

## Topic 8: Generation and Recombination

### Preface:

This section will introduce the last of the carrier transport mechanisms, Generation and Recombination; G & R. This mechanism is unlike diffusion and drift which deal with a majority or minority carrier traversing within the material. Instead, we shift our focus to the spontaneous generation of electron-hole pairs (EHP) and their recombination into covalent bonds. This section will be quite mathematically intensive. It is necessary that you go through the derivation another time to solidify not just the intuition but the mathematical descriptions provided.

### Covalent Bonds:

Recall from the beginning of the course that semiconductors form a lattice of covalent bonds with dopant atoms and the semiconductor's elements. We know that at non-absolute zero temperatures, each atom, electron, and subsequent bond will have non-zero energy. This non-zero energy results in vibrations of atoms and bonds within the lattice. If these bonds gain enough energy they can break forming an electron-hole pair. Bonds are commonly broken from thermal effects, large momentum carriers, or photons colliding with the bond.

We may wonder how an electron-hole pair is generated if covalent bonds are formed of electrons? When a bond is broken by an external force, an electron is injected into the lattice. We typically say only one electron is released since the absence of one electron is enough to break the bond. If both electrons were injected into the lattice the dopant atom would become an ion of opposite polarity; a donor atom would become a positive ion. The lack of an electron is modeled by a hole taking the place of the released electron. If an electron were to collide with a hole the covalent bond would form again. In short,

- **Generation:** The release of an electron-hole pair into the lattice due external forces.
- **Recombination:** The return of the electron-hole pair into a covalent bond.

It may seem straightforward, but generation and recombination are inverses of one another. Generation seeks to create whereas recombination destroys. Note that charges are not physically created or destroyed.

### Modified Energy Band Diagrams:

Generation and Recombination are conveniently modeled by electrons traveling between the conduction and valence bands. Holes are abundant in the conduction band and electrons are valence in the conduction band. If a bond is broken, a hole is elevated to the valence band and an electron is elevated to the conduction band. Another way to view the elevation process is that the carrier is free to move within the lattice. While holes are not necessarily free to move, they inadvertently move due to recombination with electrons. Electron-hole pairs are generated and recombined in two ways: Direct and indirect. We will discuss the two mechanisms in a moment, but more setup is necessary.

Within the silicon lattice, we characterize all the carriers within a momentum distribution. I say distribution because some particles may have high momentum, and some may have a small momentum, but the majority will have some average momentum. We can extend this idea to the bands of an energy band diagram. For example, the electrons in the conduction band have an average energy. Intuitively, it is very unlikely for a low-energy electron to remain in the conduction band. Hence it seems that plotting the energy band over momentum could provide additional insight into the mechanisms of generation and recombination.

## Direct Transition:

The general energy band diagram for a direct recombination is provided below.

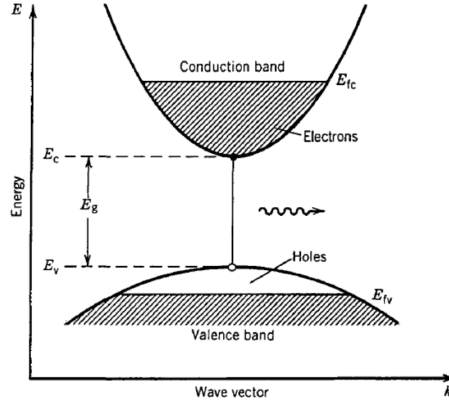


Figure 1: Direct G & R energy vs. momentum plot. [1]

From Fig. 1 we see that the peak of the valence band and the valley of the conduction band are aligned at some wave vector,  $k$ . Recall wave vector and linear momentum are related by a factor of  $\hbar$ ;  $p = \hbar k$ . We could swap the horizontal axis with linear momentum and would produce the same plot. The shaded regions of the bands represent electrons and the blank space represents holes within each band. It is interesting that the two bands are parabolic over the wave vector. This behavior is due to the bandgap representing the minimum energy to elevate an electron to the conduction band. If we have an electron closer to the nucleus, the momentum will be different from a valence electron, and will require a different amount of energy to elevate it to the conduction band. Our focus will be on middle region where the bands are separated by exactly the bandgap energy.

