

Obtaining Feasible Solution and Optimal Load Shedding in Contingency - Constrained Unit Commitment Joint With Reserve Auction

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Abstract—This paper focuses on finding the feasible and the optimal solution of security constraint unit commitment problem. The proposed day-ahead algorithm clears actual power and reserve auction simultaneously and obtains feasible solution in contingency case. In equivalent linear expression of the problem, shedding costs are used to avoid divergence and resolve congestion problem. The new terms of inequalities, which are introduced to avoid infeasibility, leads problem to obtain optimal solution. Mixed Integer Linear Programming (MILP) is used to minimize the total energy dispatch cost in 24 hours of a day. Numerical examples show that the proposed joint auction model can clear energy and ancillary services bids simultaneously while minimizing the total operating cost and satisfying transmission security constraints.

Keywords—Feasibility; Reserve power auction; Contingency; Shedding costs; Mixed Integer Linear Programming (MILP)

NOMENCLATURE

Indices:

i, j	Index for buses
m	Index for contingency
t	Index for time
α	Index for reserve interval

Variables:

$I(i, t)$	Commitment state of unit i at time t .
$P(i, t)$	Generation of unit i at time t in base case.
$P_{\alpha}^m(i, t)$	Generation of unit i at time t for reserve interval in contingency m .
$R^m(i, t)$	Called reserve of unit i at time t in contingency m .
$RD_{sys}(t)$	System ramp-down at time t .
$RU_{sys}(t)$	System ramp-up at time t .
$P_i(i, j, t)$	Power transmitted from bus i to j at time t .
$SD(i, t)$	Shutdown cost for unit i at time t .
$SU(i, t)$	Startup cost for unit i at time t .
$y(i, t)$	Binary unit startup indicator for unit i at time t .

$z(i, t)$	Binary unit shutdown indicator for unit i at time t .
$\delta P_d^m(i, t)$	shedding amount of bus i at time t in contingency m .
$\delta(i, t)$	angle of bus i at time t .

Functions:

$F_{c,i}$	Bid-based energy cost function for unit i .
$F_{R,i}$	Bid-based reserve cost function for unit i .
$F_{S,i}$	Bid-based shedding cost function for bus i .

Constants:

DR_i	Ramp-down rate limit of unit i .
N_d	Number of demand buses.
N_g	Number of units.
N_t	Number of periods under study (24 h).
P_i^{\max}	Upper limit of real power generation for unit i .
P_i^{\min}	Lower limit of real power generation for unit i .
$P_D(t)$	System load at time t .
$P_d(i, t)$	Load of bus i at time t .
$P_{l^0}(i, j)$	Lower limit of transmitted power from bus i to j .
$P_{u^0}(i, j)$	Upper limit of transmitted power from bus i to j .
R_i^{\max}	Upper limit of reserve generation for unit i .
RS_i	Both startup and shut down ramp rates.
UR_i	Ramp-up rate limit of unit i .
δ_{\max}^i	Upper limit of i^{th} bus angle.
δ_{\min}^i	Lower limit of i^{th} bus angle.

I. INTRODUCTION

Security Constraint Unit Commitment (SCUC) is one of the most unique optimization problems. It determines an optimal schedule of power generation units and tries to find the minimum generation cost. In recent years, considering all characteristics of electrical auction in several countries, Unit Commitment (UC) can be defined as the problem of finding the best economic and strategic way to balance power equations and satisfy other constraints. This objective can be defined with different formulations, but it always looks for minimum cost. As a point of mathematical view, SCUC is a non-convex, nonlinear and mixed-integer problem which can be defined as mixed integer linear programming (MILP). 24-hour markets can be modified differently, but pay as bid energy market is one of the most popular one. This structure has been settled in several countries; however, it might have some weaknesses.

Due to the nature of SCUC matter, during last these years, several combinatorial and nonlinear optimization techniques have been employed to solve SCUC problem, e.g. heuristics [1], Mixed-Integer Programming (MIP) [2], Lagrangian Relaxation (LR) [3], Benders Decomposition (BD) [4], Branch and Bound (BB) [5], Dynamic Programming (DP) [6, 7], and etc.

The hourly scheduling of UC while considering contingencies was addressed in [8]. The solution enables the real-time economic dispatch to meet contingency constraints while satisfying physical unit constraints. The proposed model introduces a co-optimization algorithm which simultaneously clears energy and ancillary services bids. The pricing algorithm in [8] and [9] considers a pre-defined ancillary services margin, which is usually a certain percentage of the system load or the largest unit capacity in the system.

