1 Lecture 2: Pattern Discovery Basic Concepts

1.1 Frequent Itemsets (Patterns)

X = itemset

- **(absolute) support (count) of X:** Frequency or the number of occurrences of an itemset X
- **(relative) support, s:** The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is **frequent** if the support of X is no less than a *minsup* threshold (denoted as σ)

1.2 Association Rules

Association rules: $X \rightarrow Y (s, c)$:

- **Support**, s: The probability that a transaction contains $X \cup Y$
- **Confidence**, c: The conditional probability that a transaction containing X also contains Y:

$$c = \frac{sup(X \cup Y)}{sup(X)}$$

1.3 Expressing Patterns in Compressed Form

Solution 1: **Closed patterns:** *A pattern (itemset) X is closed if X is frequent, and there exists no super-pattern $Y \supset X$, with the same support as X.*

Closed pattern is a lossless compression of frequent patterns.

Solution 2: **Max-patterns:** *A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern $Y \supset X$.*

Max-pattern is a lossy compression!

1.4 Recommended readings

- R. Agrawal, T. Imielinski, and A. Swami, «Mining association rules between sets of items in large databases», in Proc. of SIGMOD'93
- R. J. Bayardo, «Efficiently mining long patterns from databases», in Proc. of SIGMOD'98
- N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, «Discovering frequent closed itemsets for association rules», in Proc. of ICDT'99
- J. Han, H. Cheng, D. Xin, and X. Yan, «Frequent Pattern Mining: Current Status and Future Directions», Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

2 Lecture 3. Efficient Pattern Mining Methods

2.1 The Downward Closure Property of Frequent Patterns

The downward closure (also called «Apriori») property of frequent patterns: **Any subset of a frequent itemset must be frequent**. Apriori pruning principle: **If there is any itemset which is infrequent, its superset should not even be generated!**

Scalable mining Methods: Three major approaches

- Level-wise, join-based approach: Apriori (2.2)
- Vertical data format approach: Eclat (2.4)
- Frequent pattern projection and growth: FPgrowth (2.5)

2.2 The Apriori Algorithm

2.2.1 Algorithm pseudocode

C_k: Candidate itemset of size k
F_k : Frequent itemset of size k
TDB = transactional database

Algorithm 1 The Apriori Algorithm

```
k := 1
F_k := frequent items                                  # frequent 1-itemset
while F_k ≠ ∅ do
   C_{k+1} := candidates generated from F_k            # candidate generation
   Derives F_{k+1} by counting candidates in C_{k+1} with respect to TDB at minsup
   k := k + 1
end while
return ∪_k F_k                                  # return F_k generated at each level
```

2.2.2 How to generate candidates?

- Step1: self-joining F_k
- Step2: pruning

Algorithm 2 Step1: self-joining F_k

```
insert into C_k
select p.item_1, p.item_2, ..., p.item_{k-1}, q.item_{k-1}
from F_{k-1} as p, F_{k-1} as q
where p.item_1= q.item_1, ..., p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}
```

Algorithm 3 Step2: pruning

```
for all itemsets c in C_k do
   for all (k-1) subsets s of c do
      if s is not in F_{k-1} then
         delete c from C_k
      end if
   end for
end for
```

2.3 Extensions or Improvements of Apriori

- Reduce passes of transaction database scans
  - Partitioning
  - Dynamic itemset counting
- Shrink the number of candidates
  - Hashing
  - Pruning by support lower bounding
  - Sampling
- Exploring special data structures
  - Tree projection
  - H-miner
  - Hypecube decomposition

2.3.1 Partitioning

Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Method: Scan Database Only Twice:

- Scan 1: Partition database (how?) and find local frequent patterns
- Scan 2: Consolidate global frequent patterns (how to?)

2.3.2 Direct Hashing and Pruning (DHP)

Observation: *A k-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent*

2.4 Vertical Data Format

ECLAT - Equivalence Class Transformation
Frequent patterns are derived based on vertical intersections. To accelerate data mining you can use **diffset**: only keep track of differences of tids.