For an electron to reach the conduction band it must gain energy equal to the bandgap. Typically we associate changes in energy with a change in momentum, but Fig. 1 does not display a change in momentum at the peak and valley of the two bands. Note the tiny squiggly arrow,  $\rightsquigarrow$ , in the middle of the band diagram. The squiggly arrow represents a photon with energy,  $E = h\nu$ . Photons are the secret that allows a change in energy without a change in momentum. When a photon interacts with a bond or carrier energy is transferred without a change in momentum. If a photon with  $h\nu \geq E_g$  interacts with a bond, the energy transfer is enough to break the bond and elevate both carriers. Inversely, if recombination occurs a photon is released from the device due to the sudden change in energy.

## Applications:

Direct bandgap semiconductors are prominent throughout solar cells and light-emitting diodes. The ability to absorb and emit photons makes them perfect for optoelectrical energy conversion. Some of the more well-known direct bandgap materials are Gallium Arsenide (GaAs) and Gallium Nitride (GaN). There do exist other compound semiconductors but these two gallium-based semiconductors reveal how generation and recombination can be tuned to favor either generation or recombination. In LEDs, recombination is favored since a photon of a certain wavelength is emitted. Inversely, solar cells favor generation since incident light is absorbed by the device.

Gallium Arsenide is used heavily in high-cost, high-performance solar cells. However, silicon is the current choice in solar cells. The choice of silicon presents several downsides discussed later in these notes, but cost is always an important factor when driving technology adoption. In the past, colored plastics or films would be placed inside of LEDs that emitted white light. The type of films were selected to pass certain ranges of the visible spectrum. Blue LEDs were the most notable case where we had yet to discover a material with a bandgap able to produce blue light. Gallium Nitride, while also being explored for power electronics, forms a true-blue LED. The team who discovered that GaN produced blue led won a Nobel prize for their discovery; Professors Isamu Akasaki, Hiroshi Amano, and Shuji Nakamura.

## Indirect Transition:

A general form of indirect bandgap transitions is provided below.

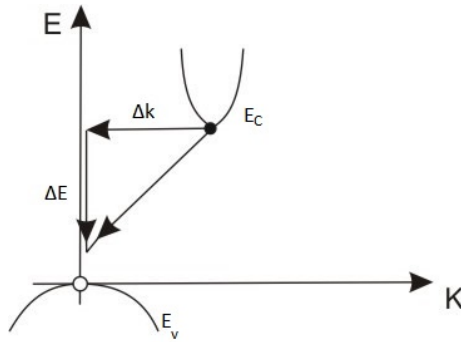


Figure 2: Indirect G & R energy vs. momentum plot. [2]

From Fig. 2 the peak of the valence band and the valley of the conduction band are not aligned. For an EHP to be generated a change in momentum and a change in energy must happen simultaneously. The interaction must occur simultaneously because a photon could interact with an electron, increasing the energy, but the energy will quickly dissipate into the lattice. The change in energy may come from several sources, but the change in momentum typically occurs from collisions within the lattice. Imagine the probability of a low-energy valence electron colliding with both a high-energy electron and a photon. Note that this is not the only mechanism to break EHPs within direct semiconductors, but only one example.

## Applications:

Many elemental semiconductors like Silicon and Germanium have indirect bandgaps. The indirect bandgap does not stop them from being used. However, a focus on temperature management is necessary. Temperature management is needed because changes in momentum and energy will manifest as heat within the device. If that heat is not effectively removed from the device then unstable behavior will occur. Nearly every device we have studied in electronics utilizes silicon. Diodes, transistors, processors, solar cells, and much more utilize silicon. The mass adoption of silicon is partly due to its abundance in nature.

## Direct Recombination:

This course will focus on direct recombination. Direct recombination is simple, and many models of indirect recombination involve a step of direct recombination. The following mathematical analysis involves carrier concentrations in thermal equilibrium before and after an excitation, such as shining light on the sample.

## Thermal Equilibrium:

Thermal equilibrium has been used several times throughout the course, but a well-defined definition was unnecessary. Up to this point, thermal equilibrium has been synonymous with  $np = n_i^2$ . The semiconductor has been essentially sitting on a table in a dark room for every calculation. If an excitation were present, carriers would be generated and the device is no longer at thermal equilibrium.

