

# Optimal Reserve Requirements and Units Schedule In Contingency Constrained Unit Commitment

Morteza Bigdeli  
Department of Electrical Engineering  
Ferdowsi University of Mashhad  
Mashhad, Iran  
mo.bigdeli@stu-mail.um.ac.ir

Ali Karimpour  
Department of Electrical Engineering  
Ferdowsi University of Mashhad  
Mashhad, Iran  
karimpor@um.ac.ir

**Abstract**—This paper presents Security Constraint Unit Commitment (SCUC) backup plan considering single contingency. The proposed method leads solution to obtain optimal units and reserve schedule. In equivalent linear expression of the problem, shedding costs are used to avoid divergence and resolve congestion problem. Obtained load shedding in contingency case are utilized to achieve an optimal reserve requirements. Mixed Integer Linear Programming (MILP) is employed to minimize the total energy dispatch cost in 24 hours of a day. Numerical results show the proposed joint auction model can clear energy and reserve market while system violation and unit outage are considered.

**Keywords**—Security Constraint Unit Commitment (SCUC); Contingency; Congestion; Shedding; Mixed Integer Linear Programming (MILP);

## NOMENCLATURE

### Indices:

|          |                            |
|----------|----------------------------|
| $i, j$   | Index for buses            |
| $m$      | Index for contingency      |
| $t$      | Index for time             |
| $\alpha$ | Index for reserve interval |

### Variables:

|                      |  |
|----------------------|--|
| $I(i, t)$            | Commitment state of unit $i$ at time $t$ .                                 |
| $P(i, t)$            | Generation of unit $i$ at time $t$ in base case.                           |
| $P_{\alpha}^m(i, t)$ | Generation of unit $i$ at time $t$ for reserve interval in contingency $m$ |
| $R^m(i, t)$          | Called reserve of unit $i$ at time $t$ in contingency $m$ .                |
| $RD_{sys}(t)$        | System ramp-down at time $t$ .   |
| $RU_{sys}(t)$        | System ramp-up at time $t$ .   |
| $P_l(i, j, t)$       | Power transmitted from bus $i$ to $j$ at time $t$ .                        |
| $SD(i, t)$           | Shutdown cost for unit $i$ at time $t$ .                                   |
| $SU(i, t)$           | Startup cost for unit $i$ at time $t$ .                                    |
| $y(i, t)$            | Binary unit startup indicator for unit $i$ at time $t$ .                   |
| $z(i, t)$            | Binary unit shutdown indicator for unit $i$ at time $t$ .                  |

|                      |   |
|----------------------|---|
| $\delta P_d^m(i, t)$ | shedding amount of bus $i$ at time $t$ in contingency $m$ . |
| $\delta(i, t)$       | angle of bus $i$ at time $t$ .                              |

### Functions:

|           |  |
|-----------|--|
| $F_{c,i}$ | Bid-based energy cost function for unit $i$ .  |
| $F_{R,i}$ | Bid-based reserve cost function for unit $i$ . |
| $F_{S,i}$ | Bid-based shedding cost function for bus $i$ . |

### Constants:

|                 |  |
|-----------------|--|
| $DR_i$          | Ramp-down rate limit of unit $i$ .                     |
| $N_d$           | Number of demand buses.                                |
| $N_g$           | Number of units.                                       |
| $N_t$           | Number of periods under study (24 h).                  |
| $P_i^{\max}$    | Upper limit of real power generation for unit $i$ .    |
| $P_i^{\min}$    | Lower limit of real power generation for unit $i$ .    |
| $P_D(t)$        | System load at time $t$ .                              |
| $P_d(i, t)$     | Load of bus $i$ at time $t$ .                          |
| $P_{l^0}(i, j)$ | Lower limit of transmitted power from bus $i$ to $j$ . |
| $P_{u^0}(i, j)$ | Upper limit of transmitted power from bus $i$ to $j$ . |
| $R_i^{\max}$    | Upper limit of reserve generation for unit $i$ .       |
| $RS_i$          | Both startup and shut down ramp rates.                 |
| $T_i^{off}$     | Minimum off time of unit $i$ .                         |
| $T_i^{on}$      | Minimum on time of unit $i$ .                          |
| $UR_i$          | Ramp-up rate limit of unit $i$ .                       |
| $X^{off}(i, t)$ | Off time of unit $i$ at time $t$ .                     |
| $X^{on}(i, t)$  | On time of unit $i$ at time $t$ .                      |

