Permitivity Synthesis

May 6, 2023

1 $\epsilon(z)$ synthesis from $\Gamma(\omega, z)\Big|_{z=0}$

[ ]:
```python
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
from scipy import integrate
from scipy import constants
from scipy import signal
```

The line is as follows

- L = 200 μ
- $Z_L = 70 + j\omega 3 \times 10^9 \Omega$
- $R_g = 50\Omega$
- W = S = 20 μ
- $L_{strip} = 3\mu$
- $S_{strip} = 1\mu$
- $W_{strip} = 30\mu$

[ ]:
```python
L = 200e-6
W = 20e-6
S = W
Lstrip = 3e-6
Sstrip = 1e-6
Wstrip = 30e-6

Rg = 50 # omega
```

We have

$$\Gamma(\omega, z) = \frac{Z_L - Z(\omega, z)}{Z_L + Z(\omega, z)}$$

We also want that $Z(\omega, L) = Z_L$

Assume we are given that $\Gamma(\omega, z)\Big|_{z=0}$ is a low pass chebyshev's filter

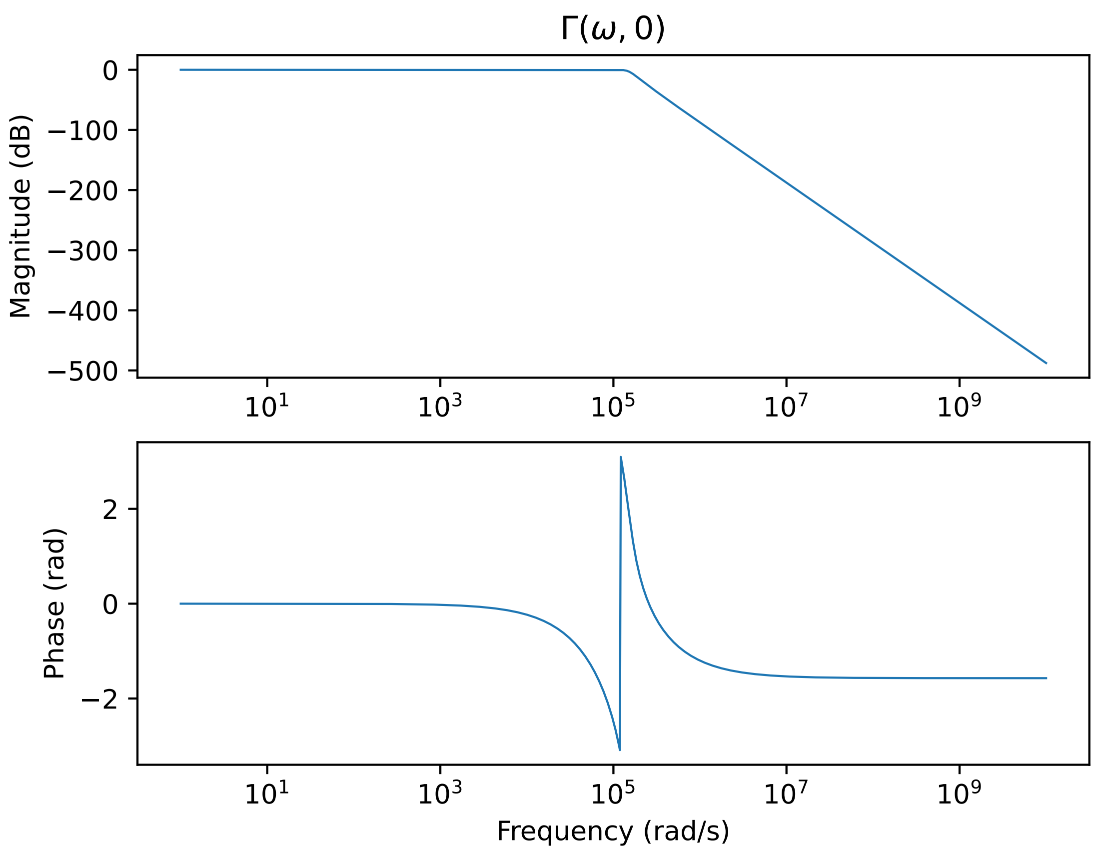
[ ]:
```python
%config InlineBackend.figure_format = 'svg'
plt.rcParams['lines.linewidth'] = 0.8
```

