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# TEXT RETRIEVAL AND SEARCH ENGINES

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The basic concepts, principles, and the major techniques in text retrieval,  
which is the underlying science of search engines.

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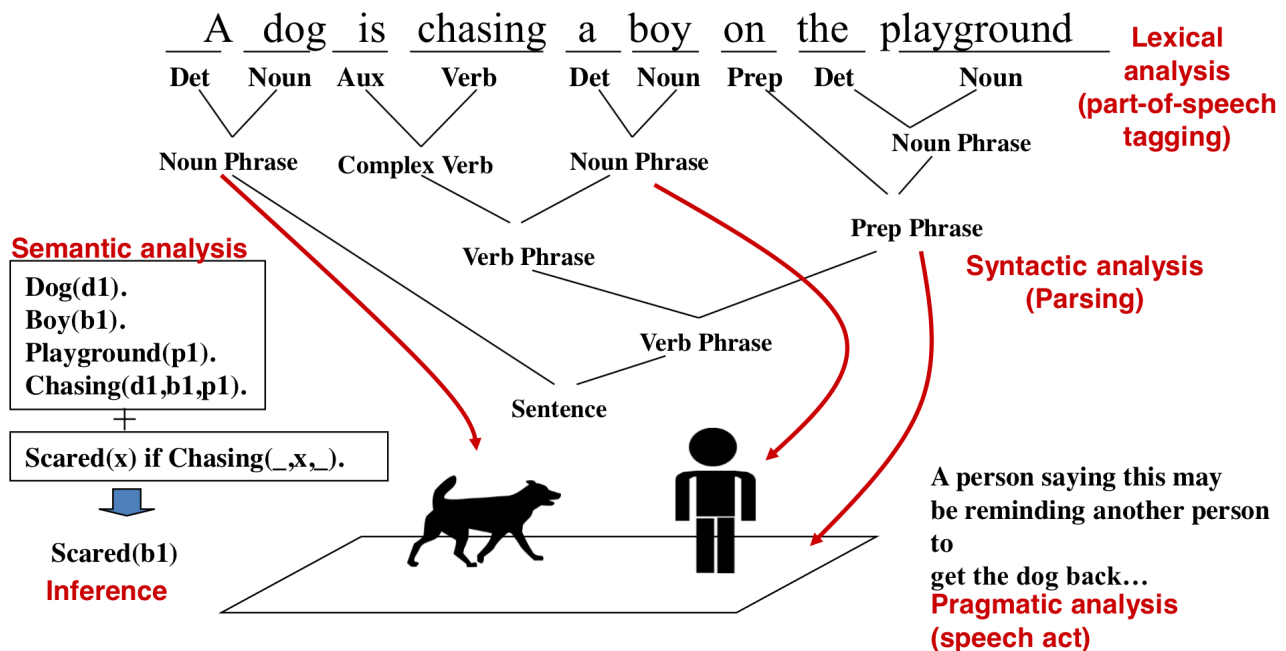
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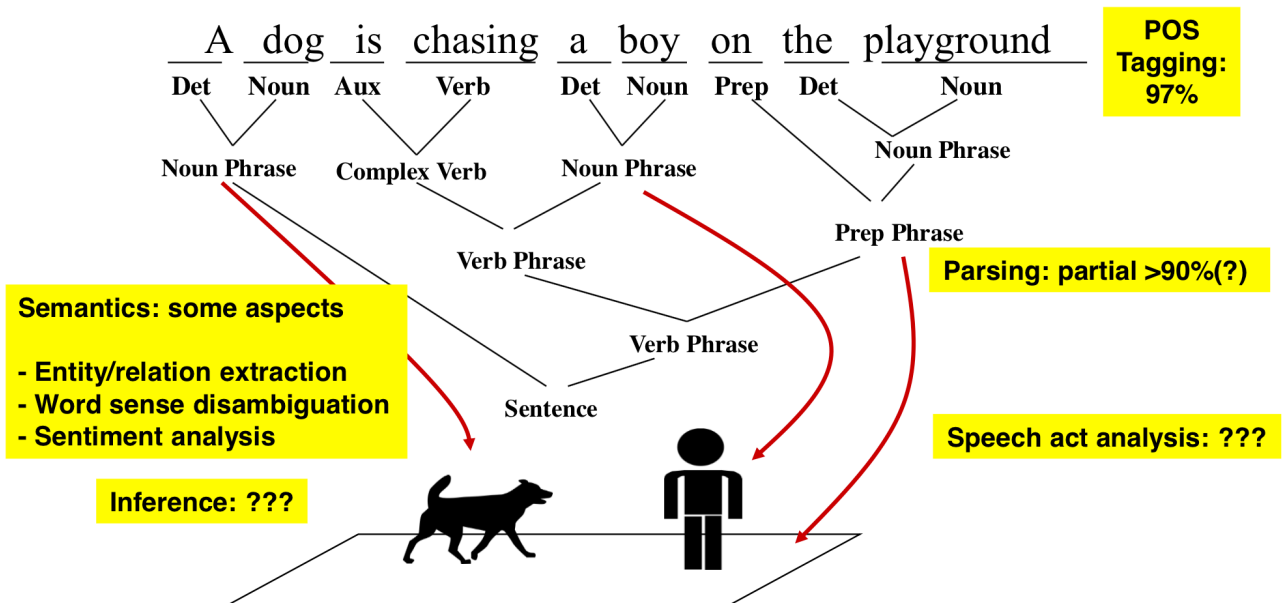
# 1 Natural Language Content Analysis

