Exercise 3: General Classification of Fixed Points

For any 2x2 matrix $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

we have the following properties:

- trace($\mathbf{A}$) = $a + d = \tau$
- det($\mathbf{A}$) = $ad - bc = \Delta$

and the characteristic equation for $\mathbf{A}$ is:

$$(a - \lambda)(d - \lambda) - bc = 0$$

$$\lambda^2 + (-a - d)\lambda + ad - bc = 0$$

which we can re-write in terms of τ and Δ as:

$$\lambda^2 - \tau\lambda + \Delta = 0$$

therefore, the quadratic formula gives us:

$$\lambda = \tfrac{1}{2}(\tau \pm \sqrt{\tau^2 - 4\Delta})$$

so now we can think of λ in terms of τ and Δ and can immediately learn about the stability of our model and types of fixed points.

Along axes of τ and Δ solve the above solution for λ and fill out the following space in terms of the sign of each eigenvalue (positive or negative) and whether λ is real or complex. Each region of the space below will correspond to different types of fixed points, with special cases on the boundaries.