|                   |   |
|-------------------|---|
| $\delta_{\max}^i$ | Upper limit of $i^{\text{th}}$ bus angle. |
| $\delta_{\min}^i$ | Lower limit of $i^{\text{th}}$ bus angle. |
| $\Delta^m$        | Desired reserve requirements.             |

## I. INTRODUCTION

System reliability and security in Contingency Constrained Unit Commitment (CCUC) should be enhanced. This enhancement in contingencies can be reached by desired reserve prediction. Security Constraint Unit Commitment (SCUC) schedule should be able to maximize security in CCUC state. Units schedule and system power flow in day-ahead auction can be overwhelmed by blind contingency consideration. However, contingency cases can be considered in SCUC schedule and enhance system security. In most cases, joint auctions have some weaknesses to cover system violations in case of contingency. Due to the nature of SCUC matter, problem formulation in these cases (joint auction and unit contingency) can be inconveniently complicated.

Contingency-constrained unit commitment and reserve auction are discussed in [1], [2], [3]–[4]. However, power balance under both normal and contingency states are separated in mentioned references.

The bid-based ancillary services auction based on SCUC and economic dispatch was presented in [5]. The proposed model clears energy and ancillary services bids simultaneously. However, the pricing algorithm in [5] and [6] consider pre-defined reserve requirements, which is usually a certain percentage of the system load or the largest unit capacity in the system.

During last these years, several optimization algorithms are employed to solve SCUC problem, e.g. heuristics [7], Mixed-Integer Programming (MIP) [8], Lagrangian Relaxation (LR) [9], Benders Decomposition (BD) [10], Branch and Bound (BB) [11], Dynamic Programming (DP) [12, 13], and etc. In [14], BD approach is employed to solve CCUC problem joint with ancillary service. However, in benders decomposition approach for each violation which may result infeasible solution, new cuts will be added to master problem to avoid infeasibility. This approach can be time consuming and in some cases would be inconclusive.

This paper develops the idea in [6], [5] and [14] for reserve auction and enhance system security while considering contingency case. The proposed algorithm presents a linear expression of CCUC while optimal reserve requirements and ancillary auction are considered.

The rest of the paper is organized as follows. Section II presents the problem formulation. Section III explains proposed algorithm. Section IV provides the numerical examples. Finally, Section V concludes the paper.

## II. PROBLEM FORMULATION

SCUC is about minimizing cost and power generation while system security is guaranteed. The objective function is cost based and can be written as (1). Note that  $m=0$  in formulation is base case with no contingency.

$$\begin{aligned} \min \sum_{i=1}^{N_g} \sum_{t=1}^{N_t} [ & F_{c,i}(P(i,t)) + F_{R,i}(R_u(i,t)) \\ & + F_{R,i}(R_d(i,t)) + SU(i,t) \\ & + SD(i,t) + F_{S,i}(\delta P_d(i,t))] \end{aligned} \quad (1)$$

Note that hourly bids are considered for real powers and daily bids are taken into account for reserves. This objective should be minimized by following constraints:

Power balance equation:

$$\sum_{i=1}^{N_g} P(i,t) \geq P_D(t) \quad (2)$$

Desired reserve requirements with load-generation balance equation:

$$\sum_{i=1}^{N_g} P_{\alpha}^m(i,t) = P_D(t) + \Delta^m \quad (3-a)$$

$$P_{\alpha}^m(i,t) = P(i,t) + R^m(i,t) \quad (3-b)$$

Power generation and reserve limits:

$$\begin{aligned} P(i,t) + R_u(i,t) & \leq P_i^{\max} I(i,t) \\ P(i,t) - R_d(i,t) & \geq P_i^{\min} I(i,t) \\ 0 & \leq R_u(i,t) \leq R_i^{\max} \\ 0 & \leq R_d(i,t) \leq R_i^{\max} \\ -R_d(i,t) & \leq R^m(i,t) \leq R_u(i,t) \end{aligned} \quad (4)$$

