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Sec : 12

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(35)

$f(x) = x/2$; $[1,4]$; right end points

Sol^n:

The length of each subinterval, $\Delta x = \frac{4-1}{n} = \frac{3}{n}$

∴ The right end point of each sub-interval:

$$x_k = a + k\Delta x$$
$$= 1 + k \cdot \frac{3}{n} = 1 + \frac{3k}{n}$$

$$f(x) = x/2$$
$$\Rightarrow f(x_k) = \frac{x_k}{2}$$
$$= \frac{1 + \frac{3k}{n}}{2}$$
$$= \frac{1}{2} + \frac{3k}{2n}$$

Now,

$$\sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \sum_{k=1}^{n} \left[\frac{1}{2} + \frac{3k}{2n}\right] \cdot \frac{3}{n}$$

$$= \sum_{k=1}^{n} \frac{3}{2n} + \frac{3 \cdot 3k}{2n^2}$$

$$= \frac{3}{2} \cdot \sum_{k=1}^{n} \frac{1}{n} + \frac{3k}{n^2}$$

$$= \frac{3}{2}\left[\sum_{k=1}^{n} \frac{1}{n} + \sum_{k=1}^{n}\left[\frac{3k}{n^2}\right]\right]$$

$$= \frac{3}{2}\left[1 + \frac{3}{n^2}\sum_{k=1}^{n} k\right]$$

$$= \frac{3}{2}\left[1 + \frac{3}{n^2} \cdot \left(\frac{1}{2} \cdot n(n+1)\right)\right]$$

$$= \frac{3}{2}\left[1 + \frac{3}{2n}(n+1)\right]$$

$$\therefore \text{Area} = \lim_{n\to\infty} \frac{3}{2}\left[1 + \frac{3}{2n}(n+1)\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1 + \frac{3}{2}\left(1 + \frac{1}{n}\right)\right]$$

$$= \frac{3}{2}\lim_{n\to\infty}\left[1 + \frac{3}{2} + \frac{3}{2n}\right]$$

$$= \frac{3}{2}\left[\lim_{n\to\infty} 1 + \lim_{n\to\infty}\frac{3}{2} + \lim_{n\to\infty}\frac{3}{2n}\right]$$

$$= \frac{3}{2}\left[1 + \frac{3}{2} + 0\right]$$

$$= \frac{3}{2} + \frac{3 \cdot 3}{4}$$

$$= \frac{9}{4} + \frac{3}{2} = \frac{9+6}{4} = \frac{15}{4} \text{ (Ans)}$$

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(40)

$f(x) = 1 - x^3$; $[-3, -1]$; right end point.

Sol^n:

The length of each subinterval, $\Delta x = \frac{-1+3}{n} = \frac{2}{n}$

The right end point of each subinterval,

$$x_k = a + k \cdot \Delta x$$
$$= -3 + k \cdot \frac{2}{n}$$
$$= -3 + \frac{2k}{n}$$

$$f(x) = 1 - x^3$$
$$\therefore f(x_k) = 1 - (x_k)^3$$
$$= 1 - \left(-3 + \frac{2k}{n}\right)^3$$
$$= 1 + \left(3 - \frac{2k}{n}\right)^3$$
$$= 1 + \left\{3^3 - \left(\frac{2k}{n}\right)^3 - 3.3.\frac{2k}{n}\left(3 - \frac{2k}{n}\right)\right\}$$
$$= 1 + \left\{27 - \frac{8k^3}{n^3} - 9.\frac{2k}{n}\left(3 - \frac{2k}{n}\right)\right\}$$
$$= 1 + \left\{27 - \frac{8k^3}{n^3} - \frac{18k}{n}\left(3 - \frac{2k}{n}\right)\right\}$$
$$= 1 + 27 - \frac{8k^3}{n^3} - \frac{54k}{n} + \frac{36k^2}{n^2}$$
$$= 28 - \frac{54k}{n} + \frac{36k^2}{n^2} - \frac{8k^3}{n^3}$$