**Thermal Equilibrium:** A semiconductor at a constant temperature which experiences zero excitation other than from heat.

Examples of excitation are light and applied voltages. To differentiate between thermal equilibrium quantities and non-thermal equilibrium quantities a subscript  $o$  is added to the concentration.

- $n_{no}$ : The thermal equilibrium electron concentration in a n-type material.
- $p_{po}$ : The thermal equilibrium hole concentration in a p-type material.

- $p_{no}$ : The thermal equilibrium hole concentration in a n-type material.
- $n_{po}$ : The thermal equilibrium electron concentration in a p-type material.
- $n_n$ : The total electron concentration in an n-type material.
- $p_p$ : The total hole concentration in a p-type material.

The mass-action law is unchanged in thermal equilibrium;  $n_{no} \cdot p_{po} = n_i^2$ . We know that excitations generate electron-hole pairs. The additional electrons and holes will increase the total concentration from equilibrium **while the excitation is present**. Implying that  $n_n \cdot p_p \neq n_i^2$ , or:

$$n_n \cdot p_p > n_i^2 \quad (1)$$

Before we can continue characterizing generation and recombination, we need to define terminology on how the electron-hole pairs are generated and recombined.

### Generation Rate:

If light is incident on a semiconductor, then some number of EHPs will be generated within in the volume of the material with respect to time. This idea composes the **generation rate**, denoted  $G$ , and has units of  $\frac{\#EHP}{V-s}$ , where  $V$  is the volume of some material. In general, the stronger an excitation the more electron-hole pairs will be produced. Implying that generation will be proportional to the excitation strength, whatever it may be. Explicitly defining an expression for the generation rate is dependent on the type of excitation, which will not be our focus. For now, we can assume an excitation produces a generation rate of  $G$ .

### Recombination Rate:

Generation is only one-half of the discussion. The **Recombination Rate** refers to the number of electron-hole pairs removed per unit time per unit volume;  $\frac{\#EHP}{V-s}$ . It is no coincidence that the generation rate and recombination rate have identical units. Typically the recombination rate is denoted by  $R$ , and is heavily dependent on the number of carriers present. If many carriers are present in a material it is likely that some will recombine. Likewise, if few carriers are present then recombination is not as likely. Since the recombination rate is proportional the number of EHPs we will let  $\beta$  describe the proportionality constant. It is difficult to express the recombination rate explicitly without delving deeper into the device physics. Other courses will provide information on the recombination rate, but we can approximate a general form.

$$R = \beta np \quad (2)$$

The subscripts were dropped to distinct that the material type does not matter. The total electron and hole concentrations influence the recombination rate.

### Thermal Equilibrium vs. Steady State:

The value of the generation and recombination rate can provide interesting information on the state of the system. In thermal equilibrium, the only excitation is due to thermal effects. The fact that  $n_i$  is nonzero implies that generation is present at thermal equilibrium. If no recombination were present then the carriers in the material would grow extremely large. Hence, in thermal equilibrium the generation and recombination rates are equal. The subscript  $Th$  will be used to denote thermal equilibrium.

$$G_{Th} = R_{Th} = \beta n_{no} p_{no} = \beta p_{po} n_{po} \quad (3)$$

It is quite incredible that without any analysis of the mechanism causing EHPs generation or recombination that we can determine the rates at thermal equilibrium. The two equations on the right side of Eq. 3 are for an n-type and p-type respectively. The analysis so far is looking at a material of one cohesive doping type. Hence,  $np = n_{no} p_{no}$  in an n-type and  $np = n_{po} p_{po}$  in a p-type; both at thermal equilibrium.