In (4),  $R_u(i,t)$  and  $R_d(i,t)$  are upward and downward reserves that are awarded at the base case. Their values are determined by the solution of the auction problem.  $R^m(i,t)$  is the actual reserve that is to be called if contingency of unit  $m$  occurs.

Unit generation ramp rates:

$$\begin{aligned} P(i,t) - P(i,t-1) & \leq UR_i \\ P(i,t-1) - P(i,t) & \leq DR_i \end{aligned} \quad (5)$$

$$\begin{aligned}
P_{\alpha}^m(i, t) - P_{\alpha}^m(i, t-1) &\leq UR_i \\
P_{\alpha}^m(i, t-1) - P_{\alpha}^m(i, t) &\leq DR_i
\end{aligned} \tag{6}$$

System ramp rates as nonlinear constraints:

$$\begin{aligned}
RU_{ys}(t) &= \sum_{i=1}^{N_g} [UR_i + P_i^{min} - RS_i] I(t-1) I(t) I(t+1) \\
&+ RS_i I(t) - P_i^{min} I(t-1) \\
RU_{ys}(t) &\geq P_D(t) - P_D(t-1)
\end{aligned} \tag{7}$$

$$\begin{aligned}
RD_{ys}(t) &= \sum_{i=1}^{N_g} [DR_i + P_i^{min} - RS_i] I(t-1) I(t) I(t+1) \\
&+ RS_i I(t-1) - P_i^{min} I(t) \\
RD_{ys}(t) &\geq P_D(t-1) - P_D(t)
\end{aligned} \tag{8}$$

System ramp rate constraints are considered as linear limitations with [15] technique.

Minimum up/down time constraints:

$$\begin{aligned}
[X^{on}(i, t-1) - T_i^{on}] * [I(i, t-1) - I(i, t)] &\geq 0 \\
[X^{off}(i, t-1) - T_i^{off}] * [I(i, t) - I(i, t-1)] &\geq 0
\end{aligned} \tag{9}$$

Linearization technique for (9) constraints have been expressed in [16].

Generalized network constraints:

$$G(P_{\alpha}^m(i, t)) \leq 0 \tag{10}$$

Generalized network constraints (10) can be conveyed as AC (nonlinear) and DC (linear) formulation. In order to avoid nonlinear formulation, generalized DC network constraints [17] are considered. Once any of these constraints violated, solution to this problem might be infeasible. However, network violations can be handled by following formulation instead of DC network flow:

$$\delta P_d^m(i, t) = P_d(i, t) + \sum_{i \neq j} B(i, j) [\delta(i) - \delta(j)] - P_{\alpha}^m(i, t) \tag{11}$$

$$\delta P_d^m(i, t) \leq P_d(i, t) \tag{12}$$

Note that  $\delta P_d^m(i, t)$  is defined as shedding amount and positive variable. This variable is weighted in minimizing object (1).

Real power balance equation (2)-(3-a) should be replaced by:

$$\sum_{i=1}^{N_g} P(i, t) \geq P_D(t) - \sum_{i=1}^{N_d} \delta P_d^m(i, t) \tag{13}$$

$$\sum_{i=1}^{N_g} P_{\alpha}^m(i, t) = P_D(t) + \Delta^m - \sum_{i=1}^{N_d} \delta P_d^m(i, t) \tag{14}$$

(13)–(14) resolve convergence difficulty and leads problem to optimal and feasible solution.

