To Be or Not To Be? The Questionable Benefits of Mutual Clearing Agreements for Derivatives.

Magdalena Tywoniuk*

Department of Finance, University of Geneva & Swiss Finance Institute

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Abstract

Recent history has seen the establishment of the first mutual clearing agreements between Central Counterparties (CCPs), but only for standard asset classes. There are existing global concerns over the unique threats and benefits which arise from such agreements, and further concern about their extension to derivatives CCPs. This is because CCP interoperability alters the financial network's interconnections and transmission of shocks; through balance sheets and through price movements. This paper applies the current mutual agreement framework to credit default swaps (derivatives) CCPs and compares this to clearing without any such agreement. Key results concern: The magnitude of price dispersion between multiple CCPs (as trading moves asset prices away from fundamental value), the magnitude of default contagion, the price impact of predation, and the disciplinary mechanism inherent in the mutual cross-margin fund (between CCPs). The work also answers a current regulatory debate; concerning the use of the default fund to meet inter-CCP shortfalls. Finally, a dynamic simulation modeling the price process (variation margin exchange) then provides real-world policy implications for a variety of market liquidity states.

Keywords: Mutual Agreement, Price Dispersion, Systemic Risk, CDS, Liquidation, Predation, Price Impact, Contagion, Financial Network, Over the Counter Markets.

JEL Classification: G00, G01, G02, G14, G10, G18, G20, G23, G33

1 Introduction

Following the 2008 financial crisis, Dodd-Frank legislation required both the standardisation of credit default swap (CDS) contracts and their clearing through central clearing counterparties (CCPs). The central clearing of this large market[1] inadvertently concentrated risk and resulted in certain CCPs being labelled as systemically important institutions, or *too big to fail*. Now, even though the CCP framework has provided undeniable benefits such as netting, trade compression[2] and adequate collateralisation [Duffie et al., 2010, Cont and Minca, 2014, Augustin et al., 2014], it has introduced trade concentration risks which could be perilous should a CCP fail. This is complicated by the fact that the current financial system is composed of multiple, independent CCPs clearing the same CDS instruments. Regulators have continuously debated whether the establishment CCP interoperability between CDS CCPs would lower system-wide risk or increase it.

Interoperable CCPs, by becoming clearing members of each other, allow their dealer members trading access to the dealer members of the other CCP(s), while only posting margin at one CCP. This

*Electronic address: `Magdalena.Tywoniuk@unige.ch`

[1] Approximately 13.3e9 in 2015 according to [Oehmke and Zawadowski, 2017]

[2] Clearing through the CCP prevents redundant trades and chains of unknown counterparties, thereby reducing counterparty risk.

allows dealers to maintain membership at only one CCP, while CCPs monitor systemic risk and exchange margin on their mutual inter-CCP positions. Interoperability naturally extends netting benefits across the network and maintains extra risk-sharing resources to protect against inter-CCP failures. However, risk-sharing also becomes a conduit for transmitting shocks. Therefore, it is important to understand the costs and benefits to a financial network with, and without, interoperability. Both a network of stand-alone, or interoperable, CCPs could uniquely transmit contagion, leading to multiple CCP failures. This would have disastrous effects on the financial network with spillover into the real economy.[Securities and Authority, 2016, McPartland and Lewis, 2016a]

CCP interoperability changes the interconnectivity in the financial network and, thereby, the transmission of shocks. As traders are connected not only by balance sheets, but also by price movements[3], CCP interoperability can introduce feedback effects which "amplify market movements, cause cascading failures and jeopardize financial stability."[Feng and Hu, 2016] The changing structure of a financial network also changes how risk is shared among constituents. Only two previous studies on CCP interoperability, by [Feng et al., 2014] and [Feng and Hu, 2016], have focused on cascading failures and this price amplification effect, respectively. However, in using a complex systems approach, they mainly study the effect of shock size on the robustness of the network. Though this is immensely valuable, it abstracts from some realistic aspects of market structure[4], which tend to be of greater concern to participants and regulators.

In light of such an important, but complex problem, this work seeks to make key contributions to the understanding of CCP interoperability and its implications for the financial stability of the CDS market, while offering novel insights into systemic risk management. By proposing a theoretical network model of multi-CCP financial systems, both with and without interoperability agreements, this work provides a structured framework to analyze the effects of these agreements on contagion and predation. The theoretical model is then integrated with a dynamic, multi-period trading simulation, calibrated to over-the-counter (OTC) market data, — exploring how interoperability impacts default rates, inter-CCP price dispersion, CCP and member losses, and dealers' margin-to-asset ratios. Crucially, the research answers a key regulatory debate: Should CCPs be allowed to use their default fund to meet other CCPs defaults or does this lead to contagion? This is resolved by demonstrating that CCP interoperability, while feared for its potential to transmit shocks, also enables an endogenous recovery mechanism by discouraging predatory trading behavior and enhancing systemic resilience. These findings are then translated into actionable insights for regulators and lenders of last resort (LoLRs), making this study one of the first to bridge theoretical modeling with real-world policy applications.

The novelty of this research lies in its ability to incorporate price-mediated contagion and predatory amplification within an interoperable CCP network under stress—elements largely unexplored in prior studies. Key contributions include demonstrating how interoperability reduces liquidation losses and contagion through mutual risk-sharing, while lowering margin-to-surplus ratios for dealers, despite increasing inter-CCP price dispersion. Moreover, it identifies a previously unconsidered benefit: interoperability's capacity to recover predatory profits via inter-CCP margin refills, which significantly curtails moral hazard. Finally, by linking these findings to liquidity scenarios ranging from stable markets to crises, the research provides innovative recommendations for LoLR interventions, including the strategic timing and targeting of liquidity injections to minimize cascading failures. These contributions not only enhance our understanding of CCP interoperability but also equip policymakers with novel tools to safeguard financial stability.

The current CCP interoperability arrangements exist only for certain derivative asset classes[5], but

[3] [Feng and Hu, 2016] note that prices "constitute imperatives for financial institutions to act through marking-to-market and risk sensitive constraints."

[4] By choosing a rigorous network model with one CDS asset and an exogenous price shock, the authors abstract away from a more realistic multi-CDS portfolio composition, from dealers motivations for trading/predating, and from any CCP constraints – all which are endogenous determinants of price shocks.

[5] As of 2016, there are five interoperability agreements in the EU, handling cash products, repos and exchange-traded derivatives.[Board, 2016]

market participants are actively pushing for an extension of interoperability to credit defaults swaps clearing. On their side, multiple regulatory bodies have expressed marked concern about the additional risks and complexities which are sure to arise since current interoperability arrangements cannot simply be transferred to this class of derivatives.[Board, 2016] The result is an active initiative to determine what are the implications of interoperability agreements for systemic risk. To date, this has consisted of numerous policy recommendations as to how such agreements should be structured[6] and a small amount of theoretical research articles. These demonstrate that significant benefits result from dealers holding their total positions at one interoperable CCP, rather than spreading their positions amongst multiple CCPs. In addition to these netting benefits, they find that CCP interoperability also brings decreased demand for collateral and avoidance of parallel defaults. However, they caution that interoperability allows for multiple CCPs to become directly exposed to one another and that, due to insufficient research, many ramifications of this new channel of contagion remain unexplored.[Board, 2016] Thus, it remains unclear what would be the full implication of CDS derivatives CCPs' interoperability on systemic risk management.

The push toward CCP interoperability for CDS is motivated by the risk of multiple failures, inherent in the current system. There exists the strong possibility of dealers' simultaneous failure across multiple CCPs, due to both the structure of the CCP and the structure of the CDS Market. Firstly, the CDS market is extremely concentrated and interconnected which increases the likelihood of multiple defaults. It is comprised of 14-20 global dealers, who own approximately 80% of the OTC market[7]. Since there are no mutual agreements between CCPs clearing CDS[8], these dealers are members in all the major CCPs.[Cont, 2010] Thus, if a large dealer were to fail in one CCP, that dealer will almost surely fail in all other independent CCPs where it holds membership. Second, the CCP resources against dealers' default are limited, which increases the likelihood of multiple CCP failures. The CCP simply serves as a central intermediary between its own members' trading contracts[9] (as in figure 1 and 2 below). To protect itself (and dealers) against counterparty risk, the CCP collateralises each side of the contract. Thus, each dealer provides an initial proprietary guarantee fund contribution and a much smaller risk-sharing default fund contribution[10]. When a dealer defaults, the CCP only has that dealer's guarantee contribution and the small default fund to protect its CCP's equity from any ensuing contagion. Therefore, given the fact that a CCP's default fund cannot cover more than two large dealers' defaults[11], and given the likely interconnectedness of dealer defaults, there exists an increased risk of CCP(s) failure.

[6] The recommending bodies are: EMIR, which provides some macro prudential policy recommendations, ESMA provides micro-prudential recommendations, the Bank of England provides a supervisory overview and the Chicago Fed provides an operational and public policy perspective on the issue.

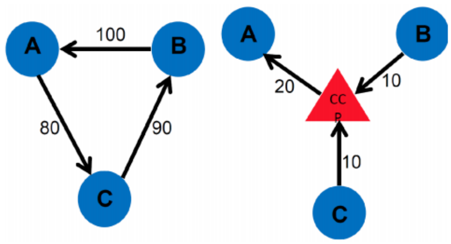
[7] where most trades are dealer-to-dealer

[8] Excluding the exchange traded derivatives agreement between the Norwegian branch of SIXX Clear AG and LCH Clearnet Ltd. for index and single Norwegian stock futures and options.[Board, 2016]

[9] One former bilateral trading contract is novated into two contracts: one between the buyer and the CCP, and one between the CCP and the seller.

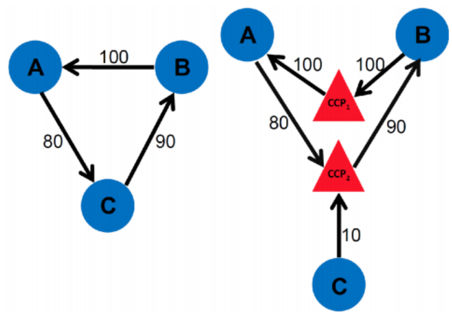
[10] This is constructed to withstand the default(s) of one or two of its largest members.[Fed, 1994] Its size is about 10 percent of the guarantee fund.

[11] This 'cover 2' systemic risk standard for CCP reserves requires that CCPs to be able to withstand the default of their two largest clearing members.[Veraart and Aldasoro, 2022]



Duffie 2015, presentation to Basel Committee

Figure 1: Bilateral Trading versus CCP Cleared Trading - In bilateral trading (left), dealers contract directly with each, leading to chains of trades. In CCP cleared trading (right), dealers contract directly with the CCP, which nets incoming and outgoing trades in a process called trade compression.



Duffie 2015, presentation to Basel Committee

Figure 2: Bilateral Trading versus Dual CCP Cleared Trading without an Inter-CCP link - With two CCPs (right), but without interoperability, dealers contract independantly with each CCP, losing some of the netting-benefits (lower margins) that come with a mono-CCP system. Dealers must also post margins at each CCP.

Another weakness of the current system is risk of under-collateralization due to *hidden illiquidity*; dealers have an incentive to spread their positions across multiple CCPs to limit risk exposure and lower collateral requirements at each CCP. However, this practice leaves the system under-collateralized[12] as a whole.[Board, 2016] Membership in multiple CCPs can also be problematic for dealers, as meeting the margin requirements at each CCP ties up collateral. Meanwhile, dealers cannot access any netting benefits when spreading their positions across these CCPs[13]. Along these lines, [Duffie and Zhu, 2011] use a theoretical model to demonstrate that multiple, isolated CCPs increase counterparty risk and decrease netting efficiency in the system. In line with this work, through simulation I illustrate two key findings: First, default contagion is decreased with CCP interoperability agreements due to the lower membership in each CCP (reducing the intensity of the default contagion cascade). Second, the ratio of surplus assets versus collateral required is higher in the case of a CCP mutual agreement (due to netting). These results hold over multiple market liquidity states: a stable market, a distressed market with waning liquidity, and an in-crisis market nearing illiquidity.

Many of these problems could be relieved with the establishment of interoperability – or a mutual

[12]This reveals itself when a dealer fails in multiple CCPs, who in turn flood the market with the same positions, creating unexpected volatility for the asset. The synergistic effect of these multiple liquidations is not covered by the sum of the initial margin contributions.

[13]This is especially problematic due to procyclicality—during times of highest risk and lowest liquidity, collateral requirements peak, potentially leading to financial distress for the dealers.[Securities and Authority, 2016]

agreement (MA)[14] – between CCPs, but at what cost? The added complexity which interoperability introduces into the system is bound to have unintended consequences. Consider that in the current system, a single CCP novates one bilateral contract (between members) into two contracts (between the CCP and each member). However, introducing interoperability would involve the novation of one bilateral trading contract into three (as in figure 3 below); one between CCP 1 and the buyer, a second between CCP 2 and the seller, and a third between CCP 1 and CCP 2.[Board, 2016] It is precisely this complexity, so hard to analyze, which has restricted regulators to citing benefits with certainty and only hypothesizing costs. It has also lead regulators to suggest that the existing static models may not be sufficient to capture the complex dynamics involved, and to advocate for the development of more dynamic and flexible modeling frameworks.[McPartland and Lewis, 2016b] Accordingly, this paper attempts to answer their call by creating a dynamic model which integrates their findings, and the stylistic features of interoperability. In doing so, it is able to identify previously unconsidered costs of interoperability and to provide solutions to some unresolved questions.

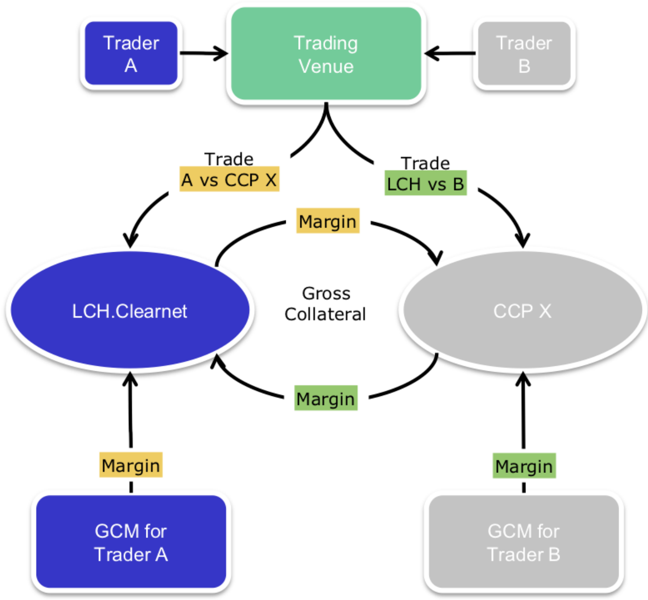


Figure 3: Trading with Interoperable CCPs -With interoperability, dealers contract with their home CCP, but can trade with dealers in the other CCP. The home CCP, being a clearing member of the other CCP, continues to net the totality of dealer trades across both CCPs. Dealers only post margins at their home CCP, while CCPs collect margins from each other.[LCH Ltd., 2017]

2 Literature Review

[Duffie and Zhu, 2011] were the first to demonstrate that a single, global CCP ensures the highest degree of multilateral netting, reduces aggregate risk, and decreases demand for collateral. Similarly, [Board, 2016, Securities and Authority, 2016, Bank of England, 2014] were the first to address the various benefits of interoperability, namely netting efficiency and decreased collateral tie-up. This work replicates and extends these findings: finding that interoperability reduces aggregate risk by reducing the number of members in each CCP, thereby minimizing the effects of contagion and predation (through decreased trading volume and opportunity). Also, my results demonstrate that collateral demand is lower with a mutual agreement (MA) versus no mutual agreement (NMA) in place. In contrast, although policy reports highlight that interoperability increases market liquidity and decreases market fragmentation,

[14]Note, interoperability is defined as providing dealers access to a local market without requiring them to becoming a local actor. Here, each CCP becomes the clearing member of the other. This is not the same as cross-margining agreement between CCPs, where dealers become members of each CCP and CCPs pools their margins. Since CCPs don't clear transactions with each other, there is no 'reciprocity'.[Securities and Authority, 2016]

this work demonstrates that these benefits do not necessarily extend to OTC markets. Instead, interoperability comes with a cost not previously considered by policymakers: increased price dispersion between CCPs. Novel results show that interoperable trading not only increases an asset's deviation from the fundamental price (price volatility), but also creates significant differences in prices achieved in each CCP (inter-CCP price dispersion/market fragmentation). To my knowledge, this is the first paper to emphasize this fact.

[Cox et al., 2013] demonstrate, through a theoretical model, that interoperability leads to a reduction in exposures. In accordance with this finding, through simulation with two linked CCPs, this work shows that the average dealer aggregate surplus is higher in the MA scenario, indicating that exposures are minimized. Like this paper, [Cox et al., 2013] also determine that this is achieved through decreased credit risk. This contrasts with [Anderson et al., 2013], which argues that CCP links increase exposures. [Board, 2016] supports the finding of decreased exposures but criticizes the [Cox et al., 2013] model for not considering network complexity or contagion risk in periods of stress. These concerns are directly addressed in this paper, which explicitly models contagion and the dynamic network complexity of predation. Furthermore, the simulation results are tested across three market liquidity scenarios: stable, distressed, and in-crisis. This work attempts to allay the concerns of [Securities and Authority, 2016], which highlights the lack of, and desperate need for, sophisticated modeling to measure netting efficiency, extra collateral requirements from interoperability, and other salient features comparing the MA and NMA scenarios. This is one of the few papers[15] to explicitly model and simulate these recommendations and concerns[16].

This theoretical model and simulation provide a unique contribution to the current literature on CCP interoperability. They incorporate the aforementioned requirements of realistic complexity, such as induced correlations between CDS in times of stress.[Board, 2016] Additionally, they describe other salient features of interoperability still under debate and provide resolutions to these issues. The first concerns the inter-CCP margining method: the method used to calculate the margin which CCPs must post to each other to absorb inter-CCP losses or mitigate the extra risk arising from the link. Some regulators, as in [McPartland and Lewis, 2016a], argue that the collection of inter-CCP margin should be in addition to that posted by clearing members/dealers to their own CCP. Others, like [Magerle and Nellen, 2011a], argue for the rehypothecation[17] of collateral. A second serious point of contention is the use of the CCP's default fund to meet any inter-CCP losses above those covered by the inter-CCP margin. One argument is that each CCP should automatically factor the other CCP into their own initial margin and default fund contribution. Others, like [Bank of England, 2014], argue that this leads to overall systemic under-collateralisation. They advocate instead for an inter-CCP margin calculated as a multiplier of the total initial margin collected from dealers, as well as the use of the default fund to meet inter-CCP losses incurred over the pre-allocated inter-CCP margin. [Magerle and Nellen, 2011b] confirm that the former model versus the latter *scalable model* results in under-collateralisation. This paper expands on this finding by employing the inter-CCP margining methodology currently used for interoperability links in the equities market.[Clearnet, b, Clearnet, a, Securities and Authority, 2016] This work finds that this scalable inter-CCP margining method does not pose inter-CCP contagion risk and leaves default funds largely untouched. Importantly, this paper also explicitly demonstrates the various mechanisms at work in the CDS market: the inter-CCP variation margin exchange, the inter-CCP collateral return, and the successful implementation of an inter-CCP default waterfall.

Finally, [Securities and Authority, 2016] address the problems inherent to the international nature of CCPs. They highlight the current lack of an explicitly defined CCP recovery plan that addresses the global nature of CDS clearing and provides a clear resolution mechanism. They underline that

[15] [Feng et al., 2014] also attempts to provide findings for national policy makers.

[16] "Balancing the collateral decrease due to the netting efficiency compared to a situation where there is no link vs. the extra collateral to cover inter-CCP risk... would require sophisticated modeling which has not been developed yet..." [Securities and Authority, 2016].

[17] Rehypothecation involves using posted initial margin/collateral to meet inter-CCP margin. According to [Magerle and Nellen, 2011a], "Margins are generally characterised as a defaulter-pays instrument. However, if a CCP rehypothecates margin [it] changes to a survivor-pays instrument."

each operating CCP is already a stand-alone international network and that linking such networks increases the complexity of the financial system. This, in turn, increases the problem of moral hazard, as the benefits of interoperability would be enjoyed internationally, while any potential costs would be faced by the home country (a Central dealer or a LoLR).[McPartland and Lewis, 2016a] This model addresses the issue of moral hazard through the predation mechanism: international participants are more likely to profit from the defaults of other dealers and the subsequent liquidation of positions by a CCP. I extend the model of [Brunnermeier and Pederson, 2005] to inter-CCP trading, illustrating how clearing members depress prices through selling and then buy up later at a profit. I then show that the margining methodology inherent to interoperability (MA scenario) provides a natural disincentive mechanism for predation (and thus moral hazard). The risk-sharing inter-CCP margin means that all participants must share the consequences of predation through mandatory margin posting for each trading day and a margin refill subsequent to their buyback strategy.

The paper is structured as follows; Section 1 provides an introduction and an overview of the literature, Section 2 outlines the theoretical model, Section 3 introduces the trading periods and model dynamics, Section 4 provides simulation details and results, and Section 5 concludes.

3 Model Background

This section introduces the modeling framework developed to analyze the dynamics of multi-CCP financial networks under both mutual interoperability agreements (MA) and no interoperability agreements (NMA). It extends the framework of [Tywoniuk, 2017b], from a mono-CCP network model, to one with multiple CCPs. This framework is structured to progressively build from fundamental trading relationships to the complex interactions that arise during distress scenarios. It begins by describing the baseline structure of the financial network, where CCPs act as central hubs connecting member dealers trading standardized credit default swap (CDS) contracts. Next, it details how these interactions evolve across multiple phases of the trading lifecycle: **Initial**, where positions and liabilities are established; **Liquidation**, where distressed positions are unwound; **Buyback**, where predatory trading behaviors emerge; and **Resolution**, where the system stabilizes.

The modeling framework also incorporates key mechanisms such as variation margin exchange, collateral requirements, and the propagation of default contagion within and across CCPs. The inclusion of interoperability agreements introduces unique features, including inter-CCP margining with additional fund contributions, which alter risk-sharing and systemic outcomes. In the subsequent sections, these foundational elements are expanded upon to develop a dynamic, multi-period simulation that quantifies the effects of CCP interoperability under varying market conditions, capturing outcomes like default cascades, inter-CCP price dispersion, and systemic stability. This framework provides a detailed roadmap for understanding the interplay between CCP design and market behavior in both stable and crisis scenarios.

The following network analysis consists of two scenarios; one with a mutual interoperability agreement between CCPs (MA) and one without any such agreement. In the case where there is no interoperability/mutual agreement (NMA), each CCP functions as its own autonomous financial network – until a member defaults. That is, a CCP u will have m member dealers, all who trade with other dealers in the same CCP. If dealer $i \in m$ in CCP u wants to trade with dealer $j \in m'$ in CCP u', then dealer i must also become a member of CCP u'. This yields a fragmented financial market. This additional CCP membership also comes along with additional collateral requirements; an initial margin and a default fund contribution in the each CCP. Yet, it also provides the ability to spread the risk of a concentrated position amongst several CCPs – lowering concentration charges in each and possibly yielding lower total collateral requirements for that dealer. [Glasserman et al., 2016] describes this phenomenon, termed hidden illiquidity[18], increases systemic risk. Therefore, each CCP is able to act

[18] The refers to the idea that if a concentrated position is spread among several CCPs, the collateral in each is lower than if the position had been placed in one CCP. Thus, if the dealer fails, he fails in all CCPs and the full concentrated

as its own entity only until a large member dealer fails, because that dealer will simultaneously fail in all CCPs, thereby linking the system. In that case, this complete system may be undercollateralised.

On the other hand, there may exist a mutual interoperability agreement (MA) between CCPs. In this case, if there are $u \in \{1...n\}$ CCP's then each CCP has m/n members. The m/n members of CCP u trade in that CCP, and the other $m\frac{n-1}{n}$ members trade in the other CCPs. It is the CCP's – rather than the dealers – that trade directly with each other as counterparties. Each provides the other with the collateral for a position, which is a multiple of what was originally collected from its own clearing member. The exact nature of the collateral exchange will be explained in further sections. The differences between clearing arrangements can be seen in figures 1, 2, and 3. The trading arrangement can be seen in figure 4.

4 Price Impact Model

4.1 Model Setup

The model consists of a multi-CCP system $u = \{1,..,n\}$. Each CCP acts as it's own structure. This consists of a star-shaped financial network, where the CCP is at the centre ($i = 0$), and from which it connects to each of its member dealer dealers ($i = \{1,..,m\}$). The dealer dealers trade in standard CDS contracts written on underlying reference entities $k = \{1,...,K\}$. Each of these member dealers clears CDS contracts through the CCP, which then acts as the counterparty to each contract. A dealer may want to trade with a dealer in another CCP: If no interoperability agreement between CCPs exists, then the dealer must also become a member of the second CCP. However, if there exists a mutual agreement between CCPs, each dealer maintains sole membership in their own CCP, whereupon the two CCPs trade with each other as counterparties – creating a larger network. For each trade, the CDS contract is represented by a position with a nominal value (X), which is positive for the *buy* side (X^+) and negative *sell* side (X^-) such that,

$$X^p \quad \text{where } p \in \{B, S\}, \quad X^B = +X \quad \text{and} \quad X^S = -X$$

Since each trading contract is held between two counterparties, this demands that if dealer i holds the buy position, dealer j must hold the sell position. Then, $X_{ij}^{u,k}$ represents the matrix of CDS positions between counterparties, for k reference entities at each CCP, u.

The portfolio of dealer i, in u different CCPs, for CDS written on k reference entities has the nominal value,

$$V_i = \sum_{u=1}^{n} V_i^u = \sum_{k=1}^{K} X_i^{u,k} S^{u,k}(t_\ell)$$

where V is the current market value of the position, X is the nominal value and S is the CDS spread. Thus, any change in the spread is denoted $\triangle S$[19], and determines the premium which must be paid by/to a counterparty at any time period (t_ℓ) . The model has multiple time periods t, each is subdivided into ℓ shorter trading periods, τ. The length of the time period t is determined by the CCP's liquidation window of five days, translating to each trading period ℓ lasting one day.

In each CCP, the net value of dealer i's position is determined by the difference between its liabilities and receivables. The amount of *variation margin* on CDS k owed by dealer i to other dealer dealers j is the payable,

$$L_i^{u,k} = \sum_{j=1}^{m} L_{ij}^{u,k} \tag{1}$$

position must be unwound at the same time, but is undercollateralised for illiquidity of market risk. This large position is hard to move all at once and is considered illiquid by the market – a fact that which was previously hidden.[Board, 2016]

[19] The change in CDS spread $\triangle S$ will start at zero and move up, taking a positive value ($\triangle S > 0$), or it will move down, taking a negative value ($\triangle S < 0$).

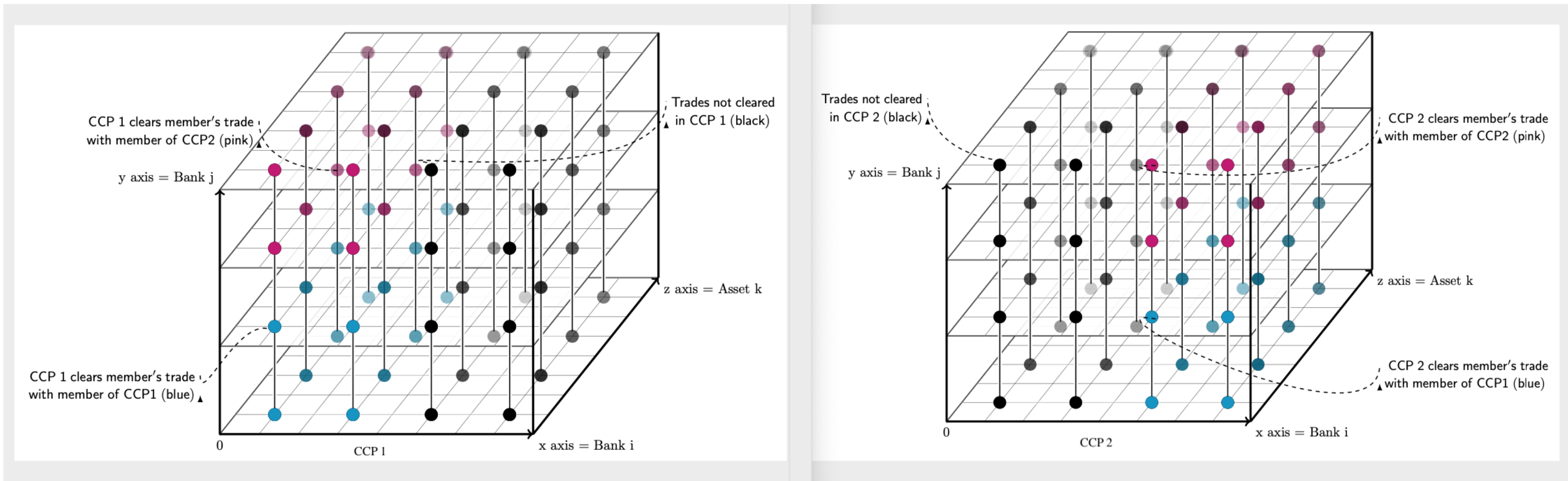


Figure 4: Illustration of network implementation of interoperability for a two CCP scenario. In the NMA scenario containing two independent CCPs, all the dealer nodes in each CCP cube would be occupied occupied, but only trades between nodes within the same cube would be executed. This figure presents an implementation of the MA scenario, containing two interoperable CCPs; each CCPs cube is half occupied by its own dealer nodes and only trades initiated by those member dealer nodes are cleared in that CCP. The remaining nodes occupying the cube represent members trades with dealers at the other CCP, which require inter-CCP clearance.

So that dealer i's cumulative daily position in each CCP for each CDS is its *net exposure*,

$$\Lambda_i^k = \sum_{u=1}^{n} \Lambda_i^{u,k} = \sum_{u=1}^{n}\sum_{j=1}^{m} L_{ji}^{u,k} - \sum_{u=1}^{n}\sum_{j=1}^{m} L_{ij}^{u,k}. \tag{2}$$

In the NMA case, where the dealer must hold membership in each CCP, the positions in CCPs u and $u' \neq u$ are independent and there are no netting benefits. Only in the MA case, where dealer i is a member of a single CCP u, will cross-cleared positions[20] be subject to inter-CCP netting. Therefore, if the variation margin is negative (positive), dealer i incurs a liability (receivable) on its account with limited liability according to,

$$[L_{ji}]^+ = \max\,[0, L_{ji}]$$
$$[L_{ij}]^- = \min\,[0, -L_{ij}]$$

Finally, the value of the net exposure for dealer i on CDS k in each CCP is determined by the size of the notional and the change in the CDS spread for k,

$$\Lambda_i^{u,k,B}(\ell\tau) = X_{ji}^{u,k,B}(\ell\tau)\,\triangle S^{u,k}(\ell\tau) - X_{ij}^{u,k,B}(\ell\tau)\,\triangle S^{u,k}(\ell\tau) \tag{3}$$

Importantly for the scope of study, this model considers only CDS transactions due to normal dealer trading activity or due to the CCPs unwinding of a dealer's position due to the dealer's default – not the default of the underlying entity on which the CDS is written. This latter case was considered as an extension in [Tywoniuk, 2017b] and still holds.

4.2 Pricing Structure and the Variation Margin

Variation margining is perhaps most salient feature of CCP clearing. This is especially important in the CDS market, where spread volatility demands the prompt exchange of daily variation margin between counterparties. Variation margin is accrued on *every* clearing members position. This is one of the 'complexities' feared by regulators in adapting interoperability agreements to OTC markets.[Board, 2016] When there is no mutual agreement, the clearing members exchange variation margin payments with their CCP; sometimes they will be liable payments to the CCP on those positions, and at other times the CCP will owe them receivables – which it has hopefully collected from the relevant counterparties. In the case of a mutual interoperability agreement, those variation margin payments may well have to be exchanged between CCPs. In this case, the CCPs act as counterparties to each other, which establishes a new channel of contagion.[Board, 2016, Bank of England, 2014] This section illustrates just how that variation structure is determined and the nuances of its exchanged in the two interoperability scenarios.

In order to determine the variation margin, this model aims to capture only the principal features of CDS spread mechanics – approximating behavioral patterns, while omitting complexities which are beyond the scope of this analysis.[21] Variation margin payments are driven by both changes in the CDS spread for an underlying k and the size of dealer i's position in that CDS contract. The largest determinant of the CDS spread is common market information about firm and industry fundamentals. This *fundamental CDS spread* component responds only to new information rather than to time. Thus, it is taken as given for each trading period, causing a permanent price impact which does not weaken over time. There are other determinants of the CDS spread which are less long-lived and cause only a temporary price impact. These, determinants such as liquidity and excess market demand, can cause large movements in CDS spreads, but with their effects often limited to a maximum of five trading days.[Tang and Yan, 2013] These temporary effects and their dissipation are incorporated into this model.[22]

[20] That is positions on behalf of dealer i which CCP u clears with CCP $u' \neq u$.

[21] The CDS spread is determined by known and unknown components, with features which are complex to model. Papers attempting an exact mathematical model of the mean-reverting CDS spread can be found in [Cont and Kan, 2011] or [O'Donoghue et al., 2014].

[22] For a thorough overview of CDS spread determinants see [Tang and Yan, 2013].

