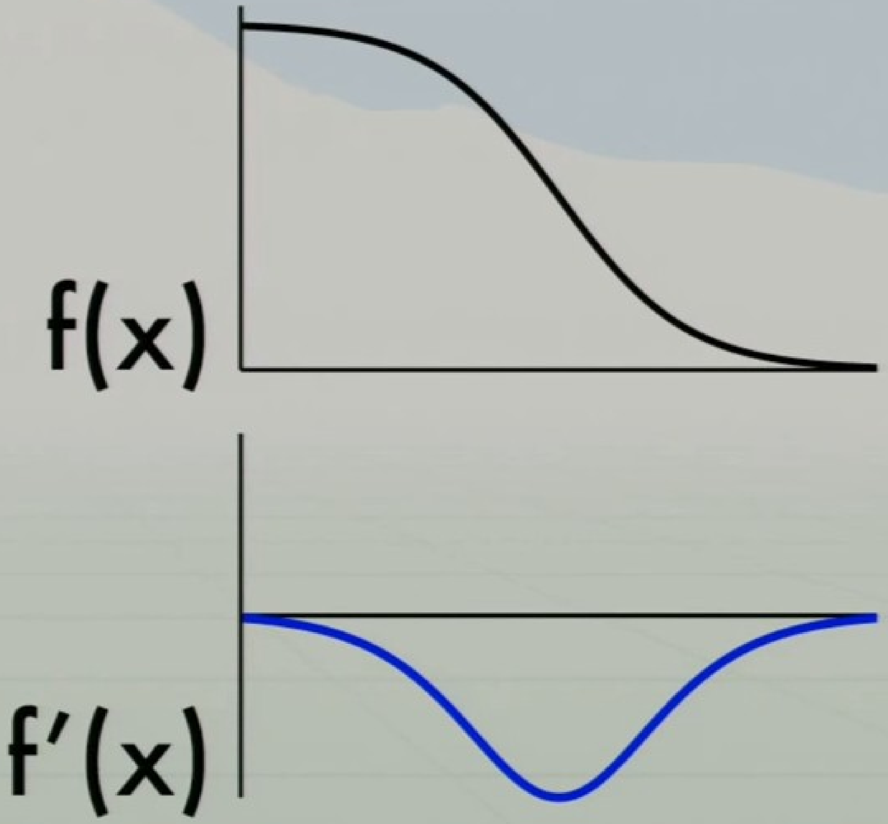
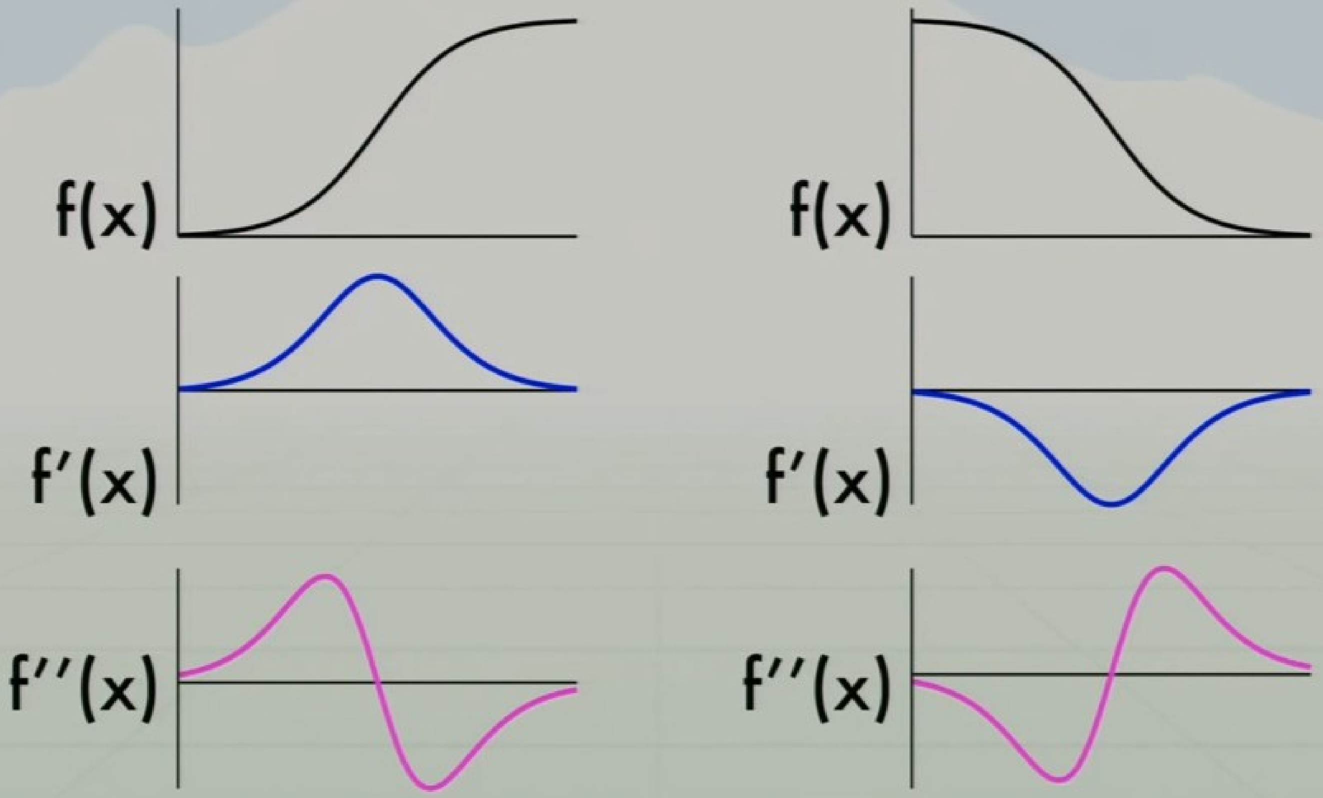
Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- But...
- Edges go both ways
- Want to find extrema