NLP = Natural Language Processing

## 1.1 An Example of NLP



## 1.2 The State of the Art



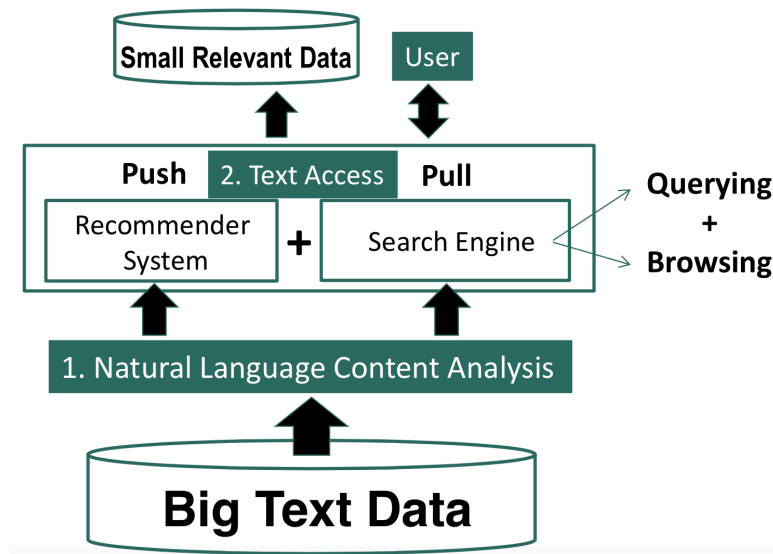
## 1.3 Recommended reading

- Chris Manning and Hinrich Schütze, «Foundations of Statistical Natural Language Processing», MIT Press. Cambridge, MA: May 1999.

## 2 Text Access

### 2.1 Two Modes of Text Access: Pull vs. Push

- Pull Mode (search engines) – Users take initiative
  - Ad hoc information need
- Push Mode (recommender systems)
  - Systems take initiative
  - Stable information need or system has good knowledge about a user's need



### 2.2 Pull Mode: Querying vs. Browsing

- Querying
  - User enters a (keyword) query
  - System returns relevant documents
  - Works well when the user knows what keywords to use
- Browsing
  - User navigates into relevant information by following a path enabled by the structures on the documents
  - Works well when the user wants to explore information, doesn't know what keywords to use, or can't conveniently enter a query

### 2.3 Recommended reading

- N. J. Belkin and W. B. Croft. 1992. «Information filtering and information retrieval: two sides of the same coin?» Commun. ACM 35, 12 (Dec. 1992), 29-38.

## 3 Text Retrieval Problem

### 3.1 What Is Text Retrieval?

TR = Text Retrieval<sup>1</sup>

- Collection of text documents exists
- User gives a query to express the information need
- Search engine system returns relevant documents to users
- Often called “information retrieval” (IR), but IR is actually much broader
- Known as «search technology» in industry

TR is an empirically defined problem:

- Can't mathematically prove one method is better than another
- Must rely on empirical evaluation involving users!

### 3.2 Formal Formulation of TR

- **Vocabulary:**  $V = \{w_1, w_2, \dots, w_N\}$  of language
- **Query:**  $q = q_1, \dots, q_m$ , where  $q_i \in V$
- **Document:**  $d_i = d_{i1}, \dots, d_{im_i}$ , where  $d_{ij} \in V$
- **Collection:**  $C = \{d_1, \dots, d_M\}$
- **Set of relevant documents:**  $R(q) \subseteq C$ 
  - Generally unknown and user-dependent
  - Query is a «hint» on which doc is in  $R(q)$
- **Task:** compute  $R'(q)$ , an approximation of  $R(q)$

### 3.3 How to Compute $R'(q)$

- Strategy 1: Document selection
  - $R'(q) = \{d \in C \mid f(d, q) = 1\}$ , where  $f(d, q) \in \{0, 1\}$  is an indicator function or binary classifier
  - System must decide if a doc is relevant or not (absolute relevance)
- Strategy 2 (generally preferred): Document ranking
  - $R'(q) = \{d \in C \mid f(d, q) > \theta\}$ , where  $f(d, q) \in \mathfrak{R}$  is a relevance measure function;  $\theta$  is a cutoff determined by the user
  - System only needs to decide if one doc is more likely relevant than another (relative relevance)

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<sup>1</sup>Retrieval - поиск

### 3.4 Theoretical Justification for Ranking

**Probability Ranking Principle [Robertson 77]:** Returning a ranked list of documents in descending order of probability that a document is relevant to the query is the optimal strategy under the following two assumptions:

- The utility of a document (to a user) is independent of the utility of any other document
- A user would browse the results sequentially

### 3.5 Recommended reading

- S.E. Robertson, «The probability ranking principle in IR». *Journal of Documentation* 33, 294-304, 1977
- C. J. van Rijsbergen, «**Information Retrieval**», **2nd Edition**, Butterworth-Heinemann, Newton, MA, USA, 1979

## 4 Overview of Text Retrieval Methods

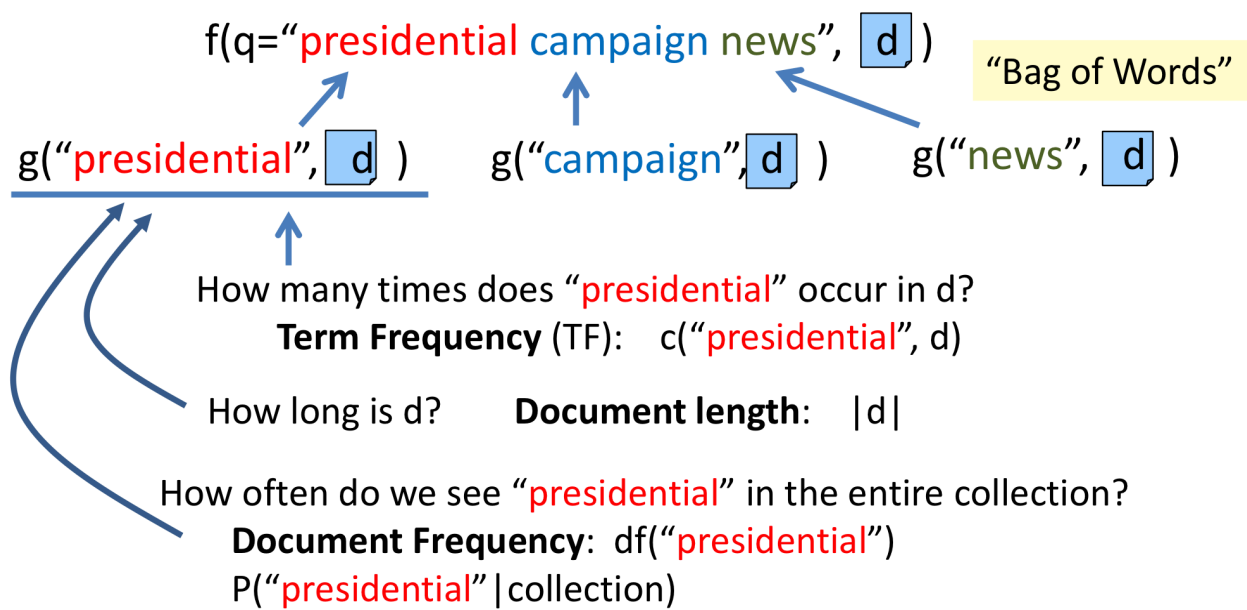
### 4.1 How to Design a Ranking Function

