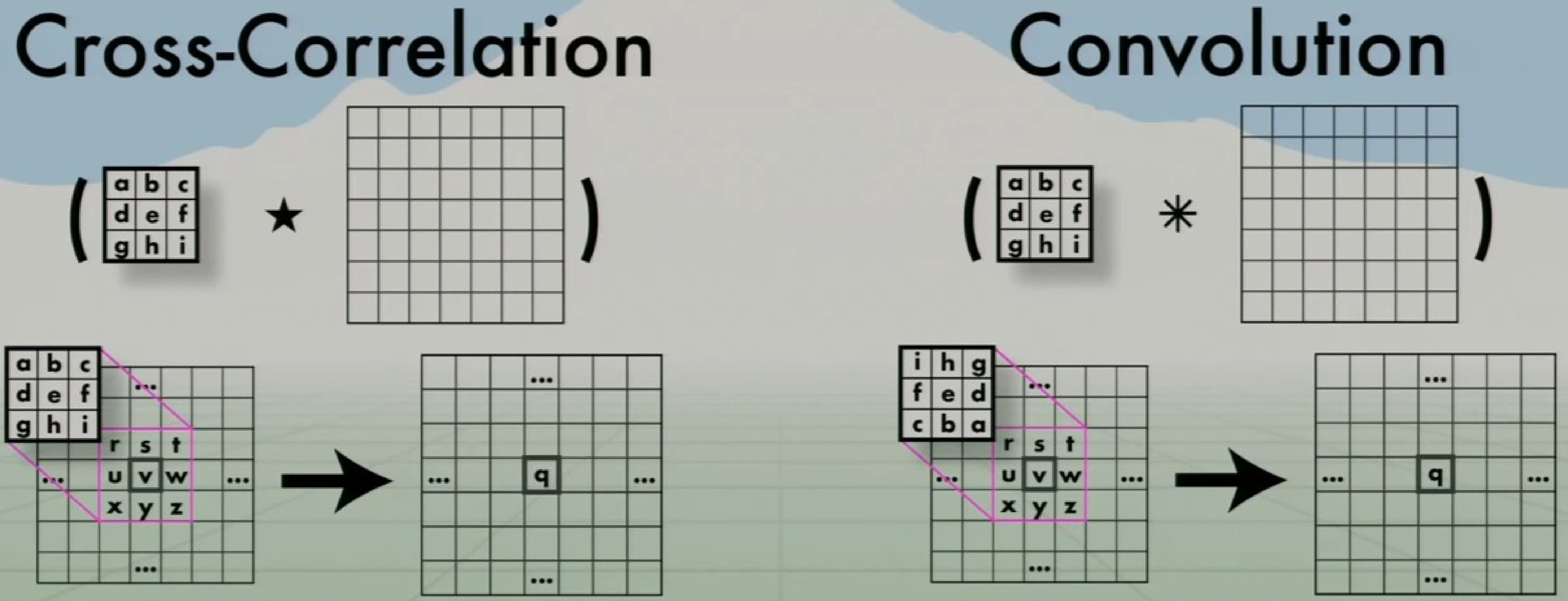
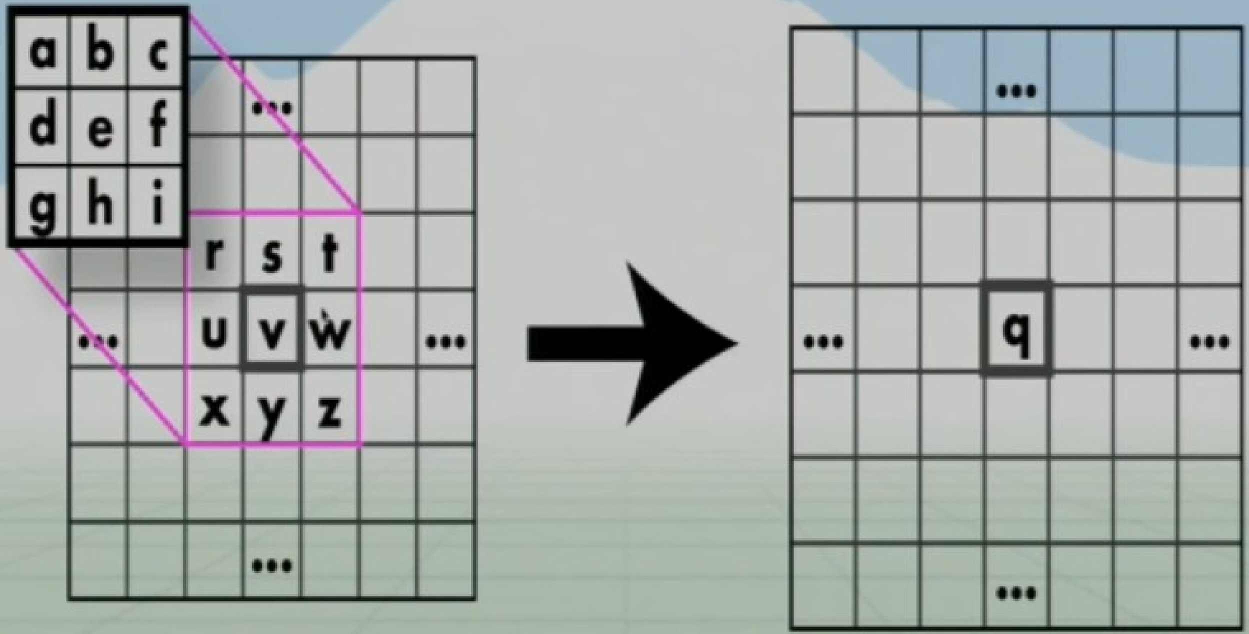
Cross-Correlation vs Convolution



$q = a{\times}r + b{\times}s + c{\times}t + d{\times}u + e{\times}v + f{\times}w + g{\times}x + h{\times}y + i{\times}z$

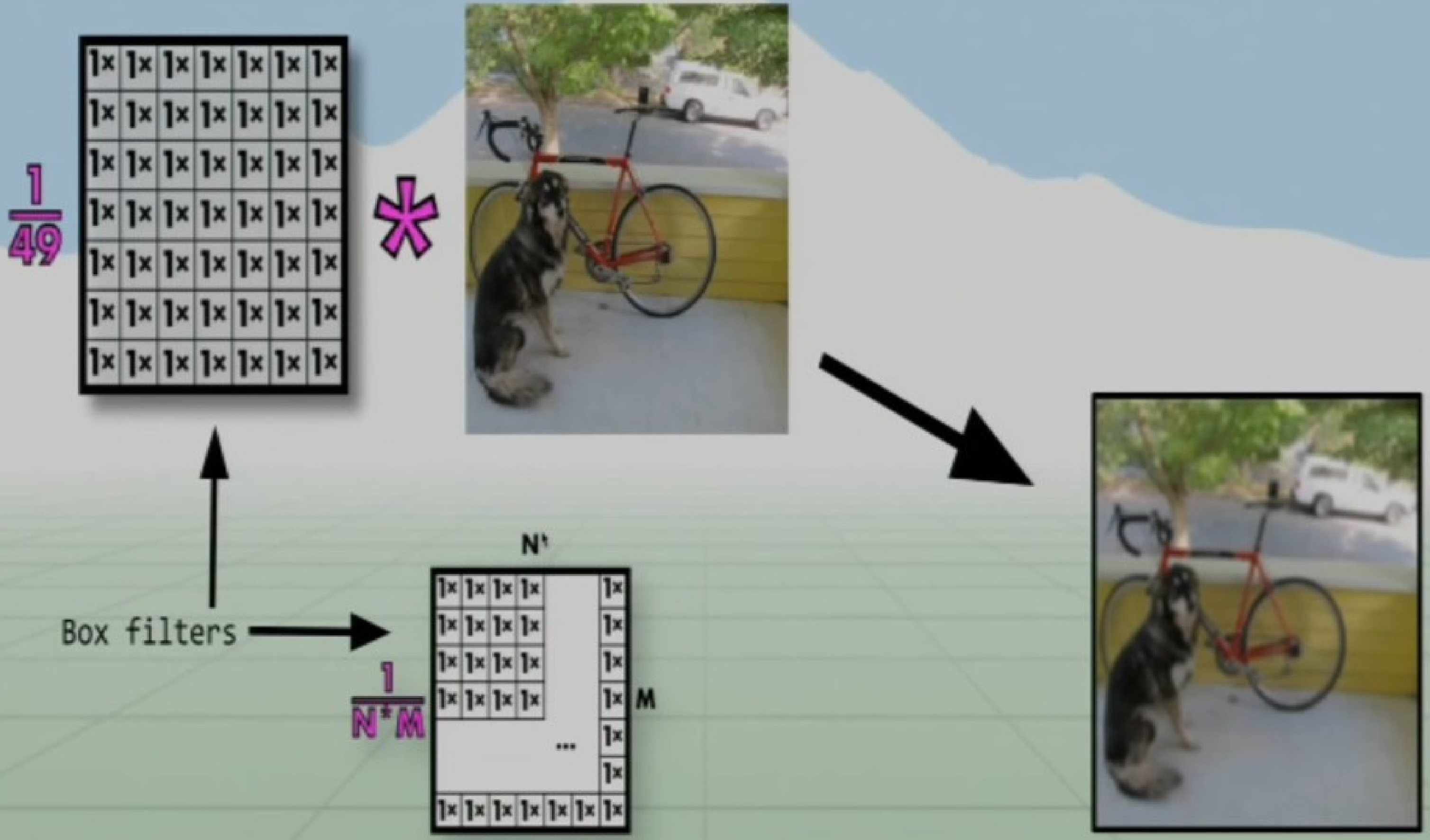
$q = i{\times}r + h{\times}s + g{\times}t + f{\times}u + e{\times}v + d{\times}w + c{\times}x + b{\times}y + a{\times}z$

Wow, so what was that convolution thing??

