1 Syntax

$$
\begin{array}{llll}
e ::= c \in \mathbb{Z} & e_e ::= \cdot +_1 e & s \in \mathit{stat} ::= \mathit{skip} & s_e ::= \mathsf{x} :=_1 \cdot \\
\quad \mid \mathsf{x} \in \mathit{Var} & \quad \mid \cdot +_2 \cdot & \quad \mid s_1 ; s_2 & \quad \mid \cdot ;_1 s_2 \\
\quad \mid e_1 + e_2 & \quad \mid @_1 (e_2) & \quad \mid \mathsf{x} := e & \quad \mid \mathit{if}_1 \, s_1 \, s_2 \\
\quad \mid \lambda \mathsf{x}.s & \quad \mid @_2 & \quad \mid \mathit{if} \ (e > 0) \ s_1 \, s_2 & \quad \mid \mathit{while}_1 \ (e > 0) \ s \\
\quad \mid e_1 \, (e_2) & \quad \mid @_3 & \quad \mid \mathit{while} \ (e > 0) \ s & \quad \mid \mathit{while}_2 \ (e > 0) \ s \\
 & & \quad \mid \mathit{return} \, e & \quad \mid \mathit{return}_1 \cdot
\end{array}
$$

2 Semantics

2.1 Expressions

$$\textsc{red-const}(c) \quad \frac{}{H_e, \ell_e, \ell_c, c \Downarrow H_e, \ell_e, c}$$

$$\textsc{red-var-local}(\mathsf{x}) \quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, \ell_c [\mathsf{x}]} \quad \mathsf{x} \in \mathit{dom} \left(H_e [\ell_c] \right)$$

$$\textsc{red-var-global}(x) \quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, E [\mathsf{x}]} \quad \mathsf{x} \in \mathit{dom} \left(H_e [\ell_e] \right) \land \mathsf{x} \notin \mathit{dom} \left(H_e [\ell_c] \right)$$

$$\textsc{red-var-undef}(x) \quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow \mathit{err}} \quad \mathsf{x} \notin \mathit{dom} \left(H_e [\ell_e] \right) \land \mathsf{x} \notin \mathit{dom} \left(H_e [\ell_c] \right)$$

$$\textsc{red-add}(e_1, e_2) \quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, \cdot +_1 e_2 \Downarrow r'}{H_e, \ell_e, \ell_c, e_1 + e_2 \Downarrow r'}$$

$$\textsc{red-add-1}(e_2) \quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad v_1, r, \cdot +_2 \cdot \Downarrow r'}{\ell_c, (H_e, \ell_e, v_1), \cdot +_1 e_2 \Downarrow r'}$$

$$\textsc{red-add-2} \quad \frac{}{v_1, (H_e, \ell_e, v_2), \cdot +_2 \cdot \Downarrow H_e, \ell_e, v_1 + v_2}$$

$$\textsc{red-lambda}(\mathsf{x}, s) \quad \frac{}{H_e, \ell_e, \ell_c, \lambda \mathsf{x}.s \Downarrow H_e, \ell_e, (\ell_c, \lambda \mathsf{x}.s)}$$

$$\textsc{red-app}(e_1, e_2) \quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, @_1 (e_2) \Downarrow r'}{H_e, \ell_e, \ell_c, e_1 (e_2) \Downarrow r'}$$

$$\textsc{red-app-1}(e_2) \quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad \ell'_c, \mathsf{x}, s, r, @_2 \Downarrow r'}{\ell_c, (H_e, \ell_e, (\ell'_c, \lambda \mathsf{x}.s)), @_1 (e_2) \Downarrow r'}$$

$$\textsc{red-app-2}(s) \quad \frac{\begin{array}{c} \ell'_c = \mathit{fresh} (H_e) \qquad C = H_e [\ell_c] \\ H_e [\ell'_c \leftarrow C [\mathsf{x} \leftarrow v]], \ell_e, \ell'_c, s \Downarrow r \qquad r, @_3 \Downarrow r' \end{array}}{\ell_c, \mathsf{x}, s, (H_e, \ell_e, v), @_2 \Downarrow r'}$$

$$\textsc{red-app-3-ret} \quad \frac{}{\mathit{ret} (H_e, \ell_e, v), @_3 \Downarrow H_e, \ell_e, v}$$

$$\textsc{red-app-3-no-ret} \quad \frac{}{H_e, \ell_e, \ell_c, @_3 \Downarrow \mathit{err}}$$

