

## Topic 7: Diffusion Current

### Introduction:

In our analysis of doping one of the physical mechanisms allowing dopant ions to penetrate the lattice was diffusion. In general, **diffusion** refers to some substance moving from a high concentration to a low concentration. Our focus before was on the physical atoms, in this section we shift our focus to the carriers introduced by these impurities.

### Concentration Plots:

The fundamental principle driving diffusion is a **varying concentration with position**. We studied semiconductors with uniform doping concentration with drift current and the next step is to allow the concentration to vary throughout the material. We can visualize a changing concentration by plotting the concentration,  $n$ , with respect to position,  $x$ . **Note** that we will only study the 1-dimensional case but the concepts apply to 2D and 3D as well.

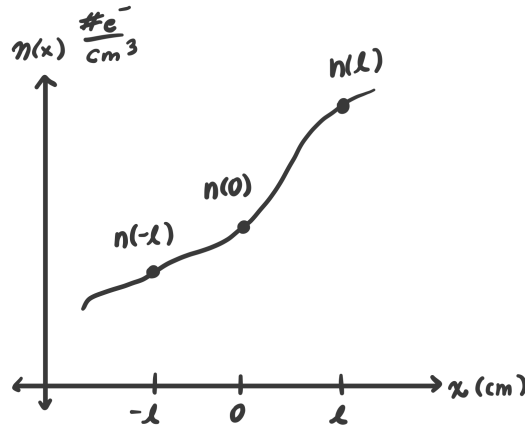


Figure 1: Arbitrary electron concentration over position plot.

The figure above is an example of an arbitrary electron concentration. The plot continues past  $\pm l$  because we are interested in the concentration for a small region of the material. At the points  $0$  and  $\pm l$  the concentration is given by  $n(-l)$ ,  $n(0)$ , and  $n(l)$ , respectively. We know these three concentrations are increasing but we may ask ourselves how the electron physically diffuses through the material? Carriers diffuse from a region of high concentration to a region of low concentration, therefore, an electron dropped anywhere on the curve will move to the left.

There is a strong analogy between carrier diffusion and a ball rolling down a hill. A ball at the top of a hill has large potential energy and will roll down the hill until it reaches the bottom or a small valley that it cannot escape from. The same is true for carriers under diffusion. The electron at a high concentration will continue to move until it reaches a low concentration. The figure below visualizes how a hole will experience the same behavior rolling along the concentration curve. Before we can start our quantitative analysis it is essential to understand intuitively where the charge will move.

The figures below visualize a hole and electron rolling down the metaphorical hill and potentially getting trapped in a valley. Carriers within the flat section of the curve do not experience diffusion. Only the carriers unlucky enough to pass by the region with the valley will have a probability of diffusing.

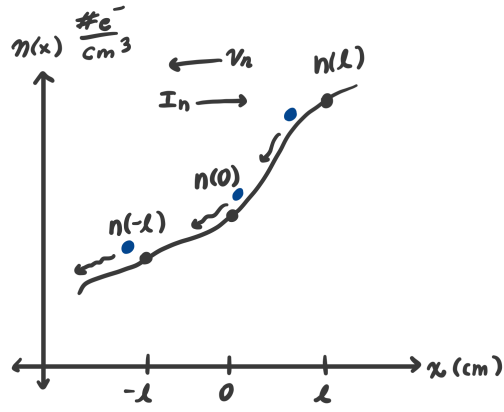


Figure 2: Electron motion due to diffusion.

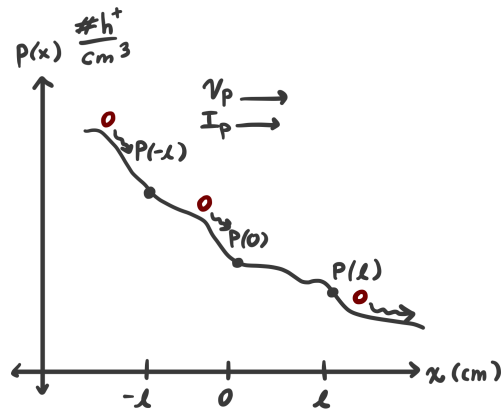


Figure 3: Hole motion due to diffusion

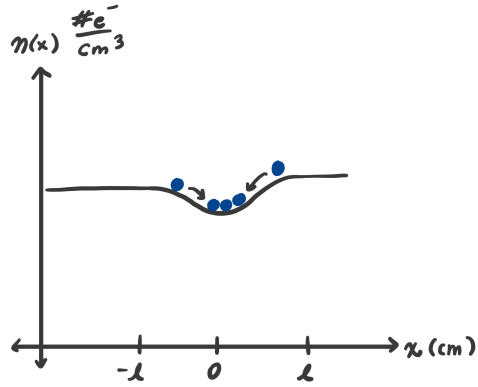


Figure 4: Electron diffusion with a local minimum plot.