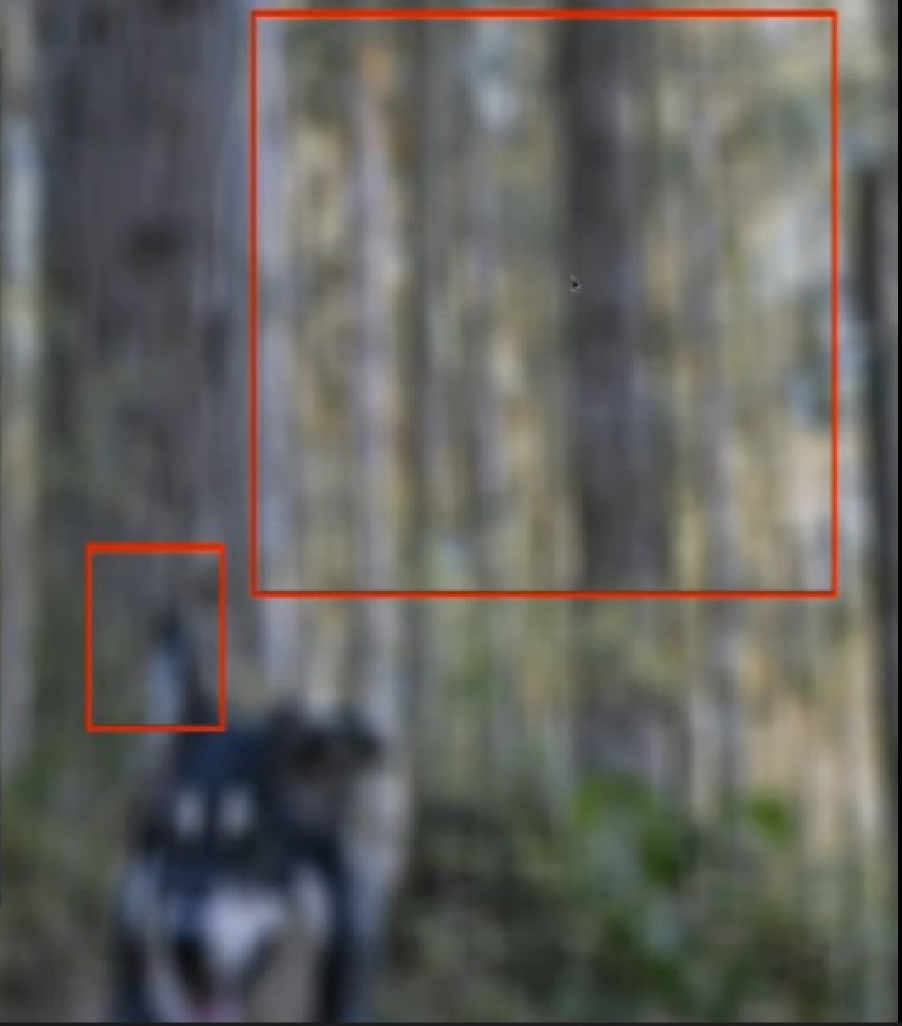


$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

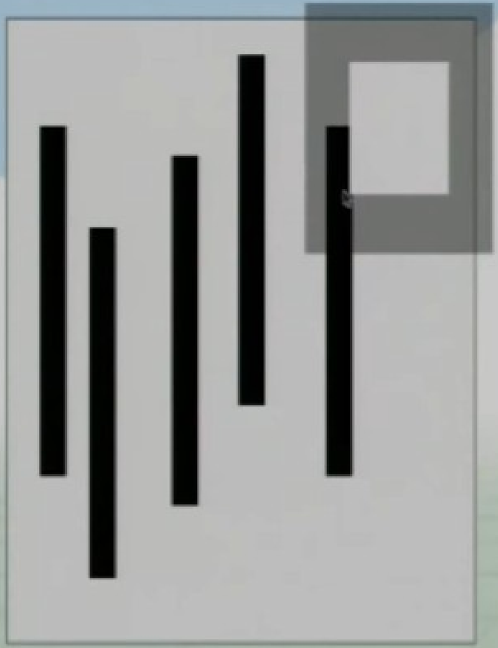
This is called box filter

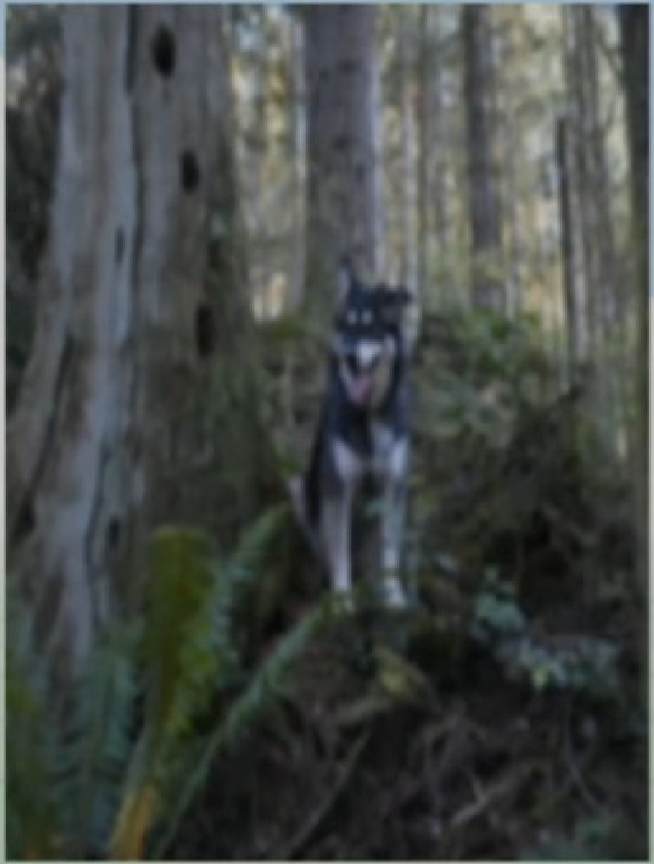


Box filters have artifacts

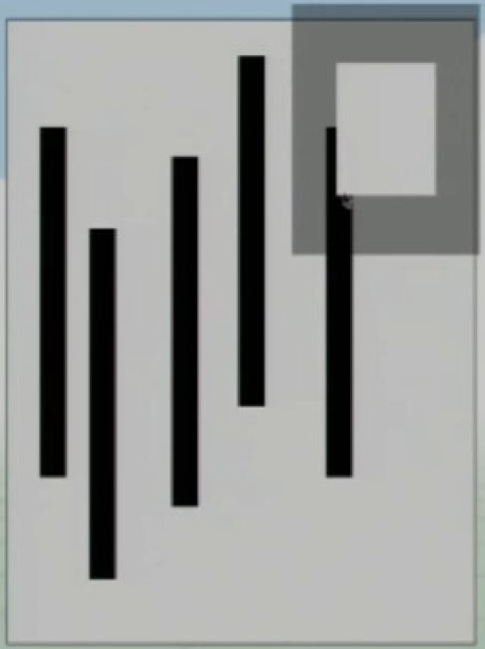


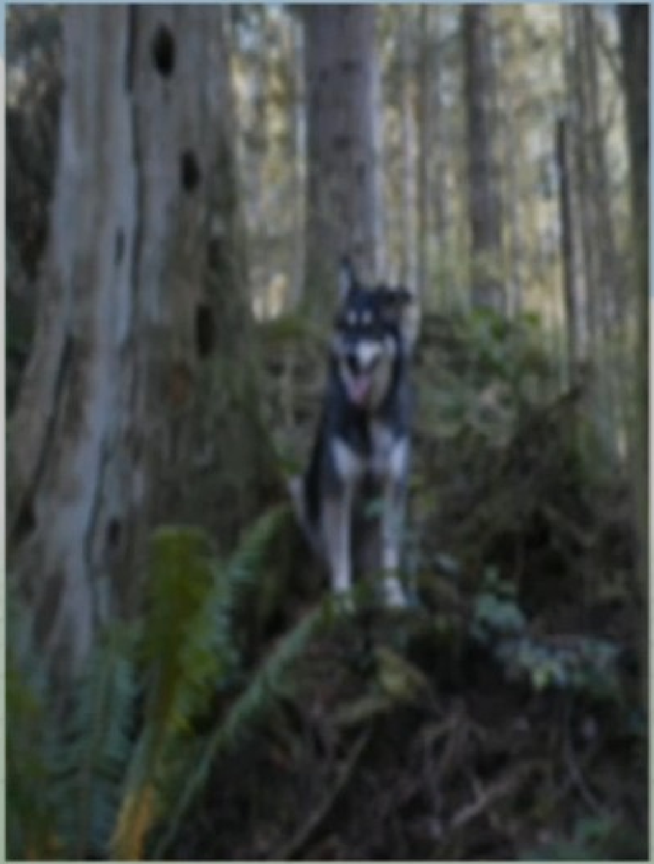
Box filters: vertical + horizontal streaking





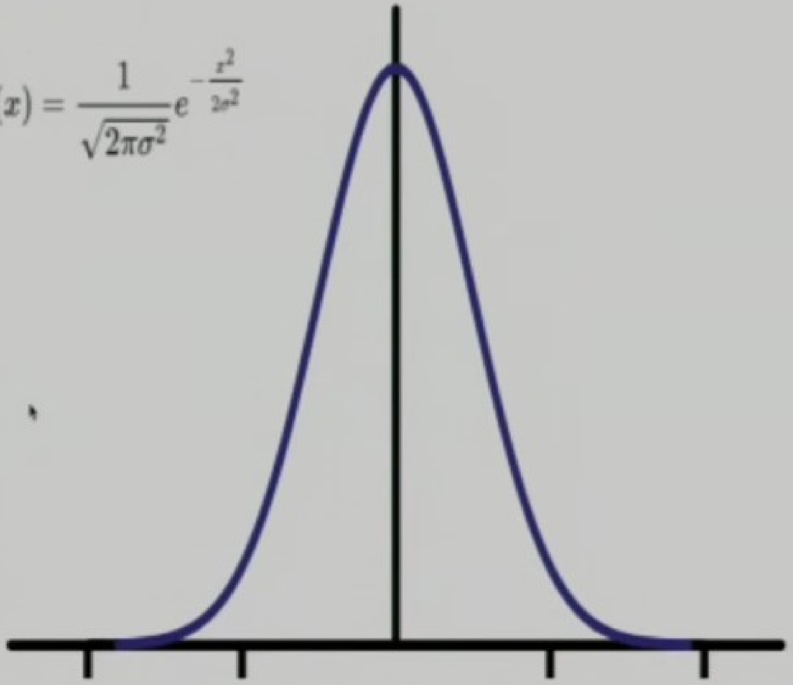
Box filters: vertical + horizontal streaking