TID	Items in the Transaction	Ordered, frequent items
100	$\{f, a, c, d, g, i, m, p\}$	$\{f, c, a, m, p\}$
200	$\{a, b, c, f, l, m, o\}$	$\{f, c, a, b, m\}$
300	$\{b, f, h, j, o, w\}$	$\{f, b\}$
400	$\{b, c, k, s, p\}$	$\{c, b, p\}$
500	$\{a, f, c, e, l, p, m, n\}$	$\{f, c, a, m, p\}$

Figure 1: Transational DB

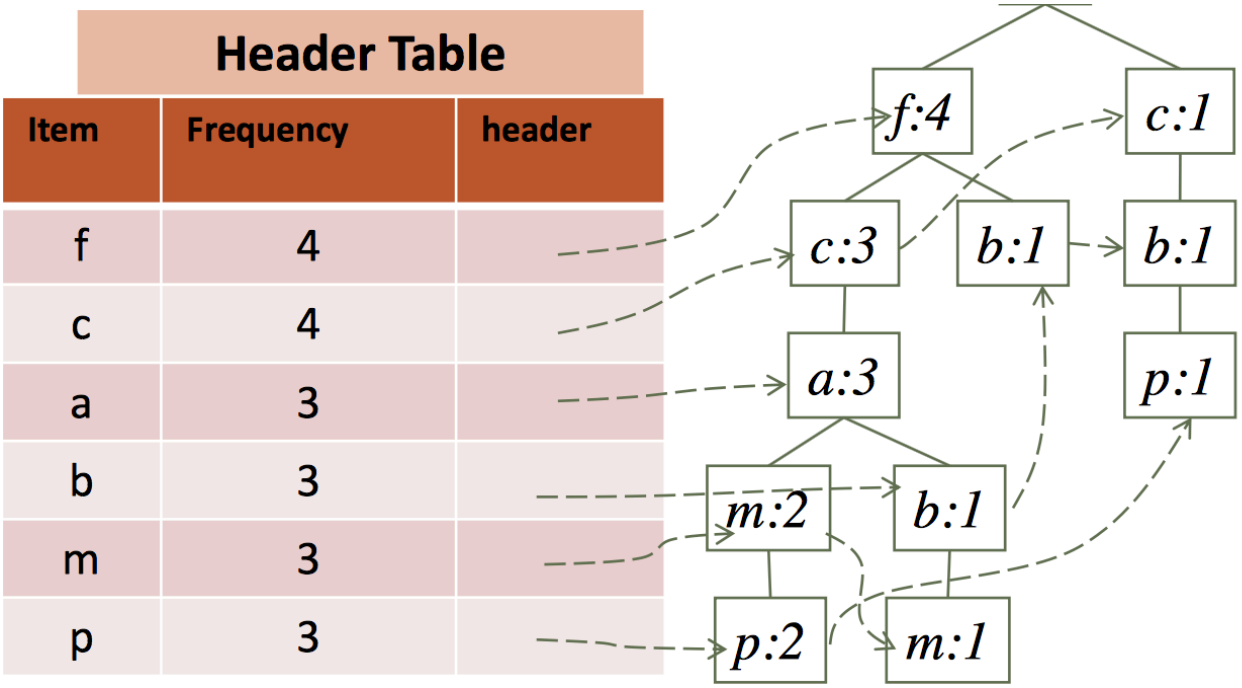

Figure 2: FP-tree

2.5 A Pattern Growth Approach

FP-tree - frequent pattern tree

2.6 CLOSET+: Mining Closed Itemsets by Pattern-Growth

Itemset merging: *If Y appears in every occurrence of X, then Y is merged with X*

2.7 Recommended readings

- R. Agrawal and R. Srikant, «Fast algorithms for mining association rules», VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, «An efficient algorithm for mining association rules in large databases», VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, «An effective hash-based algorithm for mining association rules», SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, «Integrating association rule mining with relational database systems: Alternatives and implications», SIGMOD'98

