Spatial Analysis

```
L = 0.01;
N_L = 100;
mu_0 = pi*4e-7;
eps_0 = 8.85418782e-12;
omega = 2*pi*(10^(10));
Z_L = 1e2;
vp = 0.5e8;
z = linspace(0, L, N_L)
```

```
z = 1×100
         0    0.0001    0.0002    0.0003    0.0004    0.0005    0.0006    0.0007 ···
```

Derived Solution

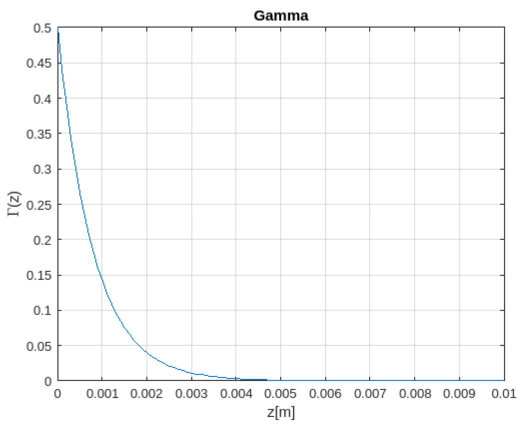
```
E = omega/vp;
```

$\Gamma(z) = \frac{1}{2}e^{-\omega/v_p z}$

```
Gamma = 0.5*exp(-E.*z);
dGamma = 0.5*(-E)*exp(-E.*z);
```

Plotting Gamma

```
plot(z, Gamma)
grid("on")
xlabel("z[m]")
ylabel("\Gamma(z)")
title("Gamma")
```



$$\epsilon(z) = \left(\frac{1}{I(L)\frac{Z_L}{\sqrt{\mu_0}}I^{-1}(z) + I^{-1}(z)\int_L^z I(x)b(x)dx} \right)^2$$

```
a = -2*dGamma.*(1./(1-Gamma.^2));
b = 4*1j*omega*sqrt(mu_0)*(Gamma./(1-Gamma.^2));
a
```

```
a = 1×100
10^3 ×
    1.6755    1.3732    1.1476    0.9722    0.8316    0.7165    0.6206    0.5396 ⋯
```

```
b
```

```
b = 1×100 complex
10^8 ×
   0.0000 + 1.8783i   0.0000 + 1.5393i   0.0000 + 1.2864i   0.0000 + 1.0898i ⋯
```

Integrating Factor $I(z) = e^{\int_L^z -a(x)dx}$

```
% Numerically integrate a from L to z
int_a = cumtrapz(a)*(L/N_L) - trapz(a)*(L/N_L);

% Calculate I(z)
I = exp(-int_a);
I
```

```
I = 1×100
    2.9760    2.5552    2.2526    2.0261    1.8514    1.7135    1.6027    1.5123 ⋯
```

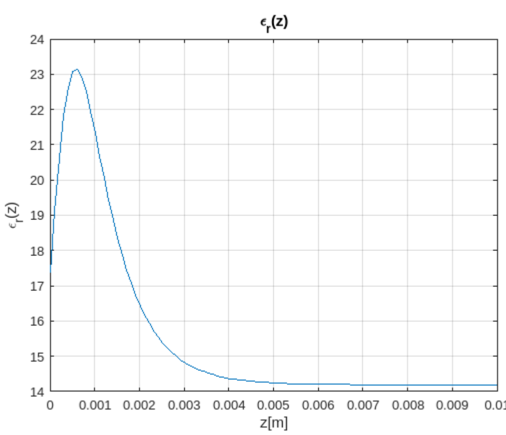
```
D = cumtrapz(I.*b)*(L/N_L) - trapz(I.*b)*(L/N_L);
```

Plotting $\epsilon(z)$

```
eps = ( I(N_L)*(Z_L/sqrt(mu_0))*I.^(-1) + I.^(-1).*D ).^(-2)
```

```
eps = 1×100 complex
10^-9 ×
  -0.1114 + 0.1064i  -0.0994 + 0.1370i  -0.0780 + 0.1651i  -0.0485 + 0.1870i ⋯
```

```
plot(z, abs(eps/(eps_0)))
grid("on")
xlabel("z[m]")
ylabel("\epsilon_r(z)")
title("\epsilon_r(z)")
```



ode45

```
clear z;
Gamma = @(z)0.5*exp(-E.*z);
dGamma = @(z)0.5*(-E)*exp(-E.*z);
a = @(z) -2*dGamma(L-z).*(1./(1-Gamma(L-z).^2));
b = @(z) 4*1j*omega*sqrt(mu_0).*(Gamma(L-z)./(1-Gamma(L-z).^2));
```