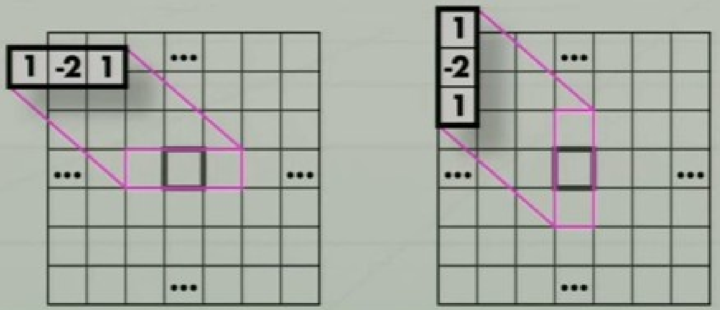
2nd derivative!

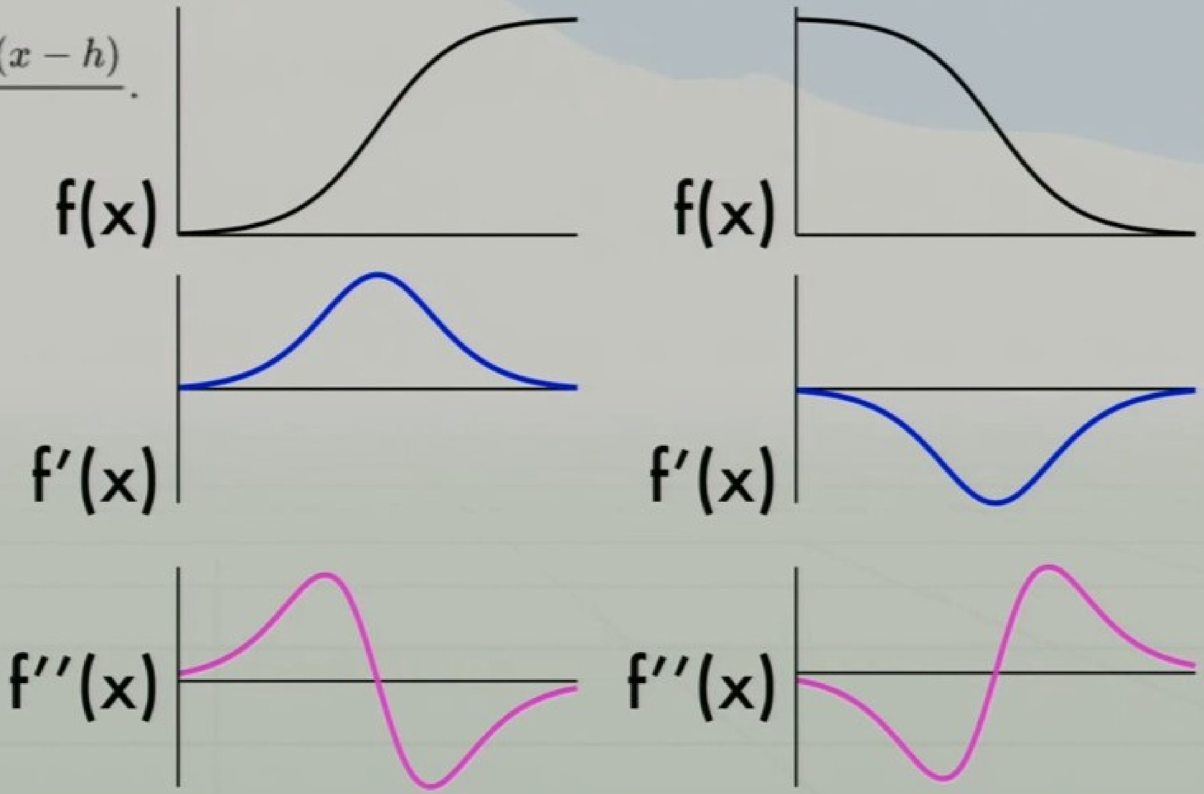
- Crosses zero at extrema



Laplacian (2nd derivative)!

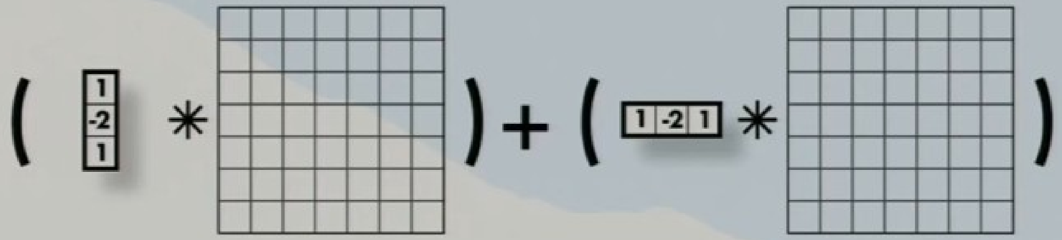
- Crosses zero at extrema
