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# TEXT RETRIEVAL AND SEARCH ENGINES

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The basic concepts, principles, and the major techniques in text retrieval,  
which is the underlying science of search engines.

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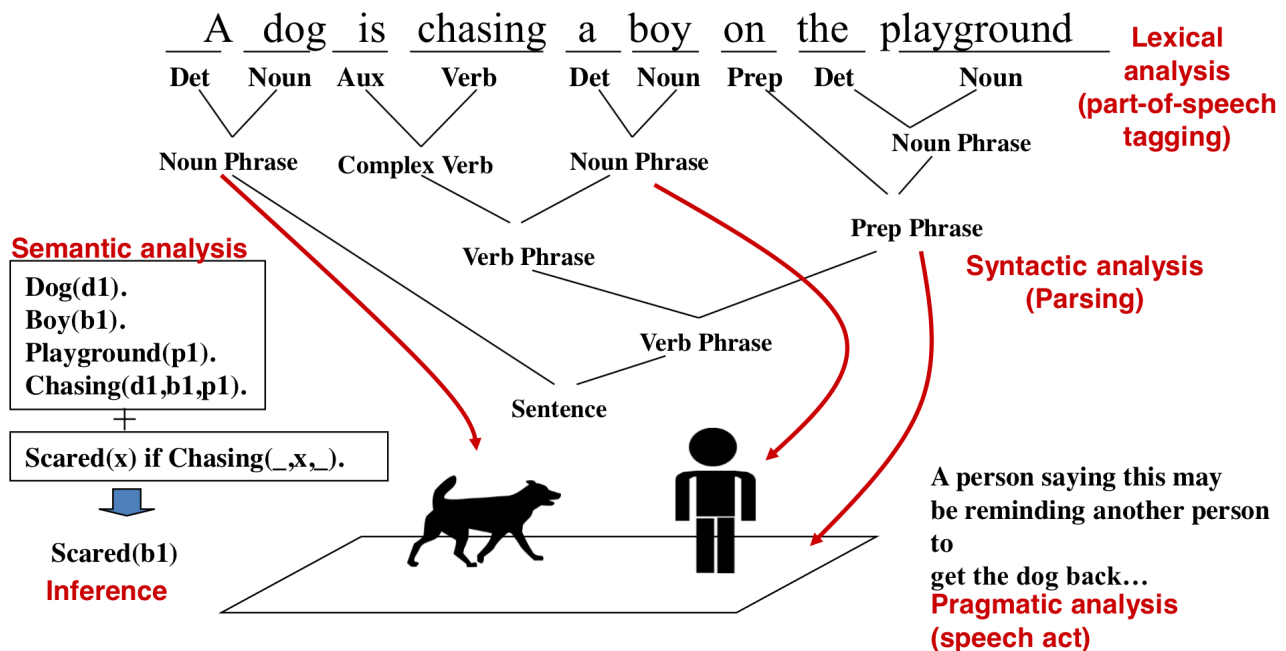
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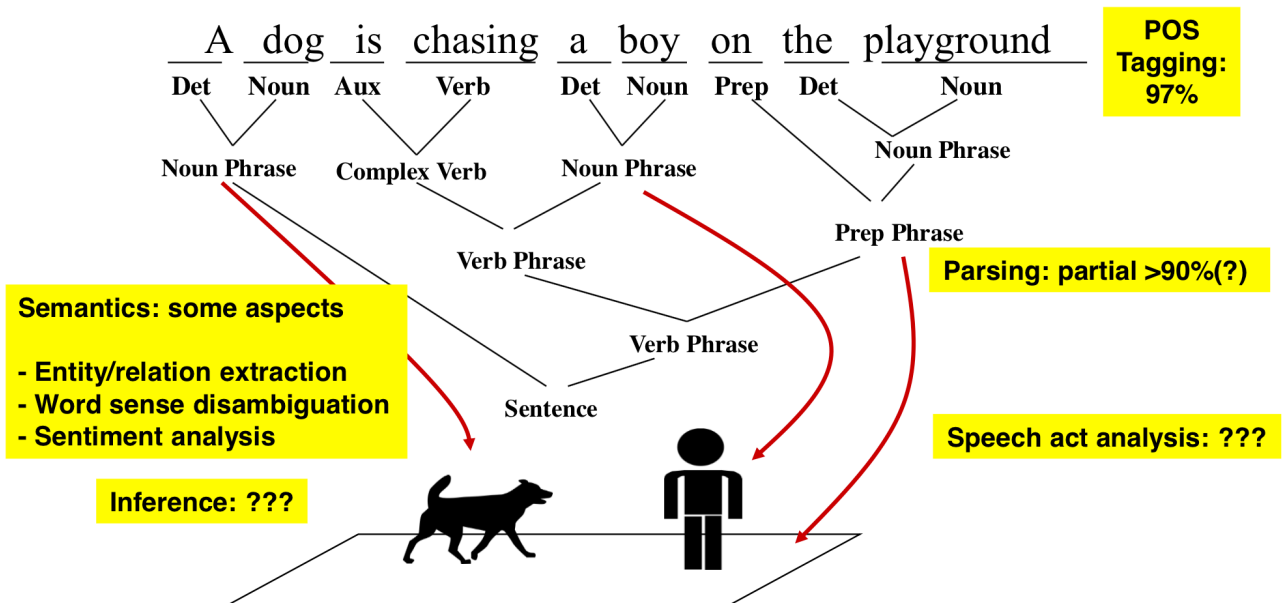
# 1 Natural Language Content Analysis

NLP = Natural Language Processing

## 1.1 An Example of NLP



## 1.2 The State of the Art



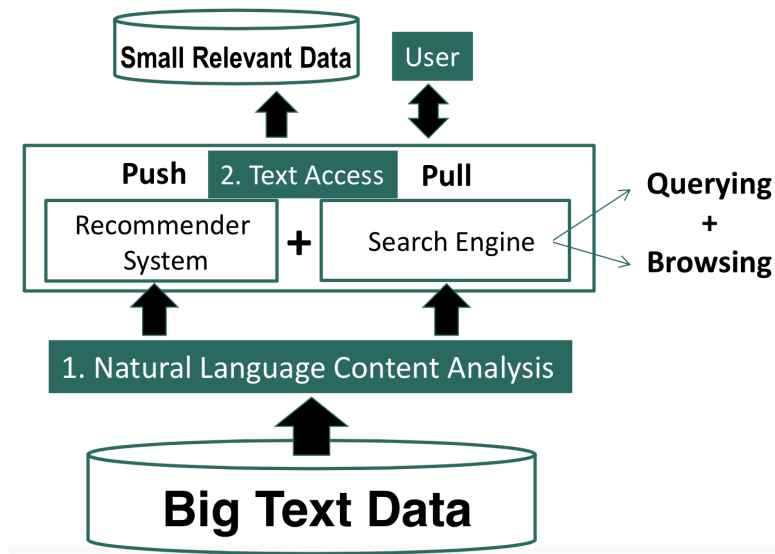
## 1.3 Recommended reading

- Chris Manning and Hinrich Schütze, «Foundations of Statistical Natural Language Processing», MIT Press. Cambridge, MA: May 1999.

## 2 Text Access

### 2.1 Two Modes of Text Access: Pull vs. Push

- Pull Mode (search engines) – Users take initiative
  - Ad hoc information need
- Push Mode (recommender systems)
  - Systems take initiative
  - Stable information need or system has good knowledge about a user's need



### 2.2 Pull Mode: Querying vs. Browsing

- Querying
  - User enters a (keyword) query
  - System returns relevant documents
  - Works well when the user knows what keywords to use
- Browsing
  - User navigates into relevant information by following a path enabled by the structures on the documents
  - Works well when the user wants to explore information, doesn't know what keywords to use, or can't conveniently enter a query

### 2.3 Recommended reading

- N. J. Belkin and W. B. Croft. 1992. «Information filtering and information retrieval: two sides of the same coin?» Commun. ACM 35, 12 (Dec. 1992), 29-38.

## 3 Text Retrieval Problem

### 3.1 What Is Text Retrieval?

TR = Text Retrieval<sup>1</sup>

- Collection of text documents exists
- User gives a query to express the information need
- Search engine system returns relevant documents to users
- Often called “information retrieval” (IR), but IR is actually much broader
- Known as «search technology» in industry

TR is an empirically defined problem:

- Can't mathematically prove one method is better than another
- Must rely on empirical evaluation involving users!

