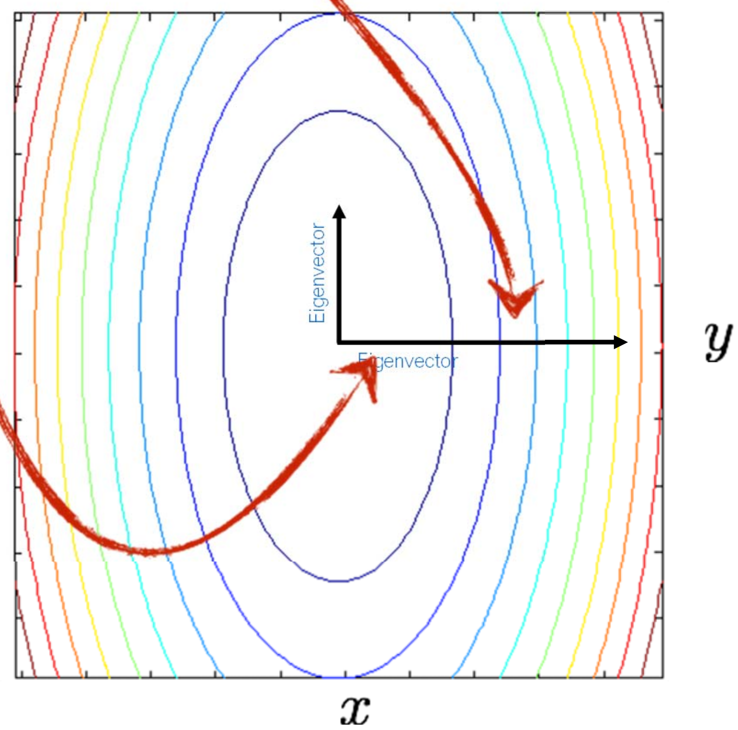
Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

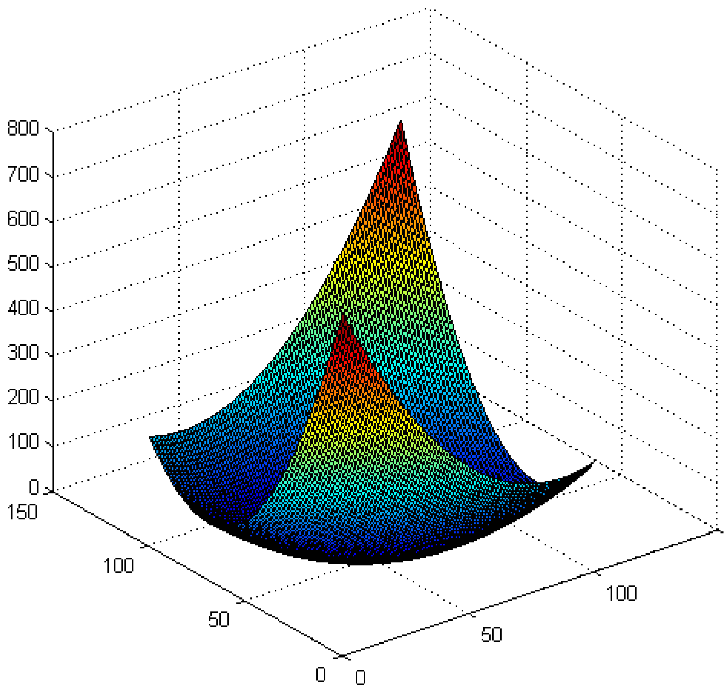
Eigenvectors Eigenvectors

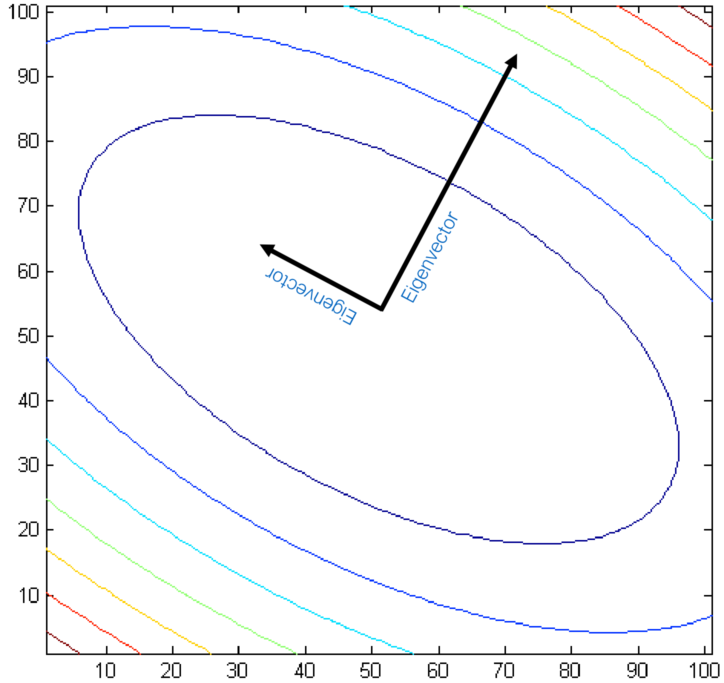
Inverse sqr of length of axis



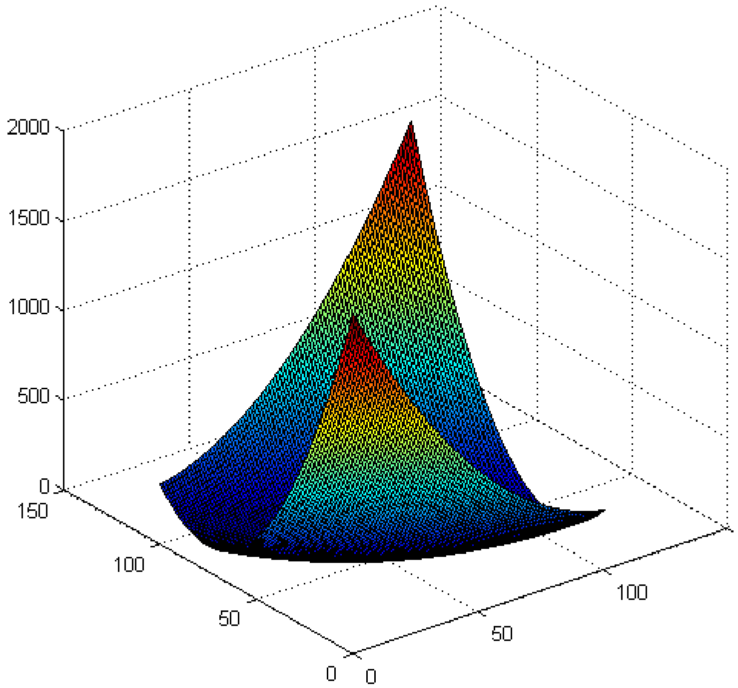


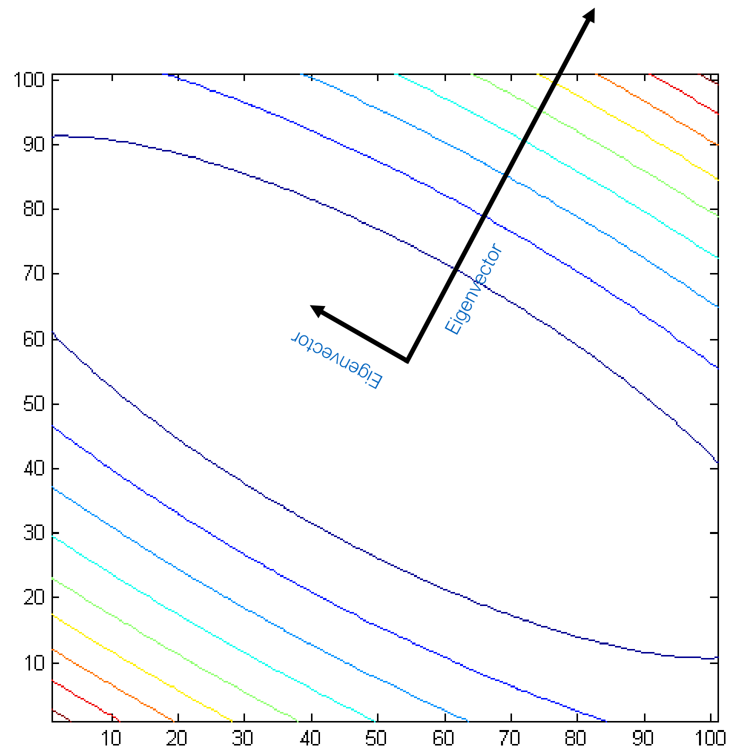
$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}}_{\text{Eigenvectors}} \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}}^{\text{Eigenvalues}} \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T}_{\text{Eigenvectors}}$$