Once the excitation is introduced,  $G$  is the sum of each excitation. In the case of only light shining on a material,  $G = G_{Th} + G_L$ , where  $G_L$  represents the generation due to light. **Thermal generation is**

**always present, regardless of excitation.** The same argument of infinite carrier generation applies. If we were to wait infinitely long, a total number of carriers would be generated. The material would saturate and not generate any additional carriers. This condition is called **steady state** and results in a net addition of  $\Delta n$  electrons and  $\Delta p$  holes. Unless stated otherwise we will assume an n-type material.

$$\begin{aligned} R &= \beta n_n p_n \\ R &= \beta (n_{no} + \Delta n)(p_{no} + \Delta p) \end{aligned} \quad (4)$$

The  $\Delta n$  and  $\Delta p$  terms represent the number of excess electrons and holes present. A clever way of describing the excitation is with the number of carriers generated. Since electron-hole pairs are generated, this forces  $\Delta n$  and  $\Delta p$  to be equal. Implying,

$$\Delta n = \Delta p \quad (5)$$

$$R = \beta [n_{no} p_{no} + n_{no} \Delta p + p_{no} \Delta n + \Delta n \Delta p] \quad (6)$$

$$R = R_{Th} + \beta \Delta n (n_{no} + p_{no} + \Delta n) \quad (7)$$

From Eq. 7 the type of the material can be used to simplify the thermal equilibrium concentrations. In an n-type material  $n_{no} \gg p_{no}$ , implying  $n_{no} + p_{no} \approx n_{no}$ .

$$R = R_{Th} + \beta \Delta n (n_{no} + \Delta n) \quad (8)$$

$$R = R_{Th} + \beta \Delta p (p_{no} + \Delta p) \quad (9)$$

## Net Rates:

Eq. 3 and 4 provide information on the material in thermal equilibrium, and in steady state. The last piece of information is the transient behavior between thermal equilibrium and steady state. Compiling the information we know so far, we can generate a concentration versus time plot of  $p_n$ . Plotting  $p_n$  was chosen because the carriers generated will have a larger influence on the minority carriers than majority carriers.

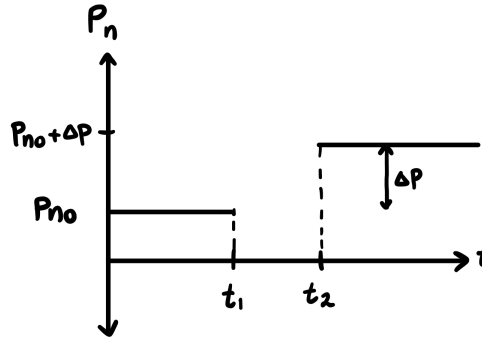


Figure 3: Thermal Equilibrium and Steady State concentration plot.

In Fig. 3 thermal equilibrium is present for time,  $t < t_1$ . A light is turned on at  $t = t_1$  and is left on for the rest of eternity. At time  $t \geq t_2$  the semiconductor has reached steady state and no additional carriers are generated. There are several ways we can connect the two curves, linear, quadratic, exponential. Typically the answer with physics is almost always exponential. However, all we can assume is that the net hole concentration will increase from  $t_1$  to  $t_2$ .

To move forward we need some information relating to  $\Delta p$ . We know at thermal equilibrium the generation and recombination rates are equal. For the semiconductor to sustain an additional  $\Delta p$  carriers the generation rate must be larger than the recombination rate. Hence, the net rate of hole generation is:

$$\frac{dp_n}{dt} = G - R = G_{Th} + G_L - R \quad (10)$$

Steady state implies  $\frac{dp_n}{dt} = 0$ ,

$$\begin{aligned}
\frac{dp_n}{dt} = 0 &= G_{Th} + G_L - R \\
G_L &= R - G_{Th} \\
G_L &= R_{Th} + \beta \Delta p (n_{no} + p_{no} + \Delta n) - G_{Th} \\
G_L &= \beta \Delta p (n_{no} + p_{no} + \Delta n)
\end{aligned} \tag{11}$$

$R_{Th}$  and  $G_{Th}$  were canceled due to steady state implying the thermal generation and recombination rates must be equal. The assumption of  $n_{no} \gg p_{no}$  was left out to show that the generation rate of light is dependent on the total number of carriers in the material. Applying the approximation and another assumption regarding  $\Delta p$  can simplify the equation further.

