Formule logica Proposizionale $\to$ Algebrico

$\neg \; \wedge \; \vee \; \to \; \leftrightarrow$ polinomi

$B_0 = \{0, 1\}$ $\quad +, \cdot \quad x+y \quad xy$

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

$B_1 = \{0, 1, x, 1+x\}$ 4 element $2^{2^1} = 4$

$B_2 = \{0, 1, x, y \dots\} = \{a \cdot 1 + b \cdot x + c \cdot y + d\, x y : a, b, c, d \in \{0,1\}\}$

Ho all'inizio polinomi x, y

$$\sum_{\substack{i \geq 0 \\ j \geq 0}}' a_{ij}\, x^i y^j \qquad i, j \in \mathbb{N} = \{0, 1, 2, \dots\}$$

$$2 + x + 3xy + 4x^2y + 5xy^2$$

$R_1 \qquad \underset{R_1}{\equiv} \quad x + xy + xy^2$

$R_2 \left(\begin{array}{l} x^2 \equiv x, \; y^2 = y \qquad x^3 = x^2 \cdot x \underset{R_2}{\equiv} x \cdot x \underset{R_2}{\equiv} x \\ x^{174} \equiv x^{172} \cdot x^2 \equiv x^{172} \cdot x \equiv x^{173} \equiv \qquad \equiv x \end{array} \right)$

$$\equiv \quad x + xy + xy = x + 2xy \underset{R_1}{\equiv} x$$

$B_n = \{$ polinomi $x_1, \; , x_n \}$ $\quad R_1$ pari $\equiv 0$, dispari $\equiv 1$ $\quad R_2 \; x_i^2 \equiv x_i$

$P_1 \qquad P_n$

$$\sum' a_{i_1 \, i_n}\, x_1^{i_1} \quad x_n^{i_n}$$

Se $i_j > 0$ $\quad x_j^{i_j} \to x_j$ $\qquad i_j = 0$ $\quad x_j$ non compare

$$x_1^{\epsilon_1} \cdots x_n^{\epsilon_n} \qquad \epsilon_1, -, \epsilon_n \in \{0, 1\}$$

$$x_{i_1} \quad x_{i_k} \qquad 1 \leq i_1 < \quad < i_k \leq n$$

$n = 3$ $\quad x_1^7 x_3^4 \underset{R_2}{\equiv} x_1 x_3$ $\quad i_1 = 1 < i_2 = 3$

$x_1^1 x_2^0 x_3^1$

$128\, x^3 y^2 z^7 \underset{R_1}{\equiv} 0$ $\qquad 111\, x^3 y^2 z^7 \underset{R_1}{\equiv} x^3 y^2 z^7 \underset{R_2}{\equiv} xyz$

Hofstaedter

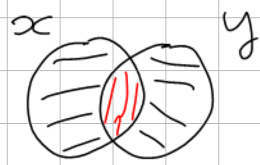
$x^2 = x$

Escher Gödel Bach

$$\neg\left(\left(\underbrace{s \wedge \neg t}_{a_1}\right) \to (t \vee q)\right) \vee \left(s \to (t \vee q)\right)$$

neg := func<x | 1+x>;

vel := func<x,y | x+y+x*y>; or



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imp := func<x,y | 1+x+x*y>;

$B_0 = \{0,1\}$ campo $\qquad$ F := GF(2); $\qquad R_1$

A := PolynomialRing(F, 3);

B<s,t,q> := quo<A | [A.i^2 - A.i : i in [1..3]]>;

$A.i^2 - A.i$ $\qquad R_2$

$x_i^2 = x_i$

a1 := s*neg(t); $\qquad s \wedge \neg t$

a2 := vel(t,q); $\qquad t \vee q$

a3 := imp(s, a2); $\qquad s \to (t \vee q)$

a4 := imp(a1, a2); $\qquad (s \wedge \neg t) \to (t \vee q)$

a5 := vel(a3, a4);

a6 := neg(a5)

vel(q, neg(q)); ottengo 1 $\quad q \vee \neg q$ tautologia

Principio III° Escluso

q * neg(q); ottengo 0 $\quad q \wedge \neg q$ contraddizione

Principio di NON

$B = B_3$ QUOZIENTE quo< A | E >

$f \equiv g \quad g = f + 2h + (x^2 - x)k$

$R_1, R_2 \qquad R_1 \qquad \ldots + (z^2 - z)l$

$x^2 \equiv x \quad x^2 - x \equiv 0$ Esercizio E equivalenza

$R_2 \qquad R_2$

Leggi di De Morgan