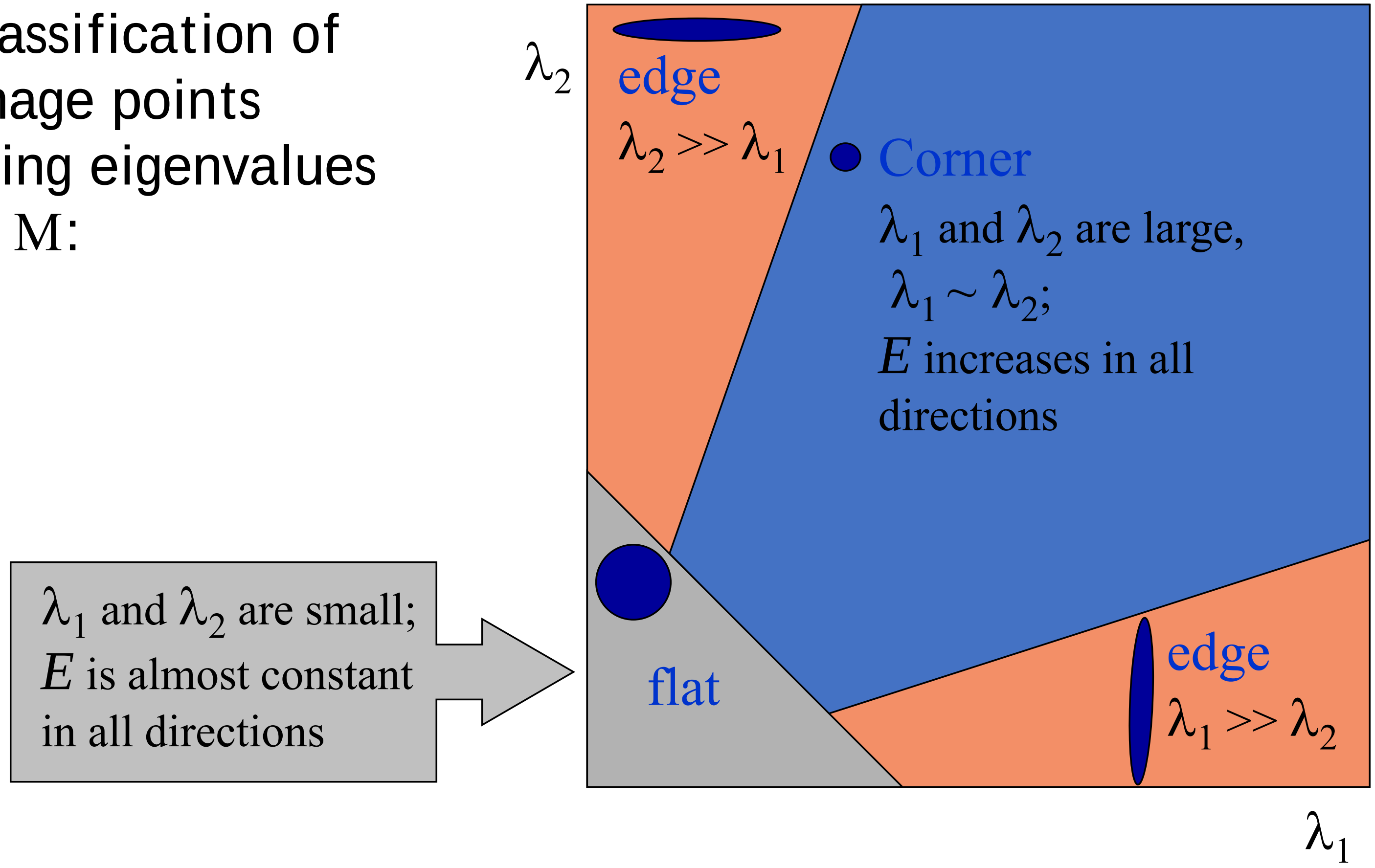
$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}}_{\text{Eigenvectors}} \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}}^{\text{Eigenvalues}} \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T}_{\text{Eigenvectors}}$$





Harris corner detector

Classification of image points using eigenvalues of M:



Harris corner detector

$$\lambda = \frac{a_{00} + a_{11} \pm \sqrt{(a_{00} - a_{11})^2 + 4a_{10}a_{01}}}{2}$$

Only for reference, you do not need them to compute R

Measure of corner response:

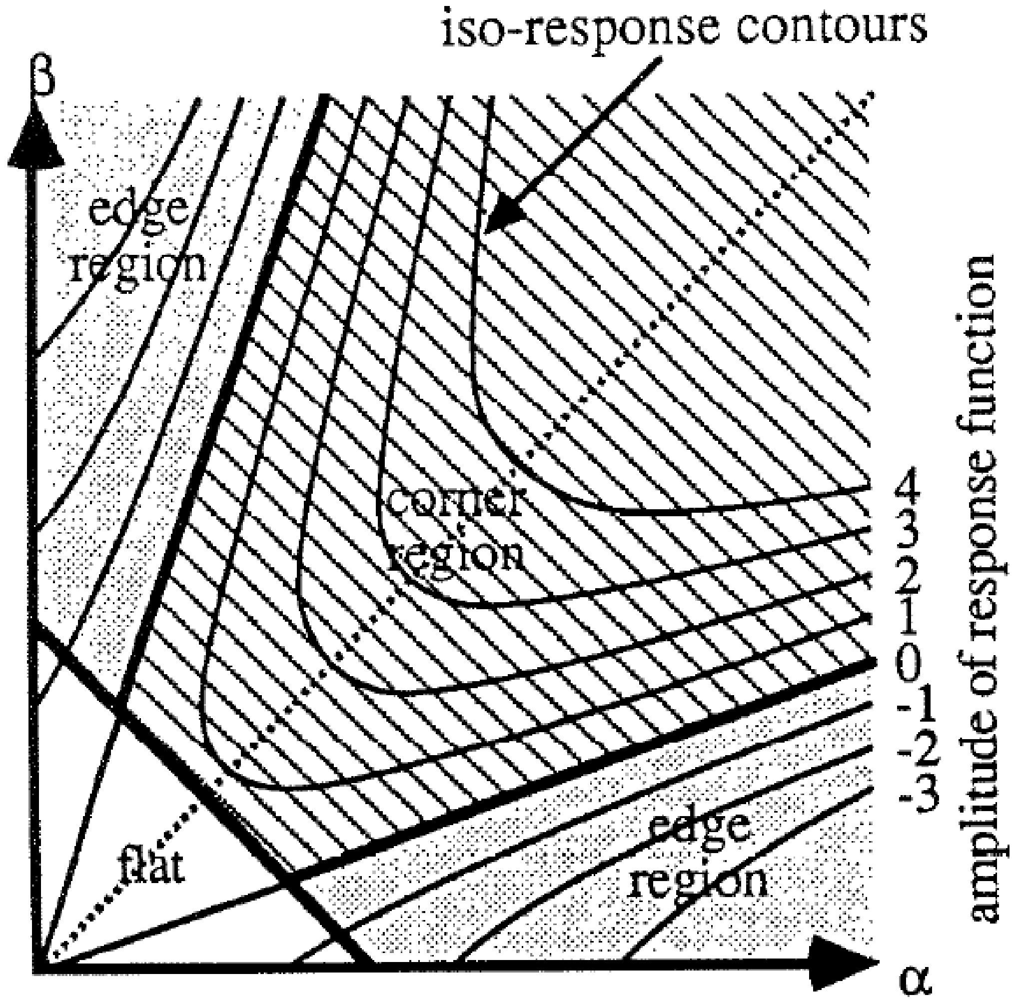
$$\boxed{R = \lambda_1 \lambda_2 - k\left(\lambda_1 + \lambda_2\right)^2}$$

