Machine Learning Notes - 2

Lipschitz Continuity

https://math.stackexchange.com/questions/2374289/understanding-lipschitz-continuity/2374305#2374305

Think about the mean value theorem and Lipschitz continuity.
Mean value theorem says if f is continuous at [a,b] and differentiable at (a,b), then

$$\exists c \in (a, b) \text{ such that } \frac{f(b) - f(a)}{b - a} = f'(c).$$

Lipschitz says that

$$\exists K > 0, \forall a, b \in D_f, \text{ such that } \frac{|f(b) - f(a)|}{|b - a|} \leq K.$$

Then if the derivative of f as a function is bounded, then f will be **Lipschitz**.

Specifically, if **K == 1**, then the function is said to be **1-Lipschitz**.

Consider the case

$$f(x) = \sqrt{x} \text{ for } x \in [0, 1]$$

One interesting fact to notice from these examples is that if a 1D function is differentiable, then its Lipschitz constant is just the maximum value of its derivative. This gives a slightly more rigorous explanation of why √x is not Lipschitz continuous: its derivative (1 / 2) • (1 / √x) is unbounded as x goes to 0.

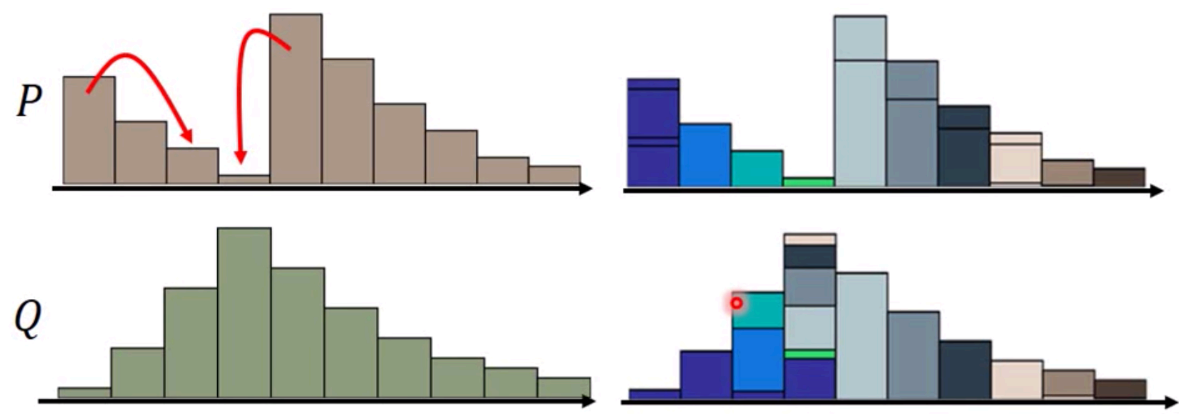
$$\sup_{x \in [0,1]} f'(x) = \lim_{x \to 0} f'(x) = +\infty.$$