This solution procedure leads problem to achieve optimal load shedding while system cannot supply demands. The lack of power supply may be caused by several reasons: 1) Transmission lines limitations; 2) system demand is more than generations; and 3) ramp rate constraints. Contingency results the last two mentioned reasons, violates system security constraints and causes inevitable load shedding. However, these shedding can be weighted in price based function (1). In this paper, the primary objective is shedding prevention and having a backup plan in case of violation. The solution procedure to this problem and suggested algorithm will be discussed in Section III.

### III. SOLUTION POROCEDURE TO AVOID IMPOSED LOAD SHEDDING

Consider Fig.1 as the flowchart of the suggested approach. The solution procedure is about finding the best schedule to cover all violations in contingency case. The solution procedure can be conveyed as following:

First, 24 hour normal schedule runs to specify all units schedule. This day-ahead auction clearing contains no contingency.

If infeasibility occurs in case of line congestion, suggested load shedding and units schedule will be found by formulation in Section II. It should be noted that, lines limitations might be handled by other approaches such as Transmission Switching (TS) [18], too. Note that suggested approach in Section II cannot specify whether this inability of finding units schedule is for the line congestion or the unit contingency. However, Section II approach handles violated cases. Line congestion can be handled by following procedure: Lines overload are priced extremely expensive to meet related constraints. The lines pricing leads the problem to optimal solution and avoid lines congestion infeasibility. It should be noted that infeasible solution in this case (lines can be overloaded) means that generations cannot supply demands. In other words, unit contingency and generation ramp rates limit real power supply to satisfy load-generation balance equation.

In CCUC run, Unit contingency is assumed to be single in each day. When unit contingency transpires infeasibility may be occurred. This infeasibility should be for two reasons: 1) system demand is more than power generations which may be caused by units ramp rates (mentioned reasons in Section II), and 2) lines congestions. If infeasibility is for the first reason; therefore, obtained load shedding should be added to demands and re-run 24hour SCUC. This procedure specifies desired reserve and schedule units to cover ramp rates violations in case of contingency. It should be noted that in this situation more demands should be supplied, but in case of same contingency, generations provide whole system

demand and load shedding could be avoided. However, added demands (obtained shedding in CCUC run) will be assumed as new desired reserve ( $\Delta^m$ ) in SCUC re-run. In the second step of the algorithm, if unit contingency leads problem to an infeasible solution, this unit considers as a risky generator. However, risky generators at each hour can be specified. These risky generators are specified as the units which should be considered in backup plan (SCUC re-run). However, in last step of the algorithm each of these important units maximum generation should be the maximum of scheduled power in CCUC run to avoid backup plan buy more supplies from this unit (note that in last step more demands should be supplied). During this procedure minimum load shedding should be reached; therefore, each bus shedding should be weighted extremely expensive. However, considering system power DC power flow and units reschedule enhance system reliability and decrease load shedding in case of contingency. Note that zero shedding must be achieved if no violation transpires (congestion or not provided demands). Numerical examples in Section IV confirm suggested approach.

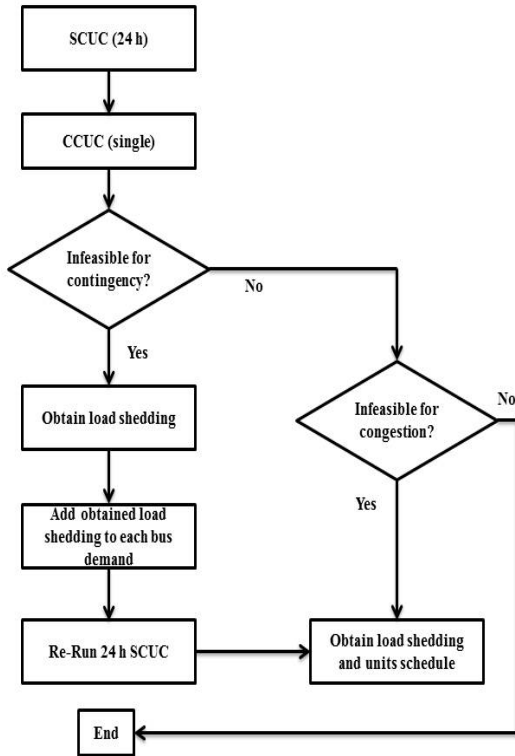


Figure 1. Flowchart of SCUC considering single contingency

#### IV. TEST SYSTEM AND NUMERICAL RESULTS

IEEE 24-bus test system (Fig.2) is employed to specify the results. Table I shows units characteristics. Table II indicates bid prices of active power, startup costs, shutdown costs, and reserves bids of each unit. Table III addresses the system power demand in each hour. Table IV shows