2.2 Statements

$$\frac{}{H_e, \ell_e, \ell_c, \mathit{skip} \Downarrow H_e, \ell_e, \ell_c}\ \textsc{red-skip}$$

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r \qquad r, \cdot ;_1 s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, s_1 ; s_2 \Downarrow r'}\ \textsc{red-seq}(s_1, s_2)$$

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{H_e, \ell_e, \ell_c, \cdot ;_1 s_2 \Downarrow r}\ \textsc{red-seq-1}(s_2)$$

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathsf{x} :=_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathsf{x} := e \Downarrow r'}\ \textsc{red-asn}(\mathsf{x}, e)$$

$$\frac{\ell'_e = \mathit{fresh}\left(H_e\right) \qquad E = H_e\left[\ell_e\right]}{\ell_c, \left(H_e, \ell_e, v\right), \mathsf{x} :=_1 \cdot \Downarrow H_e\left[\ell'_e \leftarrow E\left[\mathsf{x} \leftarrow v\right]\right], \ell'_e, \ell_c}\quad \mathsf{x} \notin \mathit{dom}\left(H_e\left[\ell_c\right]\right)\ \textsc{red-asn-1}(\mathsf{x})$$

$$\frac{\ell'_c = \mathit{fresh}\left(H_e\right) \qquad C = H_e\left[\ell_c\right]}{\ell_c, \left(H_e, \ell_e, v\right), \mathsf{x} :=_1 \cdot \Downarrow H_e\left[\ell'_c \leftarrow C\left[\mathsf{x} \leftarrow v\right]\right], \ell_e, \ell'_c}\quad \mathsf{x} \in \mathit{dom}\left(C\right)\ \textsc{red-asn-1-local}(\mathsf{x})$$

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{if}_1\ s_1\ s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{if}\ \left(e > 0\right)\ s_1\ s_2 \Downarrow r'}\ \textsc{red-if}(e, s_1, s_2)$$

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{if}_1\ s_1\ s_2 \Downarrow r}\quad v > 0\ \textsc{red-if-1-pos}(s_1, s_2)$$

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{if}_1\ s_1\ s_2 \Downarrow r}\quad v \leqslant 0\ \textsc{red-if-1-neg}(s_1, s_2)$$

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{while}\ \left(e > 0\right)\ s \Downarrow r'}\ \textsc{red-while}(e, s)$$

$$\frac{}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow H_e, \ell_e, \ell_c}\quad v \leqslant 0\ \textsc{red-while-1-neg}(e, s)$$

$$\frac{H_e, \ell_e, \ell_c, s \Downarrow r \qquad r, \mathit{while}_2\ \left(e > 0\right)\ s \Downarrow r'}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow r'}\quad v > 0\ \textsc{red-while-1-pos}(e, s)$$

$$\frac{H_e, \ell_e, \ell_c, \mathit{while}\ \left(e > 0\right)\ s \Downarrow r}{H_e, \ell_e, \ell_c, \mathit{while}_2\ \left(e > 0\right)\ s \Downarrow r}\ \textsc{red-while-2}(e, s)$$

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad r, \mathit{return}_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{return}\ e \Downarrow r'}\ \textsc{red-return}(e)$$

$$\frac{}{\left(H_e, \ell_e, v\right), \mathit{return}_1 \cdot \Downarrow \mathit{ret}\left(H_e, \ell_e, v\right)}\ \textsc{red-return-1}$$

2.3 Aborting Rules

$$\textsc{red-error-expr}(e)\quad \frac{}{\sigma, e \Downarrow err} \quad \mathbf{abort}\, \sigma \wedge \neg \mathbf{intercept}_e\, \sigma$$

$$\textsc{red-error-stat}(s)\quad \frac{}{\sigma, s \Downarrow err} \quad \mathbf{abort}\, \sigma$$

$$\frac{\sigma = C\,[\,err\,]}{\mathbf{abort}\, \sigma} \qquad \frac{}{\mathbf{intercept}_{@_3}\, ret\, (H_e, \ell_e, v)}$$