(k – empirical constant, k = 0.04-0.06)

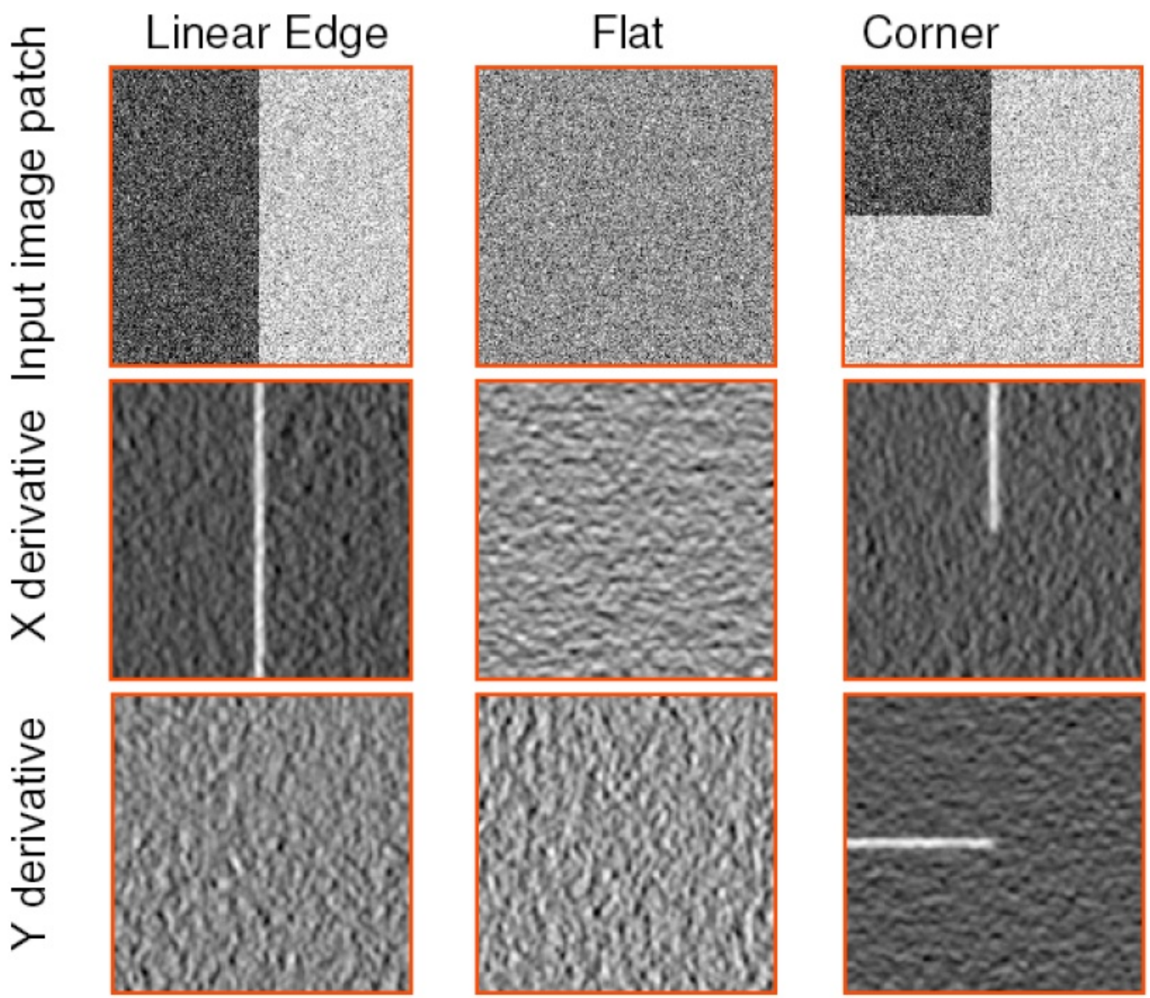
$$\boxed{\begin{aligned} \det \mathbf{M} &= \lambda_1 \lambda_2 \\ \operatorname{trace} \mathbf{M} &= \lambda_1 + \lambda_2 \end{aligned}}$$