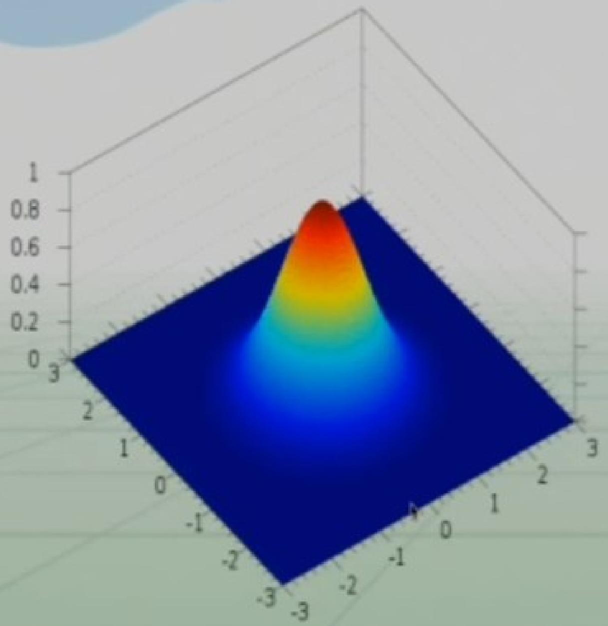
Gaussians

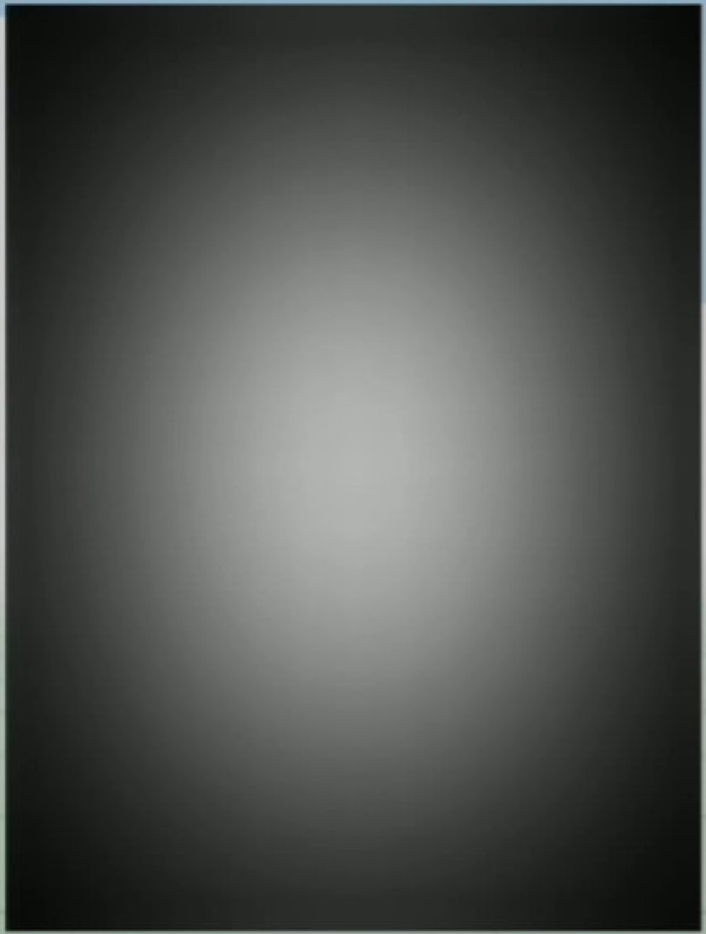
$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$



2d Gaussian

$$g(x,y) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$





Better smoothing with Gaussians

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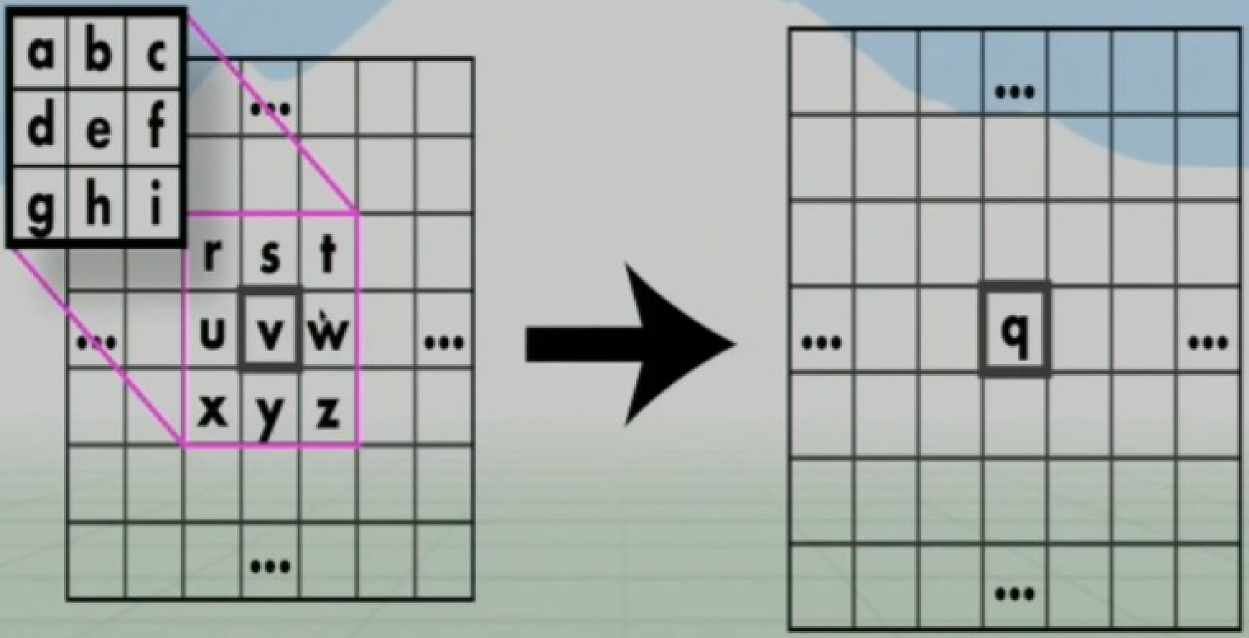
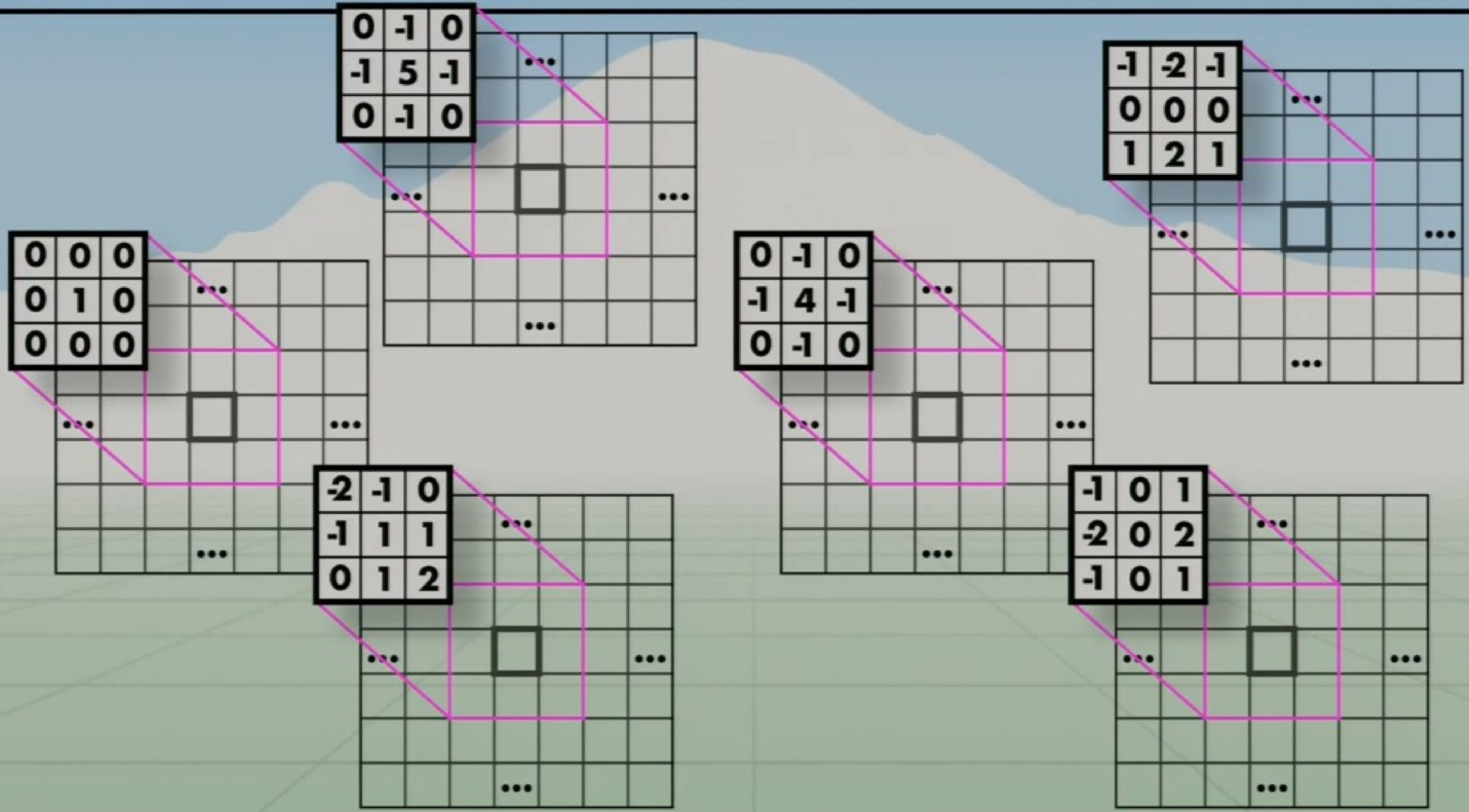
Wow, so what was that convolution thing??