- M. J. Zaki, S. Parthasarathy, M. Ogihara, and W. Li, «Parallel algorithm for discovery of association rules», Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, «Mining frequent patterns without candidate generation», SIGMOD'00
- M. J. ZakiandHsiao, «CHARM: An Efficient Algorithm for Closed Itemset Mining», SDM'02
- J. Wang, J. Han, and J. Pei, «CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets», KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, «Frequent Pattern Mining Algorithms: A Survey», in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

3 Lecture 4: Pattern Evaluation

3.1 Interestingness Measures: Lift and χ^2

3.1.1 Interestingness Measure: Lift

Lift - measure of dependent/correlated events:

$$lift(B, C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

Lift(B, C) may tell how B and C are correlated:

- $Lift(B, C) = 1$: B and C are independent
- $Lift(B, C) > 1$: positively correlated
- $Lift(B, C) < 1$: negatively correlated

3.1.2 Interestingness Measure: χ^2

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

General rules:

- $\chi^2 = 0$: independent
- $\chi^2 > 0$: correlated, either positive or negative, so it needs additional test

Too many null transactions may lead to invalid correlation result!

3.2 Null Invariance Measures

$$AllConf(A, B) = \frac{s(A \cup B)}{\max\{s(A), s(B)\}}$$

$$Jaccard(A, B) = \frac{s(A \cup B)}{s(A) + s(B) - s(A \cup B)}$$

$$Cosine(A, B) = \frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$$

$$Kulczynsky(A, B) = \frac{1}{2}\left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)}\right)$$

$$MacConf(A, B) = \max\left\{\frac{s(A)}{s(A \cup B)}, \frac{s(B)}{s(A \cup B)}\right\}$$

3.3 Imbalance Ratio

IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications:

$$IR(A, B) = \frac{|s(A) - s(B)|}{s(A) + s(B) - s(A \cup B)}$$

Kulczynski and Imbalance Ratio (IR) together present a clear picture

3.4 Recommended Readings

- C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
- T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371-397, 2010

4 Lecture 4: Mining Diverse Patterns

4.1 Mining Multi- Level Associations

Items often form hierarchies. How to set min-support thresholds? **Level-reduced min-support**: items at the lower level are expected to have lower support.

Efficient mining: **shared** multi-level mining. Use the lowest min-support to pass down the set of candidates.

Redundancy filtering: some rules may be redundant due to «ancestor[1]» relationships between items. A rule is **redundant** if:

- its support is close to the «expected» value, according to its «ancestor» rule
- it has a similar confidence as its «ancestor».

It is necessary to have customized min-support settings for different kinds of items: group-based «individualized» min-support.

4.2 Mining Multi-Dimensional Associations

Rules can be single-dimensional or multi-dimensional:

- Single-dimentional: $buys(X, \text{«milk»}) \Rightarrow buys(X, \text{«bread»})$
- Inter-dimension association rule: $age(X, \text{«18-25»}) \wedge occupation(X, \text{«student»}) \Rightarrow buys(X, \text{«coke»})$
- Hybrid-dimension association rules: $age(X, \text{«18-25»}) \wedge buys(X, \text{«popcorn»}) \Rightarrow buys(X, \text{«coke»})$

Attributes can be categorical or numerical

4.3 Mining Quantitative Associations

Methods:

- Static discretization based on predefined concept hierarchies
- Dynamic discretization based on data distribution
- Clustering: distance-based association
- Deviation analysis

4.4 Mining Negative Correlations

- Rare patterns = very low support but interesting
- Negative patterns = negatively correlated, unlikely to happen together

A support-based definition: if itemsets A and B are both frequent but rarely occur together, i.e., $\sup(A \cup B) << \sup(A) \times \sup(B)$ then A and B are negatively correlated.

The support-based definition is not null-invariant!

A Kulczynski measure-based definition: if itemsets A and B are frequent but $\frac{P(A|B)+P(B|A)}{2} < \varepsilon$, where ε is a negative pattern threshold, then A and B are negatively correlated.