### 3.2 Formal Formulation of TR

- **Vocabulary:**  $V = \{w_1, w_2, \dots, w_N\}$  of language
- **Query:**  $q = q_1, \dots, q_m$ , where  $q_i \in V$
- **Document:**  $d_i = d_{i1}, \dots, d_{im_i}$ , where  $d_{ij} \in V$
- **Collection:**  $C = \{d_1, \dots, d_M\}$
- **Set of relevant documents:**  $R(q) \subseteq C$ 
  - Generally unknown and user-dependent
  - Query is a «hint» on which doc is in  $R(q)$
- **Task:** compute  $R'(q)$ , an approximation of  $R(q)$

### 3.3 How to Compute $R'(q)$

- Strategy 1: Document selection
  - $R'(q) = \{d \in C \mid f(d, q) = 1\}$ , where  $f(d, q) \in \{0, 1\}$  is an indicator function or binary classifier
  - System must decide if a doc is relevant or not (absolute relevance)
- Strategy 2 (generally preferred): Document ranking
  - $R'(q) = \{d \in C \mid f(d, q) > \theta\}$ , where  $f(d, q) \in \mathfrak{R}$  is a relevance measure function;  $\theta$  is a cutoff determined by the user
  - System only needs to decide if one doc is more likely relevant than another (relative relevance)

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<sup>1</sup>Retrieval - поиск

### 3.4 Theoretical Justification for Ranking

**Probability Ranking Principle [Robertson 77]:** Returning a ranked list of documents in descending order of probability that a document is relevant to the query is the optimal strategy under the following two assumptions:

- The utility of a document (to a user) is independent of the utility of any other document
- A user would browse the results sequentially

### 3.5 Recommended reading

- S.E. Robertson, «The probability ranking principle in IR». *Journal of Documentation* 33, 294-304, 1977
- C. J. van Rijsbergen, «**Information Retrieval**», **2nd Edition**, Butterworth-Heinemann, Newton, MA, USA, 1979

## 4 Overview of Text Retrieval Methods

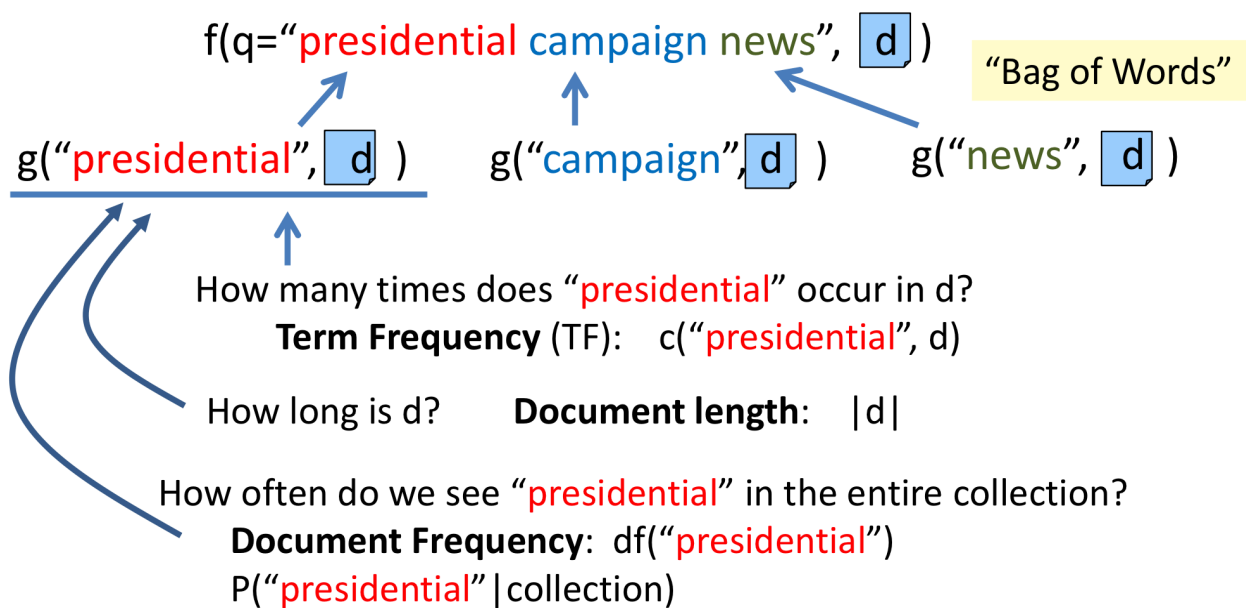
### 4.1 How to Design a Ranking Function

- **Query:**  $q = q_1, \dots, q_m$ , where  $q_i \in V$
- **Document:**  $d = d_1, \dots, d_n$ , where  $d_i \in V$
- **Ranking function:**  $f(q, d) \in \mathfrak{R}$
- **Key challenge:** how to measure the likelihood that document  $d$  is relevant to query  $q$
- **Retrieval model:** formalization of relevance (give a computational definition of relevance)

### 4.2 Retrieval Models

- **Similarity-based models:**  $f(q, d) = \text{similarity}(q, d)$ 
  - Vector space model
- **Probabilistic models:**  $f(d, q) = p(R = 1 \mid d, q)$ , where  $R \in 0, 1$ 
  - Classic probabilistic model
  - Language model
  - Divergence-from-randomness model
- **Probabilistic inference model:**  $f(q, d) = p(d \rightarrow q)$
- **Axiomatic model:**  $f(q, d)$  must satisfy a set of constraints

### 4.3 Common Ideas in State of the Art Retrieval Models



State of the art ranking functions tend to rely on:

- Bag of words representation
- Term Frequency (TF) and Document Frequency (DF) of words
- Document length

### 4.4 Which Model Works the Best?

When optimized, the following models tend to perform equally well [Fang et al. 11]:

- **Pivoted length normalization – BM25**
- Query likelihood
- PL2

### 4.5 Recommended reading

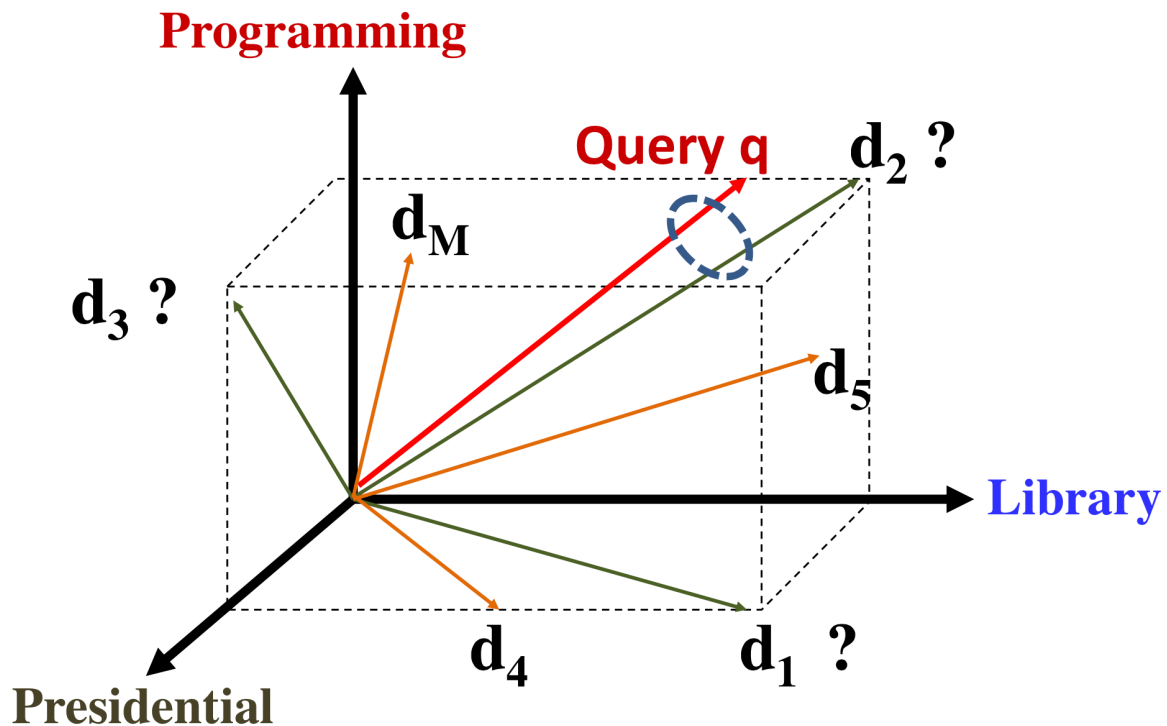
- Hui Fang, Tao Tao, and Chengxiang Zhai. 2011. «Diagnostic Evaluation of Information Retrieval Models». ACM Trans. Inf. Syst. 29, 2, Article 7 (April 2011)
- ChengXiang Zhai, «Statistical Language Models for Information Retrieval», Morgan & Claypool Publishers, 2008. (Chapter 2)



## 5 Vector Space Retrieval Model

VSM - Vector Space Model

### 5.1 Vector Space Model (VSM): Illustration

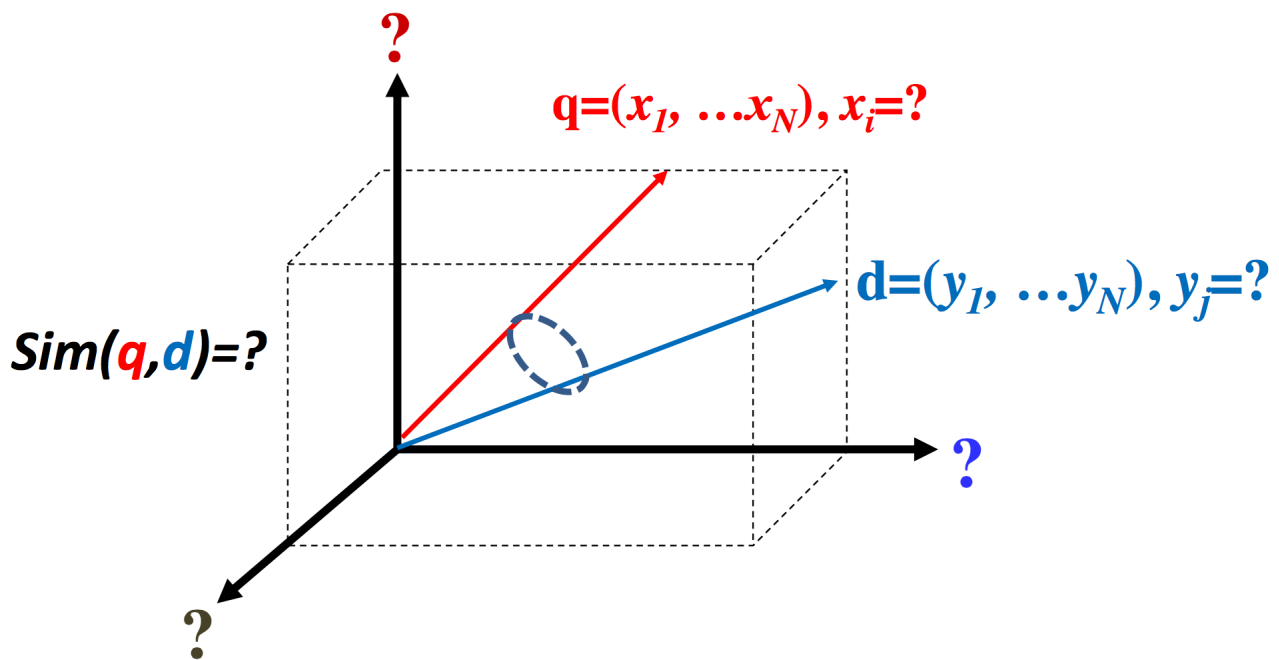


### 5.2 VSM Is a Framework

- Represent a doc/query by a term vector
  - **Term:** basic concept, e.g., word or phrase
  - Each term defines one dimension
  - N terms define an **N-dimensional space**
  - **Query vector:**  $q = (x_1, \dots, x_N)$ ,  $x_i \in \Re$  is query term weight
  - **Doc vector:**  $d = (y_1, \dots, y_N)$ ,  $y_j \in \Re$  is doc term weight
- $relevance(q, d) \propto similarity(q, d) = f(q, d)$