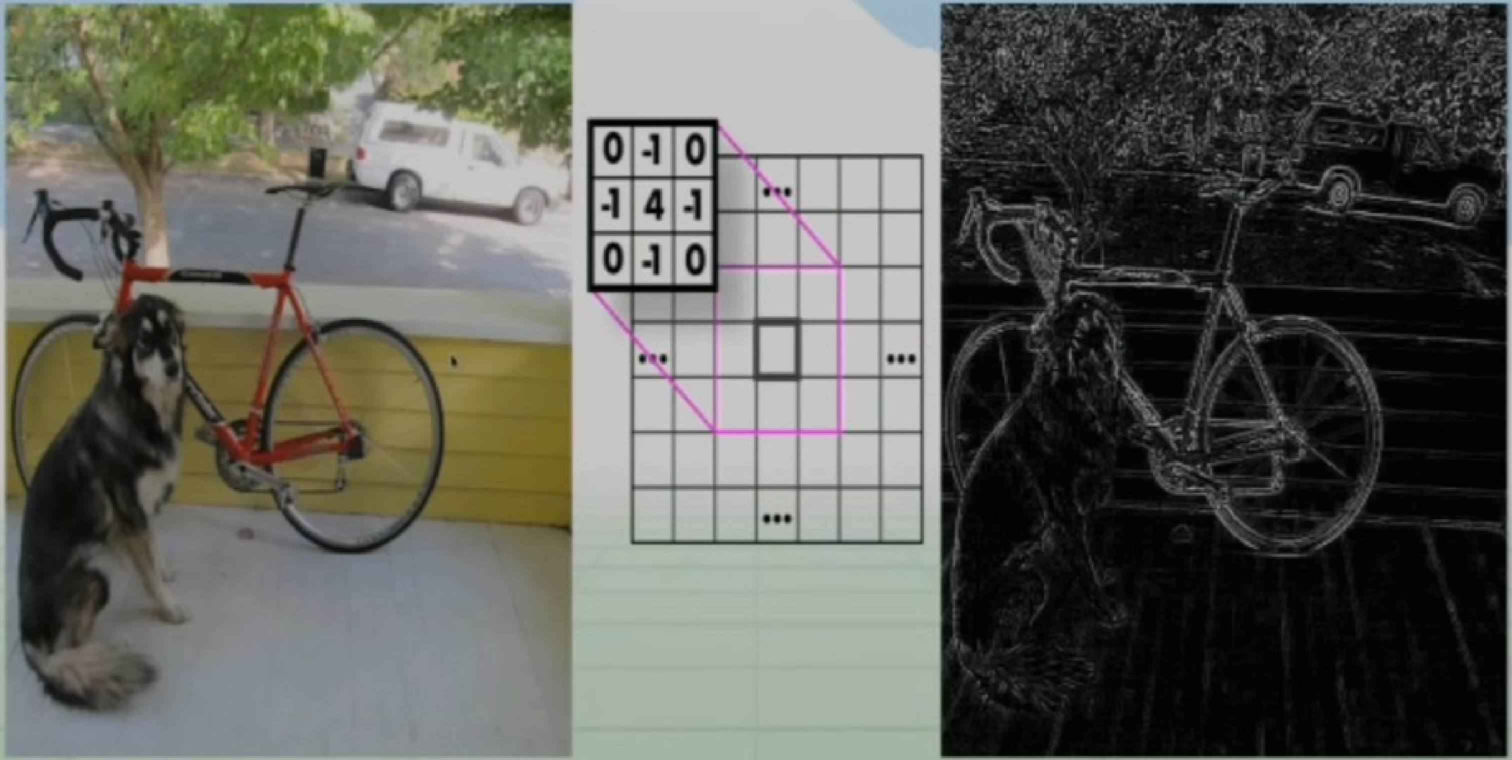
$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

Filters

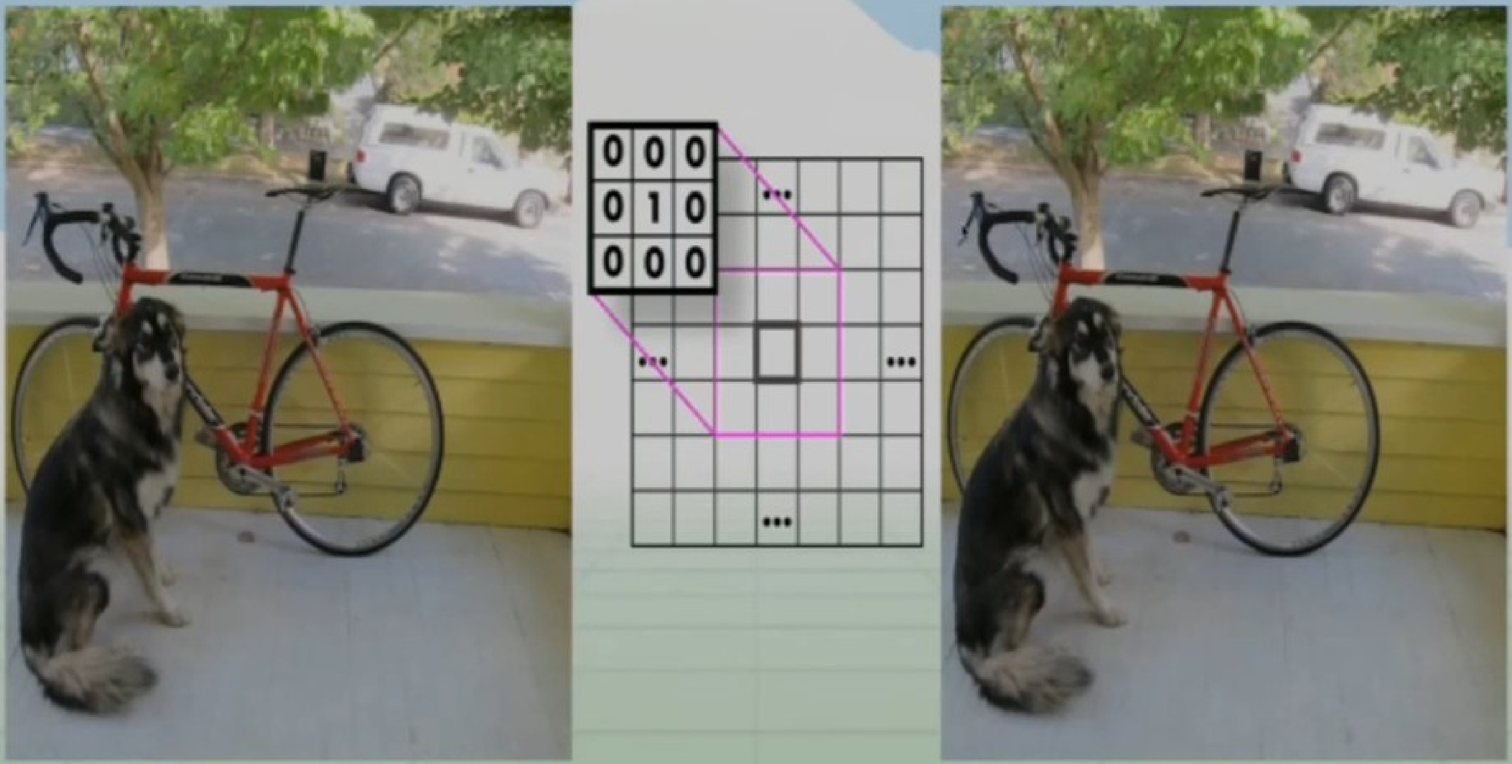


0 -1 0
-1 5 -1
0 -1 0
-1 -2 -1
0 0 0
1 2 1
0 0 0
0 1 0
0 0 0
0 -1 0
-1 4 -1
0 -1 0
-2 -1 0
-1 1 1
0 1 2
-1 0 1
-2 0 2
-1 0 1

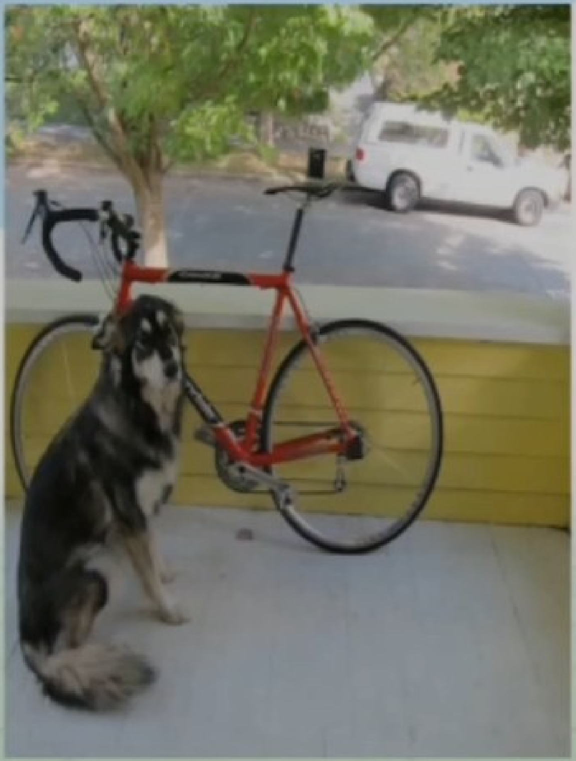
Highpass Kernel: finds edges

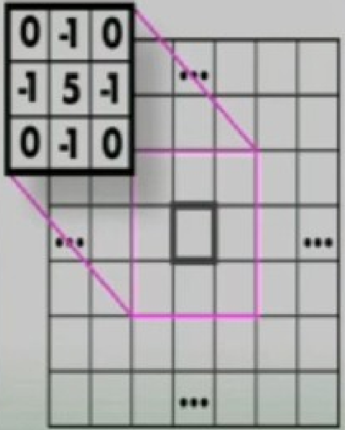


Identity Kernel: Does nothing!