## Low Level Injection:

Low-level injection is a fundamental assumption we will make regarding generation and recombination. It is defined as **the generated charge,  $\Delta p$  and  $\Delta n$ , are much less than the thermal equilibrium concentration for the respective type**. In an n-type this means that,  $\Delta n \ll n_{no}$ . Similarly, a p-type implies that  $\Delta p \ll p_{po}$ . Mathematically, the combination of low-level injection and doping type allow the parenthesis of Eq. 11 to simplify to:

$$G_L = \beta \Delta p n_{no} \tag{12}$$

$$G_L = \beta \Delta n p_{po} \tag{13}$$

Where Eq. 12 and 13 are for n and p-types respectively. Take a moment to acknowledge what Equations 12 and 13 represent. The generation rate is primarily dependent on the majority concentration in the material, why? For example, an n-type material has much more free electrons than holes. Incident light is more likely to hit a valence electron and promote it to the conduction band. Similarly, with holes for a p-type material, it all depends on the probability of a carrier experiencing the right conditions at the right time.

Returning to low-level injection, it is worthwhile to see why high-level injection is not favorable. Imagine we have a semiconductor with  $10^{15}$  donor atoms resulting in an n-type material. If the order of  $10^{13}$  electron-hole pairs were generated, what happens to  $p_{no}$ ? The  $\Delta p$  term would dominate compared to  $p_{no}$ ;  $\Delta p \gg p_{no}$ . For reference,  $p_{no} = 2.25 * 10^5$ . The high injection would certainly nullify the assumptions providing Eq. 12 and 13 but also affects the band shape in an energy band diagram. The large injection of holes would move the Fermi level, making the device less n-type. These disadvantages will be discussed with devices, but low-level injection is typically preferred.

The above analysis still has not given us an answer on how to draw the concentration between  $t_1$  and  $t_2$ . However, the approximate value of  $G_L$  does provide a start. Between  $t_1$  and  $t_2$  the concentration is change in concentration is dependent on  $G_L$ .

$$\begin{aligned}
\frac{dp_n}{dt} &= G_L + G_{Th} - R = G_L \\
&= \beta \Delta p n_{no} \\
&= \beta n_{no} (p_n - p_{no}) \\
&= \beta (n_{no} p_n - n_{no} p_{no}) \\
\Rightarrow \frac{dp_n}{dt} - \beta n_{no} p_n &= \beta n_{no} p_{no}
\end{aligned} \tag{14}$$

Hopefully, now it is clear why Eq. 12 and 13 use the opposite carrier to the majority type at thermal equilibrium. It makes no difference if  $\Delta p$  is used in Eq. 12 but is necessary to reach the differential equation in Eq. 14. Luckily Eq. 14 is a first-order nonhomogenous differential equation with constant coefficients. This set of notes will not explicitly solve the equation from scratch but will take advantage of well-known general forms. The total solution is broken into homogeneous and particular solutions.

$$\begin{aligned}
p_n(t) &= p_{n,homogeneous} + p_{n,particular} \\
p_{n,homogeneous} &= Ae^{\lambda t} \\
&\Rightarrow Ae^{\lambda t}(\lambda - \beta n_{no}) \\
&\therefore \lambda = \beta n_{no}
\end{aligned}$$

The right-hand side of Eq. 14 is a constant implying the particular solution is a constant,  $B$ . Taking the derivative of a constant is zero, producing  $-\beta n_{no}B = \beta n_{no}p_{no}$ . Canceling like terms gives  $B = -p_{no}$ .

$$p_n(t) = Ae^{\beta n_{no}t} - p_{no}. \quad (15)$$

Finding the value for  $A$  requires evaluating the differential equation at some known condition.  $p_n(t_1)$  is already known to be  $p_{no}$  but lets shift  $t_1$  to  $t = 0$  for simplicity.

$$p_n(0) = A - p_{no} \Rightarrow A = 2p_{no} \quad (16)$$

Combining each term, the final solution for  $p_n(t)$  is:

$$p_n(t) = p_{no}[2e^{\beta n_{no}t} - 1] \quad (17)$$

$$p_n(t) = p_{no}[2e^{\beta n_{no}(t-t_1)} - 1], \text{ for } t_1 \leq t \leq t_2 \quad (18)$$

Equation 17 assumed the light source is turned on at  $t = 0$  and Equation 18 accounts for a time delay  $t_1$  as given in figure 3. It is typically a good idea to check the final equation gives the same results as the steady state conditions. Plugging in  $t = t_1$  to Eq. 18 the exponential term becomes  $e^0$  and  $p_{no}(2 - 1) = p_{no}$ . To verify the condition at  $t = t_2$  would involve solving the differential equation when the light is turned off, the solution of which is provided below.

