Voltage-Controlled RF Oscillator Circuits: A Design Project

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Part I
Background and Objectives

1 Voltage-Controlled Oscillators (VCOs)

1.1 VCOs and their applications

Voltage Controlled Oscillators(VCOs) are circuits that take a DC voltage input, based off of which they output an oscillating(AC) voltage output. This can take the form of a square wave, saw-tooth wave or sinusoid. VCOs are ubiquitous in the microelectronics of handeld devices and are found abundantly in communication systems, digital and analogue circuits and RFIC in general.

Example 1. *Analog Modulation* - Modern communication systems transmit information using modulation systems, such as *Double Side-Band – Surpressed Carrier*(DSB-SC) whereby baseband signal sent using a (usually high) 'carrier frequency'. This carrier frequency is entered through a mixer along with the baseband signal and the product is what is sent along the channel. From there it is easy for a DSB-SC receiver demodulate the signal and access the signal.

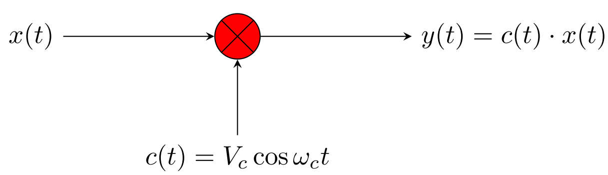

Figure 1: DSB-SC XMTR

Example 2. *Phase Locked Loops (PLL)* - PLL circuits are used to convert unstable frequency inputs into stable AC frequency outputs. It generally consists of three sequential devices connected in feedback. A VCO is used to output a wave of frequency determined by a DC input voltage. PLLs are used abundantly in communication systems.

1.2 Mathematical model of VCOs

As discussed, VCO design endeavours to implement an alternating output based on a DC voltage input. It is often required for the output to have adjustable properties, such that the frequency is malleable with the DC input (voltage *controlled*) in some specified range. It is not obvious how output frequency can be controlled by a DC input, so we may begin with a mathematical model which can then be matched to the properties of known components that could then be used in an implementation. Therefore, considering the following formal definition for a VCO.[2]

Definition. *Ideal Voltage-Controlled Oscillator Model* ∼A VCO generates an alternating output at a frequency ω_o given by a linear function of an input DC voltage,

$$\omega_o = \omega_f + K_{VCO} V_c$$

where, ω_f is some 'free running' or basis frequency, K_{VCO} [rad$s^{-1}V^{-1}$] is the gain of the VCO and V_c is the input control voltage. The inclusion of the free-running voltage is to ensure that, given our input range, the output frequency is not near DC. ω_f is the frequency about which the output is adjustable. Note that phase is given by the integral of frequency with respect to time. This gives,

$$v_{VCO}(t) = A_{VCO} \cos\left(\omega_f t + \phi_0 + K_{VCO} \int_{-\infty}^{t} V_c dt\right)$$

Where ϕ_0 denotes the initial phase.

Remark. We now can seek to orient our design approach around physical systems that will track this requirement.

1.3 Circuits that Oscillate

Due to the periodic nature of the oscillating output that we intend to produce, it is clear that oscillating circuits should employ feedback mechanisms. For this reason, RF circuits designed to produce an oscillating output will usually be analysed using techniques used for standard feedback mechanisms[2]. Therefore, we will consider a general oscillating systems as a standard feedback system that satisfies specific properties.

