4AMT322-TEST 1 SCOPE

Date: 4 September 2024 at 12:30

Venue: C12

Duration: 2 hours

Marks: 35

Scope: Interpolation, Least squares method, and numerical differentiation

Question 1 (10 marks)

For question 1, one of the following two questions will be in the test, so it is good to understand them both:

1. Suppose $f(x) = x^3$, $x_0 = 0$ and $x_1 = a$.

    i)   Write down the first-order Lagrange interpolating polynomial.

    ii)   Recall the error formula in interpolation is given by:

    $$f(x) - p_n(x) = \frac{f^{n+1}(\xi(x)) \prod_{i=0}^{n}(x-x_i)}{(n+1)!}, \forall x \in [x_0; x_n] \text{ and for some } \xi(x) \in [x_0; x_n].$$

    Verify that $\xi(x)$ exists and is given as $\xi(x) = \frac{1}{3}(x + a)$.

2. If $f(x) = \frac{1}{x^2}$; show that $f[a, b] = -\frac{(a+b)}{a^2 b^2}$ and $f[a, b, c] = \frac{ab+bc+ca}{a^2 b^2 c^2}$

Question 2 (15 marks)

For question 2, one of the following two questions will be in the test so it is good to understand them both:

1. Fit the curve $y = \frac{a_0}{x} + a_1\sqrt{x}$ to the following data

x	0.1	0.2	0.4	0.5	1	2
y	21	11	7	6	5	6

2. From the set of data below, construct a function of the type $f(x) = ae^x + be^{-x}$ using the principle of least squares

x	0.2	0.3	0.4	0.5
$f(x)$	2.0	5.0	3.5	3.0

Question 3 (10 marks)

For question 3, one of the following two questions will be in the test, so it is good to understand them both:

1. A particular finite difference formula for the first derivative of a function is

$$f'(x) = \frac{-f(x + 3h) + 9f(x + h) - 8f(x)}{6h}$$

What is the order of the truncation error? (show all your workings).

2. A particular finite difference formula for the first derivative of a function is

$$f'(x) = \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$$

What is the order of the truncation error? (show all your workings).