CHAPTER 7

Gravity Surveying

7.1 Introduction

THE aim of gravitational prospecting is to detect underground structures by means of the disturbance they produce at the surface in the earth's gravitational field. Though the method is fundamentally a simple one the field differences to be measured are so small that in practice the instruments used and the techniques employed are highly sophisticated.

The basis of the method is Newton's Law of Gravitation. This states that every particle of matter exerts a force of attraction on every other particle, this being proportional to the product of the masses and inversely proportional to the square of the distance between them, i.e.

$$F = G\frac{m_1 m_2}{r^2} \tag{7.1}$$

where F is the force between the two particles of mass m_1 and m_2, r is their separation and G the universal gravitational constant. Its accepted value, which has been obtained by direct experiment, is 6.67×10^{-11} $\mathrm{N\,m^2\,kg^{-2}}$.

By the second law of motion the acceleration a of a body of mass m_1 due to the attraction of a mass m_2 is given by

$$a = \frac{F}{m_1} = G\frac{m_2}{r^2} \tag{7.2}$$

The force per unit mass acting on the body is therefore equivalent to its acceleration and is known as the gravitational field at m_1. If we put m_2 equal to the mass of the earth and r equal to the earth's radius the resulting acceleration is roughly equal to the gravitational acceleration on the earth's surface and is directed radially downwards. However, the attraction due to, say, a small buried mass will act along a line joining the centre of the mass to the point of measurement. If the mass is dm then the acceleration

$$a = G\frac{dm}{r^2} \tag{7.3}$$

The instruments used for gravitational surveying are known as gravity meters or gravimeters and are designed to measure field differences between localities, not absolute values. To make a measurement the axis of the instrument is aligned in the field direction, i.e. perpendicular to a plane defined by spirit levels on its upper face. The gravitational field direction is, of course, the direction of the resultant of the earth field and that due to local masses. Such local fields are, however, relatively very small indeed compared with the earth's gravity so that the direction of the latter is virtually unaffected, though its magnitude changes very slightly. The direction of the "vertical", the direction of the sensitive axis of the gravimeter, is then everywhere the same for all practical purposes. Therefore, in calculating the effect on a gravimeter of a local subsurface mass it is the vertical component of its field that has to be considered. At any point on the surface this component is

$$\begin{aligned} g_z = a\cos\theta &= G\frac{dm\cos\theta}{r^2} \\ &= G\frac{zdm}{r^3} \\ &= G\frac{zdm}{(x^2+z^2)^{3/2}} \end{aligned} \tag{7.4}$$

where z, r and θ are as defined in Fig. 7.1.

If the body is a large one, then its total effect is

$$g_z = G\int\frac{zdm}{r^3} \tag{7.5}$$

where the integral sign indicates that the total field is obtained by summing the effect of all elements throughout the mass. When the body

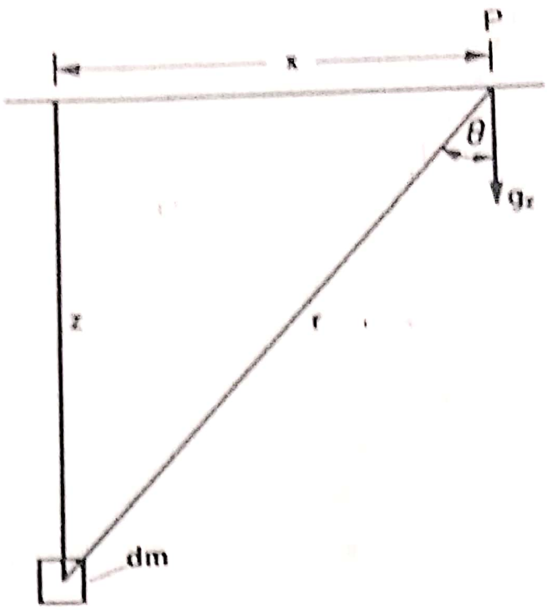


Fig. 7.1 The gravitational effect of a mass element.

has a simple geometrical shape an exact expression for the integral can be obtained. Otherwise the total is obtained numerically by dividing the body into small elements, calculating separately the attraction due to each, and then finding the sum.

In survey work modern practice is to measure anomalies in "gravity units" (μN kg^{-1} or μm s^{-2}). The older unit, the milligal, still appears in recent literature and is equal to 10 g.u.

