Alessio Mazzone CS 1571 HW 5

#1

Proposition Symbols

Y = mythical	A = mammal
I = immortal	H = horned
	M = magical

Knowledge Base

$Y \Rightarrow I$
$\neg Y \Rightarrow (\neg I \wedge A)$
$(I \vee A) \Rightarrow H$
$H \Rightarrow M$

$KB = (Y \Rightarrow I) \wedge (\neg Y \Rightarrow (\neg I \wedge A)) \wedge ((I \vee A) \Rightarrow H) \wedge (H \Rightarrow M)$

Conjunctive Normal Form

① Remove $\Rightarrow$

$Y \Rightarrow I \quad = \quad \neg Y \vee I$ ✓
$\neg Y \Rightarrow (\neg I \wedge A) \quad = \quad \neg\neg Y \vee (\neg I \wedge A) \quad = \quad Y \vee (\neg I \wedge A)$
$(I \vee A) \Rightarrow H \quad = \quad \neg (I \vee A) \vee H$
$H \Rightarrow M \quad = \quad \neg H \vee M$ ✓

② move $\neg$ inwards

$\neg (I \vee A) \vee H \quad = \quad (\neg I \wedge \neg A) \vee H$

③ Distribute

$Y \vee (\neg I \wedge A) \quad = \quad (Y \vee \neg I) \wedge (Y \vee A)$ ✓
$(\neg I \wedge \neg A) \vee H \quad = \quad (H \vee \neg A) \wedge (H \vee \neg I)$ ✓

KB in CNF $= (\neg Y \vee I) \wedge (\neg H \vee M) \wedge (Y \vee \neg I) \wedge (Y \vee A) \wedge (H \vee \neg A) \wedge (H \vee \neg I)$

Resolution By Refutation

α_1 = Unicorn is horned. $= H$
α_2 = Unicorn is magical $= M$
α_3 = Unicorn is mythical $= Y$

α_1

① negate α_1 and add to KB

$\alpha_1 = H \rightarrow \neg H$

$KB \wedge \neg H \longrightarrow (\neg Y \vee I)^{1} \wedge (\neg H \vee M)^{2} \wedge (Y \vee \neg I)^{3} \wedge (Y \vee A)^{4} \wedge (H \vee \neg A)^{5} \wedge (H \vee \neg I)^{6} \wedge \neg H^{7}$

$(H \vee \neg I)^{6}, \; (\neg Y \vee I)^{1}$
$(H \vee \neg Y), \; (Y \vee A)^{4}$
$(H \vee A), \; (H \vee \neg A)^{5}$
$H, \; \neg H^{7}$
$\times$ → $\boxed{\text{Contradiction. Thus } KB \models \alpha_1}$

α_2

negate $\alpha_2 \rightarrow \alpha_2 = M \rightarrow \neg M$

$KB \wedge \alpha_2 = (\neg Y \vee I)^{1} \wedge (\neg H \vee M)^{2} \wedge (Y \vee \neg I)^{3} \wedge (Y \vee A)^{4} \wedge (H \vee \neg A)^{5} \wedge (H \vee \neg I)^{6} \wedge \neg M^{7}$

$(\neg H \vee M)^{2}, \; (H \vee \neg I)^{6}$
$(M \vee \neg I), \; (\neg Y \vee I)^{1}$
$(M \vee \neg Y), \; (Y \vee A)^{4}$
$(M \vee A), \; (H \vee \neg A)^{5}$
$(M \vee H), \; (\neg H \vee M)^{2}$
$M, \; \neg M$
$\times$ → $\boxed{\text{Contradiction. Thus } KB \models \alpha_2}$

$\underline{\alpha_3}$

$\alpha_3 = y \rightarrow$ negate $\alpha_3 = \neg y$

$KB \wedge \neg y = (\neg y \vee I) \wedge (\neg H \vee M) \wedge (y \vee \neg I) \wedge (y \vee A) \wedge (H \vee \neg A) \wedge (H \vee \neg I) \wedge \neg y$

$$\frac{\dfrac{(y \vee A) \,,\, (H \vee \neg A)}{(y \vee H)} \,,\, (\neg H \vee M)}{(y \vee M)}$$

$\longrightarrow$ Can not add any more states. Therefore $KB \not\models \alpha_3$

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