shedding cost weights which is used to determine shedding priority.

Matlab and GAMS joint programming are used to solve this problem. CPLX 12.5 in GAMS mathematical modeling language is employed to solve MILP programming.

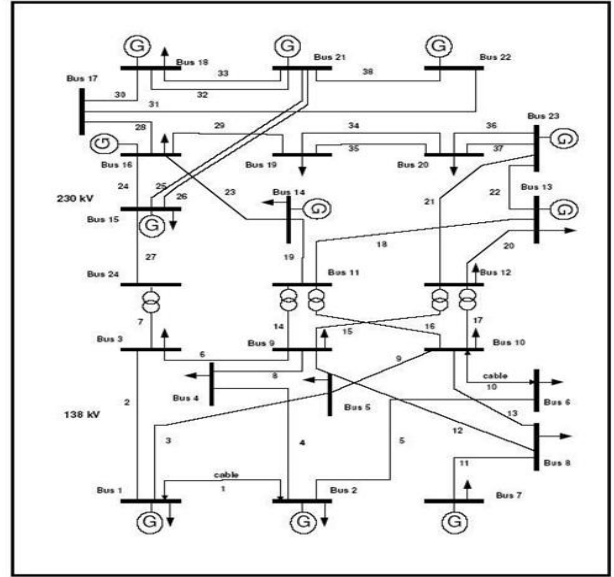


Figure 2. IEEE 24 bus test system one line diagram

TABLE I FUTURES OF GENERATING UNITS

| Units | Bus No. | $P_{max}$ (MW) | $P_{min}$ (MW) | Ini.State (h) | Min on (h) | Min off (h) | Ramp (MW/h) | Reserve <sub>max</sub> (MW) |
|-------|---------|----------------|----------------|---------------|------------|-------------|-------------|-----------------------------|
| G1    | 1       | 200            | 50             | -2            | 2          | -4          | 50          | 9.2                         |
| G2    | 2       | 200            | 20             | 2             | 4          | -3          | 70          | 15.1                        |
| G3    | 7       | 250            | 70             | -4            | 2          | -3          | 100         | 3.3                         |
| G4    | 13      | 280            | 120            | 3             | 4          | -2          | 80          | 20                          |
| G5    | 14      | 10             | 5              | 2             | 1          | -1          | 5           | 17                          |

TABLE II UNITS BIDDING DATA

| Units | Active power bids (\$/MWh) |       |       |       |       | Reserve bids (\$/MWh) | startup (SU <sub>i</sub> , \$) | Shut down (SD <sub>i</sub> , \$) |
|-------|----------------------------|-------|-------|-------|-------|-----------------------|--------------------------------|----------------------------------|
|       | $t=1$                      | $t=2$ | $t=3$ | $t=4$ | $t=5$ |                       |                                |                                  |
| G1    | 13                         | 12    | 12    | 8     | 7     | 1.2                   | 12                             | 17                               |
| G2    | 12                         | 12    | 13    | 14    | 17    | 1.4                   | 4                              | 6                                |
| G3    | 15                         | 13    | 7     | 5     | 4     | 3.05                  | 12                             | 18                               |
| G4    | 13                         | 12    | 10    | 10    | 12    | 5.2                   | 19                             | 11                               |
| G5    | 11                         | 12    | 10    | 11    | 14    | 1.7                   | 17                             | 13                               |