The earth's gravity field is roughly 10^7 g.u., gravity anomalies due to geological structures of interest varying from a few gravity units to perhaps a few thousand. It is now possible to measure spatial changes in the field to better than $1:10^8$ but before interpretation in geological terms can be considered it is necessary to separate out the gravitational effects of subsurface density changes from those with a different origin. The methods adopted for doing this are described in the next section.

7.2 The Bouguer Anomaly

That part of the difference between observed gravity and theoretical gravity at any point on the earth which is due purely to lateral variations of density beneath the surface is known as the Bouguer anomaly. To obtain this quantity observations have to be corrected to allow for changes in gravity with latitude and height and for the attraction of topography. These corrections are discussed below.

7.21 *Latitude correction*

If the earth were a homogeneous (or concentrically layered) non-rotating sphere with the same vertical density gradient everywhere, apart from local near surface density variations due to geological structure, and if it had a surface parallel to sea level, then clearly all gravity variations over the surface would be caused by geological structure. But this is not so. The best approximation to the shape of the earth for practical purposes is an ellipsoid of revolution with an equatorial radius (6378 km) about 20 km greater than the polar radius. This is known as the reference ellipsoid and is virtually the sea-level surface. Because of the flattening the poles are nearer to the centre of mass than the equator, so that gravity increases with increasing latitude. This effect is increased by the opposing acceleration due to the earth's rotation, which has a maximum equatorial value of about 1/3 per cent of the gravitational attraction. The variation of gravity with latitude over the surface of an ellipsoidal earth can be expressed in the form

$$g = g_o(1 + C_1 \sin^2\phi - C_2 \sin^2 2\phi) \tag{7.6}$$

where g_o is the value of gravity on the equator and ϕ is latitude. C_1 and C_2 are constants which depend on the earth's shape, the numerical values of which have been adjusted to give a best fit to the measured variation of gravity over the earth's surface. With these values equation (7.6) becomes

$$g = 9.780318(1 + 0.0053024 \sin^2\phi - 0.0000058 \sin^2 2\phi)\ \mathrm{m\,s^{-2}}$$

This is the 1967 International Gravity Formula. It gives the value of gravity at any latitude on the reference surface. From it can be calculated the corrections that must be applied to the measured values of a survey to allow for the increase of gravity from the equator to the poles. Since in prospecting we are often concerned only with gravity differences the correction may be applied as a difference from an arbitrarily chosen base and, as the rate of change of gravity with latitude is almost linear over a small range, provided the survey is of limited extent a constant factor can be used. In mid-latitudes this works out to be roughly 1 g.u. per 100 m north or south.

7.22 *Elevation correction*

Since gravity varies with height it is necessary to correct all observations to a datum which is usually but not always sea level (i.e., the surface of the reference ellipsoid). This correction consists of two parts, the first of which is known as the free air correction.

Provided we assume the earth to be a sphere the mass can be considered concentrated at its centre and the value of gravity at sea level is given by

$$g_o = G\frac{M}{R^2} \tag{7.7}$$

where M = mass of the earth,
R = its radius.

The value of gravity g at a height h above this is given by

$$g = G\cdot\frac{M}{(R+h)^2} = G\frac{M}{R^2}\left(1 - \frac{2h}{R}\cdot\cdot\cdot\cdot\cdot\right)$$

Therefore, ignoring higher order terms

$$g = g_o\left(1 - \frac{2h}{R}\right)$$

or

$$g_o - g = \frac{2g_o h}{R} = Fh \tag{7.8}$$

In calculating this expression the fact that the earth is ellipsoidal rather than spherical has been neglected, but the effect of this simplification is negligible. In fact in equation (7.8) we can replace g_o by g_m, the mean value of gravity over the earth's surface, and R by the mean radius R_o. The resulting constant factor F by which h is multiplied to obtain the gravity correction can now be used when working in any latitude and at all but exceptional altitudes and has the value $3.1\,\text{g.u.}\,\text{m}^{-1}$.