### 5.3 What VSM Doesn't Say

- How to define/select the “basic concept” – Concepts are assumed to be orthogonal
- How to place docs and query in the space (= how to assign term weights)
  - Term weight in query indicates importance of term
  - Term weight in doc indicates how well the term characterizes the doc
- How to define the similarity measure



#### 5.4 Simplest VSM = Bit-Vector + Dot-Product + BOW

$$q = (x_1, \dots, x_N) \quad x_i, y_i \in \{0, 1\}$$

$$d = (y_1, \dots, y_N) \quad \begin{array}{l} 1: \text{word } W_i \text{ is present} \\ 0: \text{word } W_i \text{ is absent} \end{array}$$

$$Sim(q, d) = q \cdot d = x_1 y_1 + \dots + x_N y_N = \sum_{i=1}^N x_i y_i$$

Simplest VSM:

- Dimension = word
- Vector = 0-1 bit vector (word presence/absence)
- Similarity = dot product
- $f(q, d)$  = number of distinct query words matched in d

#### 5.5 Improved Instantiation

Improved VSM:

- Dimension = word
- Vector = TF-IDF weight vector
- Similarity = dot product

## 5.6 Improved VSM with Term Frequency (TF) Weighting

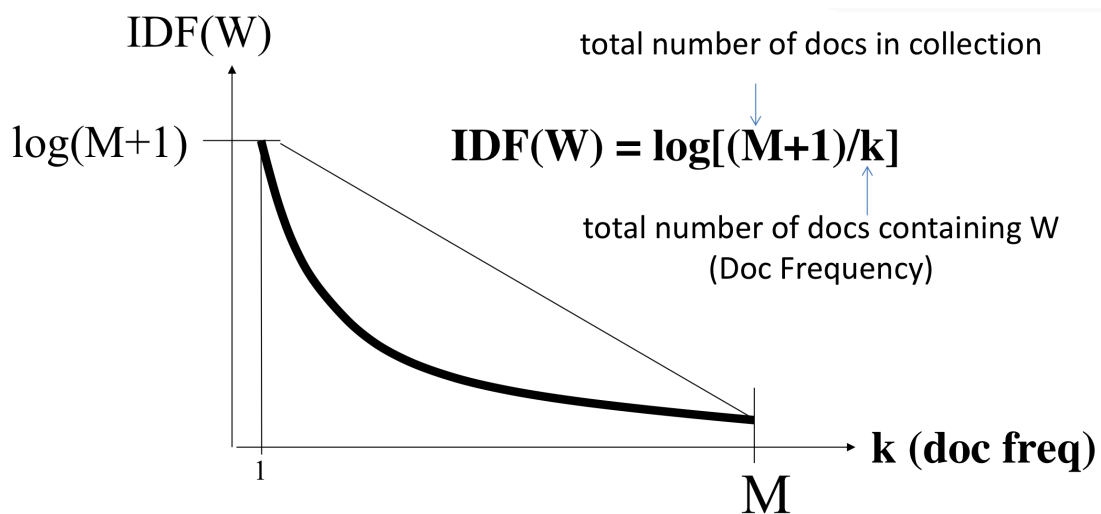
$$\mathbf{q} = (x_1, \dots, x_N) \quad \boxed{x_i = \text{count of word } W_i \text{ in query}}$$

$$\mathbf{d} = (y_1, \dots, y_N) \quad \boxed{y_i = \text{count of word } W_i \text{ in doc}}$$

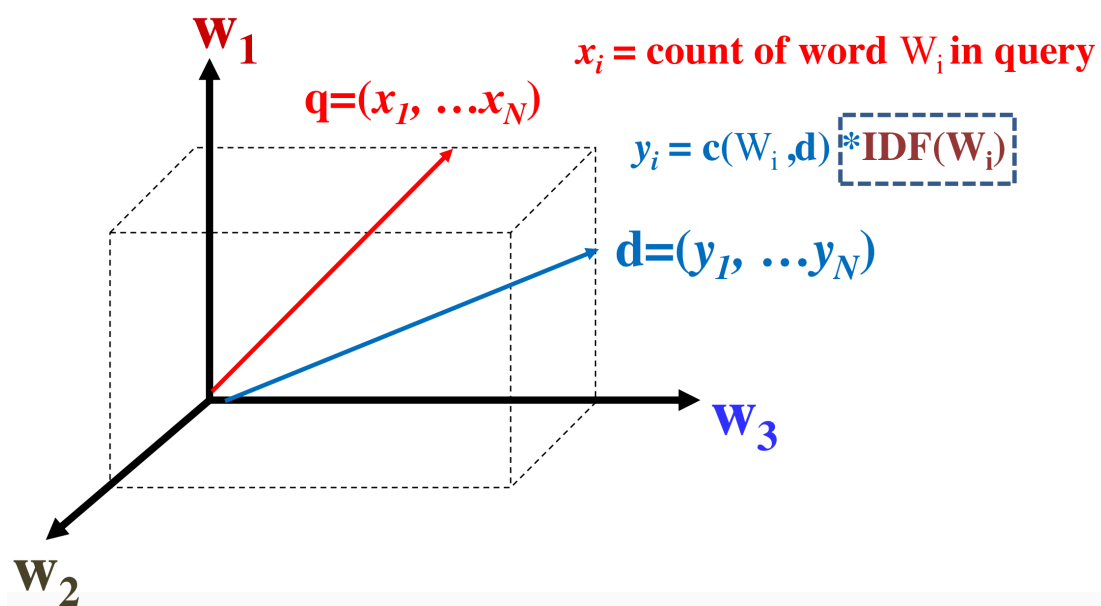
$$\text{Sim}(\mathbf{q}, \mathbf{d}) = \mathbf{q} \cdot \mathbf{d} = x_1 y_1 + \dots + x_N y_N = \sum_{i=1}^N x_i y_i$$

## 5.7 IDF Weighting: Penalizing Popular Terms

IDF — inverse document frequency



## 5.8 Adding Inverse Document Frequency (IDF)



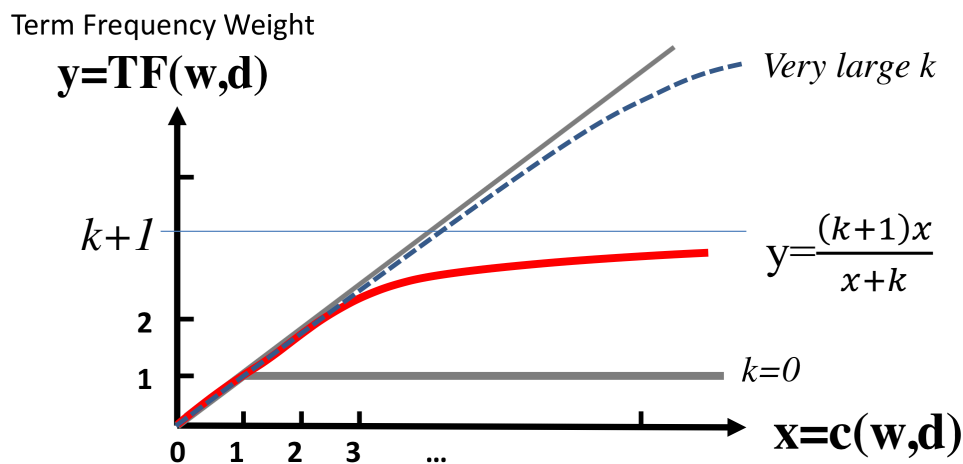
## 5.9 Ranking Function with TF-IDF Weighting

$$f(q, d) = \sum_{i=1}^N x_i y_i = \sum_{w \in q \cap d} c(w, q) c(w, d) \log \frac{M+1}{df(w)}$$

- $w \in q \cap d$  - all matched query (q) words in document (d)
- $c(w, q)$  - count of word w in document d
- $M$  - total number of documents in collection
- $df(w)$  - Doc Frequency (total number of documents containing word w)

## 5.10 TF Transformation: BM25 Transformation

BM = Best Matching



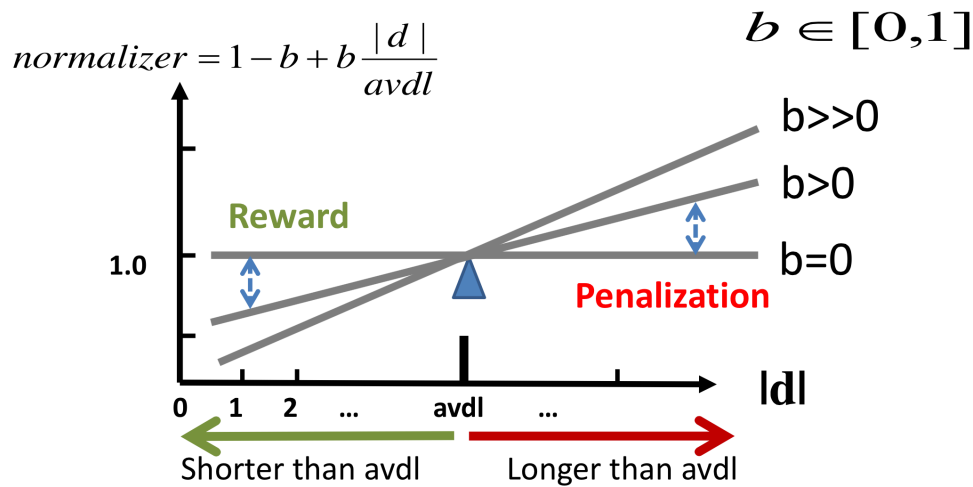
## 5.11 TF Transformation: summary

- Sublinear TF Transformation is needed to
  - capture the intuition of «diminishing return» from higher TF
  - avoid dominance by one single term over all others
- BM25 Transformation
  - has an upper bound
  - is robust and effective
- Ranking function with BM25 TF ( $k \geq 0$ ):

$$f(q, d) = \sum_{i=1}^N x_i y_i = \sum_{w \in q \cap d} c(w, q) \frac{(k+1)c(w, d)}{c(w, d) + k} \log \frac{M+1}{df(w)}$$

## 5.12 Pivoted Length Normalization

**Pivoted length normalizer:** use average doc length as «pivot»<sup>2</sup>. Normalizer = 1 if  $|d|$  = average doc length (avdl).