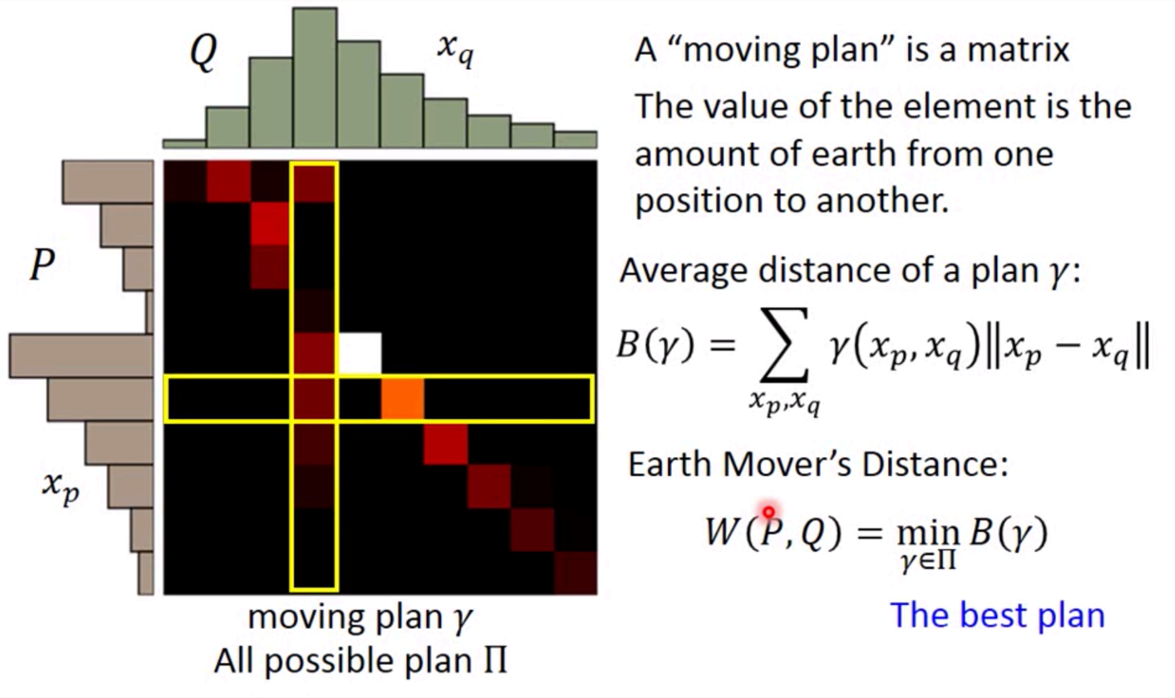
Spectral Normalization

https://christiancosgrove.com/blog/2018/01/04/spectral-normalization-explained.html

Earth Mover's Distance

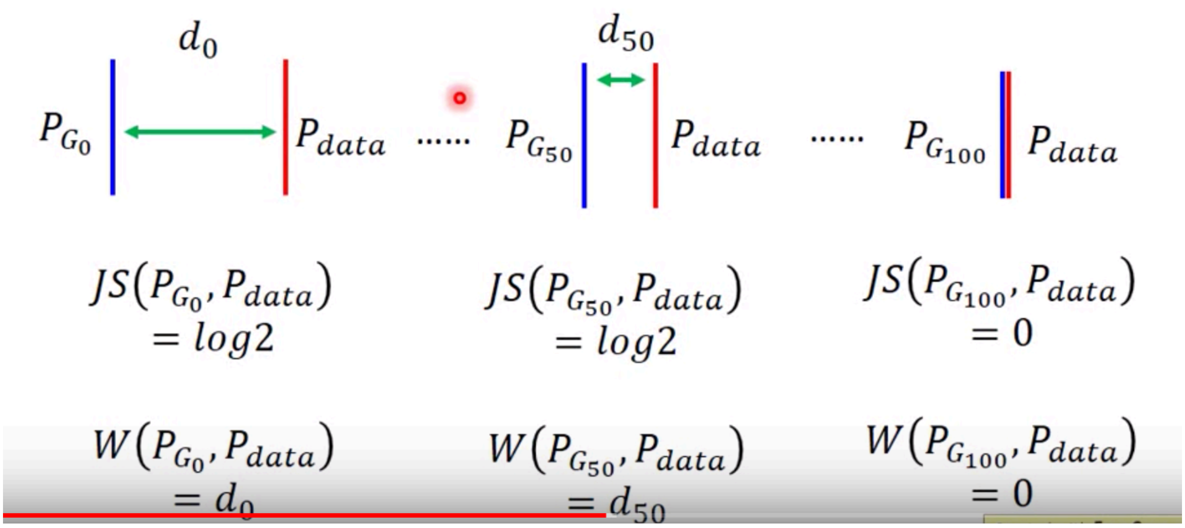
https://vincentherrmann.github.io/blog/wasserstein/
https://www.youtube.com/watch?v=KSN4QYgAtao&feature=youtu.be
Consider one distribution **P as a pile of earth**, and another distribution **Q as the target**,
Earth Mover's Distance is the average distance the earth mover has to move the earth.





Why Earth Mover's Distance ?

the comparation between **JS-Divergence** and **Earth Mover's Distance**
$\mathbf{P_G}$ denotes the distribution of Generator and $\mathbf{P_{DATA}}$ denotes the distribution of real data.
As shown below, **JSD** is less effectiveness to represent the "distance" between $\mathbf{P_G}$ and $\mathbf{P_{DATA}}$ while **Earth Mover's Distance** does.