The factor is of course the gradient of gravity in free space. As gravity decreases with height this correction has to be added to the observations to correct to sea level. The free air gradient, however, accounts for only part of the change in gravity with height. Between any elevated station and sea level there is a thickness of rock exerting a gravitational attraction at the surface which can be considered to be additional to that due to the mass of the earth below the ellipsoid. To a first approximation its attraction can be taken to be that of an infinite slab of thickness equal to the station height. The attraction of such a slab is $2\pi G\rho h$ where h = station height and ρ = density. When reducing an observation down to sea level the correction has to be subtracted since we are removing from beneath the station a slab of rock of thickness h and consequently reducing the downward attraction by an amount $2\pi G\rho h = \beta\rho h$. This correction is known as the Bouguer correction after the French geodesist who first made use of it. It is usual to combine the free air and Bouguer corrections into a single elevation correction of the form

$$g_o = g_h + (F - \beta\rho)h \tag{7.9}$$

where $\beta = 0.42$ g.u. t^{-1} m^2
g_o = gravity corrected to sea level
g_h = observed gravity at height h m.

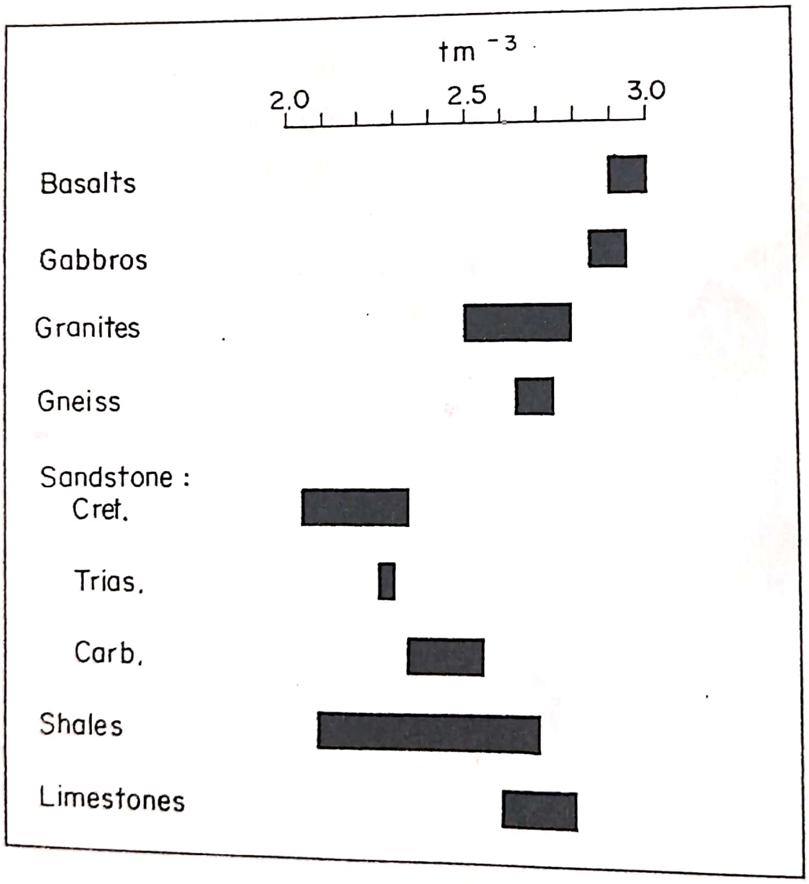
Assuming that the average rock density is 2.5 t m^{-3}* the correction amounts to roughly 2 g.u. m^{-1} so that heights must be known to better than 0.5 m to retain a relative accuracy of 1 g.u. between stations.

A further problem in making the Bouguer correction is to know what density to use. For example, an error of 0.05 t m^{-3} density for a station 500 m above sea level leads to an error of over 10 g.u. in the Bouguer correction. For a survey of any size a geological map is a necessity, for this gives the surface boundaries of the various rock formations. Densities are then found by laboratory determinations on a representative collection of rock samples and also, where possible, from a statistical analysis of field data. For example, in any area where the density of the rocks is approximately constant, a set of gravity observations may be taken over a small topographic feature. The observations are corrected

*See §1.2 for a note on the units of density.

for latitude effect and free air gradient and, assuming for the moment that there is no geological structure present causing gravity variation, the corrected values will be due only to the Bouguer effect of the topography. Thus if the corrected data are plotted against height the resulting graph will be a straight line, the gradient of which will be $\beta\rho$ g.u. m^{-1} so that the density can be determined. The range of rock densities encountered is from about 2.0 to 3.0 t m^{-3}, although exceptional materials such as peat (1.0 t m^{-3}) and ore bodies (3.0–4.0 t m^{-3}) lie outside the range. Some typical values are given in Table 7.1. In practice in any area there is almost

Table 7.1 The densities of some common rock types. Sandstone and limestone densities are to a large extent determined by porosities which may be as much as 30 per cent.



always some gravity variation due to geological structure but unless this happens to be correlated with topographic height it will produce merely a random scatter of the plotted points. Even so a reasonably accurate value of the density can usually be obtained from the straight line which best fits the data.