In this model, the fundamental CDS spread for a CDS k[23], is the same in each CCP. This reflects that the same common market information about entity fundamentals is available in both CCPs, regardless of market fragmentation. For each period, the fundamental spread is given by,

$$\triangle S^{u,k}(\ell\tau) = f\Big(\triangle S^{u,k}\big((\ell-1)\tau\big)\Big) \tag{4}$$

Definition 1: The fundamental CDS spread prices the fundamental value of dealer i's portfolio for each period and it's action is independent of the side of the CDS contract (buy/sell),

$$\begin{aligned} X_{ij}^{u,k,p}(\ell\tau)\,\triangle S^{u,k}(\ell\tau) &= X_{ij}^{u,k,p}\big((\ell-1)\tau\big)\, f\Big(\triangle S^{u,k}\big((\ell-1)\tau\big)\Big) \\ &= \Big[X_{ij}^{u,k,p}\big((\ell-1)\tau\big)\,\triangle S^{u,k}\big((\ell-1)\tau\big)\Big]^{+} \end{aligned} \tag{5}$$

Thus, the above gives the benchmark value of the net exposure of each dealer. This benchmark value is the value of its liability or receivable absent the effects of implied covariance caused by liquidation, predation and subsequent trading effects.[24]

Proposition 1: A trading system with one global CCP has a trading volume in that CCP which is greater than that in a multi-CCP system. Since, in any multi-CCP system there is less trading volume, there is also less ability to cause price movements.

$$X_{tot} = X_{tot}^{u} = \sum_{i=1}^{m} X_i = m * X$$

$$X_{tot}/n = m * [\frac{X^{NMA}}{n}] = [\frac{m}{n}] * X^{MA}$$

As will eventually be seen in the simulation in section 7, this decreased ability to induce price volatility through trading, also decreases the probability of dealer failure. All associated proofs for propositions, lemmas and corollaries are given in the appendix (A).

4.2.1 The Price Impact of Liquidation & Predation

Any member's default triggers a liquidation procedure in a CCP.[25] The CCP must then unwind that members position(s) over a pre-determined time frame of $t_\ell = T\tau$ periods. This time frame is based on the market liquidity risk of the position and the estimated time necessary to unwind this position in a way which minimises price impact. In this way, the CCP accounts for the price volatility which might result from trying to move a large position over a limited amount of time. The CCP uses this estimate to determine the initial margin contribution for each dealer. This initial margin will be higher if a position is concentrated in a certain direction, making it hard to move without incurring large price changes. However, when large positions are held in both directions, they can be netted against each other. Since trading a netted position incurs less price impact, this minimises a member's required initial margin contribution. In the case of multiple CCPs with NMA, there is no possible netting benefit

[23] Denotes that the CDS contract is written on an underlying entity k.

[24] The model works under the assumption that any covariance from firm or industry shocks is accounted for in the fundamental value.

[25] [Tywoniuk, 2017b] addresses the case where there is a default on the underlying asset and the seller must pay the buyer of the CDS. As the liquidation process is lengthy and remains unresolved during the short trading period in this model, it has been omitted for tractability.

across CCPs when holding opposing positions at each CCP. The only way to minimise concentration charges is to spread large positions among many CCPs. However, in the MA case, all cross-cleared positions are netted against each other. Since the netting effect changes the price impact created in each agreement scenario, it also induces differences for CDS spread trajectories between the MA and NMA case.

By unwinding positions and creating a price impact, the CCP introduces a volatility term which moves the CDS spread away from its fundamental value. Thus, the spread now transmits the effects of the excess demand or supply in the market. In the NMA case, as each CCP acts as an isolated entity, the spreads shift from fundamental value is based only on trading in that CCP alone. In the MA case, the CCPs are interconnected and spread dynamics are determined by both intra-CCP and inter-CCP trading. However, the CDS spread in each CCP feels only the impact of that half of the inter-CCP trade which is *cleared* at that CCP – at least in the short-term. This is a fact which will be explored further in this section.

This paper uses the pricing structure developed in [Tywoniuk, 2017b] – based on the work of [Glasserman et al., 2015] and [Cont and Wagalath, 2013][26]). Here, portfolio value has a volatility-like structure[27], which can be translated to a linear price impact formulation; this better reflects the effect of liquidating a CDS position in a particular CCP on the portfolio of dealer i – liquidation of an asset which is due to the default of a dealer j. Trading alters the market value of dealer i's portfolio by,

$$V_i^{u,k} = X_i^{u,k} \triangle S^{u,k}(t_\ell) = \underbrace{X_i^{u,k}\left[\triangle S^{u,k}(t_{\ell-1})\left(1 - \frac{1}{D_{u,k}}\sum_{j\in\mathcal{D}} X_j^{u,k}\right)\right]}_{X_i^{u,k}\,F(X_j^{u,k})} \tag{6}$$

$$\tag{7}$$

Here $D_{u,k}$ is the vector of market depths[28] and denotes the market liquidity in each CCP for CDS assets of type k.[29] $\triangle S^{u,k}(\ell\tau) = S^{u,k}(\ell\tau) - S^{u,k}\Big((\ell-1)\tau\Big)$ is the change in the CDS spread in the particular CCP where the trade is cleared. The $\triangle S$ moves in both the positive and negative directions, thus, the price impact may not always drag the CDS spread downwards.

The market value of dealer i's portfolio clearly depends on the CDS spread, whose trajectory depends on the type of interoperability arrangement. This arrangement affects the evolution of the spread in each CCP, so that spread dispersion may arise. For the NMA, this does not occur. All trades are executed only within one CCP, and each CCP acts like a microcosm. Thus, the value of dealer i's variation margin is computed independently for each CCP. If each CCP is subject to the same information flow, the CCPs are identical, the CDS spread evolves identically. However, in the MA case, the value of dealer i's variation margin is determined on portfolio held in its own CCP, but which contains inter-CCP positions. This portfolio's value is still determined by the CDS spread in that dealer's own CCP, but this spread is now affected by the trading dynamics of the other CCP. This is encapsulated in the following proposition (the proofs for propositions, lemmas and corollaries are in appendix A.2).

Proposition 2: If a set of dealers which are homogeneous in holdings, but different in trading strategies, were replicated among two CCPs to play a trading "game", only interoperability

[26] This latter work by Cont, empirically identifies the endogenous spurious correlations which arise from simultaneous liquidations of different funds.

[27] $X_{ij}^{u,k,p}\ \Sigma_{ij}\ X_{ij}^{u,k,p}$ based on the fact that CDS spreads exhibit covariance.

[28] It is bounded from below. It also means that the price of a CDS (written on k) moves by 1% when the net supply is equal to $\frac{D_{u,k}}{100}$.

[29] The model must have a parameter which adjusts for the illiquidity of the market. This illiquidity is not be caused by the default of an underlying reference entity on a CDS held by a dealer, but by the default of the large dealer itself.[Fleming and Sarkar, 2014]

(MA) would automatically value a dealer's portfolio differently in each CCP. Since the CDS spread evolves with trading dynamics, the trading interaction between CCPs would introduce the trading dynamics of one CCP (S^1) into the CDS spread of other CCP (S^2), and vice versa. This alters the evolution of the CDS spread in each CCP, even if each CCP starts with the same spread.

$$
\begin{array}{ccc}
& S^{1,k}(l\tau) = S^{2,k}(l\tau) & \\
& \swarrow \qquad\qquad \searrow & \\
\text{(MA)} & & \text{(NMA)} \\
\begin{cases} S^{1,k}\big((l+1)\tau\big) = \Big[S^{1,k}(l\tau),\ S^{12,k}(l\tau)\Big] \\ \\ S^{2,k}\big((l+1)\tau\big) = \Big[S^{2,k}(l\tau),\ S^{21,k}(l\tau)\Big] \end{cases} & & \begin{cases} S^{1,k}\big((l+1)\tau\big) = S^{1,k}(l\tau) \\ \\ S^{2,k}\big((l+1)\tau\big) = S^{2,k}(l\tau) \end{cases} \\
\downarrow & & \downarrow \\
S^{1,k}\big((l+1)\tau\big) \neq S^{2,k}((l+1)\tau) & & S^{1,k}\big((l+1)\tau\big) = S^{2,k}((l+1)\tau)
\end{array}
$$

Returning to the price impact, the benefit of transitioning to the formulation in 6 is that it explicitly admits the trading dynamics of each CCP. Asset movements and spread fluctuation are now mapped to a network and their trajectories can be followed. Inherently, this formulation also mathematically embeds the basic concepts of CDS covariance. Covariance between spreads is an important feature of CDS dynamics and tends to increase amongst all CDS instruments in times of crisis. The portfolio value (eq. 6), describes the effect of trades in CDS X_j^k by dealer j, on the holdings of CDS X_i^k by dealer i. Expanding on this formulation, it is possible to quantify all the existing covariance relationships between the CDS instruments in dealer i's portfolio and those in the CCP – especially those belonging to any defaulted dealer(s).[30] By mapping covariance to the CDS trading network, the covariance strength between portfolios will now depend on the proximity of the trading relationship – whether as direct counterparties (stronger effect), or relationships with weaker ties. These relationships are illustrated, for a single CCP, in the figure 5.

The CDS assets in a portfolio act as proxies for the trading relationships that dealers form with one other within the intermediation chain[31]. The various CDS assets, which comprise one portfolio, may each vary in the strength of their relationship to an external CDS asset. The strength of this relationship becomes important when it pertains to a defaulted CDS asset, undergoing unwinding/liquidation. As the proximity of the trading relationship decreases, covariance increases. In particular, when the trading proximity to a defaulted CDS asset decreases, negative externalities increase.[Paddrik et al., 2016]

CDS asset dynamics in turn affect the spread dynamics, and so asset unwinding will affect the CDS spread. This again depends on the interoperability agreement between CCPs. In the NMA case, asset unwinding occurs autonomously in each CCP. In the MA case, the process of asset liquidation occurs in the CCP where it was cleared, altering the spread dynamics in that CCP. Since the spread prices a CDS instrument, this results in an asymmetrical price effect on assets in each CCP. Moreover, dealers are more likely to trade with dealers in their own CCP, leading to a larger proportion of defaulter-related trades inside that CCP[32]. Thus, assets within the defaulter's CCP are more directly exposed than those traded across the interoperability link[33].

[30] As explained in [Tywoniuk, 2017b]; "As opposed to using an empirical *covariance matrix* between all assets, one can isolate the covariance effect with the defaulted assets alone, since these are the ones which will be liquidated and cause multiple price impacts."

[31] Ie. two dealers can be either direct counterparties for a CDS asset, or be customers of the same dealer for the same CDS asset, or be two unrelated holders of the same CDS asset.

[32] Intra-CCP trades are more likely for many reasons, ie. search cost, lower collateral requirement, etc.

[33] Each CCP has an asset where only a proportion is traded across the link, thus there is a different amount of the asset in each CCP. If an asset is liquidated by a counterparty which did not trade it across the link, liquidation affects

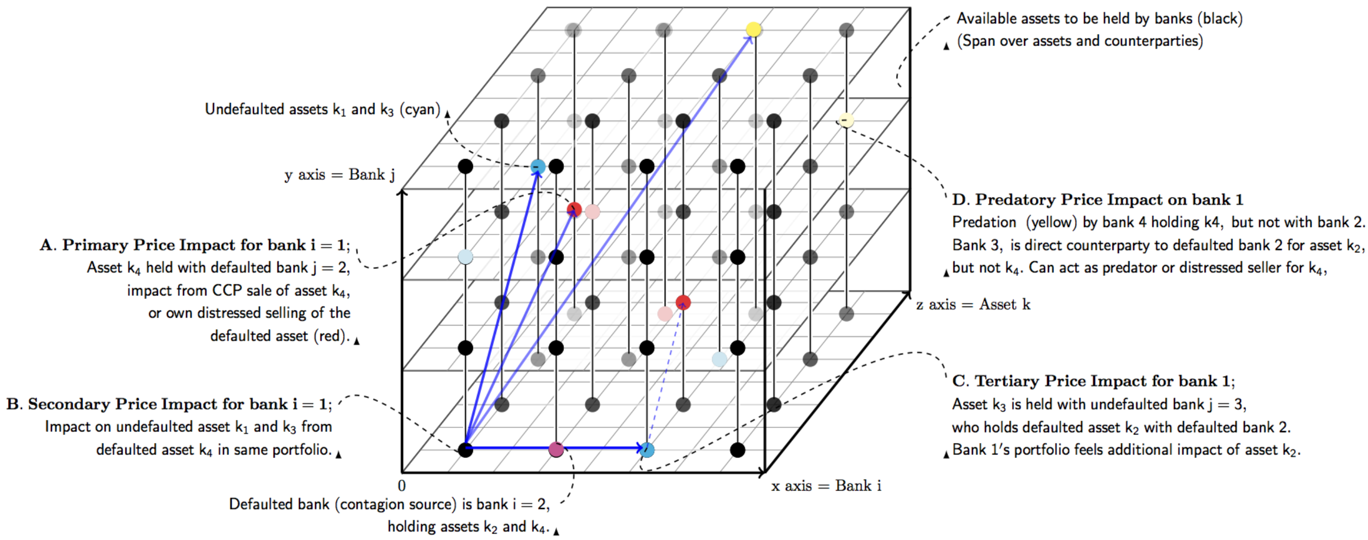


Figure 5: Map of dealers' covariance relationships in a financial network in terms of asset holdings and their relative distance from defaulted assets.

Since the trading proximity between CDS assets determines the price impact they exert on one another, price impacts can be re-written in terms of subsets of the different trading relationships. Thus, a subset of instruments which are direct counterparties will exert the strongest price impact on each other. As relationships become more distant, the price impact will weaken. A detailed explanation of each price impact is outlined in [Tywoniuk, 2017b]. The following will outline the separation of network trades into different price impacts, based on proximity to a defaulter's assets, in the context of asset unwinding/liquidation. I construct these subsets as primary, predatory, secondary and tertiary impacts, which act with weakening strength, respectively, based on their covariance relationship with a defaulted asset. This formulation is tied to the work of [Minca and Amini, 2012, Cont and Wagalath, 2014] who show that price feedback effects from a distressed liquidation (fire-sale) uncover hidden exposures between two dealers, based on common holdings in their portfolios. These exposures can be much larger than any direct exposures they have to each other because of the network of interdependent trading relationships[34].

For illustration, it is easiest to understand price impact from the perspective of dealer i. Here, an asset held in i's portfolio is viewed with respect to an asset being unwound by the CCP, on behalf of a defaulted dealer j. Assume that among all of dealer i's counterparties, j is the lone defaulted dealer. The following price impact types can occur:

The primary price impact; the effect on dealer i's holding of an asset k, as a counterparty to a defaulted dealer j, where the CCP is unwinding asset k. This term identifies the CDS spread change for CDS k caused by: the CCP's liquidation, distressed counterparty selling or selling a large position for other reasons (through the liquidation rate a_i^k assigned to each dealer).

The predatory price impact; captures the effect on dealer i's holding of an asset k from the predatory selling by another dealer i', who is also holding CDS k, but is not a *distressed* – meaning, not a direct counterparty to defaulted dealer j. This dealer can choose to liquidate along with the CCP, under the assumption that it will re-purchase these positions at a lower price later in the

counterparties in one CCP with more strength and causes more volatility. This mis-pricing between CCP's ensues because the search market permits little price transparency and little dealer opportunity to exploit competitive pricing.

[34]Thus, the assumption that this can be used as an incentive effect and cause a default cascade is validated.

future. That is, when all liquidation has ceased, during the buyback period, when it can make a profit.[35]

The secondary price impact; captures the covariance effect on dealer i's holding of an asset k, by another asset k' within the same portfolio, where k' is being unwound on behalf of the default of dealer j. Any trading decisions of dealer i for asset k' (due to its relationship with j) will affect its balance sheet, and subsequently, its trading decisions for asset k.

The tertiary price impact; captures the implied covariance effect on dealer i's holding of an asset k, due to the selling of a distant asset k' on behalf of defaulted dealer j, not held in dealer i's portfolio. This occurs through a solvent intermediary j', holding both assets $\{k', k''\}$ in its portfolio[36]. This implied covariance is observed during times of crisis (ie. fire-sale liquidation) between previously seemingly uncorrelated assets. This term, therefore, quantifies the spillover effect on the portfolio of dealer i caused by an indirect link.[Paddrik et al., 2016] Note that in the MA case, this spillover occurs on a wider scale, between CCPs – possibly spreading contagion over the link.

Each subsequent prices impact (decreasing in strength) is incorporated into the spread information flow with a time lag. The choice of four price impacts and three trading periods is not arbitrary. In [Tang and Yan, 2013], the authors show that a significant part of CDS spread movement is driven by changes in excess market demand/liquidity. This demand effect is short-lived/transitory and dissipates after, at most, three trades (an initial trade plus two time lags). In turn, four price impacts appear to sufficiently capture this temporary demand within the associated trading time. Thus, this temporary spread component, cumulatively over each period, augments the fundamental spread component.

This time lag also reflects the structure of the OTC market, which is an opaque search market[37] where participants gain information slowly; they move through the market obtaining non-binding bilateral quotes, dealer by dealer[38].[Duffie et al., 2017] This causes a delay in the speed at which distant market information is incorporated into prices. Due to this delay and price opacity, dealers also cannot readily observe the price impact of their own trading on the market. More importantly, predators remain unaware of any detrimental effects on their own portfolio relating directly to their predatory behaviour[39] – a fact which can be exploited in the recovery period to devise a disincentivise predation.

The model now accounts for price-impact, predation and covariance, but cannot yet describe the effect caused by any liquidation dynamics. This is achieved by incorporating the trading rate per discrete trading period, τ, into eq. 6. I assume that during each trading period, a dealer trades moves fraction of its position, $\Delta X/X$, at a trading rate of $a_j^k = \Delta X_j/\tau$. Then, taking a fraction of the position each period, $(a_j^k \tau/X) \cdot X$, results in the expression,

$$F(X_j^{u,k}\tau) = \triangle S^{u,k}(t_{\ell-1}) \left(1 - \frac{1}{D_{u,k}} \sum_{j\in\mathcal{D}} a_j^{u,k}\tau\right) \tag{8}$$

where $F^{u,k}$ is the general price impact function for CDS k in CCP u. The term $a^{u,k}$ for the liquidation rate per period now captures the direction and strength of trading, and its significance on price impact.

[35] A dealer that is a direct counterparty to a default, is considered *in distressed*; while the CCP liquidates the defaulted dealer's positions, the distressed dealer doesn't have a choice whether to liquidate, especially if facing illiquidity. This dealer will certainly not be in the position to engage in a buyback strategy after liquidation. However, if a healthy dealer holds asset k with a healthy counterparty, it is free to choose its strategy. Thus, it can engage in predatory liquidation.

[36] Imagine that i holds $\{k\}$, j' holds $\{k, k''\}$, j'' holds $\{k'', k'\}$ and the defaulter j holds $\{k'\}$. The liquidation of k' affects the trading decisions of j'' and j' for k'', and j' fearing negative downstream effects also moves k. Therefore, the liquidation of k' affects i's portfolio holding of k.

[37] The counterparties negotiating any CDS contract cannot see the whole market (market prices) at any one time, instead they must move through the market to gain information.

[38] [Duffie et al., 2017] show that because search is costly, traders' optimal strategy will is to obtain one quote before they commit to a trade; increasing search time increases the probability of returning to a dealer quote and finding it's no longer competitive.

[39] Since predators are as any other member dealer, they do not realise that the experienced fluctuations in CDS spread due to their predatory behaviour, and instead assume this is caused by some unexplained component of CDS spread fluctuations.

Rewriting the portfolio value in terms of a series of subsequently weaker price impacts captures the spirit of a series expansion. The portfolio value can now take on the structure,

$$V_i^{u,k} = \frac{1}{0!} X_i^{u,k}\, F(X_j^{u,k}) \;+\; \frac{1}{1!}[\, X_i^{u,k}\, F'(X_j^{u,k}) \;+\; X_i^{u,k}\, \mathcal{F}'(X_j^{u,k})] \;+\; \frac{1}{2!} X_i^{u,k}\, F''(X_j^{u,k}) \;+\; \frac{1}{3!} X_i^{u,k}\, F'''(X_j^{u,k}) \tag{9}$$

where rather than taking the series of F(X) explicitly, the weakening price impact of higher order terms is captured endogenously by network effects. A dealers portfolio is now valued as a product of network trading,

$$\triangle S^{u,k}(t_\ell) = \underbrace{[\, \triangle S^{u,k}(t_{\ell-1})\,]^+}_{P_0} + P_1\, a_j^{u,k}\tau \;+\; \mathcal{P}\, a_j^{u,k}\tau \;+\; P_2\, a_j^{u,k}\tau \;+\; P_3\, a_j^{u,k}\tau \tag{10}$$

where P_0 is the fundamental long term value of the CDS spread, and $P_1,\ P_2,\ P_3$ are the primary, secondary and tertiary price impacts, respectively. $\mathcal{P}$ is the predation impact, and a_j^k is the liquidation rate per period, τ. Equation 10 is the reduced form for the *CDS spread pricing function*[40] which acts on the portfolio holdings of dealer i – $\sum_{u=1}^n X_i^{u,k}\, \triangle S^{u,k}$ – to produce the total value of each asset k, $\sum_{u=1}^n V_i^{u,k} = \mathbf{V_i^k}$.

Proposition 3 gives the full mathematical term for $V_i^{u,k}$, accounting for the actions of dealers in the network[41]. The first term captures the position's fundamental value, approximated by the CDS spread change from the previous trading period. The second and third terms combine to give the primary price impact: the second term describes the CCP's unwinding of a defaulter's asset[42], while the third term captures trading by dealer i, as a counterparty for that asset – these are distressed trades[43].The fourth term captures any the predatory selling – selling by any non-counterparty, non-defaulted dealers j' for the asset k. The fifth term measures the secondary price impact on dealer i's holding of asset k, caused by the liquidation of any other portfolio assets k', with which this asset covaries[44]. The sixth term is the tertiary price impact, accounting for the indirect price impact on k by the trading of assets which dealer i does not hold directly, but which covary with assets inside its portfolio – a distant spillover effect from others in the network.

In valuing the portfolio, the pricing functional essentially sums over all asset trading relationships in the network. In the NMA case, the total value of a dealer's portfolio is the result of two autonomous valuations for each CCP. This is simply a summation – $V_i^k = \sum_{u=1}^n V_i^{u,k}$ – in each CCP. In the MA case, this summation will act over assets traded between multiple CCPs – for which there will be a netting effect – however, this results in both autonomous CCP valuation terms and additional inter-CCP valuation terms for the transactions made across the link(s). This important difference, detailed in future sections, requires the determination of a dealer inter-CCP margin contribution to a mutual guarantee fund, M – needed to collateralise inter-CCP transactions.

[40] An explicit form is provided in the appendix A.

[41] [Tywoniuk, 2017b] provides in-depth explanations for every term in the proposition

[42] The two terms capture how the CCP can move assets which are held between non-defaulters and defaulters or between two defaulters.

[43] When i is a counterparty to a defaulter for asset k, then i can make distressed trades for k on what it holds with that defaulter and what it holds with non-defaulters.

[44] These secondary assets, incur a stronger price impact when traded with a defaulted counterparty.

Proposition 3: The value of a dealer's portfolio is determined by the size of its holding of an asset and the various degrees of covariance relationships that asset has with liquidated assets in the market, through the pricing functional.

$$
\begin{aligned}
\sum_k X_{ij}^{u,k}(\ell\tau)\,\triangle S^{u,k}(\ell\tau) \;&=\; \sum_k \Big(X_{ij}^{u,k}((\ell-1)\tau) \;+\; a_{ji}^{u,k}\tau\Big)\,\triangle S^{u,k}(\ell\tau)\\
&= \sum_k \Bigg\{ \underbrace{[\,X_{ij}^{u,k}((\ell-1)\tau)\triangle S^{u,k}((\ell-1)\tau)\,]^+}_{fundamental\,CDSspread} \\
&\textit{primary price impact} \left\{ \begin{array}{l} + \underbrace{\Big(\sum_{j\in\mathcal{D}} \left|\frac{X_{ij}^{u,k}}{X_{ij}^{u,k}}\right| X_{ij}^{u,k} + \varepsilon \sum_{j'\notin\mathcal{D}} \left|\frac{X_{ij'}^{u,k}}{X_{ij'}^{u,k}}\right| X_{ij'}^{u,k}\Big) \sum_{i'=1}^{m} |\triangle S^{u,k}((\ell-1)\tau)| \Big(\frac{a_{ji'}^{u,k}}{D_{u,k}}\Big)}_{CCP\ liquidation} \\[2ex] + \Big(\underbrace{\sum_{j\in\mathcal{D}} \left|\frac{X_{ij}^{u,k}}{X_{ij}^{u,k}}\right| X_{ij}^{u,k}\sum_{j\in\mathcal{D}}}_{distressed\ selling} + \underbrace{\varepsilon\sum_{j'\in\mathcal{D}} \left|\frac{X_{ij'}^{u,k}}{X_{ij'}^{u,k}}\right| X_{ij'}^{u,k}\sum_{j\notin\mathcal{D}}}_{distress/predation}\Big)|\triangle S^{u,k}((\ell-1)\tau)|\Big(\frac{a_{ij}^{u,k}}{D_{u,k}}\Big) \end{array}\right.\\
&+ \underbrace{\varepsilon \sum_{j=1}^{m} \left|\frac{X_{ij}^{u,k}}{X_{ij}^{u,k}}\right| X_{ij}^{u,k} \sum_{j'\notin\mathcal{D}}\sum_{i'=1}^{m} |\triangle S^{u,k}((\ell-1)\tau)| \Big(\frac{a_{j'i'}^{u,k}}{D_{u,k}}\Big)}_{predation}\\
&+ \underbrace{\Big(\frac{1}{2!}\Big)\Big(\Big(\frac{3}{2!}\Big)\sum_{j\in\mathcal{D}} \left|\frac{X_{ij}^{u,k}}{X_{ij}^{u,k}}\right| X_{ij}^{u,k} + \sum_{j'\notin\mathcal{D}} \left|\frac{X_{ij'}^{u,k}}{X_{ij'}^{k}}\right| X_{ij'}^{u,k}\Big) \sum_{k'}\sum_{j=1}^{m} \left|\frac{X_{ij}^{u,k'}}{X_{ij}^{u,k'}}\right| \sum_{i'=1}^{m} |\triangle S^{u,k'}((\ell-2)\tau)|\Big(\frac{a_{ji'}^{u,k'}}{D_{u,k'}}\Big)}_{secondary\ price\ impact}\\
&+ \underbrace{\Big(\frac{1}{3!}\Big)\Big(\Big(\frac{9}{3!}\Big)\sum_{j\in\mathcal{D}} X_{ij}^{u,k} \sum_{k''=1}\left|1-\frac{X_{ij}^{u,k''}}{X_{ij}^{u,k''}}\right| + \sum_{j\notin\mathcal{D}} X_{ij}^{u,k} \sum_{k''=1}\left|1-\frac{X_{ij}^{u,k''}}{X_{ij}^{u,k''}}\right|\Big) \sum_{i'=1}^{m} |\triangle S^{u,k''}((\ell-2)\tau)|\Big(\frac{a_{ji'}^{u,k''}}{D_{u,k''}}\Big)}_{tertiary\ price\ impact}\Bigg\}
\end{aligned} \tag{11}
$$

Note that absent any trading the term $a_{ji'}^{k}$ is reduced to zero. The epsilon provides a dampening effect for any liquidated assets which are held with undefaulted counterparties[45]. The terminating D_k^u in each line captures the contribution of current market liquidity scenario in each CCP.

[45] The fraction terms preceding higher order terms also explicitly give a dampening effect for the purpose of the simulation. However, to capture any such effect truly would likely require empirical estimation.

4.2.2 Liquidation Rate & Predation

The following section summarizes the methods and motives for predation, in addition, it outlines how the liquidation rate is determined. This is more fully detailed in [Tywoniuk, 2017b], which adapts and expands the framework of [Brunnermeier and Pederson, 2005] to accommodate the OTC market. Unlike the original framework which assumes an exchange market with price transparency, a marked feature of the opaque OTC market is a noisy price process. This opacity delays the integration of information – about market actions and the consequences of those actions – into market prices and prevents a strategically optimal response by traders. Though this does not change the method of predation, it changes the outcome of predation; since predators cannot gauge the timely effect of their actions, they risk becoming victims of their own predation. A fact which will be described throughout the paper.

A CCP belongs to a set of I dealers which comprise the CCP network, where the CCP is dealer i_0. If the CCP becomes distressed, through a dealer default, it must unwind its positions. Any dealer counterparty to a defaulter, is by definition constrained. Any other dealer in the set, being unconstrained, can choose to engage in predation for the asset(s) being unwound. The CCP itself is part of a larger network of CCPs comprising the CDS market[46]. A CCP witnesses all trading inside its CCP network. However, it cannot see the trading activity at any other CCP in the network, and where any inter-CCP trades exist, only those inter-CCP trades are directly witnessed[47]. Thus, each network CCP sets its own market trading rate autonomously.

Considering that any dealer can default on a CDS contract, CDS k is considered to be a risky asset. In this network, each large dealer is strategic and holds a position in this risky asset k,

$$X_i^{u,k}(\ell\tau) = X_i^{u,k}(0\tau) + \sum_{\ell=1}^{T} a_i^{u,k}\ell\tau, \tag{12}$$

All the large dealers being homogeneous in their size and balance sheet capacity, are subject to the same regulatory restrictions[48]. However, since these dealers are heterogeneous in the composition of their portfolios, they will differ in their trading rates[49]. Eq.12 provides the intuition for the dealers' trading mechanism given by the following definition,

Definition 2: Trading by dealer i changes its buy/sell position in any asset k according to,

$$X_{ij}^{u,k}(\ell\tau) = X_{ij}^{u,k}\big((\ell-1)\tau\big) + \mathbf{a_{ij}^{u,k}} \quad \text{where} \quad \begin{cases} a_{ij}^{u,k,+} = +a_{ij}^{u,k} & a_{ij}^{u,k,+} > 0 \\ a_{ij}^{u,k,-} = -a_{ij}^{u,k} & a_{ij}^{u,k,-} < 0 \end{cases}$$
$$\text{where} \quad \hat{X}_{ij}^{u,k}\ (\text{NMA}) \quad \text{and} \quad \bar{X}_{ij}^{u,k} = \sum_{u'=1}^{n} X_{ij}^{uu',k}\ (\text{MA}). \tag{13}$$

At time $t_\ell = 0$, all dealer contracts are set. At time $t_\ell = 1$, all the dealers' liabilities[50] are realised. If there is a defaulter, the CCP chooses to unwind its positions at this time – initiating the liquidation phase. The CCP will always choose to unwind its positions such that it incurs minimal price impact. Therefore, it will spread its trading over all possible periods, trading for the full $T\tau$ periods[51]. This restricts the choice of its liquidation rate, setting it equal to, or below, the average trading rate of the

[46] This view maintains the idea that the CDS network is a highly fragmented market.

[47] Instead, the CDS spread in the CCP indirectly reflects the trading activity.

[48] The regulatory restriction on maximal dealer holdings prevents any dealer from holding enough of any asset k to engage in price manipulation. This implies that each dealer dealer i is restricted to hold, $X_i^k \in [-\bar{X}, \bar{X}]$.

[49] Dealers, though of similar size, can each hold a different mixture of CDS assets in their portfolios. These will be exposed to differing dynamics, and push traders to adopt different trading rates in response.

[50] Receivables are just negative liabilities.

[51] In order to balance the market risk and liquidity risk of holding a defaulters position. The CCP is contractually required to employ this prudent method.

network. This is only possible if network transactions are transparent. However, since the CCP clears every trade in the network, it obtains this information.

The total trading of all dealers, at a given CCP, is captured by the total trading rate over that network,

$$A^{u,k} = |\sum_{i}^{I} a_i^{u,k}| \tag{14}$$

So that each CCP liquidates according to the network rate,

$$a_i^{u,k,p,-}(\ell\tau) \leq -\frac{A^{u,k}}{I^{u,k}}(\text{selling}), \quad a_i^{u,k,p,+}(\ell\tau) \geq \frac{A^{u,k}}{I^u}(\text{buying}). \tag{15}$$

It liquidates at this constant speed for the full liquidation phase, over $\tau\frac{X_{t_0}^{u,k}}{A^{u,k}/I^u}$ periods. CCP regulation sets this liquidation phase at $T\tau = 5$ days, so that each trading period is $\tau = 1$ day. Thus, the CCP's smooth trading strategy and regulation determine $A^{u,k}$. Each network CCP employs this rational strategy; differences between CCP strategies differ only in the amount which each CCP must unwind, and thus, liquidations of the same CDS instrument can yield different asset values in each CCP.

Any dealer(s) who engages in a profit-maximizing strategy, at the expense of other dealers, are deemed 'predators'. These predators are simply motivated (and able) to liquidate at some point of profit[52], which restricts the predatory strategy to the set of unconstrained, non-distressed dealers. The predators' extreme profit motives mean that after liquidation, they will attempt to buyback their maximum allowable holding. In choosing their predatory strategy, predators choose that which minimizes their own price impact, given the optimal trading strategy of other dealers. Unfortunately, others trading strategies are not perfectly visible to predators, as for the CCP. In a transparent market, predators could use price information to deduce trading strategies. However, in the opaque CDS market, the noisy price process only allows the predators to pin down the fundamental value of assets. Thus, predators are forced to choose a homogeneous[53] trading rate among them. Note that all predators' inter-CCP trades are performed between CCPs, and so the predator remains blind[54]to the amount liquidated or rate of trading in other CCP(s).

Predatory attempts are highly affected by the number of predators in the market. The presence of only one monopolistic predator, or multiple predators colluding as one, results in the predators' continuous trading until the CCP has finished liquidating, at a homogeneous predatory trading rate. Since the market is opaque, predators are only able to track the CCP's trading rate. However, they assume that all others in the market are doing the same, leading to same choice of trading rate. This homogeneous trading rate then determines that predators buy back their positions at the rate $A^{u,k}$ for $\frac{\tilde{X}^{u,k}}{A^{u,k}}$ periods during $t_\ell = 2$. However, if there are multiple, independent predators, a competition effect appears: predators are forced to stop liquidating earlier than the CCP, and start buying back prematurely[55]. Thus, competitive predators are already buying back during the liquidation phase $t_\ell = 1$, rather than the buyback phase $t_\ell = 2$, reducing their possible profits. The effects and dynamics of inter-CCP predation will be discussed in later sections.