TABLE III. HOURLY SYSTEM DEMAND

| Hours                        | 1   | 2  | 3  | 4  | 5  | 6    | 7  | 8    |
|------------------------------|-----|----|----|----|----|------|----|------|
| Demand (10 <sup>2</sup> *MW) | 4.5 | 6  | 7  | 9  | 11 | 12.5 | 18 | 19   |
| Hours                        | 9   | 10 | 11 | 12 | 13 | 14   | 15 | 16   |
| Demand (10 <sup>2</sup> *MW) | 20  | 17 | 16 | 15 | 14 | 13   | 12 | 11.5 |
| Hours                        | 17  | 18 | 19 | 20 | 21 | 22   | 23 | 24   |
| Demand (10 <sup>2</sup> *MW) | 15  | 14 | 19 | 20 | 21 | 16   | 11 | 10   |

TABLE IV. SHEDDING COST WEIGHTS

| Bus No.                                     | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|---|----|----|----|----|----|----|----|----|
| Shedding Cost (10 <sup>6</sup> *\$/MWh)     | 25 | 13 | 1  | 16 | 20 | 21 | 30 | 39 |
| Bus No.                                     | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Shedding Cost*1e6 (10 <sup>6</sup> *\$/MWh) | 30 | 26 | 0  | 0  | 24 | 23 | 26 | 29 |
| Bus No.                                     | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Shedding Cost (10 <sup>6</sup> *\$/MWh)     | 29 | 25 | 50 | 25 | 0  | 0  | 0  | 0  |

Consider Section III algorithm. Several cases can be discussed to examine the proposed method. 11 units outage in 24 hours are considered in programming, but in order to specify results, unit 3 outage at hour 7 (single contingency case) is contemplated. Fig. 3 shows 4 units schedule with 1% of whole system demand as desired reserve at hours 7 to 12. Unit 3 generation at hour 7 is 210 MW which should be compensated to avoid load shedding. Not only desired reserve (1% of whole system demand) can not provide this lost power but also units ramp rates limit real power generations; therefore, shedding will be transpired. Fig.4 shows shedding amounts (consider Table V) in contingency case and Fig.5 indicates 6 units reserve schedule at mentioned hours.

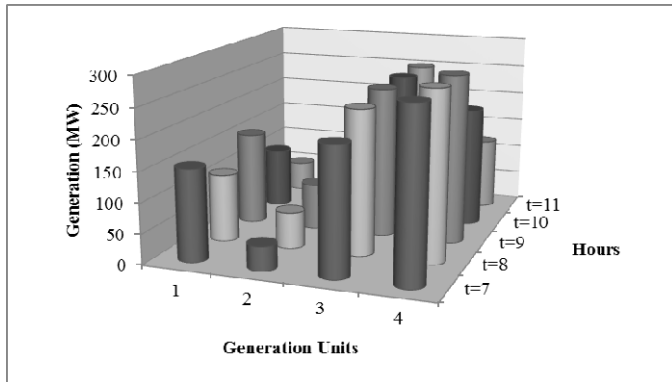


Figure 3. Four Units generation schedule with no contingency

Note that 210 MW of power generation could not be supplied and infeasible solution will be found in case of contingency, but this infeasibility is not achieved for lines congestion. Section II presented formulation lead problem to optimal schedule and obtain demands which should be excluded.