7.23 *Terrain correction*

When the topography is relatively flat the elevation correction may provide a sufficiently accurate method of reducing the data to sea level or any other convenient datum. If there are considerable irregularities of elevation, particularly in the vicinity of the station, then the simple assumption that there is an infinite slab of rock between the observation point and sea level is inadequate and a further allowance must be made for departures from this. Graphical or, more usually, computer methods are used to calculate the gravity effect of all hills above the station height, together with the mass deficiencies caused by valleys, these having been assumed to be rock filled in the Bouguer slab correction. Hills and valleys will both lead to a positive correction to the observed gravity. One approach (Bott, 1959) is to divide the topography into kilometre square columns and feed to the computer average surface elevations and positions for each column, together with gravity station heights and positions. Values of rock density are also assigned to the various rock columns. Using an appropriate formula the gravity effect of the rock columns round each station out to a pre-determined radius is calculated and the total terrain correction obtained. A hand method using specially designed transparent graticules is described by Hammer (1939). The principle is the same.

7.24 *Definition of the Bouguer anomaly*

The gravity values can now be presented as Bouguer anomalies, these being defined as the discrepancy between observed gravity and that expected on allowing for all known effects, i.e.

> Bouguer anomaly = observed gravity + elevation correction + topographic correction − theoretical gravity on the reference ellipsoid at the same latitude.

Because only gravity differences will have been measured the survey must start from a point of known gravity if the true Bouguer anomaly is to be presented. In prospecting we are often only interested in local differences of gravity anomaly, and this being so it is quite satisfactory to start from any convenient base to which an arbitrary gravity value is given. To observed differences from this point are then applied the elevation and topographic corrections and a correction for the difference in gravity due to the latitude change from base. The resulting anomaly values differ from the true Bouguer anomaly only by a constant amount.

It may also be necessary if high accuracy is sought to apply what is known as a tidal correction to the observations. The attraction of the sun and the moon results in a cyclic change in gravity which may be as much as 3 g.u. in 6 hours. The calculation is complex and the correction can be obtained from pre-published tables for the year.

7.3 Instruments

All measurements of gravity for prospecting purposes are made with gravimeters. These are designed to measure directly small differences in the strength of gravity and are quickly operated portable instruments with a sensitivity quite adequate for all survey purposes. Modern instruments are based on the same principle as the long period vertical seismograph. An approximately horizontal beam hinged at one end carries a weight as shown in Fig. 7.2. The beam is connected to the mainspring which is attached at its upper end to a support directly above the hinge. The moment the spring exerts on the beam is Sa where S is the restoring force in the spring and a is its perpendicular distance from the hinge. This balances the gravitational moment $mgl \cos \theta$, θ being the

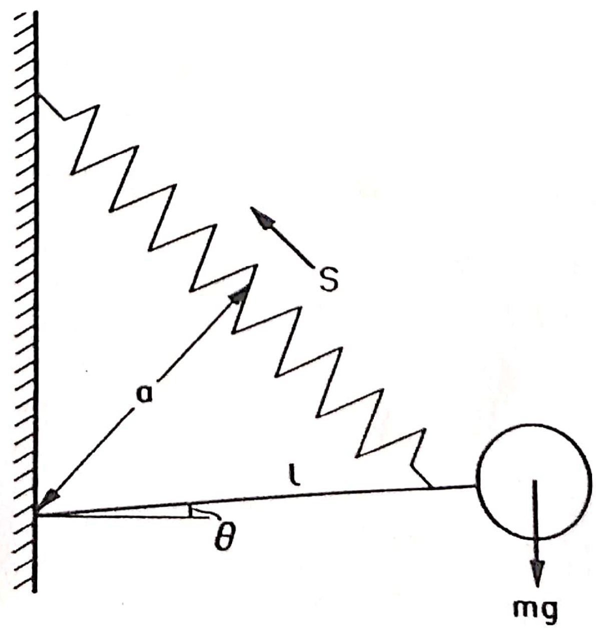


Fig. 7.2 The principle of the astatic gravimeter.

small angle the beam makes with the horizontal. If gravity increases the beam will deflect downwards until a new equilibrium position is reached. This will cause an increase in the spring length and hence of the restoring force *S* but the distance *a* will decrease. It is possible, therefore, by proper selection of the point of attachment to arrange for as small an increase of the restoring moment with increasing gravity as is desired. Using an ordinary coiled spring such an arrangement would have a very small stable working range. However, by taking advantage of the properties of pretensioned ("zero length") springs it is possible to produce an instrument with a linear and very sensitive response over a wide range.