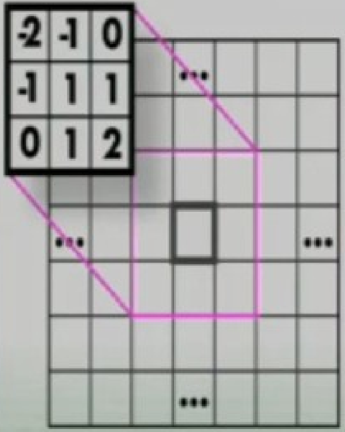
Sharpen Kernel: sharpens!



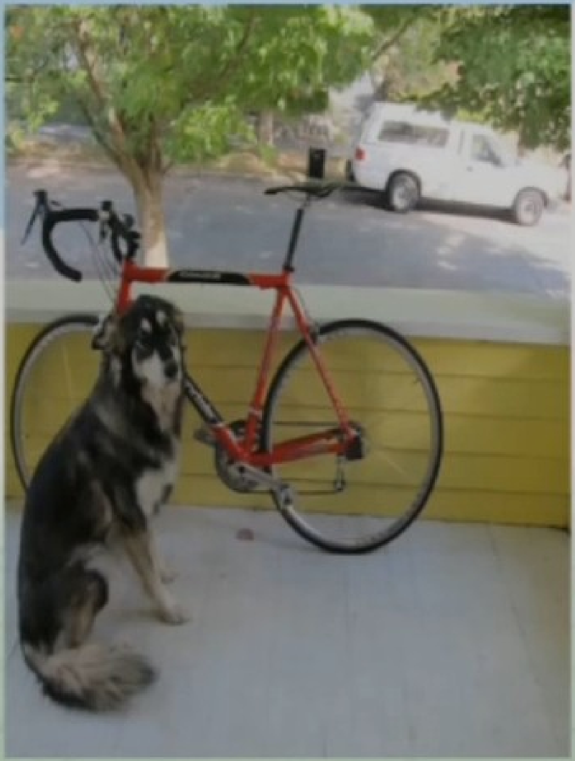


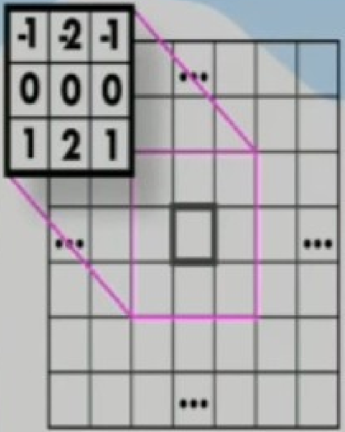
Note: sharpen = highpass + identity!

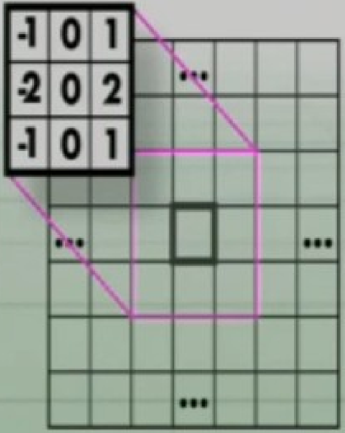
Emboss Kernel: stylin’



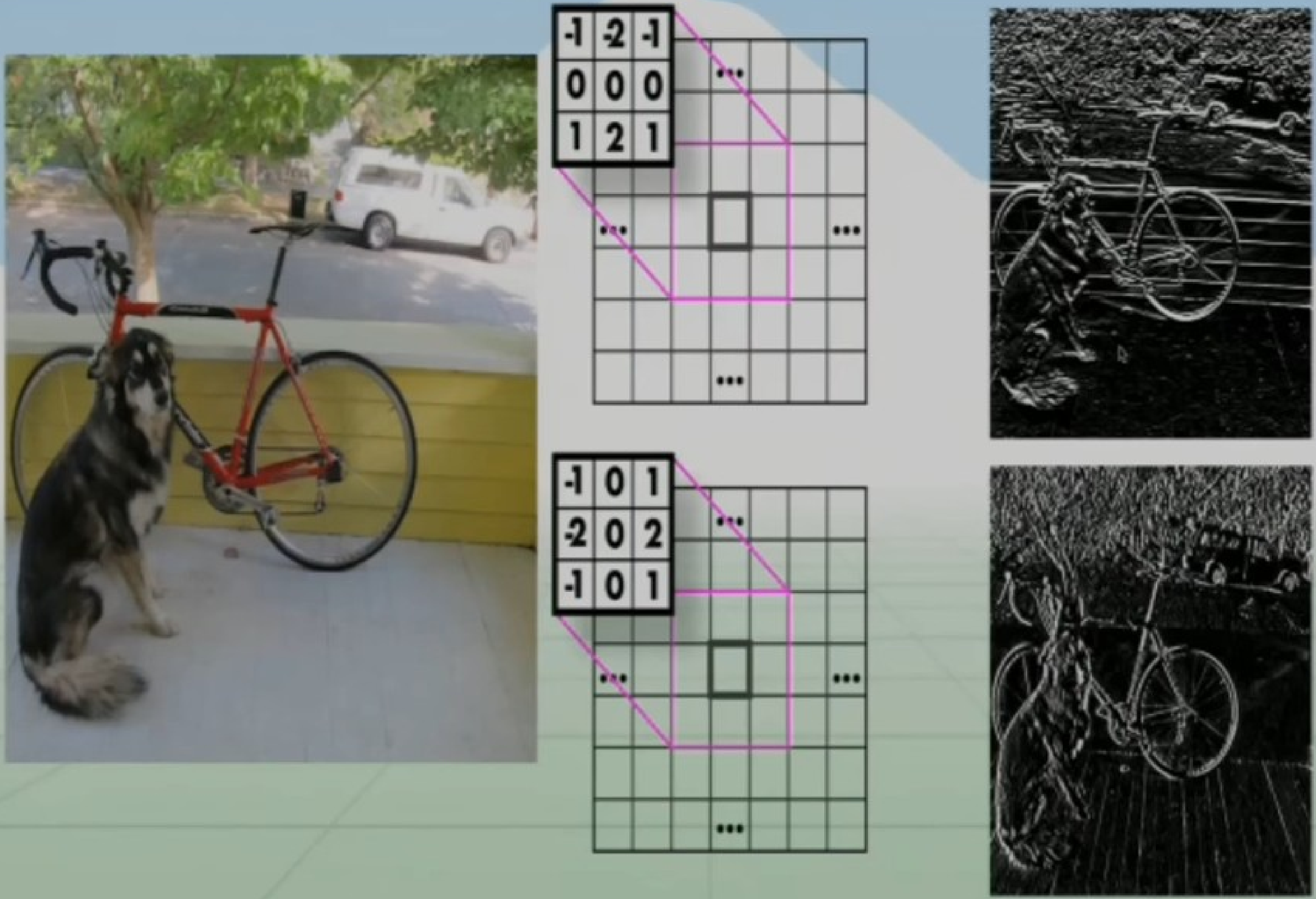
Guess those kernels!







Sobel Kernels: edges and...



So what can we do with these convolutions anyway?

Mathematically: all the nice things

- Commutative
  - A*B = B*A
- Associative
  - A*(B*C) = (A*B)*C
- Distributes over addition
  - A*(B+C) = A*B + A*C
- Plays well with scalars
  - x(A*B) = (xA)*B = A*(xB)

So what can we do with these convolutions anyway?

This means some convolutions decompose:

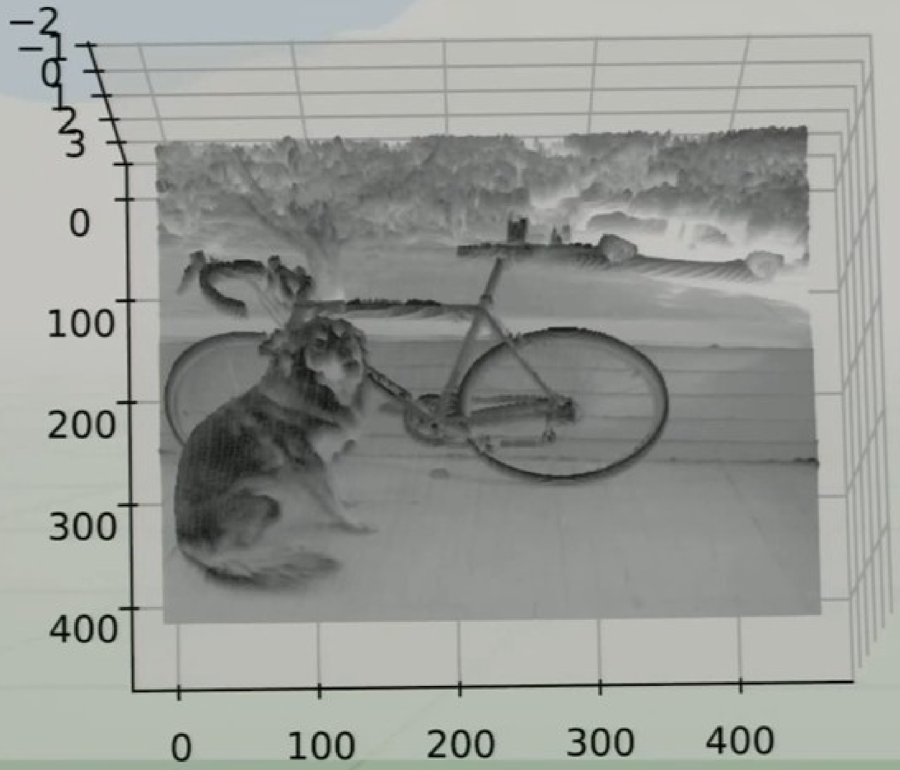
- 2d gaussian is just composition of 1d gaussians
    - Faster to run 2 1d convolutions