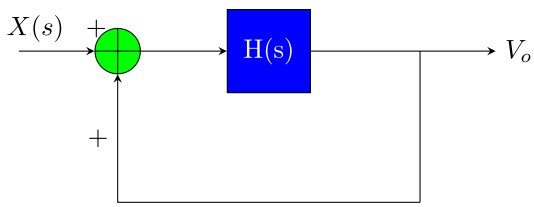

Figure 2: Feedback system

Such a positive feedback system has transfer function given by the following,

$$\frac{V_o(s)}{X(s)} = \frac{H(s)}{1 - H(s)}$$

Theorem. ***Barkhausen's Criterion*** **-** *In order for a feedback system of the form in Figure 2 to oscillate at frequency s_o, the following two criteria are necessary precursors,*

1. The loop gain magnitude satisfies: $|H(s_o)| = 1$
2. The total phase shift satisfies: $\sphericalangle H(s_o) = 0$ (or 180 degrees in the case of negative feedback)

An interesting consequence that I deduce from this is that, given analysis under a Laplace transform, sustained oscillations can only occur at purely imaginary complex frequencies. This is intuitive as it corresponds to oscillation at a particular amplitude. I discuss below several examples where it can be seen that such a criteria aligns itself with Barkhausen's Criterion.

Example 3. Non-Oscillating Circuits - The circuit in Figure 3 is fed with a DC bias voltage. Can it ever produce an oscillating signal V_{out}? No.

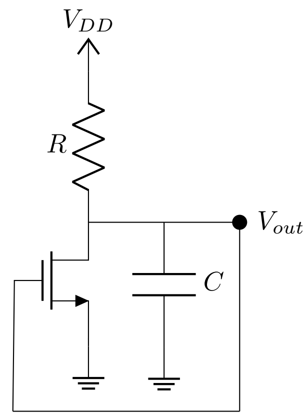

Figure 3: Single NMOS Feedback Circuit

Claim. This circuit does not support sustained oscillations
I will discuss two ways of seeing this.

1. **Necessary Oscillation Criteria -** Consider Barkhausen's Criterion. If we consider the connection from the gate to the drain as the open loop, and the connection between the drain and gate as the feedback, we can analyse the phase shifts in this circuit. The common-source configuration, biased at some source drain current, adds 180 degrees of phase. This is clear from its gain profile, given by,

$$A_{cs} = -\frac{g_m R_D}{1 + g_m R_S} \to -g_m R < 0$$

   which inverts the input signal. The capacitor adds 0 to 90 degrees of phase. Thus the total phase of 90 to 270 does not meet the necessary condition for oscillation.

2. **Small Signal Analysis -** Consider the small signal model of Figure 3, as shown in Figure 4. We are interested in considering whether or not this circuit can support small-signal oscillations, without using the above phase consideration. Consider the above SSM in the Laplace complex frequency domain. In principle the s-parameter takes the form $s = \sigma + j\omega$, $\sigma, \omega \in \mathbb{R}$. I claim, as discussed above, that for sustained oscillations, the complex frequency at which the circuit operates my be purely imaginary - we should have no exponential decay and instead a pure sinusoidal output. Can such a circuit support this?

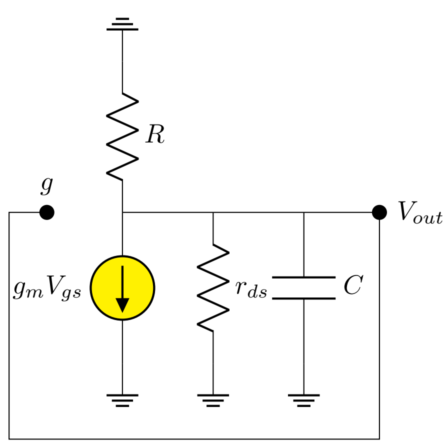

Figure 4: Small Signal Model of Figure 3

Consider the following transformation, and then Thevenin equivalent circuit,

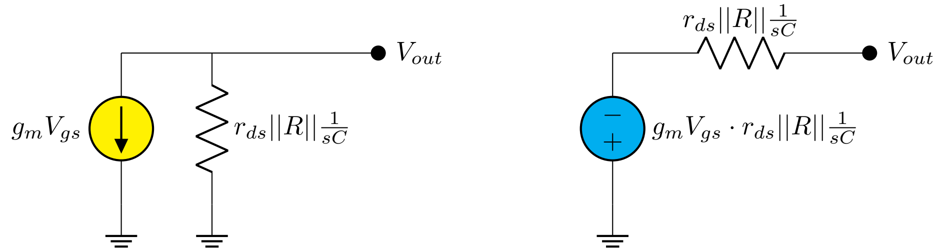

Figure 5: Figure 4 equivalents

This gives that, in the Laplace domain model, (note $V_g = V_{gs} = V_{out}$)