Such gravimeters are described as astatic. They behave as if there were two separate springs, a mainspring which balances out a fixed and large proportion of the gravity field and a fine measuring spring which responds sensitively to the remaining small and varying fraction of the field.

In practice gravimeters do not measure deflections. It is more satisfactory to return the beam to a null position and measure the force required to do so. The usual way of doing this is to attach the upper end of the mainspring to a micrometer head and measure the displacement needed to restore the beam to its null position in terms of the micrometer reading.

Temperature compensating devices have to be incorporated in all gravimeter spring systems to make extension as independent as possible of temperature change. Even so, to achieve maximum accuracy it is necessary to keep the instrument at a constant temperature by means of a small battery operated thermostatically controlled heater. Modern gravimeters have a world wide range and standard instruments can be read to an accuracy of 0.1 g.u. New high precision instruments are also now available which can be read to 0.01 g.u.

All spring systems show a slow creep and so even if a gravimeter is kept at one place the reading will change with time. This slow change in reading is called "drift". Such instruments therefore can be used only to measure differences in gravity between locations. Though the drift rate may vary a little from day to day and even during the course of a single day the amount of drift is approximately proportional to time over a limited period. The rate may be found by returning to a base station after making a series of measurements. Provided the times were noted at which

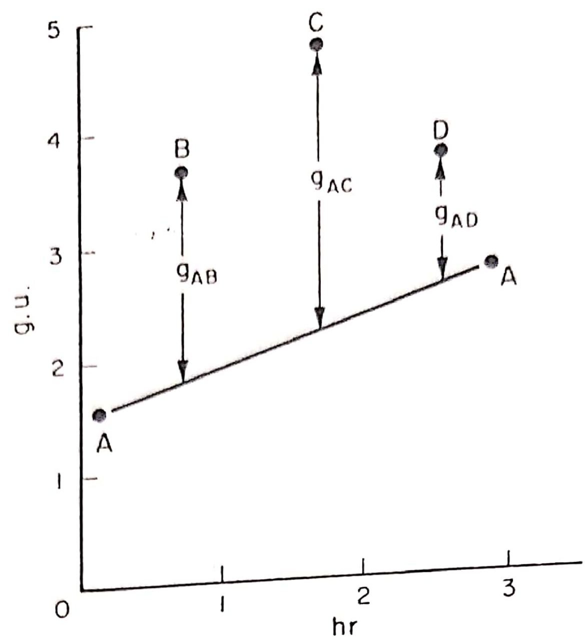


Fig. 7.3 The drift correction: g_{AB}, g_{AC}, and g_{AD} are the gravity differences between the base station A and the stations B, C and D.

the measurements were made a drift correction at each station may be made by linear interpolation as shown in Fig. 7.3. Drift rates for different instruments vary widely from 10 g.u. per day to 10 g.u. per month or less and depend to some extent on the conditions under which the instrument is being used. Provided that the returns to base are not more than a few hours apart tidal changes in gravity can be considered linear and treated as part of the drift correction.

Instruments in common use are the Worden and the La Coste and Romberg gravimeters. Both are portable and extremely sensitive instruments. With thermostat battery and case the La Coste meter weighs less than 10 kg. It has an accuracy over the full range of 70,000 g.u. of a few tenths of a gravity unit and drift, which is not sensitive to vibration, is usually less than 5 g.u. per month. Though the drift properties of the Worden are not as good, it also has high sensitivity and is lighter and more portable.

Standard gravimeters have been adapted for shallow under-water work. The instrument is housed in a watertight container lowered to the sea or lake bed and there levelled, either automatically or by remote control. An electrical signal proportional to scale reading is passed up the control cable to enable the instrument to be read at the surface.

CHAPTER 8

Magnetic Surveying

8.1 Introduction

THOUGH there are certain marked similarities between magnetic and gravitational methods, both in the field techniques and the presentation of data there are important differences.

