Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Stock Market Prediction Based on Time-frequency Analysis and Convolutional Neural Network

To cite this article: Dandi Jia *et al* 2022 *J. Phys.: Conf. Ser.* **2224** 012017

Stock Market Prediction Based on Time-frequency Analysis and Convolutional Neural Network

Dandi Jia[1,*], Qiang Gao[1,2] and Hui Deng[1]

[1] Department of Electronic and Information Engineering, Beihang University, Beijing 100191, China

[2] Hangzhou Innovation Institute, Beihang University, Hangzhou, 310051, China

* Email: dandyjia@buaa.edu.cn

Abstract. Recently, researchers have shown an increased interest in stock market prediction with neural networks. Stock market is affected by a multiplicity of factors with different active periods, thus financial time series possess multiscale frequency characteristics, which can be exploited to facilitate prediction of stock market. In this paper, we propose a stock market prediction model combining time-frequency analysis and convolutional neural network (CNN), in which the influence extent of different frequency components has been considered. We transform original financial time series into the spectrogram reflecting time-localized frequency information by short-time Fourier transform (STFT). The 2-dimensional time-frequency feature is obtained from the spectrogram by frequency bands extraction, which is then pre-weighted and input into CNN to forecast the future price change. The frequency bands extraction and pre-weight are set according to the frequency influence. The results of experiments on Shanghai Composite Index show that the proposed model with frequency bands extraction considering frequency influence achieves a 4% relative decrease in mean absolute error (MAE) compared with that does not consider the frequency influence. Moreover, the pre-weight gives an additional 3% relative decrease of MAE.

1. Introduction

Stock market prediction is an important issue to researchers and investors, which can help to forecast future market environment and make investment decisions. It is also a challenging task because of the non-linear and fluctuated nature of stock price. Various neural networks have been applied for stock market prediction with their different non-linear modelling capabilities [1-3]. Recurrent neural network (RNN) possesses memory characteristics, which can deal with the before–after associated data [4,5]. Long short-term memory (LSTM) is capable of selectively remembering patterns for long duration of time [6,7]. CNN can extract features from original data, recognize data patterns and dependencies, which has an excellent capacity in sequent data analysis [8-10].

On the other hand, stock price series possess multiscale frequency characteristic, as financial market is affected by a multiplicity of different factors with different active periods. The impact of economic cycles on financial market may last from several months to several years. The impact of seasonal factors such as weather, supply and demand may last several months. Trading activities usually have weekly or daily patterns. Exploitation of the frequency characteristics can facilitate the stock market prediction with neural networks.

Some researchers have combined frequency decomposition with neural networks for stock market prediction. The empirical mode decomposition (EMD) and the complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) are combined with LSTM to establish a hybrid time series forecasting method which display a good performance in one-step-ahead forecasting of financial time series [11]. EMD and complete ensemble empirical mode decomposition (CEEMD) are combined with CNN-LSTM to present a hybrid algorithm which is applied to one-step-ahead prediction to enhance the prediction accuracy [12]. Wavelet transform (WT) is combined with stacked autoencoder (SAE) and LSTM to generate a hybrid model WT-SAE-LSTM for U.S. electricity prices forecasting [13]. A stock prices forecasting model combines the empirical wavelet transforms (EWT) with deep RNN is effective when evaluated on the S&P500 stock index and Mackey-Glass time series [14].

Different frequency characteristics of financial time series will play different roles in forecasting tasks with different time-scales. Daily and weekly frequency characteristics are more resulted from trading patterns. Weekly trading patterns are more significant for financial time series forecasting of 2-day than that of 1-day, while it is opposite for daily trading patterns. To our knowledge, the influence extent of different frequency components for financial time series forecasting with different time-scales has not been considered in those previous researches.

In this paper, we propose a model combining time-frequency analysis and CNN to forecast stock market, in which the different influences of frequency components have been considered. The original time series is firstly mapped to spectrogram by STFT. And the 2-dimensional time-frequency feature is extracted from the spectrogram by the filter bank considering frequency influence, which is weighted and then passed as the input to CNN to forecast the future price change. Our experiments are conducted on Shanghai Composite Index, and the results show that frequency bands extraction considering frequency influence in the proposed forecasting model achieves a 4% relative decrease in MAE, and the pre-weight gives a 3.0% relative decrease in MAE, respectively, compared with that does not consider the frequency influence. The gains from frequency bands extraction and weight assignment are complementary.