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[52] A point of profit refers to a point where predators can sell at a high price, and expect to buy back at a low price.

[53] Predators can only see CPPs conservative network trading rate. As well, they can only see the same fundamental value of assets from which to deduce the same price/trading information. This shared information constrains them to choose the same trading rate.

[54] Since the predator cannot adjust his trading rate using information in the other CCP, he cannot truly predate in that CCP.

[55] For a deeper explanation of this effect see [Brunnermeier and Pederson, 2005]

5 Model Mechanisms

To capture the outcomes of a network model, the static model of the previous section must be extended to reflect the dynamics of the CDS market; both trading dynamics and interactions with the market structure. Thus, this section will define the flow of liabilities and receivables within and between[56] CCPs. In turn, these balance-sheet flows capture trading outcomes and determine the effects of predation.

The CDS market dynamics are comprised of a number of subtle features. Of fundamental importance, is the interplay between the CDS spread and market demand. This interplay determines the variation margin, and in turn, the liabilities for each dealer. This then, will also have implications for the possible appearance of price dispersion between CCPs.

The CDS spread for an instrument (k) is determined by the combination of the underlying firm's fundamentals and market demand for CDS k. Demand is the result of dealer quotes for any buy or sell-side positions. A quote for each side of the position has different effects on the price movements of the CDS spread and resulting trading dynamics.

For a sell-side position, a decrease in quotes or price implies a decreased demand for insurance on the underlying reference entity, k. This decreased demand, or drop in credit spread, reflects the markets decreasing belief in the creditworthiness of the underlying. If a dealer holds the sell-side, a decrease in CDS price increases the dealers liabilities, and in turn, its variation margin. The submission of a sell quote signifies decreased demand, but the submission of a buy quote increases demand for sell-side positions.

On the other side, demand for buy-side positions has the same effect, except for one small difference. The submission of a buy quote shows an increased demand for insurance on the underlying and temporarily lowers the CDS spread of that reference entity. At the same time, a buy position can be sold off (indicating increased confidence in the creditworthiness of the underlying, which increase the CDS spread and the liability of any dealer that still holds the buy position.

5.1 Liabilities, net exposure, and shortfall

Having established the fundamentals of the model, the following introduces the manner by which dealers interact with, and within, the market structure. This addition, captures the ongoing effects of trading on CCPs' and dealers' net-worths. The result is a multi-period, dynamic trading model which accommodates multiple assets, variation margin exchange, price impact effects and distress amplification (caused by predation). Moreover, the model captures the influence of the CCP interoperability agreement type on each of these aspects.

The CDS market is largely defined by the structure of its CCPs and the type of interoperability agreements in place. Thus, the CCPs fund structure critically determines the evolution of network trading dynamics. In this model, each CCP possesses the current real-world guarantee fund structure, regardless of the agreement scenario. A standard guarantee fund (G_{tot}) is comprised of the proprietary initial margin contributions (g_i) of each member-dealer such that $G^u_{tot} = \Sigma^M_{i=1} g_i$. This initial margin charge is calculated based on the size, direction and risk of a dealer's position. A further smaller contribution (d_i) is required for a common, risk-sharing default fund (D_{tot}). This means that each default fund contribution (d_i) can be applied against any other dealers shortfall. Both these contributions are mandatory for membership. If the MA framework is in place, a dealer must provide one additional mandatory contribution. This margin contribution (m_i), placed in the mutual guarantee fund (M_{tot}), is meant to secure inter-CCP trades; these dynamics will be discussed in the next section.

In the model, as in reality, dealers interact with the gurantee fund in a dynamic way. A dealer may

[56] In the case of a mutual agreement (MA).

need to access the guarantee fund if its liabilities exceed its assets[57]. However, the dealer must first liquidate its total external assets, (Q_i), after which it can access its initial guarantee contribution (g_i). If these funds prove insufficient, the dealer can further access its own default fund contribution (d_i). These actions represent the first steps in a CCP's Default Waterfall.[Group, 2015] The CCP must also contribute to this waterfall. The CCP charges a clearing fee to dealers, and with this flat fee (f^u), it earns a profit on the volume cleared – this comprises its equity. A small tranche (ϵ_0^u) of the CCP's equity becomes the next step in the CCP waterfall.

The inclusion of these and the following trading mechanics, progresses the model from one which simply describes dealers' one-sided liabilities, to one which can determine dealers' overall net-worths. The aim of the model is to capture the evolution of network dynamics, trade-by-trade, dealer-by-dealer. It also relies on an extension of the framework from [Tywoniuk, 2017b, Amini et al., 2015], which consists of four trading phases: *Initial* (t=0), *Liquidation* (t=1), *Buyback* (t=2) and *Resolution* (t=3). Using this framework, the model evolution is best described by tracing the trading actions of a dealer which holds a the sell-side CDS position. This side of the position is more illustrative, as it is subject to wrong-way risk[58].

Consider a CDS market where CDS are written on $k \in \{1...K\}$ different references entities. Dealer i holds a mixture of buy and sell side positions on different CDS k. Naturally, the other side of these positions must be held by some other dealer(s) $j \neq i$. These trading contracts must be established during the initial phase, before trading start. The liabilities and receivables, which are the outcome of trading in these contracts, are then realised during the liquidation period. At this time, each dealer's account is comprised of assets ($\mathcal{A}_i$) and liabilities ($\mathcal{L}_i$), which can be represented by a stylised balance sheet[59] as,

$$\mathcal{A}_i\ (t_{1\tau} = 1) = \sum_{u=1}^{N} \sum_{k=1}^{K} \left(\underbrace{\gamma_i}_{cash} + \underbrace{\sum_{j=1}^{m} L_{ji}^{u,k}}_{receivables} + \underbrace{Q_i}_{external\ assets} \right) \tag{16}$$

$$\mathcal{L}_i\ (t_{1\tau} = 1) = \sum_{u=1}^{N} \sum_{k=1}^{K} \left(\underbrace{\sum_{j=1}^{m} L_{ij}^{u,k}}_{L_i{}^{u,k}} + \underbrace{\left(\gamma_i + \sum_{j=1}^{m} L_{ji}^{u,k} - L_i^{u,k}\right)}_{nominal\ net\ worth} \right) \tag{17}$$

During each trading period, in each CCP, dealer i's total net exposure is calulated. The net exposure is the sum of all the dealer's receivables minus the sum of all it's liabilities,

$$\Lambda_i^S(\ell\tau) = \sum_{u=1}^{N} \sum_{k=1}^{K} \Lambda_i^{u,k,S}(\ell\tau) = \sum_{u=1}^{N} \sum_{k=1}^{K} \left(\sum_{j=1}^{m} L_{ji}^{u,k,S}(\ell\tau) - \sum_{j=1}^{m} L_{ij}^{u,k,S}(\ell\tau) \right) \tag{18}$$

Each dealer's liabilities and receivables remain CCP specific, especially when no mutual agreement between CCPs exists. Thus, dealer may have shortfalls in multiple CCPs. When this occurs, it still has only a single pool of cash and external assets (Q_i) from which to settle its debts. A dealer becomes *illiquid* if its liabilities exceed its receivables, but it becomes *insolvent* if its liabilities further exceed its external assets and guarantee fund contribution. An insolvent dealer is in default; that dealer loses all trading privileges and the CCP take on its positions, to be unwound/liquidated. The resulting constraints expose the CCP the possibility of predatory trading by liquid dealers. Furthermore, the CCPs constraints may be tightened by the possibility of illiquid (though solvent) dealers engaging in distressed selling. Once again, interoperability agreements affect the evolution of outcomes. In

[57]This can occur suddenly as a trading outcome. Positive liabilities represent variation margin which must be paid by the following morning, at the latest.

[58]The risk of incurring asymmetrically large liabilities in the event of default of the underlying entity on that CDS. The seller must then settle claims for nominal value of the buy-sides position.

[59]This form is commonly used in this literature and detailed in ([Amini et al., 2015, Cont et al., 2013, Minca and Amini, 2012]).

the NMA case, dealer i's trading is fragmented between two CCPs, limiting contagion to each CCP. In the MA case, where distressed or predatory trading can occur across the inter-CCP link, in effect, makes each CCP a possible conduit of contagion. The link becomes a pathway for distress amplification.

Constrained asset sales in a network can become a dangerous incentive for predation. As distressed sales of a CDS k begin to lower the value of that asset, rational dealers holding the asset may choose to limit their exposures by also selling that asset. This further lowers the price. If some dealer chooses to hold onto k while others sell, it remains fully exposed to the price impact of that network's liquidation (proof available appendix A.2). Thus, if dealers choose their optimal strategy given the optimal strategy of other dealers, then when others are selling, a dealer's optimal strategy is to also sell. The following proposition extends this idea to predation.

> *Proposition 4: If one dealer chooses to predate for asset k, then any dealer holding the same assets is always better off by predating for that same asset.*

Trading occurs during the Liquidation phase, at $t_\ell = 1$, consisting of multiple daily trading periods, $\ell \in \{1, ..., T\}$. At the end of each trading period, dealer i's liabilities and receivables are realised, and the net exposure is calculated. As liquidation phase evolves, this net exposure cumulates over multiple trading days; thus, the dealer can become illiquid or insolvent at any point. A trading period can leave a dealer i with a positive net exposure. That dealer, having cleared those positions in through a chosen CCP(s), must pay a its volume-based flat fee. This CCP now has a liability to dealer i on the cleared value given by,

$$L_{0i}^u = (1 - f^u)\Lambda_i^{+,u} \quad \text{s.t.} \quad L_0^u = (1 - f^u)\sum_{i=1}^{m} \Lambda_i^{+,u} \tag{19}$$

which it must return (net of its fee) at the end of the relationship. Where no interoperability exists (NM), the dealer is charged a clearing fee in each CCP. However, CCP inter-operability (MA) allows the dealer only pay a fee on what is cleared in his own CCP.

If a trading period leaves a dealer i with negative net exposure ($\Lambda_i^-(\ell\tau)$), liabilities exceed receivables, that dealer also has a liquidity problem. It owes a liability to the CCP, $L_{i0}(\ell\tau) = \Lambda_i^-(\ell\tau)$, which the dealer must meet by first liquidating its total external asset, Q_i, for which it receives a reduced liquidation value R_i[60]. If insufficient, the dealer further uses its initial margin, g_i. Interoperability affects how much the dealer needs to liquidate. In an NMA structure, when the combined initial margin and liabilities from each CCP exceed the cash holdings of dealer i, it must liquidate the minimum proportion of its external asset that satisfies this debt (with limited liability). The total that amount that dealer i needs to liquidate is,

$$\hat{Z}_i = \frac{\left(\gamma_i - \sum_{u=1}^{N}(g_i^u + \Lambda_i^{-,u})\right)^-}{R_i} \wedge 1 \tag{20}$$

I continue to employ the notation the hat notation, $\hat{Z}$ for a system without CCP interoperability (NMA) and the bar notation $\bar{Z}$ for interoperability (MA). This is not the case in the MA structure, as rather than multiple initial margins, the dealer must instead pay a single initial margin g_i and an additional mutual margin, m_i for any inter-CCP transactions. Thus, in the MA structure, the amount which dealer i needs to liquidate is,

$$\bar{Z}_i = \frac{\left(\gamma_i - (g_i^u + m_i^u + \Lambda_i^{-,u})\right)^-}{R_i} \wedge 1 \tag{21}$$

where u denotes only the particular CCP to which dealer i belongs.

Dealer i's ultimate share in the guarantee fund of the CCP which holds its liability, is dependent

[60]For simplicity, one can make the assumption that the external asset is a different asset class, and that all dealers clear/sell it in a separate but common CCP fetching a common reserve price

on the realised value of its net exposures and its external asset. Therefore, its share will differ between the two inter-operability structures. In the NMA structure, each CCP maintains the standard initial guarantee fund denoted as $\hat{G}_tot$, Thus, with trading the NMA guarantee fund contribution changes according to,

$$\hat{G}_i^u = \Big(\sum_{u=1}^{N} \Lambda_i^u + \gamma_i + R_i - \sum_{u'=1}^{N-1} g_i^{u'}\Big)^+ - \Big(\sum_{u=1}^{N} \Lambda_i^u + \gamma_i + R_i - \sum_{u'=1}^{N-1} g^{u'} - g_i^u\Big)^+ \quad \text{(NMA)} \tag{22}$$

Here, $\hat{G}$ depends on the net exposures and initial margins charged at *other* CCPs, which can deplete the cash or external margin reserves of dealer i. With interoperability, the initial guarantee fund structure is a combination of the standard fund and the mutual inter-CCP fund, denoted respectively as $\bar{G}_{tot}$ and M_{tot}. In turn, the MA guarantee fund contribution changes according to,

$$\bar{G}_i^u = \Big(\Lambda_i^u + \gamma_i + R_i\Big)^+ - \Big(\Lambda_i^u + \gamma_i + R_i - g_i^u\Big)^+ \quad \text{(MA)} \tag{23}$$

It differs because dealer i's guarantee fund contribution is made well before the mutual margins are determined – before trading is initiated. Thus, the guarantee fund contribution remains independent of the future mutual margins that must be paid out to other CCPs u'.

It is not readily apparent which system allows for a larger guarantee fund contribution for dealer i to the CCP. In the NMA scenario, a dealer's concentrated CDS positions can be spread among multiple CCPs in order to minimize the margin (g_i^u) charged in each CCP. In the MA scenario the positions are netted and can lead to a lower margin charge. If in both the NMA and MA systems the dealer has the same initial open exposures (net), and each CCP charges the same margin, then spreading positions brings the same benefit as netting positions. However, once trading occurs, net exposures are no longer equivalent due to the differing trading dynamics which determine CDS prices in each scenario. Thus, the equilibrium margin contribution of dealer i is heavily dependant on the system, which allows fewer liquidations.

Proposition 5: Given an initial margin that is homogenous in all CCPs, there is no difference in the initial margin charged with or without operability. That is spreading (NMA) becomes equivalent to netting (MA) in decreasing margin charges. Inhomogeneity in margin charges makes the spreading more favourable (lower total margins charges) or less favourable (higher total margin charges) than netting.

$$\hat{g}_i = \bar{g}_i = g_i^u$$

5.2 Interoperability and Mutual Inter-CCP Margin Exchange

CCP interoperability necessitates the establishment of a mutual clearing agreement between CCPs, whereby, they become trading partners. This effectively positions each CCP as a counterparty to the other – potentially its largest. Where CCPs begin to interact, they largely maintain their standard structure, but trading dynamics must be altered. Each CCP still clears all own members' intra-CCP positions, but two partner CCPs must split the clearing of any inter-CCP positions[61]. Therefore, each CCP clears their respective members' payments within their own facility, but also provides one lump payment to the other CCP for their inter-CCP trades. To secure the risk of its own member's trading positions, a CCP demands the standard collateral – an initial margin, g_i and a default contribution, d_i. On its members' cross-traded positions, the CCP collects an additional mutual clearing margin $M_{tot} = \sum_{i=1}^{M} m_i$ demanded by its partner CCP. In turn, it makes the same demand of its partner CCP(s). In this way, each CCP secures its exposure to its own members and to other CCPs[62].

[61] This effectively makes the position's volume appear smaller in the context of margining.

[62] Triple-margining collateralises the counterparty risk arising from own members' positions, and the additional risk arising from its 'extra' members positions, represented by its partner CCP(s).

At first glance, it would seem sufficient for a CCP to simply rehypothecate its members' initial margins as collateral for cross-margined positions. In this case, it would collect an initial margin on the home-cleared portion of a cross-cleared position, and post it at the partner CCP.[Board, 2016] This logical solution is subtly misleading by neglecting systemic risk. Consider that margining is meant to secure the full risk of a dealer's portfolio position; applying charges[63] for concentration risk and reductions for netting. Since the risk of a concentrated position grows non-linearly with its size, proper margining requires that concentration premiums scale progressively with position volume. This implies that for any concentrated position, the premium charged on the total position will be larger than the sum of premiums charged on two half positions. This has large implications for cross-cleared positions, where each CCP clears and margins only one half of the original position. If each CCP were to simply rehypothecate the initial margins charged on their members' positions, they would jointly fail to fully collateralise the concentration risk of their cross-traded positions. Should a dealer now default, the *system* would be undercollateralised. In an effort to allay this systemic risk, CCPs collect cross-margin on top of initial margin from their members.

Exact cross-margining is a complex procedure. In order to simplify cross-margining, partner CCPs net their exposures to one another. As a result, the CCP with the net positive exposure posts a cross-margin to the other CCP. The cross-margin is applied as a multiplier of the initial margin; usually $1.5-2$ times that of the initial margin, with higher charges applied to portfolios with asset correlations. Theoretically, the margin charged to each member should be based on the exact volume it cleared over the link. Unfortunately, the exact tracking of each member's cleared volume is prohibitively difficult. As a practical solution, the CCP spreads its total cross-margin payment proportionally amongst its members, using the initial margin charge as a proxy for cleared volume. Thus, members with larger net open positions[64] also pay a higher proportion of the cross-margin, accounting for the higher risk of their positions. Reassuringly, data shows this to be an equitable solution; it reveals that the amount of inter-CCP margin charged is highly correlated with the actual trading volume cleared across the link.[Securities and Authority, 2016]

Any rational dealer will certainly choose to be part of a CCP with a mutual agreement if this proves more incentive compatible than being a member of multiple CCPs. Incentive compatibility could be fulfilled by the payment of equal or lower overall margins. Assuming that there are no extra, unquantifiable benefits[65] of belong to multiple CCPs this gives,

$$g_i + d_i + u * m_i \leq u * (g_i + d_i) \tag{24}$$

where m_i is the inter-CCP cross-margin guarantee contribution, and can be further denoted by $m_i = \bar{m} g_i$. This implies that dealer dealer i will choose to belong to a CCP with a mutual agreement when,

$$m_i \leq \frac{u-1}{u} * (g_i + d_i) \tag{25}$$

For a two CCP system, this implies that a dealer i's incentive compatible inter-CCP margin contribution cannot exceed a threshold of 1/2 the sum of its initial and default margin contributions.

As before, dealer i's share in CCP u's cross-margin guarantee fund, posted to its partner CCP u', is dependant on the realisation of its net exposures, the realised value of its external asset, as well as, its cross-margin payments to $N-2$ other partner CCPs, such that,

$$M_i{}^u = \Big(\Lambda_i^u + \gamma_i + R_i - g_i^u - d_i^u - \sum_{u'=1}^{n-2} m_i^{uu'}\Big)^+ - \Big(\Lambda_i^u + \gamma_i + R_i - g_i^u - d_i^u - \sum_{u'=1}^{n-1} m_i^{uu'}\Big)^+ \tag{26}$$

[63]A concentration charge addresses the risk of holding (and liquidating) a large amount of one asset. A concentration premium is charged on large positions as a multiplier on the total margin contribution.

[64]An open positioning k signifies an unhedged position; that is, no corresponding position in the other direction which would mitigate its risk.

[65]This is a strong assumption as many CCPs maintain membership in CCPs with both a mutual and no mutual agreement which may suggest some added benefit to maintaining both types of arrangements.[Securities and Authority, 2016]

where dealer i's total cross-margin payment across all partner CCP's is the sum $\bar{m}_i^u = \sum_{u'=1}^{n-1} m_i^{uu'}$.

5.3 Interoperability and Price Dispersion between CCPs

One plausible consequence of interoperability is yet to be considered in this literature – the appearance of price dispersion between CCPs. The possibility of price dispersion is suggested by certain market features. Previous sections have already demonstrated how trading dynamics within CCPs move an asset's CDS spread away from its fundamental value. Naturally, amongst multiple CCPs, these trading dynamics are bound to be heterogeneous. Furthermore, CDS trading occurs in an opaque search market (OTC), making the price impact effects of these heterogeneous trading patterns largely invisible between CCPs. These combined features suggest the following scenario: Given a group of heterogeneous CCPs with the same fundamental value for asset k, at the start of a trading period, each CCP will begin by trading the asset's fundamental value. However, as trading evolves, in each CCP heterogeneous trading strategies will uniquely move traded price away from the fundamental value. Since market opacity results in the fragmentation of price information, the CCPs cannot readily correct for price differences. It follows that at the end of a trading day, there will exist some discrepancy in the CDS spread for asset k amongst the CCPs.

The possibility of price dispersion gives rise to an interesting question: How large of an inter-CCP price discrepancy might arise between the two possible operability scenarios (MA and NMA)? The answer is interesting because the scenario which produces the larger price discrepancy might be systemically undesirable; indicating that it produces a less efficient market with reduced price transparency. In such a market, dealers may be paying vastly different prices in each CCP for the same asset.

To answer this question, I investigate the effect of trading mechanics on price in each interoperability scenario for a two CPP system, consisting of m dealers and one traded asset k. I first consider the NMA scenario, where a lack of interoperability leads CCPs to trade autonomously. Given m dealers, each dealer will gain access to all other market dealers only if all trade in both CCPs. Consequently, each dealer i, with a holding of asset k, will trade a portion in CCP 1 and a portion in CCP 2. If dealers split their trades equally amongst CCPs, homogeneous trading patterns should yield the same price in each CCP[66]. However, if dealers split their trades unequally, identical trading patterns will now generate a different final price in each CCP. Therefore, price dispersion is not an unavoidable consequence of the NMA scenario. In the MA scenario, interoperability places half of the members ($\frac{m}{2}$) into CCP 1 and half of the members into CCP 2. Each dealer $i = 1, ..., \frac{m}{2}$ will first trade a portion their portfolio in CCP 1, with counterparties $j = 1, ..., \frac{m}{2}$. Each dealer will subsequently trade their remaining portion in CCP 2, using an interoperability link to gain access to counterparties $j = \frac{m}{2}, ..., m$. By trading across the link, a dealer now affects the price process in both CCPs. Furthermore, any trade across the link breaks homogeneity in CCP trading patterns[67]. As a result, an interoperability link can create varying degrees of price dispersion for asset k. The most extreme example would have all of CCP 1's members place only buy-side trades across the link and all of CCP 2's members place only sell-side trades across the link; one can imagine that, barring any compensatory trades by a CCP's own dealers, the resulting final prices would differ vastly between CCPs[68]. The following proposition attempts to formalise these ideas.

> *Proposition 6: All things being equal, the MA scenario renders information integration into prices less efficient. That is, when members in each interoperability scenario follow the homogeneous trading strategies, CPP interoperability (MA) will result in a larger inter-CCP price discrepancy for a CDS asset k.*

[66] The price impact on the CDS spread depends on the volume of the traded asset.

[67] With just one dealer trading over the link, one CCP adds one buy-side trade for CDS k, while the other CCP adds one sell-side trade for CDS k. Each side of the trade has an opposite effect on the the price of CDS k. Since only one half of the trade determines the CDS spread in that CCP, the CDS price in each CCP instantaneously moves in the opposite direction.

[68] As buy-side trades raise CDS spreads and sell-side trades lower CDS prices, relative to the volume traded.

This outcome is given by difference between each period's pricing equations10 for any single interoperability framework,

$$\dot{S}^{u,k} = \Big| |\Delta S^{u,k}(0)| - |\Delta S^{u,k}(l\tau)| \Big| \tag{27}$$

$$\Delta \dot{S}^{k}(l\tau) = |\dot{S}^{u,k}(l\tau) - \dot{S}^{u',k}(l\tau)| \tag{28}$$

Equation 27 calculates the difference of an asset's current price from it's fundamental starting value, in each CCP. Equation 28 provides a measure of the price dispersion between the CCPs for an asset. The result being that,

$$\Delta \dot{S}_t^{NMA} \leq \Delta \dot{S}_t^{MA} \tag{29}$$

The mean final inter-CCP price dispersion[69] between the two interoperability scenarios is the result of,

$$\Delta \bar{\dot{S}}(l\tau)_{tot} = \sum_{k=1}^{K} \frac{\Delta \bar{\dot{S}}^{k,NMA}(l\tau)}{K} - \sum_{k=1}^{K} \frac{\Delta \bar{\dot{S}}^{k,MA}(l\tau)}{K} \tag{30}$$

The proofs for the above can be found in the appendix (A.2). The results of the simulation exercise in section 7 support eq.29, demonstrating that increased price dispersion between CCPs is an unfortunate by-product of a mutual agreement scenario.

Corollary 1: Given homogeneous holdings among dealers, and equivalent trading strategies between the NMA and MA scenarios, the inter-CCP price dispersion between CCPs is the same for each scenario. However, the drivers of price dispersion differ between the two scenarios. In an interoperable market (MA), the inter-CCP price dispersion emerges as a direct consequence of inter-CCP trading. Conversely, in a fragmented market (NMA), price dispersion arises due to either inhomogeneity in the dealers' volume of trades within individual CCPs, or the variation in dealers' trading strategies across CCPs.

Lemma 1: For identical trading conditions across MA and NMA scenarios (all else being equal), a dealer's liabilities (or negative net exposures) are more pronounced under interoperability (MA) given unfavorable changes in the CDS spread, and reduced when these changes are favorable. This result is driven by the mutual risk-sharing mechanism inherent in interoperability agreements.

$$\hat{\Lambda}_i^{k,-} < \bar{\Lambda}_i^{k,-} \quad \text{if} \quad \Delta S_t^{u',k} < 0$$

Lemma 1 arises out of the fact that interoperability exposes dealers to interconnected price adjustments and additional margin obligations through inter-CCP trades, amplifying the impact of adverse market conditions. Conversely, favorable CDS spread changes allow the same interconnections to distribute gains across the network, mitigating liabilities. This reflects the dual-edged nature of risk-sharing in an interoperable system.

Corollary 2: Interoperability (MA) amplifies the sensitivity of a dealers' equilibrium ($\star$) guarantee fund contributions to changes in the CDS spread, compared to the fragmented (NMA) scenario. This heightened sensitivity is a consequence of the additional inter-CCP price dispersion introduced by interoperability (MA), which magnifies the dependence of the guarantee fund requirements on CDS spread dynamics. All else being equal, overall favorable CDS spread changes translate to greater contributions with interoperability (MA) than without, as the former requires additional margin to account for inter-CCP trades. Conversely, in unfavorable scenarios, the same interconnected system absorbs losses more effectively, resulting in lower contributions than in the fragmented case.

$$\hat{G}_i^{\star,\,u} > \bar{G}_i^{\star,\,u} \quad \text{if} \quad \bar{\Lambda}_i^{k} < 0$$

This result highlights how interoperability intensifies the coupling between CDS spread volatility and risk management requirements. Furthermore, since equilibrium guarantee fund contributions are determined by outcomes of net exposures, corollary 2 provides a measure for dealers' incentive compatibility between the two operability scenarios. The details of the clearing equilibrium and equilibrium contributions are the focus of the next section.

[69] Multiplication by 1000 will give this dispersion in bps.

5.4 Clearing Payments & Defaults

The above sections have described the network in terms of *dealers'* net exposure and guarantee payments. However, network dynamics only become apparent when considering both dealer and CCP interactions, and the corresponding changes in exposures and payments. In this way, the realisation of all member and CCP liabilities establishes a clearing equilibrium. As a result, dealers may be left with a surplus or a shortfall. Dealers with an shortfall must make an equilibrium clearing payment to the CCP. Occasionally, a dealer cannot meet this payment, possibly triggering default[70] in one or more CCPs.

As in [Amini et al., 2015], each CCP begins the liquidation phase (t=1), with the following nominal balance sheet composed of assets and liabilities[71],

$$\mathcal{A}_0^u(t_{\ell\mathcal{T}}=1) = \underbrace{\gamma_0^u}_{cash} + \sum_{i=1}^{m} g_i^u + \begin{cases} \sum_{k=1}^{K}\sum_{i=1}^{m} L_{i0}^{u,k} & \text{(NMA)} \\ \sum_{u'=1}^{n}(\sum_{k=1}^{K}\sum_{i=1}^{m} L_{i0}^{u'u,k} + \sum_{i=1}^{m} m_i^{u'u}) & \text{(MA)} \end{cases}$$

$$\mathcal{L}_0^u(t_{\ell\mathcal{T}}=1) = \begin{cases} \sum_{k=1}^{K}\sum_{i=1}^{m} L_{0i}^{u,k} + \left(\gamma_0^u + f^u\sum_{k=1}^{K}\sum_{i=1}^{m}\Lambda_i^{u,k,+}\right) + \sum_{i=1}^{m}\hat{G}_i^u & \text{(NMA)} \\ \sum_{u'=1}^{n}\Big(\underbrace{\sum_{k=1}^{K}\sum_{i=1}^{m} L_{0i}^{uu',k}}_{L_0^{uu'}} + \underbrace{\big(\gamma_0^u + f^u\sum_{k=1}^{K}\sum_{i=1}^{m}\Lambda_i^{u'u,k,+}\big)}_{nominal\ net\ worth}\Big) + \underbrace{\sum_{i=1}^{m}\bar{G}_i^u}_{\bar{G}_{tot}^u} + \sum_{i=1}^{m}\sum_{u=1}^{n} M_i^u & \text{(MA)} \end{cases}$$

The MA case (bar) differs from the NMA case (hat) in that the CCP must consider liabilities (receivables) that are outgoing (incoming) to (from) its partner CCP(s), on behalf of its own members. As a result, the nominal net worth (equity) of the CCP includes the fees charged for clearing inter-CCP receivables owed to its own members. For all interoperability scenarios, the value of each CCP's balance sheet is determined by the realisation of dealers' clearing payments – defining the clearing equilibrium.

If at any time, dealer i cannot make payments due to insufficient cash[72], it must liquidate Z_i of its external assets. This determines its equilibrium clearing payment to each CCP,

$$L_i^{u,\star} = \begin{cases} \hat{L}_{i0}^u \quad \wedge \quad (\gamma_i - \sum_{u=1}^{n} g_i^u + R_i - \sum_{u'=1}^{n-1}\hat{L}_{i0}^{u'}) + g_i^u & \text{(NMA)} \\ \underbrace{\bar{L}_{i0}^u}_{\sum_{u'=1}^{n} L_{i0}^{uu'}} \quad \wedge \quad (\gamma_i - g_i^u - \sum_{u'=1}^{n-1} m_i^{uu'} + R_i) + g_i^u & \text{(MA)} \end{cases} \tag{31}$$

In each agreement scenario, a dealer's clearing payment to one CCP depends on its liabilities to all other CCPs where it is a member.

Clearing payments from dealers alter a CCP's equilibrium nominal balance sheet; its equilibrium assets and receivables change according to,

$$\mathcal{A}_0^u(t_{\ell\mathcal{T}}=1) = \begin{cases} \gamma_0^u + \sum_{i=1}^{m} g_i^u + \sum_{i=1}^{m}\hat{L}_i^{u,\star}(t_{\ell\mathcal{T}}=1) & \text{(NMA)} \\ \gamma_0^u + \sum_{i=1}^{m} g_i^u + \sum_{i=1}^{m}\bar{L}_i^{u,\star}(t_{\ell\mathcal{T}}=1) + \sum_{i=1}^{m} m_i^{u'u} & \text{(MA)} \end{cases} \tag{32}$$

[70] Default occurs when dealer's receivables and the liquidation value its outside asset is lower than the total liabilities which must be met.

[71] Cross-liabilities in the MA case; $\bar{L}_{i0}^{u,k}$ which originates from $\Lambda_{i0}^{k,-} = \sum_{u=1}^{n}\Lambda_{i0}^{u,k,-} = \sum_{u=1}^{n}[\sum_{u=1}^{n}\Lambda_{i0}^{uu',k,-}]$. This accounts for what dealer i owes in its own CCP ($\Lambda_{i0}^{uu,k,-}$) and what it technically owes to the other CCP ($\Lambda_{i0}^{uu',k,-}$).

[72] Insufficient cash in the NMA case is $\gamma_i - \sum_{u=1}^{n} g_i^u \leq L_{i0}^u$ while in the MA case this is $\gamma_i - g_i^u\sum_{u'=1}^{n-1} m_i^{uu'} \leq L_{i0}^u$.

$$L_0^{u,\star}(t_{\ell\tau}=1) = \begin{cases} \hat{\mathcal{A}}_0^u(t_{\ell\tau}=1) \wedge \hat{L}_0^u(t_{\ell\tau}=1) & \text{(NMA)} \\ \\ \bar{\mathcal{A}}_0^u(t_{\ell\tau}=1) \wedge \bar{L}_0^u(t_{\ell\tau}=1) & \text{(MA)} \end{cases} \tag{33}$$

Taking inspiration from previous work [Eisenberg and Noe, 2001, Amini et al., 2015], each dealer's payments to a CCP are governed by the proportionality rule ($L_0^{u,\star} \times \Pi_{0i}^u$). Each dealer's share of payments or contributions is,

$$\Pi_{0i}^u = \begin{cases} \frac{L_{0i}^u}{L_0^u} = \frac{\Lambda_i^{u,+}}{\sum_{j=1}^m \Lambda_j^{u,+}} & \text{if} \quad L_0^u \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

with the following Π_{0i}^u equation for each scenario: the NMA ($\hat{\Pi}_{0i}^u$, $\hat{L}_{0i}^u$, $\hat{L}_0^u$) and the MA ($\bar{\Pi}_{0i}^u$, $\bar{L}_{0i}^u$, $\bar{L}_0^u$) case.