From these three figures, we can make a few observations that will guide our derivations:

1. Carrier velocity is in the direction of lower concentration.
2. Electron and hole current point in opposite directions.
3. Diffusion does not occur for constant concentrations.
4. Carriers can get trapped in regions of low concentration.

## Net-Flow Rate:

The above observations provide a better understanding of how the current can be modeled. The current density,  $J_n$ , is the instantaneous rate of electrons flowing per unit area. If we relax the condition of instant flow to average flow, we can approximate the diffusion current. Let  $F_L$  be the average number of electrons passing  $x = 0$  from  $x = -\ell$ ;  $[F_L] = \frac{\#e}{s \cdot cm^2}$ . Analyzing the units of  $F_L$  we need to gain a distance term in the numerator and a temporal term in the denominator. Let us assume the electrons will move a distance,  $\ell$ , over a time,  $\tau$ .

$$F_L = n(-\ell) \frac{\ell}{2\tau} = \frac{n(-\ell)v_{th}}{2} \quad (1)$$

The concentration at  $x = -\ell$  is given by  $n(-\ell)$  and these electrons are moving with some velocity,  $v_{th} = \frac{\ell}{\tau}$ . The subscript th stands for thermal velocity. Diffusion occurs without an external voltage, so the thermal parameters of the material will determine how fast carriers can diffuse. The only perplexing part of the formula is the factor of two. While we know electrons will diffuse the left in Fig. 1 some may momentarily pass to the right of the boundary. Introducing the factor of 2 removes the bias associated with assuming carriers move in one direction, giving a 50% chance for the electron to move to the left or to the right of  $x = -\ell$ . Repeating with the carriers passing  $x = 0$  from  $x = \ell$

$$F_R = n(\ell) \frac{\ell}{2\tau} = n(\ell) \frac{v_{th}}{2} \quad (2)$$

The net flow of electrons from left to right, as  $x = 0$  is given by  $F_n = F_L - F_R$ .

$$F_n = \frac{F_L - F_R}{2} = \frac{v_{th}(n(-\ell) - n(\ell))}{2} \quad (3)$$

If  $\ell$  is large, the net rate in Eq. 3 will be inaccurate. We have chosen  $\ell$  to be some arbitrary point, so we can make it as small as we want. Taking the difference of a function at two points over a small interval sounds a bit like calculus. To better justify our approximation Taylor Series will be used to approximate our arbitrary  $n(x)$ .

## Taylor Series:

The general form of Taylor Series is:

$$f(x) = f(a) + \frac{d}{dx}f(a)(x-a) + \frac{d^2}{dx^2}f(a)\frac{(x-a)^2}{2} + \frac{d^3}{dx^3}f(a)\frac{(x-a)^3}{3} + \dots$$

$$f(x) = f(a) + \sum_1^{\infty} \frac{d^n}{dx^n}f(a) \frac{(x-a)^n}{n}$$

Many applications Taylor series involve hand analysis or the simplification of higher-order terms. We will use Taylor series to linearly approximate  $n(x)$  and  $p(x)$ . A linear approximation makes the system linear and much easier to analyze by hand. There is nothing stopping us from using a nonlinear approximation. It is just much easier to restrict our analysis to linear systems by hand. The general form of carrier concentration will be:

$$\begin{aligned} n(\ell) &= n(0) + \frac{dn(\ell)}{dx}\ell \\ n(-\ell) &= n(0) - \frac{dn(-\ell)}{dx}\ell \\ F_n &= \frac{v_{th}}{2}(n(-\ell) - n(\ell)) \\ &= -v_{th}\ell \frac{dn}{dx} \\ \therefore F_n &= -D_n \frac{dn}{dx}, D_n = v_{th}\ell \end{aligned} \quad (4)$$

The quantity  $F_n$  is in terms of the **number** of electrons within a current density, not the charge. To convert from the number of electrons to charge we multiply by  $F_n$  by  $-q$ . Therefore, the current density due to electrons is:

$$J_n = qD_n \frac{dn}{dx} \quad (5)$$

$$I_n = qAD_n \frac{dn}{dx} \quad (6)$$

We can use either the current density or the current through the cross-sectional area,  $A$ . The only term we have not discussed is  $D_n$  called the **diffusion constant**. From the analysis above  $D_n = v_{th}\ell$  which has units of  $[D_n] = \frac{cm^2}{s}$ . Both time and squared distance must be positive, implying that  $D_n$  is a strictly positive constant dependent on material properties. Intuitively the diffusion constant states how fast the carriers diffuse through a cross-sectional area. The fundamental charge and area are also positive constants. Hence, the only signed term is  $\frac{dn}{dx}$ .