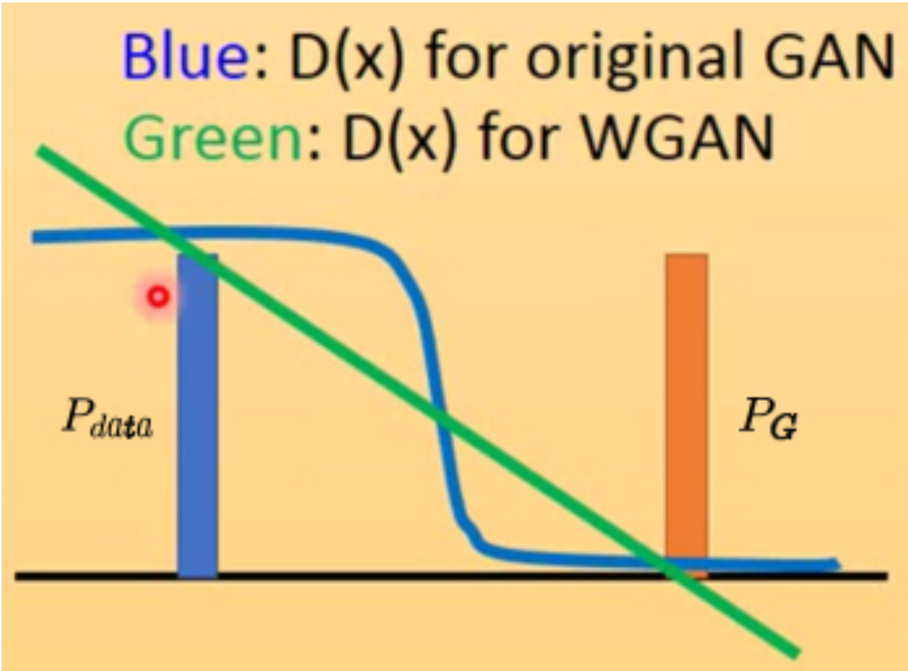
Back to the GAN framework

$$D_f(P_{data}||P_G) \Rightarrow W(P_{data}, P_G)$$

$$= \max_D \{E_{x\sim P_{data}}[D(x)] - E_{x\sim P_G}[f^*(D(x))]\}$$

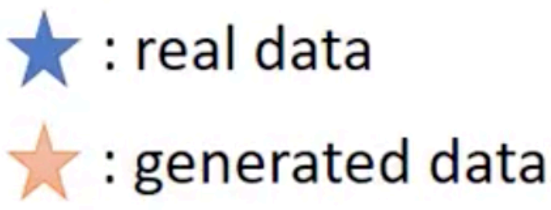
$$W(P_{data}, P_G)$$

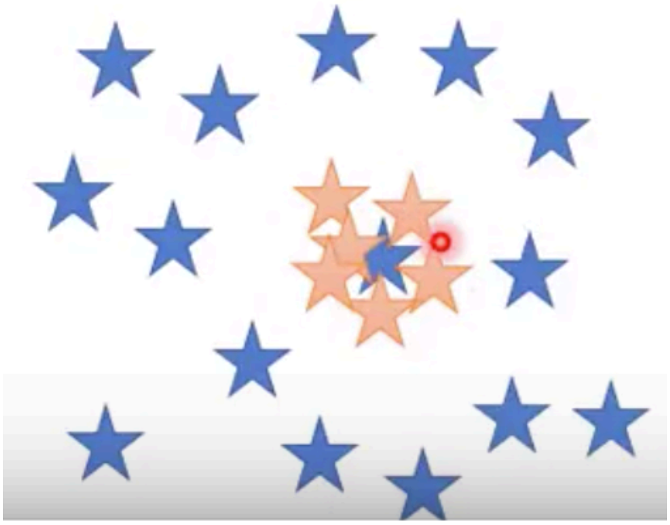
$$= \max_{D\in 1-Lipschitz} \{E_{x\sim P_{data}}[D(x)] - E_{x\sim P_G}[D(x)]\}$$



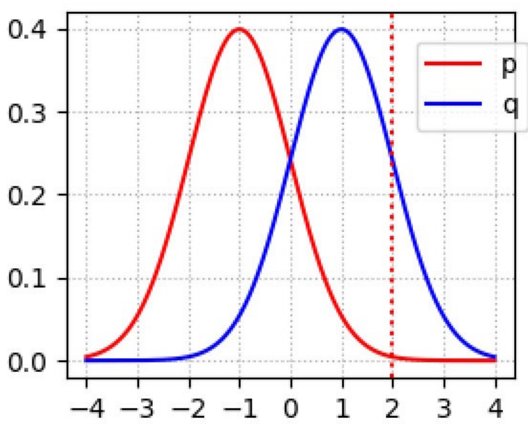
Mode Collapse

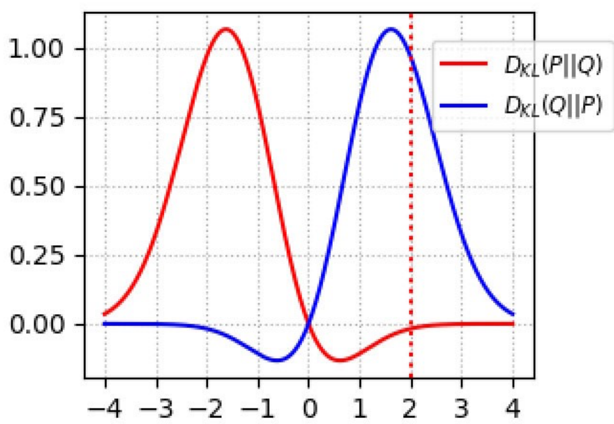
https://www.youtube.com/watch?v=av1bqilLsyQ