Gravity surveys depend for their effectiveness on subsurface density differences and magnetic surveys make use of the variation in magnetization of rocks. Density is a bulk property of rocks and tends to be consistent throughout a formation. Rocks, however, owe their magnetic properties to minor constituents which can be very variable in their distribution, and thus less diagnostic of the formation.

In igneous and metamorphic rocks the main carriers of the magnetization are the mixed oxides of iron and titanium, of which the iron rich member magnetite is the best known. Sediments, in general less magnetic, contain magnetic particles derived originally from igneous and metamorphic rocks and also oxidation products, haematite being particularly important. The iron sulphide pyrrhotite contributes significantly to the magnetization in a limited number of rock types, occurring for example in association with the valuable nickel ore pentlandite.

As a result of the presence of the earth's field rocks containing magnetizable minerals show an induced magnetization. The constant of proportionality between the inducing field and the magnetization is known as the susceptibility. The inducing field, however, is the field within the magnetized material, though in rocks, which can normally be classed as weakly magnetic bodies, this does not differ much from the earth's field. It is therefore convenient to define an apparent susceptibility k relating the magnetization to the undisturbed applied field. This relationship has the form

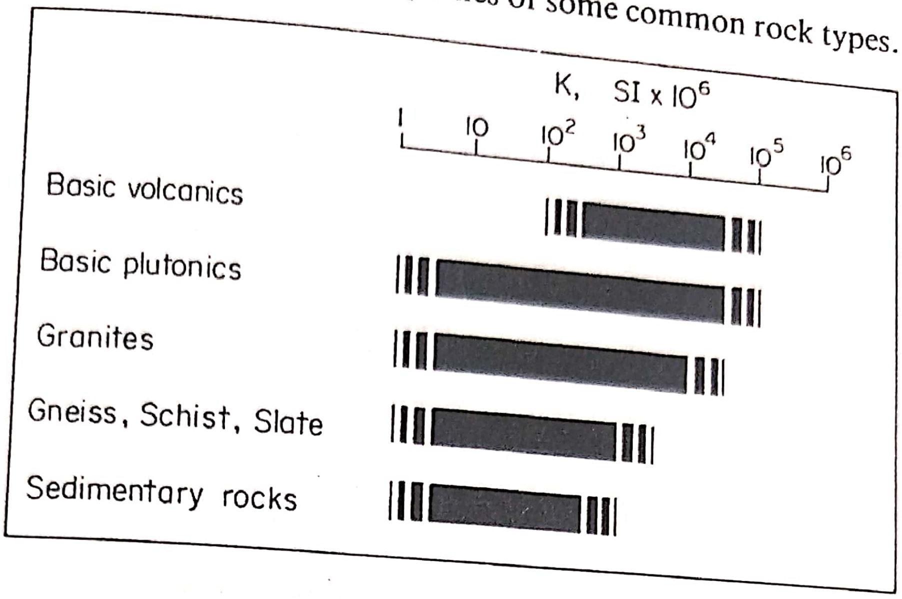
$$\mu_o J = kB \tag{8.1}$$

where J is the induced magnetization (in A m^{-1}), B the field strength measured in teslas (T) and μ_o a constant with dimensions of H m^{-1} (see §1.2). It holds for most rocks (i.e. magnetic content <10 per cent) but for very magnetic bodies the value of k will depend on body shape and field direction. With the exception of certain strongly magnetized bodies the induced magnetization lies in the direction of the inducing field. Rocks may also show a remanent or permanent magnetization. This can be acquired in a number of different ways. Igneous rocks become magnetized on cooling in the earth's field and so this phase of the magnetization may date from the time of their formation if it is sufficiently permanent or "hard". Sedimentary rocks may acquire a remanence during deposition in water, this being due to the alignment of magnetic particles on settling in the earth's field. Chemical changes during diagenesis may also produce a remanence. Whatever the cause the direction of the remanence in rocks is, with some exceptions, close to that of the geomagnetic field at the time of formation of the magnetization. Its direction today may be very different, for there may have been changes in the position of the magnetic poles relative to the rock mass, and also faulting and folding of the formation. The stability of this remanence in both magnitude and direction depends on both the mineralogy and the texture of the magnetic minerals. In some rocks, as a result of age and instability of the magnetic minerals, its magnitude may be insignificant (see Tarling, 1971).

Induced and remanent magnetism add vectorially to give a resultant, the direction of which depends on the strength and direction of the remanence relative to that of the induced component.