Comparing previous works combining frequency decomposition with neural networks in financial time series forecasting in [11-14], the contributions of our model combing time-frequency analysis with CNN considering frequency influence are as follows:

1) The original time series is transformed into a 2-dimensional time-frequency feature reflecting time-localized frequency information, rather than multiple 1-dimensional sub-series representing different frequencies in other works.

2) Different granularities and weights are used for different frequency components when extracting the time-frequency feature, while the sub-series of different frequencies are treated equally in other works.

3) The extracted 2-dimensional time-frequency feature is learned by a 2-dimensional CNN, thus the relationships between adjacent frequency components can be exploited for forecasting, while the sub-series of different frequencies are learned separately by multiple neural network models in other works.

The rest of this paper is as follows. In Section 2 we present and describe the fore-casting model combining time-frequency analysis and CNN. The experimental design and results are presented in Section 3. Finally, Section 4 gives the conclusion of this study.

2. Forecasting Model Combining Time-frequency Analysis and Convolutional Neural Network

In order to exploit the frequency features to forecast stock market, we propose a forecasting model combining time-frequency analysis and CNN shown in figure 1. The input of forecasting model is time series sample $[x_{t-T+1}, \cdots, x_t]$ split by a sliding window of length T. x_t is the price at time t. The output $\Delta\hat{x}_{t+K}$ is the prediction of price change of the Kth data point ahead, and the true value of the prediction is $\Delta x_{t+K} = x_{t+K} - x_t$. The proposed forecasting model mainly consists of time-frequency representation and CNN prediction.

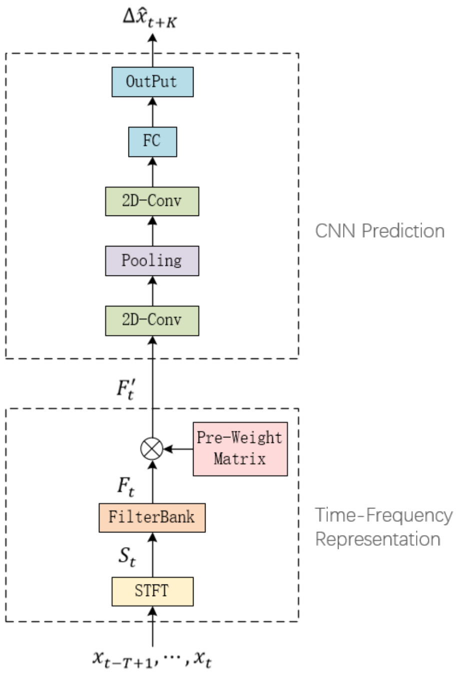

Figure 1. The forecasting model combining time-frequency analysis and CNN.

2.1. Time-frequency Representation

First of all, the original time series is mapped into time-frequency domain. STFT provides the time-localized frequency information for situations in which frequency components of a signal vary over time, whereas the standard Fourier transform provides the frequency information averaged over the entire signal time interval [15,16]. Financial time series is no-stationary signals, thus STFT is used in time-frequency representation, which can be expressed as

$$X_t(m,k) = \sum_{n=0}^{T-1} x_{n+t-T+1} w(n-mR) e^{-j2\pi\frac{k}{M}n} \tag{1}$$

where x_n is a data point of the time series $[x_{t-T+1}, \cdots, x_t]$ and $X_t(m,k)$ is the time-localized frequency information, $0 \le m \le T/R$, $0 \le k \le \mathrm{M}/2$. $w(n)$ is Hann window with length of M. R is the amount of shift for $w(n)$ called hop size. To improve the computational efficiency, STFT is implemented by calculating as the fast Fourier transform (FFT) of a series of windowed signal in which the window slides over times.

The spectrogram of time series which is the time-frequency power spectrum can be expressed as

$$S_t(m,k) = \log(|X_t(m,k)|^2) \tag{2}$$

Then, the time-frequency feature F_t is extracted from spectrogram S_t by filter bank, in which the influence extent of different frequency components for financial time series forecasting with different time-scales has been considered. Afterwards F_t are pre-weighted to obtain the 2-dimensional time-frequency feature F_t' which is the input feature of CNN prediction model. The pre-weight matrix is determined according to the frequency influence rule. It is different to other works, in which different frequency components are treated equally.