$$\therefore f(x_k) \cdot \Delta x$$
$$= \frac{2}{n}\left[28 - \frac{54k}{n} + \frac{36k^2}{n^2} - \frac{8k^3}{n^3}\right]$$

$$\therefore \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$
$$= \sum_{k=1}^{n} \frac{2}{n}\left[28 - \frac{54k}{n} + \frac{36k^2}{n^2} - \frac{8k^3}{n^3}\right]$$
$$= \frac{2}{n} \cdot \left[\sum_{k=1}^{n} 28 - \frac{54}{n}\sum_{k=1}^{n} k + \frac{36}{n^2}\sum_{k=1}^{n} k^2 - \frac{8}{n^3}\sum_{k=1}^{n} k^3\right]$$

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$$= \frac{2}{n}\left[28n - \frac{54}{n} \cdot \frac{n(n+1)}{2} + \frac{36}{n^2} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{8}{n^3} \cdot \left(\frac{n(n+1)}{2}\right)^2\right]$$

$$= \frac{2}{n}\left[28n - 27(n+1) + \frac{6(n+1)(2n+1)}{n} - 2 \cdot \frac{(n+1)^2}{n}\right]$$

Now,

$$\text{Area} = 2\lim_{n \to \infty^+}\left[28 - 27\left(1+\frac{1}{n}\right) + 6\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 2\left(1+\frac{1}{n}\right)^2\right]$$

$$= 2(28 - 27 + 12 - 2) = 22$$

(41)

$f(x) = x/2$; $[1, 4]$; left end point

Soln:

the length of each subinterval, $\Delta x = \frac{4-1}{n} = \frac{3}{n}$

The left end point of each subinterval, $x_k = a + k\,\Delta x$

$$= 1 + (k-1)\frac{3}{n}$$

$$= 1 + \frac{3(k-1)}{n}$$

$f(x) = x/2$

$\therefore f(x_k) = \frac{x_k}{2}$

$$= \frac{1 + \frac{3(k-1)}{n}}{2}$$

$\therefore f(x_k) \cdot \Delta x$

$$= \frac{1}{2} \cdot \left(1 + \frac{3(k-1)}{n}\right) \cdot \frac{3}{n}$$

$$= \frac{1}{2}\left[\frac{3}{n} + \frac{9}{n^2}(k-1)\right]$$

$\therefore \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$$= \sum_{k=1}^{n} \frac{1}{2}\left[\frac{3}{n} + \frac{9}{n^2}(k-1)\right] = \frac{1}{2}\sum_{k=1}^{n}\left[\frac{3}{n} + \frac{9}{n^2}(k-1)\right]$$

$$= \frac{1}{2}\left[3\sum_{k=1}^{n}\frac{1}{n} + \frac{9}{n^2}\sum_{k=1}^{n}(k-1)\right]$$

$$= \frac{1}{2}\left[3 + \frac{9}{n^2} \cdot \frac{1}{2}(n-1)n\right]$$

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$= \frac{3}{2} + \frac{9}{4} \cdot \frac{n-1}{n}$

$\therefore \text{Area} = \lim_{n \to \infty^+} \left[\frac{3}{2} + \frac{9}{4} \cdot \frac{n-1}{n}\right]$

$= \lim_{n \to \infty^+} \left[\frac{3}{2} + \frac{9}{4}\left(1 - \frac{1}{n}\right)\right]$

$= \lim_{n \to \infty^+} \left[3/2 + 9/4 \left(1 - \frac{1}{n}\right)\right]$

$= \frac{3}{2} + \frac{9}{4} = \frac{15}{4}$ (Ans)

(47) $f(x) = x^2$; $[0, 1]$, mid point.

