Topic 5: Carrier Transport & Drift

Preface:

Our study has introduced techniques to increase the number of charge carriers within a material. For a material to produce a substantial current, these free carriers must move. We call these phenomena **Carrier Transport**, and this set of notes will focus on the first of these mechanisms; drift. Throughout this section, I will swap to and from vector notation. The vector notation will explicitly state the direction that a carrier, or current, will be due to some interaction. Vector notation will not be necessary once we gain an intuitive understanding of the mechanism.

Carrier Transport:

There are six carrier transport mechanisms common throughout semiconductor physics.

1. Drift
2. Diffusion
3. Generation and Recombination
4. Tunneling
5. Space charge Effect
6. High Field Effect

This course will look at the first three mechanisms: Drift, Diffusion, and Generation & Recombination. The remaining phenomena are beyond the scope of this course, but if time permits, we will discuss them briefly.

Drift Current:

Formally, **Drift is the flow of charge carriers due to the presence of an electric field.** An electric field is the result of a potential difference within the material, implying we can view carrier motion either from an electric field or voltage perspective.

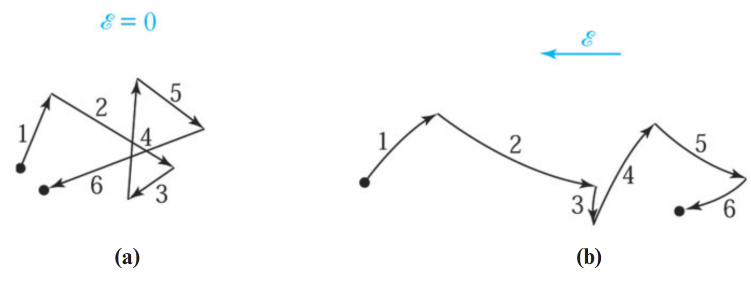


Figure 1: Electron motion for $\mathscr{E} = 0$ (a) and $\mathscr{E} \neq 0$ (b).

The figure above from the (Sze S. M., (2012) text aims at realizing the randomness associated with carrier motion from thermal effects; Fig. 1 Ch. 2. Imagine a piece of silicon sitting on a table with nothing attached to it, can there be a net current? No. There is no way for a net current to leave the device. We may also describe this effect by observing a single carrier, let's choose an electron. Carriers are constantly colliding with one another, changing direction and momentum. Each interaction will result in a non-zero net motion, but on average the displacement will be zero. Hence, there can are tiny currents associated with each electron. However, the cumulative interaction of carriers within the lattice results in a zero net current.

The moment an electric field is applied to the device a current is developed. The most common technique to generate this field is to attach a voltage source across the device. Recall that an electric field has units of $[\mathscr{E}] = \frac{V}{cm}$, implying that a voltage applied over a distance results in an electric field. A non-zero electric field will force the carriers to experience a net displacement. Holes will move in the direction of the field, and electrons opposite the field. This net motion does not mean that collisions stop, but that the carrier's motion is not purely dependent on random collisions. If I hadn't mentioned the field direction in the figure above, you could verify it because the electron is moving opposite the applied field.

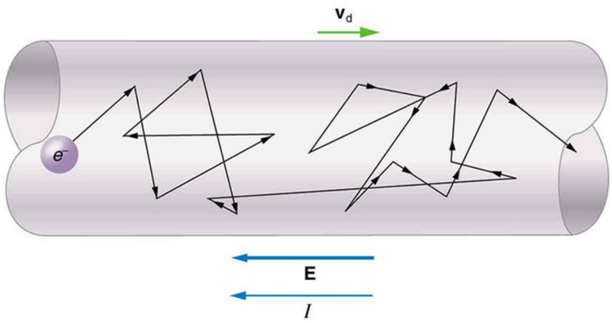


Figure 2: Electron drift visualization within a material. (Lumen, Unknown)

I have included this figure from Lumen Learning to show how a non-zero displacement may look within the material. It also serves to show that a carrier may occasionally move backward. The net motion is dependent on how a carrier interacts with the lattice. The change in energy with each interaction is not our focus, but having these visualizations help represent the phenomenon.

Physics Review & Terminology:

This section will discuss the terminology and physics concepts associated with electron motion, the electric field, and potential. If you feel comfortable with Electrodynamics, please feel free to skip this section.