To see if Eq. 5 is valid, we can compare it to the intuitive analysis of Fig. 1. An electron will move to the left which results in a current to the right;  $J_n > 0$ . From Eq. 5 a positive current requires  $\frac{dn}{dx}$  to be positive. Inspection of Fig. 1 confirms that the derivative is positive at every point in the curve. Hence, Eq. 5 passes our first inspection of the system.

The derivation for holes is identical to electrons. The final expressions are provided below but repeat the derivation to convince yourself it is true.

$$J_p = -qD_p \frac{dp}{dx} \quad (7)$$

$$I_p = -qAD_p \frac{dp}{dx} \quad (8)$$

The only difference in the hole diffusion current is an additional negative. Approximating  $p(x)$  with Taylor Series gives the expression  $F_n = -v_{th}\ell \frac{dp}{dx}$ . When we convert the flow rate to current density the charge of the electron is multiplied,  $-q$ . The charge of a hole is positive which prevents the negative from canceling out. A convenient way to remember the signs is that hole current is negative. Electron flow is opposite conventional current, so another negative is multiplied with the hole current. As well as switching the  $p$  variables to  $n$ . The net diffusion current is the sum of hole and electron components.

$$J_T = qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx} \quad (9)$$

It is worth noting that the only way to increase the diffusion current is to increase the doping gradient.  $D_p$  and  $D_n$  are constants that cannot be changed unless we change materials. Methods to vary the doping gradient is not the scope of this course, but acknowledging the steepness of the slope is proportional to the current.

## Problem Setup:

There are several ways a concentration curve may be given: An explicit function of concentration, a set of data points, or even two data points. Be aware that if data for some change in concentration occurs over some distance that we cannot assume the concentration plot. For example, say the concentration of electrons varies from  $10^{18}$  to  $7 \cdot 10^{17}$  over a distance of 0.1 cm. It requires an assumption to say the concentration increased from  $7 \cdot 10^{17}$  to  $10^{18}$ , or that the concentration decreased from  $10^{18}$  to  $7 \cdot 10^{17}$ . The diffusion current magnitude will be the same in either case but the direction will be different. If a curve were given as in Fig. 1 there is no doubt over the direction of the resulting current.

## Simulation:

Of the carrier transport mechanisms, diffusion is the most difficult to visualize. At first, diffusion may seem similar to transient circuit analysis. Implying we only need to consider diffusion while the concentrations stabilize but this is not true. Once we start studying devices, diffusion current compose

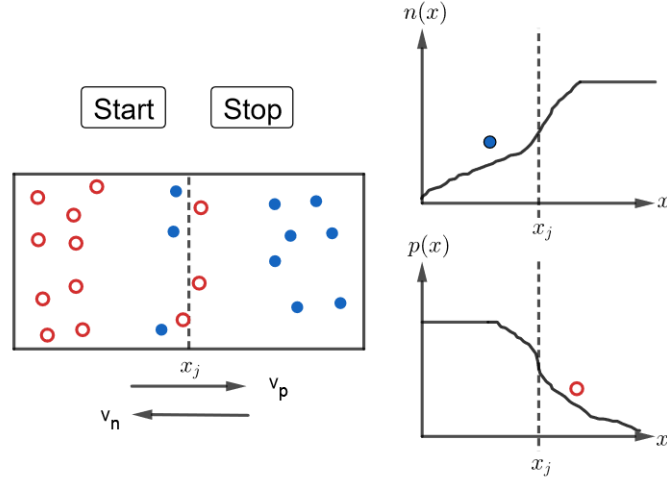


Figure 5: Diffusion geogebra simulation.

the most significant terms of the device's current. I have made the following Geogebra simulation to help visualize diffusion current; <https://www.geogebra.org/m/uycuhr2w>

In the simulation I have plotted the electron and hole concentrations for an n and p-type material. The plots are similar to those discussed at the beginning of this file. The two sections are placed together to form a PN junction. The concentration curves do not accurately describe diffusion within a PN junction. Instead, these curves are aimed at just visualizing how the current exists within the device. This simulation is not interactive, the only free parameter is the concentration in each region. The carrier concentration is proportional to carrier velocity, which is difficult to implement well for the moment.