$$\boxed{R = \det\mathbf{M} - k\left(\operatorname{trace}\mathbf{M}\right)^2}$$

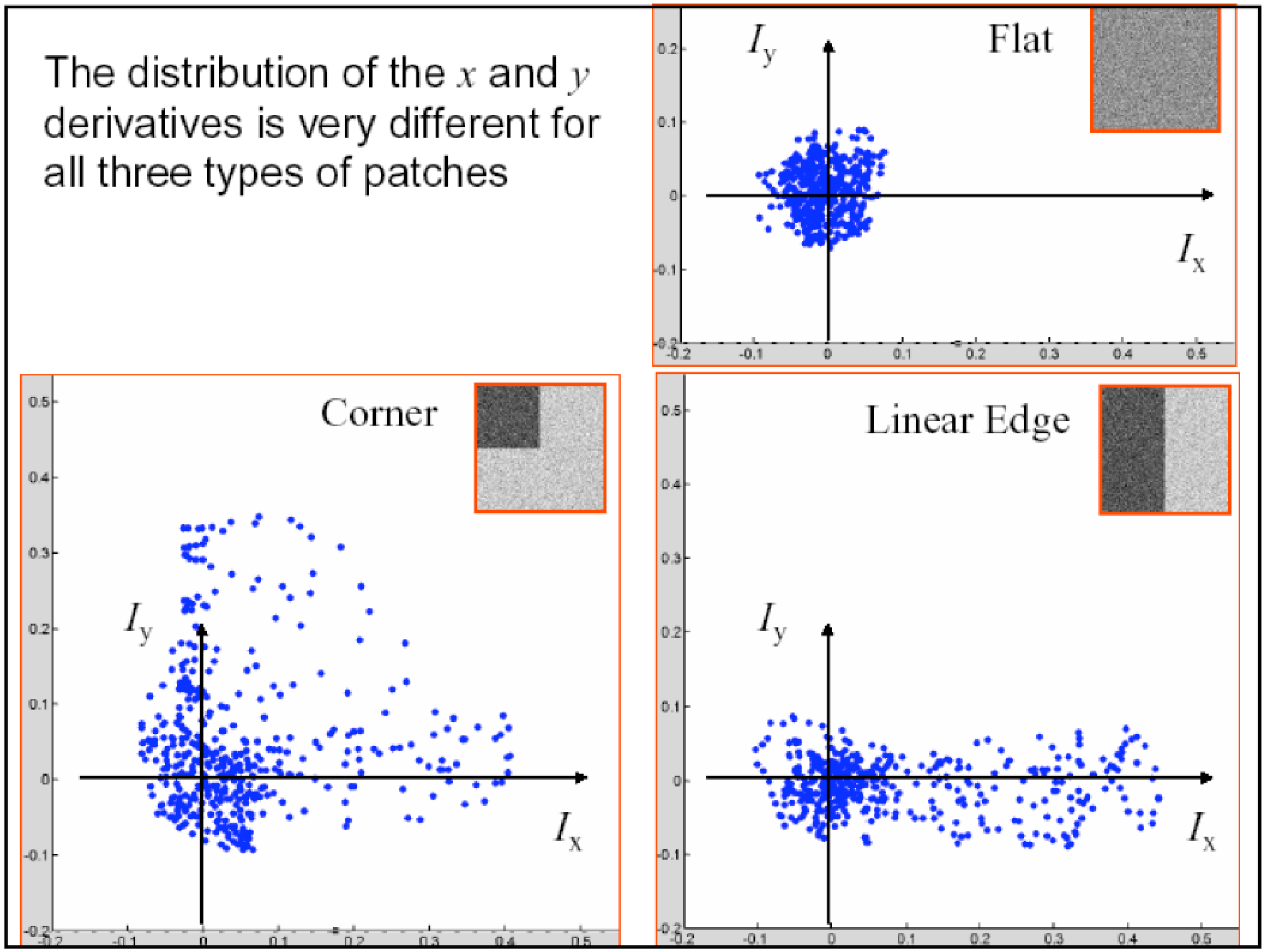
Harris corner detector



Another view

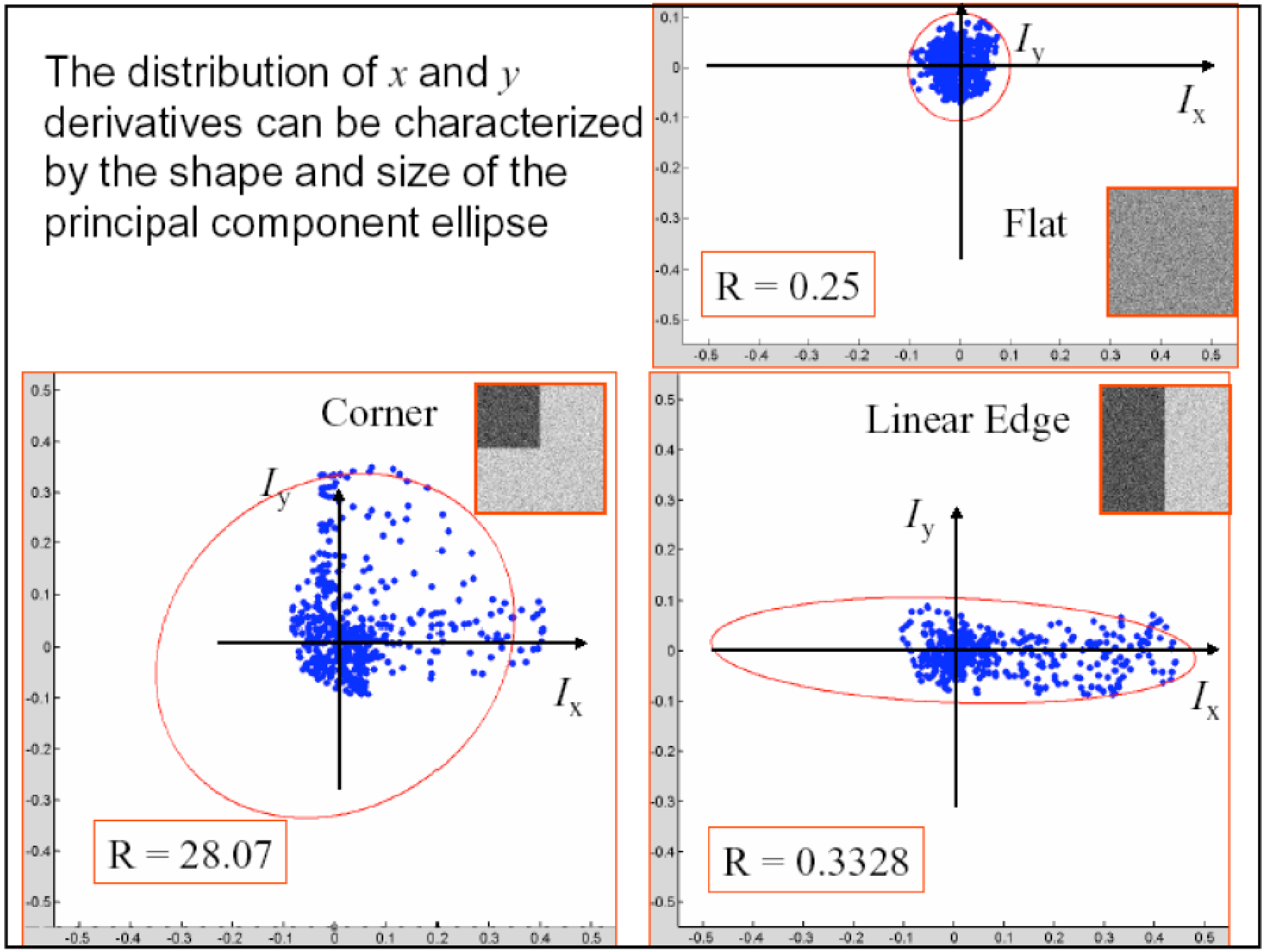


Another view



Another view

The distribution of x and y derivatives can be characterized by the shape and size of the principal component ellipse



Summary of Harris detector

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

Summary of Harris detector

4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

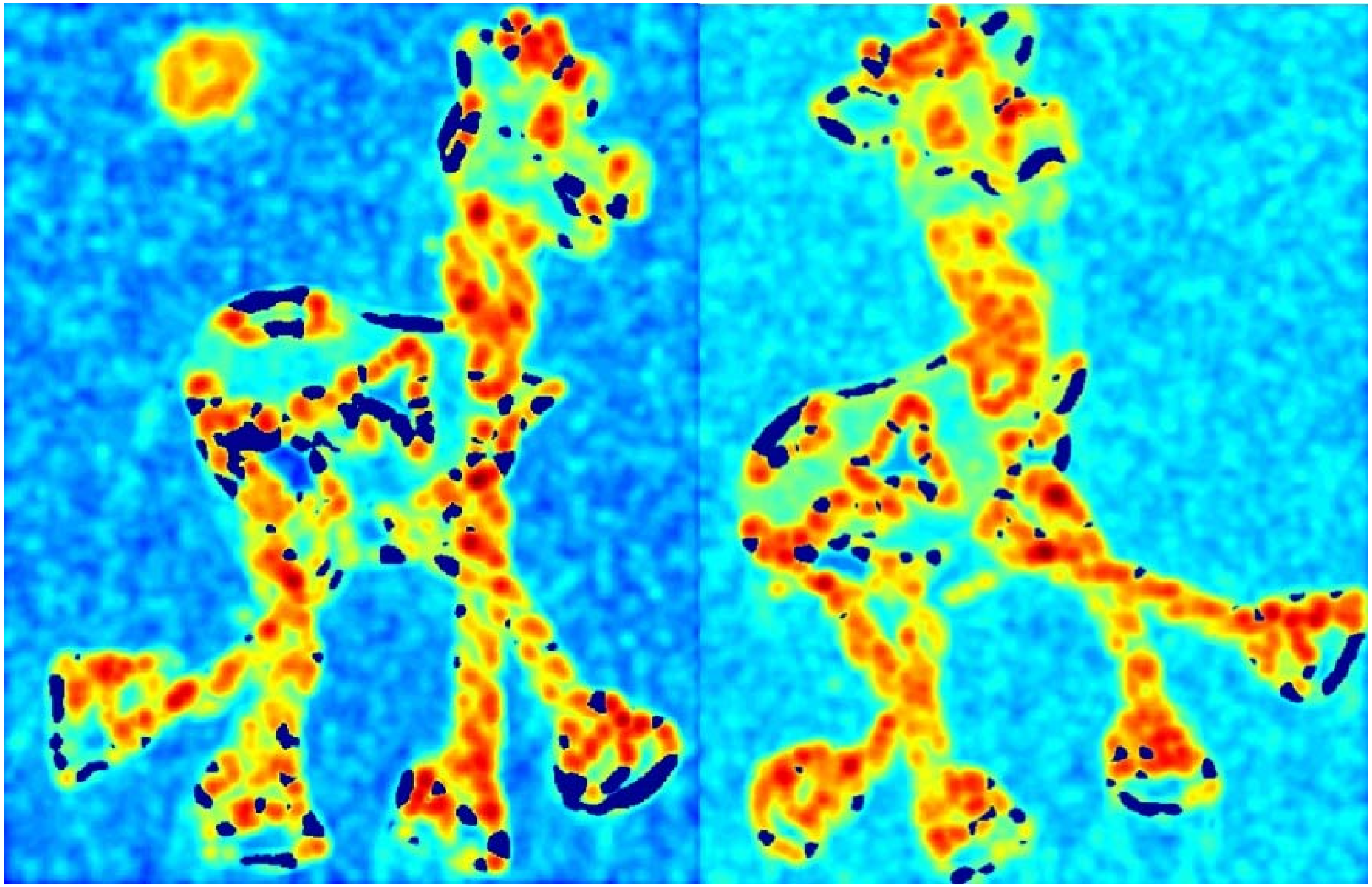
5. Compute the response of the detector at each pixel