- Recall:
  - $f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Again, have to estimate f''(x):





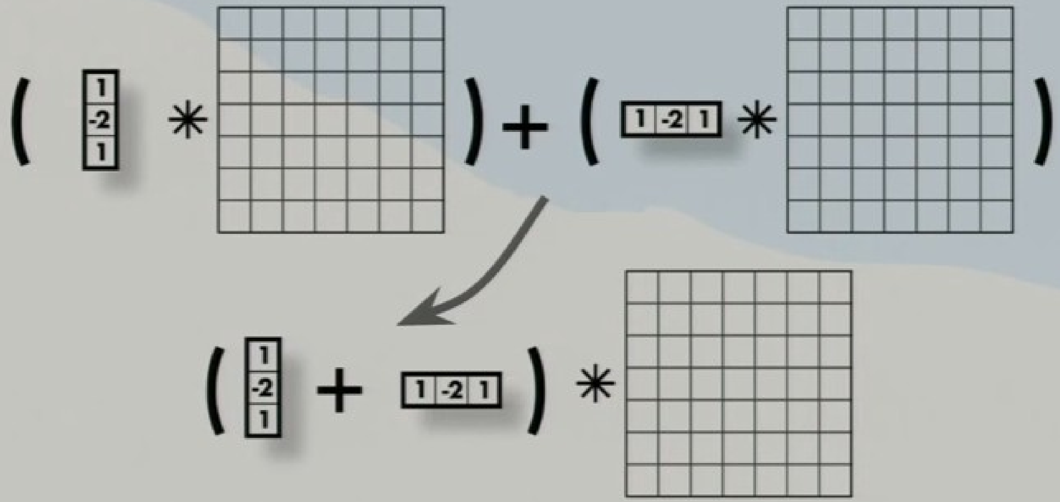
Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$