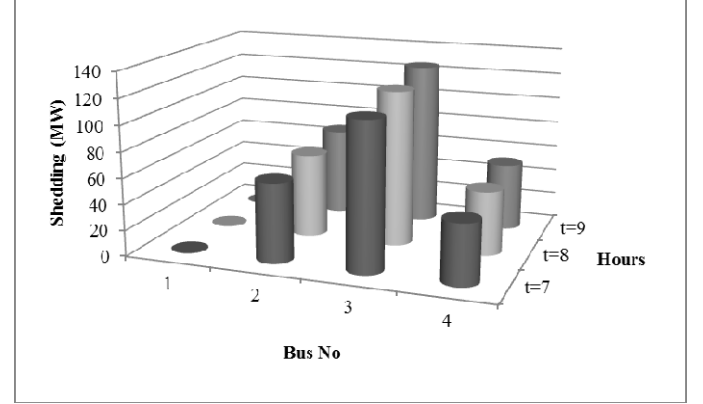


Figure 4. Shedding amounts in contingency case

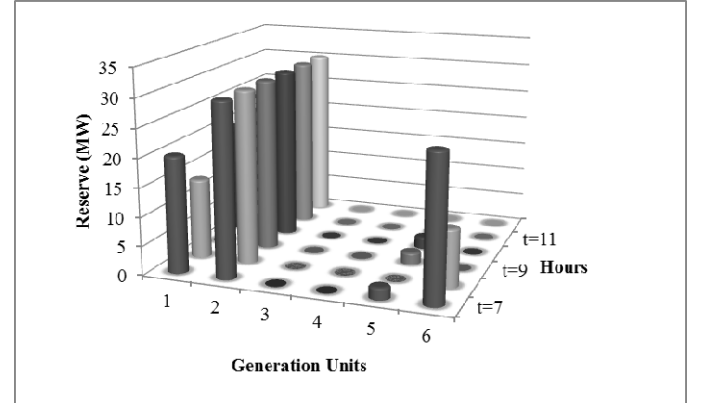


Figure 5. Six units reserve Schedule in contingency case (1% of whole system demand as desired reserve)

Following Section III algorithm in scheduling provide more demands in case of contingency. Fig.6 shows reserve schedule of 6 units in SCUC 24 hour re-run. In this situation desired reserve is achieved by summation of shedding amount in case of unit 3 outage (single contingency). It should be noted that more reserves should be considered in this case. Fig.7 indicates shedding amount in case of allocated extra demand. Compare to Fig.4, less demand should be excluded in case of contingency. However, no shedding should be considered if no outage transpires. In fact, Fig.7 shows excluded demands in case of contingency with new schedule and reserve consideration.

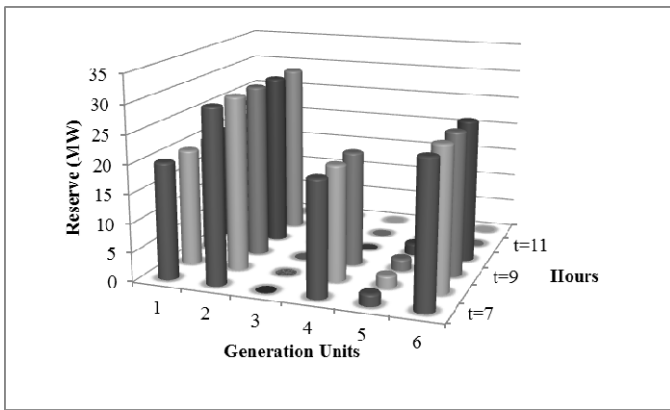


Figure 6. Six units reserve Schedule (using Section III algorithm for specifying desired reserve)

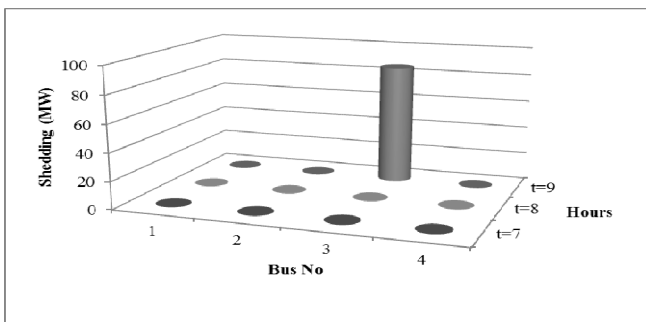


Figure 7. Shedding amounts in SCUC re-run

## V. CONCLUSION

In the presented paper security constraint unit commitment is formulated as a linear problem while considering contingencies. The proposed method considers contingency cases and find optimal reserve schedule with reference to electricity market. Reserve auction and backup plan present a fine units schedule to handle violations and lost generation. Numerical example and comparison on IEEE modified test system shows that proposed algorithm looks forward to system dispatch in case of violation. Reduced shedding in contingency case and results confirms optimality and effectiveness of presented method.