- **Query:**  $q = q_1, \dots, q_m$ , where  $q_i \in V$
- **Document:**  $d = d_1, \dots, d_n$ , where  $d_i \in V$
- **Ranking function:**  $f(q, d) \in \mathfrak{R}$
- **Key challenge:** how to measure the likelihood that document  $d$  is relevant to query  $q$
- **Retrieval model:** formalization of relevance (give a computational definition of relevance)

### 4.2 Retrieval Models

- **Similarity-based models:**  $f(q, d) = \text{similarity}(q, d)$ 
  - Vector space model
- **Probabilistic models:**  $f(d, q) = p(R = 1 \mid d, q)$ , where  $R \in \{0, 1\}$ 
  - Classic probabilistic model
  - Language model
  - Divergence-from-randomness model
- **Probabilistic inference model:**  $f(q, d) = p(d \rightarrow q)$
- **Axiomatic model:**  $f(q, d)$  must satisfy a set of constraints

### 4.3 Common Ideas in State of the Art Retrieval Models



State of the art ranking functions tend to rely on:

- Bag of words representation
- Term Frequency (TF) and Document Frequency (DF) of words
- Document length

### 4.4 Which Model Works the Best?

When optimized, the following models tend to perform equally well [Fang et al. 11]:

- **Pivoted length normalization – BM25**
- Query likelihood
- PL2

### 4.5 Recommended reading

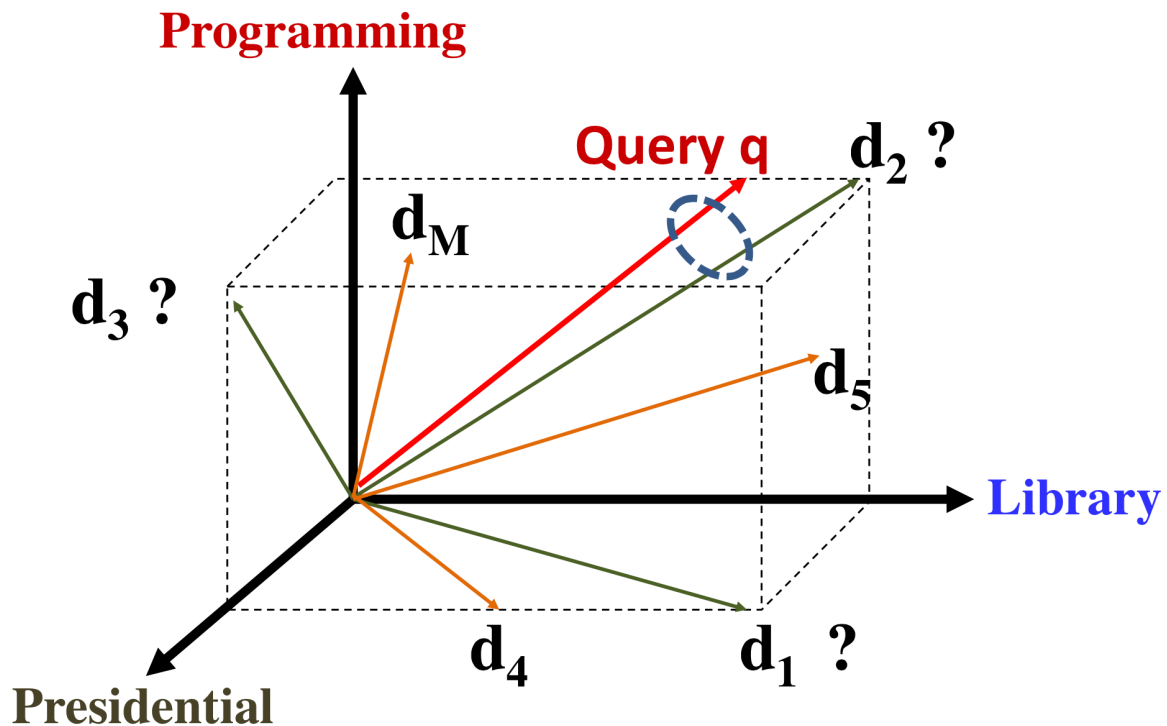
- Hui Fang, Tao Tao, and Chengxiang Zhai. 2011. «Diagnostic Evaluation of Information Retrieval Models». ACM Trans. Inf. Syst. 29, 2, Article 7 (April 2011)
- ChengXiang Zhai, «Statistical Language Models for Information Retrieval», Morgan & Claypool Publishers, 2008. (Chapter 2)