## 5.13 State of the Art VSM Ranking Functions

Pivoted Length Normalization VSM [Singhal et al 96]:

$$f(q, d) = \sum_{w \in q \cap d} c(w, q) \frac{\ln[1 + \ln(1 + c(w, d))]}{1 - b + b \frac{|d|}{avdl}} \log \frac{M + 1}{df(w)}$$

**BM25/Okapi** [Robertson & Walker 94]:

$$f(q, d) = \sum_{w \in q \cap d} c(w, q) \frac{(k + 1) c(w, d)}{c(w, d) + k \left(1 - b + b \frac{|d|}{avdl}\right)} \log \frac{M + 1}{df(w)}$$

## 5.14 Further Improvement of VSM?

- Improved instantiation of dimension?
  - stemmed words, stop word removal, phrases, latent semantic indexing (word clusters), character n-grams, ...
  - bag-of-words with phrases is often sufficient in practice
  - Language-specific and domain-specific tokenization is important to ensure “normalization of terms”
- Improved instantiation of similarity function?
  - cosine of angle between two vectors?
  - Euclidean?
  - dot product seems still the best (sufficiently general especially with appropriate term weighting)

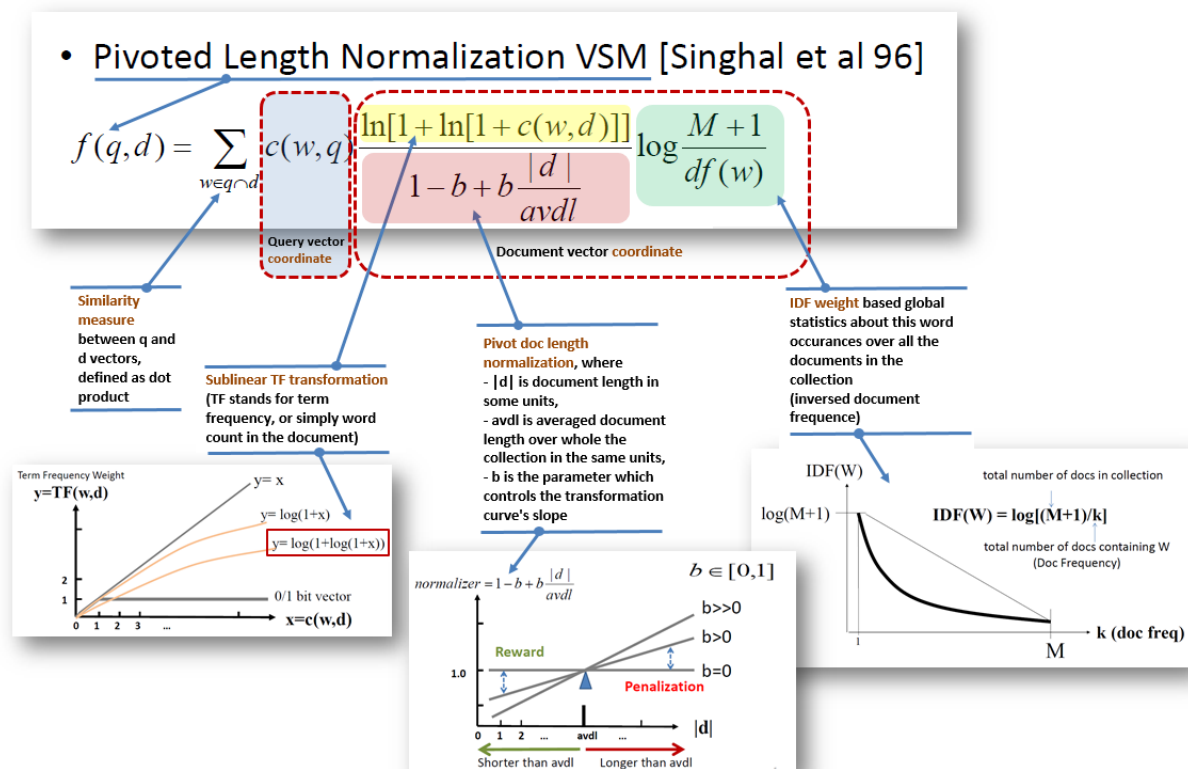
<sup>2</sup>Pivot - стержень; точка опоры, вращения

## 5.15 Further Improvement of BM25

- BM25F [Robertson & Zaragoza 09]
  - Use BM25 for documents with structures («F»=fields)
  - Key idea: combine the frequency counts of terms in all fields and then apply BM25 (instead of the other way)
- BM25+ [Lv & Zhai 11]
  - Address the problem of over penalization of long documents by BM25 by adding a small constant to TF
  - Empirically and analytically shown to be better than BM25

## 5.16 Summary of Vector Space Model

- $\text{Relevance}(q,d) = \text{similarity}(q,d)$
- Query and documents are represented as vectors
- Heuristic<sup>3</sup> design of ranking function
- Major term weighting heuristics
  - TF weighting and transformation
  - IDF weighting
  - Document length normalization
- BM25 and Pivoted normalization seem to be most effective



<sup>3</sup>Heuristic - эвристический

• BM25/Okapi [Robertson & Walker 94]

$$b \in [0,1]$$

$$k \in [0, +\infty)$$

$$f(q,d) = \sum_{w \in q \cap d} c(w,q) \cdot \frac{(k+1)c(w,d)}{c(w,d) + k(1-b + b \frac{|d|}{avdl})} \cdot \log \frac{M+1}{df(w)}$$

Query vector coordinate

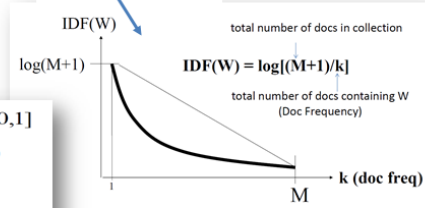
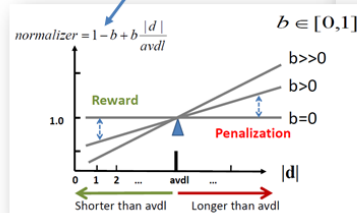
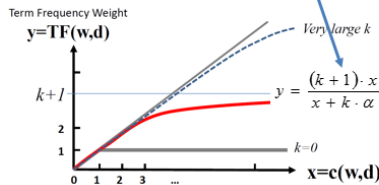
Document vector coordinate

Similarity measure between q and d vectors, defined as dot product

Sublinear TF transformation (TF stands for term frequency, or simply word count in the document)

Pivot doc length normalization, where  
-  $|d|$  is document length in some units,  
-  $avdl$  is averaged document length over whole the collection in the same units,  
-  $b$  is the parameter which controls the transformation curve's slope

IDF weight based global statistics about this word occurrences over all the documents in the collection (inversed document frequency)

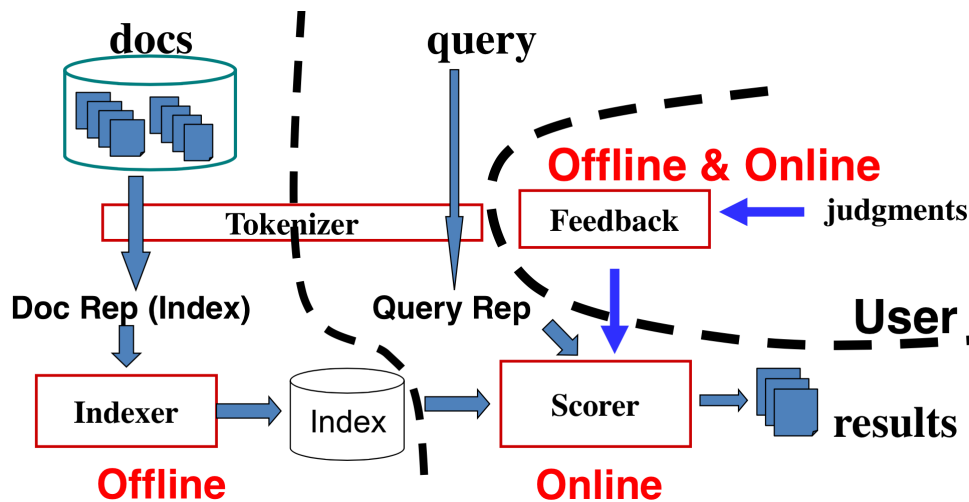


## 5.17 Recommended reading

- A.Singhal, C.Buckley, and M.Mitra. «Pivoted document length normalization». In Proceedings of ACM SIGIR 1996.
- S. E. Robertson and S. Walker. «Some simple effective approximations to the 2-Poisson model for probabilistic weighted retrieval», Proceedings of ACM SIGIR 1994.
- S. Robertson and H. Zaragoza. «The Probabilistic Relevance Framework: BM25 and Beyond», Found. Trends Inf. Retr. 3, 4 (April 2009).
- Y. Lv, C. Zhai, «Lower-bounding term frequency normalization». In Proceedings of ACM CIKM 2011.