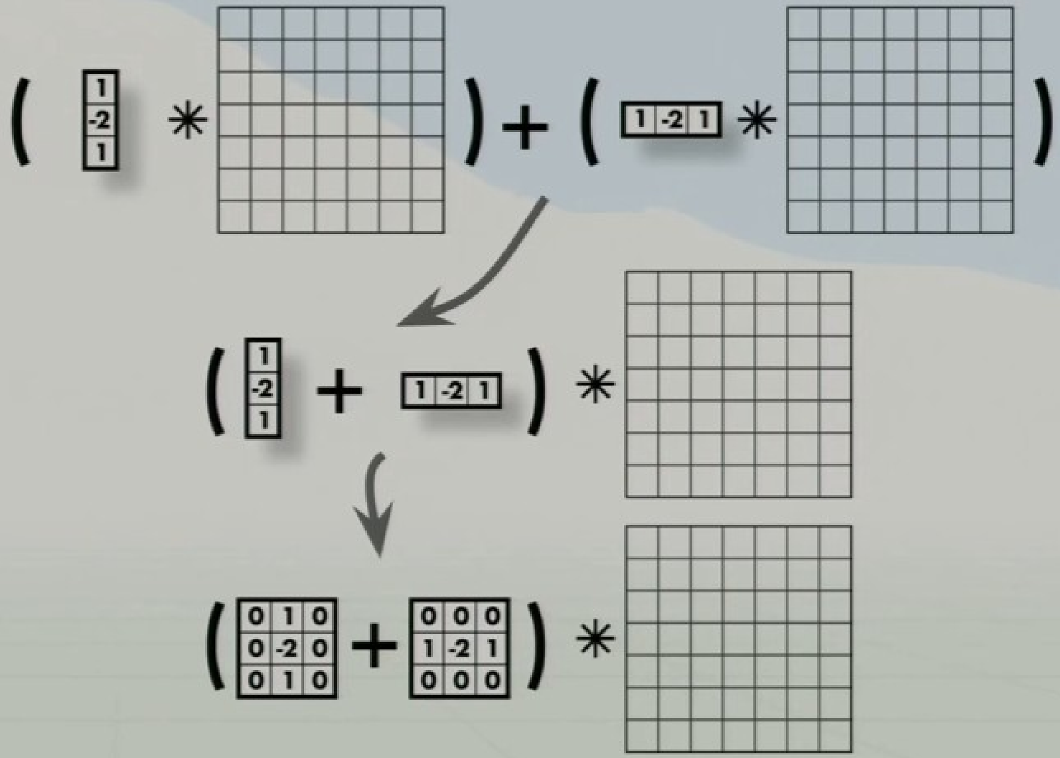
Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$