If a dealer still cannot meet its obligations, it is placed in default. This means its total liabilities (to all CCPs) outweigh its cash, liquidated assets ($Z_i = 0$), and guarantee contribution(s). Each guarantee contribution g_i^u can only be applied to liabilities in the same CCP and only after a dealer depletes its external assets. A dealer defaults if,

$$\hat{L}_{i0}^u \geq \gamma_i - \sum_{u=1}^{n-1} g_i^u + g_i^u + R_i - \sum_{u'=1}^{n-1} \hat{L}_{i0}^{u'} \qquad \text{(NMA)}$$

$$\bar{L}_{i0}^u \geq \gamma_i + g_i^u + R_i - \sum_{u'=1}^{n-1} m_i^{uu'} \qquad \text{(MA)}$$

Since a dealer benefits from limited liability, a dealer's remaining shortfall becomes the liability of the CCP so that $L_{0\ i}^{u,\mathbb{D}} = \sum_{i=1}^m L_{0\ i}^{u,\star,-}$. The liability is cumulative, varying over the trading periods which define the liquidation phase. Therefore, a CCP's total equilibrium liability consists of,

$$L_{0,\ (\ell\tau\in[1\tau,T\tau])}^{u,\star} = L_0^{u,1-\mathbb{D}} + L_0^{u,\mathbb{D}}$$

Finally, there may be many cases where a dealer does not default, but is rendered illiquid; where in attempting to meet its obligations, it has totally exhausted its assets. This dealer has a remaining lifeline in its CCP-bound guarantee fund contribution(s). However, by drawing on its initial margin account, the value drops below the required threshold. Thus, in order to continue membership in the CCP, this dealer will have to replenish this initial margin account. Replenishment occurs once a trading period has ceased. At that time, the relevant CCP will make a margin call on dealer i and demand $\mathcal{G}_i^u = G_i^u - g_i^u$.

Replenishment occurs in both agreement scenarios, however, with interoperability (MA) a dealer must additionally replenish the inter-CCP margin $\mathcal{M}_i^u$. Given that this is a pooled fund, it can be applied to all members inter-CCP shortfalls, and so one dealer could potentially consume all dealers' contributions. This has great implication for the deterrence of self-serving behaviors because regardless of which dealers consumed the fund, each dealer must still replenish their own contribution. More concretely, predation by unconstrained dealers may indirectly result into distressed dealers consumption of predators' margin contributions. This will be discussed further in section 6.4.

6 Model Dynamics

This section explores the dynamic processes underlying financial networks, focusing on how central counterparties (CCPs) and their members respond to market shocks and evolving trading conditions. By breaking down network activity into four distinct phases—Initial, Liquidation, Buyback, and Resolution—the section provides a comprehensive framework for understanding the lifecycle of trading and risk management in multi-CCP systems. These phases allow us to capture the critical interplay between trading strategies, collateral requirements, and risk transmission in scenarios with and without interoperability agreements. More concretely, they provide a lens through which to investigate the interactions between dealers, CCPs, and market liquidity in both interoperable (MA) and fragmented (NMA) clearing frameworks.

This section introduces important findings about how CCP interoperability (MA) alters the dynamics of risk and trading behavior. For example, cross-CCP linkages in interoperable systems create feedback loops that amplify price dispersion and interconnect CDS spreads across CCPs. These interactions reshape collateral requirements, dealer liabilities, and price stability in ways that differ substantially from fragmented markets. Moreover, the analysis highlights the dual nature of interoperability: while it enhances risk-sharing and reduces systemic liquidity demands, it also increases sensitivity to market fluctuations, particularly through price dispersion and changes in guarantee fund contributions. Together, these insights set the stage for understanding the trade-offs inherent in interoperability agreements and their impact on market resilience.

Specifically, this section highlights how the risk-sharing mechanisms intrinsic to interoperability (MA) ensure that all dealers collectively bear the direct losses resulting from predatory behavior, while notably excluding them from sharing in the corresponding predatory profits.[73] Further analysis reveals that applying default funds to address inter-CCP shortfalls can play a pivotal role in curbing such predatory strategies, offering a practical resolution to a longstanding regulatory debate. Finally, the section illustrates several ways in which interoperability (MA) mitigates the downward price pressures typically associated with CCP liquidations, thereby enhancing overall market stability.

6.1 Liquidation Phase

This section utilizes the previously described mechanisms to trace the transition from initial network formation through dealer default and the unwinding of defaulted dealer assets. In the initial phase, ($t_0 = 0\tau$), the establishment of trading contracts in the preceding period results in network formation. Due to the mechanics of standard CDS contracts, outlined in [Tywoniuk, 2017b], all contracts are initially set to have zero value, $\Lambda_i^{u,k,S}(0\tau) = 0$. At the beginning of the liquidation phase ($t = 1$), liabilities are realised and any defaults become evident. In this model, the first dealer default is exogenous[74].

The default of any dealer $i' \in \mathbb{D}_0^u$ at t=1 triggers a liquidation mechanism for the CCP. In the NMA scenario, the dealer i' independently defaults in each CCP where it holds membership, while in the MA scenario it only fails in its own CCP. As part of the liquidation process, the CCP must unwind the defaulted dealer's assets. Using proper risk management, the CCP has protected itself against the liquidity risk associated with position closeout, by setting its members' initial margin charge (g_i^u) to cover approximately five days of liquidation, such that $\ell\tau \in [1\tau, T\tau = 5]$. Unfortunately, by setting this charge, the CCP has revealed its time constraint to its members. Common knowledge of this constraint becomes the source of predation.

Each CCP begins liquidating any defaulted positions on the first trading day ($t_{1\tau} = 1$), decreasing its holdings by $X_i^{u,k}(\ell\tau) = X_i^{u,k}\big((\ell-1)\tau\big) + \mathbf{a}_{\mathbf{ij}}^{\mathbf{u},\mathbf{k},\pm}$. This causes fluctuations in the associated assets' CDS prices. In turn, this changes the dealers' variation margins on those assets, and thus, variation

[73] Dealers are excluded from *directly* benefiting from predatory profits.

[74] Exogenous denotes that the dealer fails due to insolvency and not due to default of the underlying on CDS in its portfolio.

margin exchange. The spread changes are due to the effect of the various price (and predation) impacts associated with trading (from sec.4.2.1). As shown in [Tywoniuk, 2017b], these impacts act cumulatively over each trading period – transmitted through the pricing function – such that unfavourable CDS spread movements are amplified and positive CDS spread movements are dampened over time. This greatly affects the value of buy and sell positions in dealer portfolios and their net exposures (Λ_i).

The cumulative effect of predation and price impact means that they act as amplifiers of distress, which can lead to default contagion in the system. In the NMA scenario, no interoperability means that each CCP's default cascade would proceed independently of the other. Thus, even if a dealer defaults in every CCP (and it may not), these outcomes from defaults are not linked. This also results in a containment of the amplification mechanism to each CCP, potentially dampening its effects. In the MA scenario, a dealer default (and subsequent liquidation) in one CCP affects its positions in the partner CCP. As a result, the amplification mechanism spills over between CCPs as inter-CCP trading continues. This occurs because the partner CCP's distressed dealers (non-liquidating) react to a linked default by also selling the defaulted CDS k, causing a similar drop in the asset's price in the partner CCP. Regrettably, the appearance of distressed trading in the partner CCP now motivates predatory sales, amplifying the inter-CCP contagion.

In an effort to mitigate inter-CCP contagion, regulators ([Board, 2016]) have stated definitively that CCPs should not contribute to each other's default funds. However, there is currently no consensus whether a CCP should use its own default fund to meet inter-CCP shortfalls; that is, to protect its own viability (and reimburse its members). In an effort to provide some guidance on this issue, this work suggests that the use of the default fund against inter-CCP shortfalls (after M_u is depleted) can be highly beneficial. This results from its risk-pooling structure, which can strengthen efforts to penalise predatory behavior. This again will be explored in detail in sec. 6.4.

As outlined above, anytime during trading a dealer may be unable to meet its shortfall and so defaults. This defaulted dealer joins the set $i \in \mathbb{D}_m(t_{\ell\tau})$. There can be at most m defaults before the CCP is non-viable. The defaulted dealer's shortfall now goes onto the balance of the CCP. In each operability scenario, the shortfall of this dealer i is described by,

$$C_i^{u,-}(\ell\tau) = L_{i0}^u(\ell\tau) - L_i^{u,\star}(\ell\tau) \tag{35}$$

where $\bar{C}_i^{u,-}$ (MA) occurs uniquely in a single CCP for each dealer. Contrastingly, $\hat{C}_i^{u,-}$ (NMA) is independent to each CCP, but still incorporates a dealer's liabilities to other CCPs – liabilities which may have equally contributed to this dealer's shortfall. Therefore, even though CCPs are technically independent in the NMA case, the risk from multiple CCPs can still be transmitted through the balance sheet of a dealer.

During any trading period $\ell\tau$, a defaulted dealer's shortfall increases the liabilities of its home CCP,

$$C_0^{u,-}(\ell\tau) = L_0^u(\ell\tau) - L_0^{u,\star}(\ell\tau) = \big(A_0(\ell\tau) - L_0(\ell\tau)\big)^{u,-} \tag{36}$$

with $\hat{C}_0^{u,-}$ and $\bar{C}_0^{u,-}$ in the respective NMA and MA scenarios. Thus, at any point in time, a CCP's current liability is composed of the sum of its own liability and the previous period's defaulter shortfall (which it has attempted to liquidate), $L_0^u\big((\ell+1)\tau\big) = L_0^{u,1-\mathbb{D}}\big((\ell+1)\tau\big) + L_0^{u,\mathbb{D}}\big(\ell\tau\big)$. As a result, liquidation can either increase or decrease the CCP's total realised liability for a trading period. In fact, the more/faster the CCP liquidates, the more it exposes itself to predation, yielding lower possible profits from liquidating defaulted positions. Therefore, the CCP's final profits will depend heavily on its liquidation rate and its containment of predation amongst its members.

The clearing payment equilibrium continues to evolve throughout the liquidation phase, over each trading period. The CCPs' nominal balance sheets change according to,

$$\mathcal{A}_0^u\,(t_{\ell\tau}) = \begin{cases} \hat{\mathcal{A}}_0^u\big(\ell\tau\big) \;+\; \sum_{k=1}^{K}\sum_{j\in\,\mathbb{D}}^{m} \Bigg(\underbrace{X_{ij}^{u,k}\big((\ell-1)\tau\big) + \mathbf{a}_{\mathbf{ij}}^{\mathbf{u},\mathbf{k},\pm}\big((\ell-1)\tau\big)}_{\hat{X}_{ij}^{u,k}\left(\ell\tau\right)} \;\triangle S^{u,k}(\ell\tau) & \text{(NMA)} \\ \\ \bar{\mathcal{A}}_0^u\big(\ell\tau\big) \;+\; \sum_{k=1}^{K}\sum_{j\in\,\mathbb{D}}^{m} \Bigg(\sum_{u'=1}^{n} \underbrace{X_{ij}^{uu',k}\big((\ell-1)\tau\big) + \mathbf{a}_{\mathbf{ij}}^{\mathbf{uu'},\mathbf{k},\pm}\big((\ell-1)\tau\big)}_{\bar{X}_{ij}^{u,k}\left(\ell\tau\right)} \;\triangle S^{u,k}(\ell\tau) & \text{(MA)} \end{cases} \tag{37}$$

$$\mathcal{L}_0^u\,(t_{\ell\tau}) = \begin{cases} \hat{\mathcal{L}}_0^u\big(\ell\tau\big) \;+\; \underbrace{C_0^{u,-,\star}\big((\ell-1)\tau\big)}_{\hat{L}_0^{u,\star}\left(\ell\tau\right)} & \text{(NMA)} \\ \\ \bar{\mathcal{L}}_0^u\big(\ell\tau\big) \;+\; \underbrace{\sum_{u'=1}^{n} C_0^{u'u,-,\star}\big((\ell-1)\tau\big)}_{\bar{L}_0^{u,\star}\left(\ell\tau\right)} & \text{(MA)} \end{cases} \tag{38}$$

Equation 37 indicate that a CCP's assets change with the profits yielded from liquidation. In turn, a CCP's liabilities (eq. 38) increase with the defaults of its members. The MA equations also reveal that the interconnectedness of CCPs create a new avenue of contagion – because a CCP's shortfall now also incorporates the shortfall of its defaulted members to all its partner CCPs. For both the NMA and MA case, these equations also reveal where distress amplification take place. Amplification acts through liquidation revenue because it is highly affected by the price impact resulting from predation; now predatory trading can come from members of multiple CCPs [75].

6.2 Buyback Phase and Margin Funds

The liquidation phase terminates at $t_{5\tau} = 1$, when each CCP has finished selling its defaulted positions. The next trading phase commences at $t_{1\tau} = 2$, when dealers are free to buyback their positions. The buyback phase has no CCP limitation on the allowable number of trading days. Instead, it is limited by predator competition, available funds, and the regulated maximal allowable holdings. As with the liquidation phase, each trading day brings an evolution in CCP and dealer assets and liabilities. In turn, each trading phase closes with the establishment of a clearing equilibrium. This determines the state of CCP and dealer terminal net worths – either surplus or shortfall – and the state of the margin funds. This accounting becomes most consequential at the end of the buyback phase, when predatory trading ceases.

Following the clearing equilibrium outcomes of the liquidation phase, any financially healthy dealer can engage in trading during the buyback phase. The result of this strategic trading will be a change in dealer i's balance sheet – leading to a change in its nominal assets,

$$\mathcal{A}_i\,(t_{\ell\tau}=2) = \;\gamma_i \;+\; Z_i R_i \;+\; (1-Z_i)\,Q_i \;+\; \begin{cases} \sum_{u=1}^{n}\,(\hat{L}_0^{\star}\times\hat{\Pi}_{0i})^u \;+\; \sum_{u=1}^{n}\,(\hat{G}_i^{\star}-\hat{g}_i)^u & \text{(NMA)} \\ \\ (\bar{L}_0^{\star}\times\bar{\Pi}_{0i})^u \;+\; (\bar{G}_i^{\star}-\bar{g}_i)^u + (M_i^{\star}-m_i)^u & \text{(MA)} \end{cases}$$

The changes which have been the result of earlier liquidation will vary across interoperability scenarios because they depend on the liquidation amount Z_i, which is different in each scenario. It remains that dealer i's assets are now composed of its cash, liquidation proceeds, remaining external assets,

[75]This occurs because the effect is driven by the traded volume. It is interesting to consider the size of the predatory effect between the NMA and MA cases: Given equal amounts of predators in each CCP, predation will yield greater price impact in the MA case given that more predatory dealers have access to each CCP (in the MA versus the NMA network).

proportion of receivable and unused portions of guarantee fund(s). Dealer i's liabilities continue as in the liquidation phase. The resulting net worth of dealer i is,

$$\mathbf{C_i}\ (t_{\ell\mathcal{T}}=2)=\begin{cases}\sum_{u=1}^{n}\hat{\mathcal{A}}_i^u-\sum_{u=1}^{n}\hat{L}_{i0}^u & \text{(NMA)}\\ \\ \bar{\mathcal{A}}_i^u-\bar{L}_{i0}^u & \text{(MA)}\end{cases}\tag{39}$$

with the caveat that, in the (MA) case, the dealer belongs to only one[76] CCP (u). A dealer's terminal net worth may indicate a shortfall or surplus for that dealer. The shortfall of a dealer i is the result of,

$$\mathbf{C_i}^-\ (t_{\ell\mathcal{T}}=2)=\begin{cases}\sum_{u=1}^{n}(\hat{L}_{i0}^u-\hat{L}_i^{u,\star})=\Big(\sum_{u=1}^{n}\ \hat{\Lambda}_i^u+\gamma_i+R_i\Big)^- & \text{(NMA)}\\ \\ \bar{L}_{i0}^u-\bar{L}_i^{u,\star}=\Big(\bar{\Lambda}_i^u+\gamma_i+R_i\Big)^- & \text{(MA)}\end{cases}\tag{40}$$

In contrast, the possible surplus of dealer i is the result of,

$$\mathbf{C_i}^+\ (t_{\ell\mathcal{T}}=2)=\ \gamma_i\ +\ Z_iR_i\ +\ (1-Z_i)\,Q_i\tag{41}$$

which again differs across interoperability scenarios due to Z_i. Equation 41 is important because, through a dealer's possible surplus, one can determine its incentive compatibility for a given scenario. From this equation one can also obtain dealers' aggregate wealth ($\mathbf{C}_{tot}^+=\sum_{u=1}^{n}\mathbf{C}_i^+$), which is integral to determining the general consequences of predation; setting general dealer wealth against the profits made by predators.

Dealers' equilibrium shortfalls are of particular interest because they affect the state of the guarantee fund. Any shortfalls result in the consumption of dealers' guarantee fund contributions (and possible defaults), which changes the equilibrium state of a CCP's guarantee fund. The equilibrium guarantee fund value is given by,

$$G_{tot}^{u,\star}=\sum_{i=0}^{m}G_i^{u,\star}=G_{tot}^u\ \wedge\ \left(A_0^u-L_0^{u,\star}-\gamma_0^u-\begin{cases}f^u\sum_{k=1}^{K}\sum_{i=1}^{m}\Lambda_i^{u,k,+} & \text{(NMA)}\\ \\ f^u\sum_{u'=1}^{n}\sum_{k=1}^{K}\sum_{i=1}^{m}\Lambda_i^{u'u,k,+} & \text{(MA)}\end{cases}\right)^+\tag{42}$$

where each dealer cannot access more than its own contribution, $\hat{G}_i^{u,\star}=\big(\hat{G}_i^u-\Lambda_i^{u,-}\big)^+$. This equilibrium portion, which will eventually be returned to dealer i, is the outcome of the fund's remaining value and the application of the proportionality rule, $G_i^{u,\star}=\frac{G_i^u}{G_{tot}^u}\times G_{tot}^{u,\star}$ ($=0$ if $G_{tot}^u=0$).

The equilibrium guarantee fund level can now be explicitly re-written in terms of dealer shortfalls, $G_{tot}^{u,\star}=\Big(G_{tot}^u-\sum_{i=1}^{m}C_i^{u,-}\Big)^+$. This reveals that, regardless of interoperability scenario, each guarantee fund only incorporates the shortfalls from its own CCP (u). This is sufficient where CCPs are autonomous (NMA), but doesn't cover inter-CCP shortfalls for the interoperable CCPs, where $C_i^{u,-}=C_i^{uu,-}$. Therefore, for interoperable CCPs, the termination of buyback trading must bring a corresponding change in the equilibrium cross-margin guarantee fund level, $M_{tot}^{u,\star}=\Big(M_{tot}^u-\sum_{u=1}^{n-1}\sum_{i=1}^{m}C_i^{uu',-}\Big)^+$, which comparatively solely accommodates shortfalls in inter-CCP positions.

From this one arrives at dealer i's terminal net worth,

$$C_{i\ (t_{1\mathcal{T}}=2)}=\gamma_i+Q_i-Z_i(Q_i-R_i)+\begin{cases}\sum_{u=1}^{n}\Big(\Lambda_i^u-\hat{\Pi}_{0i}^u\hat{C}_0^{u,-}-f^u\Lambda_i^{u,+}-\frac{\hat{G}_i^u}{\hat{G}_{tot}^u}(\hat{G}_{tot}^u-\hat{G}_{tot}^{u,\star})\Big) & \text{(NMA)}\\ \\ \Lambda_i^u-\bar{\Pi}_{0i}^u\bar{C}_0^{u,-}-f^u\Lambda_i^{u,+} & \\ \\ \qquad-\frac{\bar{G}_i^u}{\bar{G}_{tot}^u}(\bar{G}_{tot}^u-\bar{G}_{tot}^{u,\star})-\frac{M_i^u}{M_{tot}^u}(M_{tot}^u-M_{tot}^{u,\star}) & \text{(MA)}\end{cases}\tag{43}$$

[76] Even though the dealer can contract trades in across CCPs, it is still a clearing member in only one CCP.

The equation now shows that a dealer's positive net worth is composed of cash, external assets and receivables. However, it also demonstrates that this may be diminished by: the loss incurred on liquidating any fraction of external assets, the loss of a cleared receivable from the CCP (due to default), the clearing fee charged on positions, and the loss from using any of its guarantee fund contribution to meet shortfalls. Additionally, in the MA scenario, there is the loss of dealer i's contribution to the cross-margin fund due to *any* other dealer's shortfall. This means that dealer i will directly share the losses incurred from any predatory behaviour – whether or not it, itself, is a predator.

The same clearing calculations are made for the CCP. The state of the equilibrium fund is also important in determining a CCP's net worth, which is a crucial determinant of CCP viability. For each CCP, the end of trading results in a calculation of its terminal net worth[77] such that,

$$\mathbf{C_0^u}\ (t_{\ell\tau}=2) = \mathcal{A}_0^u - L_0^u - \begin{cases} G_{tot}^{u,\star} & \text{(NMA)} \\ \\ \left(G_{tot}^{u,\star} + \sum_{u'=1}^{n-1} M_{tot}^{uu',\star}\right) & \text{(MA)} \end{cases} \tag{44}$$

which varies across the interoperability scenarios. This clearing calculation can result in a CCP's shortfall according to,

$$\mathbf{C_0^{u,-}}\ (t_{\ell\tau}=2) = L_0^u\ _{(t_{\ell\tau}=2)} - L_0^{u,\star}\ _{(t_{\ell\tau}=2)} = \left(\mathcal{A}_0^u\ _{(t_{\ell\tau}=2)} - L_0^u\ _{(t_{\ell\tau}=2)}\right)^- \tag{45}$$

A CCP shortfall at the end of the buyback stage signals a CCP in need of recovery. Finally, the CCP's terminal net worth from eq.44 can be rewritten in order to explicitly capture the effect of dealer shortfalls,

$$\mathbf{C}_0^u(t_{\ell\tau}=2) = \gamma_0^u + \sum_{i=1}^{m} f\Lambda_i^{u,+} - \begin{cases} \left(\hat{G}_{tot}^u - \sum_{i=1}^m C_i^{u,-}\right)^- & \text{(NMA)} \\ \\ \left(\bar{G}_{tot}^u - \sum_{i=1}^m C_i^{uu,-}\right)^- + \left(M_{tot}^u - \sum_{u=1}^{n-1}\sum_{i=1}^m C_i^{uu',-}\right)^- & \text{(MA)} \end{cases} \tag{46}$$

Now one can see that in a situation where all things are equal, and the inter-margin fund is greater than zero, $\bar{C}_0\ _{(t_{\ell\tau}=2)} \geq \hat{C}_0\ _{(t_{\ell\tau}=2)}$.

With the clearing equilibrium in place, it is now possible to translate the different interoperability mechanisms into network outcomes. During the buyback phase, financially healthy dealers alter their strategy from selling CDS to buying them back at lower spreads. Given that the CCP has ceased unwinding assets, the predatory buy-side trades now dominate, which slowly raises CDS spreads in the network. If predators can buy when CDS spreads are lowest, then this is a profitable strategy. However, increasing competition forces predators to start buying earlier (at higher prices), yielding lower profits.[Brunnermeier and Pederson, 2005] Therefore, increasing competitive pressure diminishes profits. Profits also depend on the amount of distressed dealers on which to prey. Therefore, strategies which increase predators' profits increase shortfalls. The above equations for the MA case demonstrate how increased shortfalls deplete margin funds in a way which affects all network members. This can have grave implications for predators final profits, which will be demanded to replenish default and inter-CCP margin funds (to maintain membership priveleges or CCP viability). Thus, when predatory profits don't outweigh the losses incurred in replacing their margin contributions, predators become victims of their own strategy. Hence, this is one way that interoperability diminishes the incentives for predation.

The buyback phase give rise to another differenece in the predation effect between scenarios. In the NMA scenario, where all large dealers are members in multiple CCPs, a member's default in any CCP becomes common information among participants. Now predators can divert their selling to the CCP

[77] Note that we use nominal assets, but actual total liabilities (L_0) versus nominal liabilities ($\mathcal{L}_0$). Nominal liabilities take into account the nominal net worth of the CCP which cannot be used until further in the default waterfall.

which must unwind the defaulter's assets, reaping the lowest price. In this way, the NMA scenario with multiple CCPs, can amplify the existing downward price pressure for CDS k in the liquidating CCP – driving it more quickly towards failure.

This effect looks quite different in the MA scenario; here the members of the non-liquidating CCP cannot directly see the liquidation process in the other CCP(s). The partial inobservability of the inter-CCP liquidation means that dealers possess less market information in the MA scenario than they would in the NMA scenario. Given this search market, where one CCP's predators are not privy to the other's default events, predators cannot gather timely information so as to exploit a liquidation. To illustrate this handicap, consider dealers' possible actions when an inter-CCP failure is known and yet opaque. First, a predatory dealer can sell assets in their home CCP, but without the help of a CCP liquidation, cannot meaningfully depress market prices. Therefore, this does not yield significant profit. Second, predatory dealers can still sell across CCPs, but this demands extra collateral $m_i^{uu',k}$ which increases with the price volatility produced from the influx of sales. This again reduces the profits of the dealer. Third, the decreasing price for *buy* CDS k in the liquidating CCP may become attractive to dealers in the other interoperable CCPs'. These unsuspecting dealers may bolster the price of k and hold up the market in the liquidating CCP. Through these three actions, the MA scenario may attenuate the existing downward price pressure for CDS k in the liquidating CCP - possibly saving it from failure.

The above is a rich result; though the dynamic complexity makes it difficult to quantify mathematically, it can be illustrated through simulation. The simulation results – provided in section 7 – reveal that what was anticipated here.

6.3 Default Waterfall

The previous section outlined how buyback trading, small shortfalls and subsequent clearing equilibrium affected the terminal net worths of the CCP and dealers. In this section, I further show how larger shortfalls demand the use of dealer contributions and CCP equity, as well as, how these alter their the clearing equilibrium. Thus, in the presence of significant member shortfalls, the CCP will progress through the mitigation measures outlined by the Default Waterfall. The risk-sharing default fund composes an important portion of this waterfall, but the steps governing its use differ between operability scenarios. Indeed, the safe use of the default fund is the subject of much regulatory debate. The current policy on interoperable frameworks states clearly that interoperable CCPs should not contribute to each other's default funds due to contagion risk. Yet, it remains uncertain whether allowing CCPs to use their own default funds to meet inter-CCP shortfalls (beyond what can be covered by the mutual inter-CCP margin) does the same.[Bank of England, 2014, Board, 2016] This paper is the first to explore this inter-CCP use of the default fund, and determine whether its use is actually detrimental to the overall health of the CCP.

At the end of the buyback phase, all dealers' (and CCPs) shortfalls are fully realised. In each operability scenario, the CCP will meet these shortfalls by progressing through its own unique default waterfall. In the existing (NMA) default waterfall, the CCP first meets a dealer shortfall by applying the dealer's own initial margin contribution (G_i^u) against that dealer's liabilities. Next, the CCP liquidates that dealer's positions, and follows with the use of that dealer's default contribution (D_i^u) to meet remaining shortfalls. For any remaining liability overhang, the CCP must then use a small tranche (ϵ_i^u) of it's own equity, followed by the full remaining (risk-sharing) default fund (D_{tot}^u). If it is still not be able to meet these liabilities, a CCP's only recourse is to declare failure or to seek a bail-out from a Lender of Last Resort (LoLR).

The MA default waterfall follows the standard version except that it presents with an extra requirement before the full default fund can be accessed. This step requires that inter-CCP shortfalls be met by a dealer's guarantee fund contribution (g_i), default contribution (d_i), inter-CCP margin contribution(m_i), and full risk-sharing inter-CCP margin ($M_{tot}^{uu'}$) before the rest of the default fund can be applied. Once the default fund can be accessed, I assume that it can be applied to both own-CCP and inter-CCP shortfalls.

In each operability scenario, a dealer's own contribution to the default fund is given by,

$$\hat{D}_i^u = \Big(\sum_{u=1}^{N} \Lambda_i^u + \gamma_i + R_i - \sum_{u'=1}^{N-1} g_i^{u'}\Big)^+ - \Big(\sum_{u=1}^{N} \Lambda_i^u + \gamma_i + R_i - \sum_{u'=1}^{N-1} g^{u'} - g_i^u\Big)^+ \quad \text{(NMA)}$$

$$\bar{D}_i^u = \Big(\Lambda_i^u + \gamma_i + R_i - g_i^u\Big)^+ - \Big(\Lambda_i^u + \gamma_i + R_i - g_i^u - d_i^u\Big)^+ \quad \text{(MA)} \tag{47}$$

Then, the total default fund is given by the combined contributions of all dealers; $D^u_{tot} = \sum_{i=1}^m D^u_i$. As before, the clearing reveals the equilibrium total default fund levels,

$$D^\star_{tot} = \sum_{i=1}^{m} D^\star_i = \begin{cases} \hat{D}^u_{tot} \wedge \sum_{i=1}^m \Big(\hat{D}^u_i - \big(\hat{G}^u_i - \hat{C}^{u,-}_i\big)^-\Big)^+ & \text{(NMA)} \\ \\ \bar{D}^u_{tot} \wedge \sum_{i=1}^m \Big(\bar{D}^u_i - \big(\bar{G}^u_i + \bar{M}^u_i - \bar{C}^-_i\big)^-\Big)^+ & \text{(MA)} \end{cases} \tag{48}$$

In the NMA case, there are no special provisions to help dealer's meet shortfalls to multiple CCPs. Instead, a dealer's ability to meet shortfalls in one CCP is highly conditional on its remaining cash and liquidation proceeds *after* meeting shortfalls in its other CCPs. In the NMA scenario, a dealer's ability to meet its shortfalls in any CCP is determine by,

$$\hat{C}_i^{u,-} = \big(\hat{\Lambda}_i^u + (\gamma_i + R_i - \Big(\sum_{u'=1}^{n-1} \hat{\Lambda}_i^{u',-}\Big))^+\big)^- \tag{49}$$

In the MA case, however, a dealer maintains additional financial resources to service inter-CCP liabilities with its margin fund. Thus, in the MA scenario the dealer is less likely to utilise the default fund. With interoperability the dealer's ability to meets its shortfall in any CCP is a consequence of,

$$\big(\bar{G}_i^u + \bar{M}_i^u - \bar{C}_i^{u,-}\big)^- = (\bar{G}_i^u + (-\bar{C}_i^{uu,-} + \sum_{u'=1}^{n-1} (\bar{M}_i^{uu'} - \bar{C}_i^{uu',-}))\,)^- \tag{50}$$

In the cases where the guarantee fund contribution(s) and default fund contribution(s) – and cross-margin fund in the MA scenario – are insufficient to meet the shortfall(s) of a defaulted member, the CCP be forced to apply a small tranche of its equity. However, each CCP's equity tranche is only a fraction[78] of it's nominal worth,

$$(1-\epsilon)\Big(\gamma_0^u + f^u \sum_{i=1}^{m} \Lambda_i^{u,+}\Big) \tag{51}$$

At the end of the buyback phase, applying the equity tranche changes the resulting terminal net worth of the CCP according to,

$$C_0^u(t_{T_\mathcal{T}} = 2) = \gamma_0^u + f^u \sum_{i=1}^{m} \Lambda_i^{u,+} - \begin{cases} \Big[\epsilon\big(\gamma_0^u + f^u \sum_{i=1}^m \hat{\Lambda}_i^{u,+}\big) - \sum_{i=1}^m \big(\hat{G}_i^{u,\star} + \hat{D}_i^{u,\star} + \hat{C}_i^{u,-}\big)^-\Big]^- & \text{(NMA)} \\ \\ \Big[\epsilon\big(\gamma_0^u + f^u \sum_{i=1}^m \bar{\Lambda}_i^{u,+}\big) - \sum_{i=1}^m \big(\bar{G}_i^{u,\star} + M_i^{u,\star} + \bar{D}_i^{u,\star} + \bar{C}_i^{u,-}\big)^-\Big]^- & \text{(MA)} \end{cases} \tag{52}$$

[78]The CCPs equity is not large enough to meet any sizeable defaults (by large members), and so only a tranche of that is likely insufficient to meet most shortfalls. However, it is immensely important that each CCP's equity is on the line so that each CCP has *skin-in-the-game*. That is, the threat to a CCP's equity prevents it from engaging in/or permitting excessively risky behaviour. An ongoing criticism of CCPs regards their equity as being too far down the default waterfall to make them truly accountable. Thus, strong and effective regulation and policy is particularly important for this profit-making institution.

with the result that CCP equity is depleted when the bracketed term is greater than zero, $[\cdot]^- \geq 0$.

Given that a dealer has also used its contributions, the resultant change in the terminal worth for a dealer i is,

$$C_i(t_{T\tau}=2) = (\gamma_i + Q_i + \Lambda_i) - \begin{cases} \Big[\hat{Z}_i(Q_i - R_i) + \sum_{u=1}^{n}(\hat{\Pi}^u_{0i}\hat{C}^{u,-}_0 + f^u\Lambda^{u,+}_i)\Big] \quad + & \text{(NMA)} \\ \qquad \sum_{u=1}^{n}\Big[\frac{\hat{G}^u_i}{\hat{G}_{tot}}(\hat{G}^u_{tot} - \hat{G}^{u,\star}_{tot}) + \frac{\hat{D}^u_i}{\hat{D}^u_{tot}}(\hat{D}^u_{tot} - \hat{D}^{u,\star}_{tot})\Big] & \\ \Big[\bar{Z}_i(Q_i - R_i) + \bar{\Pi}^u_{0i}\bar{C}^{u,-}_0 + f^u\Lambda^{u,+}_i\Big] \quad + & \text{(MA)} \\ \qquad \Big[\frac{\hat{G}^u_i}{\hat{G}_{tot}}(\hat{G}^u_{tot} - \hat{G}^{u,\star}_{tot}) + \frac{\hat{D}^u_i}{\hat{D}^u_{tot}}(\hat{D}^u_{tot} - \hat{D}^{u,\star}_{tot}) + \sum_{u' \neq u}^{n}\frac{M^{uu'}_i}{M^{uu'}_{tot}}(M^{uu'}_{tot} - M^{uu',\star}_{tot})\Big] & \end{cases} \tag{53}$$

Thus, the above equations show that both scenarios restrict the CCP's use of the full default fund (to meet remaining shortfalls) to the end of the buyback phase; when all trading is complete and the equilibrium is fully realised. This is in addition to the use of the cross-margin fund for inter-CCP positions in MA scenario. The replenishment of these three funds and its effect is detailed in the next section on CCP Recovery.