[ ]:

```
# Define filter parameters
order = 5
cutoff_freq = 1e5
ripple = 1e-3  # in dB
# Create filter
b, a = signal.cheby1(order, ripple, cutoff_freq, 'low', analog=True)
Gamma0 = signal.TransferFunction(b, a)
w = np.logspace(0, 10, num=1000)
# Plot frequency response
w, Gamma0 = signal.freqresp(Gamma0, w)
plt.subplot(2, 1, 1)
plt.semilogx(w, 20 * np.log10(abs(Gamma0)))
plt.title('$\Gamma(\omega, 0)$')
plt.ylabel('Magnitude (dB)')

plt.subplot(2, 1, 2)
plt.semilogx(w, np.angle(Gamma0))
plt.xlabel('Frequency (rad/s)')
plt.ylabel('Phase (rad)')

plt.show()
```

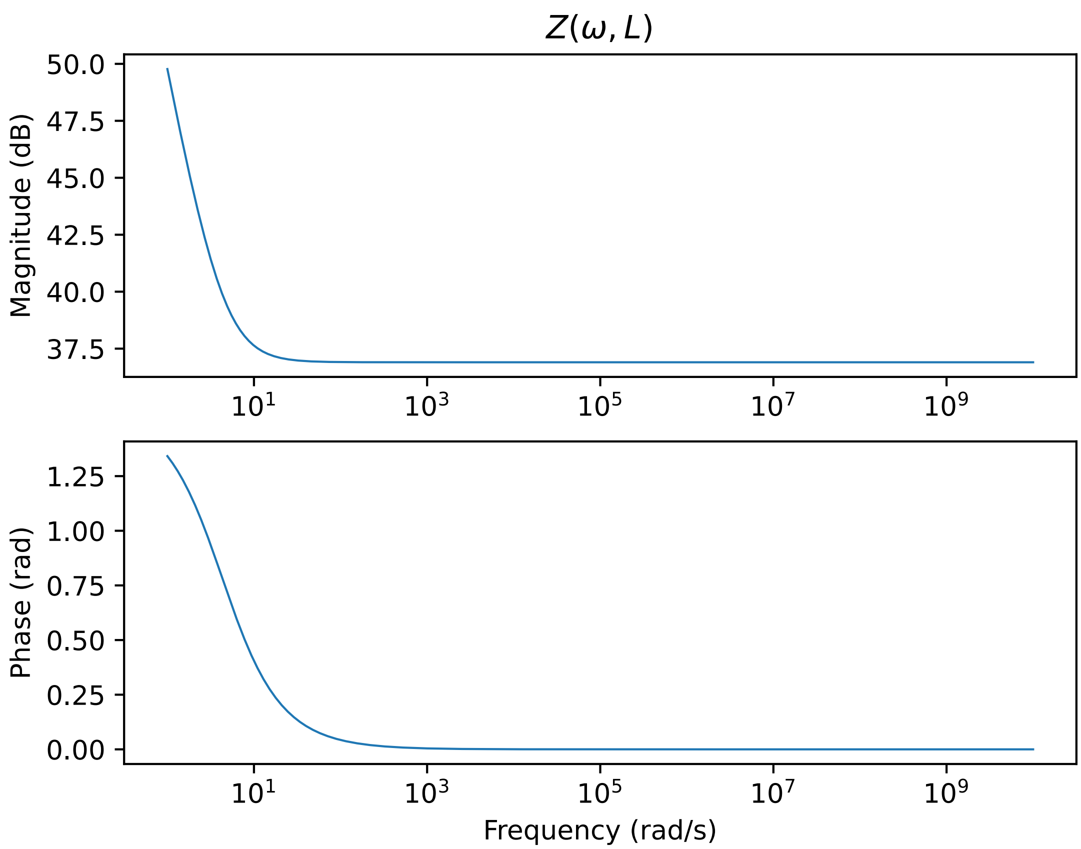


Now that ω is defined, we can define Z_L

```
[ ]: ZL = 70 + 3e2j/w
```

```
[ ]: plt.subplot(2,1,1)
     plt.semilogx(w, 20 * np.log10(abs(ZL)))
     plt.title('$Z(\omega, L)$')
     plt.ylabel('Magnitude (dB)')

     plt.subplot(2,1,2)
     plt.semilogx(w, np.angle(ZL))
     plt.xlabel('Frequency (rad/s)')
     plt.ylabel('Phase (rad)')
     plt.show()
```



Using $\Gamma(\omega, 0)$, we can find $Z(\omega, 0)$ given by `Z0`

$$\Gamma(\omega,0) = \frac{R_g - Z(\omega,0)}{R_g + Z(\omega,0)} \implies Z(\omega,0) = R_g\frac{1-\Gamma(\omega,0)}{1+\Gamma(\omega,0)}$$