We use the filter bank comprising a set of triangular filters for frequency bands extraction, in which different bandwidth are set considering frequency influence, to make it being more discriminative at frequencies with greater influence and less discriminative at frequencies with smaller influence. The value of entries in pre-weight matrix is in proportion to the influence extent of corresponding frequency, which changes with frequency but does not change with the time.

2.2. Convolutional Neural Network (CNN) Prediction

In CNN prediction, the 2-dimensional time-frequency feature F_t' obtained in time-frequency representation is passed as the input, while the output is the future price change prediction $\Delta\hat{x}_{t+K}$. We apply a 2-dimentional CNN to forecast stock market, which consists of two convolutional layers, pooling layer, fully connected layer and output layer. Here the 2-dimensional convolutional operation is used and the relationships between adjacent frequency components can be exploited for forecasting, while different frequency components are learned separately by multiple neural network models in other works.

The two convolutional layers are used for extracting the features from the inputs, each with 32 feature maps. To keep the consistency of the size of feature maps, the boundary of the input matrix is filled by zeros before the convolution operation. We use one 3×5 time-frequency filter for each convolutional layer, and these two filters are shared across the entire time-frequency space. Weights of filters are updated during the process of model training. The non-linear transformation is used after the convolutional operation, which is realized by the Leaky ReLU activation function.

The pooling layer is mainly responsible for subsampling for feature maps, which is only used after the first convolutional layer. It not only can reduce the computational overhead caused by redundant parameters, but also is an effective means to avoid overfitting. Non-overlapping max pooling with size of 3×7 is used in our model, in which the maximum of value in the pooling window is chosen.

To generate the final results from the extracted features, a fully connected layer which has 16 hidden units is utilized. The predicted value of future price change $\Delta\hat{x}_{t+K}$ is given by the output layer which has only 1 unit.

3. Experiments and Results

To investigate the proposed forecasting model, we conduct experiments on Shanghai Composite Index in 20 years from 2000 to 2019 which is presented in the figure 2.

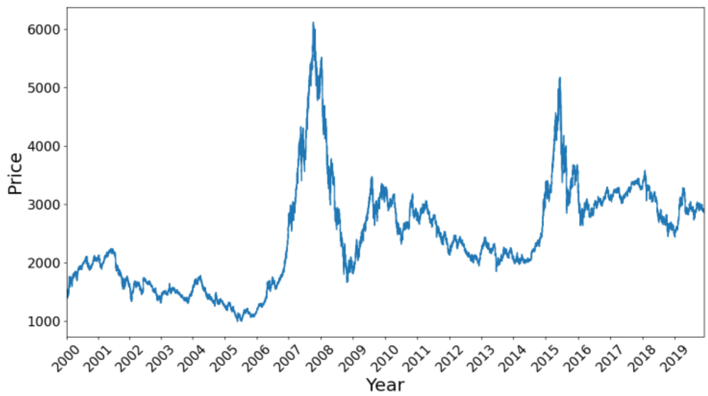

Figure 2. Shanghai Composite Index from 2000 to 2019.

We use the sampled data of Shanghai Composite Index with sampling period of 1 minute. The time series sample $[x_{t-T+1}, \cdots, x_t]$ is used to predict the price change of the Kth data point ahead $\Delta\hat{x}_{t+K}$. Here, T is set as 2400, that is, the historical data of two weeks is used for prediction. The prediction time-scales are chosen as 1-day and 2-day, this is, K is set as 240 and 480, respectively. All of the samples are split into training set, validation set and test set, which are applied for model training, selection and evaluation.

To evaluate the performance of proposed forecasting model, we use MAE as criteria. As the name suggests, MAE is an average of the absolute errors, which can be expressed as

$$MAE = 1/m \sum_{i=1}^{m} |y_i - \hat{y}_i| \tag{3}$$

where m is the sample size, $\hat{y}_i$ denotes the predicted value while y_i denotes its true value. In our study, y_i is a future price change and $\hat{y}_i$ is its prediction by the forecasting model.

3.1. Frequency Influence Investigation

In our forecasting model, the frequency influence has been considered. In order to investigate the frequency influence, the time-frequency features are passed as the input of CNN prediction and one of the frequency bands is excluded each time when training CNN. The MAE of CNN prediction reflects the influence of frequency component excluded in some extent.