## 6 Implementation of TR Systems

### 6.1 Typical TR System Architecture



### 6.2 Tokenization

- Normalize lexical units: words with similar meanings should be mapped to the same indexing term
- Stemming: mapping all inflectional forms of words to the same root form
- Some languages (e.g., Chinese) pose challenges in word segmentation

### 6.3 Inverted Index

Dictionary (or lexicon)			Postings		
Term	# docs	Total freq	Doc id	Freq	Position
news	3	3	1	1	p1
campaign	2	2	2	1	p2
presidential	1	2	3	1	p3
food	1	1	2	1	p4
...	...	...	3	1	p5
			3	2	p6,p7
			2	1	p8
			...	...	
			...	...	

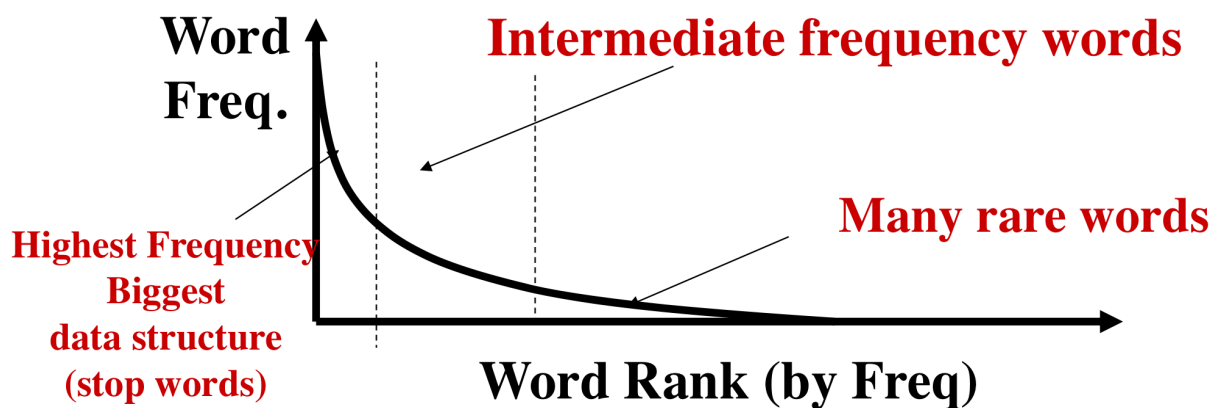
### 6.4 Empirical Distribution of Words

There are stable language-independent patterns in how people use natural languages:

- A few words occur very frequently; most occur rarely. E.g., in news articles:
  - Top 4 words: 10 15% word occurrences
  - Top 50 words: 35 40% word occurrences
- The most frequent word in one corpus may be rare in another



## 6.5 Zipf's Law



$$F(w) = \frac{C}{r(w)^\alpha}, \alpha \approx 1, C \approx 0.1$$

rank  $\times$  frequency  $\approx$  constant:

- $F(w)$  - word frequency
- $r(w)$  - word rank

## 6.6 Data Structures for Inverted Index

- Dictionary: modest size
  - Needs fast random access
  - Preferred to be in memory
  - Hash table, B-tree, trie, ...
- Postings: huge
  - Sequential access is expected
  - Can stay on disk
  - May contain docID, term freq., term pos, etc
  - Compression is desirable

## 6.7 Constructing Inverted Index

Sort-based method:

- Step 1: Collect local (termID, docID, freq) tuples from documents
- Step 2: Sort local tuples by termID (to make «runs») and save to files
- Step 3: Pair-wise merge runs
- Step 4: Output inverted file

## 6.8 Inverted Index Compression

In general, leverage skewed distribution of values and use variable-length encoding:

- TF compression:
  - Small numbers tend to occur far more frequently than large numbers (Zipf's law)
  - Fewer bits for small (high frequency) integers at the cost of more bits for large integers
- Doc ID compression:
  - «d-gap» (store difference):  $d_1, d_2 - d_1, d_3 - d_2, \dots$
  - Feasible due to sequential access

## 6.9 Integer Compression Methods

- **Binary**: equal-length coding
- **Unary**:  $x \geq 1$  is coded as  $x - 1$  one bits followed by 0, e.g.,  $3 \Rightarrow 110$ ;  $5 \Rightarrow 11110$
- **$\gamma$ -code**:  $x \Rightarrow$  unary code for  $1 + \lfloor \log x \rfloor$  followed by uniform code for  $x - 2^{\lfloor \log x \rfloor}$  in  $\lfloor \log x \rfloor$  bits, e.g.,  $3 \Rightarrow 101$ ,  $5 \Rightarrow 11001$
- **$\delta$ -code**: same as  $\gamma$ -code, but replace the unary prefix with  $\gamma$ -code. E.g.,  $3 \Rightarrow 1001$ ,  $5 \Rightarrow 10101$

## 6.10 General Form of Scoring Function

$$f(q, d) = f_a \left( h \left( g(t_1, d, q), \dots, g(t_k, d, q) \right), f_d(d), f_q(q) \right)$$

- $f_d(d), f_q(q)$  - adjustment factors of document and query
- $g(t_i, d, q)$  - weight of a **matched** query term  $t_i$  in  $d$
- $h()$  - weights aggregation function
- $f_a()$  - final score adjustment function

## 6.11 A General Algorithm for Ranking Documents

- $f_d(d)$  - can be precomputed at index time,  $f_q(q)$  - at query time
- Maintain a score accumulator for each  $d$  to compute  $h$
- For each query term  $t_i$ 
  - Fetch the inverted list  $\{(d_1, f_1), \dots, (d_n, f_n)\}$
  - For each entry  $(d_j, f_j)$ , compute  $g(t_i, d_j, q)$ , and update score accumulator for doc  $d_i$  to incrementally compute  $h$
- Adjust the score to compute  $f_a$ , and sort

## 6.12 Further Improving Efficiency

- Caching (e.g., query results, list of inverted index)
- Keep only the most promising accumulators
- Scaling up to the Web-scale? (need parallel processing)

## 6.13 Some Text Retrieval Toolkits

- [Lucene](#)
- [Lemur/Indri](#)
- [Terrier](#)
- [MeTA](#)
- More can be found [here](#)

## 6.14 Summary of System Implementation

- Inverted index and its construction
  - Preprocess data as much as we can
  - Compression when appropriate
- Fast search using inverted index
  - Exploit inverted index to accumulate scores for documents matching a query term
  - Exploit Zipf's law to avoid touching many documents not matching any query term
  - Can support a wide range of ranking algorithms
- Further scaling up using distributed file system, parallel processing, and caching

## 6.15 Recommended reading

- Ian H. Witten, Alistair Moffat, Timothy C. Bell: «Managing Gigabytes: Compressing and Indexing Documents and Images», Second Edition. Morgan Kaufmann, 1999.
- Stefan Büttcher, Charles L. A. Clarke, Gordon V. Cormack: «Information Retrieval - Implementing and Evaluating Search Engines». MIT Press, 2010.

## 7 Evaluation of Text Retrieval Systems

### 7.1 The Cranfield Evaluation Methodology

A methodology for laboratory testing of system components developed in 1960s. General idea is to build reusable test collections and define measures. A test collection can then be reused many times to compare different systems.

- A sample collection of documents (simulate real document collection)
- A sample set of queries/topics (simulate user queries)
- Relevance judgments (ideally made by users who formulated the queries) => ideal ranked list
- Measures to quantify how well a system's result matches the ideal ranked list

### 7.2 Evaluating a Set of Retrieved Docs

	Retrieved	Not Retrieved
Relevant	a	b
Not Relevant	c	d

- Precision: are the retrieved results all relevant?

$$Precision = \frac{a}{a + c}$$

- Recall: have all the relevant documents been retrieved?

$$Recall = \frac{a}{a + b}$$

- In reality, high recall tends to be associated with low precision

### 7.3 Combine Precision and Recall: F-Measure

$$F_{\beta} = \frac{1}{\frac{\beta^2}{\beta^2 + 1} \frac{1}{R} + \frac{1}{\beta^2 + 1} \frac{1}{P}} = \frac{(\beta^2 + 1) \cdot P \cdot R}{\beta^2 \cdot P + R}$$

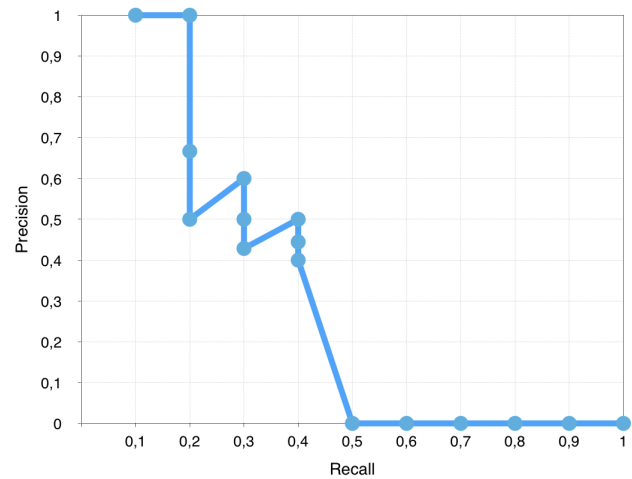
- $P$  - precision
- $R$  - recall
- $\beta$  - parameter, often set to 1:

$$F_1 = \frac{2 \cdot P \cdot R}{P + R}$$

### 7.4 Evaluating Ranking: Precision-Recall (PR) Curve

- Total number of relevant documents in collection:  $a + b = 10$
- Number of retrieved documents:  $a + c = 10$