Sol^n:

The length of each subinterval, $\Delta x = \frac{1-0}{n} = \frac{1}{n}$

The ~~mid~~ mid end point of each subinterval,

$x_K = a + (K - \frac{1}{2}) \cdot \Delta x$

$= 0 + (K - \frac{1}{2}) \cdot \frac{1}{n}$

$= \frac{K - \frac{1}{2}}{n} = \frac{2K-1}{2n}$

$f(x) = x^2$

$\therefore f(x_K) = x_K^2$

$= \frac{(K - 1/2)^2}{n^2}$

$= \left(\frac{2K-1}{2n}\right)^2$

$\therefore f(x_K) \cdot \Delta x$

$= \left(\frac{2K-1}{2n}\right)^2 \cdot \frac{2}{n} = \frac{K^2}{n^3} - \frac{K}{n^3} + \frac{1}{4n^3}$

$\therefore \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$= \frac{1}{n^3} \sum_{k=1}^{n} k + \frac{1}{4n^3} \sum_{k=1}^{n} 1 .$

$\therefore \text{Area} = \lim_{n \to \infty^+} \left(\frac{1}{n^3} \sum_{k=1}^{n} k + \frac{1}{4n^3} \sum_{k=1}^{n} 1 \right)$

$= \frac{1}{3} + 0 + 0 = \frac{1}{3}$. (Ans)

(27)

$f(x) = 3x + 1 \quad ; [2, 6]$

$n = 4$

$\therefore \Delta x = \frac{b-a}{n} = \frac{6-2}{4} = 1$

a) left end points are, $x_1 = a + (k-1)\Delta x$

$= 2 + (1-1) \cdot \Delta x$

$= 2 + 0 = 2$

$x_2 = 3, \; x_3 = 4, \; x_4 = 5$

$\sum_{k=1}^{4} f(x_k) \cdot \Delta x = [f(x_1) + f(x_2) + f(x_3) + f(x_4)] \cdot \Delta x$

$= 46$

b) mid end points, $x_1 = a + (k - \frac{1}{2}) \Delta x$

$= 2 + (1 - \frac{1}{2}) \cdot 1 = 5/2$

$x_2 = 7/2$

$x_3 = 9/2$

$x_4 = 11/2$

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$$\sum_{k=1}^{4} f(x_k)\cdot \Delta x = [f(x_1)+f(x_2)+f(x_3)+f(x_4)]\cdot \Delta x$$
$$= 52$$

c) Right end points, $x_1 = a + k\Delta x$
$$= 2+1 = 3$$
$$x_2 = 4$$
$$x_3 = 5$$
$$x_4 = 6$$

$$\sum_{k=1}^{n} f(x_k)\cdot \Delta x = [f(x_1)+f(x_2)+f(x_3)+f(x_4)]\cdot \Delta x$$
$$= 10+13+16+19 = 58$$

(28)
$f(x) = \frac{1}{x}$; $[1,9]$

$n = 4$
$\therefore \Delta x = \frac{9-1}{4} = 2$

a) left end points, $x_k = a + (k-1)\cdot\Delta x$
$$\therefore x_1 = 1+(1-1)\cdot 2$$
$$= 1$$
$$\therefore x_2 = 3$$
$$\therefore x_3 = 5$$
$$\therefore x_4 = 7$$
$$\therefore \sum_{k=1}^{n} f(x_k)\cdot \Delta x = [f(x_1)+f(x_2)+f(x_3)+f(x_4)]\cdot \Delta x$$
$$= \left[1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\right]\cdot 2 = \frac{352}{105}$$

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b) Mid end points , $x_k = a + (k - \frac{1}{2}) \cdot \Delta x$

$\therefore x_1 = 2$

$\therefore x_2 = 4$

$\therefore x_3 = 6$

$\therefore x_4 = 8$

$\therefore \sum_{k=1}^{4} f(x_k) \cdot \Delta x = [f(x_1) + f(x_2) + f(x_3) + f(x_4)] \cdot \Delta x$