$$R = \det M - k(\mathrm{trace} M)^2$$

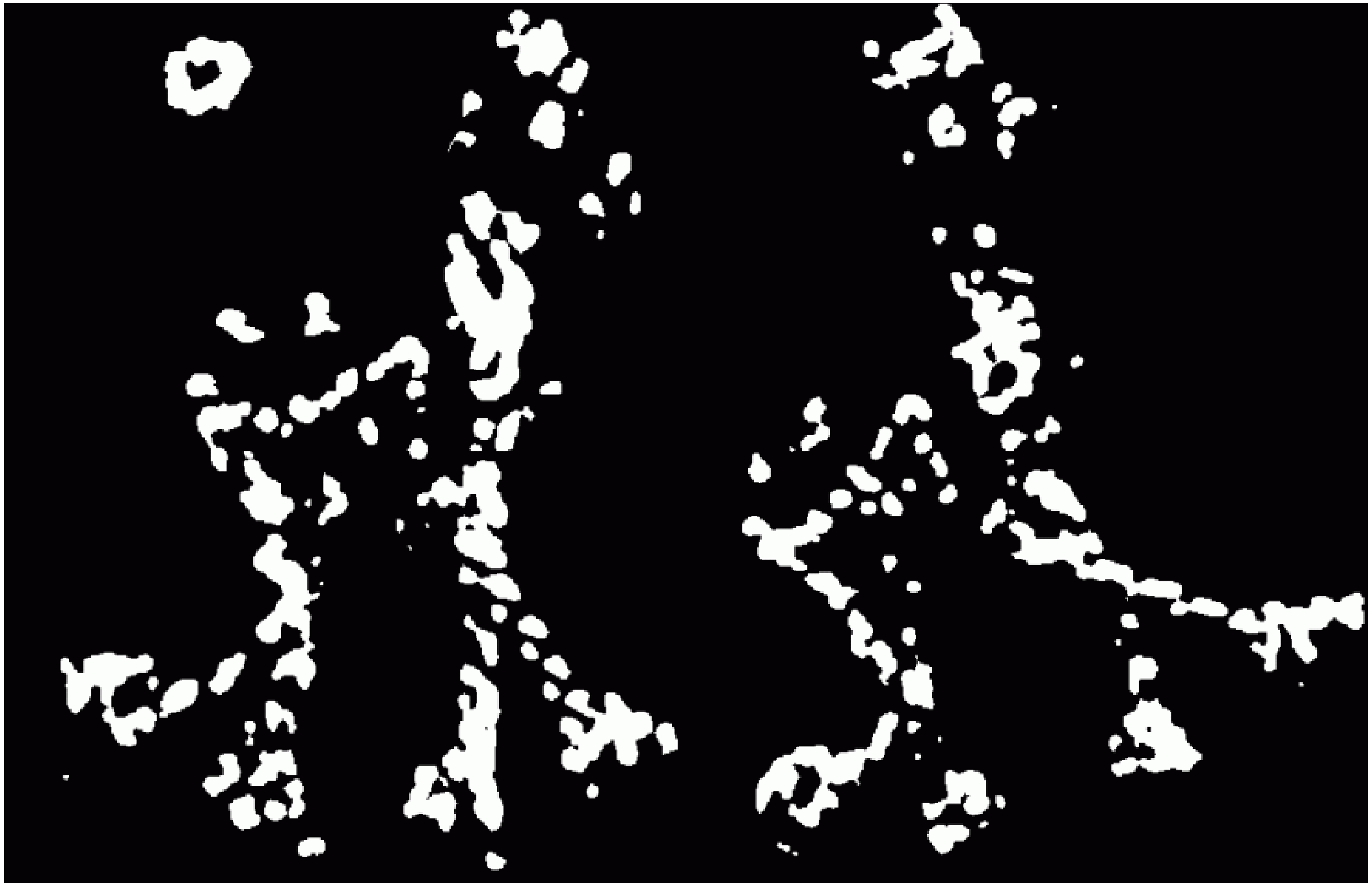
6. Threshold on value of R; compute nonmax suppression.

Harris corner detector (input)