$$\begin{aligned}
p_n(t) &= p_{no} + Ae^{-\beta n_{no}t} \\
p_n(t) &= p_{no} + Ae^{-\beta n_{no}(t-t_3)} \\
A &= p_n(t_3) - p_{no} = \Delta p \\
p_n(t) &= p_{no} + \Delta p e^{-\beta n_{no}t} \\
p_n(t) &= p_{no} + \Delta p e^{-\beta n_{no}(t-t_3)}, \text{ for } t_3 \leq t \leq t_4
\end{aligned} \quad (19)$$

The moment  $t = t_3$  represents when the light is turned off, starting the second transient response.  $A = \Delta p$  is known since the steady state condition keeps a constant  $\Delta p$  carriers in the material. As  $t$  becomes large, the exponential term will decay to zero leaving the concentration at  $p_{no}$ . Evaluating Eq. 20 at  $t = t_3$  gives the final condition of Eq. 18 as  $p_{no} + \Delta p$ ;  $p_n(t_3) = p_n(t_2)$ . With initial and final conditions  $\Delta p$  could be found explicitly from practical values of thermal equilibrium concentration, duration of excitation, the recombination proportionality. The additional steps are aimed at showing what is possible when you take a thorough look at the solutions for each stage. With the transient response, it is possible to connect the two curves in Fig. 3. The figure below shows the response due to the light turning on and off.

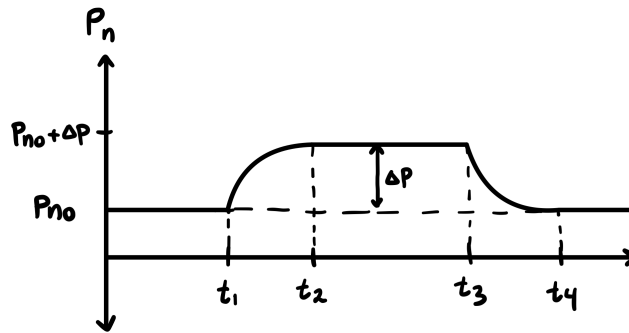


Figure 4: Complete transient and steady state concentration plot.

A good way to remember the curve is it looks identical to the output of an RC filter being driven by a square wave.

## Carrier Lifetime:

The argument of each exponential term contained a  $\beta n_{no}$  term. To reduce the amount we have to write, we can combine the constant into a quantity called carrier lifetime, denoted by  $\tau$ . For an n-type material we use  $\tau_p$  and  $\tau_n$  for a p-type material. Note the lifetime subscript is the opposite of the material type. Substituting into equations 18 and 20:

$$\tau_n = \frac{1}{\beta p_{po}} \quad (21)$$

$$\tau_p = \frac{1}{\beta n_{no}} \quad (22)$$

$$p_n(t) = p_{no} [2e^{\frac{(t-t_1)}{\tau_p}} - 1], \text{ for } t_1 \leq t \leq t_2 \quad (23)$$

$$p_n(t) = p_{no} + \Delta p e^{-\frac{(t-t_3)}{\tau_p}}, \text{ for } t_3 \leq t \leq t_4 \quad (24)$$

Regardless of the physical description of  $\tau$  we can make several observations. The argument of an exponential must be unitless, implying  $\tau$  has units of seconds. How does an increase in  $\tau$  affect equations 23 and 24? For Eq. 23, increasing  $\tau$  makes the denominator larger, implying the exponential grows more slowly. Likewise, the exponential in Eq. 24 will decay slower. If the exponential term, which drives the transient response, is slower then the length of the response must increase. If more time is required to return to steady state then the material's recombination rate is not very effective. Generated carriers will be present, or live, within the material longer for a long  $\tau$ .

The idea of a carrier being alive aligns with the terminology of carrier lifetime. Generated carriers have a time they are expected to live within the material. Holes are the minority carrier in an n-type material, hence the hole concentration is dependent on how long generated carriers live. Does it make sense for the majority concentration to determine the lifetime of a minority carrier? If there are  $10^{15}$  electrons in one material and  $10^{20}$  in another material, which material would likely have a shorter lifetime? The larger concentration means that more free carriers are available to recombine with minority carriers, so the  $10^{20}$  sample will have a shorter lifetime. This description is why lifetime is inversely proportional to the majority carrier concentration. The same argument can be made with  $\beta$  with a larger  $\beta$  representing a higher probability of recombination, which reduces the lifetime.