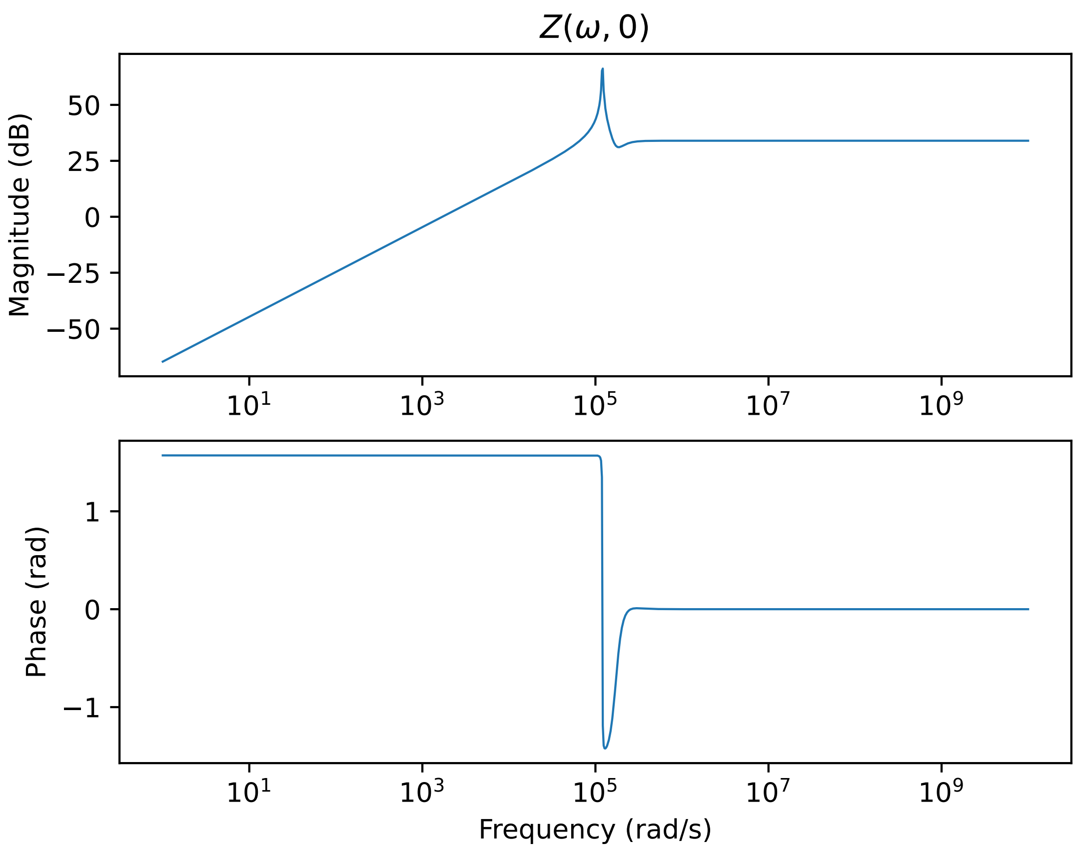
[ ]:
```
Z0 = Rg*((1-Gamma0)/(1+Gamma0))
```

[ ]:
```
plt.subplot(2, 1, 1)
plt.semilogx(w, 20 * np.log10(abs(Z0)))
plt.title('$Z(\omega, 0)$')
plt.ylabel('Magnitude (dB)')

plt.subplot(2, 1, 2)
plt.semilogx(w, np.angle(Z0))
plt.xlabel('Frequency (rad/s)')
plt.ylabel('Phase (rad)')

plt.show()

print('rad(Z0) = [', max(np.angle(Z0)), min(np.angle(Z0)), "]")
```




```
rad(Z0) = [ 1.5707963143905022 -1.4222099989654222 ]
```

Task : to synthesize $Z(\omega, z)$ and thus $\epsilon(\omega, z)$ across the line **Idea** 1. Divide the TL into N parts where $N = \lfloor \frac{L}{L_{strip}+S_{strip}} \rfloor$ 2. Each part of the line can either have an open or a closed switch, thus each part has a constant $Z(\omega)$ 3. We denote the i^{th} part by $Z(\omega, i) \forall i \in [0, N-1]$ 4. Denote by

Z_{in_i} the input impedance seen by the ith element. Thus $Z_{in_N} = Z_L$.
Also denote Z_{o_i}. Thus, $Z_{o_0} = R_g$ 5. Also denote Γ_{L_i} and Γ_{0_i}

We start at the source. Then, we find the Γ at the start of the element. Using this, we can find Γ at the end of the element, which will be the Γ at the start for the next element

[ ]:
```
l = Lstrip+Sstrip
N = int(L/l)
```

2 $\epsilon(t)$ synthesis from $\Gamma(\omega, t)\Big|_{t=0}$

$\Gamma(\omega, 0)$ and $Z(\omega, L)$ are still the same