$= (\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}) \cdot 2$

$= \frac{25}{12}$

c) Right end points , $x_k = a + k \Delta x$

$\therefore x_1 = 3$

$\therefore x_2 = 5$

$\therefore x_3 = 7$

$\therefore x_4 = 9$

$\therefore \sum_{k=1}^{4} f(x_k) \cdot \Delta x = [f(x_1) + f(x_2) + f(x_3) + f(x_4)] \cdot \Delta x$

$= [\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}] \cdot 2$

$= \frac{496}{315}$

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(29) $f(x) = \cos x \; ; \; [0, \pi]$

$n = 4$

$\Delta x = \frac{\pi - 0}{4} = \pi/4$

(a) left end points, $x_k = a + (k-1) \cdot \Delta x$

$\Rightarrow x_1 = 0 + 0 = 0$

$\Rightarrow x_2 = \pi/4$

$\Rightarrow x_3 = \pi/2$

$\Rightarrow x_4 = \frac{3}{4}\pi$

$\therefore \sum_{k=1}^{n} f(x_k) \cdot \Delta x = \left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right] \cdot \Delta x$

$= \left[1 + \frac{1}{\sqrt{2}} + 0 + \left(-\frac{1}{\sqrt{2}}\right)\right] \cdot \pi/4 \quad = \pi/4$

(b) Mid end points, $x_k = a + (k - \frac{1}{2}) \cdot \Delta x$

$\Rightarrow x_1 = \pi/8$

$\Rightarrow x_2 = \frac{3\pi}{8}$

$\Rightarrow x_3 = \frac{5\pi}{8}$

$\Rightarrow x_4 = \frac{7\pi}{8}$

$\therefore \sum_{k=1}^{n} f(x_k) \cdot \Delta x = \left[\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8}\right] \cdot \pi/4$

$= 0$

ⓒ Right end points, $x_k = a + k\Delta x$

$\Rightarrow x_1 = \pi/4$

$\Rightarrow x_2 = \pi/2$

$\Rightarrow x_3 = \frac{3\pi}{4}$

$\Rightarrow x_4 = \pi$

$\therefore \sum_{k=1}^{4} f(x_k).\Delta x = \left(\frac{1}{\sqrt{2}} + 0 - \frac{1}{\sqrt{2}} - 1\right).\pi/4$

$= -\pi/4$

(30) $f(x) = 2x - x^2$; $[-1, 3]$;

$n = 4$, $\Delta x = \frac{3+1}{4} = 1$

ⓐ left end point, $x_k = a + (k-1).\Delta x$

$= -1 + (k-1).1$

$= -1 + k - 1$

$= k - 2$

$\therefore x_1 = -1$

$\therefore x_2 = 0$

$\therefore x_3 = 1$

$\therefore x_4 = 2$

$\therefore \sum_{k=1}^{4} f(x_k).\Delta x = -3 + 0 + 1 + 0 = -2$

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(b) Mid end points, $x_k = a+(k-\frac{1}{2}).\Delta x$

$$\Rightarrow x_k = -1+(k-\tfrac{1}{2}).1$$

$$\therefore x_1 = -\tfrac{1}{2}$$

$$\therefore x_2 = \tfrac{1}{2}$$

$$\therefore x_3 = \tfrac{3}{2}$$

$$\therefore x_4 = 5/2$$

$$\therefore \sum_{k=1}^{n} f(x_k).\Delta x = -\tfrac{5}{4}+\tfrac{3}{4}+\tfrac{3}{4}-\tfrac{5}{4}$$