## 5 Vector Space Retrieval Model

VSM - Vector Space Model

### 5.1 Vector Space Model (VSM): Illustration

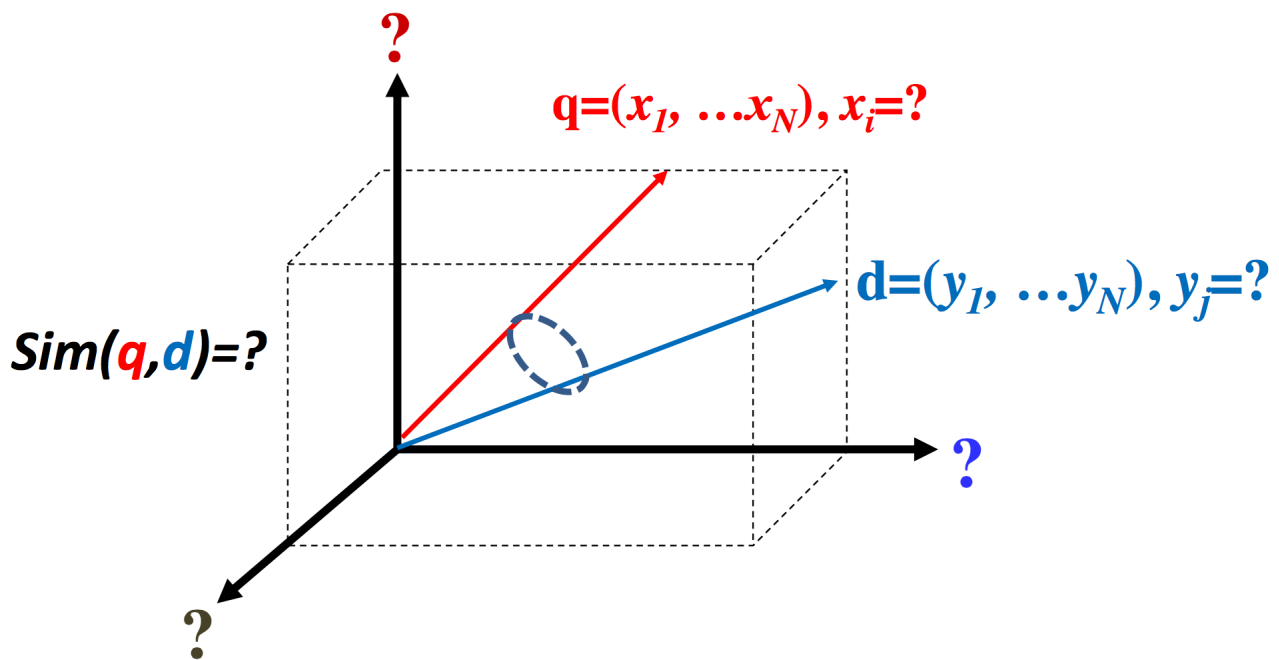


### 5.2 VSM Is a Framework

- Represent a doc/query by a term vector
  - **Term**: basic concept, e.g., word or phrase
  - Each term defines one dimension
  - N terms define an **N-dimensional space**
  - **Query vector**:  $q = (x_1, \dots, x_N)$ ,  $x_i \in \Re$  is query term weight
  - **Doc vector**:  $d = (y_1, \dots, y_N)$ ,  $y_j \in \Re$  is doc term weight
- $relevance(q, d) \propto similarity(q, d) = f(q, d)$

### 5.3 What VSM Doesn't Say

- How to define/select the “basic concept” – Concepts are assumed to be orthogonal
- How to place docs and query in the space (= how to assign term weights)
  - Term weight in query indicates importance of term
  - Term weight in doc indicates how well the term characterizes the doc
- How to define the similarity measure



#### 5.4 Simplest VSM = Bit-Vector + Dot-Product + BOW

$$q = (x_1, \dots, x_N) \quad x_i, y_i \in \{0, 1\}$$

$$d = (y_1, \dots, y_N) \quad \begin{array}{l} 1: \text{word } W_i \text{ is present} \\ 0: \text{word } W_i \text{ is absent} \end{array}$$

$$Sim(q, d) = q \cdot d = x_1 y_1 + \dots + x_N y_N = \sum_{i=1}^N x_i y_i$$

Simplest VSM:

- Dimension = word
- Vector = 0-1 bit vector (word presence/absence)
- Similarity = dot product
- $f(q, d)$  = number of distinct query words matched in d

#### 5.5 Improved Instantiation

Improved VSM:

- Dimension = word
- Vector = TF-IDF weight vector
- Similarity = dot product

## 5.6 Improved VSM with Term Frequency (TF) Weighting

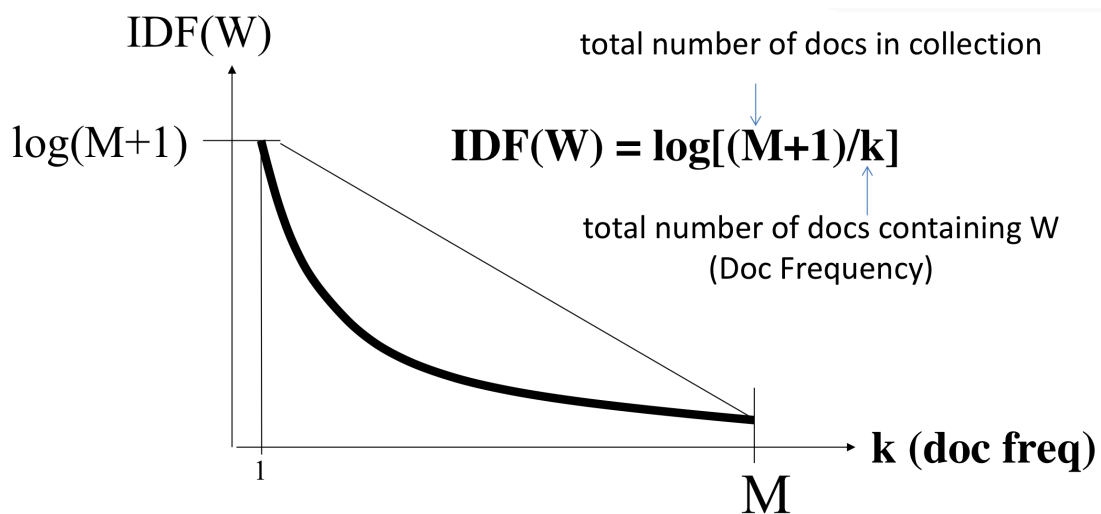
$$\mathbf{q} = (x_1, \dots, x_N) \quad \boxed{x_i = \text{count of word } W_i \text{ in query}}$$

$$\mathbf{d} = (y_1, \dots, y_N) \quad \boxed{y_i = \text{count of word } W_i \text{ in doc}}$$