In considering local anomalies in the earth's magnetic field we have to take into account not only the magnetization of the rock but the important fact, later discussed, that for a magnetic body of a given shape the form of the anomaly is governed not only by this shape but also by the inclination of the present direction of the earth's field and by the body orientation. We are thus faced, in dealing with magnetic anomalies, with a far more variable and thus less diagnostic feature than we have to deal

Table 8.1 Magnetic properties of some common rock types.



with in gravity interpretation. Though it is certainly possible to make broad distinctions between weakly and strongly magnetized rocks (prominent anomalies in fact most often arise as a result of the juxtaposition of the members of the two types) interpretation in terms of body size and shape is far less easy. For this reason much magnetic survey tends to be of a reconnaissance nature and interpretation is qualitative or at best semi-quantitative.

8.2 The Earth's Magnetic Field

The reduction of magnetic field data for the purpose of producing an anomaly map generally requires a knowledge of the normal variation of the geomagnetic field in space and time in the area of the survey. A brief discussion of the relevance of this subject to prospecting is therefore given here.

To a rough approximation the form of the magnetic field at the earth's surface is that which would be produced by a small but powerful magnet placed at the earth's centre with its north magnetic pole pointing southwards and inclined at 11° to the rotation axis (Fig. 8.1). If the field were perfectly regular the lines of force would be vertical at the pole of the magnetic axis and horizontal on the magnetic equator, this being a great circle inclined at 11° to the true equator. All isodynamic lines, or lines of equal force, would be small circles round the earth parallel to the

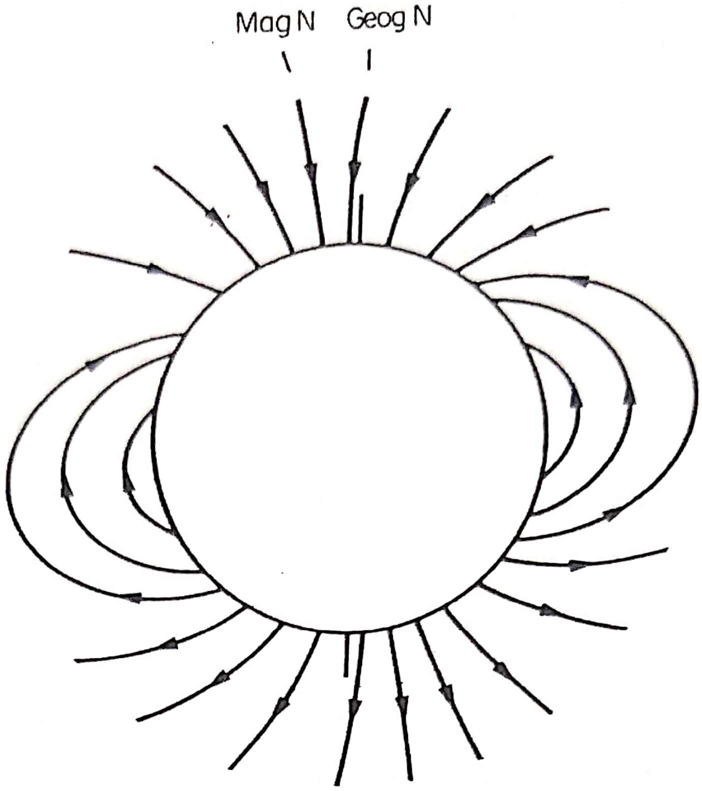


Fig. 8.1 The form of the earth's magnetic field.

magnetic equator. In fact the actual pattern of the field is only roughly of this form as there are regions of up to continental size where there are marked departures from this simple picture. One result of the added complexity is that the dip poles, where the inclination at the surface is 90°, do not lie on the magnetic axis. In addition both the strength and form of the field change slowly by amounts which are quite easily measured over a period of a year. Despite the complex form of the field and its variation with time it can be described mathematically with adequate accuracy, though the expression used contains a large number of terms. The field so calculated is known as the International Geomagnetic Reference Field (1976). At the poles the strength of the field is about 6×10^{-5} T. Magnetic anomalies have amplitudes that are only a small fraction of this and are measured in nanoteslas (1 nT = 10^{-9} T), more commonly known as gammas (γ).

Figure 8.2 shows the conventions and nomenclature adopted in describing the field. The total field vector is denoted by F, H and Z being the horizontal and vertical components. The angle of inclination of the field is I and D, the angle the horizontal component makes with the meridian, is the declination.