ODE is $\xi'(z)+a(z)\xi(z)+b(z)\xi^2(z)=0$

Let $z_r=L-z$. Then we have

$$\xi'(L-z_r)+a(L-z_r)\xi(L-z_r)+b(L-z_r)\xi^2(L-z_r)=0$$

Define $\xi(L-z_r)=\xi_r(z_r)$, $a(L-z_r)=a_r(z_r)$, and $b(L-z_r)=b_r(z_r)$

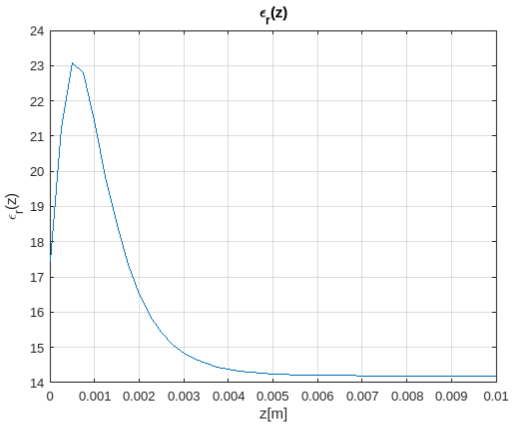
We solve for $\xi_r(z_r)$ and by substition we have

$$-\xi_r'(z_r)+a_r(z_r)\xi_r(z_r)+b_r(z_r)\xi_r(z_r)^2=0$$

```
odefun = @(z, v) -a(z)*v - b(z);
[z, v] = ode45(odefun, [0, L], (Z_L/sqrt(mu_0))^(1));
eps_ode = flip((1./v).^2)
```

```
eps_ode = 41×1 complex
10^-9 ×
  -0.1124 + 0.1059i
  -0.0664 + 0.1759i
   0.0187 + 0.2035i
   0.0932 + 0.1792i
   0.1336 + 0.1349i
   0.1483 + 0.0945i
   0.1500 + 0.0647i
   0.1470 + 0.0443i
   0.1429 + 0.0307i
   0.1390 + 0.0214i
     ⋮
```

```
plot(z, abs(eps_ode/(eps_0)))
grid("on")
xlabel("z[m]")
ylabel("\epsilon_r(z)")
title("\epsilon_r(z)")
```



Temporal Analysis

```
clear Gamma dGamma
T = 1e-10;
N_T = 100;
t = linspace(0, T, N_T)
```

```
t = 1×100
10^-10 ×
        0    0.0101    0.0202    0.0303    0.0404    0.0505    0.0606    0.0707 ···
```

Derived Solution

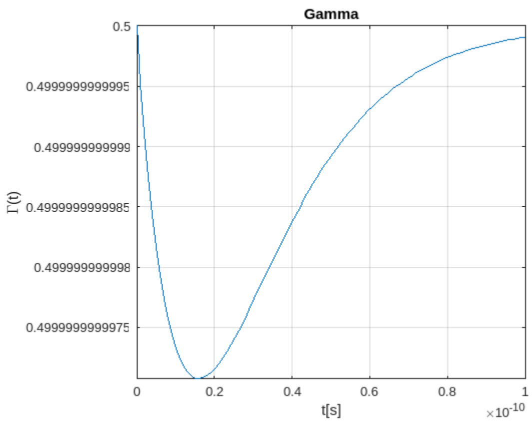
```
F = omega;
```

$$\Gamma(t) = \frac{1}{2} - \frac{1}{2} t e^{-\omega t}$$

```
Gamma = 0.5-0.5*t.*exp(-F.*t);
dGamma = -0.5*exp(-F.*t) - 0.5*(-F)*exp(-F.*t);
```