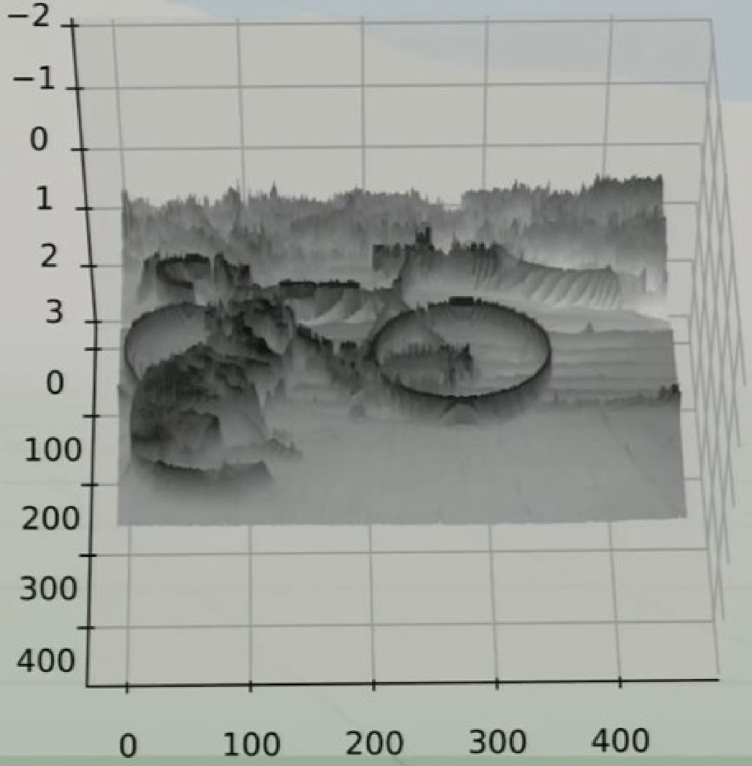
So what can we do with these convolutions anyway?

- Blurring
- Sharpening
- Edges
- Features
- Derivatives
- Super-resolution
- Classification
- Detection
- Image captioning
- ...

What's an edge?

- Image is a function
- Edges are rapid changes in this function





What's an edge?

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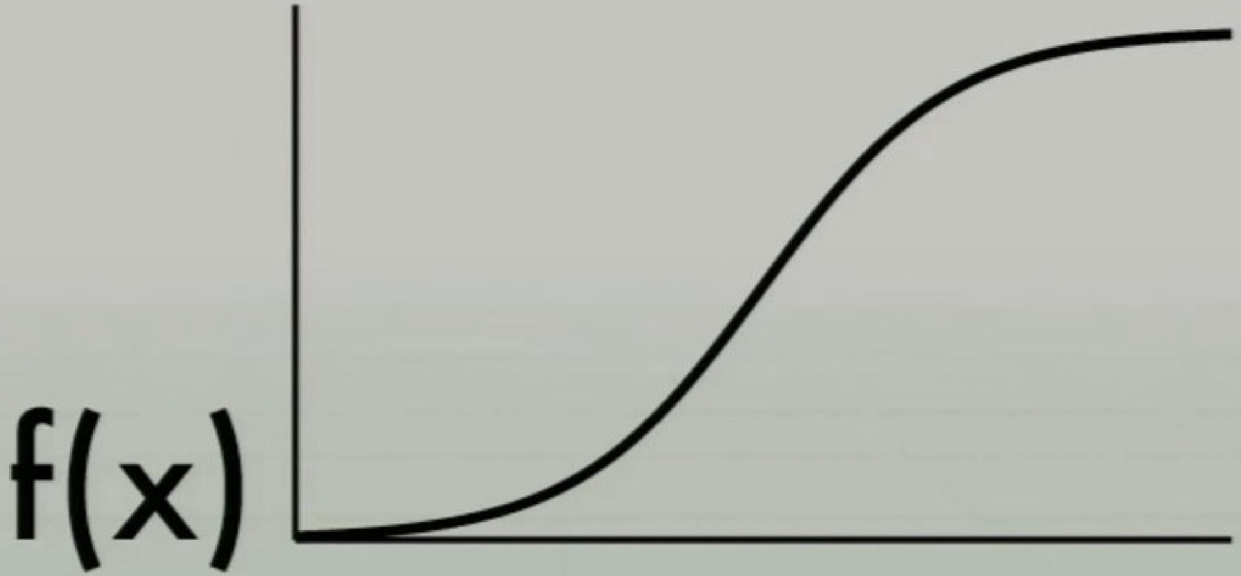


Finding edges

- Could take derivative
- Edges = high response

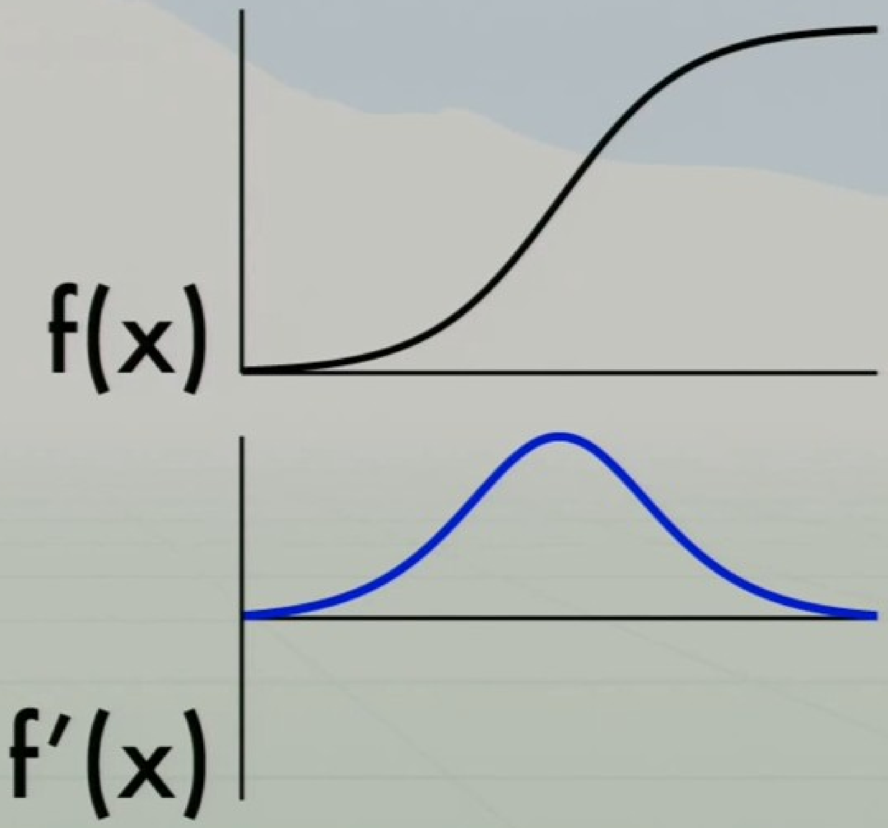
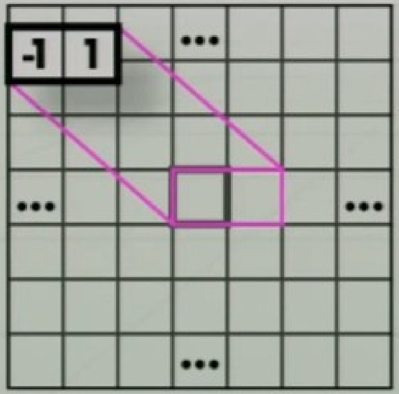


Image derivatives

- Recall:
  - $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$
- We don’t have an “actual” Function, must estimate
- Possibility: set h = 1
- What will that look like?