In [10], benders decomposition approach is employed to solve Contingency Constrained Unit Commitment (CCUC) problem joint with ancillary service market. However, in benders decomposition approach for each violation which may result infeasible solution, new cuts will be added to master problem to avoid infeasibility. This approach can be time consuming and in some cases would be inconclusive.

This paper expands the idea developed in [8], [9] and [10] for the deployment of ancillary services while maintaining the system security. While system security is guaranteed, violations may be transpired which needs to be handled. In this paper, a linear expression for SCUC problem is obtained and a 0/1 mixed integer linear programming is used to solve the problem. The proposed algorithm handles emergency and violation situation in SCUC problem and obtains optimal and feasible solution to this problem.

The rest of this paper is organized as follows. Section II presents the problem formulation. Section III explains the solution methodology. Section IV provides the numerical examples. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

The objective function (1) is to minimize cost and power generation of units. The constant startup and shut down cost of each unit can be modified by, $SU(i, t) = SU_i \times y(i, t)$ and $SD(i, t) = SD_i \times z(i, t)$, respectively. It should be noted that reserve can be conveyed as spinning reserve and operating reserve [10]; however, hourly unit commitment formulation may not be able to separate these two types. Nevertheless, reserve power, which is determined in auction problem, is considered as upward reserve and downward reserve in formulation (4). It is assumed that upward and downward reserve bids are the same in 24 hours. These bids are determined daily for each generator and actual reserve should be limited with upward and downward reserve. Note that $m=0$ means base case with no contingency transpired.

The hourly unit commitment constraints include power balance equation (2) at base case with no contingency, desired reserve requirements with load-generation balance equation (3), power generation and reserve limits (4), unit generation ramp rates (5)-(6), system ramp rates as nonlinear constraints (7)-(8), minimum up time and down time constraints (9), and network constraints (10). In (3), Δ^m is desired reserve which determines required reserves in unit scheduling. However, reserve requirements scheduling expresses by (4) in which $R_u(i, t)$ and $R_d(i, t)$ are upward and downward reserves that are awarded at the base case. Their values are determined by the solution of the auction problem. $R^m(i, t)$ is the actual reserve that is to be called if contingency of unit m occurs. Constraints (7) – (9) are nonlinear and should be linearized in order to employ MILP methods. Linearization technique for constraints (7) – (9) have been expressed in [11] and [12]. Generalized network constraints (10) can be conveyed as DC or AC. In this paper, DC network constraints are considered in order to avoid nonlinear formulation. DC network power flow equations have been used to specify load shedding, obtaining feasible solution and handle lines congestion. Generalized network constraints (10) can neglect reserve outputs during emergency because the transmission line capacity can be overloaded for a short period of time when there are multiple capacity ratings for a transmission line. However; line congestion may avoid MILP to reach an optimal and feasible solution and line capacities cannot be overloaded for several hours; therefore, the solution procedure to this point of view will be explained in Section III.

$$\begin{aligned} \min \sum_{i=1}^{N_g} \sum_{t=1}^{N_t} [& F_{c,i}(P(i, t)) + F_{R,i}(R_u(i, t)) \\ & + F_{R,i}(R_d(i, t)) + SU(i, t) \\ & + SD(i, t) + F_{S,i}(\delta P_d(i, t))] \end{aligned} \quad (1)$$