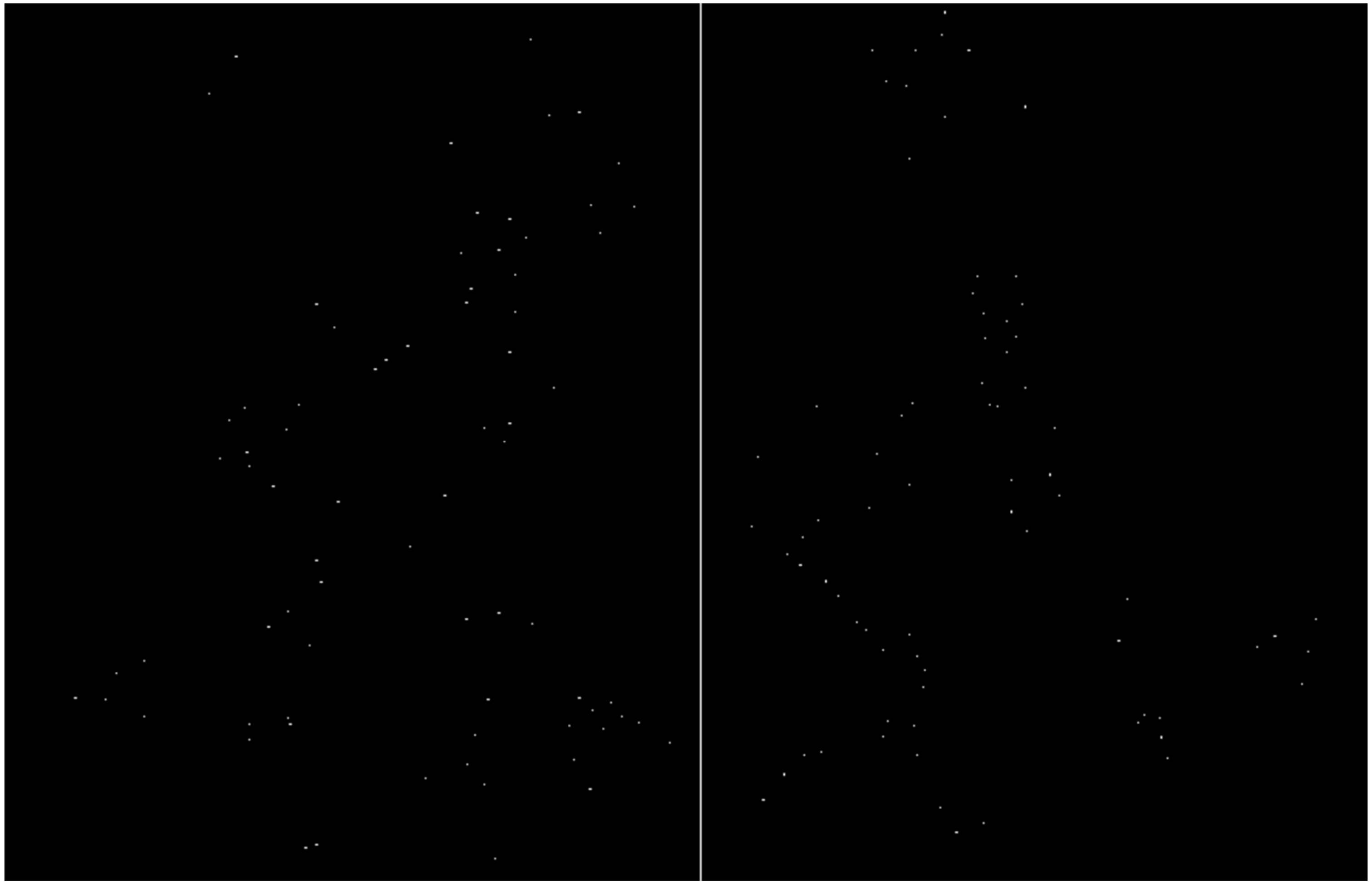
Corner response R



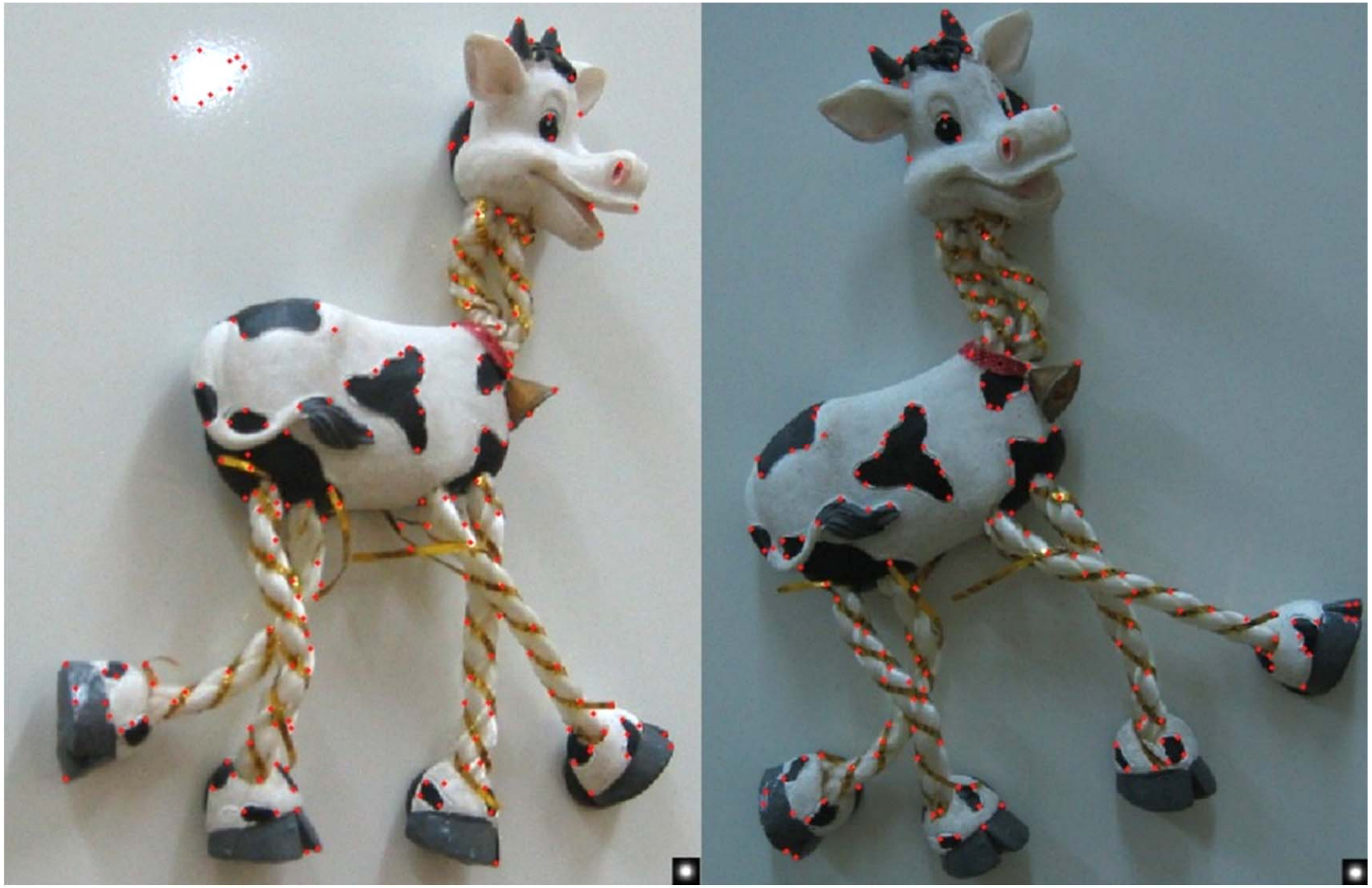
Threshold on R



Local maximum of R



Harris corner detector



Harris detector: summary

- Average intensity change in direction [u,v] can be expressed as a bilinear form:

$$E(u,v) \cong [u,v]\,\mathbf{M}\begin{bmatrix} u \\ v \end{bmatrix}$$

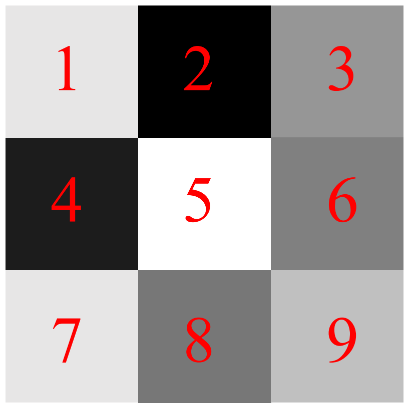
- Describe a point in terms of eigenvalues of M:
  measure of corner response

$$R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$

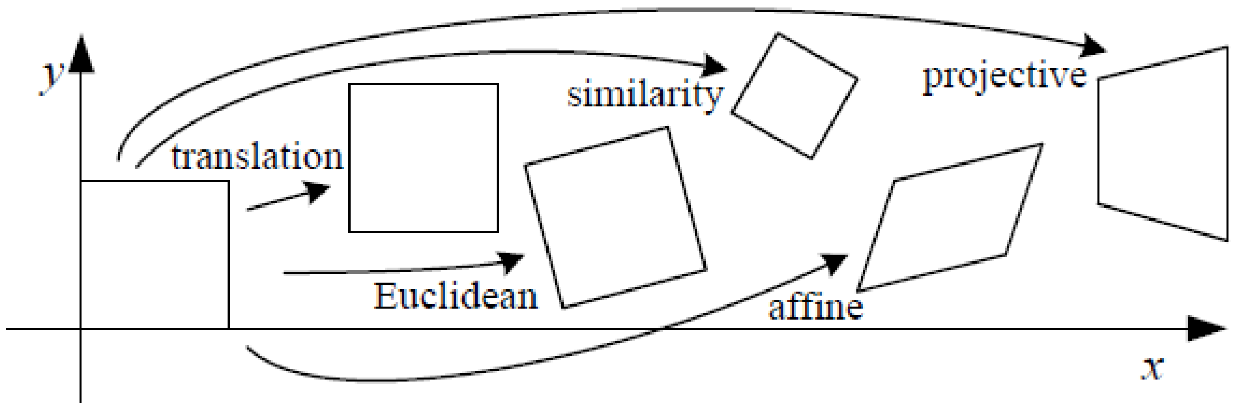
- A good (corner) point should have a *large intensity change* in *all directions*, i.e. R should be large positive

Now we know where features are

- But, how to match them?
- What is the descriptor for a feature? The simplest solution is the intensities of its spatial neighbors. This might not be robust to brightness change or small shift/rotation.