Relevance	Precision	Recall
D1 +	1/1	1/10
D2 +	2/2	2/10
D3 -	2/3	2/10
D4 -	2/4	2/10
D5 +	3/5	3/10
D6 -	3/6	3/10
D7 -	3/7	3/10
D8 +	4/8	4/10
D9 -	4/9	4/10
D10 -	4/10	4/10



## 7.5 How to Summarize a Ranking

**Average Precision** is sensitive to the rank of each relevant document:

$$AveP = \frac{\sum_{k=1}^{a+c} P(k) \cdot rel(k)}{a+b} = \sum_{k=1}^{a+c} P(k) \cdot \Delta r(k)$$

- $a + c$  - number of retrieved documents
- $a + b$  - total number of relevant documents in collection
- $P(k)$  - the precision at cut-off  $k$  in the list
- $rel(k)$  - indicator function equaling 1 if the item at rank  $k$  is a relevant document, zero otherwise
- $\Delta r(k)$  - the change in recall that happened between cut-off  $k - 1$  and cut-off  $k$

In special case, when there's only one relevant document in the collection (e.g., known item search):

- Average Precision = **Reciprocal Rank** =  $1/r$ , where  $r$  is the rank position of the single relevant doc

## 7.6 Mean Average Precision (MAP)

In case of multiple queries:

- MAP = arithmetic mean of average precision over a set of queries

$$MAP = \frac{\sum_{q=1}^N AveP(q)}{N}, \text{ where } N \text{ is the number of queries}$$

- **gMAP** = geometric mean of average precision over a set of queries

$$gMAP = \sqrt[N]{\prod_{q=1}^N AveP(q)} = \exp \frac{\sum_{q=1}^N \log(AveP(q))}{N}$$

## 7.7 Summary on Average Precision

- Precision-Recall curve characterizes the overall accuracy of a ranked list
- The **actual** utility of a ranked list depends on how many top-ranked results a user would examine
- Average Precision is the standard measure for comparing two ranking methods
  - Combines precision and recall
  - Sensitive to the rank of **every** relevant document

## 7.8 Multi-level Relevance Judgments

**Discounted cumulative gain (DCG)** is a measure of ranking quality. Two assumptions are made in using DCG and its related measures:

- Highly relevant documents are more useful when appearing earlier in a search engine result list (have higher ranks)
- Highly relevant documents are more useful than marginally relevant documents, which are in turn more useful than irrelevant documents.

For a rank position  $p$ :

- **Cumulative Gain:**  $CG_p = \sum_{i=1}^p rel_i$ , where  $rel_i$  is the graded relevance of the result at position  $i$
- **Discounted Cumulative Gain:**  $DCG_p = rel_1 + \sum_{i=2}^p \frac{rel_i}{\log_2 i}$
- Alternative version of **Discounted Cumulative Gain:**  $DCG_p = \sum_{i=1}^p \frac{2^{rel_i} - 1}{\log_2(i + 1)}$
- **Normalized DCG:**  $nDCG_p = \frac{DCG_p}{IDCG_p}$ , where  $IDCG_p$  is an Ideal DCG (the maximum possible DCG till position  $p$ )

For example, each document is to be judged on a scale of 0-3 with 0 meaning irrelevant, 3 meaning completely relevant, and 1 and 2 meaning «somewhere in between»

Document	$rel_i$	$\frac{rel_i}{\log_2 i}$
D1	3	–
D2	2	2
D3	3	1.892
D4	0	0
D5	1	0.431
D6	2	0.774

- $DCG_6 = rel_1 + \sum_{i=2}^6 \frac{rel_i}{\log_2 i} = 3 + (2 + 1.892 + 0 + 0.431 + 0.774) = 8.10$
- $IDCG_6 = 8.69 \quad (rel_i = 3, 3, 2, 2, 1, 0)$
- $nDCG_6 = \frac{DCG_6}{IDCG_6} = \frac{8.10}{8.69} = 0.932$

## 7.9 Statistical Significance Tests

Query	System A	System B	Sign Test	Wilcoxon
1	0.02	0.76	+	+0.74
2	0.39	0.07	-	- 0.32
3	0.16	0.37	+	+0.21
4	0.58	0.21	-	- 0.37
5	0.04	0.02	-	- 0.02
6	0.09	0.91	+	+0.82
7	0.12	0.46	+	+0.34
Average	0.20	0.40	$p=1.0$	$p=0.9375$

**Нулевая гипотеза** - гипотеза об отсутствии взаимосвязи или корреляции между исследуемыми переменными, об отсутствии различий (однородности) в распределениях (параметрах распределений) двух и/или более выборках.

- $H_0$ : median difference between the pairs is zero
- $H_1$ : median difference is not zero.

### 7.9.1 Sign test

**Критерий знаков** используется при проверке нулевой гипотезы о равенстве медиан двух непрерывно распределенных случайных величин

Рассмотрим две непрерывно распределенные случайные величины  $X$  и  $Y$ , и пусть нулевая гипотеза выполняется, то есть их медианы равны. Тогда  $p = \mathbb{P}(X > Y) = 0.5$ . Иными словами, каждая из случайных величин равновероятно больше другой.

Рассмотрим пару связанных выборок  $\{(x_1, y_1), \dots, (x_n, y_n)\}$ . Будем считать, что в выборке нет элементов, для которых  $x_i = y_i$  (иначе уберем эти элементы из выборки). Построим статистику  $w$ , равную числу элементов в выборке, при которых  $x_i > y_i$ . При выполнении нулевой гипотезы, эта величина имеет биномиальное распределение:  $w \sim B(n, 0.5)$  с функцией вероятности

$$p_Y(k) \equiv \mathbb{P}(Y = k) = \binom{n}{k} p^k q^{n-k}, \quad k = 0, \dots, n,$$

где  $\binom{n}{k} = C_n^k = \frac{n!}{(n-k)! k!}$  — биномиальный коэффициент.

Для применения критерия необходимо вычислить «левый хвост» биномиального распределения до  $w$ :

$$b = 2^{-n} \sum_{i=0}^w \binom{n}{i}$$

Согласно критерию, при уровне значимости  $\alpha$ : если  $b \notin [\alpha/2, 1 - \alpha/2]$ , то нулевая гипотеза  $p \neq 0.5$  отвергается.

### 7.9.2 Wilcoxon signed-rank test

**The Wilcoxon signed-rank test** is a non-parametric statistical hypothesis test used when comparing two related samples or repeated measurements on a single sample to assess whether their population mean ranks differ.

Let  $N$  be the sample size, the number of pairs. Thus, there are a total of  $2N$  data points. For  $i = 1, \dots, N$ , let  $x_{1,i}$  and  $x_{2,i}$  denote the measurements.

- For  $i = 1, \dots, N$ , calculate  $|x_{2,i} - x_{1,i}|$  and  $\text{sign}(x_{2,i} - x_{1,i})$ .
- Exclude pairs with  $|x_{2,i} - x_{1,i}| = 0$ . Let  $N_r$  be the reduced sample size.
- Order the remaining  $N_r$  pairs from smallest absolute difference to largest absolute difference,  $|x_{2,i} - x_{1,i}|$ .
- Rank the pairs, starting with the smallest as 1. Ties receive a rank equal to the average of the ranks they span. Let  $R_i$  denote the rank.
- Calculate the test statistic  $W$ , the absolute value of the sum of the signed ranks:

$$W = \left| \sum_{i=1}^{N_r} [\text{sign}(x_{2,i} - x_{1,i}) \cdot R_i] \right|$$

- As  $N_r$  increases, the sampling distribution of  $W$  converges to a normal distribution. Thus,
  - For  $N_r \geq 10$ , a z-score can be calculated as  $z = \frac{W-0.5}{\sigma_W}$ ,  $\sigma_W = \sqrt{\frac{N_r(N_r+1)(2N_r+1)}{6}}$ . If  $z > z_{critical}$  then reject  $H_0$
  - For  $N_r < 10$ ,  $W$  is compared to a critical value from a reference table. If  $W \geq W_{critical, N_r}$  then reject  $H_0$

Example:

			$x_{2,i} - x_{1,i}$	
$i$	$x_{2,i}$	$x_{1,i}$	sgn	abs
1	125	110	1	15
2	115	122	-1	7
3	130	125	1	5
4	140	120	1	20
5	140	140		0
6	115	124	-1	9
7	140	123	1	17
8	125	137	-1	12
9	140	135	1	5
10	135	145	-1	10

order by absolute difference

			$x_{2,i} - x_{1,i}$			
$i$	$x_{2,i}$	$x_{1,i}$	sgn	abs	$R_i$	$\text{sgn} \cdot R_i$
5	140	140		0		
3	130	125	1	5	1.5	1.5
9	140	135	1	5	1.5	1.5
2	115	122	-1	7	3	-3
6	115	124	-1	9	4	-4
10	135	145	-1	10	5	-5
8	125	137	-1	12	6	-6
1	125	110	1	15	7	7
7	140	123	1	17	8	8
4	140	120	1	20	9	9

Notice that pairs 3 and 9 are tied in absolute value. They would be ranked 1 and 2, so each gets the average of those ranks, 1.5.

$$N_r = 10 - 1 = 9, W = |1.5 + 1.5 - 3 - 4 - 5 - 6 + 7 + 8 + 9| = 9$$

$$W < W_{\alpha=0.05,9} = 39 \therefore \text{fail to reject } H_0$$



## 7.10 Pooling: Avoid Judging all Documents

Pooling strategy:

- Choose a diverse set of ranking methods (TR systems)
- Have each to return top-K documents
- Combine all the top-K sets to form a pool for human assessors to judge
- Other (unjudged) documents are usually assumed to be non-relevant (though they don't have to)

Pooling strategy is okay for comparing systems that contributed to the pool, but problematic for evaluating new systems.