## Energy Band Diagram

With expressions representing the minority concentration over time, it would be preferable to find some way to encode the information within the energy band diagram. We know what the EBD at thermal equilibrium looks like, every band diagram drawn up to this point has been at thermal equilibrium. With an excitation, the definition of the Fermi level needs a slight revision. Recall that  $\frac{dE_F}{dx} = 0$  at thermal equilibrium. The generation of charges causes a non-zero slope in the Fermi level. To take advantage of prior analysis, we define quasi-Fermi levels which behave nearly identical. By definition:

$$n = n_i e^{\frac{E_{F_n} - E_i}{kT}} \quad (25)$$

$$p = n_i e^{\frac{E_i - E_{F_p}}{kT}} \quad (26)$$

The terms  $E_{F_n}$  and  $E_{F_p}$  represent the quasi-Fermi level for an n and p-type, respectively.

**Example:** A GaAs sample with  $n_{no} = 10^{10} \text{ cm}^{-3}$  is illuminated with light producing  $10^{11}$  electron-hole pairs per  $\text{cm}^{-3}$  every microsecond. Given  $\tau_p = \tau_n = 2 \text{ ns}$  find the quasi-Fermi level at room temperature.  $n_i$  for Gallium Arsenide is  $2 \cdot 10^6 \text{ cm}^{-3}$ .

**Solution:** At thermal equilibrium,  $n_{no} = 10^{10} \text{ cm}^{-3}$ .  $n_{no} p_{no} = n_i^2$  implying:

$$\begin{aligned} p_{no} &= \frac{n_i^2}{n_{no}} \\ &= \frac{4 \cdot 10^{12}}{10^{10}} \\ &= 400 \text{ cm}^{-3} \end{aligned}$$



$$E_F - E_i = kT \ln\left(\frac{n}{n_i}\right) = 0.22 \text{ eV}$$

When the light is turned on we need to first determine the number of carriers generated,  $\Delta n$  and  $\Delta p$ . Throughout our several pages of derivations, surely there is one equation we can use? The device is an n-type, but is the injection low? We can assume the injection is low and see if we arrive at a contradiction. With low-level injection,

$$G_L = \beta \Delta p n_{no}$$

$$\Delta p = \frac{G_L}{\beta n_{no}} = G_L \tau_p$$

$\tau_p$  is given in the problem, and  $G_L$  may be found by taking the given rate of  $10^{11} \frac{\text{\#EHP}}{\text{cm}^3 \cdot 10^{-6} \text{s}}$ . Plugging into the above equation,

$$\Delta p = 2 \cdot 10^{-9} \cdot 10^{17}$$

$$= 2 \cdot 10^8 \frac{\text{\#EHP}}{\text{cm}^3}$$

$$n_{no} = 10^{10} \text{cm}^{-3}$$

$$\Rightarrow \frac{n_{no}}{\Delta p} = 50$$

A 50 times increase from  $\Delta p$  to  $n_{no}$  somewhat validates the low-level injection. The larger this ratio the more accurate the assumption is. There is not a guaranteed condition where if you are X times below the thermal equilibrium concentration then low-level injection is satisfied. However, a 50 times difference will give an error of about 2% which is more than fine for our needs. Plugging in  $\Delta p$  for  $n_n$  and  $p_n$  we have:

$$n_n = n_{no} + \Delta p = 10 \cdot 10^{10} + 2 \cdot 10^8 = 1.02 \cdot 10^{10}$$

$$p_n = p_{no} + \Delta p = 400 + 2 \cdot 10^8 \approx 2 \cdot 10^8$$

The final step involves solving for the Quasi Fermi level from  $n_n$  and  $p_n$ , respectively.

$$E_{F_n} - E_i = \frac{kT}{q} \ln\left(\frac{n_n}{n_i}\right) \approx 0.221 \text{ eV}$$

$$E_i - E_{F_p} = \frac{kT}{q} \ln\left(\frac{p_n}{n_i}\right) \approx 0.119 \text{ eV}$$

The resultant energy band diagram is provided below, but take note of the distance between  $E_i$  and  $E_{F_p}$ . If the distance is greater, it implies more holes are being generated by the light. As the intensity of the light decreases both  $E_{F_n}$  and  $E_{F_p}$  will converge to  $E_i$ .