[1] Ancestor – предок

4.5 Mining Compressed Patterns

4.5.1 Mining Compressed Patterns

Pattern distance measure:

$$Dist(P_1, P_2) = 1 - \frac{\left|T(P_1) \cap T(P_2)\right|}{\left|T(P_1) \cup T(P_2)\right|}$$

δ-clustering. For each pattern P, find all patterns which can be expressed by P and whose distance to P is within δ (δ-cover). All patterns in the cluster can be represented by P = compressed patterns.

Method for efficient, direct mining of compressed frequent patterns: Xin et al., VLDB'05.

4.5.2 Redundancy-Aware Top-k Patterns

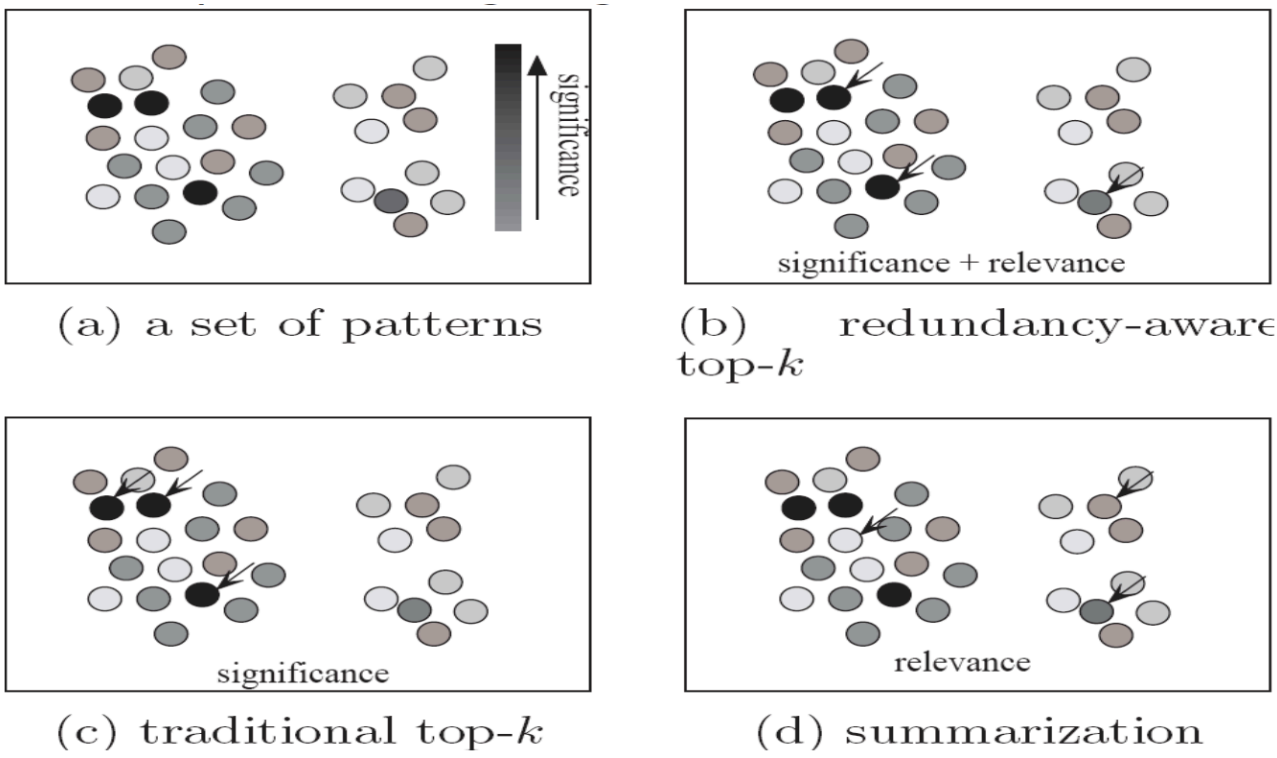

Figure 3: Desired patterns: high significance & low redundancy

Use **MMS (Maximal Marginal Significance)** for measuring the combined significance of a pattern set: Xin et al., Extracting Redundancy[2]-Aware Top-K Patterns, KDD'06.

4.6 Mining Colossal Patterns

4.6.1 Pattern-Fusion

Pattern fusion strategy: fuse small patterns together in one step to generate new pattern candidates of significant sizes.