## 7.11 Summary of TR Evaluation

Evaluation is extremely important:

- TR is an empirically defined problem
- Inappropriate experiment design misguides research and applications
- Make sure to get it right for your research or application

Cranfield evaluation methodology is the main paradigm:

- MAP and nDCG: appropriate for comparing ranking algorithms
- Precision@10docs is easier to interpret from a user's perspective

Not covered:

- A-B Test [Sanderson 10]
- User studies [Kelly 09]

## 7.12 Recommended reading

- Donna Harman, «Information Retrieval Evaluation. Synthesis Lectures on Information Concepts, Retrieval, and Services», Morgan & Claypool Publishers 2011
- Mark Sanderson, «Test Collection Based Evaluation of Information Retrieval Systems». Foundations and Trends in Information Retrieval 4(4): 247-375 (2010)
- Diane Kelly, «Methods for Evaluating Interactive Information Retrieval Systems with Users». Foundations and Trends in Information Retrieval 3(1-2): 1-224 (2009)

## 8 Probabilistic Model

### 8.1 Basic Idea of Probabilistic Model

- Probabilistic models ranking function:

$$f(d, q) = p(R = 1 \mid d, q), \quad R \in \{0, 1\}$$

- **Query Likelihood:** if a user likes document  $d$ , how likely would the user enter query  $q$  (in order to retrieve  $d$ )?
- Assumption: a user formulates a query based on an «imaginary relevant document»:

$$p(R = 1 \mid d, q) \approx p(q \mid d, R = 1)$$

- Basic idea based on user clicks ( $R = 1$ ):

$$f(q, d) = p(R = 1 \mid d, q) = \frac{\text{count}(q, d, R = 1)}{\text{count}(q, d)}$$

- How to compute  $p(q \mid d)$ ? How to compute probability of text in general? → **Language Model**

### 8.2 Language Model

The term language model (LM) refers to a probabilistic model of text (i.e., it defines a probability distribution over sequences of words).

Uses of a Language Model:

- Representing topics
- Discovering word associations

#### 8.2.1 The Simplest Language Model: Unigram LM

Unigram Language Model = word distribution

- Generate text by generating each word **independently**
- Thus,  $p(w_1, w_2 \dots w_n) = p(w_1)p(w_2) \dots p(w_n)$
- Parameters:  $\{p(w_i)\} : p(w_1) + \dots + p(w_N) = 1$  ( $N$  is vocabulary size)
- Text = sample drawn according to this **word distribution**

Maximum Likelihood (ML) Estimator:

$$p(w \mid \theta) = p(w \mid d) = \frac{c(w, d)}{|d|}$$

- $\theta$  - document language model
- $c(w, d)$  - count on word  $w$  in document  $d$
- $|d|$  - length of document  $d$

### 8.2.2 Recommended reading

- Chris Manning and Hinrich Schütze, «Foundations of Statistical Natural Language Processing», MIT Press. Cambridge, MA: May 1999.
- Rosenfeld, R., «Two decades of statistical language modeling: where do we go from here?», Proceedings of the IEEE , vol.88, no.8, pp.1270,1278, Aug. 2000

## 8.3 Ranking based on Query Likelihood

- Query:  $q = w_1 w_2 \dots w_n$
- Vocabulary of the language of the documents:  $V = \{w_1, \dots, w_{|V|}\}$
- $c(w, q)$  - count of word  $w$  in query  $q$

How likely would we observe this query from this document model?

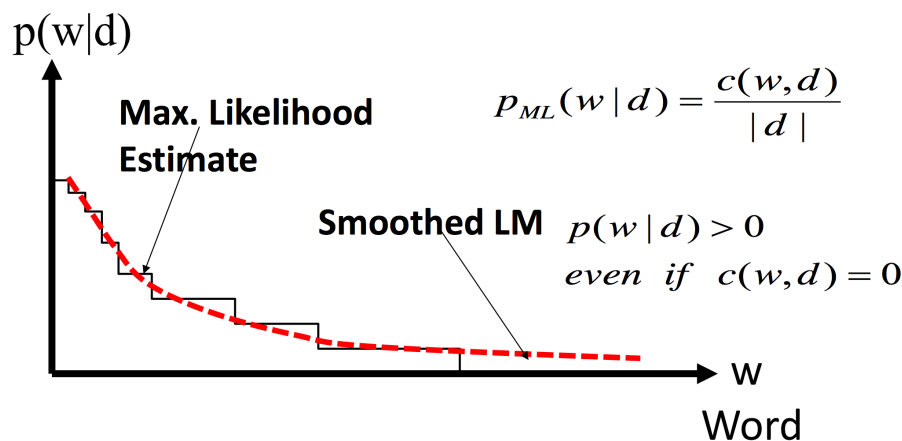
$$p(q | d) = p(w_1 | d) \times \dots \times p(w_n | d)$$

Retrieval problem  $\rightarrow$  estimation of  $p(w_i | d)$

$$f(q, d) = \log p(q | d) = \sum_{i=1}^n \log p(w_i | d) = \sum_{w \in V} c(w, q) \cdot \log p(w | d)$$

### 8.3.1 How to Estimate $p(w | d)$

Smoothing of  $p(w | d)$  is necessary for query likelihood:



Key question: what probability should be assigned to an unseen word? Let the probability of an unseen word be proportional to its probability given by a reference LM. One possibility: Reference LM = Collection LM:

$$p(w | d) = \begin{cases} p_{Seen}(w | d) & \text{Discounted ML estimate} \\ & \text{if } w \text{ is seen in } d \\ \alpha_d p(w | C) & \text{otherwise} \\ & \text{Collection language model} \end{cases}$$

### 8.3.2 Rewriting the Ranking Function with Smoothing

$$\begin{aligned}
 f(q, d) &= \sum_{w \in V} c(w, q) \log(p(w|d)) \\
 &= \sum_{\substack{w \in V, c(w, d) > 0 \\ \text{Word found in document}}} c(w, q) \log(\underbrace{p_{\text{seen}}(w|d)}_{\text{Modified } p(w|d)}) + \sum_{\substack{w \in V, c(w, d) = 0 \\ \text{Word NOT found in document}}} c(w, q) \log(\underbrace{\alpha_d p(w|C)}_{\text{Weighted probability of word in the collection}}) \\
 &= \sum_{w \in V} c(w, q) \log(\alpha_d p(w|C)) - \sum_{w \in V, c(w, d) > 0} c(w, q) \log(\alpha_d p(w|C)) + \sum_{w \in V, c(w, d) > 0} c(w, q) \log(p_{\text{seen}}(w|d)) \\
 &= \sum_{w \in V} c(w, q) \log(\alpha_d) + \sum_{w \in V} c(w, q) \log(p(w|C)) - \sum_{w \in V, c(w, d) > 0} c(w, q) \log(\alpha_d) + \sum_{w \in V, c(w, d) > 0} c(w, q) \log(p_{\text{seen}}(w|d)) \\
 &= \sum_{w \in V, c(w, d) > 0} c(w, q) \log\left(\frac{p_{\text{seen}}(w|d)}{\alpha_d p(w|C)}\right) + |q| \log(\alpha_d) + \sum_{w \in V} c(w, q) \log(p(w|C))
 \end{aligned}$$

The diagram illustrates the derivation of the smoothed ranking function. It starts with the initial formula  $f(q, d) = \sum_{w \in V} c(w, q) \log(p(w|d))$ . This is split into two parts: words found in the document and words not found. The second part is then rewritten using the collection probability  $p(w|C)$  and a smoothing factor  $\alpha_d$ . The final formula combines the log-likelihood of the seen words, the log of the smoothing factor, and the log-likelihood of the words in the collection.

Smoothing with  $p(w|d)$  leads to a general ranking formula for query likelihood with TF-IDF weighting and document length normalization:

$$\log p(q|d) = \sum_{\substack{w_i \in d \\ w_i \in q}} \left[ \log \frac{p_{\text{seen}}(w_i|d)}{\alpha_d p(w_i|C)} \right] + n \log \alpha_d + \sum_{i=1}^n \log p(w_i|C)$$

**TF weighting** points to the fraction  $\frac{p_{\text{seen}}(w_i|d)}{\alpha_d p(w_i|C)}$ .  
**Doc length normalization** points to  $n \log \alpha_d$ .  
**matched query terms** points to the summation over  $w_i \in q$ .  
**IDF weighting** points to  $p(w_i|C)$ .  
**Ignore for ranking terms** points to the final summation  $\sum_{i=1}^n \log p(w_i|C)$ .