$$\sum_{i=1}^{N_g} P(i, t) \geq P_D(t) \quad (2)$$

$$\sum_{i=1}^{N_g} P_{\alpha}^m(i, t) = P_D(t) + \Delta^m \quad (3-a)$$

$$P_{\alpha}^m(i, t) = P(i, t) + R^m(i, t) \quad (3-b)$$

$$P(i, t) + R_u(i, t) \leq P_i^{\max} I(i, t)$$

$$P(i, t) - R_d(i, t) \geq P_i^{\min} I(i, t)$$

$$0 \leq R_u(i, t) \leq R_i^{\max}$$

$$0 \leq R_d(i, t) \leq R_i^{\max}$$

$$-R_d(i, t) \leq R^m(i, t) \leq R_u(i, t) \quad (4)$$

$$P(i, t) - P(i, t-1) \leq UR_i$$

$$P(i, t-1) - P(i, t) \leq DR_i \quad (5)$$

$$P_{\alpha}^m(i, t) - P_{\alpha}^m(i, t-1) \leq UR_i$$

$$P_{\alpha}^m(i, t-1) - P_{\alpha}^m(i, t) \leq DR_i \quad (6)$$

$$RU_{gs}(t) = \sum_{i=1}^{N_g} [UR_i + P_i^{\min} - RS_i] I(t-1) I(t) I(t+1)$$

$$+ RS_i I(t) - P_i^{\min} I(t-1)]$$

$$RU_{gs}(t) \geq P_D(t) - P_D(t-1) \quad (7)$$

$$RD_{gs}(t) = \sum_{i=1}^{N_g} [DR_i + P_i^{\min} - RS_i] I(t-1) I(t) I(t+1)$$

$$+ RS_i I(t-1) - P_i^{\min} I(t)] \quad (8)$$

$$RD_{gs}(t) \geq P_D(t-1) - P_D(t)$$

$$[X^{on}(i, t-1) - T_i^{on}] * [I(i, t-1) - I(i, t)] \geq 0 \quad (9)$$

$$[X^{off}(i, t-1) - T_i^{off}] * [I(i, t) - I(i, t-1)] \geq 0$$

$$G(P_{\alpha}^m(i, t)) \leq 0 \quad (10)$$

III. Solution Procedure to Find a Feasible Solution and Optimal Load Shedding Values

SCUC problem is about minimizing the operation cost to meet the network demand and guarantee the system reliability. The suggested approach uses linearized power flow equations to resolve convergence difficulty. This convergence difficulty may be caused by power generation and transmission lines capacities limitations. The presented modeling proposes a new approach to avoid divergence in optimization problem.

Sometimes, in a load bus, local regional demand cannot be supplied. This lack of supply may cause by three reasons: 1) transmission lines limitations, 2) system demand is more than generations, and 3) ramp rates constraints. Ramp rates violation may cause by sudden hourly demand changes, these changes may be caused by rapid load increase or sudden load decrease; therefore, ramp rates constraints would not allow generating units to meet load-generation balance equation. Note that in most cases, each of mentioned reasons is caused by unit contingencies. Generalized network constraints (10) in DC formulation are needed to explain the procedure; so, linearized power flow formulation can be written as:

$$P_{\alpha}^m(i, t) - P_d(i, t) = \sum_{i \neq j} B(i, j) [\delta(i, t) - \delta(j, t)] \quad (11)$$

$$\delta_{\min}^i \leq \delta(i, t) \leq \delta_{\max}^i$$

$$P_L(i, j, t) = B(i, j) \times [\delta(i, t) - \delta(j, t)]$$

$$-P_{\alpha}^m(i, j) \leq P_L(i, j, t) \leq P_{\alpha}^m(i, j) \quad (12)$$

Once any of these constraints (11) – (12) violated, infeasible solution may be found. Each bus angle must be limited with 1 and -1 (in radians, in per unit calculation). Not only these constraints (11)-(12), consider power flow in DC and linearized form, but also check the whole system load-generation balance equations. Note that in (12), $B(i, j)$ is $-Y_{bus}(i, j) / \sqrt{-1}$ which is calculated by lines reactance ($X(i, j)$) in per unit [13].

In order to find a feasible solution, in DC network constraints (11) – (12), a new positive variable δP_{α}^m is introduced in (13). Consider objective function (1), on the right hand side of minimizing object, this new variable is added to cost terms. This new variable is named the amount of load shedding and its cost is conveyed as load shedding cost. So, in order to importance of load shedding, each load bus shedding can be weighted. Notice that these cost weights must be extremely expensive to avoid shedding. However, objective function is looking for minimum and optimum cost, which should obtain feasible solution, zero shedding in normal situation (no violation) must be achieved.

Replacing (13-14) by (11), in case of infeasibility for mentioned reasons, each bus load shedding cost and feasible solution are found.

$$\delta P_d^m(i, t) = P_d(i, t) + \sum_{i \neq j} B(i, j) [\delta(i) - \delta(j)] - P_a^m(i, t) \quad (13)$$

$$\delta P_d^m(i, t) \leq P_d(i, t) \quad (14)$$

Finally, load-generation balance equation (2)-(3-a) should be replaced by:

$$\sum_{i=1}^{N_g} P(i, t) \geq P_D(t) - \sum_{i=1}^{N_d} \delta P_d(i, t) \quad (15-a)$$

$$\sum_{i=1}^{N_g} P_a^m(i, t) = P_D(t) + \Delta^m - \sum_{i=1}^{N_d} \delta P_d^m(i, t) \quad (15-b)$$

In this paper, unit contingency assumed to be single in each day. It should be noted that the specific unit will not be able to commit again until the end of the day (24th hour). Therefore, temporary overloading of lines assumption is not true and lines limitations with reserves generating should be considered as (13) – (14). As soon as a unit contingency transpired, reserves auction will commit to the problem anxiously to fill in the lost generated power, but in some cases of contingency the required power may not provide by neither other units real power nor reserves. In these cases some loads must be excluded to find a feasible solution; therefore, Constraints (13)-(15), specify the amount of shedding cost and guide the problem to avoid infeasibility. This feasibility is synchronized by finding and optimal solution, units schedule, and optimal shedding by all means of unit commitment problem.