[2]Redundancy - избыточность

Subpatterns α_1 to α_k cluster tightly around the colossal pattern α by sharing a similar support. Such subpatterns are **core patterns** of α. A colossal pattern can be generated by merging a set of core patterns.

4.6.2 Robustness of Colossal Patterns

For a frequent pattern α, a subpattern β is a τ-core pattern of α if β shares a similar support set with α, i.e.,

$$\frac{\left|D_\alpha\right|}{\left|D_\beta\right|} \geqslant \tau, 0 < \tau \leqslant 1,$$

where $\square$ is called the **core ratio**.

(d, τ)**-robustness**[3]: a pattern α is (d, τ)-robust if d is the maximum number of items that can be removed from α for the resulting pattern to remain a τ-core pattern of α. For a (d, τ)-robust pattern α, it has $\Omega(2^d)$ core patterns.

Robustness of Colossal Patterns: a colossal pattern tends to have much more core patterns than small patterns. Such core patterns can be clustered together to form «dense balls» based on pattern distance defined by

$$Dist(\alpha, \beta) = 1 - \frac{\left|D_\alpha \cap D_\beta\right|}{\left|D_\alpha \cup D_\beta\right|}$$

4.6.3 The Pattern-Fusion Algorithm

- Initialization (Creating initial pool): Use an existing algorithm to mine all frequent patterns up to a small size, e.g., 3
- Iteration (Iterative Pattern Fusion):
  - At each iteration, K seed patterns are randomly picked from the current pattern pool
  - For each seed pattern thus picked, we find all the patterns within a bounding ball centered at the seed pattern
  - All these patterns found are fused together to generate a set of super-patterns
  - All the super-patterns thus generated form a new pool for the next iteration
- Termination: when the current pool contains no more than K patterns at the beginning of an iteration

4.7 Recommended Readings

- R. Srikant and R. Agrawal, «Mining generalized association rules», VLDB'95
- Y. Aumann and Y. Lindell, «A Statistical Theory for Quantitative Association Rules», KDD'99

[3]Robustness - прочность

- D. Xin, J. Han, X. Yan and H. Cheng, «On Compressing Frequent Patterns», Knowledge and Data Engineering, 60(1): 5-29, 2007
- D. Xin, H. Cheng, X. Yan, and J. Han, «Extracting Redundancy-Aware Top-K Patterns», KDD'06
- F. Zhu, X. Yan, J. Han, P. S. Yu, and H. Cheng, «Mining Colossal Frequent Patterns by Core Pattern Fusion», ICDE'07
- J. Han, H. Cheng, D. Xin, and X. Yan, «Frequent Pattern Mining: Current Status and Future Directions», Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

5 Constraint-Based Pattern Mining

5.1 Meta-Rule Guided Mining

In general, (meta) rules can be in the form of

$$P_1 \wedge P_2 \wedge ... \wedge P_l \Rightarrow Q_1 \wedge Q_2 \wedge ... \wedge Q_r$$

Method to find meta-rules:

- Find frequent (l + r) predicates (based on min-support)
- Push constants deeply when possible into the mining process
- Also, push min_conf, min_correlation, and other measures as early as possible (measures acting as constraints)

5.2 Kinds of Constraints

- Pattern space pruning constraints
  - Anti-monotonic: If constraint c is violated, its further mining can be terminated
  - Monotonic: If c is satisfied, no need to check c again
  - Succinct[4]: if the constraint c can be enforced by directly manipulating the data
  - Convertible: c can be converted to monotonic or anti-monotonic if items can be properly ordered in processing
- Data space pruning constraints
  - Data succinct: Data space can be pruned at the initial pattern mining process
  - Data anti-monotonic: If a transaction t does not satisfy c, then t can be pruned to reduce data processing effort

Constraint c is **anti-monotone**: if an itemset S violates constraint c, so does any of its superset. That is, mining on itemset S can be terminated. For example, constraint $\sup(S) \geqslant \sigma$ is anti-monotone.

[4]Succinct - краткий