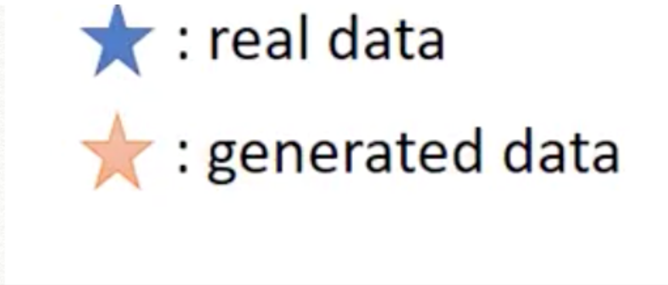
e.g., Reversed KL-Divergence
https://medium.com/@jonathan_hui/gan-why-it-is-so-hard-to-train-generative-advisory-networks-819a86b3750b
The reverse KL-divergence ***LKD(q, p)*** penalizes the generator if the images does not look real: high penalty if *p(x)* → *0* but *q(x)* > *0*. But it explores less variety: low penalty if *q(x)* → *0* but *p(x)* > *0*. **(Better quality but less diverse samples)**





Mode Dropping

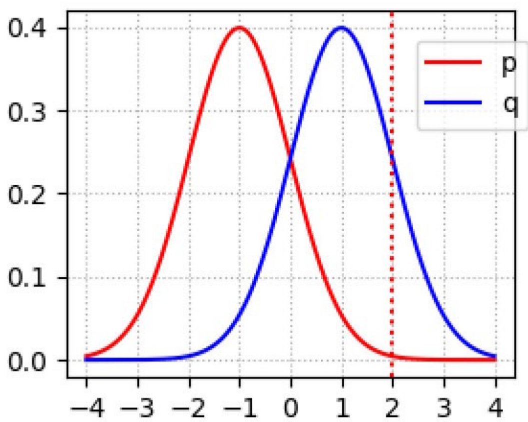
https://www.youtube.com/watch?v=av1bqilLsyQ

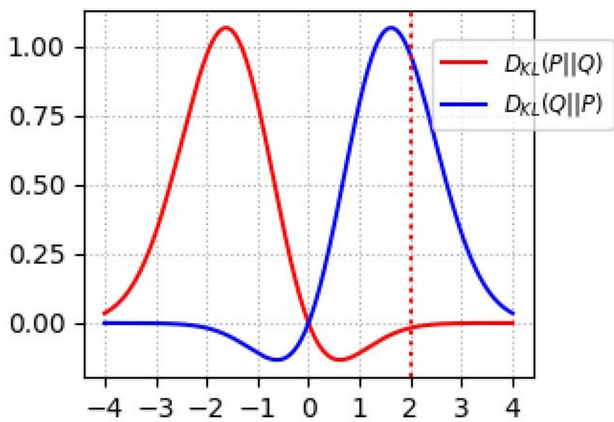


e.g., KL-Divergence

https://medium.com/@jonathan_hui/gan-why-it-is-so-hard-to-train-generative-advisory-networks-819a86b3750b

The KL-divergence ***KLD(p, q)*** penalizes the generator if it misses some modes of images: the penalty is high where *p(x) > 0* but *q(x)* → *0*. Nevertheless, it is acceptable that some images do not look real. The penalty is low when *p(x)* → *0* but *q(x)>0*. **(Poorer quality but more diverse samples)**