IV. TEST SYSTEM AND NUMERICAL RESULTS

IEEE 24-bus modified system (Fig.1) is employed to specify the results. Table I shows units characteristics. Table II is network data. Table III indicates bid prices of active power, startup costs, shutdown costs, and reserves bids of each unit. Table IV addresses the system power demand in each hour. Table V present weighted shedding costs of each bus.

Matlab and GAMS joint programming are used to solve this problem. CPLX 12.5 in GAMS mathematical modeling language is employed to solve MILP programming.

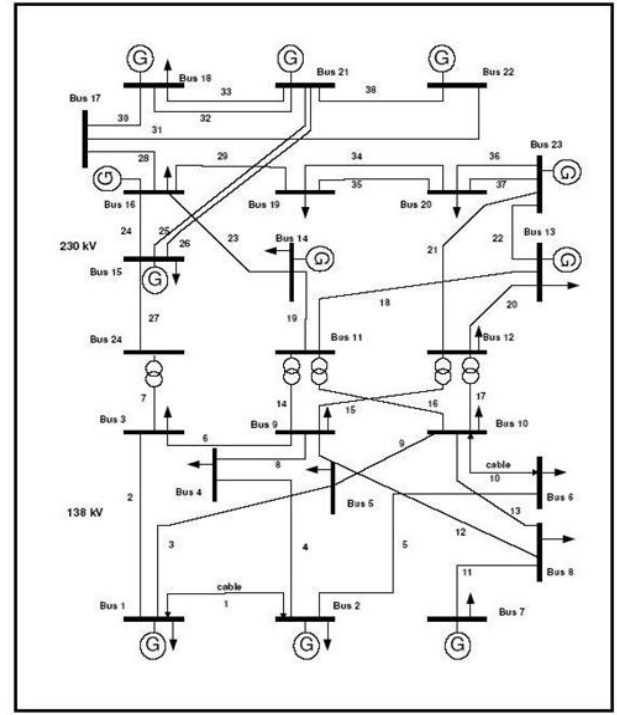


Figure1. IEEE 24 bus test system one line diagram

TABLE I FUTURES OF GENERATING UNITS

Units	Bus No.	P _{max} (MW)	P _{min} (MW)	Ini.State (h)	Min on (h)	Min off (h)	Ramp (MW/h)	Reserve _{max} (MW)
G1	1	200	50	-2	2	-4	50	9.2
G2	2	200	20	2	4	-3	70	15.1
G3	7	250	70	-4	2	-3	100	3.3
G4	13	280	120	3	4	-2	80	20
G5	14	10	5	2	1	-1	5	17
G6	15	230	110	-3	3	-2	110	4
G7	16	180	50	5	2	-4	55	3.5
G8	18	450	100	2	5	-4	120	16
G9	21	450	150	3	5	-6	170	17
G10	22	300	80	-2	3	-5	100	20
G11	23	200	70	-2	2	-2	110	18

TABLE II TRANSMISSION LINES DATA

Line number	From	to	X (p.u.)	line capacity (i to j)	line capacity (j to i)
1	1	2	0.014	200	120
2	1	3	0.211	100	70
3	1	5	0.085	90	200
4	2	4	0.127	150	220
5	2	6	0.192	200	150
6	3	9	0.119	100	120
7	3	24	0.084	170	140
8	4	9	0.104	180	170
9	5	10	0.088	220	200
10	6	10	0.061	130	160
11	7	8	0.061	140	180
12	8	9	0.165	150	190
13	8	10	0.165	230	250
14	9	11	0.084	170	200