The 2-dimensional time-frequency feature F_t used for frequency influence investigation is obtained from time series sample shown as figure 3(a). However, multiple 1-dimensional sub-series representing different frequencies are used in other works. The time series sample is firstly mapped to spectrogram S_t shown as figure 3(b) by STFT with the Hann window of length 1024 and hop size 16. It can be seen from figure 3(b) that the spectrogram S_t has high energy and variable characteristics in low frequency range, while in high frequency range has low energy and similar characteristics. To represent the time-frequency variation patterns more clearly with less data, we utilize a filter bank with linearly increasing bandwidth as shown in figure 4(a) instead of equal spaced bandwidth for frequency bands extraction. figure 3(c) shows time-frequency feature F_t extracted from S_t by the filter bank with linearly increasing bandwidth, which is then passed into CNN prediction without pre-weight.

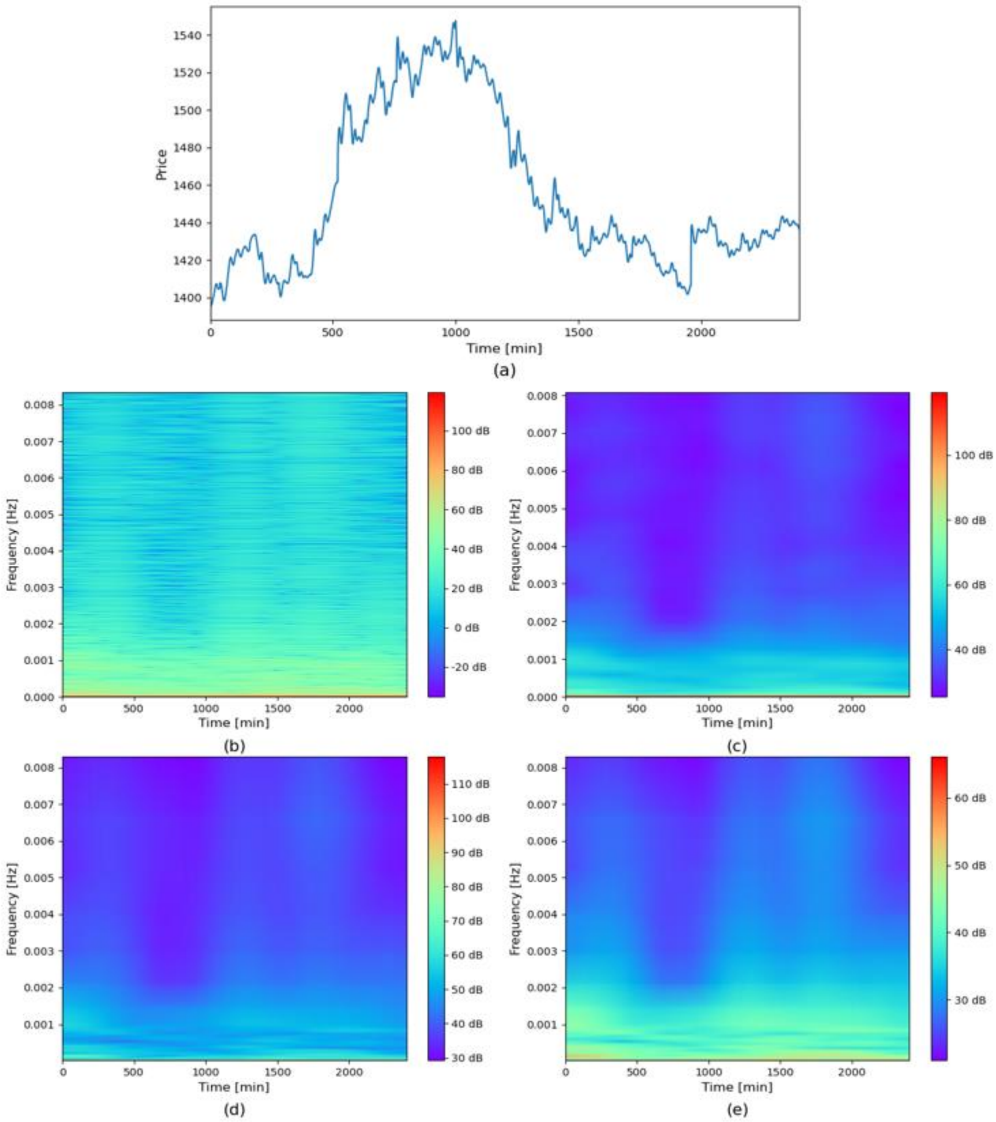

Figure 3. Time series sample and its 2D time-frequency representations. (**a**) Time series sample; (**b**) Spectrogram S_t; (**c**) 2D time-frequency feature F_t extracted by filter bank with linearly increasing bandwidth; (**d**) 2D time-frequency feature F_t extracted by filter bank considering frequency influence; (**e**) 2D time-frequency feature F_t' obtained from (**d**) by pre-weight.

