Damping of oscillations of a string by using multiple point dampers

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The oscillations of a string are described by the equation:

$$y_{tt} = a^2 y_{xx} + g(t,x), \quad 0 <= t <= T, \quad 0 <= x <= l, \quad a = \text{const}.$$

The initial conditions: deviation and velocity – are known:

$$y(0,x) = h_0(x), \quad y_t(0,x) = h_1(x). \tag{1}$$

On the boundary of a string imposes fixing conditions:

$$y(t,0) = y(t,l) = 0.$$

The problem of damping is: to find the control function $g(t,x)$, which allows to get the state of a string from initial state (1) to final state:

$$y(T,x) = 0, \quad y_t(t,x) = 0.$$

This problem also was considered by Lagness [1], Rassel [2], Butkovsky [3]. All these works used the condition of Levinson [4], from which follows theoretical time of damping of oscillations $T = 2l/a$. These approaches show the problem has the solutions, however they are difficult to use in practical applications. In this regard, one considered another approaches including usage of point damper which moves on small part of a string [5].

In this report we consider the damping of oscillations of a string by using multiple point dampers. The function $g(t,x)$ is considered in the form:

$$g(t,x) = \sum_{i=1}^{n} W_i(t)\delta(x - x_i),$$

where x_i – the points on a string where dampers are placed,
$\delta(x - x_i) = \begin{cases} 1, & x = x_i \\ 0, & x \neq x_i \end{cases}$, $W_i(t)$ – control functions.

To find the required control function we use method of gradient descent, to compute gradient we use a method for fast automatic differentiation proposed by Evtushenko [6].

E x a m p l e 1. Let us consider a damping of oscillations of a string with $l = 1$, $a = 1$ by using two dampers, which are placed at $x_1 = 3l/16$, $x_2 = 10l/16$. The initial conditions $h_0(x) = \sin(2\pi x/l)$, $h_1(x) = 0$, $\epsilon = 10^{-4}$. The problem was solved by time $T = 1.2$. Figures 1 and 2 show process of damping oscillations of $y(t, x)$ and the control functions $W_1(t)$ and $W_2(t)$ correspondingly.

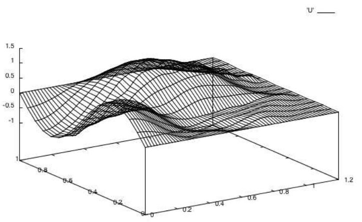

Figure 1: Changing of oscillations by time

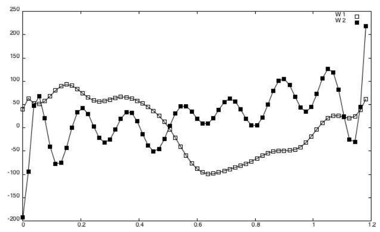

Figure 2: The control functions

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