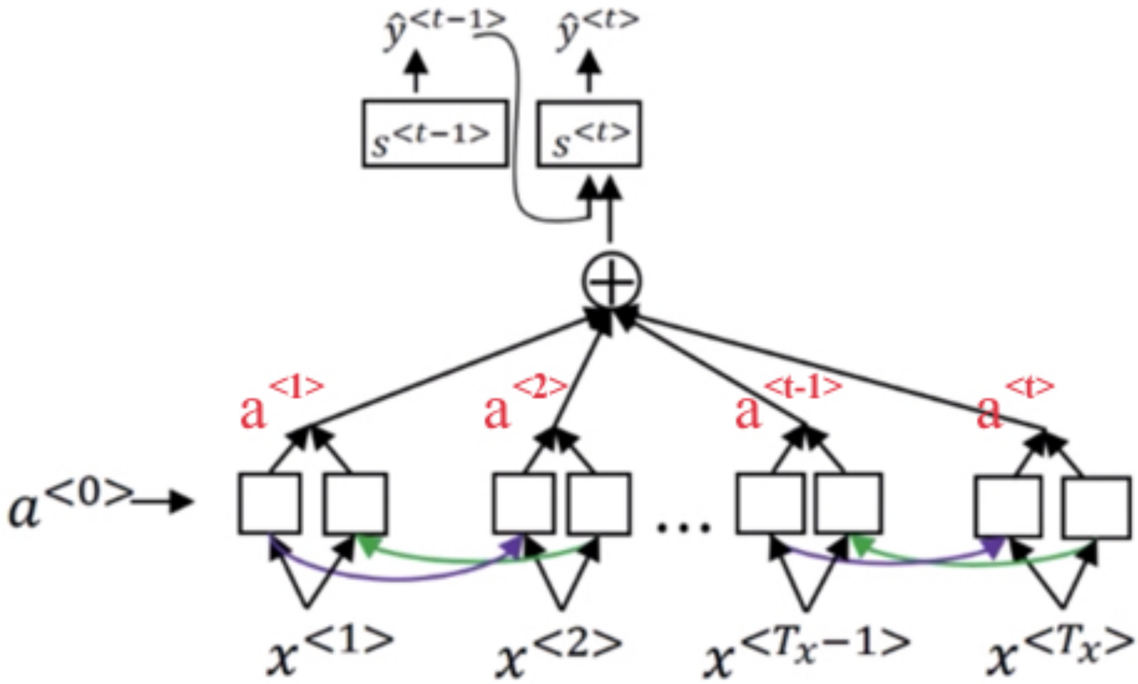




Some generative models (other than GANs) use MLE (a.k.a KL-divergence) to create models. It was originally believed that KL-divergence causes poorer quality of images (blurry images). But be warned that some empirical experiments may have disputed this claim.

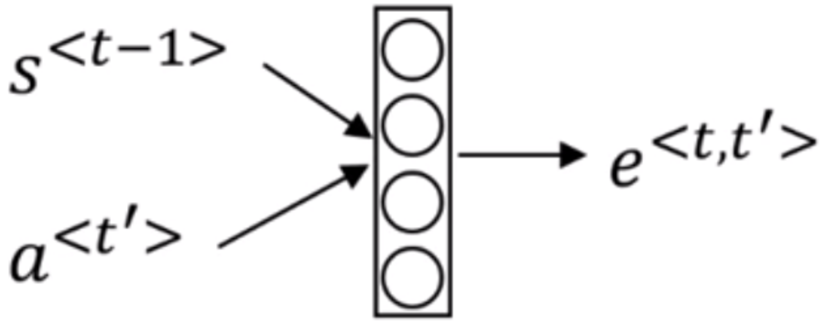
Attention Model

Attention Model is usually being applied into a machine translation model where generally you have a **Encoder-Decoder** structure to transform a sequence into another sequence.



Say you have a sequence of input **($a^{<0>}$, $x^{<1>}$, $x^{<2>}$ ... $x^{<T^x-1>}$, $x^{<T^x>}$)**, **with Attention Model, $y^{<t>}$ will pay attention to all of the ouputs ($a^{<1>}$, $a^{<2>}$ ... $a^{<t-1>}$, $a^{<t>}$) generated by the encoder**.

Essentially, attention is the relationship between the previous hidden state of decoder **$s^{<t-1>}$** and each of the **($a^{<1>}$, $a^{<2>}$ ... $a^{<t-1>}$, $a^{<t>}$)**, which is approximated by a simple neural network.



Once all of the relationships $\mathbf{e^{<t,t'>}}$ are calculated, normalizing them with softmax function,

$$\alpha^{<t,t'>} = \frac{\exp(e^{<t,t'>})}{\sum_{t'=1}^{T_x} \exp(e^{<t,t'>})}$$

($\partial^{<t,1>}, \partial^{<t,2>} \ldots \partial^{<t,t-1>}, \partial^{<t,t>}$) would be used as the weights w.r.t. each **($a^{<1>}, a^{<2>} \ldots a^{<t-1>}, a^{<t>}$)**,

In another words,

$$y^{<t>} = f(\partial^T a, y^{<t-1>})$$