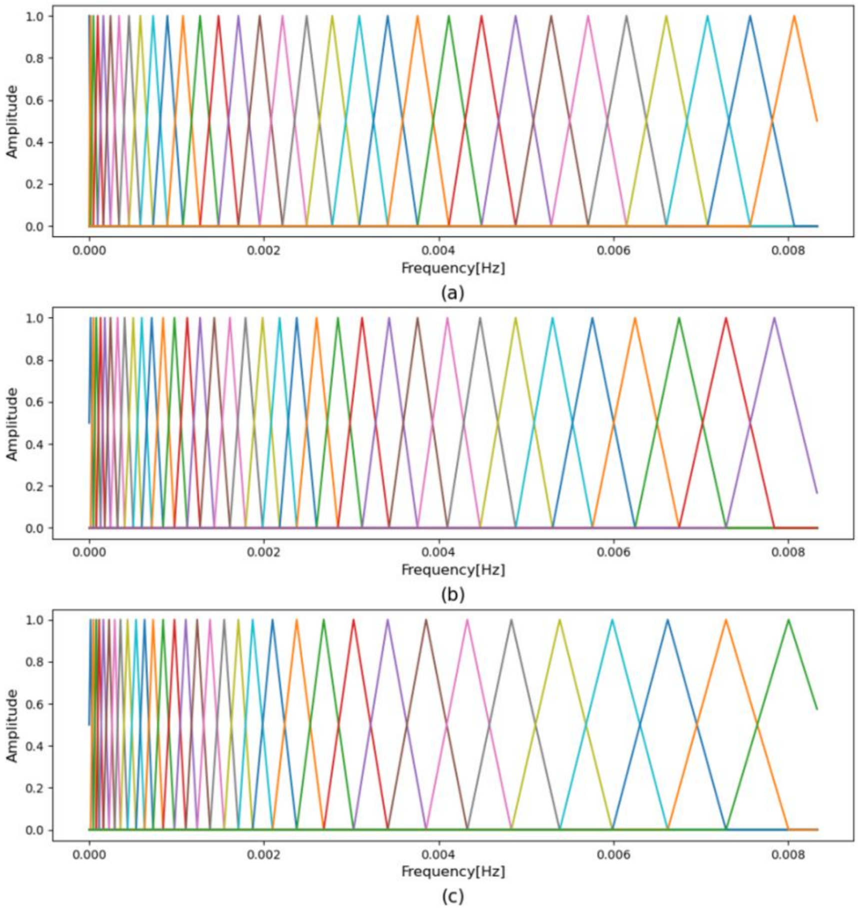

Figure 4. Filter banks for frequency bands extraction. (**a**) Filter bank with linearly increasing bandwidth; (**b**) Filter bank considering frequency influence for 1-day prediction; (**c**) Filter bank considering frequency influence for 2-day prediction.

Figure 5 shows the MAE of CNN prediction on validation set for 1-day, 2-day and 4- day prediction, where one of the frequency bands is excluded each time. To better reveal the variation patterns of frequency influence, the linear fitting of experiment results is also given in figure 5. It can be clearly seen in this figure that the MAE decreases as the frequency of discarded band increases for predictions with all three different time-scales. And the MAE of prediction with longer time-scale decreases faster with frequency increasing than that with shorter ones. Different frequency components of financial time series play different roles in forecasting with different time-scales. This suggests that the frequency influence should be considered for stock market prediction.