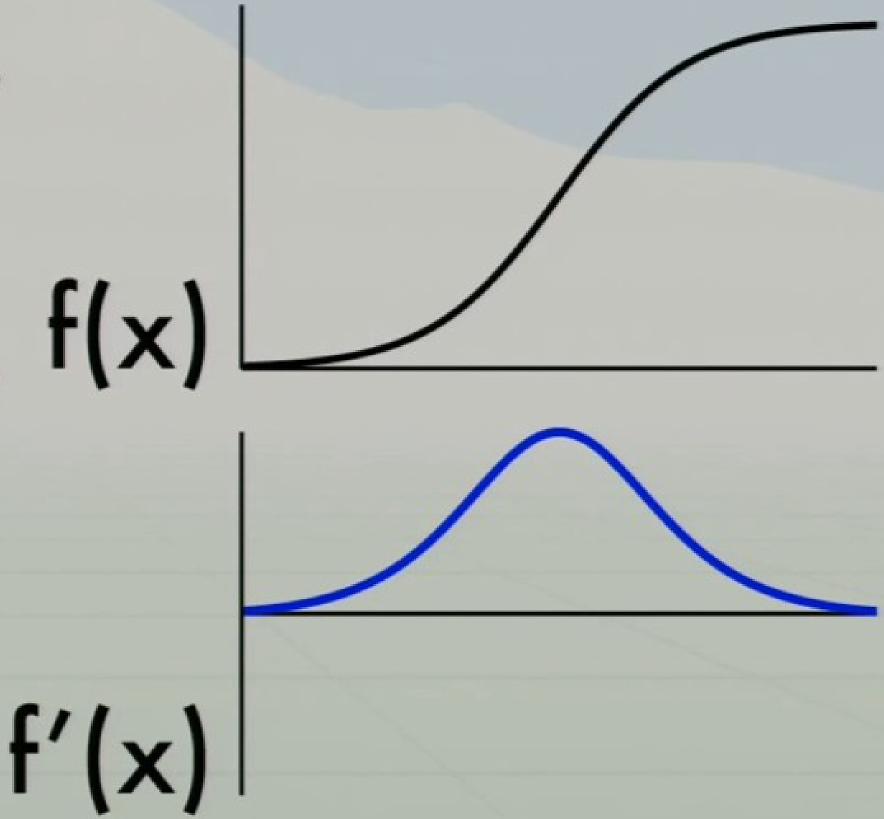
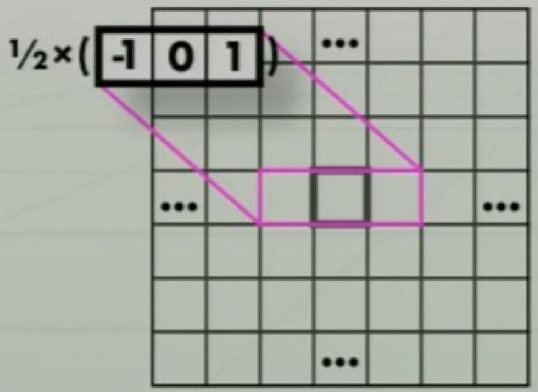
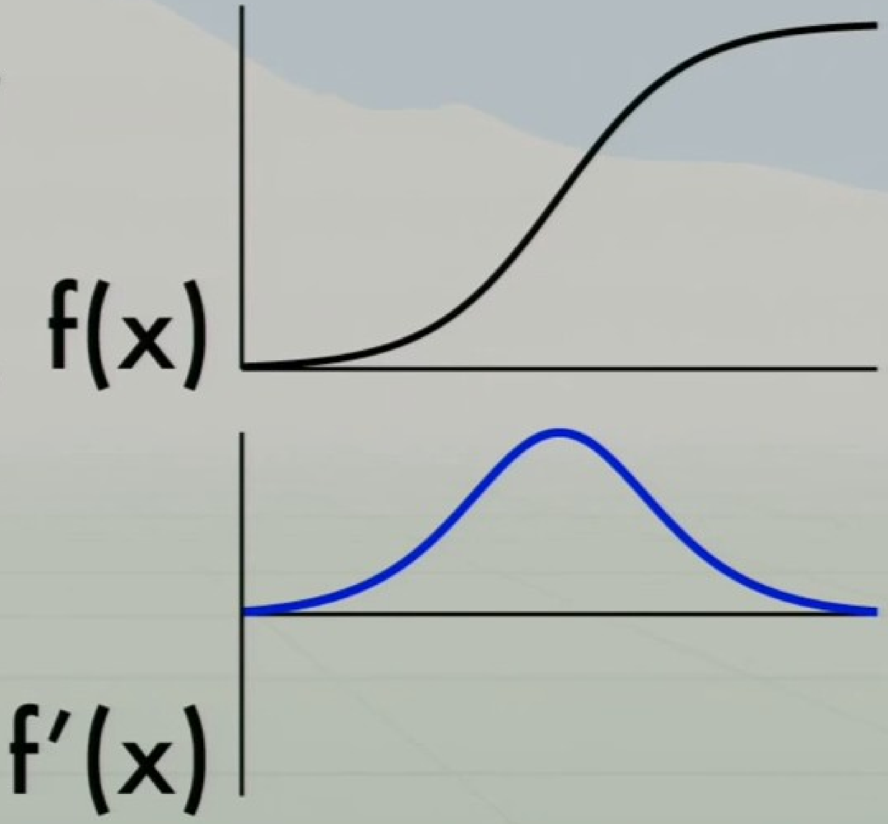


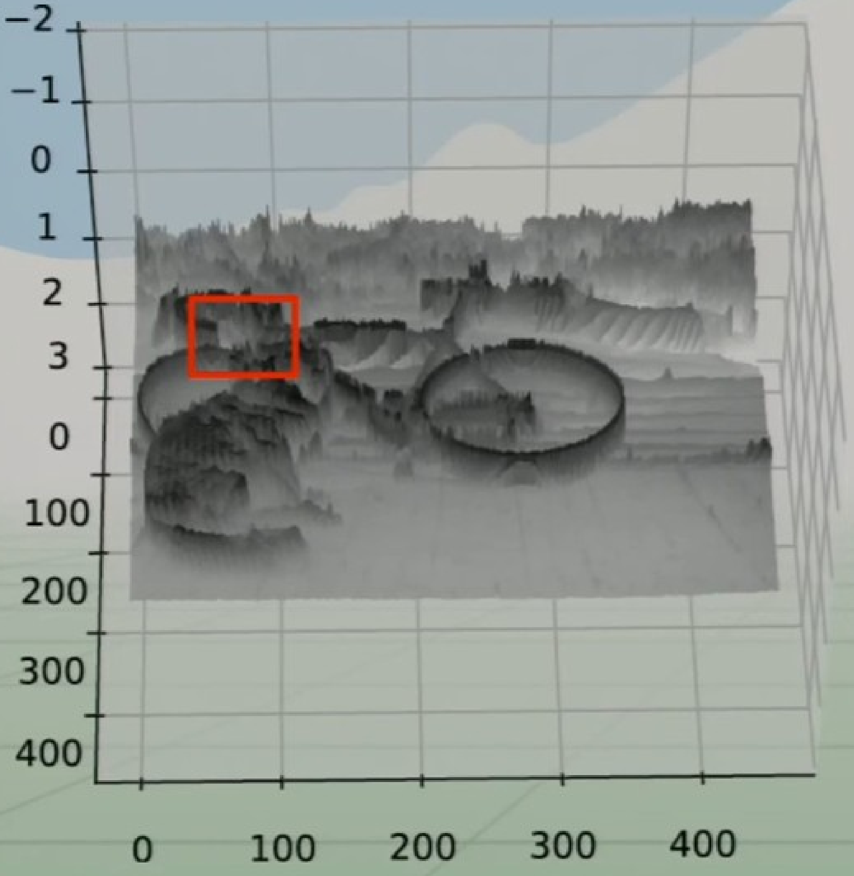
Image derivatives

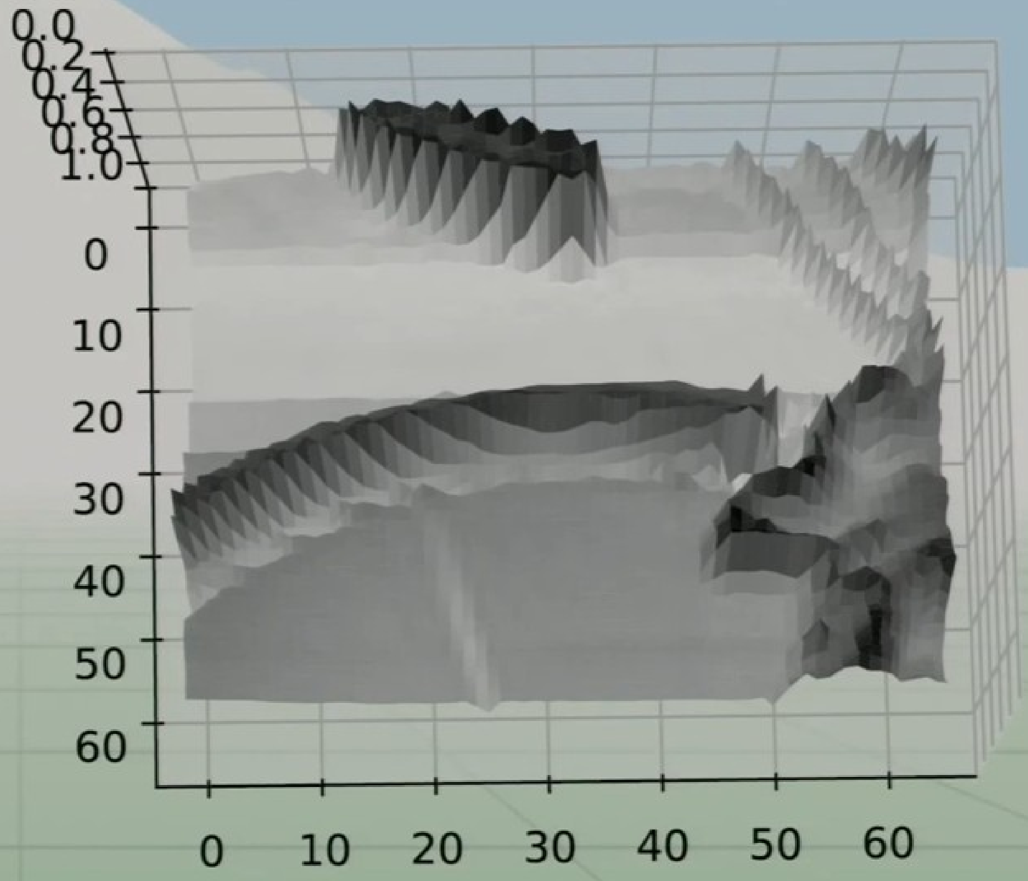
- Recall:
  - $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$
- We don't have an "actual" Function, must estimate
- Possibility: set h = 2
- What will that look like?



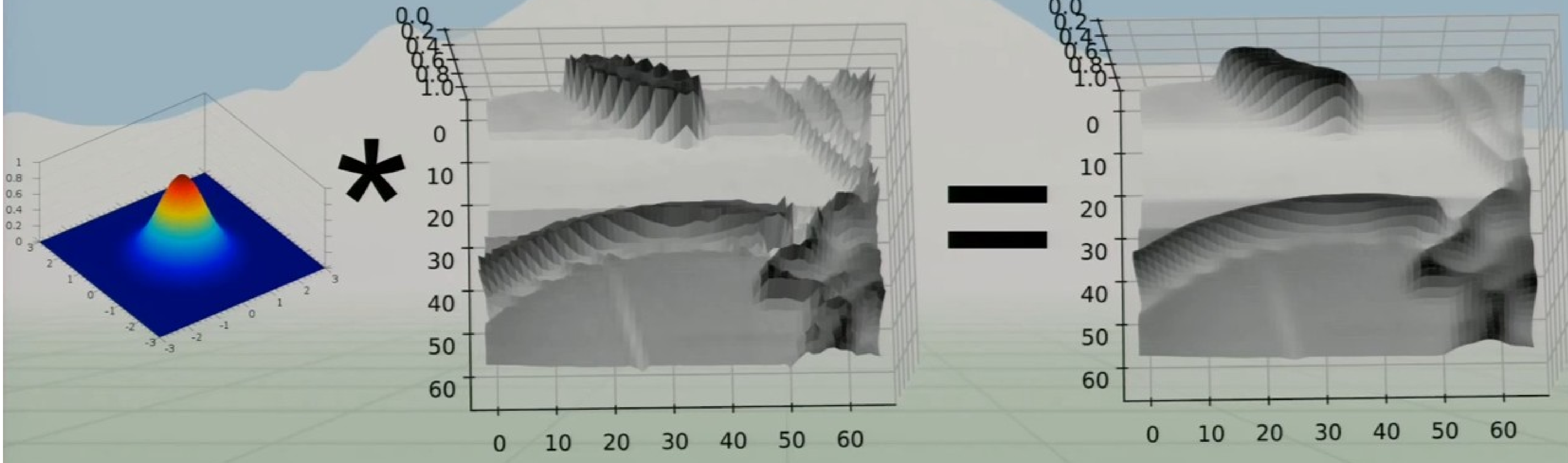


Images are noisy!