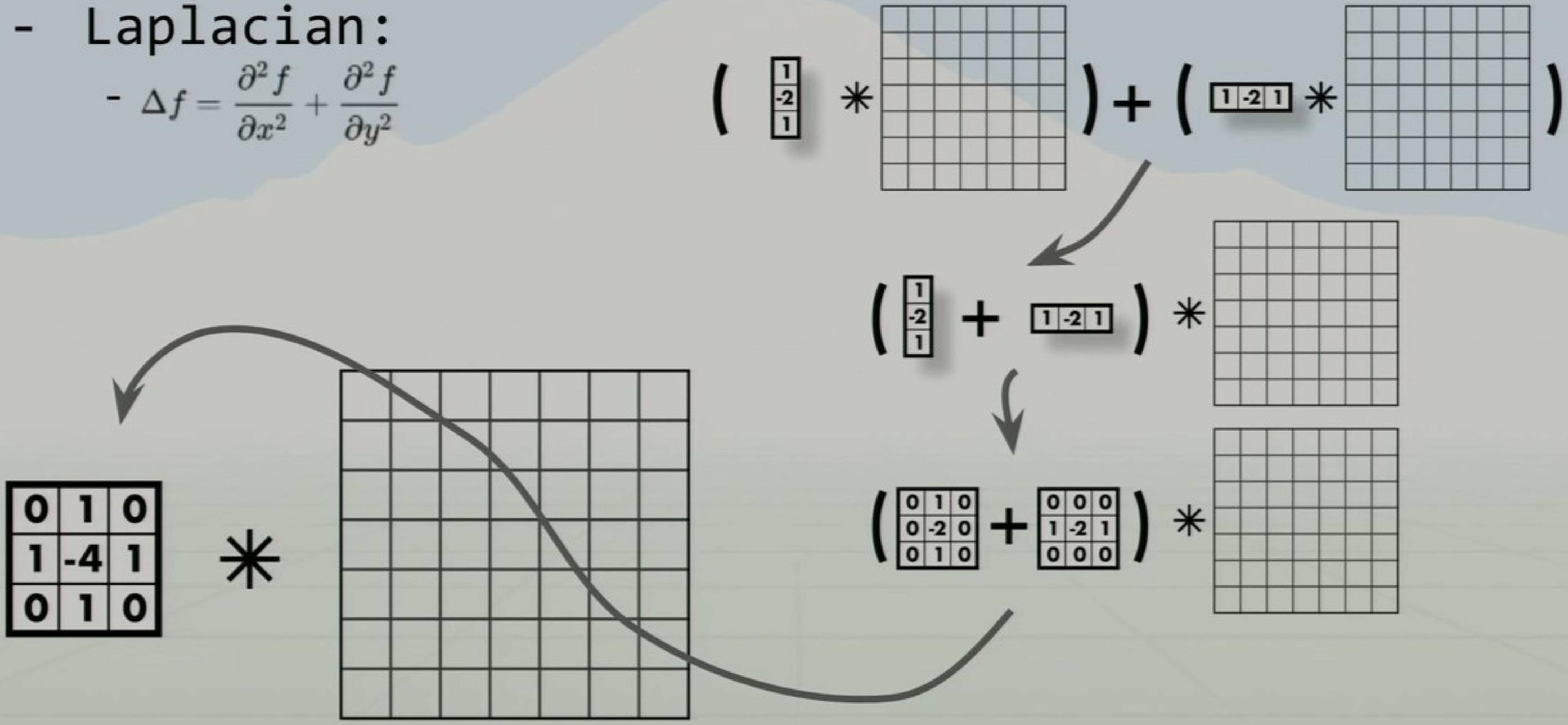
Laplacians

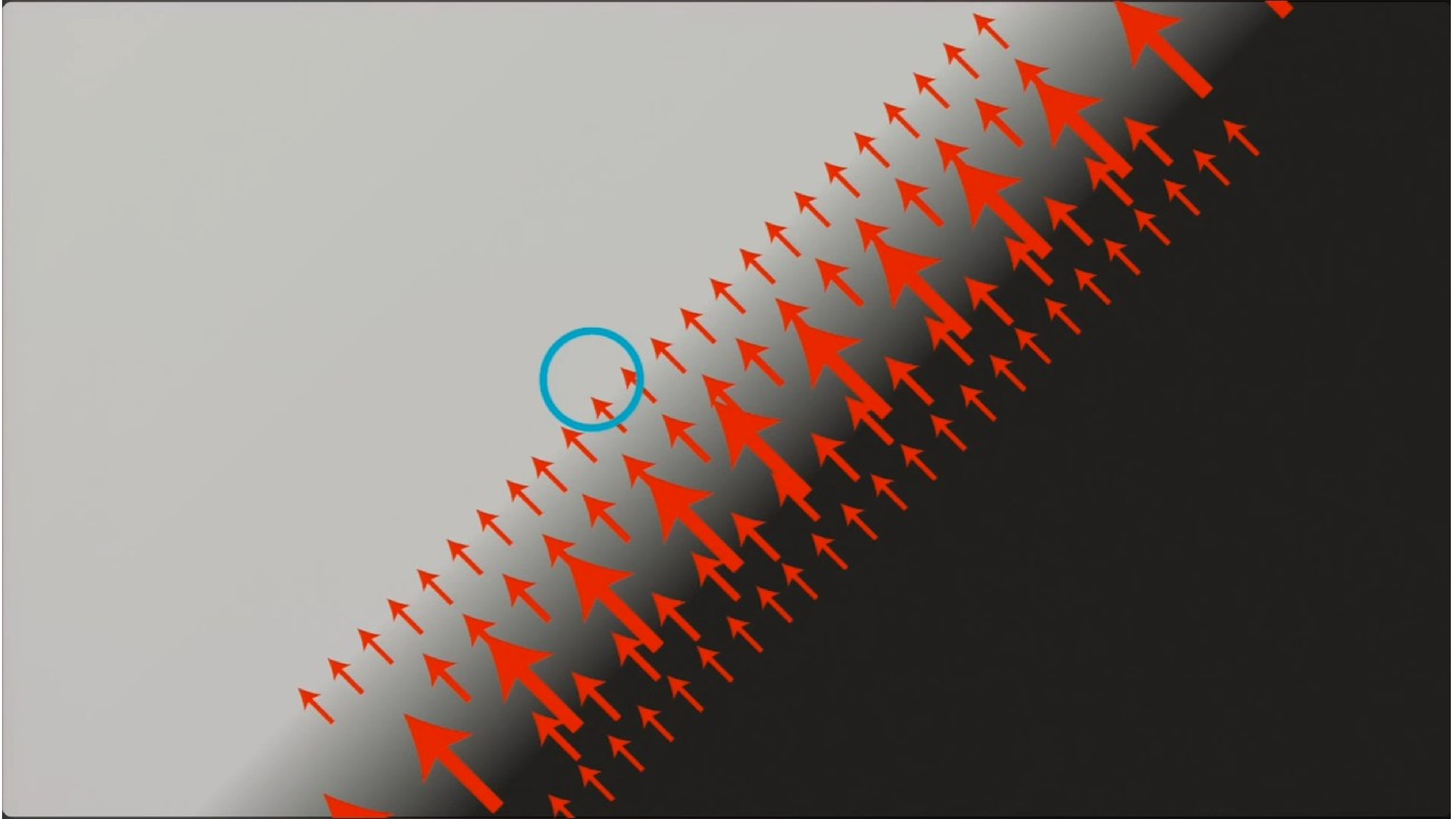
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