2D Geometric Transformations

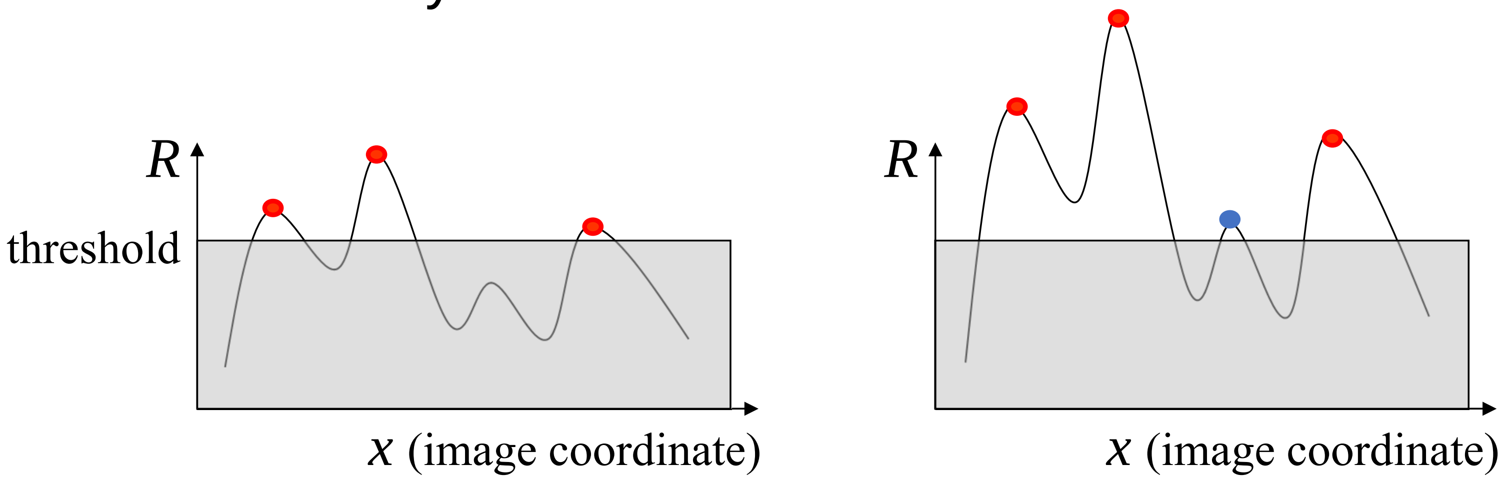


Harris detector: some properties

- Partial invariance to affine intensity change

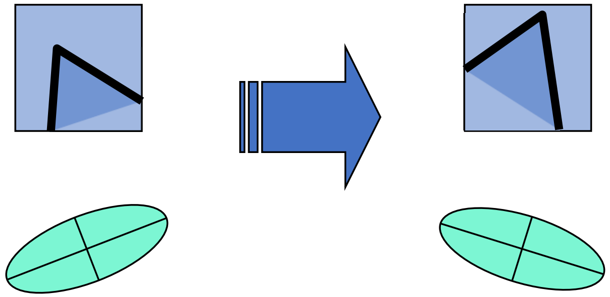
  ✓ Only derivatives are used =>
  invariance to intensity shift $I \rightarrow I + b$

  ✓ Intensity scale: $I \rightarrow a\,I$



Harris Detector: Some Properties

- Rotation invariance



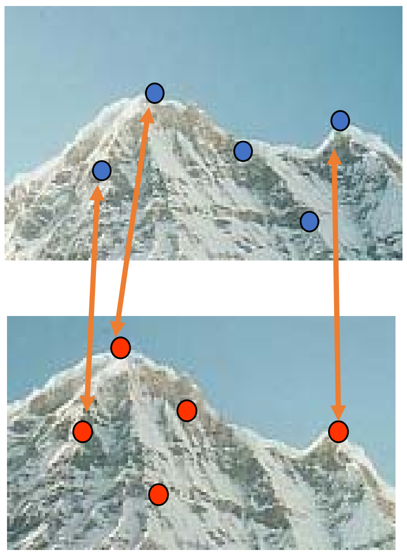
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

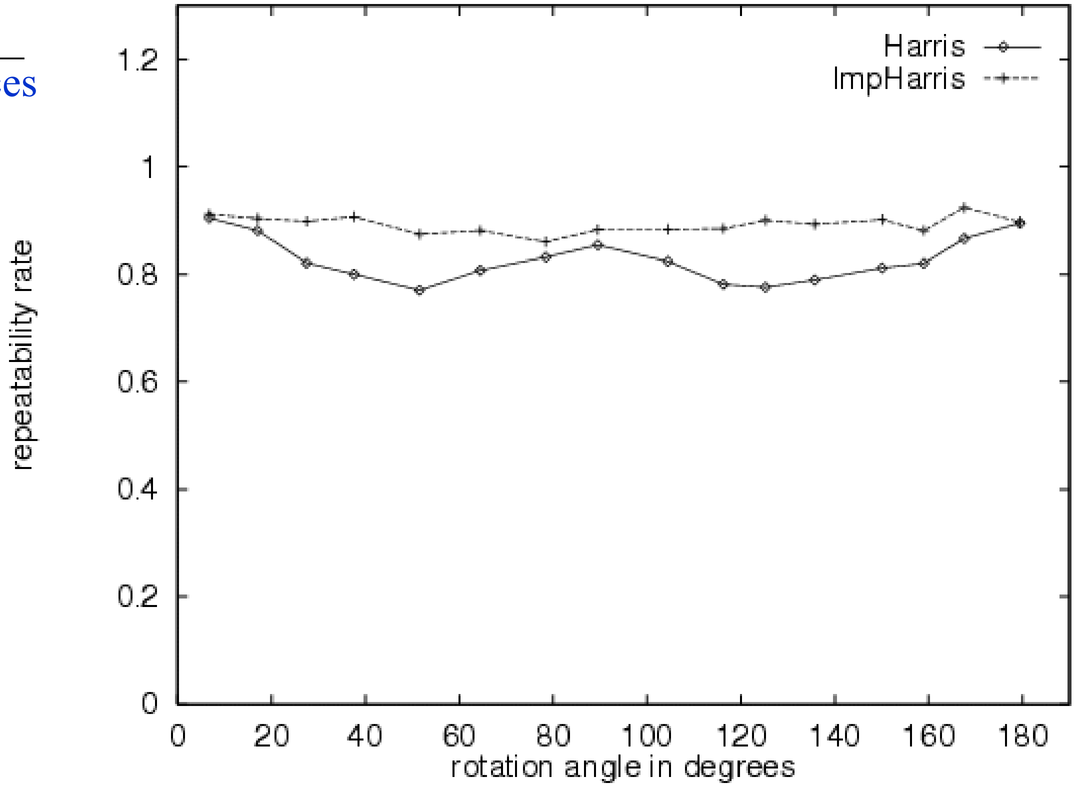
Corner response R is invariant to image rotation