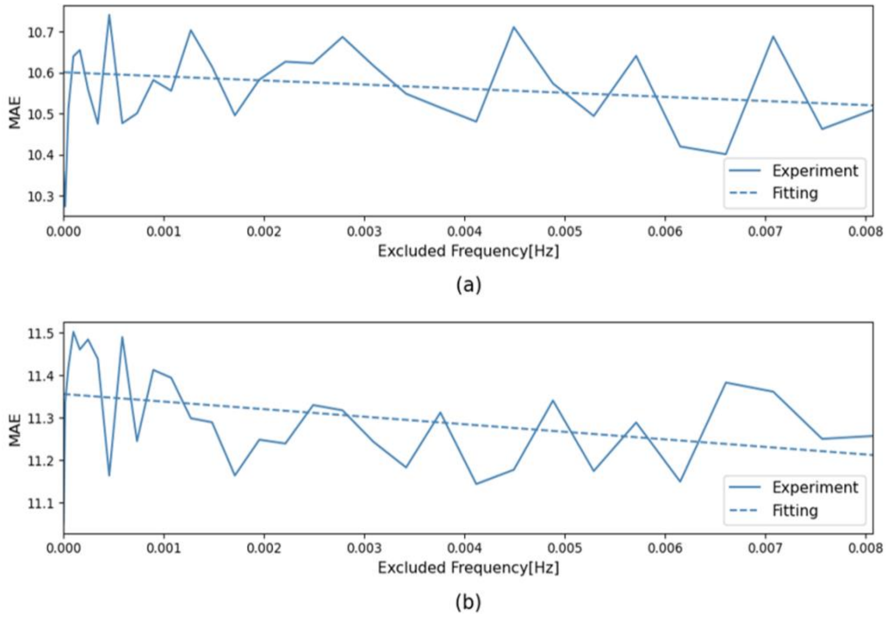

Figure 5. MAE of the CNN prediction varies with frequency excluded. (**a**) 1-day prediction; (**b**) 2-day prediction.

In order to exploit the frequency features to predict stock market, both of the filter bank and the pre-weight matrix in time-frequency representation of our model is determined according to the frequency influence rule.

Figure 4(b) shows the filter bank designed for 1-day prediction, while figure 4(c) show that for 2-day prediction. Filters in low frequency range have narrower bandwidth, while those in high frequency range have wider bandwidth. It is due to heavier influence of low bands and weaker influence of high frequency bands for future price change prediction. Besides, filters in high frequency range have wider bandwidth in the filter bank for 2-day prediction than that for 1-day prediction, as high frequency bands have weaker influence on longer time-scale forecasting than on shorter one. Figure 3(d) gives an example of time-frequency feature F_t extracted from S_t by the filter bank considering frequency influence for 1-day prediction.

We conduct the pre-weight to F_t, and the time-frequency feature F_t' shown in figure 3(e) is obtained as the input of CNN prediction in our model. The value of entries in pre-weight matrix is in proportion to the influence extent of corresponding frequency, this is, greater weights are given to frequency bands that have heavier influence and less weights are given to those have weaker influence.

3.2. Price Change Prediction

We perform experiments on Shanghai Composite Index to predict the future price change. The frequency influence is considered in proposed forecasting model. The filter banks for frequency bands extraction are shown as figure 4(b)(c) for 1-day and 2-day prediction. Pre-weight mentioned before is also used.

Table 1 shows the performance of proposed forecasting model on prediction time-scales of 1-day and 2-day, which is evaluated by MAE. As can be seen from the table, comparing with prediction which does not consider frequency influence, the frequency bands extraction considering frequency influence in the proposed forecasting model achieves a 3.9% and 4.1% relative decrease in MAE for 1-day and 2-day prediction. The pre-weight considering frequency influence gives a 3.6% and 3.0% relative decrease in MAE. The MAE of proposed forecasting model with both frequency bands extraction and pre-weight has decreased by about 6.8% in total for both 1-day and 2-day prediction. This indicates that frequency influence consideration in forecasting can improve the performance and the gains from frequency bands extraction and pre-weight are complementary.

Table 1. MAE of proposed forecasting model with different time-frequency representation methods for different time-scales pre-diction.

Time-frequency Representation Methods	Time-scale of Prediction	MAE	Relative Decrease in MAE
Frequency bands extraction without frequency influence consideration, no Pre-weight	1-day	10.5899	-
	2-day	11.2810	-
Frequency bands extraction considering frequency influence, no Pre-weight	1-day	10.1795	3.9%
	2-day	10.8198	4.1%
Frequency bands extraction without frequency influence consideration, Pre-weight	1-day	10.2086	3.6%
	2-day	10.9394	3.0%
Frequency bands extraction considering frequency influence, Pre-weight	1-day	**9.8692**	6.8%
	2-day	**10.5125**	6.8%

4. Conclusions

In this paper, we propose a forecasting model combining time-frequency analysis and CNN to exploit time-frequency features for stock market prediction, in which the influence extent of different frequency components has been considered. To investigate the proposed forecasting model, we perform experiments on Shanghai Composite Index in 20 years to forecast the future price change on different prediction time-scales.