$$= -1$$

(c) Right end points, $x_k = a+k\Delta x$

$$= -1+k$$

$$\therefore x_1 = 0$$

$$\therefore x_2 = 1$$

$$\therefore x_3 = 2$$

$$\therefore x_4 = 3$$

$$\therefore \sum_{k=1}^{n} f(x_k).\Delta x = 0+1+0-3 = -2$$

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④

$$\int_{-1}^{+\infty} \frac{x}{x^2+1}\, dx$$

$$= \lim_{t \to +\infty} \int_{-1}^{t} \frac{x}{1+x^2}\, dx$$

let,
$1+x^2 = p$
$\Rightarrow \frac{dp}{dx} = 2x$
$\Rightarrow \frac{dp}{2} = x\,dx$

$$= \lim_{t \to +\infty} \int_{-1}^{t} \frac{\frac{dp}{2}}{p}$$

$$= \frac{1}{2} \lim_{t \to \infty^+} \int_{-1}^{t} \frac{1}{p}\, dp$$

$$= \frac{1}{2} \lim_{t \to \infty^+} \left[\ln|p|\right]_{-1}^{t}$$

$$= \frac{1}{2} \lim_{t \to \infty^+} \left[\ln|1+x^2|\right]_{-1}^{t}$$

$$= \frac{1}{2} \lim_{t \to \infty^+} \left[\ln|1+t^2| - \ln 2\right]$$

$= +\infty$; divergent

⑦ $\int_{e}^{\infty^+} \frac{1}{x \ln^3 x}\, dx$

let,
$\ln x = p$
$\Rightarrow \frac{1}{x} = \frac{dp}{dx}$
$\Rightarrow \frac{1}{x}\, dx = dp$

Now,

$$\int \frac{1}{p^3}\, dp$$

$$= \int p^{-3}\, dp$$

$$= \frac{p^{-2}}{-2} = -\frac{1}{2} \cdot \frac{1}{(\ln x)^2} = \frac{-1}{2\ln^2 x}$$

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$$\therefore \int_e^{\infty^+} \frac{1}{x\ln^3 x}\,dx$$

$$= \lim_{t\to\infty^+} -\frac{1}{2\ln^2 x}\Big]_e^t$$

$$= \lim_{t\to+\infty}\left[-\frac{1}{2\ln^2 t}+\frac{1}{2}\right]$$

$$= \frac{1}{2}$$; convergent

(8) $$\int_2^{+\infty} \frac{1}{x\sqrt{\ln x}}\,dx$$

$$\therefore \int \frac{1}{x\sqrt{\ln x}}\,dx$$

let, $\ln x = p$

$\Rightarrow \frac{dp}{dx} = \frac{1}{x}$

$\Rightarrow \frac{1}{x}dx = dp$

$$= \int \frac{1}{\sqrt{p}}\,dp$$

$$= \int p^{-1/2}\,dp$$

$$= \frac{p^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{p^{1/2}}{1/2} = 2p^{\frac{1}{2}} = 2\sqrt{p} = 2\sqrt{\ln x}$$

$$\therefore \int_2^{+\infty} \frac{1}{x\sqrt{\ln x}}\,dx$$

$$= \lim_{t\to+\infty}\left[2\sqrt{\ln x}\right]_2^t$$

$$= \lim_{t\to\infty}\left[2\sqrt{\ln t} - 2\sqrt{\ln 2}\right]$$

$= +\infty$; divergent

(12) $\int_{-\infty}^{0} \frac{e^{x}\,dx}{3-2e^{x}}$

Now,

$\int \frac{e^{x}dx}{3-2e^{x}}$

let,

$3-2e^{x}=P$

$\Rightarrow \frac{dP}{dx}=0-2\cdot e^{x}$

$\Rightarrow \frac{dP}{dx}=-2e^{x}$

$\Rightarrow \frac{dP}{-2}=e^{x}dx$

$=-\frac{1}{2}\int \frac{dP}{P}$

$=-\frac{1}{2}\cdot \ln|P| = -\frac{1}{2}\ln|3-2e^{x}|$

Now,

$\int_{-\infty}^{0} \frac{e^{x}dx}{3-2e^{x}}$

$=\lim_{t\to-\infty}\left[-\frac{1}{2}\ln|3-2e^{x}|\right]_{t}^{0}$

$=\lim_{t\to-\infty}\left[-\frac{1}{2}\ln|3-2|+\frac{1}{2}\ln|3-2e^{t}|\right]$

$=\lim_{t\to-\infty}-\frac{1}{2}\ln|3-2e^{t}|$

$=-\frac{1}{2}\cdot\ln|3-2\cdot e^{-\infty}|$

$=-\frac{1}{2}\ln 3$ Ans

$=$ convergent

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(16)