Harris Detector is rotation invariant

Repeatability rate:

$$\frac{\#\ \text{correspondences}}{\#\ \text{possible correspondences}}$$

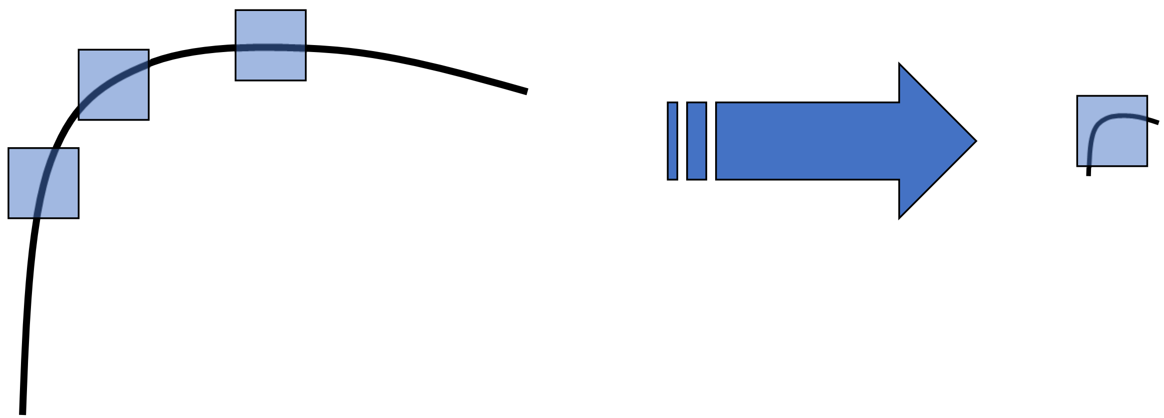




C.Schmid et.al. "Evaluation of Interest Point Detectors". IJCV 2000

Harris Detector: Some Properties

- But: not invariant to *image scale*!



All points will be classified as edges

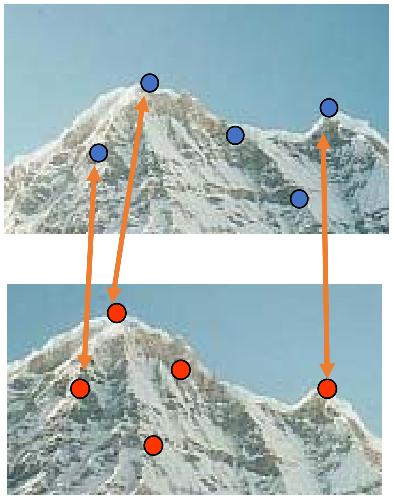
Corner !

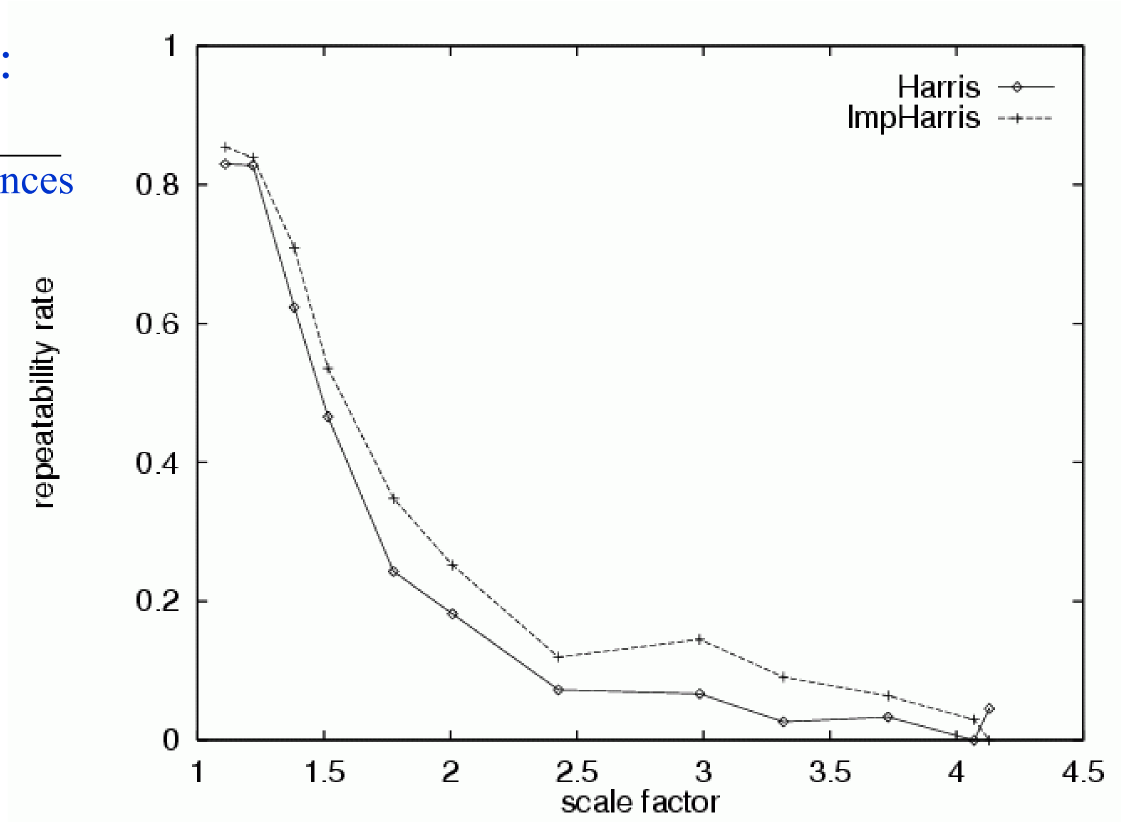
Harris detector: some properties

- Quality of Harris detector for different scale changes

Repeatability rate:

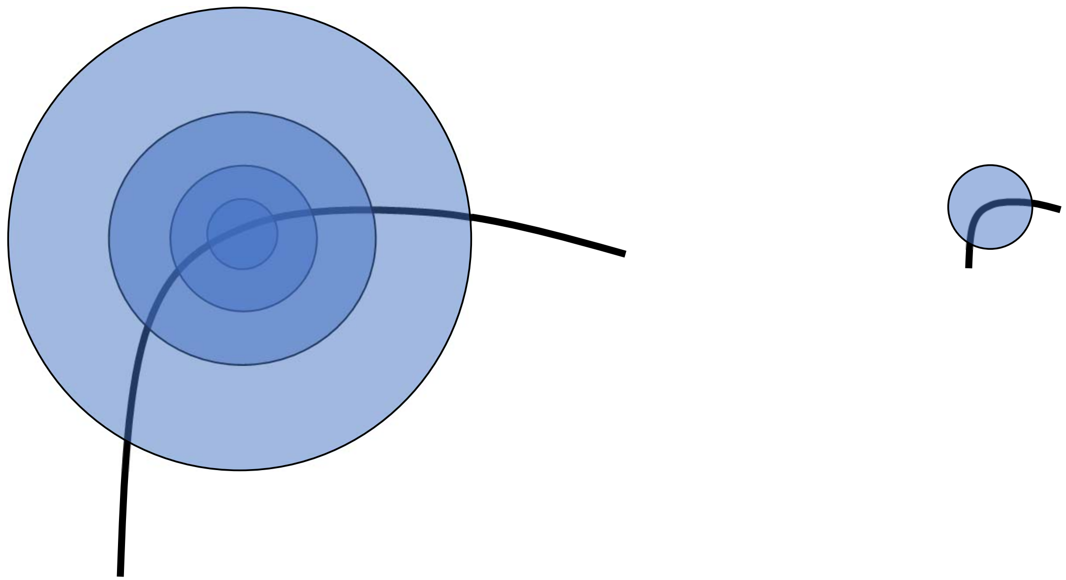
$$\frac{\text{\# correspondences}}{\text{\# possible correspondences}}$$





Scale invariant detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



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- Cordelia Schmid, Roger Mohr and Christian Bauckhage, “Evaluation of Interest Point Detectors”. International Journal of Computer Vision, Volume 37, Number 2, pp. 151-172, 2000.

- Steffen Gauglitz, Tobias Höllerer and Matthew Turk, “Evaluation of Interest Point Detectors and Feature Descriptors for Visual Tracking ”. International Journal of Computer Vision, Volume 94, Number 3, pp. 335-360, 2011.