6.4 Recovery Phase and Fund replenishment

After the buyback phase come a final *Recovery* phase, at $t_{l\tau} = 3$. This is a resolution period where each CCP determines the final state of accounts and any member margin contributions needed to setup future trading. It is in this phase where I identify a potential tool for predation disincentivisation. It exists only in the MA scenario, due to its particular inter-CCP margin fund structure.

At the beginning of the Recovery phase, all trading has finished and the default waterfall has been applied: All dealer shortfalls have been met by their own proprietary contributions to the guarantee, default and cross-margin (in the MA scenario) funds. Any shortfalls resulting from dealer defaults, having been transferred to the CCP, have been met by the remaining total default and cross-margin funds – where all contributions can be used to meet fellow dealers' shortfalls. Thus, with the default waterfall applied, the CCP is left with $D^{u,\star}_{tot}$ and $M^{u,\star}_{tot}$ (in the MA scenario).

Given that in both scenarios, NMA and MA, each dealer's guarantee fund contribution is proprietary, $G^{u,\star}_i$, so it must be returned during the Recovery phase. In order that a dealer may remain a member of a CCP for the future, it must repost a guarantee margin. This means that the member must top-up their margin account if portfolio composition carries more risk or if any amount was used to meet shortfalls in the previous phases. Thus, each member dealer must post the following guarantee fund top-up to the CCP,

$$G^{u,\Re}_i(t_{T\tau}=3) = \Big(g^u_i - G^{u,\star}_i\Big)$$

This remains true for the default and cross-margin contributions as well. However, since these are risk sharing funds, dealer i's proportion of the remaining fund may be further reduced according to $\Big(\frac{D^u_i}{D^u_{tot}}(D^u_{tot} - D^{u,\star}_{tot})\Big)$. This means that in recovery, a dealer i may be forced to top up more default fund contribution than it used, such that,

$$D^{u,\Re}_i(t_{T\tau}=3) = \left(d^u_i - D^u_i\Big(1 - \frac{\sum_{i=1}^{m} D^{u,\star}_i}{D^u_{tot}}\Big)\right)$$

Thus, when this fund is depleted, a dealer doesn't necessarily have limited liability and in extreme scenarios, the CCP can make a call for additional default funds. One might think that these two facts would be sufficient to disincentivise predatory behaviors. However, in both scenarios the default fund is only 10% the size of the guarantee fund, resulting in a miniscule upper limit for any single contribution. As well, in past extreme cases when margin calls have demanded larger contributions, legal enforcement has been difficult resulting largely in dealer non-compliance. Therefore, replenishment of the default fund could only be a mild determinant to unwanted behaviour.

In the MA case consisting of the cross-margin fund, the amount depleted is augmented by any predation in a dealer's own CCP – as positions get more volatile and variation margin owed increases, the cross-margin increases as well. This means that home-CCP predation can severely deplete the total cross-margin fund, and risk every members personal contribution. Therefore, members may have to refill more than what they themselves used to meet liabilities. As this margin is set at approximately 1.2 times the size of the guarantee fund contribution, this could serve as a highly effective predation disincentive tool. The amount that members would need to refill in recovery is given by,

$$M_i^{u,\mathfrak{R}}(t_{T_\mathcal{T}} = 3) = \left(m_i^u - \left(M_{tot}^u - \sum_{i' \neq i}^{m-1} M_{i'}^{u,\star} - C_{tot}^{u,-} \right) \right)$$

There is also no reason to assume limited liability; if the current total cross-margin delivered to a partner CCP is insufficient to cover inter-CCP shortfalls, the partner CCP can make a margin call for more funds. Those funds would then have to be covered by the original CCP's members; either through cash or through the remaining default fund. If this is not met, then the original CCP is considered to be in default, or failure – which can be disastrous for its member dealers. Thus, in the MA scenario the replenishment of $M_i^{u,\mathfrak{R}} \geq m_i^u$ (and to a lesser extent $D_i^{u,\mathfrak{R}}$) can become quite a severe punishment for predatory behaviour. Since predatory traders are likely to be the healthiest members of their own CCP, it is they who will have to use their profits to refill the various funding schemes. Essentially, in this scenario, the predators become their own prey.

7 Simulation

The Simulation section builds upon the theoretical framework by empirically testing its most critical predictions through a dynamic, multi-period trading environment calibrated to real-world OTC market data. This section specifically examines how CCP interoperability (MA) influences systemic risk, market stability, and trading dynamics compared to fragmented clearing systems (NMA). The simulation evaluates the theoretical predictions about key mechanisms, including the effects of interoperability on default cascades, inter-CCP price dispersion, predatory trading, and collateral efficiency. By doing so, it not only validates the theoretical model but also translates its findings into actionable insights for policymakers and regulators.

A core focus of this section is the regulatory debate over the use of CCP default funds to address inter-CCP shortfalls. The theoretical framework suggests that applying these funds under interoperability agreements can significantly curb predatory behavior by forcing dealers to internalize the systemic consequences of their actions. At the same time, interoperability reduces the downward price pressure associated with CCP liquidations, mitigating the contagion effects seen in fragmented systems. Another key insight tested here is whether the mutual risk-sharing structure of interoperability penalizes predatory trading by requiring margin replenishments and increasing the sensitivity of contributions to CDS spread dynamics.

Through the simulation, these theoretical results are subjected to rigorous testing under a variety of market conditions, ranging from stable liquidity to crisis scenarios. The results aim to provide concrete guidance to regulators, including the conditions under which default fund utilization and interoperability agreements could enhance systemic stability without introducing unacceptable risks. This section bridges the gap between abstract theory and practical regulatory applications, offering a comprehensive analysis of the costs and benefits of interoperability in financial markets.

7.1 Setup and Technical Details

In order to illustrate the real-world implications of this theoretical framework and to discern its regulatory implication, the model is tested in a dynamic, multi-period simulation. The parameters are based on those provided in [Tywoniuk, 2017b]; they are provided in appendix A.3.1 and adapted to accommodate a two-CCP (rather than a mono-CCP) system, which may or may not, have a mutual agreement between CCPs. The simulation tests multiple default scenarios, varying the market liquidity and the number of distressed dealers vs. the number of predatory dealers. The same holdings[79], trading partners and trading decisions[80] are replicated between the NMA and the MA scenarios, with differences only in the membership (CCP location) of each dealer. The results reveal the effect of the above on the price process.

The simulation incorporates the data showing that 14 large global dealers dominating the CDS market. In the NMA scenario, each CCP contains the same 14 dealers; therefore, the situation of one CCP is mirrored in the other. In the MA case, the first set of seven dealers ($m^{u=1} = [1, ..., 7]$) belongs to CCP 1 and the second set of seven dealers ($m^{u=2} = [8, ..., 14]$) belongs to CCP 2. Thus, the single set of holdings, trades, and clearing are split between two CCPs. There are 100 different CDS instruments where each dealer can hold $k \in [1, .., 100]$. The current market data [Oehmke and Zawadowski, 2017, Duffie et al., 2015, Amini et al., 2015] shows that dealers hold an average of approximately 184 single-name CDS. However, since this analysis consists of liquidations over a very short-term, I assume that dealers would be able to move a smaller number of CDS on average. Therefore, I equally assume that it is valid that each dealer will only be able to trade about 100 CDS, on average. Furthermore, the data also supports that these large dealers do, in fact, hold CDS positions in multiple CCPs.

This simulation analyses the following scenario: A liquidation which takes place over one business week, followed by an indefinite buyback phase which caps predator holdings at the maximum of their previous holdings. The assignment of CDS positions to dealers is random, with trading partners assigned randomly as well. To achieve a realistic simulation of portfolios, dealers do not hold every CDS, nor do they have partnerships with every other dealer. This method of position assignment aligns with dealer portfolio attributes given by the data and is further detailed in [Tywoniuk, 2017b]. Particularly, data is used to the calibrate the market size, market depth, market share and size of dealer holdings. One slight amendment is made for the NMA scenario; the data for market share must be split amongst CCPs. As well, appendix A.3.1 also details the method used to set dealer margin contributions, the fundamental value for each CDS, and each dealers liquidation matrix (a_i).

The simulation also analyses various market liquidity scenarios. The first scenario explored is a stable, normally functioning market (*stable*) with a steady market depth, D_k^u, of $221e9$ – this market volume is given by Bloomberg (2017) and [Oehmke and Zawadowski, 2017]. The second scenario simulates a financial crisis (*crisis*) liquidity which is defined by a liquidity dry-up and has a market depth of $12e9$. The third scenario is one of varying liquidity (*decreasing*) – decreasing liquidity with distressed selling and increasing liquidity with bullish buying. During the liquidation period the market depth decreases from the stable level to the crisis level and increases again during the buyback period – this gives depth increments of approximately $38e9$.

Multiple trading strategies are also explored, including those where each CCP can and cannot trade during the buyback round. As well, the ability of distressed dealers to trade in the buyback round is also explored. Further scenarios investigate varying the amount of dealers and predators, as well as simply keeping predators or distressed dealers stable. By keeping the levels of one dealer type stable, one is able to tease out the effects of predation, competition, and distressed selling. This gives important insight where to focus monetary effort during a crisis, in order to achieve the most effective recovery or bailout plan (ie. *where* and *when* to inject funds in the financial network).

[79] In the NMA case, total holding is spread equally between CCPs

[80] This can easily be varied in the simulation by setting a different random seed for each CCP. However, keeping the same trading pattern normalises the results.

Once the holdings and trading relationships are established, the simulation sets the fundamental starting value of assets. Then the external default of one dealer initiates the liquidation cascade and the simulation tracks the price evolution, variation margin payments, CCP and dealer net worth, defaults, and margin application for each period, according to the equations outlined in previous sections. For the MA scenario, the simulation additionally calculates the cross-margin exchange between CCPs each period. It also tracks the continuously evolving difference in CDS prices (price dispersion) between CCPs and also between interoperability scenarios. At the end of each trading period, any dealer shortfalls from defaults are moved to the CCP's account; these assets are then progressively liquidated until the end of the liquidation phase – 5 trading periods. The buyback period focuses primarily on the asset purchases of the designated as predatory dealers; profit calculations are based on current versus starting asset prices. During the recovery, any predatory profits are used to refill any outstanding margin funds as previously outlined – the refill is then subtracted from profit, and a percentage change in assets is obtained. Main results are then plotted for both scenarios, and over all variations of trading strategies, however, only the results of non-trivial strategies are outlined below.

Using [Tywoniuk, 2017b], appendix A.3.2 and A.3.3 clearly outline the technical details used to validate the model, as well as, its shortcomings. It gives details and results of robustness tests and sensitivities. It also demonstrates that the model holds up to a Monte-Carlo-like simulation which tests 50 different dealer holding assignments (and thus, trading relationships/patterns) for the different trading strategy scenarios. The results do not vary greatly based on how holdings are assigned, and a general pattern for all the results emerges. The above technical details still hold with the addition of another CCP, and calibrated market values are distributed amongst the CCPs (approximately the global market, where a few global CCPs dominate).

7.2 Results & Policy Implications

This following presents the most relevant results from the simulation with corresponding references to the corresponding plots (found in the figures section). The discussion focuses on liquidity scenarios/trading strategies whose results are most revealing, or whose results provide important contrast.

I use a convention in the figures to simplify the designation of different scenarios. The 4 trading scenarios employed are; *Collusion, No Collusion, Stable Distressed* and *Stable Predators*. *Collusion* refers to a scenario with a single predator, or multiple predators colluding to predate as one. *No Collusion* refers to a scenario where predators must compete with each other for profits. In each of these scenarios *No. of dealers* refers to increasing numbers of predatory dealers (and thus, numbers of distressed dealers are decreasing inversely). *Stable Distressed (Stable Predators)* means that the number of distressed (predatory) dealers stays constant and low with 2 distressed dealers; here, as before, the numbers of predatory (distressed) dealers is increasing.

The first important result pertains to the number of dealer defaults which result from the liquidation and predation process. This result is roughly similar for both the NMA and MA scenarios, and is more revealing when contrasted with the single CCP scenario first presented in [Tywoniuk, 2017b]. Figure 6 demonstrates that, regardless of the amount of distressed dealers[81] in the market or the level of market liquidity, both multi-CCP operability (NMA and MA) scenarios result in a low level of dealer defaults during liquidation. In contrast, figure 7 shows that with only a single CCP, the number of defaults increases with the amount of distressed dealers in the system. Furthermore, with a single CCP, low liquidity seems to exacerbate defaults (figure 8). This result strongly supports *proposition 1* (sec. 4.2). Thus, it appears that the default mechanism is driven by the concentration, or volume, of positions in any CCP; since higher concentration translates to a higher trading volume, this amplifies the fire sale discount of an asset. Instead, in a system where positions are split amongst multiple CCPs, the amount which can be liquidated and predated on, is decreased; this attenuates the fall in asset prices. It is surprising that although the NMA multi-CCP system may decrease bilateral netting

[81]Where no. of distressed dealers is inverse of the number of predators for 14 dealers.

efficiency, still, the ability to split positions over multiple CCPs is protective of dealer default. Perhaps, this is one explanation for the perplexing fact that even in asset classes where inter-CCP agreements exist, some still dealers opt to maintain multi-CCP memberships.[McPartland and Lewis, 2016a]

The possibility of price dispersion between CCPs has not yet been considered by regulators. Yet, since each CCP presents with different dealer trading patterns and the market is OTC, the resulting price process for CDS will differ in each CCP. Thus, an asset's price will differ between CCPs, with the degree of difference dependent on the particular operability scenario. From the simulation results, we see that the type of liquidity scenario also affects the size of the price dispersion between CCPs. During normal liquidity (figure 9), the NMA structure shows very little price dispersion between CCPs; depending on the amount of distressed vs. predatory dealers, this fluctuates between 0.16 - 1.44 bps. In contrast the MA scenario shows marked price dispersion, between 36 - 59 bps. During crisis liquidity (figure 10) this is becomes much more accentuated in the MA scenario, where price dispersion jumps to 183 - 503 bps vs. 1.61-17.86 bps in the NMA scenario – that is an increase of 11 - 20% (MA) vs. 8 - 9% (NMA). Thus, all things being equal, mutual agreements yield less inter-CCP price transparency for dealers. This result is in-line with the premise put forth in *proposition 6* (sec. 5.3).

Figures 10 and 11 compare the inter-CCP price dispersion between the two agreement scenarios across different trading strategies. For all types of liquidity, when the number of distressed dealers is kept low, the price dispersion remains constant, regardless of the number of predators present. This demonstrates that price dispersion is likely driven by the number of prey[82] in the market rather than just the number of predators. Furthermore, only in the NMA scenario does a low number of distressed dealers results in a consistently low price dispersion. Yet, as the number of distressed dealers increases in the NMA scenario, the CCP price dispersion quickly increases to approach the MA scenario. Thus, in truly difficult financial times both operability scenarios can result in rapidly decreasing price transparency.

Figures 12 and 13 display the the liquidity and buyback surplus of dealers. This is the aggregate surplus over all the dealers in the system, regardless of whether they are predator or prey. The surplus is greater for the MA scenario in almost all cases. The difference in surplus between operability scenarios is small when there are a high numbers of distressed dealers. However, when the number of distressed dealers is low (and number of predators is high), there is a marked difference between operability scenarios, regardless of market liquidity. This may be an important indicator of dealer incentive compatibility to partake in either agreement scenario; the guaranteed surplus makes it more favorable for dealers to belong to a CCP with a mutual agreement. This result also indicates the profitability for all dealers to ensure that no dealer engages in actions that put other dealers in distress.

The aggregate surplus of dealers (in each type of CCP agreement) during normal and crisis liquidity is then compared in Figures 14 and 15. The figures provide the surplus difference between the NMA and MA scenarios: The negative result indicates that, in the liquidation phase, the MA scenario always yields a higher aggregate surplus for dealers. However, this is the opposite during the buyback phase, where the NMA scenario wins out. In all cases, these surplus differences are minimised when levels of distressed dealers are low, and markedly increase as the number of distressed dealers increases.[83] For the MA scenario, the increased liquidation surplus may indicate that the attenuation of any fire sale and predatory selling effects when dealers are split amongst two CCPs. This is likely caused by the fewer number of predators in a CPP who can actively witness (and take advantage of) the distressed selling of that CCP. As well, in the MA scenario, there is the advantage that any selling by the other CCP's predators affects that CCP, and cross-CCP buying holds up the price of assets during liquidation. Yet, since a CCP has more predators in the NMA scenario, it yields a larger number of commissions from predatory trade profits, given the larger profitable trade volume.

An important indicator of the dealers' incentive compatibility of each operability system, is given

[82] Dealers who are prey would be engaging in distressed selling.
[83] Any lack of data on the graph indicates a CCP failure, and thus no comparison is possible.

by the ratio of *surplus gained vs. the total margin charged*[84] to each dealer – as a function of the number of banks in the system. The MA scenario is the clear winner for both the liquidation and buyback phases, as shown in figures 17 and 18. Here dealers tend to have lower equilibrium margin charges in the MA scenario. However, in figure 18 there are many dips and spikes, which are the result of the predator competition effect. The competition effect (seen in figure 16) is most powerful when predators cannot make sufficient profit from the low number of prey (distressed dealers) available[85]. This occurs because when predator's face competition for a finite number of prey, the resulting margin replenishment charges may outweigh the available predatory surplus – which is exacerbated by low market liquidity. Thus, in the MA scenario, the margin replenishment charges incurred from predation can severely diminish predators' anticipated profits. Thus, the operability structure exerts a large influence on the competition effect; it peaks and falls at different points in the NMA vs MA scenarios.

It is equally as important to look at the CCPs' incentive compatibility for one operability scenario over the other. For this one can investigate a CCP's total equilibrium account over all phases and across multiple liquidity scenarios. This determines one scenario's superior robustness to CCP failure and/or recovery. Figure 18 demonstrates that the NMA scenario is more appealing and incentive compatible for the CCP; the MA scenario shows greater trading losses per period with a lower equilibrium margin to cushion CCP equity. This appears to a greater degree when liquidity is unstable (figure 19). It is interesting to note that the greatest positive profits occur for the CCP when the level of predators is kept reasonably low, or when there are low levels of distressed dealers to prey on. Figure 20 suggests that predators, often, cannot provide sufficient surplus to the CCP – fees on clearing – through predatory trading in order to offset the margin draw-downs which result from predation. This is especially evident during a time of crisis and best illustrated in figure 21. However, in each scenario, the CCP can seemingly mitigate losses by keeping the levels of distressed dealers low. This may be an indication to regulators, CCPs and an LoLR, that: First, considerable efforts should be made to generally dissuade predation. Second, and more important, monetary aid should be provided to distressed dealers at the first signs of systemic stress, in order to halt a subsequent default cascade.

It is possible to do the same analysis for the aggregate accounts of dealers, for each agreement scenario and for all the phases. This provides nuances for the above results. Figures 23 and 24 compare how dealers fare under stable and variable liquidity. Note that the first panel shows the terminal equilibrium accounts of dealers in aggregate. A pattern begins to emerge; in the MA scenario, predatory dealers make lower buyback profits, but sustain similar levels of profit and loss as compared to the NMA scenario. This supports our earlier supposition that a mutual agreement (MA) provides increased opacity to a CCP conducting liquidation - thus, preventing predator efficiency. As well, it is evident that recovery measure in the MA scenario make it more resilient over all trading strategies. Unstable liquidity conditions in panel 3 of figure 23 display the average dealer's account; it displays wealth including predation profits and contrasts this with wealth when margins must be replenished. Although the NMA scenario allows predators to gain greater profits from predation, the larger mutual losses which must be covered by all dealers mean that, on average, all dealers are wealthier in the MA scenario. Therefore, the MA is more incentive compatible for each dealer.

Finally, one can answer the question about the proper use of the default fund by investigating the state of the combined margin funds at the end of each trading period. Recall that in the NMA scenario, the combined fund is just the guarantee and default fund, and in the MA scenario, the inter-CCP margin exchange fund is added. At the end of the liquidation period, a CCP can only use the guarantee fund and cross-margin funds to meet dealers' shortfalls while in the buyback phase, the default fund may finally be used. Figure 24 displays the case for stable liquidity: It shows that the equilibrium guarantee fund ($G^{\star}$) is always larger in the NMA case, although in the MA case, it is more stable over trading strategies. Note that the fund level is 2 times higher in the NMA case, as there are twice the participants in each CCP; this is good for the CCPs, but has large costs for dealers. Once liquidity sees

[84] Incentive compatibility does not include replenishment margin charged, which would not be known ex-anti, at the time the dealer makes this calculation.

[85] This is especially evident with only 4 or 6 banks in the market.

instability (figure 25), the much larger NMA-type fund sees more volatility and greater depletion over certain scenarios. The NMA scenario displays a greater dependence on the risk-sharing default fund, suggesting it is less resilient to volatility in market liquidity. During crisis levels of liquidity, figure 26 shows that the NMA scenario also results in greater CCP distress under certain trading strategies. Firstly, the guarantee fund is depleted when there are large numbers of competing predators with few distressed dealers to prey on – predators increasingly prey on themselves. Second, during the buyback period, the guarantee fund and default funds are severely depleted when there are high levels of distressed dealers, but levels of predators are too low to profit sufficiently (as seen previously in figure **??**). For these case, the smaller MA funds shows greater resilience to depletion. This provides evidence that a mutual agreement between CCPs makes them less sensitive to liquidity risk; thus, giving strength to the financial network in times of crisis. As well, these results allay the current worry of regulators; using the default fund to meet the shortfalls from a partner CCP does not increase the risk of contagion and seems warranted. Thus, this might be another strong indication to regulators that there is increased resiliency in releasing CCPs' default funds to meet inter-CCP shortfalls.

8 Conclusions

This paper provides a framework for investigating the risks and benefits of CCP multilateral clearing agreements for the CDS market. The theoretical model examines the effects of dealer actions—including guarantee fund contributions, variation margin exchange, and trading—on price dynamics within and across multiple CCPs. It then evaluates the robustness of interoperable and fragmented systems to default cascades, contagion, and predatory amplification effects. The integration of stylized features of interoperability offers novel insights, such as the sufficiency of the inter-CCP margin fund in mitigating contagion and the benefits of applying default funds to address inter-CCP shortfalls—a key point in ongoing regulatory debates. The framework also uncovers the surprising impact of interoperability on increasing price dispersion between CCPs due to fragmented clearing, while providing an endogenous mechanism to discourage predatory trading through the structure of inter-CCP margins. These findings shed light on the trade-offs inherent in CCP interoperability and offer actionable insights for regulators and lenders of last resort (LoLRs).

The theoretical framework yields several key results: (i) interoperability reduces aggregate systemic risk by attenuating the likelihood and severity of default contagion, (ii) it increases collateral efficiency through netting but introduces inter-CCP price dispersion and amplified asset volatility, (iii) interoperable margining inherently discourages predatory trading by enforcing mutual risk-sharing among participants, and (iv) interoperability can serve as an endogenous recovery mechanism during crises by curbing moral hazard and stabilizing volatility driven by predatory behavior. Novel contributions include quantifying inter-CCP price dispersion—an overlooked source of market fragmentation—and demonstrating the systemic benefits of prudent interoperability frameworks, which can provide resilience under financial distress. These findings offer practical recommendations to policymakers and LoLRs, especially concerning the timing and targeting of interventions in different liquidity scenarios.

The simulation, calibrated to OTC market data, substantiates the theoretical results and provides additional evidence for the trade-offs of interoperability. It demonstrates that a multi-CCP system attenuates fire-sale losses, reduces contagion among dealers, and increases overall systemic stability. Notably, the interoperable (MA) scenario proves more incentive-compatible for dealers by generating higher ratios of surplus to overall margin contributions, while fragmented (NMA) systems appeal more to CCPs seeking larger guarantee funds to protect their equity. In the context of predatory behavior, interoperability provides clear benefits: forced margin replenishments penalize predatory traders, leading to higher average surpluses for all dealers. The simulation also shows that reducing dealer distress and minimizing predation consistently enhance CCP profits and network stability. Additionally, interoperability reduces CCP liquidation losses and diminishes predator buyback profits, supporting the theoretical conclusion that predatory trading is less efficient in opaque, interoperable networks.

This work provides actionable recommendations for regulators and LoLRs to stabilize financial networks during stress. First, targeted liquidity injections should be directed at dealers whose defaults pose systemic risks, ideally early in the crisis cycle to prevent cascading failures and mitigate contagion. Second, funding should prioritize CCPs with mutual clearing agreements, as their endogenous risk-sharing mechanisms better absorb shocks and discourage predatory behavior. Third, the use of CCP default funds to cover inter-CCP shortfalls is strongly supported, as this mechanism not only avoids increased risk but also enhances CCP resilience to failure. Together, these recommendations advocate for prudent regulation of derivatives interoperability, highlighting how its benefits can outweigh its risks when carefully managed.

While this paper provides important insights into the dynamics of CCP interoperability, it also acknowledges several limitations. The model and simulation, while capturing key features of OTC market dynamics, are necessarily simplified to ensure tractability. Certain real-world complexities, such as nuanced dealer behaviors and broader macroeconomic feedback loops, remain outside the scope of this analysis. Additionally, the results are based on simulations calibrated to OTC data but are not validated against extensive empirical data, which was limited in availability. Therefore, this work should be viewed as a starting point for further empirical investigation into the costs and benefits of interoperability agreements. Future research could expand upon these findings by incorporating richer datasets, exploring additional market scenarios, and further refining the mechanisms modeled here.

In summary, this paper answers regulators' calls for a rigorous analysis of CCP interoperability by providing a large-scale, multi-CCP, multi-dealer, and multi-asset trading simulation. It captures critical dynamics under various liquidity scenarios, including stable, distressed, and crisis conditions. The results offer valuable insights into the trade-offs of interoperability, equip policymakers with evidence-based recommendations, and provide precise guidance for LoLRs on the timing and targeting of liquidity interventions. By addressing both the benefits and risks of CCP interoperability, this work serves as an important foundation for future research and policy development in this critical area of financial stability. ((iv) Finally, the paper recommends the use of the default fund to meet inter-CCP shortfalls. In most cases, the default fund remains intact highlighting how its use not only does not increase risk, but actually increases CCP resilience to failure. Thus, this work then firmly supports derivatives interoperability; although interoperability poses unique risks, with prudent regulation, its benefits clearly outweigh its risks.

References

[Amini et al., 2015] Amini, H., Filipovic, D., and Minca, A. (2015). Systemic risk and central clearing counterparty design. *Swiss Finance Institute Research Paper*, 13(34).

[Anderson et al., 2013] Anderson, S., Dion, J.-P., and Saiz, H. P. (2013). To link or not to link? netting and exposures between central counterparties. *Bank of Canada*.

[Augustin et al., 2014] Augustin, P., Subrahmanyam, M. G., Tang, D. Y., and Wang, S. Q. (2014). Credit default swaps: A survey. *Foundations and Trends in Finance*, 9(1-2):1–196.

[Bank of England, 2014] Bank of England (2014). Implementation by the bank of england of ESMA's guidelines and recommendations on CCP interoperability arrangements. *Consultation Paper*.

[Board, 2016] Board, E. S. R. (2016). ESRB report to the european commission on the systemic risk implications of CCP interoperability arrangements.

[Brunnermeier and Pederson, 2005] Brunnermeier, M. K. and Pederson, L. H. (2005). Predatory trading. *Journal of Finance*, 60(4):1825–1863.

[Clearnet, a] Clearnet, L. Frequently asked questions regarding interoperability for equities.

[Clearnet, b] Clearnet, L. Interoperability.

[Cont, 2010] Cont, R. (2010). Credit default swaps and financial stability. *Banque de France - Financial Stability Review*, No. 14:35–43.

[Cont and Kan, 2011] Cont, R. and Kan, Y. H. (2011). Statistical modeling of credit default swap portfolios. *Working Paper*.

[Cont and Minca, 2014] Cont, R. and Minca, A. (2014). Credit default swaps and systemic risk. *Annals of Operations Research*.

[Cont et al., 2013] Cont, R., Moussa, A., and Santos, E. B. (2013). *Handbook of Systemic Risk*, chapter Network structure and systemic risk in banking systems, pages 327–368. Cambridge University Press.

[Cont and Wagalath, 2013] Cont, R. and Wagalath, L. (2013). Running for the exit: Distressed selling and endogenous correlation in financial markets. *Mathematical Finance*, 23(4):718–741.

[Cont and Wagalath, 2014] Cont, R. and Wagalath, L. (2014). Fire sales forensics: Measuring endogenous risk. *Mathematical Finance*, pages 1–32.

[Council, 2012] Council, F. S. O. (2012). 2012 FSOC annual report. Appendix A: Designation of systemically important financial market utilities, Unites States Treasury.

[Cox et al., 2013] Cox, N., Garvin, N., and Kelly, G. (2013). Central counterparty links and clearing system exposures. *Reserve Bank of Australia*.

[Duffie et al., 2017] Duffie, D., Dworczak, P., and Zhu, H. (2017). Benchmarks in search markets. *Journal of Finance*, 72.

[Duffie et al., 2010] Duffie, D., Li, A., and Lubke, T. (2010). Policy perspectives on OTC derivatives market infrastructure. *Federal Reserve Bank of New York Staff Reports*, (424).

[Duffie et al., 2015] Duffie, D., Scheicher, M., and Vuillemey, G. (2015). Central clearing and collateral demand. *Journal of Financial Economics*, 116(2):237–256.

[Duffie and Zhu, 2011] Duffie, D. and Zhu, H. (2011). Does central counterparty clearing reduce counterparty risk. *Review of Asset Pricing Studies*.

[Eisenberg and Noe, 2001] Eisenberg, L. and Noe, T. H. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

[Fed, 1994] Fed, C. (1994). Chicago Fed letter. *Essays On Issues*, (87).

[Feng and Hu, 2016] Feng, X. and Hu, H. (2016). CCP interoperability and system stability. *International Journal of Modern Physics C*, 27(6):1–18.

[Feng et al., 2014] Feng, X., Hu, H., and Pritsker, M. (2014). Network effects, cascades and CCP interoperability. *International Journal of Modern Physics C*, 25(9):1–13.

[Fleming and Sarkar, 2014] Fleming, M. J. and Sarkar, A. (2014). The failure resolution of Lehman Bothers. *Federal Reserve Bank of New York Economic Policy Review*, pages 175–206.

[Glasserman et al., 2015] Glasserman, P., Moallemi, C. C., and Yuan, K. (2015). Hidden illiquidity. *Office of Financial Research*.

[Glasserman et al., 2016] Glasserman, P., Moallemi, C. C., and Yuan, K. (2016). Hidden illiquidity with multiple central counterparties. *Operations Research*, pages 1143–1158.

[Group, 2015] Group, L. S. E. (2015). CC&G risk disclosure.

[IOSC, 2012] IOSC (2012). The credit default swap market. Report 5/12, The International Organization of Securities Commissions.

[LCH Ltd., 2017] LCH Ltd. (2017). CCP interoperability.

[Magerle and Nellen, 2011a] Magerle, J. and Nellen, T. (2011a). Interoperability between central counterparties. *Swiss National Bank Working Papers*.

[Magerle and Nellen, 2011b] Magerle, J. and Nellen, T. (2011b). Interoperability between central counterparties. *Swiss National Bank*.

[McPartland and Lewis, 2016a] McPartland, J. and Lewis, R. (2016a). The challenges of derivatives CCP interoperability arrangements. *The Journal of Financial Market Infrastructures*.

[McPartland and Lewis, 2016b] McPartland, J. and Lewis, R. (2016b). The challenges of derivatives CCP interoperability arrangements. *Journal of Financial Market Infrastructures*, 4(4).

[Minca and Amini, 2012] Minca, A. and Amini, H. (2012). *Advances In Network Analysis and Its Applications*, volume 18 of *Mathematics in Industry*, chapter Mathematical Modeling of Systemic Risk, pages 3–26. Springer Verlag.

[O'Donoghue et al., 2014] O'Donoghue, B., Peacock, M., Lee, S., and Capriotti, L. (2014). Mean-reverting model for credit spread dynamics. *International Journal of Theoretical and Applied Finance*, 17(3):14.

[Oehmke and Zawadowski, 2017] Oehmke, M. and Zawadowski, A. (2017). The anatomy of the CDS market. *The Review of Financial Studies*, 30(1):80–119.

[Paddrik et al., 2016] Paddrik, M., Rajan, S., and Young, H. P. (2016). Contagion in the cds market. *Office of Financial Research Working Paper*, 16(12):1–28.

[Securities and Authority, 2016] Securities, E. and Authority, M. (2016). Final report: Possible systemic risk and cost implications of interoperability arrangements.

[Tang and Yan, 2013] Tang, D. Y. and Yan, H. (2013). What moves cds spreads. *Working Paper*.

[Terhune, 2010] Terhune, C. (2010). ICE's jeffrey sprecher: The sultan of swaps. Technical report, Bloomberg.

[Tywoniuk, 2017a] Tywoniuk, M. (2017a). CDS central counterparty clearing default measures: Road to recovery or invitation to predation? *SSRN: Working Paper version 2017*.

[Tywoniuk, 2017b] Tywoniuk, M. (2017b). CDS counterparty clearing: Road to recovery or invitation to predation. *Working Paper*.

[Veraart and Aldasoro, 2022] Veraart, L. A. M. and Aldasoro, I. (2022). Systemic risk in markets with multiple central counterparties. *BIS Working Papers*, (1052).

9 Figures

List of Figures

Unless otherwise indicated, the number of distressed dealers decreases as the predator number increases. This a constraint due to a finite number of dealers in the market. As well, numerics of the order of 10^9 can be interpreted as in billions and 10^{12} as trillions. They are shown in scientific notation to underline the magnitude of losses which can be incurred.