$$V_{out} + g_mV_{out}\cdot r_{ds}||R||\frac{1}{sC} = 0$$

And therefore,

$$V_{out}(1 + g_m\cdot r_{ds}||R||\frac{1}{sC}) = 0$$

Let us assume for now(without basis) that the circuit can product some small signal non-zero output. This may, in any case, be false, but given that we are trying *to disprove* the propensity of this circuit to oscillate, it suffices. Thus we need to analyse the frequencies at which the parenthesised quantity vanishes. I claim that there exists no $\omega_o \in \mathbb{R}$such that $s_o = j\omega_o$. This is clear from the below steps.

$$1+g_m\cdot r_{ds}||R||\frac{1}{sC} = 0 \implies 1+\frac{g_m}{\underbrace{\frac{1}{R}+\frac{1}{r_{ds}}}_{\boldsymbol{Def}\cdot k>0}+sC} = 0 \implies \underbrace{(g_m+k)}_{>0}+Cs = 0 \implies s_0 = \frac{-(g_m+k)}{C}\in\mathbb{R}^-$$

Hence, there exists no viable oscillatory frequency at which the output can be sustained. This lines up with that obtained by Barkhausen's Criterion. For intuition, let us consider how the circuit could be modified so as to ensure it can support sustained oscillations? Could an additional common source stage and idential capacitor output connection in the open loop suffice? No. Let us, again, consider why. The common source stages cancel each other out as one inverts the sign of the signal and the the second then inverts this inversion. Both capacitors add up to an extra ± 180 degrees of phase, leaving this is as the total, so, as in the previous example, the phase shift analysis shows that this circuit cannot sustain oscillations. I have ommited the small signal analysis of this case.

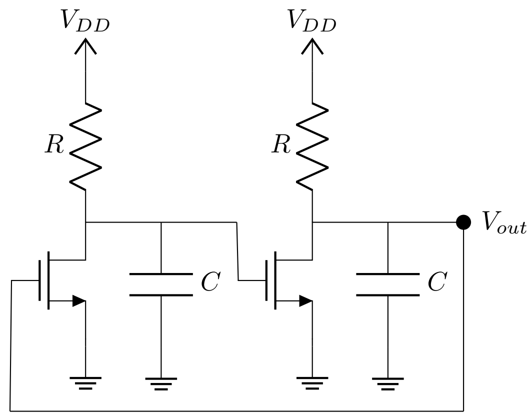

Figure 6: Double NMOS feedback circuit

Example 4. An Oscillating Circuit - The circuit in Figure 7 is fed with a DC bias voltage. Can it ever produce an oscillating signal V_{out}? Yes. It can be seen that it satisfies the phase oscillation criterion. How can this been seen with small signal analysis?