15	9	12	0.084	180	210
16	10	11	0.084	220	100
17	10	12	0.084	130	170
18	11	13	0.048	100	120
19	11	14	0.042	150	115
20	12	13	0.048	130	110
21	12	23	0.097	350	300
22	13	23	0.087	180	100
23	14	16	0.059	300	270
24	15	16	0.017	260	300
25	15	21	0.049	250	280
26	15	21	0.049	300	270
27	15	24	0.052	270	300
28	16	17	0.026	180	150
29	16	19	0.023	100	50
30	17	18	0.014	220	200
31	17	22	0.105	350	300
32	18	21	0.026	200	250
33	18	21	0.026	150	250
34	19	20	0.04	30	50
35	19	20	0.04	20	30
36	20	23	0.022	180	150
37	20	23	0.022	80	170
38	21	22	0.068	200	250

TABLE III UNITS BIDDING DATA

Units	Active power bids (\$/MWh)					Reserve bids (\$/MWh)	startup (\$/SU _i)	Shut down (\$/SD _i)
	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$			
G1	13	12	12	8	7	1.2	12	17
G2	12	12	13	14	17	1.4	4	6
G3	15	13	7	5	4	3.05	12	18
G4	13	12	10	10	12	5.2	19	11
G5	11	12	10	11	14	1.7	17	13
G6	13	11	13	12	15	3.2	18	15
G7	12	10	12	11	8	6.8	11	14
G8	13	13	11	10	13	1.9	23	22
G9	14	12	9	11	5	5.5	14	15
G10	11	10	12	13	12	3.2	10	7
G11	11	10	8	4	13	1.4	17	15

TABLE IV HOURLY SYSTEM DEMAND

Hours	1	2	3	4	5	6	7	8
Demand (10 ² *MW)	7	8	9	11	13	13.5	14	15.5
Hours	9	10	11	12	13	14	15	16
Demand (10 ² *MW)	17	20	30	32	29	25	24.5	16
Hours	17	18	19	20	21	22	23	24
Demand (10 ² *MW)	14.5	15	14	11	11.5	12	9	7

TABLE V. SHEDDING COST WEIGHTS

Bus No.	1	2	3	4	5	6	7	8
Shedding Cost (10 ⁶ *\$/MWh)	25	13	1	16	20	21	30	39
Bus No.	9	10	11	12	13	14	15	16
Shedding Cost*1e6 (10 ⁶ *\$/MWh)	30	26	0	0	24	23	26	29
Bus No.	17	18	19	20	21	22	23	24
Shedding Cost (10 ⁶ *\$/MWh)	29	25	50	25	0	0	0	0

A. Base case ($m=0$)

At base case with no contingency constraints (1) – (10) is taken into mathematical programming. 1% of whole system load is considered for desired reserve in this case. However, in order to avoid infeasibility for mentioned reasons in Section III, generalized network constraints (10) are replaced by (13) – (15).

According to Table V, the cheapest shedding cost is bus 3. Shedding amount in Fig.2 proves that shedding weights specify the exclaimed demands. It should be noted that in hours 11 to 13, the whole demand power of the system is more than the maximum summation of generation units; so, shedding must be transpired to avoid infeasibility. The optimum shedding cost is obtained according to Table V by shedding weights (consider Section III formulation). The same explanations would be conveyed for lines congestions. Consider bus 19 for instance (Fig.3), lines capacities connected to this bus do not allow generation units to provide the demand in hours 8 to 17. Although in some hours whole generation would not provide the demand, but at 9th hour generations can supply the demands equally; however, lines limitations do not allow generations to supply the demand (Table II). Fig.3 shows demands and imposed shedding of this bus. Sometimes shedding must be imposed to avoid buses angle violations, too. These load losses are calculated with the same explained procedure.