Plot of Gamma

```
plot(t, Gamma)
grid("on")
xlabel("t[s]")
ylabel("\Gamma(t)")
title("Gamma")
```



$$\epsilon(t) = \left(\frac{1}{I(0)\frac{1}{\xi(0)}I^{-1}(t) + I^{-1}(t)\int_0^t -I(\tau)b(\tau)d\tau} \right)^2$$

```
a = -2*dGamma.*(1./(1-Gamma.^2));
b = 4*1j*omega*sqrt(mu_0)*(Gamma./(1-Gamma.^2));
a
```

```
a = 1×100
10^10 ×
   -8.3776   -7.8624   -7.3789   -6.9251   -6.4993   -6.0996   -5.7245   -5.3725 ···
```

```
b
```

```
b = 1×100 complex
10^8 ×
   0.0000 + 1.8783i   0.0000 + 1.8783i   0.0000 + 1.8783i   0.0000 + 1.8783i ···
```

```
% Numerically integrate a from L to z
int_a = cumtrapz(a)*(T/N_T) - cumtrapz(a(1))*(T/N_T);

% Calculate I(z)
I = exp(int_a);
I
```

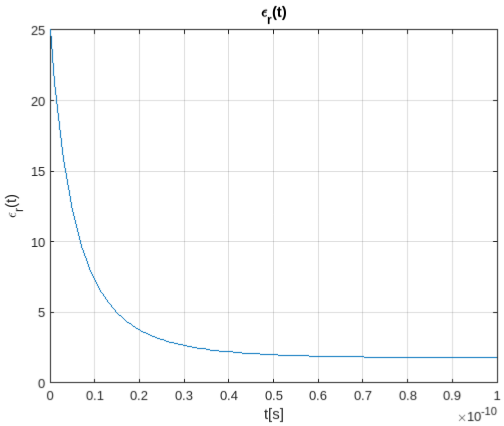
```
I = 1×100
    1.0000    0.9220    0.8544    0.7954    0.7438    0.6983    0.6583    0.6227 ···
```

```
D = cumtrapz(I.*b)*(T/N_T) - cumtrapz(I(1)*b(1))*(T/N_T);
eps = ( I(1)*(1/(5*sqrt(eps_0)))*I.^(-1) + I.^(-1).*D ).^(-2)
```

```
eps = 1×100 complex
10^-9 ×
   0.2214 + 0.0000i   0.1882 - 0.0000i   0.1616 - 0.0000i   0.1400 - 0.0000i ···
```

```
plot(t, abs(eps/(eps_0)))
grid("on")
xlabel("t[s]")
ylabel("\epsilon_r(t)")
```

```
title("\epsilon_r(t)")
```



ode45

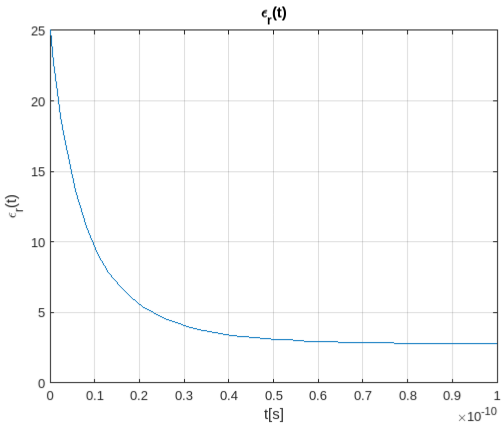
```
clear t;
Gamma = @(t) 0.5-0.5*exp(-F*t);
dGamma = @(t) -0.5*(-F)*exp(-F*t);
a = @(t) -2.*dGamma(t).*(1./(1-Gamma(t).^2));
b = @(t) 4*1j*omega*sqrt(mu_0).*(Gamma(t)./(1-Gamma(t).^2));
```

ODE is

```
odefun = @(t, v) -a(t)*v - b(t);
[t, v] = ode45(odefun, [0, 1e-10], (5*sqrt(eps_0))^(-1));
eps_ode = (1./v).^2
```

```
eps_ode = 45×1 complex
10^-9 ×
   0.2214 + 0.0000i
   0.2007 + 0.0000i
   0.1828 + 0.0000i
   0.1673 + 0.0000i
   0.1536 + 0.0000i
   0.1207 + 0.0000i
   0.0979 + 0.0000i
   0.0815 + 0.0000i
   0.0695 + 0.0000i
   0.0605 + 0.0000i
      ⋮
```

```
plot(t, abs(eps_ode/(eps_0)))
grid("on")
xlabel("t[s]")
ylabel("\epsilon_r(t)")
title("\epsilon_r(t)")
```



$\epsilon_r(t)$
25
20
15
10
5
0
$\epsilon_r(t)$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t[s]
$\times 10^{-10}$