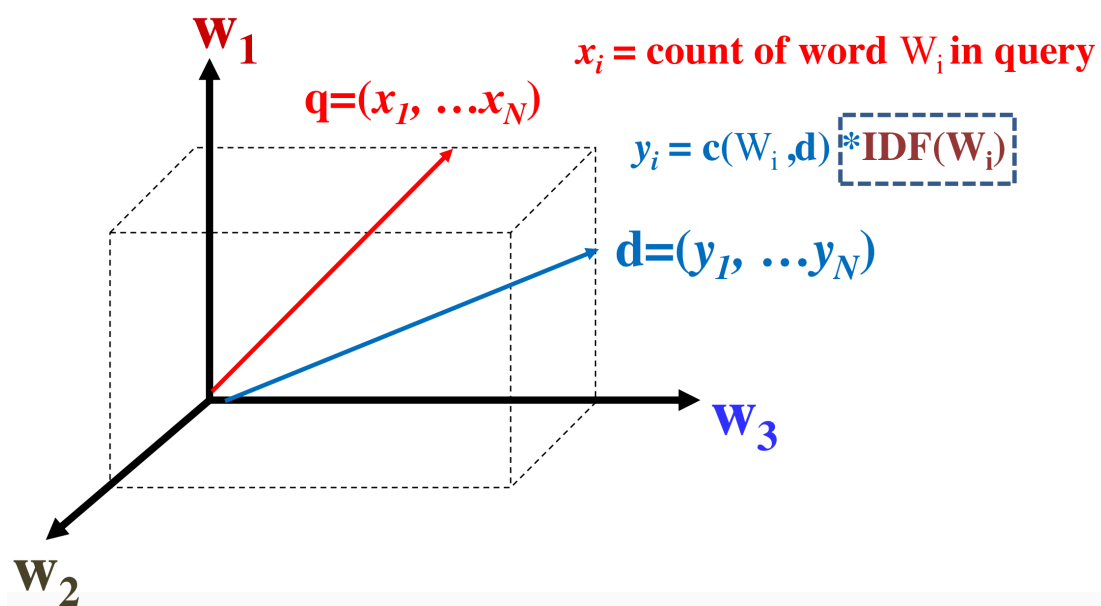
$$\text{Sim}(\mathbf{q}, \mathbf{d}) = \mathbf{q} \cdot \mathbf{d} = x_1 y_1 + \dots + x_N y_N = \sum_{i=1}^N x_i y_i$$

## 5.7 IDF Weighting: Penalizing Popular Terms

IDF — inverse document frequency



## 5.8 Adding Inverse Document Frequency (IDF)



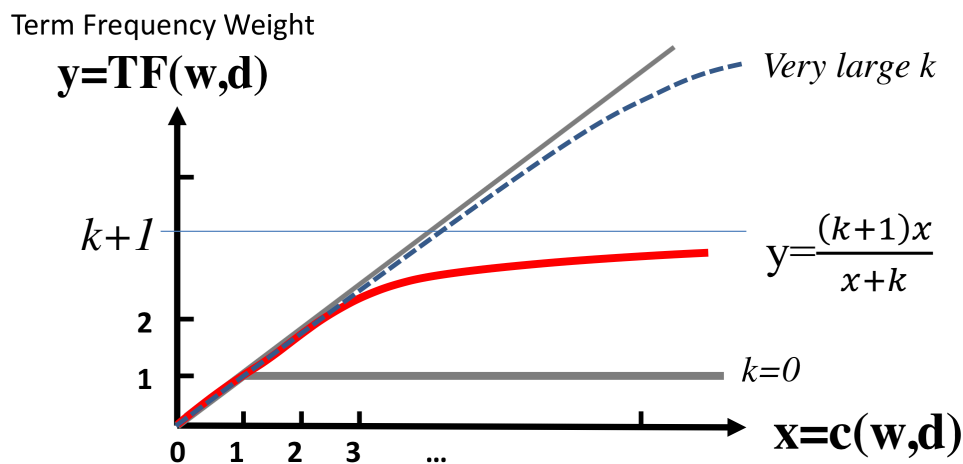
## 5.9 Ranking Function with TF-IDF Weighting

$$f(q, d) = \sum_{i=1}^N x_i y_i = \sum_{w \in q \cap d} c(w, q) c(w, d) \log \frac{M+1}{df(w)}$$

- $w \in q \cap d$  - all matched query (q) words in document (d)
- $c(w, q)$  - count of word w in document d
- $M$  - total number of documents in collection
- $df(w)$  - Doc Frequency (total number of documents containing word w)

## 5.10 TF Transformation: BM25 Transformation

BM = Best Matching



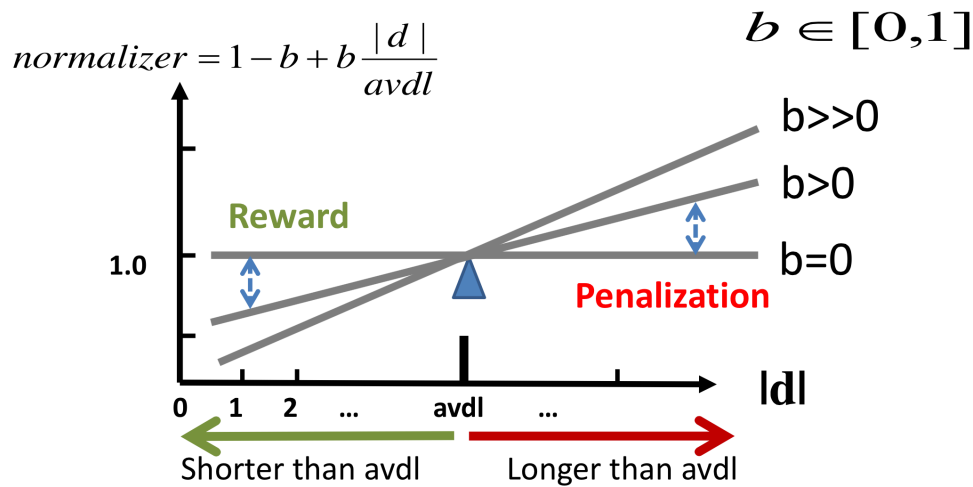
## 5.11 Summary

- Sublinear TF Transformation is needed to
  - capture the intuition of «diminishing return» from higher TF
  - avoid dominance by one single term over all others
- BM25 Transformation
  - has an upper bound
  - is robust and effective
- Ranking function with BM25 TF ( $k \geq 0$ ):

$$f(q, d) = \sum_{i=1}^N x_i y_i = \sum_{w \in q \cap d} c(w, q) \frac{(k+1)c(w, d)}{c(w, d) + k} \log \frac{M+1}{df(w)}$$

## 5.12 Pivoted Length Normalization

**Pivoted length normalizer:** use average doc length as «pivot»<sup>2</sup>. Normalizer = 1 if  $|d|$  = average doc length (avdl).