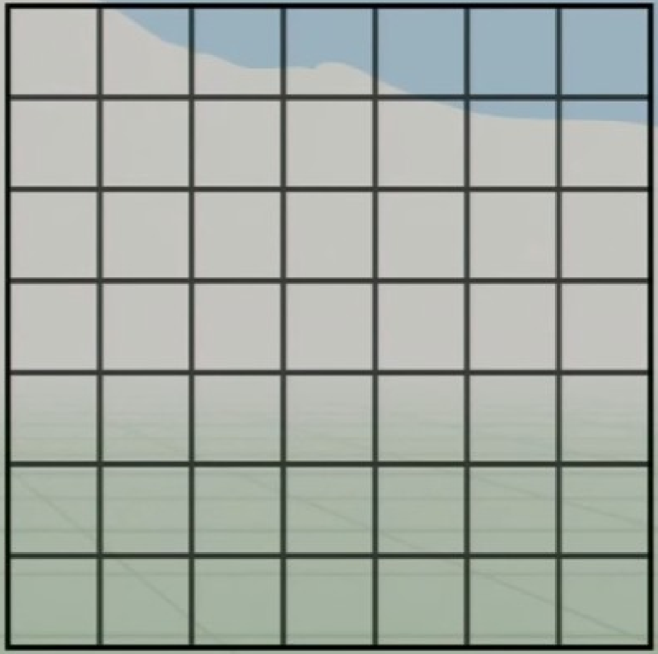


But we already know how to smooth



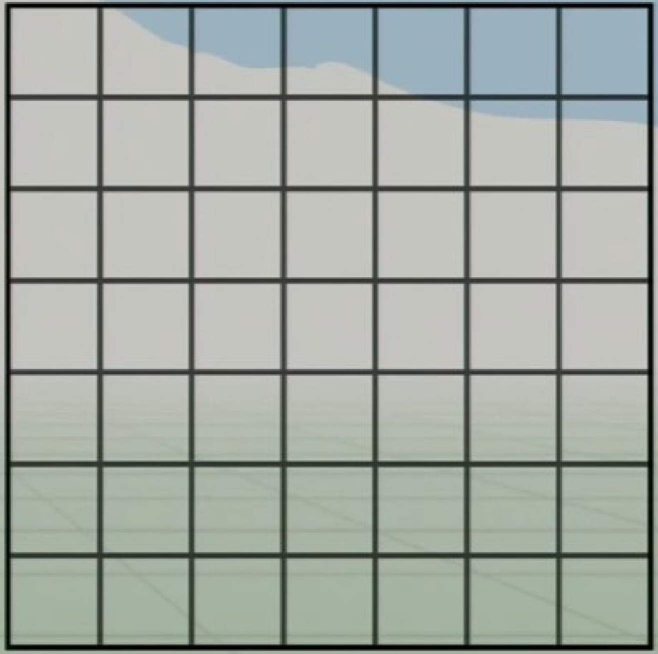
Smooth first, then derivative

$$\tfrac{1}{2}\times\left(\boxed{\begin{array}{c|c|c}-1&0&1\end{array}}\right) * \left(\boxed{\begin{array}{c|c|c}1&2&1\\ \hline 2&4&2\\ \hline 1&2&1\end{array}} * \qquad \right)$$



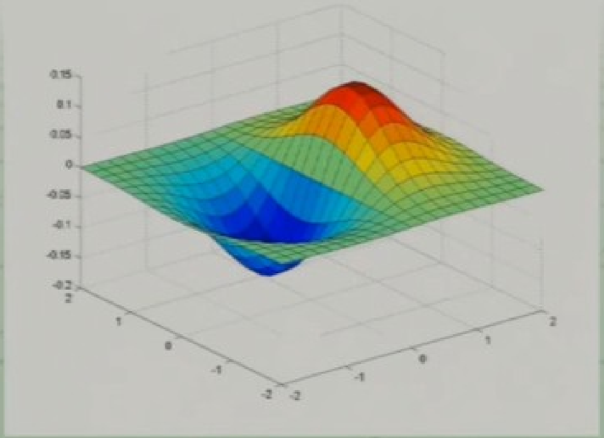
Smooth first, then derivative

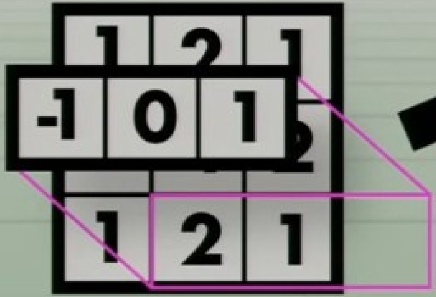
$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right) *$$



Sobel filter! Smooth & derivative

$$\tfrac{1}{2}\times\left(\begin{bmatrix}-1 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}\right)$$

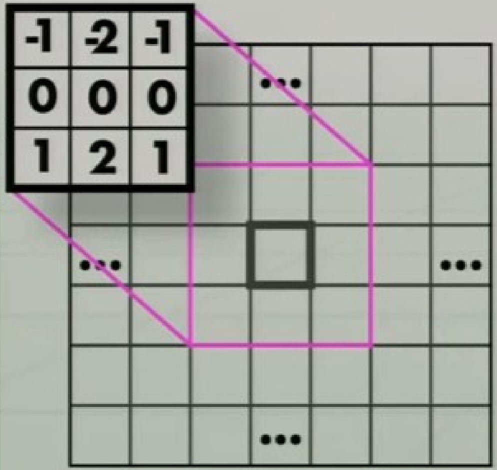


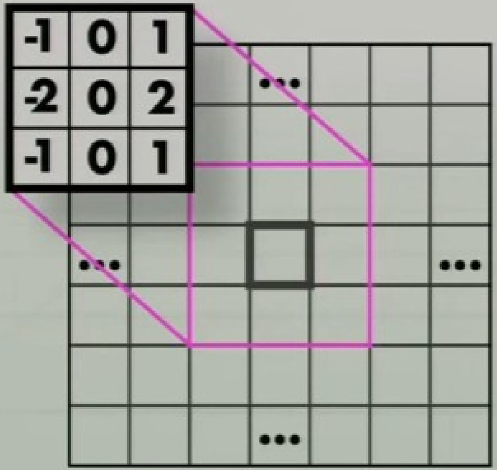


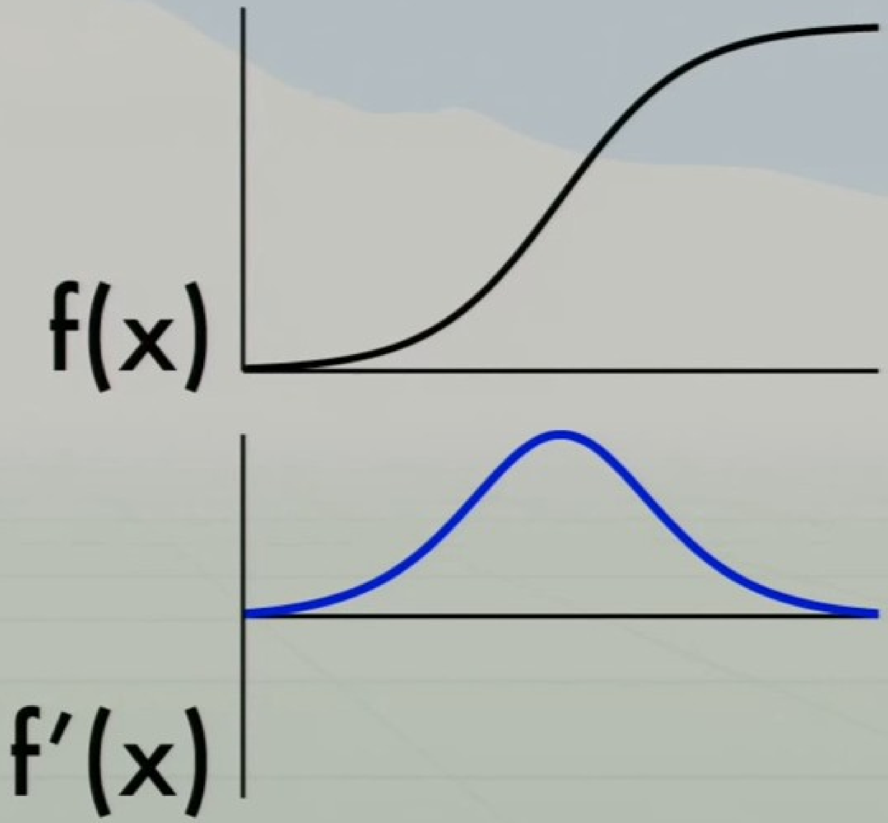
$$\longrightarrow \begin{bmatrix}1 & 0 & -1\\ 2 & 0 & -2\\ 1 & 0 & -1\end{bmatrix}$$

Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- But...







Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- But...
- Edges go both ways
- Want to find extrema