9.1 Comparison of 2 CCP System with a 1 CCP System

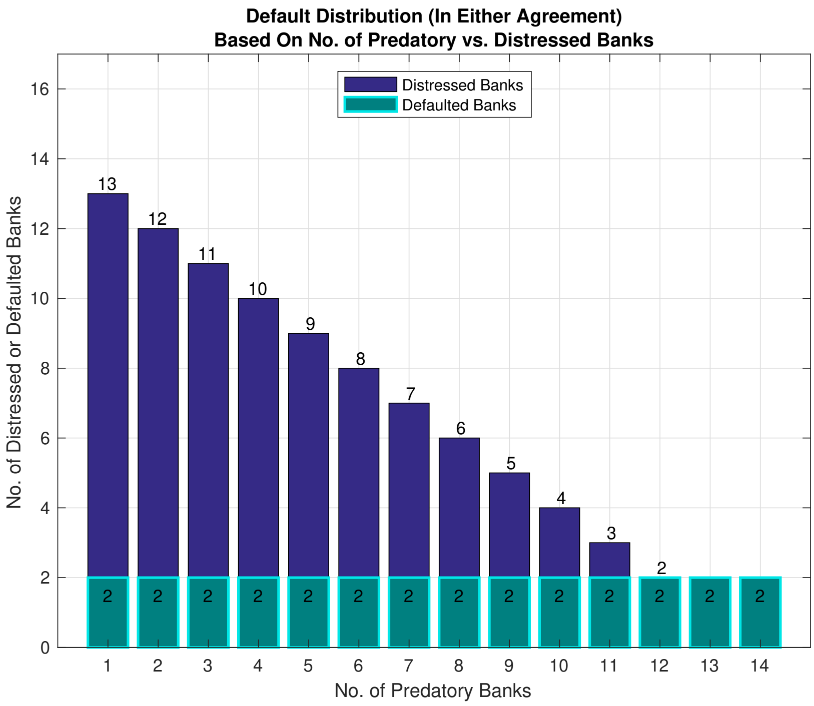

Figure 6: Number of distressed dealers as drivers of defaults (no collusion, increasing predators) for two CCPs in all liquidity states.

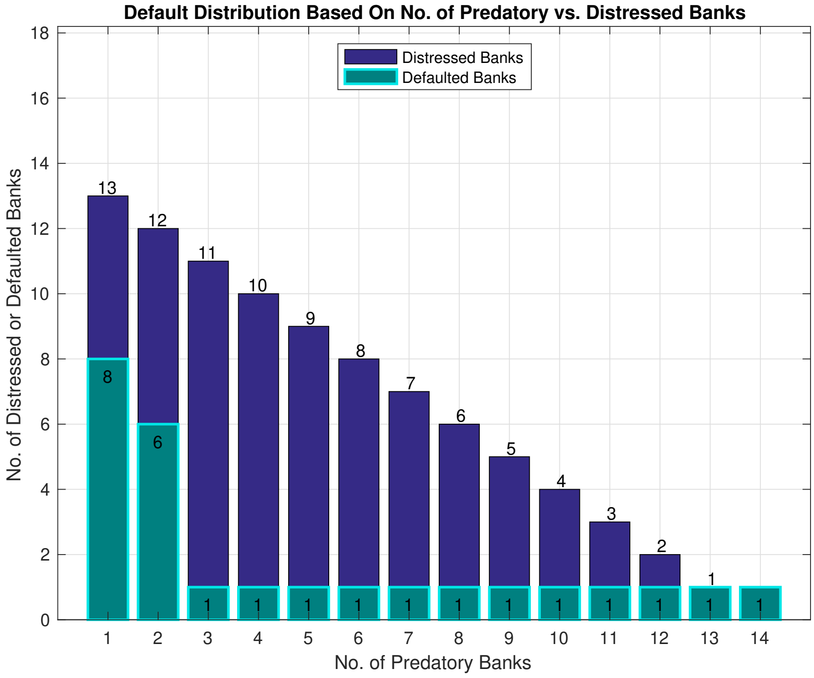

Figure 7: Number of distressed dealers as drivers of defaults (no collusion, increasing predators) for 1 CCP for Normal liquidity

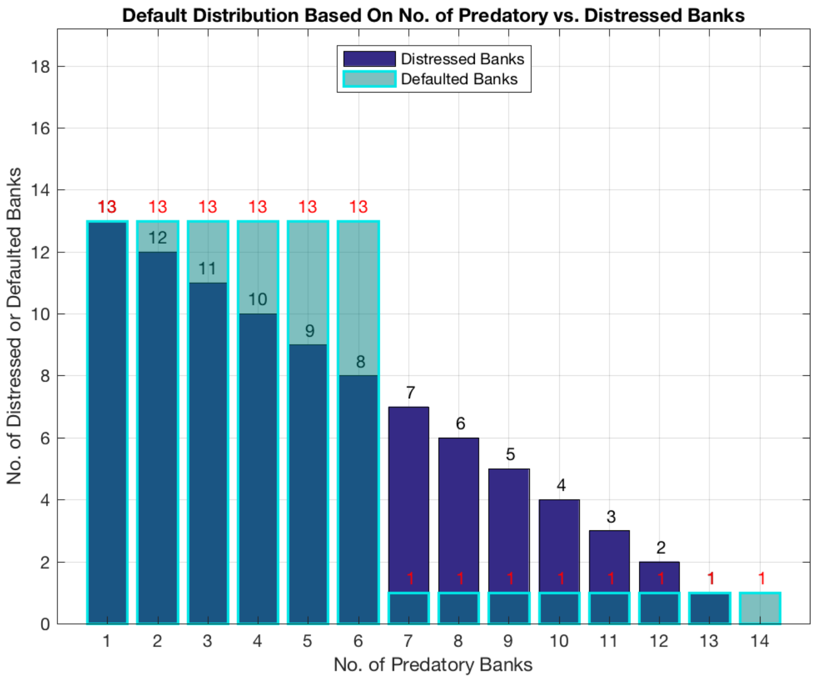

Figure 8: Number of distressed dealers as drivers of defaults (no collusion, increasing predators) for 1 CCP in a crisis liquidity state.

9.2 CCP Mutual Agreements under Normal, Volatile/Variable, Crisis Market Liquidity

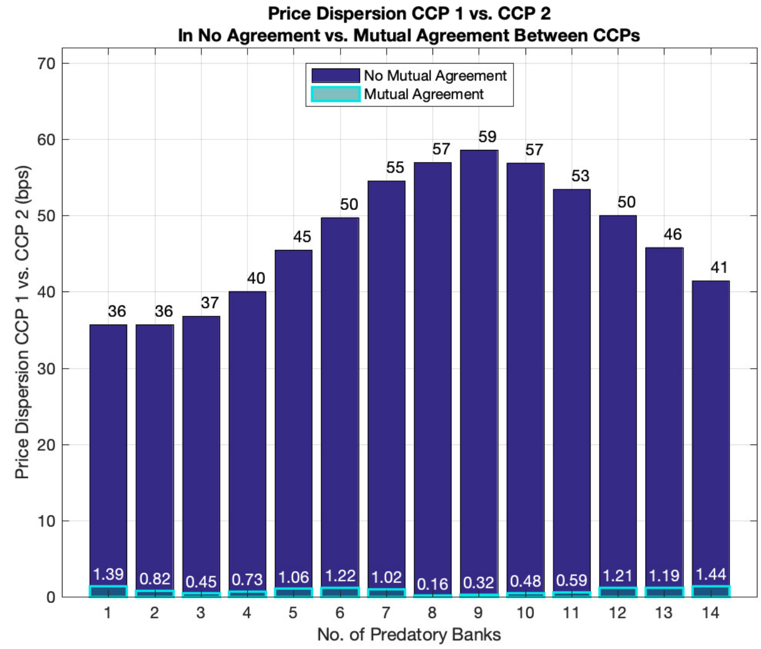

Figure 9: Price dispersion between CCP 1 and CCP 2 in each type of CCP agreement structure under normal liquidity.

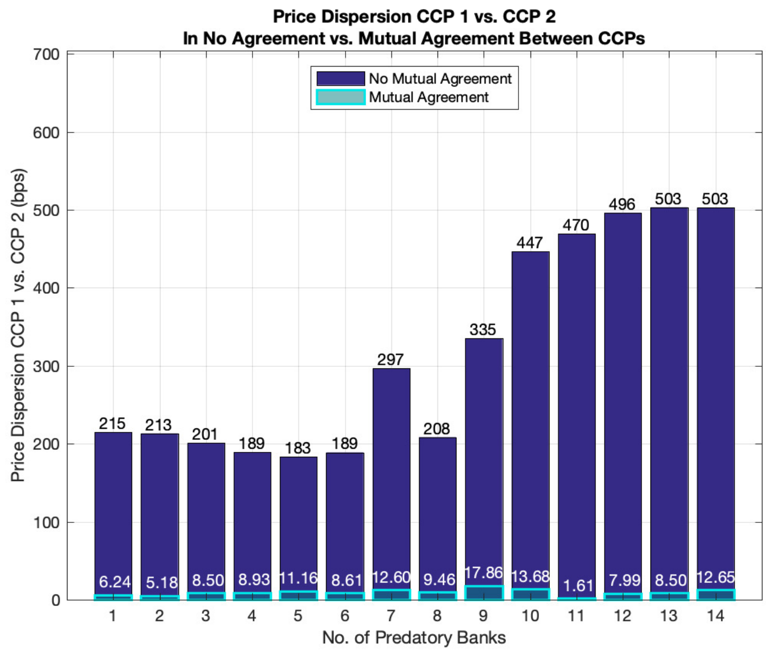

Figure 10: Price dispersion between CCP 1 and CCP 2 in each type of CCP agreement structure under crisis liquidity.

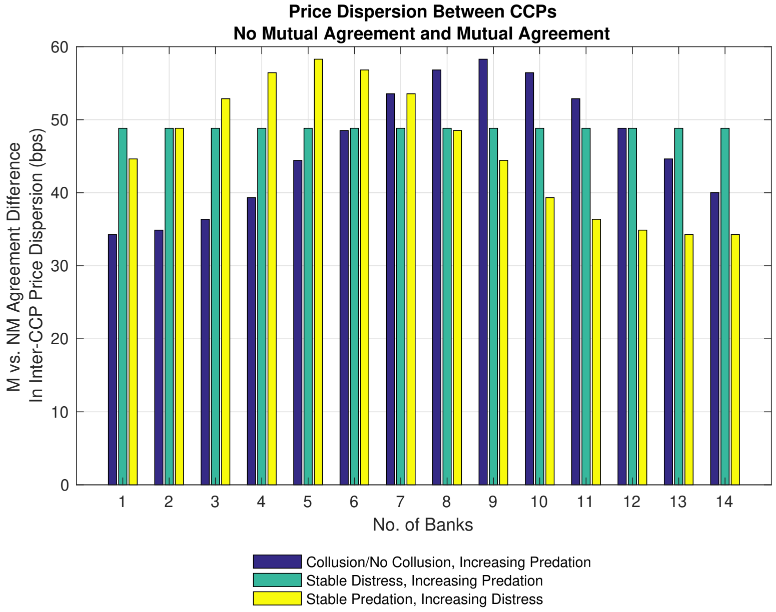

Figure 11: Difference in Inter-CCP Price Dispersion between No Mutual Agreement vs. Mutual Agreement between CCP's during normal liquidity

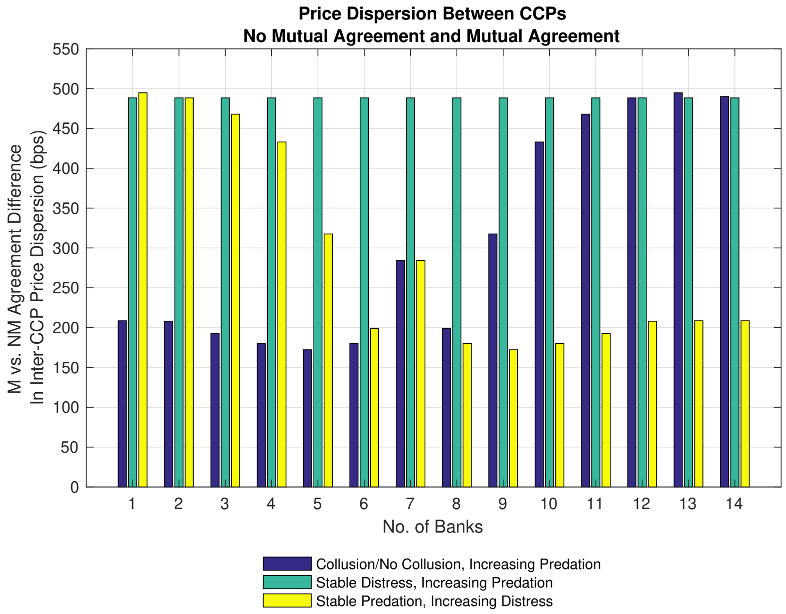

Figure 12: Difference in Inter-CCP Price Dispersion between No Mutual Agreement vs. Mutual Agreement between CCP's during crisis liquidity

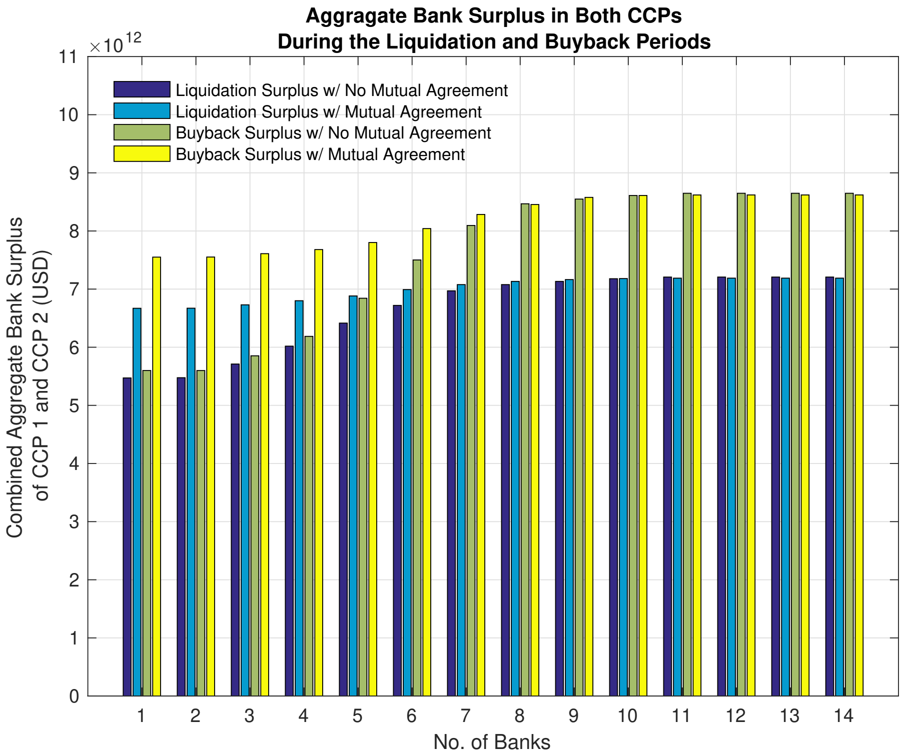

Figure 13: The aggregate surplus of dealers in both CCP's during the liquidation and buyback periods during normal market liquidity.

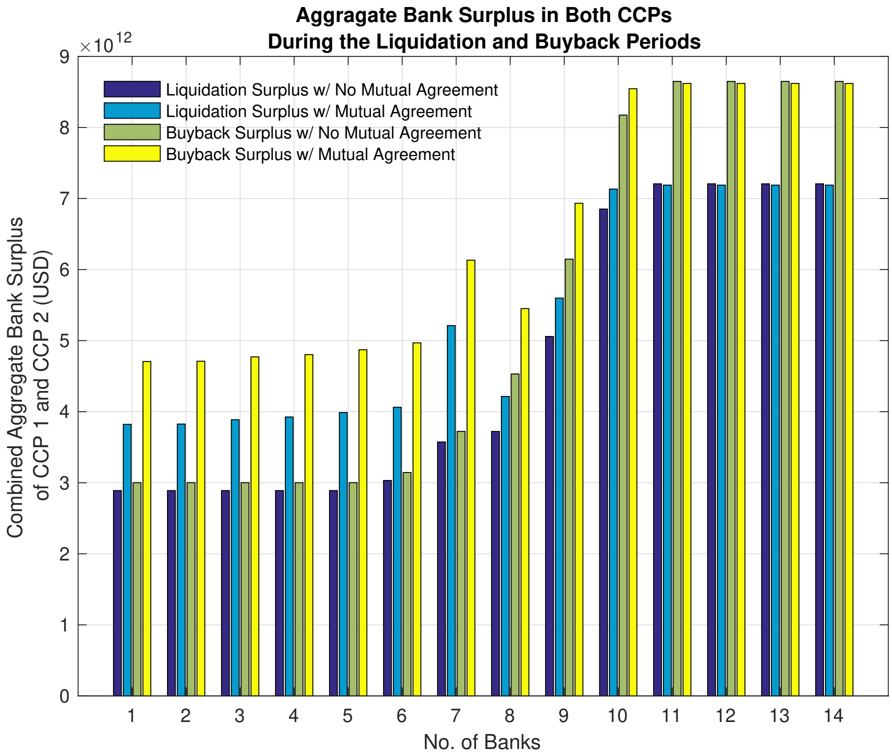

Figure 14: The aggregate surplus of dealers in both CCP's during the liquidation and buyback periods during crisis liquidity.

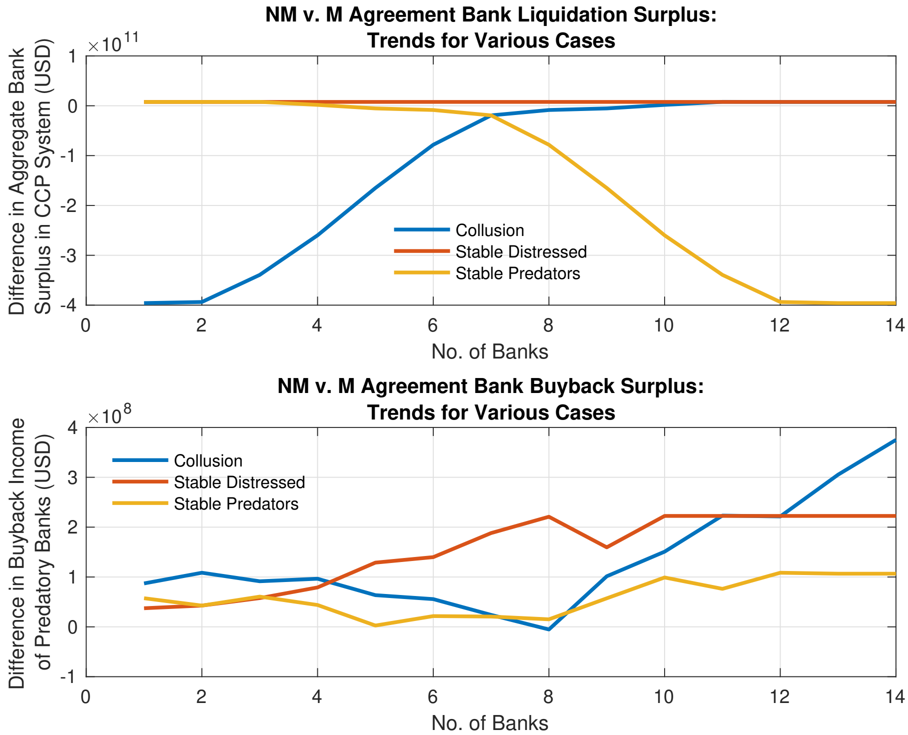

Figure 15: The liquidation and buyback surplus for dealers in each type of CCP agreement during normal liquidity.

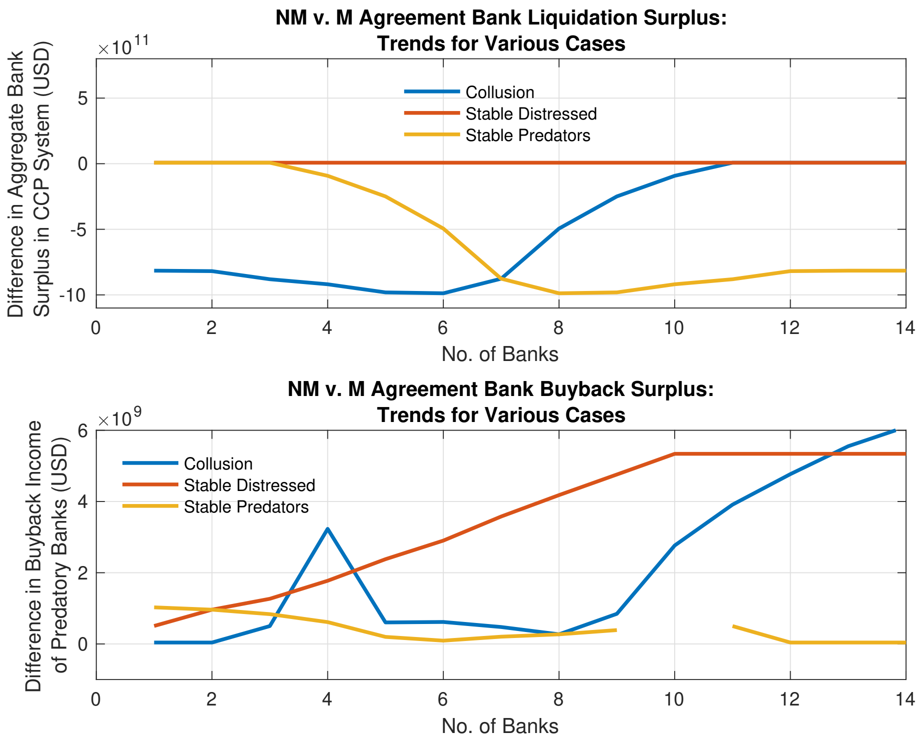

Figure 16: The liquidation and buyback surplus for dealers in each type of CCP agreement during crisis liquidity.

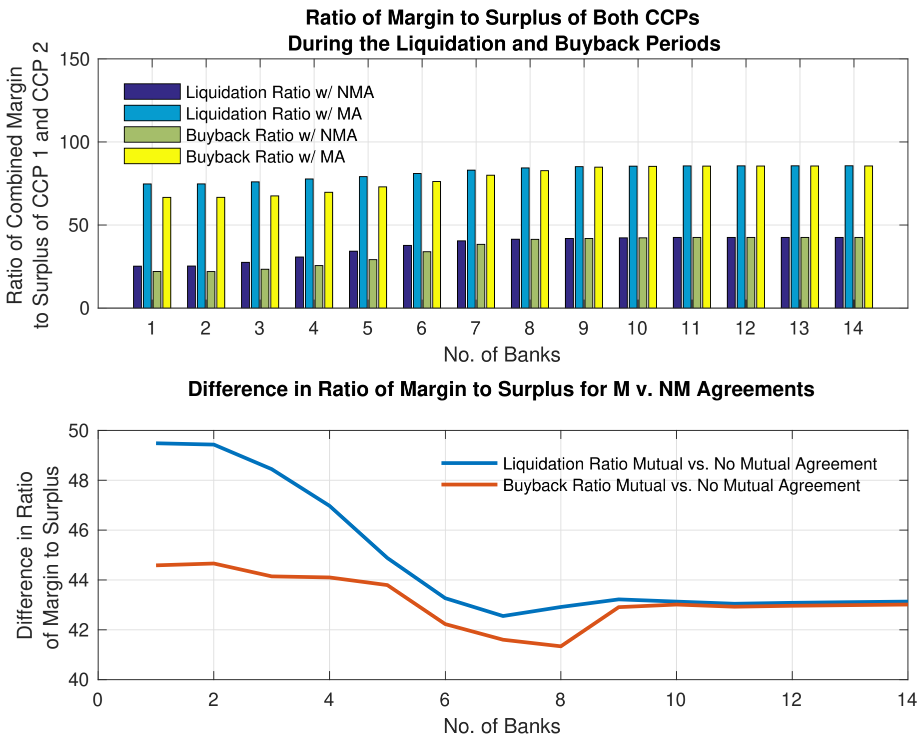

Figure 17: Ratio of Margin-to-Surplus for dealers in each CCP agreement system under normal liquidity.

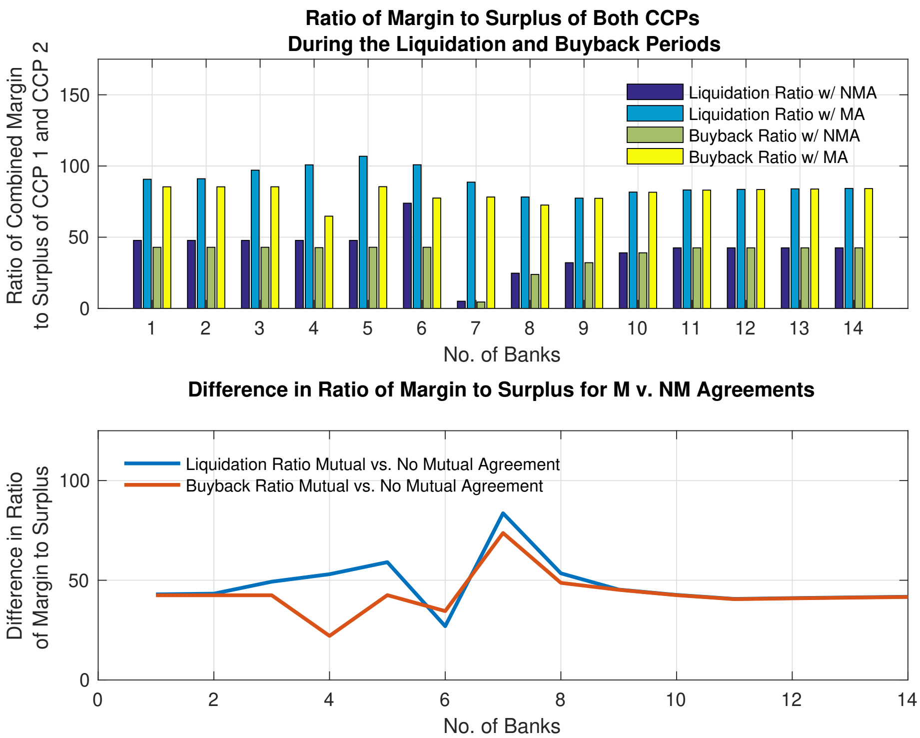

Figure 18: Ratio of Margin-to-Surplus for dealers in each CCP agreement system under crisis liquidity.

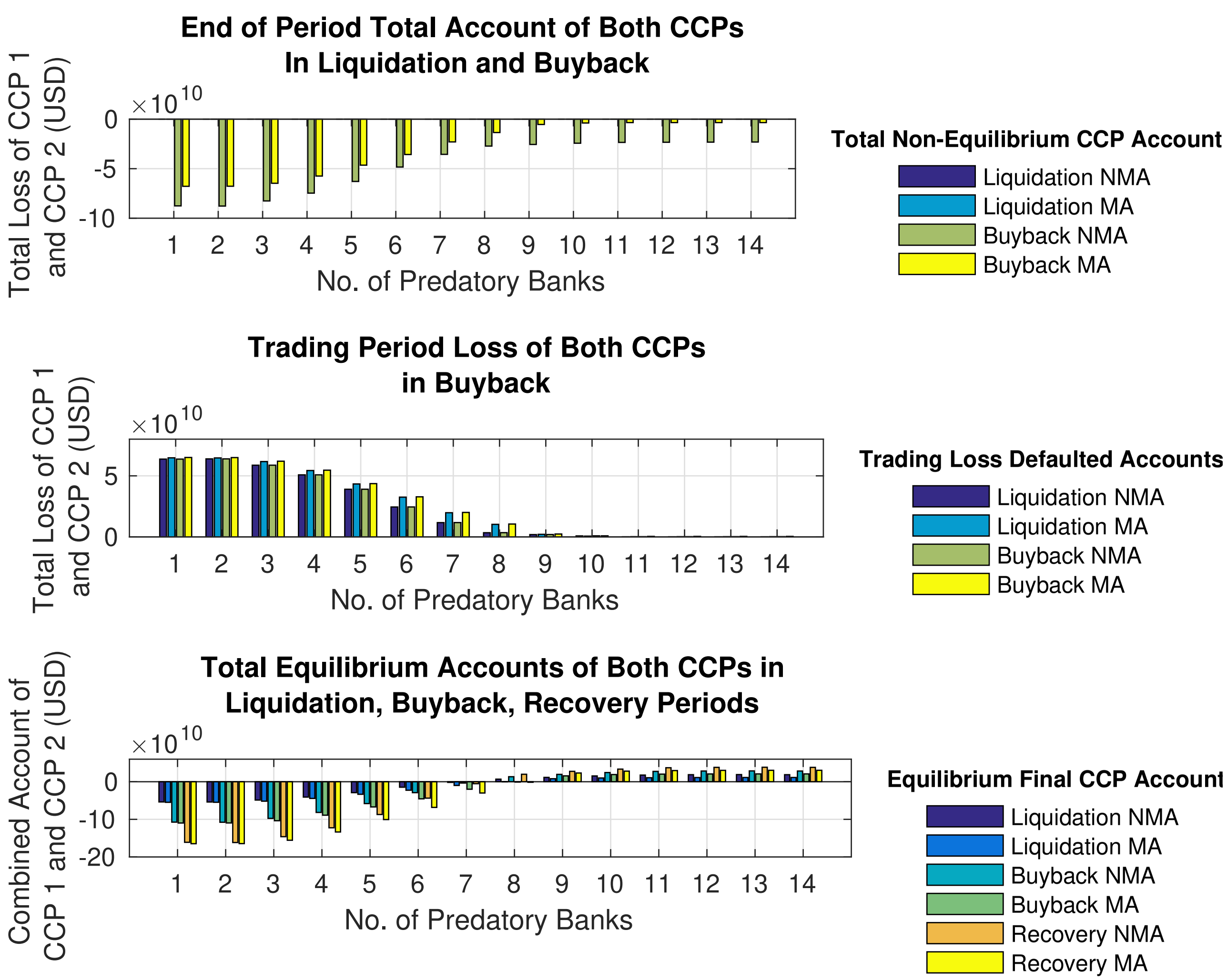

Figure 19: Accounts and trading losses during the liquidation, buyback and recovery periods in each type of CCP agreement during normal liquidity.

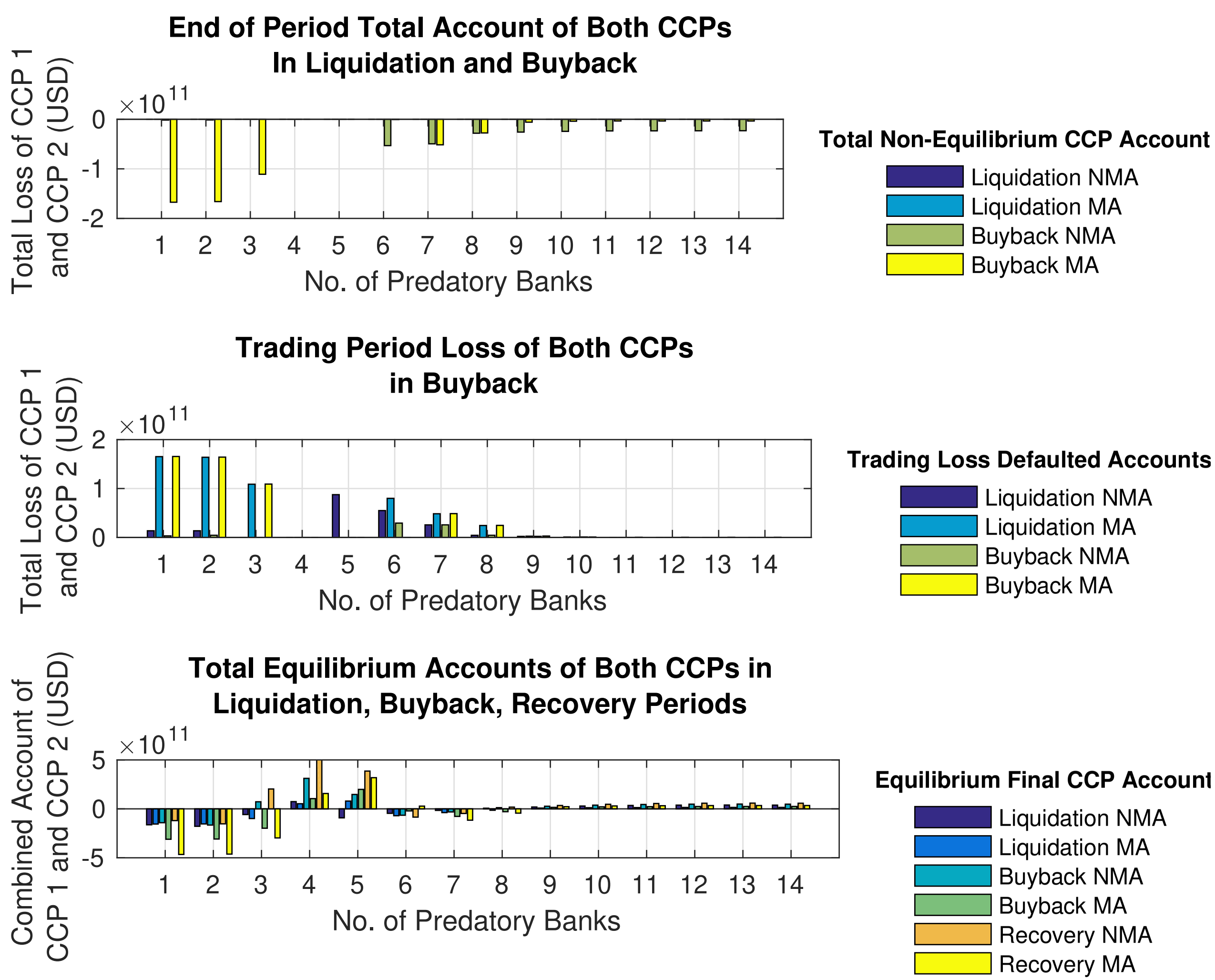

Figure 20: Accounts and trading losses during the liquidation, buyback and recovery periods in each type of CCP agreement during variable liquidity.