## 5.13 State of the Art VSM Ranking Functions

Pivoted Length Normalization VSM [Singhal et al 96]:

$$f(q, d) = \sum_{w \in q \cap d} c(w, q) \frac{\ln[1 + \ln(1 + c(w, d))]}{1 - b + b \frac{|d|}{avdl}} \log \frac{M + 1}{df(w)}$$

BM25/Okapi [Robertson & Walker 94]:

$$f(q, d) = \sum_{w \in q \cap d} c(w, q) \frac{(k + 1) c(w, d)}{c(w, d) + k \left(1 - b + b \frac{|d|}{avdl}\right)} \log \frac{M + 1}{df(w)}$$

## 5.14 Further Improvement of VSM?

- Improved instantiation of dimension?
  - stemmed words, stop word removal, phrases, latent semantic indexing (word clusters), character n-grams, ...
  - bag-of-words with phrases is often sufficient in practice
  - Language-specific and domain-specific tokenization is important to ensure “normalization of terms”
- Improved instantiation of similarity function?
  - cosine of angle between two vectors?
  - Euclidean?
  - dot product seems still the best (sufficiently general especially with appropriate term weighting)

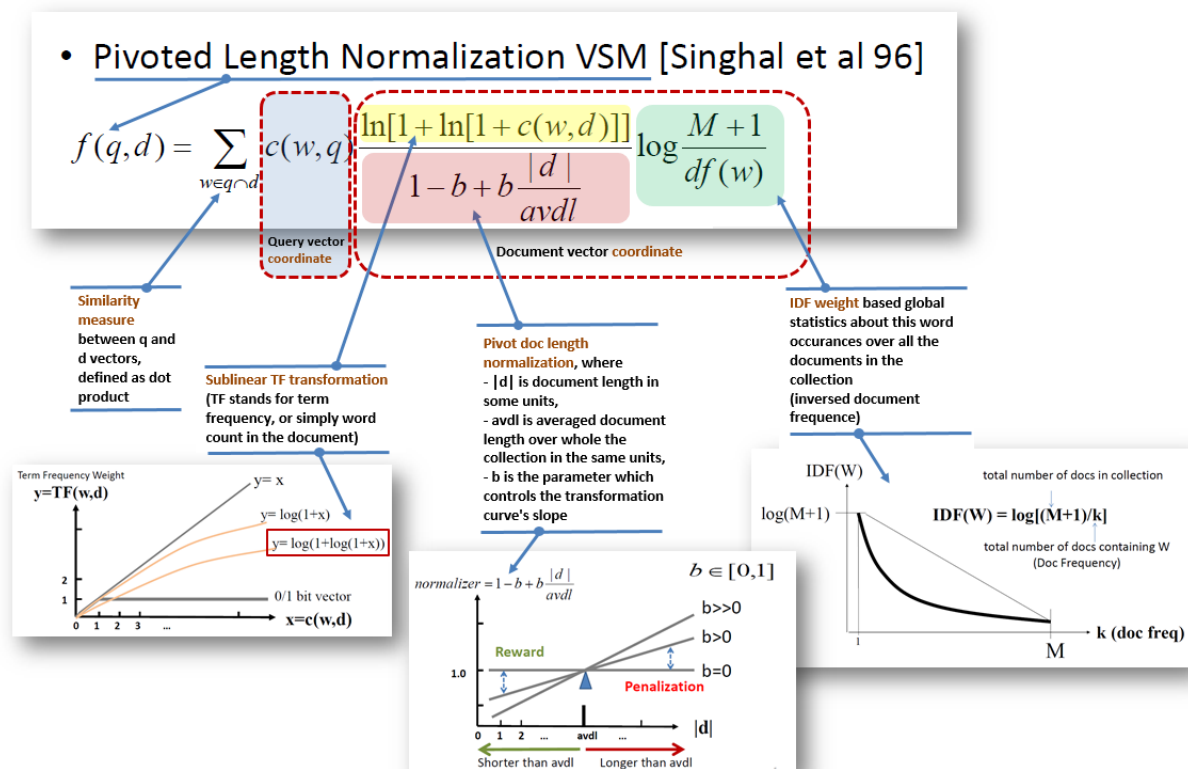
<sup>2</sup>Pivot - стержень; точка опоры, вращения

## 5.15 Further Improvement of BM25

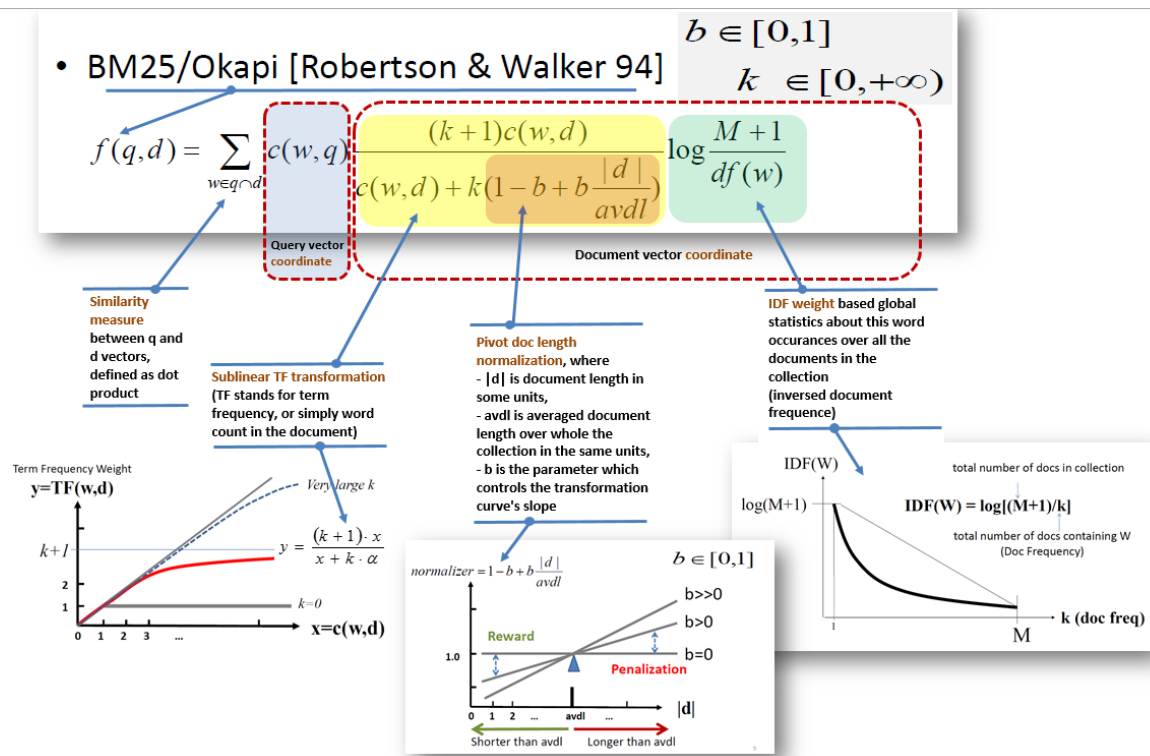
- BM25F [Robertson & Zaragoza 09]
  - Use BM25 for documents with structures («F»=fields)
  - Key idea: combine the frequency counts of terms in all fields and then apply BM25 (instead of the other way)
- BM25+ [Lv & Zhai 11]
  - Address the problem of over penalization of long documents by BM25 by adding a small constant to TF
  - Empirically and analytically shown to be better than BM25