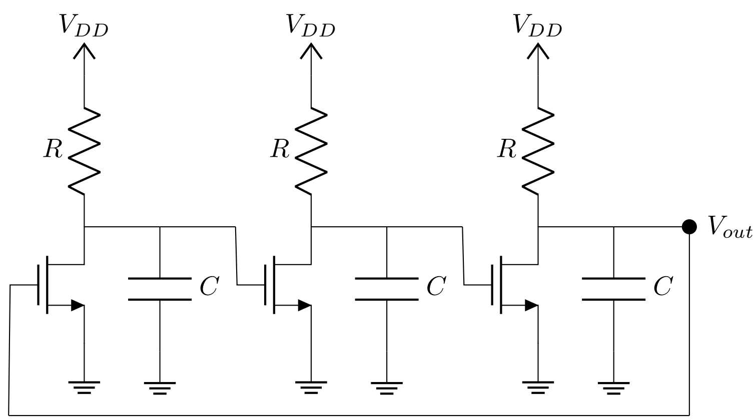

Figure 7: 3 stage NMOS feedback circuit

1.4 Properties of oscillating circuits

1.4.1 Resonance effects

Resonance is a physical property (not exclusive to electronics) that exists in systems with the propensity to store energy in various components. When such systems are excited with an input, these systems can start to undergo oscillations at frequencies that are determined by the physical properties of the system itself, and that are sustained by energy moving between the system's components. Note that energy tends to be lost as oscillations occur, so, in the absence of continuous power input, such oscillations often slowly come to a stop. Resonance occurs when the input frequency is matched to the components in such a way that allows to for near-complete power-transfer between the components in an oscillatory manner.

Example 5. *Ideal LC Pair* - A critical form of resonance that is exploited in the design of many oscillators is that of a parallel inductor-capacitor pair. We can begin to understand how it works by first qualitatively considering the function of inductors and capacitors . Both inductors and capacitors are storage elements, but store energy in different ways, and therefore exhibit different characteristic behaviours when connected in circuits. Inductors stored energy in the magnetic field that is induced in their core when a changing current passes. Inductors resultingly have a property of resistance to a change in current through them. Capacitors store energy in an electric field between plates that are charged when current passed. The phenomenon of capactor charging is intrinsically one which entails increases and decreases in current. We can demonstrate this, to begin with, with a thought experiment.

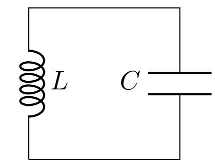

Figure 8: Ideal LC Pair

Consider the a charged capacitor connected to an inductor. The The capacitor begins to discharge, as current flows through the inductor. This is a time-varying current, which therefore produces a magnetic field in the inductor. As the discharging advances, the current begins to decrease, which is resisted by the inductor. This recharges the capacitor. In a system without losses, this would, in theory, continue to repeat itself.

We would like to model LC resonance more realistically, and therefore consider a model which accounts for the internal resistance which would be an inevitable property of the wire and components.

Example 6. *Parallel Resonance Circuit*[2]- Consider the model in Figure . R_L and R_i are taken to be the parasitic inductance of the inductor and current source respectively. The series resistance of the capacitance is not denoted with a subscript as in most such circuits, the parasitic impedance is not attributed to the plate capacitor[2].

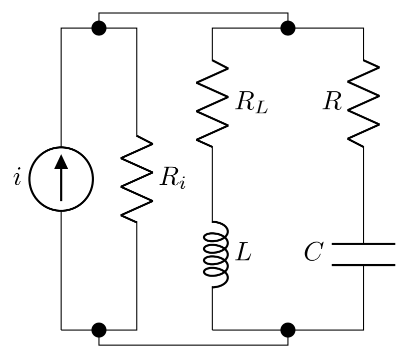

Figure 9: Parallel Resonance Circuit

Definition. *The Quality Factor of a Resonance Circuit*∼ The Q factor of a resonanting system is defined as the the product of 2π and the ratio between the total energy of the system and that lost in a single period

Varactor-based resonance regulation - Razavi

1.4.2 Phase noise

1.4.3 Tuning range

1.4.4 DC to RF efficiency

1.5 LC Oscillators

1.5.1 Feedback Analysis of LC Oscillators

Here, we will discuss how the study of LC oscillators can be understood via the study of their feedback mechanisms.

1.6 Existing oscillator models and their properties

1.6.1 Collpitts Oscillators

1.6.2 Cross Coupled Oscillators

1.6.3 Hartley Oscillators

1.6.4 RC Oscillators

1.6.5 Quartz Crystal technologies

1.6.6 Ring Oscillators

Part II
Design

Part III
Testing

References

[1] Behzad Razavi. RF Microelectronics. 2008, pp. 206–210.

[2] Duran Leblebici. High Frequency CMOS Analog Integrated Circuits. 2009