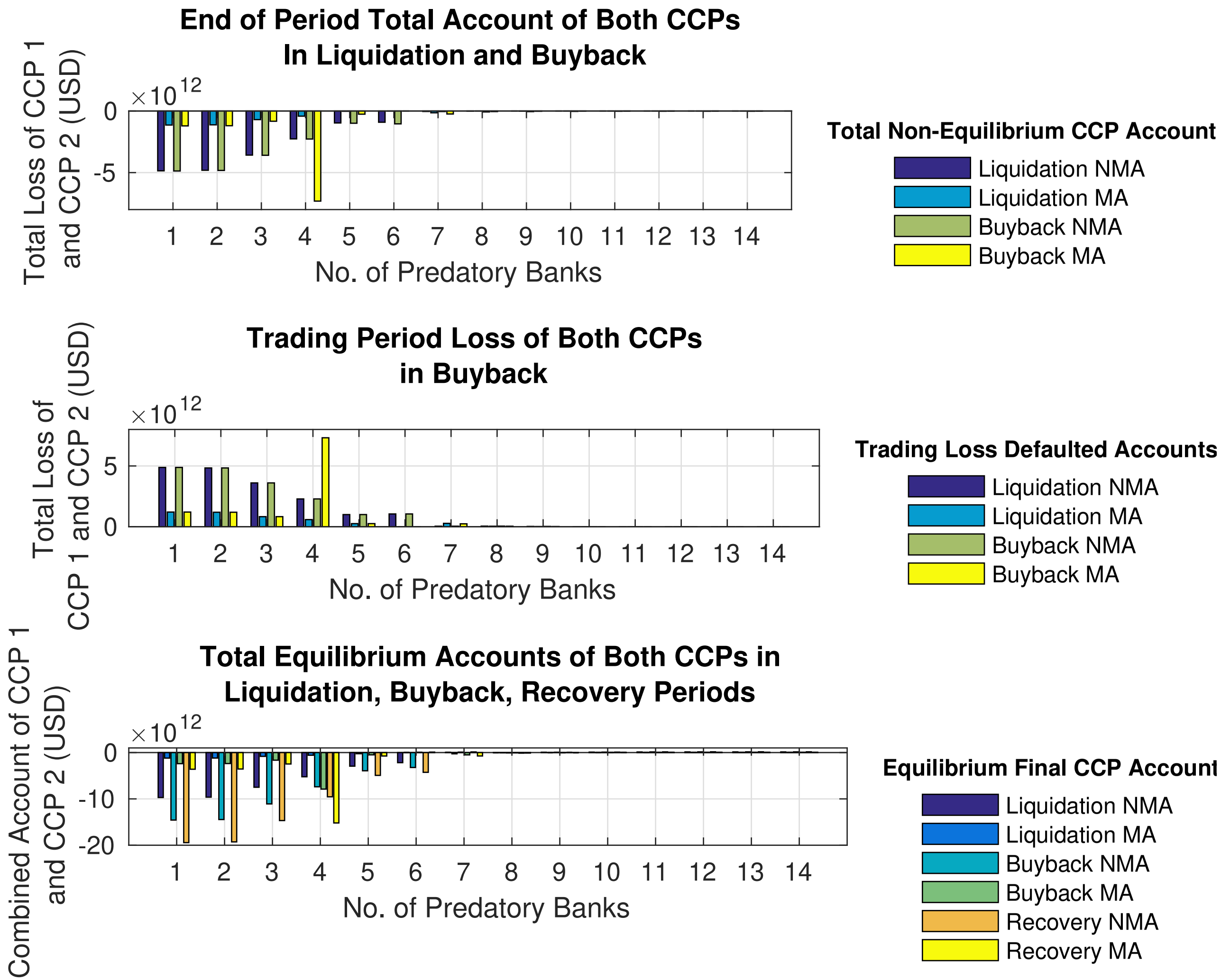

Figure 21: Accounts and trading losses during the liquidation, buyback and recovery periods in each type of CCP agreement during crisis liquidity.

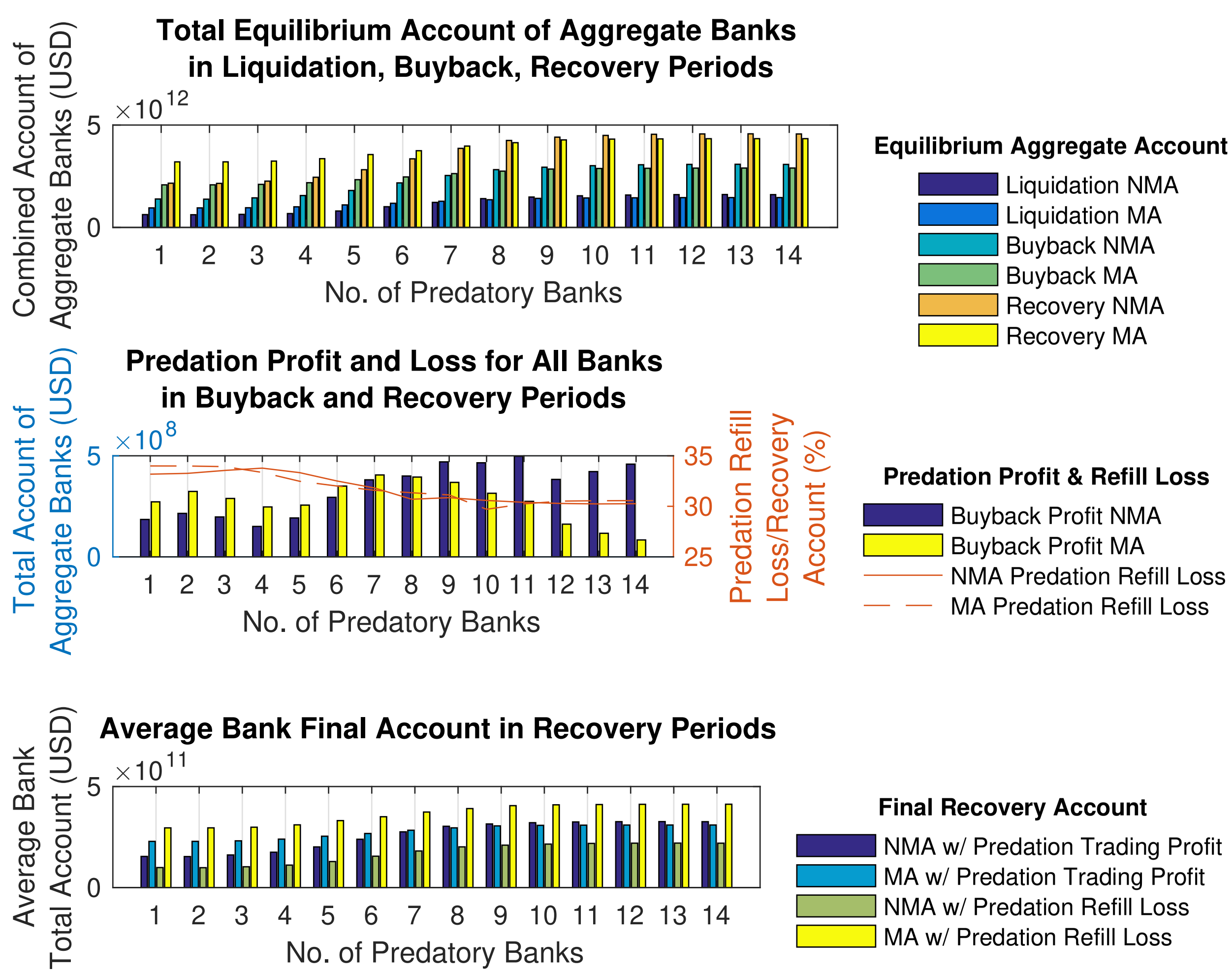

Figure 22: The accounts, predation profits and guarantee fund refill losses for dealers in both CCP agreement systems during the liquidation, buyback and recovery periods under normal market liquidity.

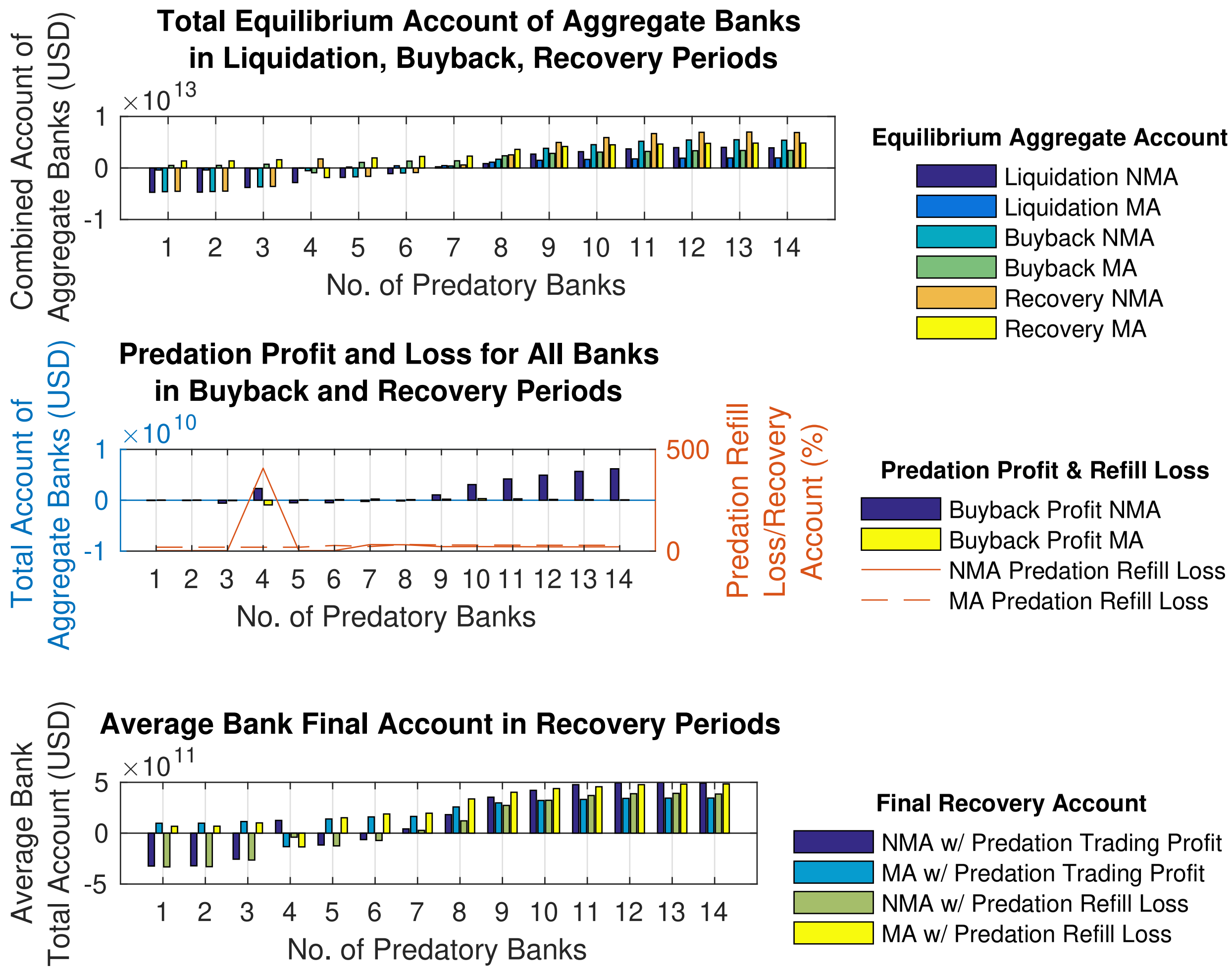

Figure 23: The accounts, predation profits and guarantee fund refill losses for dealers in both CCP agreement systems during the liquidation, buyback and recovery periods under crisis market liquidity.

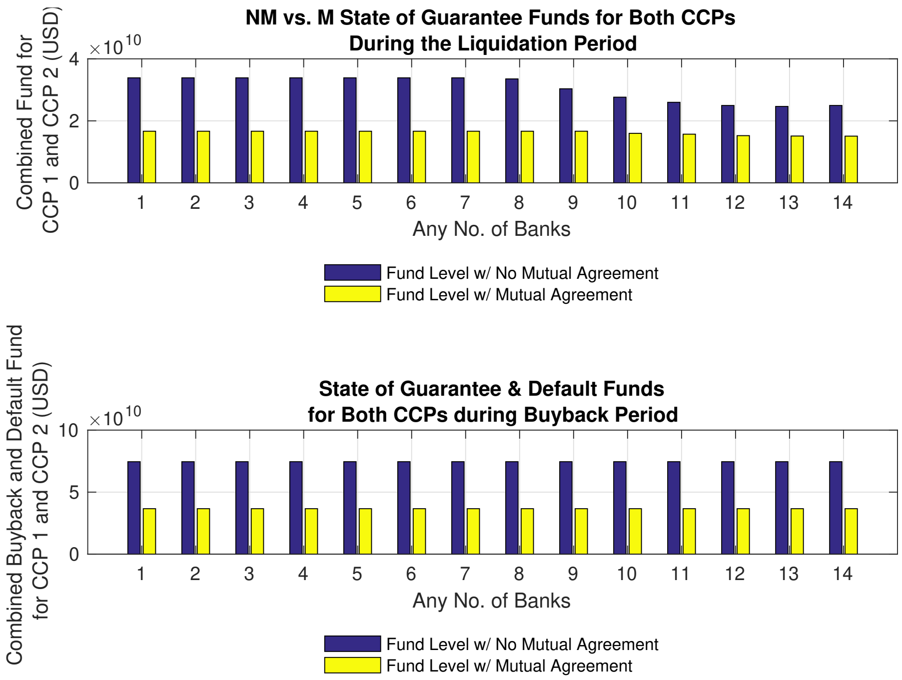

Figure 24: The state of the Guarantee and Default Funds after the liquidation and buyback trading periods in the two CCP agreement systems under normal liquidity.

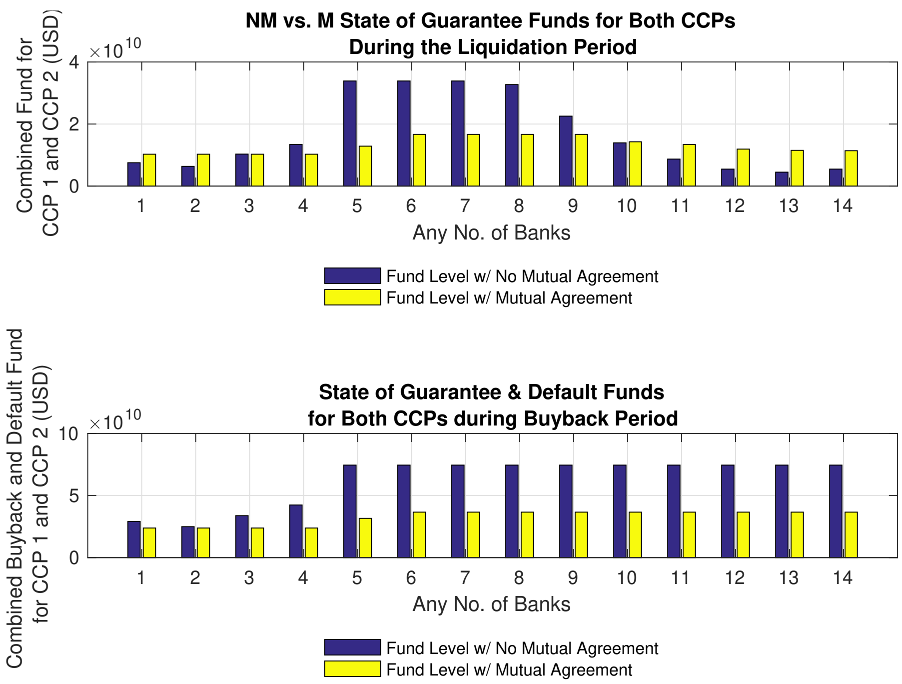

Figure 25: The state of the Guarantee and Default Funds after the liquidation and buyback trading periods in the two CCP agreement systems under variable liquidity.

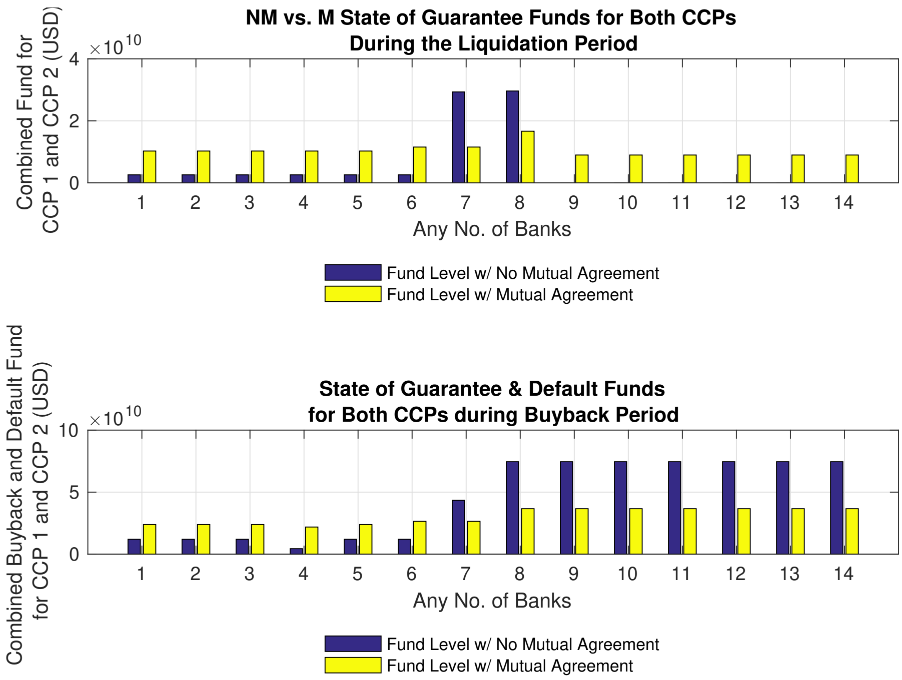

Figure 26: The state of the Guarantee and Default Funds after the liquidation and buyback trading periods in the two CCP agreement systems under crisis liquidity.

A Appendix: Technical Details and Proofs

A.1 Technical Details

Technical Detail 1: dealer defaults and liabilities from other CCPs

Liabilities at other CCPs create a draw on dealer assets and affect both thet dealers cash and external assets. Thus, a dealers shortfall in one CCP can be a caused by liabilities in other CCPs through Z_i and R_i. Dealers default when,

$$\begin{cases} \gamma_i^u - \sum_{u=1}^{n} g_i^u - R_i < \hat{L}_{i0}^u + \sum_{u' \neq u}^{n-1} \hat{L}_{i0}^{u'} & \text{(NMA)} \\ \\ \gamma_i^u - g_i^u - R_i - \sum_{u' \neq u}^{n-1} m_i^{uu'} < \bar{L}_{i0}^u = \sum_{u' \neq u}^{n-1} \bar{L}_{i0}^{u'u} & \text{(MA)} \end{cases}$$

Technical Detail 2: dealer exposures in mutual agreement and cross-margin

The following term explains dealer i's net exposure in the mutual agreement structure for two CCPs. Assuming that dealer i is a member of CCP 1:

$$\sum_{u=1}^{n} \bar{\Lambda}_i^{u,k} = \sum_{u'=1}^{n} \big[\sum_{u'=1}^{n} \Lambda_i^{u'u,k,} \big] = \Lambda_i^{11} + \Lambda_i^{12} + \Lambda_i^{21} + \Lambda_i^{22}$$

$\Lambda_i^{11} = \Lambda_{i0}^{11} + \Lambda_{0i}^{11}$ presents dealer i's net exposure to its own CCP 1, $\Lambda_{i0}^{12,-}$ is what CCP 1 would owe CCP 2 on behalf of dealer i.

$\Lambda_i^{21} = \Lambda_{0i}^{21,-}$ is positive if CCP 2 owes CCP1 dealer i a receivable, and $\Lambda_{i0}^{22,-}$ is zero for dealer i as it doesn't trade in CCP2.

Cross-margin moves from CCP 1 to CCP 2 iff $\Lambda_i^{12} \geq \Lambda_i^{21}$. Thus margin is,

$$m_i^{12} = 1.2 g_i * (\Lambda_i^{12} - \Lambda_i^{21})^+$$

A.2 Proofs

Proof 1: Proposition 1 - Decreased price volatility in homogenous multi-CCP system

It is important to see how dealers and dealer positions are split amongst CCPs in each mutual agreement. This gives insight as to why trading is less volatile and leads to less defaults when there are more CCPs in the system. In the financial system there are m dealers (of which each is i) and the system contains n CCPs (of which each is i). These dealers have a total position X_i in the system. Thus, in each mutual agreement, the total amount traded in each system is,

$$X_{tot} = \sum_{u=1}^{n} X_{tot}^u = \begin{cases} \sum_{i=1}^{m} \frac{X_i}{\sum_{u=1}^{n} u} & \text{(NMA)} \\ \\ \sum_{i=\left(1+(u-1)*\lfloor \frac{m}{n} \rceil\right)}^{m'=u*\lfloor \frac{m}{n} \rceil} X_i & \text{(MA)} \end{cases}$$

If all dealer holdings X_i are homogenous, then a one CCP system has a $X_{tot} = X_{tot}^u = \sum_{i=1}^{m} X_i = m * X$

possible trading volume in one CCP. However, any multi-CCP system which contains homogenous holdings and even number of dealers among CCPs has the possible trading volume of,

$$X^u_{tot} = \begin{cases} m * [\frac{X}{n}] & \text{(NMA)} \\ \\ [\frac{m}{n}] * X & \text{(MA)} \end{cases}$$

Thus, only X_{tot}/n traded in each CCP. Assuming that all positions are traded, any multi-CCP system has less possible trading volume and, thus, is less able to cause price movements and induce price volatility through trading. Note that although dealers have homogenous holdings, they do not have homogenous trading strategies, and, thus, price volatility between between CCPs still differs, even for the same trading volume.

Proof 2: Price Impact Formulation

The price impact is assumes a volatility-like structure due the covariance exhibited by CDS,

$$X^{u,k,p}_{ij} \, \Sigma_{ij} \, X^{u,k,p}_{ij}$$

where Σ is the covariance matrix between assets and X is the portfolio of CDS contracts dealer i holds with various dealers j (in either CCP). This is extended to a linear price impact formulation,

$$X^{u,k,p}_{ij} \, F(X^{u,k,p}_{ij}) \quad \text{with} \quad F(X^{u,k,p}_{ij}) = \triangle S^{u,k}(\ell\tau)\left(\frac{X^{u,k,-p}_{ij}}{D_{u,k}}\right)$$

where $F(X)$ is the is the *change* in the closing market value of the CDS in a dealers portfolio. Thus, S is the CDS spread and $\triangle S^{u,k}(\ell\tau) = S^{u,k}\big(\ell\tau\big) - S^{u,k}\Big((\ell-1)\tau\Big)$ is the change in the CDS spread in the particular CCP where the trade is cleared.

It has a linear price impact formulation which reflects the effect on the portfolio of dealer i of liquidating a CDS position in a particular CCP – where the liquidation is due to a defaulted dealer j for asset k and is given by,

$$\triangle S^{u,k}(t_\ell) = \triangle S^{u,k}(t_{\ell-1}) \left(1 - \frac{1}{D_{u,k}} \sum_{j \in \mathcal{D}} X^{u,k}_j\right)$$

Here $D_{u,k}$ is the vector of market depths/market liquidity in each CCP for each CDS.

Proof 3: Proposition 2 - Difference in inter-CCP CDS Spread and Portfolio Value

In each CCP a portfolio's value is determined by the CDS spread seen in that CCP. It is possible that in the homogenous NMA system – homogenous in trading rates and trading patterns – there will be no difference in CDS spread between CCPs. That is each dealer is homogenous in holdings, but heterogenous in trading strategy. However, each dealers trading strategy and holdings are mirrored in both CCP's. Then the value of dealer i's portfolio is,

$$\begin{aligned} V^k_i &= V^{1,k}_i + V^{2,k}_i \\ &= X^{1,k}_i S^{1,k}(l\tau) + X^{2,k}_i S^{2,k}(l\tau) \end{aligned}$$

The CDS spread is determined by the pricing function[86]. It is worthwhile to look only at the variables which have the most effect on the pricing functional - the previous CDS spread, distressed selling, and predation.

$$
\begin{aligned}
S^{1,k}(l\tau) &= (S^{1,k}((l-1)\tau)), \sum_{j\in\mathcal{D}^1}\sum_{i=1}^{m}\frac{a^{1,k}_{ji'}\tau}{D_{1,k}}, \sum_{j'\notin\mathcal{D}^1}\sum_{i=1}^{m}\frac{a^{1,k}_{j'i}\tau}{D_{1,k}})\\
S^{2,k}(l\tau) &= (S^{2,k}((l-1)\tau)), \sum_{j\in\mathcal{D}^2}\sum_{i=1}^{m}\frac{a^{2,k}_{ji'}\tau}{D_{2,k}}, \sum_{j'\notin\mathcal{D}^2}\sum_{i=1}^{m}\frac{a^{2,k}_{j'i}\tau}{D_{2,k}})
\end{aligned}
$$

where $a^{u,k}_{ji'}$ is the amount of asset liquidated or predated in each CCP, and $D_{u,k}$ is the market depth or liquidity in that CCP and $\mathcal{D}^u$ is the set of defaulted dealers in that CCP. Thus, if all m dealers are members of both CCPs, with the same holdings and trading in each CCP is mirrored. Then $S^{1,k}(l\tau) = S^{2,k}(l\tau)$ and both CCP's see the same evolution of the CDS spread in $S^{u,k}((l+1)\tau)$ – reaping the same portfolio value in each CCP.

This is not the case in the MA case for the same m dealers, which are divided into $m/2$ dealers in each CCP where dealers $1,...,m/2$ in the first CCP, have different trading strategies to dealers $m/2+1,....,m$. These dealers makes inter-CCP trades, as well as, intra-CCP trades and each half of the trade isserparately in each CCP. dealer i's portfolio value becomes,

$$
\begin{aligned}
V^k_i &= V^{11,k}_i + V^{12,k}_i + V^{21,k}_i + V^{22,k}_i\\
&= X^{11,k}_i S^{1,k}(l\tau) + X^{12,k}_i S^{12,k}(l\tau) + X^{21,k}_i S^{21,k}(l\tau) + X^{22,k}_i S^{2,k}(l\tau)
\end{aligned}
$$

The CDS spread for intra-CCP trades becomes,

$$
\begin{aligned}
S^{1,k}(l\tau) &= (S^{1,k}((l-1)\tau)), \sum_{j\in\mathcal{D}^1}\sum_{i=1}^{m/2}\frac{a^{11,k}_{ji'}\tau}{D_{1,k}}, \sum_{j'\notin\mathcal{D}^1}\sum_{i=1}^{m/2}\frac{a^{11,k}_{j'i}\tau}{D_{1,k}})\\
S^{2,k}(l\tau) &= (S^{2,k}((l-1)\tau)), \sum_{j\in\mathcal{D}^2}\sum_{i=m/2+1}^{m}\frac{a^{22,k}_{ji'}\tau}{D_{2,k}}, \sum_{j'\notin\mathcal{D}^2}\sum_{i=m/2+1}^{m}\frac{a^{22,k}_{j'i}\tau}{D_{2,k}})
\end{aligned}
$$

The CDS spread for inter-CCP trades becomes,

$$
\begin{aligned}
S^{12,k}(l\tau) &= (\underbrace{\frac{S^{1,k}((l-1)\tau)) + S^{2,k}((l-1)\tau))}{2}}_{S^{3,k}((l-1)\tau)}, \sum_{j\in\mathcal{D}^2}\sum_{i=1}^{m/2}\frac{a^{12,k}_{ji'}\tau}{D_{2,k}}, \sum_{j'\notin\mathcal{D}^2}\sum_{i=1}^{m/2}\frac{a^{12,k}_{j'i}\tau}{D_{1,k}})\\
S^{21,k}(l\tau) &= (S^{3,k}((l-1)\tau)), \sum_{j\in\mathcal{D}^1}\sum_{i=m/2+1}^{m}\frac{a^{21,k}_{ji'}\tau}{D_{1,k}}, \sum_{j'\notin\mathcal{D}^1}\sum_{i=m/2+1}^{m}\frac{a^{21,k}_{j'i}\tau}{D_{1,k}})
\end{aligned}
$$

Of course this term is simplified, but it is easy to see that even trading the same price $S^{12,k}(l\tau) \neq S^{23,k}(l\tau)$ due to interactions of heterogenous members interacting with the other CCPs trading strategies and market depth. Since the CDS spread evolution depends on the previous CDS spreads such that,

$$
\begin{aligned}
S^{1,k}((l+1)\tau) &= (S^{1,k}((l)\tau)), S^{12,k}((l)\tau)))\\
S^{2,k}((l+1)\tau) &= (S^{2,k}((l)\tau)), S^{21,k}((l)\tau)))
\end{aligned}
$$

[86] One can use CDS spread or the change in the spread with no difference in the proof

Thus $S^{1,k}((l+1)\tau) \neq S^{2,k}((l+1)\tau)$ and dealer portfolios will be priced differently between CCPs.

Proof 4: Proposition 3 - CDS Pricing Functional

$$\sum_k \triangle S^{u,k}(\ell\tau) = \sum_k \Bigg\{ [\, \triangle S^k((\ell-1)\tau)\,]^+ + \sum_{j\in\mathcal{D}} \sum_{i'=1}^{m} |\triangle S^{u,k}((\ell-1)\tau)| \left(\frac{a^{u,k}_{ji'}\tau}{D_{u,k}}\right)$$

$$+ \sum_{j\in\mathcal{D}} \sum_{i'=1}^{m} |\triangle S^{u,k}((\ell-1)\tau)| \left(\frac{a^{u,k}_{i'j}\tau}{D_{u,k}}\right) + \varepsilon \sum_{j'\notin\mathcal{D}} \sum_{i'=1}^{m} |\triangle S^{u,k}((\ell-1)\tau)| \left(\frac{a^{u,k}_{j'i'}\tau}{D_{u,k}}\right)$$

$$+ \left(\frac{1}{2!}\right) \sum_{k'} \sum_{j=1}^{m} \sum_{i'=1}^{m} |\triangle S^{u,k'}((\ell-2)\tau)| \left(\frac{a^{u,k'}_{ji'}\tau}{D_{u,k'}}\right) + \left(\frac{1}{3!}\right) \sum_{j\in\mathcal{D}} \sum_{k''} \sum_{i'=1}^{m} |\triangle S^{u,k''}((\ell-2)\tau)| \left(\frac{a^{u,k''}_{ji'}\tau}{D_{u,k''}}\right) \Bigg\} \tag{54}$$

Proof 5: Proposition 4 - Predatory incentive and motivation

Where there is any predatory selling which causes falling prices, all predators are better off predating. Consider a system with two predators i and i', holding a homogenous amount $X_i(t=0) = X_{i'}(t=0)$ of an asset k, such that,

$$X_i(t=0)\; S_0^k = X_{i'}(t=0)\; S_0^k$$

$$V_i(t=0) = V_{i'}(t=0)$$

If trading starts at period t=1 where the price moves to $S^k_{l\tau}(t=1) = S^k_0$ and if i decides to liquidate asset k at $X_i(l\tau) < X_i(t=0) - \delta X_i(l\tau)$ for a cash profit of $\gamma_i = \delta X_i(l\tau) S^k_{l\tau}$,

$$X_i(l\tau)\; S^k_{l\tau} + \gamma_i = X_{i'}(l\tau)\; S^k_{l\tau}$$

$$\text{when} \quad X_i(t=0) < X_{i'}(t=0)$$

$$V_i(l\tau) = V_{i'}(l\tau)$$

If the CCP liquidation and the predation liquidation lowers the price such $S^k_{(l+1)\tau}(t=1) < S^k_{l\tau}(t=1)$ and $\triangle S^k_{(l+1)\tau} < 0$ then,

$$\gamma_i + X_i((l+1)\tau)\; S^k_{(l+1)\tau} > X_{i'}((l+1)\tau)\; S^k_{(l+1)\tau}$$

$$V_i((l+1)\tau) > V_{i'}((l+1)\tau)$$

Such that if holdings and prices become $\delta X_i(l\tau) = \frac{1}{2} X_i(0) = \frac{1}{2} X_{i'}(0)$ and $S^k_{(l+1)\tau} = \frac{1}{2} S^k_0$,

$$\delta X_i(l\tau)\ S^k_{l\tau} + (X_i(0) - \delta X_i(l\tau))\ S^k_{(l+1)\tau} > \left[X_{i'}(0)\ S^k_{(l+1)\tau} \right]$$

$$\left[\frac{1}{2} X_i(0) \times \frac{1}{2} S^k_0\right] + \left[\frac{1}{2} X_i(0) \times S^k_0\right] > X'_i(0) \times \frac{1}{2}\ S^k_0$$

$$\frac{3}{4} X_i(0)\ S^k_0 > \frac{1}{2} X_i(0)\ S^k_0 \quad \text{since} \quad X_i(0) = X'_i(0)$$

$$V_i\ ((l+1)\tau) = 1.5 * V_{i'}\ ((l+1)\tau)$$

Thus, the falling prices due to liquidations give all predators both incentive and motivation to predate.

Proof 6: Proposition 5 - equality of total initial margin charges for spreading (NMA) vs. netting (MA)

Consider a dealer with a holding of asset k, where that asset can be divided into equal fractions, $X^k_q = \{x^k_1, ..., x^k_Q\}$ so that $X^k_q = \sum_{q=1}^{Q} x^k_q$. Suppose now that each CCP charges the same of margin percentage p on the value of the portfolio such that,

$$g^u_i = p * X^{u,k}_q\ S^{u,k}_{t=0} = p * \sum_{q=1}^{Q} x^k_q\ S^{u,k}_0$$

Given that at $t = 0$ all CCPs start with the same fundamental value $S^{u,k}_0$ for CDS spread which has not yet been altered by trading.