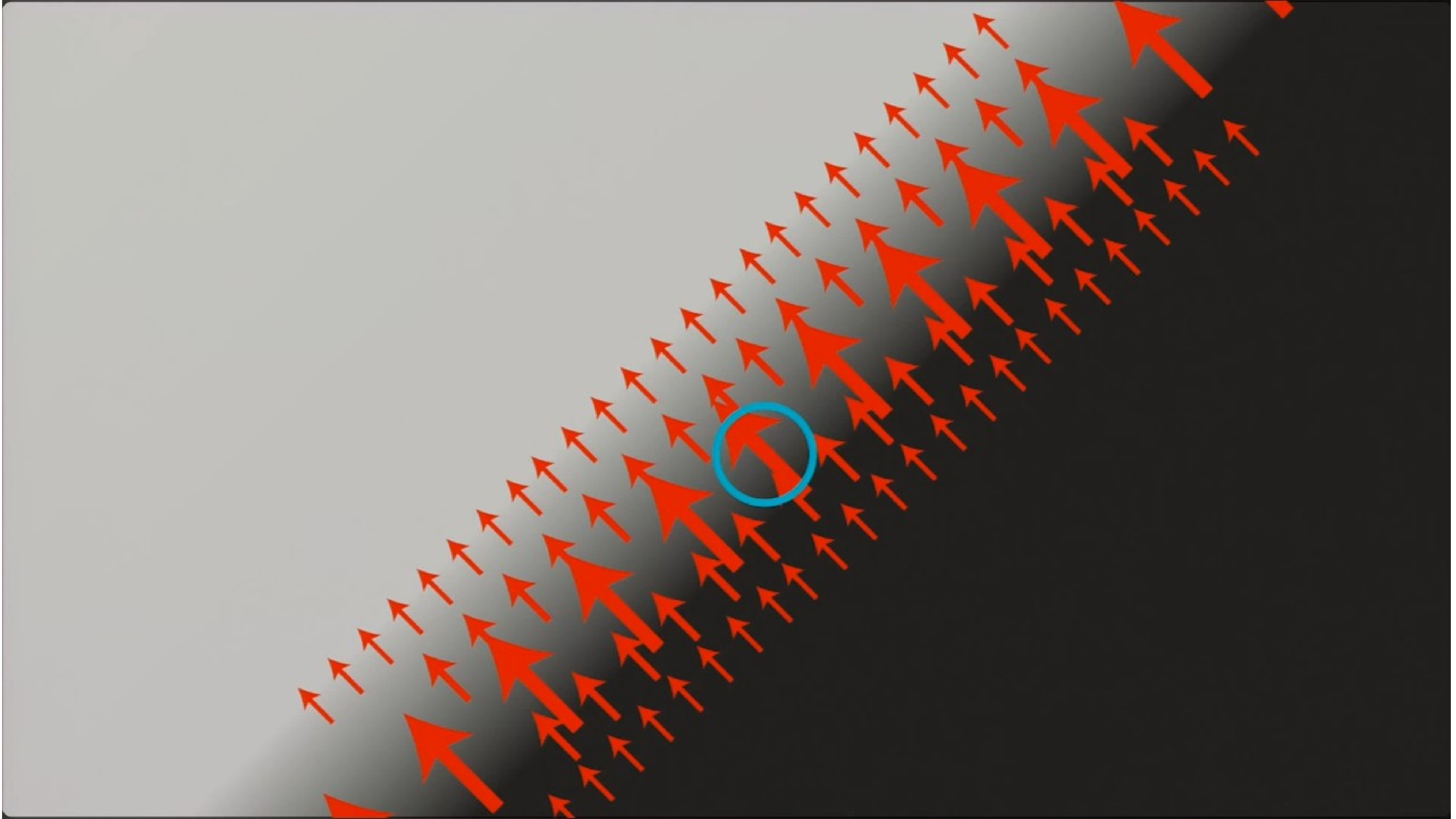


Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

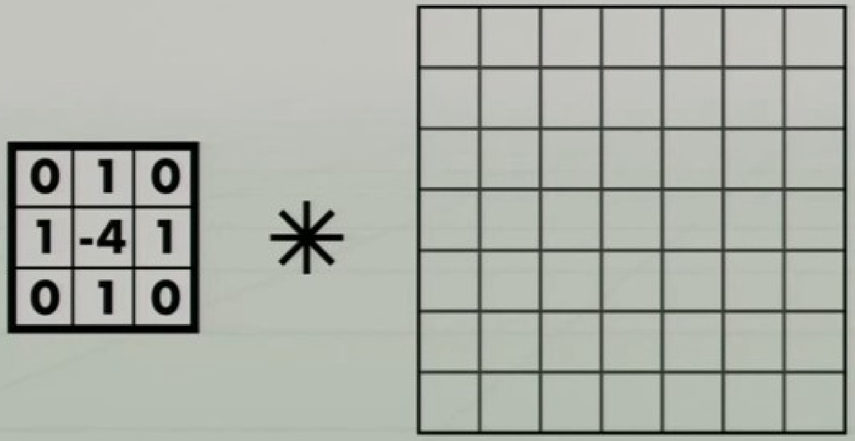


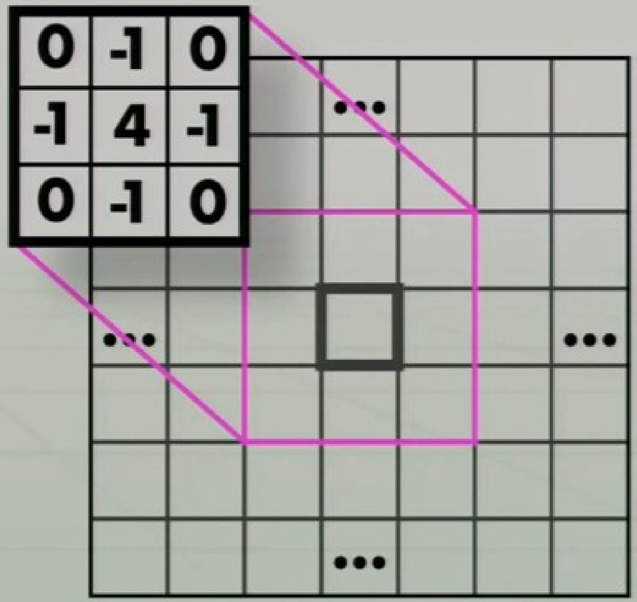




Laplacians

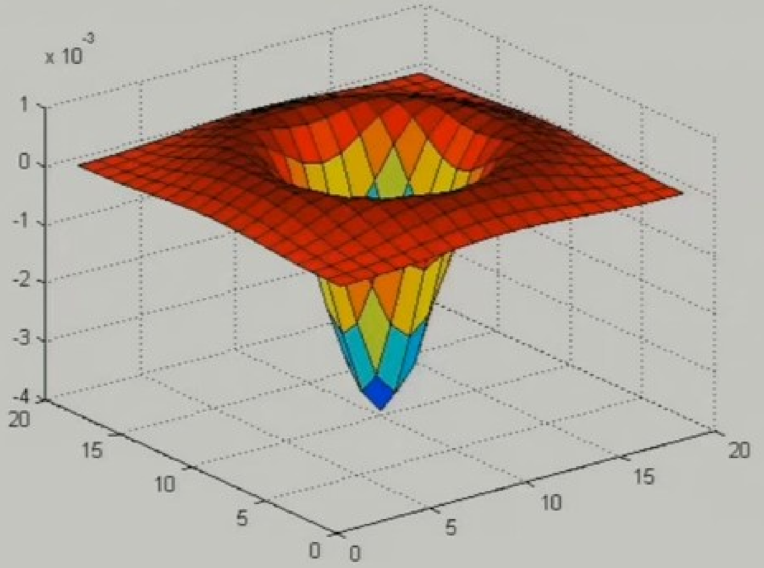
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Negative Laplacian, -4 in middle
- Positive Laplacian --->





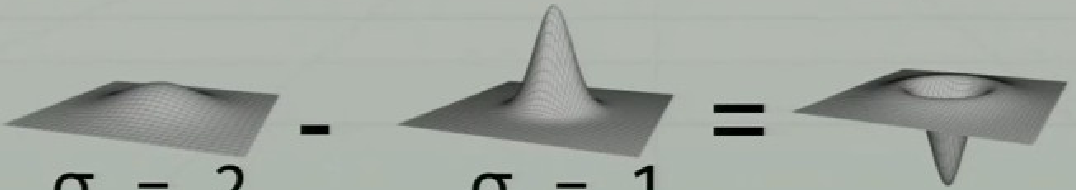
Laplacians also sensitive to noise

- Again, use gaussian smoothing
- Can just use one kernel since convs commute
- Laplacian of Gaussian, LoG
- Can get good approx. with
  5x5 - 9x9 kernels



Difference of Gaussian (DoG)

- Gaussian is a low pass filter
- Strongly reduce components with frequency $f < \sigma$
- $(g*I)$ low frequency components
- $I - (g*I)$ high frequency components
- $g(\sigma 1)*I - g(\sigma 2)*I$
  - Components in between these frequencies
- $g(\sigma 1)*I - g(\sigma 2)*I = [g(\sigma 1) - g(\sigma 2)]*I$



$\sigma = 2$ $\sigma = 1$

DoGs