## 5.16 Summary of Vector Space Model

- $\text{Relevance}(q,d) = \text{similarity}(q,d)$
- Query and documents are represented as vectors
- Heuristic<sup>3</sup> design of ranking function
- Major term weighting heuristics
  - TF weighting and transformation
  - IDF weighting
  - Document length normalization
- BM25 and Pivoted normalization seem to be most effective



<sup>3</sup>Heuristic - эвристический

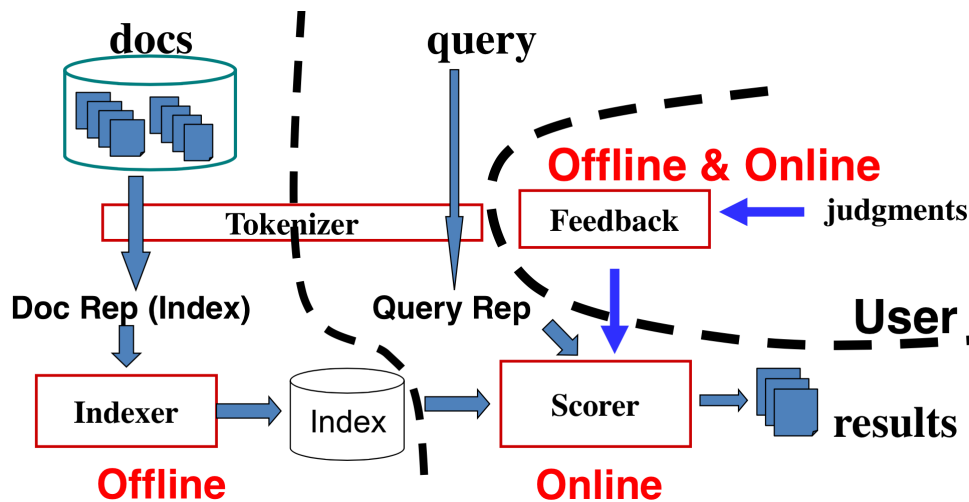


## 5.17 Recommended reading

- A. Singhal, C. Buckley, and M. Mitra. «Pivoted document length normalization». In Proceedings of ACM SIGIR 1996.
- S. E. Robertson and S. Walker. «Some simple effective approximations to the 2-Poisson model for probabilistic weighted retrieval», Proceedings of ACM SIGIR 1994.
- S. Robertson and H. Zaragoza. «The Probabilistic Relevance Framework: BM25 and Beyond», Found. Trends Inf. Retr. 3, 4 (April 2009).
- Y. Lv, C. Zhai, «Lower-bounding term frequency normalization». In Proceedings of ACM CIKM 2011.

## 6 Implementation of TR Systems

### 6.1 Typical TR System Architecture



### 6.2 Tokenization

- Normalize lexical units: words with similar meanings should be mapped to the same indexing term
- Stemming: mapping all inflectional forms of words to the same root form
- Some languages (e.g., Chinese) pose challenges in word segmentation

### 6.3 Inverted Index

Dictionary (or lexicon)			Postings		
Term	# docs	Total freq	Doc id	Freq	Position
news	3	3	1	1	p1
campaign	2	2	2	1	p2
presidential	1	2	3	1	p3
food	1	1	2	1	p4
...	...	...	3	1	p5
			3	2	p6,p7
			2	1	p8
			...	...	
			...	...	

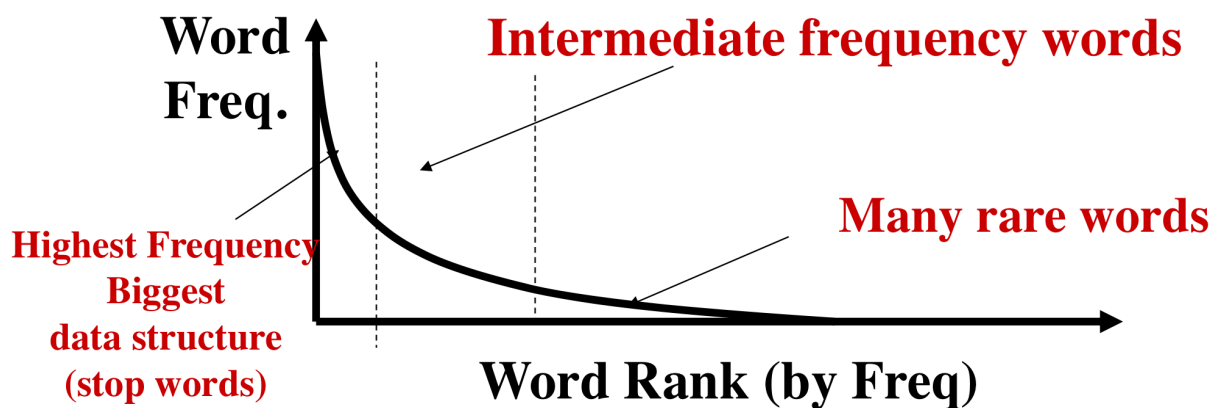
### 6.4 Empirical Distribution of Words

There are stable language-independent patterns in how people use natural languages:

- A few words occur very frequently; most occur rarely. E.g., in news articles:
  - Top 4 words: 10 15% word occurrences
  - Top 50 words: 35 40% word occurrences
- The most frequent word in one corpus may be rare in another



## 6.5 Zipf's Law



$$F(w) = \frac{C}{r(w)^\alpha}, \alpha \approx 1, C \approx 0.1$$

rank  $\times$  frequency  $\approx$  constant:

- $F(w)$  - word frequency
- $r(w)$  - word rank

## 6.6 Data Structures for Inverted Index

- Dictionary: modest size
  - Needs fast random access
  - Preferred to be in memory
  - Hash table, B-tree, trie, ...
- Postings: huge
  - Sequential access is expected
  - Can stay on disk
  - May contain docID, term freq., term pos, etc
  - Compression is desirable

## 6.7 Constructing Inverted Index

Sort-based method:

- Step 1: Collect local (termID, docID, freq) tuples from documents
- Step 2: Sort local tuples by termID (to make «runs») and save to files
- Step 3: Pair-wise merge runs
- Step 4: Output inverted file

## 6.8 Inverted Index Compression

In general, leverage skewed distribution of values and use variable-length encoding:

- TF compression:
  - Small numbers tend to occur far more frequently than large numbers (Zipf's law)
  - Fewer bits for small (high frequency) integers at the cost of more bits for large integers
- Doc ID compression:
  - «d-gap» (store difference):  $d_1, d_2 - d_1, d_3 - d_2, \dots$
  - Feasible due to sequential access

## 6.9 Integer Compression Methods

- **Binary**: equal-length coding
- **Unary**:  $x \geq 1$  is coded as  $x - 1$  one bits followed by 0, e.g.,  $3 \Rightarrow 110$ ;  $5 \Rightarrow 11110$
- **$\gamma$ -code**:  $x \Rightarrow$  unary code for  $1 + \lfloor \log x \rfloor$  followed by uniform code for  $x - 2^{\lfloor \log x \rfloor}$  in  $\lfloor \log x \rfloor$  bits, e.g.,  $3 \Rightarrow 101$ ,  $5 \Rightarrow 11001$
- **$\delta$ -code**: same as  $\gamma$ -code, but replace the unary prefix with  $\gamma$ -code. E.g.,  $3 \Rightarrow 1001$ ,  $5 \Rightarrow 10101$

## 6.10 General Form of Scoring Function

$$f(q, d) = f_a \left( h \left( g(t_1, d, q), \dots, g(t_k, d, q) \right), f_d(d), f_q(q) \right)$$

- $f_d(d), f_q(q)$  - adjustment factors of document and query
- $g(t_i, d, q)$  - weight of a **matched** query term  $t_i$  in  $d$
- $h()$  - weights aggregation function
- $f_a()$  - final score adjustment function

## 6.11 A General Algorithm for Ranking Documents

- $f_d(d)$  - can be precomputed at index time,  $f_q(q)$  - at query time
- Maintain a score accumulator for each  $d$  to compute  $h$
- For each query term  $t_i$ 
  - Fetch the inverted list  $\{(d_1, f_1), \dots, (d_n, f_n)\}$
  - For each entry  $(d_j, f_j)$ , compute  $g(t_i, d_j, q)$ , and update score accumulator for doc  $d_i$  to incrementally compute  $h$
- Adjust the score to compute  $f_a$ , and sort

## 6.12 Further Improving Efficiency

- Caching (e.g., query results, list of inverted index)
- Keep only the most promising accumulators
- Scaling up to the Web-scale? (need parallel processing)

## 6.13 Some Text Retrieval Toolkits

- [Lucene](#)
- [Lemur/Indri](#)
- [Terrier](#)
- [MeTA](#)
- More can be found [here](#)

## 6.14 Summary of System Implementation

- Inverted index and its construction
  - Preprocess data as much as we can
  - Compression when appropriate
- Fast search using inverted index
  - Exploit inverted index to accumulate scores for documents matching a query term
  - Exploit Zipf's law to avoid touching many documents not matching any query term
  - Can support a wide range of ranking algorithms
- Further scaling up using distributed file system, parallel processing, and caching

## 6.15 Recommended reading

- Ian H. Witten, Alistair Moffat, Timothy C. Bell: «Managing Gigabytes: Compressing and Indexing Documents and Images», Second Edition. Morgan Kaufmann, 1999.
- Stefan Büttcher, Charles L. A. Clarke, Gordon V. Cormack: «Information Retrieval - Implementing and Evaluating Search Engines». MIT Press, 2010.

## 7 Evaluation of Text Retrieval Systems

### 7.1 The Cranfield Evaluation Methodology

A methodology for laboratory testing of system components developed in 1960s. General idea is to build reusable test collections and define measures. A test collection can then be reused many times to compare different systems.

- A sample collection of documents (simulate real document collection)
- A sample set of queries/topics (simulate user queries)
- Relevance judgments (ideally made by users who formulated the queries) => Ideal ranked list
- Measures to quantify how well a system's result matches the ideal ranked list

### 7.2 Evaluating a Set of Retrieved Docs

	Retrieved	Not Retrieved
Relevant	a	b
Not Relevant	c	d

- Precision: are the retrieved results all relevant?

$$Precision = \frac{a}{a + c}$$

- Recall: have all the relevant documents been retrieved?

$$Recall = \frac{a}{a + b}$$

- In reality, high recall tends to be associated with low precision

### 7.3 Combine Precision and Recall: F-Measure

$$F_{\beta} = \frac{1}{\frac{\beta^2}{\beta^2 + 1} \frac{1}{R} + \frac{1}{\beta^2 + 1} \frac{1}{P}} = \frac{(\beta^2 + 1) \cdot P \cdot R}{\beta^2 \cdot P + R}$$

- $P$  - precision
- $R$  - recall

- $\beta$  - parameter, often set to 1:  $F_1 = \frac{2 \cdot P \cdot R}{P + R}$