Now suppose that the dealers holding is composed of buy side and sell side exposures; these can be divided into fractions which can be spread amongst CCPs or netted in one CCP,

$$X^k_q = X^k_q = \{x^{k,+}_1, x^{k,+}_2, x^{k,+}_3, x^{k,-}_4, x^{k,-}_5, x^{k,-}_6, x^{k,-}_7\}$$

In the NMA scenario, the dealer can spread the position in netted pairs $\{x^{u,k,+}_q, x^{u,k,-}_q\}$ so that at each CCP it has no open exposures $X^{u,k}_q = 0$.

However, there is now an open exposure of which can be placed in any CCP such that,

$$g^{u'}_i = p * x^{u',k,-}_7\ S^{u',k}_0$$
$$g_i = \sum_{u=1}^{n} g^u_i$$
$$\bar{g}_i = g_i = 3 * g^u_i + g^{u'}_i = g^{u'}_i$$

If $n > Q/2$, this can be spread to endless CCPs, if $n > Q/2$ this exposure must be concentrated to one CCP. The less CCPs the less places there are to spread and the higher the margin charged in the CCP since one way exposures become larger. In the MA scenario all positions are netted in the same CCP such that,

$$\hat{g}_i = g_i^u = p * x_7^{u,k,-} S_0^{u,k}$$

Therefore, all things being equal, the initial margin charged is the same with and without interoperability.

$$\hat{g}_i = \bar{g}_i = g_i^u$$

Decreases in the number of CCPs available make spreading less achievable and inhomogeneity in margins, so that some can make interoperability agreements more appealing.

Proof 7: Proposition 6 - Increased price dispersion for interoperability

Consider the difference between the pricing functional at the end of the trading period. That is,

$$\dot{S}^{u,k} = \Big| |\Delta S^{u,k}(0)| - |\Delta S^{u,k}(l\tau)| \Big| \tag{55}$$

$$\Delta \dot{S}^{k}(l\tau) = |\dot{S}^{u,k}(l\tau) - \dot{S}^{u',k}(l\tau)| \tag{56}$$

The first equation calculates how far an asset's current price is from its fundamental starting value, in each CCP. The second equation provides a measure of the price dispersion between the CCP's for that asset, in a particular agreement framework. The claim is that,

$$\Delta \dot{S}_t^{NMA} \leq \Delta \dot{S}_t^{MA} \tag{57}$$

Furthermore, mean final inter-CCP price dispersion[87] between two agreement scenarios can be calculated,

$$\Delta \bar{\dot{S}}(l\tau)_{tot} = \sum_{k=1}^{K} \frac{\Delta \bar{\dot{S}}^{k,NMA}(l\tau)}{K} - \sum_{k=1}^{K} \frac{\Delta \bar{\dot{S}}^{k,MA}(l\tau)}{K} \tag{58}$$

The proof is contained in the following example below, and is later illustrated by the large-scale simulation. Consider a system of four dealers $i = 1, ..., 4$ and two CCPs $u = 1, 2$ who all trade asset k with each other.

	No Mutual Agreement (NMA)		Mutual Agreement (MA)	
CCP	**u = 1**	**u = 2**	**u = 1**	**u = 2**
Dealer(i)	1	1	1	
	2	2	2	
	3	3		3
	4	4		4

They are then connected through their liabilities $L_i = \Delta X_i^u \Delta S^u$ with $\Delta S^u = f(\Delta S_{t-1}^u) = \pm\$1$ as the change from fundamental value, and a quantity of $\Delta X_i^{u,\pm} = (X_i^{u,\pm} + a_i^{u,\pm}) = \pm 1$. Note that the final price in each CCP is affected by trading induced by a_i and so in this first simplified back-of-the-envelope

[87] Multiplication by 1000 will give this dispersion in bps.

calculation, we can simply calculate $\Delta S_t^u = f(\Delta S_{t-1}) + \sum_i^u L_i^u$ in each CCP. The price difference between CCP's will then be $\Delta \bar{S}_t$ for each type of agreement.

$\mathbf{u=1}$	$\mathbf{u=2}$	$\mathbf{u=1}$	$\mathbf{u=2}$
$L_{12}^1 = +1 \quad L_{12}^1 = -1$	$L_{34}^2 = +1 \quad L_{43}^2 = -1$	$L_{12}^1 = +1 \quad L_{21}^1 = -1$	$L_{34}^2 = +1 \quad L_{43}^2 = -1$
$L_{13}^1 = +1 \quad L_{31}^1 = -1$		$L_{13}^1 = +1$	$L_{31}^2 = -1$
	$L_{23}^2 = +1 \quad L_{23}^2 = -1$	$L_{12}^1 = +1$	$L_{32}^2 = -1$

$$\begin{aligned} \Delta S_t^1 &= f(\Delta S_{t-1}) + 1 - 1 + 1 - 1 \qquad & \Delta S_t^1 &= f(\Delta S_{t-1}) + 1 - 1 + 1 + 1 \\ \Delta S_t^2 &= f(\Delta S_{t-1}) + 1 - 1 + 1 - 1 \qquad & \Delta S_t^2 &= f(\Delta S_{t-1}) + 1 - 1 - 1 - 1 \end{aligned}$$

$$\Delta \dot{S}_t^{NMA} = |0 - 0| = 0 \qquad \Delta \bar{S}_t^{MA} = |2 - (-2)| = 4$$

This simple example is illustrative of what is seen in the final large scale simulation, which finds that the unfortunate by-product of a mutual agreement scenario is increased price dispersion between CCP's.

Proof 8: Corollary 1 - Price Dispersion in any non-homogenous system

The theoretical model and simulation show that with homogenous positions and homogenous trading strategies between CCPs, the NMA case shows very low price dispersion compared to the MA case. It is in fact the case that any movement away from homogeneity in either positions or trading strategies will increase price dispersion between CCPs.

It is important to understand that low price dispersion between CCPs can only be maintained in the NMA case if dealers employ the same trading strategy amongst CCPs; the CCPs act as mirrors of each other, each with half the dealer positions. In reality, this also comes with a wedge between lower concentration charges vs. less netting benefit in each CCP. The first lowers margins charged and the second increases them.

In the MA scenario, there are m/n dealers in each CCP, acting as each others trading partners and take their full holdings to each CCP, and thus, do not have identical trading patterns in each (though identical size total holdings). Thus, if in NMA scenario each CCP than have CCPs mirror each other, they instead mirror the trading strategy of the CCPs in the MA scenario, with a duplication of $n * \frac{m}{n}$ dealers in each scenario and $\frac{X^{u,MA}}{n}$ smaller holdings. Such that,

$$X_{tot}^{u,NMA} = m * \frac{X^{NMA,u}}{n} = n * \frac{m}{n} * \frac{X^{MA,u}}{n} = \frac{m}{n} X^{MA,u} = X_{tot}^{MA,u}$$

Thus, proving that if all holdings are homogenous amongst dealers, and trading strategies between the NMA and MA scenarios are made equivalent, then the price dispersion between CCPS is the same in each scenario. This gives the result that in a fragmented market, price dispersion increases with inhomogeneity in dealer position size size and inhomogeneity in dealer trading strategies between CCPs.

Proof 9: Lemma 1 - Effect of Price Dispersion on Dealer's Net Exposures

Following from proposition 1 and corollary 1, consider that at $t = 0$ all CCPs start with the same fundamental value $S_0^{u,k}$. We have shown that the net exposures are such that;

$$\hat{\Lambda}_i^k = \sum_{u=1}^{n} L_i^{u,k} \qquad \text{NMA}$$

$$\bar{\Lambda}_i^k = \sum_{u=1}^{n} \sum_{u'=1}^{u'} L_i^{uu',k} \quad \text{MA}$$

Consider that at $t = 1$ the CDS spread moves to $S_0^{u,k} = S_1^{u,k} > S_1^{u',k}$.

From the corollary 1 concerning price dispersion between CCPs, we see that all else equal price dispersion $\Delta \dot{S}_t^{MA,k} > 0$ given $\Delta \dot{S}_t^{NMA,k} = 0$.

Thus, all CCPs the cds evolution for NMA is the same in each CCP so that each CCP has $S_0^{u,k} = S_1^{u,k}$ giving $\Delta S_1^{u,k} = 0$.

For the MA case, price dispersion means that CCP u has $S_1^{u,k}$ and CCP u' has $S_1^{u',k}$ so that $\Delta S_1^{u',k} < 0$. Consider a sell (buy) position for the dealer, such that when the change in the CDS spread is negative (positive) the seller (buyer) pays the buyer[88]. Thus,

$$\begin{aligned} \hat{\Lambda}_i^k &= 2 * (X_i^{u,k}\, \Delta S_1^{u,k}) = 0 && \text{NMA} \\ \bar{\Lambda}_i^k &= (X_i^{u,k}\, \Delta S_1^{u,k}) + (X_i^{u',k}\, \Delta S_1^{u',k}) < 0 = \bar{\Lambda}_i^{k,-} && \text{MA} \end{aligned}$$

Thus,

$$\hat{\Lambda}_i^k < \bar{\Lambda}_i^k \quad \text{if } \Delta S_t^{u',k} < 0$$

So that net exposure for a dealer is greater with interoperability due to price dispersion if if it is a net seller (net buyer) and the overall CDS spread changes are negative (positive).

Proof 10: Corollary 2 - Effect of Price Dispersion on Dealer's Equilibrium Guarantee Fund Contribution

We have that,

$$\hat{G}_i^u = \Big(\sum_{u=1}^{N} \Lambda_i^u + \gamma_i + R_i - \sum_{u'=1}^{N-1} g_i^{u'}\Big)^+ - \Big(\sum_{u=1}^{N} \Lambda_i^u + \gamma_i + R_i - \sum_{u'=1}^{N-1} g^{u'} - g_i^u\Big)^+ \quad \text{(NMA)} \tag{59}$$

$$\bar{G}_i^u = \Big(\Lambda_i^u + \gamma_i + R_i\Big)^+ - \Big(\Lambda_i^u + \gamma_i + R_i - g_i^u\Big)^+ \quad \text{(MA)} \tag{60}$$

As established before that $\hat{\gamma} = \hat{\gamma}_i$ and $\hat{R}_i = \hat{R}_i$. As well, consider Proposition 5 ($\hat{g}_i = \bar{g}_i = g_i^u$) and Corollary 2 and that,

$$\hat{\Lambda}_i^{k,-} < \bar{\Lambda}_i^{k,-} \quad \text{if } \Delta S_t^{u',k} < 0$$

[88] Premium on standard CDS is set at 100/500 bps for the buyer, and so the payment is the credit spread-premium. There is an up-front payment that gives the contract zero value and so only the change in CDS spread determines variation payments that make up exposures. Negative payment values mean the seller pays the buyer.

Then we have that,

$$\hat{G}_i^u > \bar{G}_i^u \quad \text{if } \bar{\Lambda}_i^k < 0$$

So that a dealer's guarantee fund contribution is dependant on the net exposure condition on the side of the position, price dispersion and change in CDS spread. Therefore, the net exposure is lower with interoperability if if it is a net seller (net buyer) and the overall CDS spread changes are negative (positive).

A.3 Simulation Calibration

A.3.1 Simulation Parameters

This section outlines the parameters and method used to calibrate the simulation based on the work in [Tywoniuk, 2017a]. This method is adapted to a dual CCP scenario, with and without interoperability. The following parameters are used to study the theoretical CCP system's resilience to certain market mechanisms. The method consists of a dynamic agent-based network simulation with the objective to perturb the system with an exogenous shock, and investigate how exogenous trade rules and CCP architecture react to contagion and amplification. The simulation is calibrated to OTC financial market data, both in order to gauge the magnitudes of the theoretical results and to capture nuanced network effects. To obtain realistically meaningful results, I investigate multiple scenarios – varying both the market liquidity and the number of distressed vs. predatory dealers. The simulation, in general, replicates the global CDS market by approximating it with 14 large core dealer banks. Each dealer is set to trade in 100 different types of CDS with the other 13 dealers. I rely on data from four reputable sources (below) to obtain suitable market parameters. First, to model recent historical CDS market features and liquidity, I obtained data from Bloomberg (2017). Then to ascertain the historical market depth, average CDS spread movements, market turnover, as well as, gross and net CDS holdings for dealers, I used three sources; [Oehmke and Zawadowski, 2017, Duffie et al., 2015, Amini et al., 2015]. Since each of these explores the CDS market from a different angle and considers CDS positions held at multiple CCPs, I attempt to extract data which is consistent with the mono-CCP market scenario (size and data granularity). For brevity, an in-depth explanation of the simulation design and magnitudes of variable outputs is given in appendix C[89]. First, to recreate key trading features, I employ the summary statistics for daily CDS spread and trade data from January 2008 until December 2011 given by [Duffie et al., 2015]. This data consists of 184 single-name CDS reference entities comprised of 31 sovereigns and 153 global financial entities. To obtain trade statistics, the researchers include the trades of any counterparty which holds a CDS on at least 1/184 of the reference entities. [Oehmke and Zawadowski, 2017] specifies the trade characteristics of 1000 single-name CDS where a reference entity is only included when it is traded by at least three dealers. I focus on a smaller subset of their data, given for a sub-sample of 97 reference entities, which provides a floor for the data from [Duffie et al., 2015]. I then validate the general market size and market volume against data from [Amini et al., 2015] which also considers a mono-CCP system. I then divide this mono-CCP system into two for comparison.

The scenario of interest is one where CDS positions are held at at two identical global CCPs, therefore we first investigate the scenario for one. I proceed as if there is one large CCP (ICE Clear Credit) which dominates the CDS market, and initially split positions in a mirrored way across CCPs to examine operability.[?] In this simulation, CDS are traded amongst 14 large dealer banks meant to represent the 14 core global dealers. If one (or more) of these large dealers defaults, their positions are consistently unwound over a 5-day trading period, which triggers distressed liquidations (i.e. by counterparties) or predatory selling by other dealers. The simulation is triggered with one external default while any further defaults occur organically out of network dynamics. The number of predatory and distressed dealers is set externally for each simulation run, and I explore their different combinations in order to pin-down the underlying drivers of contagion. In the next trading period, the predatory (unconstrained) dealers buyback their original positions over a period ranging from one day to a month, depending on

[89]Further results (ie. different calibrations and multiple runs) are also available in the appendix, sec. ??.

scenario characteristics.[90] I then obtain a realised profit or loss for predatory dealers. An important consideration in the simulation is the number of CDS which dealers trade. Since, it is unlikely that any dealer will hold the full universe of possible CDS in any one CCP over a short period of time, I set a maximum of 100 contracts for each dealer. This approximates the CDS samples given in the above statistics and its suitability is tested in the robustness section below. The types and sizes of positions are assigned randomly, as are the trade counterparties for each dealer. This is done position by position, ensuring each position is distributed autonomously. To ensure a realistic distribution of trade relationships, the multi-dimensional array of bilateral contracts is not saturated; with zeros in the array representing no trade contract/position for that particular pair of dealers. The number and identity of un-matched pairs is also determined randomly. The characteristics of positions sizes which I assign to trades are again obtained from the sources above. This is done by examining the notional value of all the market positions provided in the data, and distributing the value amongst dealers and trades. Using a similar approach to [Amini et al., 2015], I set position size (X_i^k) using a normal probability distribution with parameters obtained from data. As they have done, I use the gross (rather than net) notional position size to capture both buy and sell positions. They provide a total notional value of positions in the CCP to be \$19e12 USD (2010 BIS data), however this is yearly data and represents the whole CDS market. Instead, I rely more heavily on two other notional market size values; \$4.91e12 USD from [Duffie et al., 2015] based on daily data for single-name global CDS over a period including the 2008 financial crisis, and \$13.41e12 USD from [Oehmke and Zawadowski, 2017] over a similar period.[91] The former is a suitable sub-sample which accounts for 31.5% of the global single-name CDS market, and 18.9% of the total CDS market. This data also encompasses all the core dealers making it suitable for use in the simulation. The latter, [Oehmke and Zawadowski, 2017], provides less granular data, but which captures useful aspects of trade activity. They use the Trade Information Warehouse of the Depository Trust & Clearing Corporation (DTCC) which represents 95% of the total globally traded single-name CDS market. They obtain monthly data which describes notional positions for October 2008 to December 2012. They analyse standard 5-year CDS referencing 1,000 different entities. As the CDS market is very concentrated and dominated by the group of core dealers, it is reasonable to consider a notional of this size. Furthermore, as it is of the same order of magnitude as the previous data source, I use it as a lower bound when setting and validating the notional size in the simulation. To set values for the total CDS market size and depth for market stability, I use Bloomberg data from February 21, 2017. This data captures CDS on 1,241 reference entities. I also use this data to evaluate the magnitude of a dealer bank's holdings as outlined below.

As these data-sets look at different compositions of the world of CDS, I verify all the data against each other to construct the most realistic values for a one CCP system. As outlined above, the values for the gross notional position size – given in ascending order (all in USD) – by [Oehmke and Zawadowski, 2017],
[Duffie et al., 2015] and [Amini et al., 2015] are respectively \$13.41e12, \$4.91e12 and \$19.0e12. For simulating the outstanding global dealer positions in only one CCP, I conservatively start with the lowest data value from [Oehmke and Zawadowski, 2017] of \$13.41e12 USD. To calibrate the amount traded in each CDS, I use the statistics provided for the net-notional position on a single-named CDS from this data since it is drawn from the smallest sub-sample of CDS. However, in cross-referencing this data set with available CCP data, key values appear to be magnitude larger. With caution, I augment the [Oehmke and Zawadowski, 2017] values by a factor of 10 to more closely match the values available in the CCP statistics (and the other data sets). This allows me to simulate the net-notional for each of the model's 100 CDS by drawing it from a normal distribution with mean \$13.3e9 USD and standard deviation \$12.19e9 USD.

The available CCP data is sparse and mostly proprietary. However, the CDS market is dominated by ICE (with ICE Clear Credit and ICE Europe), followed by its smaller rival CME, and two barely

[90] Other possible scenarios, constructed with the above data, are given in the robustness section. These may produce extended time periods of buyback, into months and years.

[91] Data for the former encompasses a period of January 2008 to December 2011, and for the latter a period of October 2008 to December 2012 for notional data, but August 2010 to December 2012 for trading data.

competitive CCPs, LCH CDS Clear, and JSCC. I used clearing statistics from [IOSC, 2012] to determine the proportion of single-named CDS that would be cleared through one large global CCP like ICE. I obtain multiple statistics to adjust or verify the use of the data above. [IOSC, 2012] provides the single-name CDS market share for all CDS cleared by these CCPs on March 23, 2015. ICE Clear Credit clears the largest share of the single-name CDS market with approx. 77.1%, followed by 18.8% for ICE Europe, 3.69% for CME, 0.369% for LCH CDS Clear and 0.0369% for JSCC. This provides a mean CCP market share of 20%. Since this is one day of data, it is only enough to determine that ICE is the largest CCP and should be the object of the calibration. ICE is designated an GSII[92] which also supports the consideration of its statistics for the simulation. According to [Council, 2012], ICE has 27 clearing members, the largest 14 of which are financial firms; supporting the simulation of only 14 core dealers. In 2011, ICE cleared contracts on approx. $1,145$ single-name reference entities,[93] which is similar to the larger [Oehmke and Zawadowski, 2017] set. According to [Terhune, 2010], ICE cleared 29% of the total single-named CDS worth \$$24e12$ USD market in 2010, or \$$6.96e12$ USD. This is higher than what [Oehmke and Zawadowski, 2017] provides, but lowering this percentage to match the mean CCP market share of 20% (as above) gives approx. \$$4e12$ USD – approaching values in the previous data. As well, though ICE clears CDS on 200 of the most liquid reference entities on a daily basis, it handles only 15% of market contracts on any one reference entity. These values provide rough estimates which I use to reduce the global CDS data statistics to a magnitude which more closely resembles a one CCP network.

In an effort to further support the inflation of the [Oehmke and Zawadowski, 2017] values and to settle on the right magnitude for the net-notional in each CDS, I employ a second top-down approach. I use both the above CCP data and the gross notional from [Duffie et al., 2015] – which more closely represents the statistics given for a smaller number of CDS cleared. I consider only 29%[94] of $4.9e12$ as the market share of one CCP. Again I take only 15%[95] of that value to account for the actual cleared market share in only the most liquid CDS, which provides an approx. 4% market share for one CCP. With this I obtain the most realistic value for the net notional of single-name reference entities cleared, \$$16e10$ USD, which is fairly close to the \$$13.3e10$ USD inflated value of [Oehmke and Zawadowski, 2017]. This supports augmenting the values in this data for mean and standard deviation and its use with values from the other data sets. With this I am able to approximate one average CCP – rather than the mean outstanding positions in each asset for the total market. To ensure that the simulation properly distributes this value among each contract, I use a normal distribution. However, the mean and the standard deviation must be standardised by the number of dealers (m), accounting for the number counterparties holding the other side of the positions, and the number of CDS in the market. Therefore, by dividing by the number of banks (m=14), number of counterparties (14) and the number of assets available (k=100), I obtain the parameters for the normal distribution with mean= \$$13.3e10/m(m-1)k$ USD and standard deviation= \$$12.19e10/m(m-1)k$ USD. I use this normal distribution to assign both sides of the positions.

I then verify that the above manner of distributing the net-notional amongst contracts will sum to reproduce the values in the data. Taking a total sum over all positions and over all banks, gives the value of \$$1.50e11$ USD, which is close to data for daily market size of \$$2.21e11$ USD (Bloomberg 2017). It also agrees with the [Oehmke and Zawadowski, 2017] sub-data set[96] of \$$4e11$ USD. Taking 31.5% of the full market (1000 reference entities) in [Oehmke and Zawadowski, 2017] gives \$$1.11e11$ USD, which approximates the size of the market sample cited in [Duffie et al., 2015]. This produces a range of \$$1.11e11$ – \$$12.6e11$ USD for a market with 100 CDS; thus, validating each data set against the other reveals a credible snapshot of a one CCP network[97].

[92] FOSC designation of Global Systemically Important Institution

[93] As well as, 821 index and 397 sovereign, and have a daily trading volume of approx. $300e9$ for all contracts. The peak daily cleared is $14,708$ contracts.

[94] I use this rather than 29% to more closely match the CCP average and not necessarily to replicate ICE.

[95] Taking the market share of ICE as 29% plus its clearing in any one of the most liquid CDS, 15%, and multiplying these gives approx. 4%

[96] It is further from their value for the top 1000 reference entities, $12.6e11$, and for $4.91e12$ from [Duffie et al., 2015].

[97] A further investigation for value robustness and sensitivities is provided in appendix D.

Using margin data from [Amini et al., 2015], I set the initial margin at g=11.2e9 – the maximum which is incentive compatible for dealers – though this gives 0.09% margin on each banks holdings. In [Duffie et al., 2015] the system wide collateral demands on gross positions is 0.78% and dealer-to-dealer collateral demand is 1.37% of the gross notional. Over 14 dealers, the dealer-to-dealer collateral demand of the gross notional is 1.26% using the margin level of [Amini et al., 2015]. The default contribution of banks is then set at 10% of the initial margin, as is often given in the literature. The cash and external asset holdings are set in accordance with [Amini et al., 2015] at $\gamma_0 = 5e9$, $\gamma_i = 10e10 - g_i - d_i$ (endowment - initial and default fund contributions), $Q_i = 1.1e10$, respectively. This is in close agreement with [Oehmke and Zawadowski, 2017] which gives average banks asset holdings at $1.06e10$.

When setting the fundamental spread for each CDS, S_0^k, I again draw it from a normal distribution, with mean and standard deviation provided by CDS spread movement values from the trade data in [Oehmke and Zawadowski, 2017][98]; this normal distribution has mean=249.2 bps (basis points) and standard deviation=269 bps. A novel feature of this work is the creation of a function which calculates the CDS spread; it uses this simulated fundamental value and incorporates temporary effects from trading. The spread is then used to price the variation margin of each bank, from which the net exposure follows. The liquidation matrix is drawn from a normal distribution based on mean and standard deviation from CDS turnover data ([Oehmke and Zawadowski, 2017]). In order to distribute the CDS turnover to trades in the CCP, the mean=\$0.516e9 USD and standard deviation=\$0.646e9 USD need to be adjusted by the no. of banks (m) and available CDS reference entities (k) as was done above for net-notional.

To create a complete simulation of a default scenario, I model three realisations of market liquidity: normal, decreasing, and crisis market state. In a normal market, healthy levels of liquidity prove most resilient to the destabilising effects of default. Decreasing market liquidity illustrates a downturn leading to large-scale instability; liquidity decreases proportional to the amount of liquidation and increases during the buyback period. Finally, financial crisis liquidity replicates a market dry-up[99]. The simulation requires two final assumptions[100]. First, that the CCP continues trading[101] as a counterparty to its members, past the unwinding window. Second, that distressed traders are no longer able to trade during the buyback period. These assumptions are removed and explored in two extra simulation runs.

In setting market liquidity, I use market volume data from both Bloomberg (2017) and [Oehmke and Zawadowski, 2017]; a healthy market depth (D_k) is set at at \$221e9 USD, and at \$12e9 USD for a financial crisis. Changing market liquidity is obtained by decreasing normal liquidity, incrementally over the unwinding window, until it reaches crisis liquidity. I permit liquidity to rebound with purchases during buyback – these change increments are set at \$38e9 USD per trading day. The simulation also outputs the period and trading day in which the CCP defaults – either during the unwinding and/or buyback round. During buyback, the model stops once predators have reached their maximum allowable holding. Various runs of the simulation accommodate different CCP trading strategies[102]. As well, certain runs examine the network effects on CCP losses of allowing buyback round trading by distressed banks. Finally, the simulation runs allow variation in the maximum number of predators vs. distressed banks.

Given these parameters, I then calculate a final worth/value of the CCP and members for each period and track both the losses/gains from period to period. The simulation tracks the evolution of dealer default(s), identifying whether failure(s) occurs in the liquidation or buyback periods. Starting with

[98]The normal distribution is a rational assumption as this is only used for the fundamental part of the CDS spread, which is determines by the integration of public information into prices and accounts for 40% of the explainable movement in the CDS spread.

[99]Similar to that which occurred during the 2008 great financial crisis.

[100]I explore the effect of removing these in two of the simulation runs.

[101]However, the CCP has finished unwinding the holdings of the original defaulter within the liquidation window.

[102]Trading strategies are, both, the CCP moving the original defaulted position at the end T=5 day/period window (for the original defaulted asset), and when it continues to unwind into buyback round.

one exogenous dealer default, its gains and losses (and those of all subsequent defaulting banks) go on the account of the CCP. Gains and losses across CCPs are accounted for according to the particular operability scenario. Without interoperability, gains and losses are account for at each CCP in a mono-CCP way. With interoperable CCPs, inter-CCP gains and losses are accounted for in the normal way, but adding each CCP as the clearing member of the other. During each trading period, each subsequent defaulted dealer's CDS are added to the pool of defaulted assets, $k \in \mathbb{D}$. Their unwinding is accounted for in the spread and pricing functions, through the covariance and price impact terms. Thus, the effects of liquidation and predation are transmitted through each period. Finally, the trading function ($a_i^{k,\pm}$) accommodates both selling (-) and buying (+) – assuring that CDS spreads on positions move according to all types of market trading and are not biased in any one direction. The robustness checks on various parameters and features of the simulation are explained are given in [Tywoniuk, 2017a].

A.3.2 Robustness and sensitivities

This section explores the validity of assumptions and calibration values, previously outlined. This is applied to the mono-CCP model used in [Tywoniuk, 2017a] which is replicated with modification for this paper. As the robustness check produced results similar to that above, I do not plot this section. However, I do compare features of this sections simulated outputs to the results of the previous section. I chose the original specified simulation values as they provide the most realistic OTC financial market for the model scenario. However, I acknowledge that arguments can be made for other calibration values. An investigation of alternative specifications can give good insights on the possibilities/limits of the simulation, and the sensitivities of certain values. These supplementary cases appear below along with any novel features/problems.[103] I outline the first case for all three liquidity scenarios, then focus on one particular scenario for the other two cases[104].

CASE 1: Larger pool of CDS traded (150 CDS), but same market size.

In the case when I increase the number of CDS traded, this specification produces a total market gross notional of \$1.51e11 USD, which is approximately the same as the original simulation. For a normal market depth, more assets increase the number of defaultsin the presence of a high number of distressed dealers, but this effect flattens out quickly. The CCP shows great losses in this case, but these are greatly minimised when levels of distress are low– showing greater sensitivity to this factor. Increased assets also increase the volatility of predation profits. Profits are maximised by lack of collusion amongst predators, but minimised when the low dealer distress creates insufficient prey.Crisis liquidity makes the CCP liquidation and final losses further sensitive to dealer distress; losses are maximised with half of the dealers in distress. In most cases, the pure and hybrid funds yield increasingly similar outcomes, except at the lowest point of dealer distress. As market depth is decreases, more assets minimise the CCPs overall final loss. As well, there is a minimisation of the burden caused by increasing levels of dealer distress. This is most likely due to the increased profits accrued to predators and subsequent ability to replenish the guarantee fund. Indeed, in most cases the predator losses to the recovery margin refill are increased. This refill loss proves disastrous with low predation levels and half of dealers in distress– with fewer predators to shoulder the debt burden. Profits are equally volatile. Predation profits are clearly increased with more predators buying back original positions and raising the spread. Overall, the average margin refill required of each predatory dealer is diminished if the CCP can continue unwinding positions recently defaulted positions into the buyback round – as spreads rise it can recover higher liquidation value. Furthermore, there is an increasing advantage for the CCP with a hybrid fund when it comes to its liquidation lossas the fund can meet shortfalls which would be delegated to a smaller default fund. The hybrid fund loss minimisation also extends to banks overall and for the predators aloneas it reduces early liquidation. Preventing the CCP from selling in the buyback round and allowing this same selling for distressed banks has less dire consequences with increased assets.

[103]Note we call the case used in the paper, the *Official Case: 100 CDS and Mean net notional exposure=\$13.3e10/m(m-1)* USD.

[104]All additional tables and figures can be requested from the author.

CASE 2: Larger market size but same number of CDS traded (100)

This model produces \$$1.50e12$ USD as the total market gross notional size, which is a factor of 10 larger than the original case. This approaches the [Oehmke and Zawadowski, 2017] scenario which expands the number of available CDS from 100 towards 1000 reference entities. Note, that in the data, the matrix of holdings between dealer banks may be incredibly sparse (ie. 3 dealers holding 1/1000 CDS with each other). As market depth decreases and larger positions of the same number of assets lead to lower diversification, the dealer default rate is maximal for all scenarios.Losses are increased in all scenarios for the CCPas it unwinds larger positions into an increasingly smaller market. Predator profits are tiny in all cases and the dealers as a whole experience large losses. Clearly larger holdings make an attempt to sell high and buy low increasingly time-consuming and difficult. Predator profits and losses become larger and increasingly more volatile, but percent margin lost to the margin is much lower, as predators can make larger profits. The margin refill required by the CCP becomes more volatile as well, largely due to higher sensitive to the competition vs. prey effect mentioned earlier. For low levels of dealer distress, the CCP loses increasingly more in the pure fund – larger positions yield larger liquidation loss, but smaller pool of prey for predators. Interestingly, increasing the market size (positions) reverses the network configuration for which the CCP sees the hybrid fund as an advantage. The liquidation surplus for the banks is much higher in the hybrid fund, but there is no difference during buyback. There is a shift in the effects of the CCP and distressed dealers' selling strategies. There is no longer the possibility of an extreme loss for the economy (CCP and banks) due to one configuration of the financial network, but losses have increases overall. Furthermore, increased market volatility means that dealers no longer see any final profit.Finally, crisis market liquidity creates a clear benefit to having a hybrid fund over a pure fund (fig.12, **??**).

CASE 3 - Larger pool of CDS traded (184) CDS, and larger market.

This scenario produces a total gross notional market size \$$2.75e12$ USD, approaching the [Duffie et al., 2015] scenario. It includes a slightly larger market with multiple CCPs and admits CDS on sovereigns. This scenario produces the same affects as the previous scenarios, but increased in magnitude. Thus, increasing assets has the same augmentation effect in both a smaller and a larger market.

A.3.3 Monte-Carlo-type Simulation

This section provides further robustness testing of the [Tywoniuk, 2017a] model which is the basis of our investigation. In this type of simulation is it most definitely certain that the random endowment or distribution of assets, and the random assignment of the liquidation rate among banks determines, to some degree, the results. However, a robust simulation, should show patterns and similarities in the results over many iterations with a randomised seed. In this section, average results from 50 separate randomised trials are provided. Over all trials, the majority of results remain similar, but present with more granularity. In the runs under variable market liquidity, there are a few interesting and surprising features.

Under normal market liquidity, the number of distressed banks remain largely the same, at 7.44, 6.16 and 1.32 defaulted banks for 1, 2, 3 predatory banks and 13,12, 11 distressed banks, respectively. The final CCP loss is increased by -2e11 with a new pronounced loss of -6e11 for the non-collusive case for 2 predatory banks. The final predation profits remain volatile, and keep largely the same pattern, except that profits are now largely positive for the collusive case; this is owing to the lack of competition and predators acting as a monopoly.

Under decreasing market liquidity. There is much more fine grained detail. The default distribution decreases more gradually with defaults jumping from 1 to 4.08 banks around 7 predatory banks and 7 distressed banks. When predatory banks are stable at 2 banks, at 7 distressed banks the default number jumps to 3.48. This gives an idea of the augmentation effect of competition on defaults, which is about 0.6 banks at this turning point. For 2 distressed banks, the default distribution remains unchanged. For the final CCP loss, in the case of collusion and non-collusion, the pattern remains the same except for the predatory case which takes on the pattern of the former two cases. This reinforces the idea of this threshold point where predation profit cannot overtake price impact. The average profit and loss for banks is extremely stable, as do predator profits and losses, though buyback values are depressed. Under financial crisis liquidity, all results are stable.