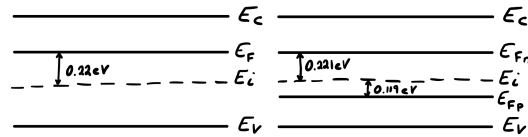


Figure 5: Energy band diagram at thermal equilibrium (left) and under excitation (right).

## Indirect Recombination

Indirect recombination will not be covered extensively. This section is aimed at providing more of an introduction instead of in-depth analysis like indirect recombination. Within direct recombination there exist no states within the bandgap. The intrinsic and Fermi levels do exist but an electrons will not exist at that state. Indirect recombination relies on the fact that states within the bandgap do exist. These states can capture holes and electrons moving between the valence and conduction band.

A good analogy is a layover when taking a long flight. To travel from the valence band to the conduction band an electron metaphorically takes one flight to the distribution center, waits for a little bit, and then takes the next flight to the conduction band. These energy levels go by names of: **Recombination center**, **Distribution center**, and **Trap**. Recombination and Distribution centers refer to energy levels near the middle of the bandgap, whereas traps are levels near the conduction and valence bands. These levels trap any carriers the moment before they reach either band. The figure below visualizes the recombination and each of the processes is given a name. A carrier, hole or electron, can be captured or released (emission). If a carrier is captured it now resides in the energy level. Likewise, emission removes the carrier from the energy level. These four rates compose the Shockley-Reed-Hall (SRH) Recombination, which is outside the scope of this course.

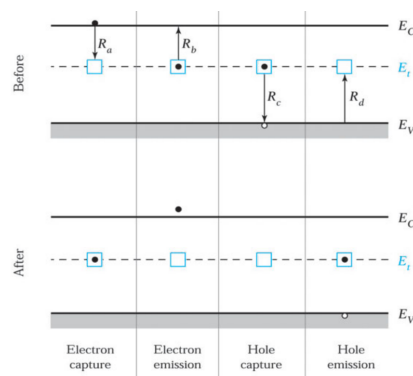


Figure 6: Indirect recombination energy band diagram [3]

## Conclusion

This set of notes is incredibly dense, it is the nature of Generation and Recombination. Recall we started with an intuitive definition that Generation and Recombination are a byproduct of the breaking and formation of covalent bonds. Depending on if a material is a direct or indirect bandgap material the underlying processes are fairly different. Our focus on direct recombination is not only easier mathematically but common throughout several compound semiconductors. The analysis focused heavily on separating thermal equilibrium concentrations and energy levels from those under excitation. I cannot understate how important it is to know what each term represents. It is incredibly easy to write  $p_{po}$  instead of  $p_{no}$  which leads to several contradictions throughout a problem.

Thermal equilibrium is the reference allowing us to analyze a semiconductor under some excitation. Initial conditions, thermal generation, and thermal recombination are all necessary to solving the differential equations. After deriving expressions for generation and recombination,  $G_L$  was found from the minority carrier net rate of change. Some assumptions were made along the way, such as low-level injection, and the concept of carrier lifetime was introduced. Explicit expressions are nice but relating all the above information into the energy band diagram prompted the creation of quasi-Fermi levels to replicate the constant slope of the Fermi Level in thermal equilibrium. We ended with a not-so-brief example and a short introduction of indirect recombination. The next set of notes will start to bridge our cumulative knowledge to more interesting cases such as the PN junction.

## References

- [1] FOSCO. *Semiconductor Laser Physics*. <https://www.fiberoptics4sale.com/blogs/wave-optics/semiconductor-laser-physics>. 2020.
- [2] Dr. Anjan Ghosh. *Chapter 19: Band to Band Generation*. NPTEL. [https://nptel.ac.in/content/storage2/courses/117104071/topic19/19\\_3.html](https://nptel.ac.in/content/storage2/courses/117104071/topic19/19_3.html). 2009.
- [3] Lee M. K. Sze S. M. *Semiconductor Devices: Physics and Technology 3rd edition*. Wiley. (2012).