

Probability theory and statistics

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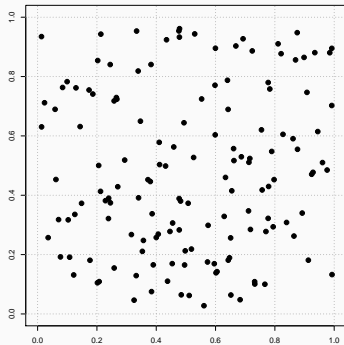
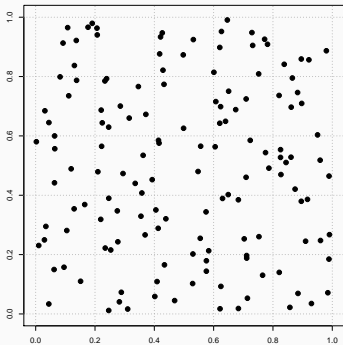
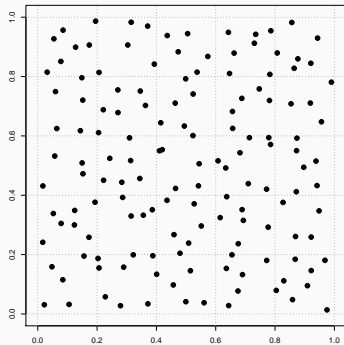
Contents

Probability theory

Statistics

Probability theory

'Random' points



What is probability?

‘The extent to which something is likely to happen’

— Oxford English Dictionary

Examples

- Probability that it will rain tomorrow
- Probability that you will win the lottery

Sources of uncertainty



Imperfect information

Current predictive tools can only assign a number indicating our degree of certainty



Stochastic process

The experiment is designed to produce uncertain results (because it's fun)

Probability theory

What?

The branch of mathematics concerned with **random phenomena**

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How?

Using mathematical **abstractions** of non-deterministic events

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How?

Using mathematical **abstractions** of non-deterministic events

Why?

To identify **patterns** in (apparently) random occurrences

We cannot predict with certainty
if it's going to rain tomorrow

but

we can predict 'average behaviour'

Statistical regularity

In summary...

- Probability theory describes the behaviour of random phenomena **in the long run**
- If this information is useful, probability theory can be a valuable tool for **decision-making**

Random variables 'encapsulate' random events

Notation

- X, Y, \dots (upper case) are **random variables**
- $X = x$ (lower case) is a value (**realisation**) of X
- $\Pr(X = x)$ is the **probability** that $X = x$

Example

- X represents the ('archetypal') outcome of a coin toss
- $X = \text{'head'}$ represents one (actual) outcome
- $\Pr(X = \text{'head'})$ is the long-term probability of the outcome 'head'

Maximum of two fair dice

A fair die

x	$\Pr(X = x)$
1	$1/6$
2	$1/6$
3	$1/6$
4	$1/6$
5	$1/6$
6	$1/6$

Maximum of two fair dice

- How many outcomes?
- $\Pr(X = 1)$?
- $\Pr(X = 6)$?

Simplified approximations to reality

- Detailed enough to capture important characteristics and serve as **prediction tools**
- Simple enough to be usable in practice

Characterising probability distributions

Measures of central tendency

- (Arithmetic) mean or average
- Median
- Mode

Characterising probability distributions

Measures of central tendency

- (Arithmetic) mean or average
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Measures of dispersion

- Variance
- Minimum and maximum \rightarrow range
- Quantiles (a.k.a. order statistics)

Characterising probability distributions

1, 8, 16, 30, 32, 37, 53, 80, 86, 91, 93

- Mean?
- Median?
- Mode?
- Variance and standard deviation?
- Minimum and maximum, and range?
- Quartiles?

Statistics

What?

The science of collecting and analysing numerical data

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How?

By planning studies, exploring and modelling the data using the tools of **probability theory**

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Why?

To infer properties of a **population** from a **sample**

Probability or statistics?

You have a fair coin. You toss it 100 times.
How likely is it to land heads 60 times or more?

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Probability

- Random process is known (or assumed): 'fair coin'
- Objective: **find probability of a certain outcome**

Probability or statistics?

I give you a coin. You toss it 100 times and count 60 heads.
Is the coin fair?

Probability or statistics?

I give you a coin. You toss it 100 times and count 60 heads.
Is the coin fair?

Statistics

- Outcome is known (or measured): '60/100 heads'
- Objective: **characterise the random process**

Probability theory and statistics

Probability theory

- Defines the model
- ...and often its **parameters**

Statistics

- Collects the data
- 'Fits' the model
(estimates its parameters)
- Makes **inferences**