We firstly investigate the influence of different frequency components in stock market prediction on different prediction time-scales. We find that higher frequency bands have smaller influence while lower bands have more powerful influence, and higher frequency bands have much weaker influence when forecasting on longer time-scale. Then, considering the frequency influence, the time series are converted to 2-dimensional time-frequency features within the time-frequency representation which assigns different granularities and weights for different frequency bands. Finally, the extracted 2-dimensional features are applied for CNN modelling to forecast future price change.

Results of experiments show that frequency bands extraction considering frequency influence in the proposed forecasting model achieves a 4% relative decrease in MAE, and the pre-weight gives a 3.0% relative decrease in MAE, respectively, compared with that does not consider the frequency influence. And the MAE of proposed forecasting model with both frequency bands extraction and pre-weight has decreased by about 6.8% in total. This demonstrates that considering frequency influence in forecasting can improve its performance and the gains from frequency bands extraction and pre-weight are complementary.

The proposed model which exploit time-frequency features considering frequency influence can be studied for its potentials and applied in other fields as well.

References

[1] Guresen E, Kayakutlu G and Daim T U 2011 Using artificial neural network models in stock market index prediction *Expert Systems with Applications* **38** 10389–97

[2] Ma W, Wang Y and Dong N 2010 Study on stock price prediction based on BP Neural Network *2010 IEEE International Conference on Emergency Management and Management Sciences (IEEE)*

[3] Persio L and Honchar O 2016 Artificial Neural Networks architectures for stock price prediction: comparisons and applications *International Journal of Circuits, Systems and Signal Processing* **10** 403–13

[4] Xie X-K and Wang H 2016 Recurrent neural network for forecasting stock market trend *International Conference on Computer Science, Technology and Application (CSTA2016)* (Changsha, China: WORLD SCIENTIFIC) pp 397–402

[5] Anbazhagan S and Kumarappan N 2013 Day-ahead deregulated electricity market price forecasting using recurrent neural network *IEEE Systems Journal* **7** 866–72

[6] Bao W, Yue J and Rao Y 2017 A deep learning framework for financial time series using stacked autoencoders and long-short term memory ed B Podobnik *PLoS ONE* **12** e0180944

[7] Nelson D M Q, Pereira A C M and de Oliveira R A 2017 Stock market's price movement prediction with LSTM neural networks *2017 International Joint Conference on Neural Networks (IJCNN)* (Anchorage, AK, USA: IEEE) pp 1419–26

[8] Sezer O B and Ozbayoglu A M 2018 Algorithmic financial trading with deep convolutional neural networks: Time series to image conversion approach *Applied Soft Computing* **70** 525–38

[9] Cao J and Wang J 2019 Stock price forecasting model based on modified convolution neural network and financial time series analysis *Int J Commun Syst* **32** e3987

[10] Hoseinzade E and Haratizadeh S 2019 CNNpred: CNN-based stock market prediction using a diverse set of variables *Expert Systems with Applications* **129** 273–85

[11] Cao J, Li Z and Li J 2019 Financial time series forecasting model based on CEEMDAN and LSTM *Physica A: Statistical Mechanics and its Applications* **519** 127–39

[12] Rezaei H, Faaljou H and Mansourfar G 2021 Stock price prediction using deep learning and frequency decomposition *Expert Systems with Applications* **169** 114332

[13] Qiao W and Yang Z 2020 Forecast the electricity price of U.S. using a wavelet transform-based hybrid model *Energy* **193** 116704

[14] Althelaya K A, Mohammed S A and El-Alfy E-S M 2021 Combining deep learning and multiresolution analysis for stock market forecasting *IEEE Access* **9** 13099–11

[15] Kehtarnavaz N 2008 Frequency domain processing *Digital Signal Processing System Design* (Elsevier) pp 175–96

[16] Rajoub B 2020 Characterization of biomedical signals: Feature engineering and extraction *Biomedical Signal Processing and Artificial Intelligence in Healthcare* (Elsevier) pp 29–50