## REFERENCES

- [1] J. M. Arroyo and F. D. Galiana, "Energy and reserve pricing in security and network-constrained electricity markets," *Power Systems, IEEE Transactions on*, vol. 20, pp. 634-643, 2005.
- [2] F. D. Galiana, F. Bouffard, J. M. Arroyo, and J. F. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves," *Proceedings of the IEEE*, vol. 93, pp. 1970-1983, 2005.
- [3] J. Wang, N. E. Redondo, and F. D. Galiana, "Demand-side reserve offers in joint energy/reserve electricity markets," *Power Systems, IEEE Transactions on*, vol. 18, pp. 1300-1306, 2003.
- [4] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Co-optimization of generation unit commitment and transmission switching with N-1 reliability," *Power Systems, IEEE Transactions on*, vol. 25, pp. 1052-1063, 2010.
- [5] Z. Li and M. Shahidehpour, "Security-constrained unit commitment for simultaneous clearing of energy and ancillary services markets," *Power Systems, IEEE Transactions on*, vol. 20, pp. 1079-1088, 2005.
- [6] Y. Fu, M. Shahidehpour, and Z. Li, "AC contingency dispatch based on security-constrained unit commitment," *Power Systems, IEEE Transactions on*, vol. 21, pp. 897-908, 2006.
- [7] N. H. Kjeldsen and M. Chiarandini, "Heuristic solutions to the long-term unit commitment problem with cogeneration plants," *Computers & Operations Research*, vol. 39, pp. 269-282, 2012.
- [8] X. Guan, Q. Zhai, and A. Papalexopoulos, "Optimization based methods for unit commitment: Lagrangian relaxation versus general mixed integer programming," in *Power Engineering Society General Meeting, 2003, IEEE*, 2003.
- [9] X. Guan, S. Guo, and Q. Zhai, "The conditions for obtaining feasible solutions to security-constrained unit commitment problems," *Power Systems, IEEE Transactions on*, vol. 20, pp. 1746-1756, 2005.
- [10] Y. Fu and M. Shahidehpour, "Fast SCUC for large-scale power systems," *Power Systems, IEEE Transactions on*, vol. 22, pp. 2144-2151, 2007.
- [11] P. Somol, P. Pudil, and J. Kittler, "Fast branch & bound algorithms for optimal feature selection," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 26, pp. 900-912, 2004.
- [12] C. Wang and S. Shahidehpour, "Optimal generation scheduling with ramping costs," in *Power Industry Computer Application Conference, 1993. Conference Proceedings, 1993*, pp. 11-17.
- [13] N. Padhy, "Unit commitment using hybrid models: a comparative study for dynamic programming, expert system, fuzzy system and genetic algorithms," *International Journal of Electrical Power & Energy Systems*, vol. 23, pp. 827-836, 2001.
- [14] J. Wang, M. Shahidehpour, and Z. Li, "Contingency-constrained reserve requirements in joint energy and ancillary services auction," *Power Systems, IEEE Transactions on*, vol. 24, pp. 1457-1468, 2009.
- [15] T. Niknam, A. Khodaei, and F. Fallahi, "A new decomposition approach for the thermal unit commitment problem," *Applied Energy*, vol. 86, pp. 1667-1674, 2009.
- [16] N. Zendehele, A. Karimpour, and M. Oloomi, "Optimal unit commitment using equivalent linear minimum up and down time constraints," in *Power and Energy Conference, 2008. PCon 2008. IEEE 2nd International*, 2008, pp. 1021-1026.
- [17] J. J. Grainger, *Power system analysis*: Tata McGraw-Hill Education, 2003.
- [18] A. Khodaei and M. Shahidehpour, "Transmission switching in security-constrained unit commitment," *Power Systems, IEEE Transactions on*, vol. 25, pp. 1937-1945, 2010.