## Current Components:

With drift and diffusion current expressions, it is possible to have up to four terms of current. To get the net current we take a sum over each term:

$$J_{n,total} = qD_n \frac{dn}{dx} + qn\mu_n \mathcal{E} \quad (10)$$

$$J_{p,total} = -qD_p \frac{dp}{dx} + qp\mu_p \mathcal{E} \quad (11)$$

$$\Rightarrow J_{total} = qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx} + q(n\mu_n + p\mu_p) \mathcal{E} \quad (12)$$

Eq. 12 may seem daunting but the parameters of a problem can help simplify the expression. If there is no electric field the drift terms become zero. Similarly, if the doping concentration is constant the diffusion terms become zero.

## Einstein Relation:

Keeping track of the diffusion and mobility constants in Eq.10-12 is quite inconvenient. It would be nice if there was some way to convert between diffusion constant and mobility to factor out the constant. Luckily, Einstein has already helped theorize this relation. One of Einstein's many contributions is the **equipartition of energy** theorem. The theorem states that the average kinetic energy of a set of particles is partitioned equally to each mode, or degree of freedom, of movement. Note the angled brackets refer to the average of the quantity enclosed.

$$\langle KE \rangle = \frac{m_n v_{th}^2}{2} = \frac{3}{2} kT$$

Each degree of freedom contributes a term of  $\frac{kT}{2}$ . The three degrees of freedom are encoded in movement in the  $\hat{x}, \hat{y},$  and  $\hat{z}$  direction, respectively. However different representations of this freedom

exist, such as translational, rotational, and vibrational. For more information check out this set of notes by Dr. Claire Vallance from the University of Oxford (<http://vallance.chem.ox.ac.uk/pdfs/Equipartition.pdf>). Our study is focused on the 1-dimensional case, therefore, the factor of three is dropped.

$$\begin{aligned}\frac{m_n v_{th}}{2} &= \frac{1}{2} kT \\ \Rightarrow v_{th}^2 &= \frac{kT}{m_n}\end{aligned}\tag{13}$$

From our definition of  $D_n = v_{th} \ell$ , the length an electron travels is the thermal velocity multiplied by time;  $D_n = v_{th} * (v_{th} \tau) = v_{th}^2 \tau$ . Recall the definition for mobility being  $\mu_n = \frac{q\tau}{m_n}$ . With  $\tau$  and  $\mu_n$  being common to both constant it seems we may be able to relate, or equate, them.

$$\begin{aligned}D_n &= v_{th}^2 \tau \\ v_{th}^2 &= \frac{kT}{m_n} \\ \Rightarrow D_n &= \frac{kT\tau}{m_n} \\ \frac{\tau}{m_n} &= \frac{\mu_n}{q} \\ \therefore D_n &= \frac{kT}{q} \mu_n\end{aligned}\tag{14}$$

$$D_p = \frac{kT}{q} \mu_p\tag{15}$$

Eq. 14 and 15 are formally called the Einstein relation. The two expressions allow us to convert between diffusion constant and carrier mobility. There is no need to worry about the sign since we have already stated that both terms are positive. At room temperature,  $300K$ ,  $V_{th} = \frac{kT}{q} = 25.9 \text{ mV} \approx 26 \text{ mV}$ . The thermal voltage at room temperature will come up often in candidate problems, quizzes, and exam questions that remembering **26 mV at room temperature** will save some time. When practicing this derivation be aware of the notation of  $v_{th}$  being thermal velocity, but  $V_{th}$  is also used for thermal voltage.

## Conclusion:

With the addition of drift current, we now have four types of currents within semiconductor materials. We used Taylor Series to make a linear approximation of a concentration curve to simplify our sets of equations. After finding the net flow through a section of the semiconductor we arrived at current density expressions involving a new constant  $D_n$  and  $D_p$ . The diffusion constant is heavily related to the material type and is a constant for our purposes. Lastly, the Einstein relation was used to develop a relationship between diffusion and mobility. It was not done in the notes, but you can rearrange Eq. 12 to include only the diffusion or mobility constants respectively. This set of notes is shorter compared to others but diffusion is without a doubt the most important of the current mechanisms. The next set of notes will look at the final carrier transport, generation and recombination.