$$\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$\therefore \int \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$= \int \frac{-dp}{1+p^2}$$

$$= -\tan^{-1}(p)$$

$$= -\tan^{-1}(e^{-t})$$

let,

$$e^{-t} = p$$

$$\Rightarrow \frac{dp}{dt} = \frac{d}{dt}(e^{-t})$$

$$= e^{-t} \cdot -1$$

$$= -(e^{-t})$$

$$\Rightarrow dp = -e^{-t}\,dt$$

$$\Rightarrow -dp = e^{-t}\,dt$$

$$\therefore \lim_{m\to\infty^+} \int_0^m \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$= \lim_{m\to\infty^+} \left[-\tan^{-1}(e^{-t})\right]_0^m$$

$$= \lim_{m\to\infty^+} \left[-\tan^{-1}(e^{-m}) + \tan^{-1} 0\right]$$

$$= \pi/4 \quad \text{—— (I)}$$

And,

$$\lim_{l\to\infty^-} \int_l^0 \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$= \lim_{l\to\infty^-} \left[-\tan^{-1} e^{-t}\right]_l^0$$

$$= \lim_{l\to\infty^-} \left[-\tan^{-1} e^0 + \tan^{-1} e^{-l}\right]$$

$$= \pi/4 \quad \text{—— (II)}$$

$\therefore$ eq(I) + eq(II),

$$\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt$$

$$= \pi/4 + \pi/4 = \pi/2 \; ;$$

convergent

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(23)

$$\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\, dx$$

Now,

$$\int \frac{\sin x}{\sqrt{1-2\cos x}}\, dx$$

let,

$1-2\cos x = p$

$\Rightarrow +\frac{1}{2} dp = \sin x\, dx$

$$= +\frac{1}{2}\int \frac{dp}{\sqrt{p}}$$

$$= +\frac{1}{2}\left[\left(\frac{1}{2}\right)^{-1} \cdot p^{\frac{1}{2}}\right]$$

$$= +\frac{1}{2} \cdot 2 \cdot \sqrt{p}$$

$$= +\sqrt{p}$$

$$= +\sqrt{1-2\cos x}$$

Now,

$$\lim_{t \to \pi/3^+} \sqrt{1-2\cos x}\Big]_t^{\pi/2}$$

$$= \lim_{t \to \pi/3^+}\left[\sqrt{1-2\cos \pi/2} - \sqrt{1-2\cos t}\right]$$

$$= \sqrt{1} - \sqrt{1-2\cdot\frac{1}{2}}$$

$$= 1 - \sqrt{1-1}$$

$= 1 - 0 = 1$; convergent

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(24)

$$\int_0^{\pi/4} \frac{\sec^2 x}{1-\tan x}\,dx$$

Now,

$$\int \frac{\sec^2 x\,dx}{1-\tan x}$$

Let,

$1-\tan x = p$

$\Rightarrow \frac{dp}{dx} = 0-\sec^2 x$

$\Rightarrow -dp = \sec^2 x\,dx$

$$= -\int \frac{dp}{p}$$

$$= -\ln p$$

$$= -\ln|1-\tan x|$$

Now,

$$\lim_{m\to \pi/4^+} \Big[-\ln|1-\tan x|\Big]_0^m$$

$$= \lim_{m\to\pi/4^+} \Big[-\ln|1-\tan m| + \ln|1-\tan 0|\Big] = \infty + \ln|1|$$

$= +\infty$; divergent.

