- The programme `Euclid.m` implements Euclid's algorithm for the Bernstein basis polynomials $f = f(y)$ and $g = g(y)$.

The algorithm is described in the paper:

Algorithm 812:BPOLY: An object-oriented library of numerical algorithms for polynomials in Bernstein form, Y. Tsai and R. Farouki, ACM Transactions on Mathematical Software, volume 27, number 2, June 2001, pp. 267-296.

To run the programme `Euclid.m`, type

```
Euclid(n,ec,tolerance)
```

where

> `n` is an integer that defines the polynomials $f(y)$ and $g(y)$ in the programme `ex.m`
>
> `ec` is the ratio
>
> $$\frac{\text{noise level}}{\text{signal level}}$$
>
> measured in the componentwise sense.
>
> `tolerance` is the stopping criterion for the termination of Euclid's algorithm

Examples: Four examples of executing the programme `Euclid.m` are

`Euclid(8,0,1e-5)`, `Euclid(14,0,1e-5)`, `Euclid(19,1e-8,1e-2)`, `Euclid(20,1e-10,1e-3)`

□

Note: The database `ex.m` is exactly the same as the database `ex.m` for the Sylvester and Bézout matrices.

The method requires that a linear algebraic equation of the form $Ax = b$, where $A \in \mathbb{R}^{m\times m}$ is non-singular, is solved at each stage of Euclid's algorithm. This equation is solved by applying the QR decomposition to the augmented matrix $[A \;\; b]$,

$$[A \;\; b] = QR, \qquad Q \in \mathbb{R}^{m\times m}, \quad R^{m\times(m+1)}.$$

Since $Ax = b$ can be written as

$$[A \;\; b]\begin{bmatrix} x \\ -1 \end{bmatrix} = 0,$$

it follows that

$$QR\begin{bmatrix} x \\ -1 \end{bmatrix} = Q\,[R_1 \;\; r]\begin{bmatrix} x \\ -1 \end{bmatrix} = 0,$$

where $R_1 \in \mathbb{R}^{m\times m}$ is upper triangular and r is the $(m+1)$th column of R. Since Q is non-singular, the equation $Ax = b$ is transformed to the equation $R_1 x = r$.