### 8.3.3 Smoothing Methods

$$f(q, d) = \sum_{w_i \in d, w_i \in q} c(w_i, q) \log \frac{p_{\text{seen}}(w_i|d)}{\alpha_d p(w_i|C)} + n \log \alpha_d$$

## Jelinek-Mercer Smoothing

Jelinek-Mercer: Fixed coefficient linear interpolation

$$p_{seen}(w|d) = (1 - \lambda) \frac{c(w, d)}{|d|} + \lambda p(w|\mathcal{C}), \quad \alpha_d = \lambda, \quad \lambda \in [0, 1]$$

Ranking Function:

$$f_{JM}(q, d) = \sum_{w \in d, w \in q} c(w, q) \log \left( 1 + \frac{1 - \lambda}{\lambda} \frac{c(w, d)}{|d| p(w|\mathcal{C})} \right)$$

## Dirichlet Prior (Bayesian) Smoothing

Dirichlet Prior: Adding pseudo counts; adaptive interpolation

$$p_{seen}(w|d) = \frac{c(w, d) + \mu p(w|\mathcal{C})}{|d| + \mu} = \frac{|d|}{|d| + \mu} \cdot \frac{c(w, d)}{|d|} + \frac{\mu}{|d| + \mu} \cdot p(w|\mathcal{C})$$

$$\alpha_d = \frac{\mu}{|d| + \mu}, \quad \mu \in [0, +\infty)$$

Ranking Function:

$$f_{DIR}(q, d) = \sum_{w \in d, w \in q} c(w, q) \log \left( 1 + \frac{c(w, d)}{\mu p(w|\mathcal{C})} \right) + n \log \frac{\mu}{\mu + |d|}$$

### 8.3.4 Summary of Query Likelihood Probabilistic Model

- Effective ranking functions obtained using pure probabilistic modeling
  - Assumption 1:  $Relevance(q, d) = p(R = 1 | q, d) \approx p(q | d, R = 1) \approx p(q | d)$
  - Assumption 2: Query words are generated independently
  - Assumption 3: Smoothing with  $p(w|\mathcal{C})$
  - Assumption 4: JM or Dirichlet prior smoothing
- Less heuristic compared with VSM
- Many extensions have been made [Zhai 08]

### 8.3.5 Recommended reading

- ChengXiang Zhai, «Statistical Language Models for Information Retrieval» (Synthesis Lectures Series on Human Language Technologies), Morgan & Claypool Publishers, 2008.

## 9 Feedback in Text Retrieval

Feedback = learn from examples

### 9.1 Types of Feedback

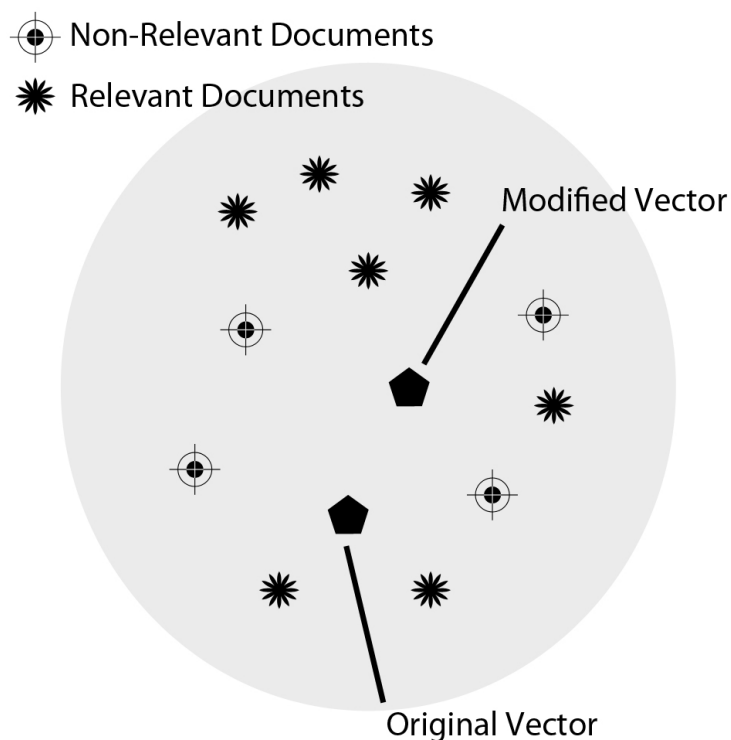
- **Relevance Feedback**  
Users make explicit relevance judgments on the initial results. Judgments are reliable, but users don't want to make extra effort.
- **Pseudo/Blind/Automatic Feedback.**  
Top-k initial results are simply assumed to be relevant. Judgments aren't reliable, but no user activity is required.
- **Implicit Feedback.**  
User-clicked docs are assumed to be relevant; skipped ones non-relevant. Judgments aren't completely reliable, but no extra effort from users.

### 9.2 Feedback in Vector Space Model

How can a TR system learn from examples to improve retrieval accuracy? General method - query modification:

- Adding new (weighted) terms (query expansion)
- Adjusting weights of old terms

#### 9.2.1 Rocchio Feedback



$$\overrightarrow{Q_m} = (\alpha \cdot \overrightarrow{Q_o}) + \left( \frac{\beta}{|D_r|} \cdot \sum_{\overrightarrow{D_j} \in D_r} \overrightarrow{D_j} \right) - \left( \frac{\gamma}{|D_{nr}|} \cdot \sum_{\overrightarrow{D_k} \in D_{nr}} \overrightarrow{D_k} \right)$$

- $\overrightarrow{Q_m}$  - Modified Query Vector
- $\overrightarrow{Q_o}$  - Original Query Vector
- $\overrightarrow{D_j}$  - Related Document Vector
- $\overrightarrow{D_k}$  - Non-Related Document Vector
- $\alpha$  - Original Query Weight
- $\beta$  - Related Documents Weight
- $\gamma$  - Non-Related Documents Weight
- $D_r$  - Set of Related Documents
- $D_{nr}$  - Set of Non-Related Documents

### 9.2.2 Rocchio in Practice

- Negative (non-relevant) examples are not very important
- Often truncate the vector (i.e., consider only a small number of words that have highest weights in the centroid vector) (efficiency concern)
- Avoid «over-fitting» (keep relatively high weight on the original query weights)
- Can be used for relevance feedback and pseudo feedback ( $\beta$  should be set to a larger value for relevance feedback than for pseudo feedback)
- Usually robust and effective

## 9.3 Feedback in Language Model

### 9.3.1 The KL-divergence measure

Given two probability mass functions  $p(x)$  and  $q(x)$ , the Kullback-Leibler divergence (or relative entropy) between  $p$  and  $q$  is defined as:

$$D(p \parallel q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

### 9.3.2 Kullback-Leibler (KL) Divergence Retrieval Model

KL-divergence (cross entropy):

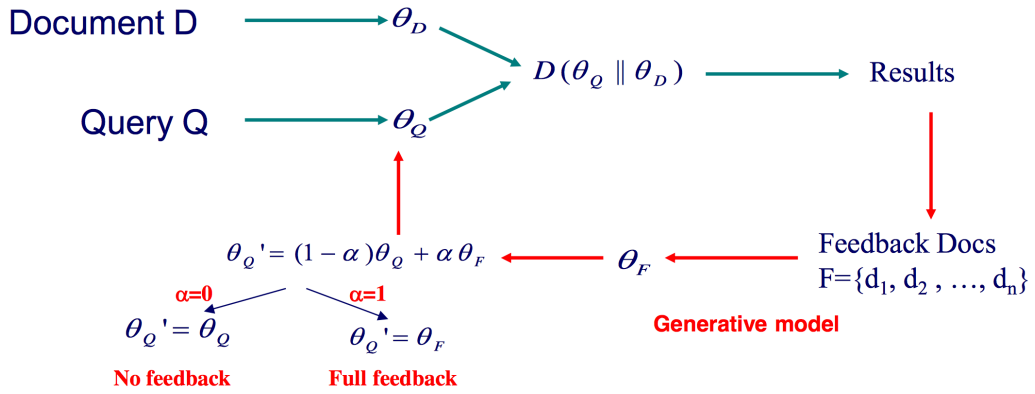
$$f(q, d) = \sum_{\substack{w \in d, \\ p(w|\hat{\theta}_Q) > 0}} p(w | \hat{\theta}_Q) \log \frac{p_{seen}(w_i | d)}{\alpha_d p(w_i | \mathcal{C})} + \log \alpha_d$$

- $\hat{\theta}_Q$  - estimated query unigram language model

If query language model is maximum likelihood  $p_{ml}(w | \hat{\theta}_Q) = \frac{c(w, q)}{|q|}$ , then KL-divergence becomes a query likelihood retrieval formula:

$$f(q, d) = \sum_{\substack{w_i \in d, \\ w_i \in q}} c(w_i, q) \log \frac{p_{seen}(w_i | d)}{\alpha_d p(w_i | \mathcal{C})} + n \log \alpha_d$$

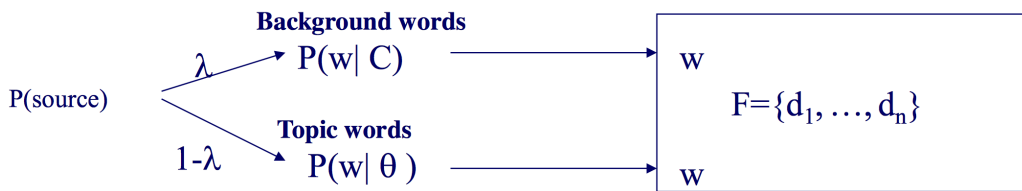
### 9.3.3 Feedback as Model Interpolation



$$p(w | \hat{\theta}_Q) = (1 - \alpha) p_{ml}(w | \hat{\theta}_Q) + \alpha p(w | \theta_F)$$

- $\alpha$  is a parameter that needs to be set empirically
- $p(w | \theta_F)$  is a feedback model

### 9.3.4 Generative Mixture Model



$$\log p(\mathcal{F} | \theta_F) = \sum_{i=1}^n \sum_w c(w, d_i) \log \left( (1 - \lambda) p(w | \theta_F) + \lambda p(w | \mathcal{C}) \right)$$

- $F = \{d_1, \dots, d_n\}$  - the set of feedback documents
- $\lambda$  - parameter that indicates the amount of «background noise» in the feedback documents

Maximum likelihood estimate of  $\hat{\theta}_F$ :

$$\hat{\theta}_F = \underset{\theta_F}{\operatorname{argmax}} \log p(\mathcal{F} | \theta_F)$$



## 9.4 Recommended reading

- ChengXiang Zhai, «Statistical Language Models for Information Retrieval» (Synthesis Lectures Series on Human Language Technologies), Morgan & Claypool Publishers, 2008.
- Victor Lavrenko and W. Bruce Croft. 2001. «Relevance based language models». In Proceedings of ACM SIGIR 2001.