Degree of an AGCD of two Bernstein polynomials

This note describes the programs

`o_roots_Univariate.m` and `o_gcd_Univariate_2Polys.m`

1. The first file is used in the computation of a polynomial's roots and corresponding multiplicities, where the given polynomial is in Bernstein form.
2. The second file is used in teh computation of the Greatest Common Divisor (GCD) of two polynomials in Bernstein form.

The programs are executed by typing

```
o_roots_Univariate(ex_num, emin, emax, mean_method, bool_alpha_theta,
     low_rank_approx_method, apf_method, sylvester_build_method)
o_gcd_2Polys_Univariate(ex_num, emin, emax, mean_method, bool_alpha_theta,
     low_rank_approx_method, apf_method, sylvester_build_method)
```

where

`ex_num` A String typically containing an integer, which defines the example to be run.

`emin` Minimum signal : noise ratio

`emax` Maximum signal : noise ratio

`mean_method` Method used to compute the mean of the entries of the partitions of the Sylvester subresultant matrix

`None` No mean method used

`Geometric Mean My Method` : Fast method

`Geometric Mean Matlab Method` : Standard Matlab method

`Arithmetic Mean` :

`bool_alpha_theta`

`true` : Preprocess polynomials

`false` : Exclude preprocessing

`low_rank_approx_method`

`None`

`Standard SNTLN`

`Standard STLN`

`Root Specific SNTLN`

`apf_method`

`None`

`Standard Linear APF`

`Standard NonLinear APF`

`Sylvester_Build_Method`

`T` : The matrix $T_k\left(f(x), g(x)\right)$

`DT` : The matrix $D^{-1}_{m+n-k}T_k\left(f(x), g(x)\right)$

`DTQ` : The matrix $D^{-1}_{m+n-k}T_k\left(f, g\right)\hat{Q}$

`TQ` : The matrix $T_k\left(f, g\right)\hat{Q}$

`DTQ Rearranged` : The matrix $S_k\left(f(x), g(x)\right) = D^{-1}_{m+n-k}T_k\left(f(x), g(x)\right)\hat{Q}$ where the entries are computed in a rearranged form.

`DTQ Rearranged Denom Removed` : $\tilde{S}\left(f(x), g(x).\right)$

Examples of executing the programs are

`o_gcd_Univariate_2Polys('1', 1e-10, 1e-12, 'Geometric Mean Matlab Method', true, 'None', 'None', 'DTQ')`

`o_roots_Univariate('1', 1e-12, 1e-10, 'Geometric Mean Matlab Method', true, 'None', 'None', 'DTQ')`

The programs produce the following output:

1 Points of Interest

1.1 Limits

The code makes frequent use of variables `t_limits` and `k_limits`.

`t_limits` : In the computation of the factorisation of $\hat{f}_0(x)$, many GCD computations are required to generate the sequence $\hat{f}_i(x) = GCD\left(\hat{f}_{i-1}(x), \hat{f}'_{i-1}(x)\right)$. The degree of $GCD\left(\hat{f}_i(x), \hat{f}'_i(x)\right)$ is bound by the number of distinct roots of $\hat{f}_i(x)$, and the number of distinct roots is always less than or equal to the number of distinct roots of $\hat{f}_{i-1}(x)$.

`k_limits` : This variable defines the range of Sylvester subresultant matrices $S_k\left(\hat{f}_i(x), \hat{f}'_i(x)\right)$ considered in the computation of the degree of the GCD. By default this range is set between 1 and $\min(m, n)$, however `limits_t` can also be used since it is known that all subresultant matrices outside this range are known to be singular.

1.2 Computing the Degree of the GCD

There are several methods considered for the computation of the degree of the GCD. A global variable `SETTINGS.RANK_REVEALING_METRIC` defined in the file `SetGlobalVariables.m` determines which method is used.

Singular Values :

R1 Row Norms :

R1 Row Diagonals :

Residuals :