(36)

$$\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$$

Now, $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x$

Now,

$\therefore \int_t^{a} \frac{dx}{x\sqrt{x^2-1}} + \int_a^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$, where $a>1$, let we take $a=2$.

$$\therefore \int_1^{2} \frac{dx}{x\sqrt{x^2-1}} = \lim_{t\to 1^+}\left[\sec^{-1}x\right]_t^2$$

$$\lim_{t\to +1} \sec^{-1}2 - \sec^{-1}t$$

$$= \pi/3$$

And, $\int_2^{+\infty} \frac{dx}{x\sqrt{x^2-1}} = \lim_{t\to\infty^+} \sec^{-1}x\Big]_2^t$

$$= \pi/2 - \pi/3$$

$$\therefore \int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}} = \pi/3 + \pi/2 - \pi/3$$

$= \pi/2$; convergent Ans

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$$\int_0^1 \frac{dx}{\sqrt{x}(x+1)}$$

let,

$\sqrt{x} = p$

$\Rightarrow p^2 = x$

$\Rightarrow \frac{dp}{dx} = \frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dp}{dx} = \frac{1}{2p}$

$\Rightarrow 2p\,dp = dx$

Now,

$2\int \frac{p\,dp}{p(p^2+1)}$

$= 2\int \frac{dp}{p^2+1}$

$= 2\tan^{-1}p + c$

$= 2\tan^{-1}\sqrt{x} + c$

Now,

$\int_0^1 \frac{dx}{\sqrt{x}(x+1)}$

$= 2\lim_{l\to 0^+} \left.\tan^{-1}\sqrt{x}\right]_l^1 = 2\left[\tan^{-1}1 - \tan^{-1}\sqrt{l}\right]$

$= 2\left[\tan^{-1}1 - \tan^{-1}\sqrt{0}\right] = 2\left[\pi/4 - 0\right]$

$= 2.\pi/4$

$= \pi/2$; convergent.

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$$\int_0^{+\infty} \frac{dx}{\sqrt{x}(x+1)}$$

Now,

$$\int \frac{dx}{\sqrt{x}(x+1)}$$

$$= 2\int \frac{p\,dp}{p(p^2+1)}$$

$$= 2\int \frac{dp}{p^2+1}$$

$$= 2\cdot \tan^{-1} p$$

$$= 2\tan^{-1}\sqrt{x}$$

let,

$\sqrt{x} = p$

$\Rightarrow x = p^2$

$\frac{1}{2\sqrt{x}} = \frac{dp}{dx}$

$\Rightarrow \frac{1}{2p} = \frac{dp}{dx}$

$\Rightarrow dp\cdot 2p = dx$

$\Rightarrow dx = 2p\,dp$

Now,

$$\int_1^{+\infty} \frac{dx}{\sqrt{x}(x+1)} = \lim_{a\to\infty^+}\left[2\tan^{-1}\sqrt{x}\right]_1^a$$

$$= \lim_{a\to\infty^+}\left[2\tan^{-1}\sqrt{a} - 2\tan^{-1}\sqrt{1}\right]$$

$$= 2\cdot\pi/2 - 2\cdot\pi/4$$

$$= \pi - \pi/2 = \pi/2$$

$$\int_0^1 \frac{dx}{\sqrt{x}(x+1)} = 2\lim_{a\to 0^+}\left[\tan^{-1}\sqrt{x}\right]_a^1$$

$$= 2\lim_{a\to 0^+}\left[\tan^{-1}\sqrt{1} - \tan^{-1}\sqrt{a}\right]$$

$$= 2\left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= 2\cdot\pi/4$$

$$= \pi/2$$

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$$\therefore \int_0^{+\infty} \frac{dx}{\sqrt{x}(x+1)}$$

$$= \pi/2 + \pi/2$$

$$= \pi \; ; \text{ convergent (Ans)}$$

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