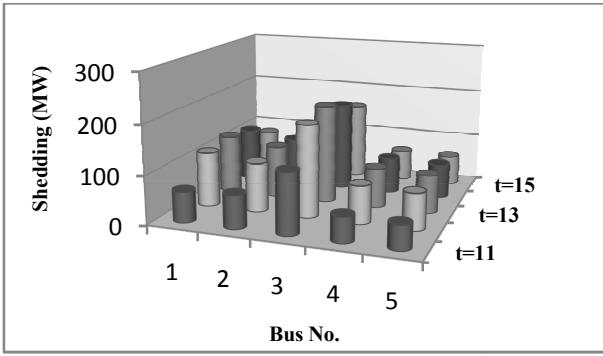


Figure2. Shedding amounts of 5 buses in hours 11-15

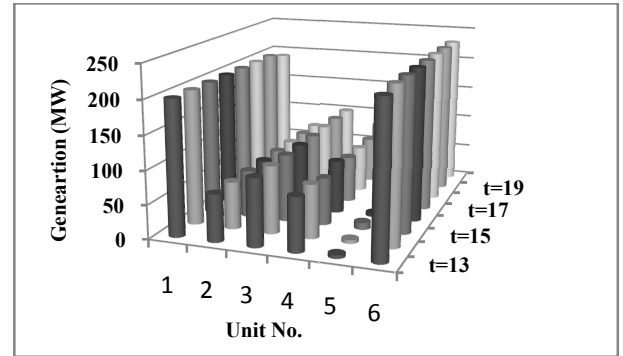


Figure4. Six units generation schedule in hours 13-19 – Base case

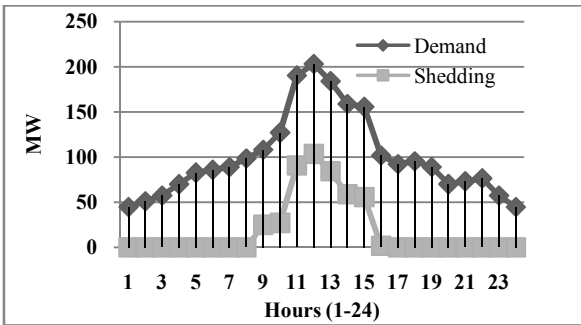


Figure3. Demand and shedding schedule of bus 19

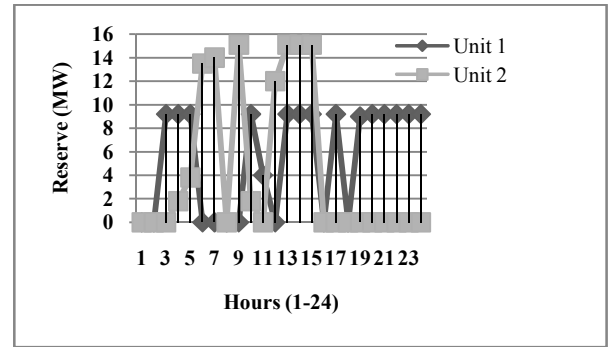


Figure5. Reserve schedule of two units

B. Single Contingency of Unit 6

In comparison with case A, unit 6 cannot commit in hours 13-24. The first 13 hours of unit commitment is as same as case 4.1. Fig.4 show generation schedule in case 4.1 with no contingency in hours 13-20. 5% of whole system demand is considered as desired reserve in contingency case. It should be noted that unit 6 is generating 226 MW at hour 13 (Fig.3). Contingency occurrence at hour 13 makes network loose this amount of generation. This loss of generation should make the solution infeasible; however, implementing the proposed method bridge infeasibility and specify the shedding amount of each bus in this situation. It should be noted that Case B requires additional ancillary service because of unit 6 outage. Fig.5 shows two units reserve schedule in contingency case. It should be noted that reserves are cheaper than actual power to supply the considered reserve.

Fig.6 shows shedding amount in case B. These shedding are achieved by considering Table V and 226 MW lost power at hour 13th. Note that bus 3 shedding amount is more than other buses which proves this allocation is based on defined weights.

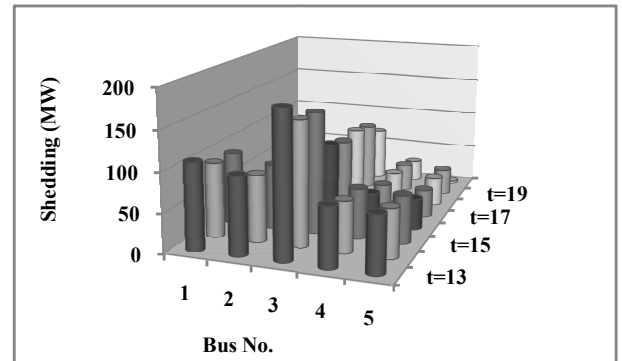


Figure6. Shedding amounts of 5 buses in hours 13-20 – Contingency case

V. CONCLUSION

In this work the unit commitment problem is formulated as linear problem with reference to the electricity market environment. This formulation tries to find an optimal solution while violations transpired. The proposed formulations handle violations and unit contingency. The defined objective is to find optimal

schedule by minimizing the energy dispatch cost and obtain the loss of load value if it exist. These losses can be allocated to cheaper load buses and avoid random shedding. Mixed integer programming technique is selected and employed on test system. The test system is considered to be congested and unable to supply the whole demand in some hours. Results from considered cases prove the optimality and effectiveness of the method.

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