Terminology:

1. m_n: The effective mass of an electron in kg.
2. m_p: The effective mass of a hole in kg.
3. v_n: The drift velocity of an electron in $\frac{cm}{s}$.
4. v_p: The drift velocity of a hole in $\frac{cm}{s}$.
5. τ: The mean free time between carrier collisions within a material in s.
6. μ_n: The electron mobility of the material in $\frac{cm^2}{V-s}$. $\mu_n \leq 1350$ in Si at 300K
7. μ_p: The hole mobility of the material in $\frac{cm^2}{V-s}$. $\mu_p \leq 350$ in Si at 300K

Concepts:

- Electric Force: $\vec{F_e} = \pm q\vec{\mathscr{E}}$, where the $\pm$ accounts for the polarity of the carrier.
- Momentum: $\vec{p} = m\vec{v}$
- Impulse: $\Delta P = F\Delta\tau$

This set of equations should recall memories of object collisions from classical physics. These concepts and the conversation of momentum will be used to derive the drift velocity for a hole. An electric field applies a force in the direction of the field; $\vec{F} = q\vec{E}$. The momentum of the hole at any moment is given by $\vec{p} = m_p\vec{v_p}$, where $\vec{v_p}$ is the desired hole velocity. The mean time between collisions represents how before a hole experiences another collision. Therefore, it represents the amount of time the force may increase the momentum of the hole. Therefore, the impulse relates to how much the momentum, specifically the drift velocity, will change between collisions. Therefore,

$$\begin{aligned}
\vec{p} &= \Delta\vec{p} \\
m_p v_p &= q\tau E \\
\therefore v_p &= \frac{q\tau E}{m_p} \\
v_p &= \mu_p E && (1) \\
\mu_p &= \frac{q\tau}{m_p} && (2)
\end{aligned}$$

The term $\frac{q\tau}{m_h}$ is so commonly used, we abstracted the quantity into a constant called mobility; μ_n & μ_p for electrons and holes respectively. There are two approaches the analyzing the units of mobility with only one providing useful units. Rearranging the final line above, $\mu = \frac{v}{\mathscr{E}} = \frac{cm*cm}{V*s} = \frac{cm^2}{V-s}$. If we analyze the mobility units in terms of the constant composing it, we have $\frac{C-s}{kg}$. These two quantities of units are equal and algebra can be used to convert from one another. We will use the $\frac{cm^2}{V-s}$ notation exclusively. Note from Eq. 2 that if $\uparrow \tau$ or $\downarrow m$ then mobility must increase. If a particle becomes lighter, the mobility must increase. If the time between collisions increases, the particle is more mobile because it does not collide as much; maintaining or increasing its velocity.

Holes vs. Electrons:

The analysis until this point has been with the hole as the carrier. The analysis is identical for electrons, except for an additional negative. This negative is due to the force on an electron being opposite the direction of the electric field. In short,

$$\begin{aligned}
\vec{F} &= -q\vec{\mathscr{E}} \\
m_n\vec{v_n} &= -q\tau_n\vec{\mathscr{E}} \\
\Rightarrow \vec{v_n} &= \frac{-q\tau_n E}{m_n} \\
\vec{v_n} &= -\mu_n\vec{\mathscr{E}} && (3)
\end{aligned}$$

In terms of structure, the drift velocity equations are nearly identical aside from the negative and the mobility for the respective carrier. An important observation is that **hole mobility is a purely positive constant**.

Lattice Scattering:

Compared to free space, a carrier traveling through the lattice does not have the same freedom. The lattice is a big mess of ions and atoms, charge carriers in motion, and covalent bonds. Each of these structures will affect the path of the electron due to either Coulombic forces or by a collision. We call these phenomena **scattering** since it includes any action which deflects, or scatters, a carrier from its intended motion. **Scattering affects carrier mobility.**

We will consider two scattering mechanisms: lattice and impurity scatting.

- **Lattice Scattering**: $\uparrow T \downarrow \mu$ Increasing temperature increases the energy of lattice atoms, resulting in more collisions which decreases τ.
- **Impurity Scattering**: $\uparrow T \uparrow \mu$ The charge of dopant ions will exert a Coulombic force on the carrier, deflecting it toward or away from the ion; depending on the polarity of ion and carrier. Increasing temperature increases the energy of the carrier which decreases the time spent near the ion, and hence decreases the total deflection on the carrier.

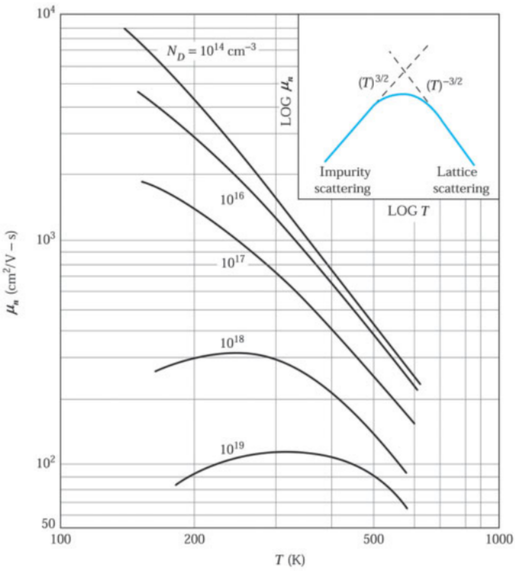


Figure 3: Mobility versus doping concentration and temperature. Ch. 2. Fig. 2 of (Sze S. M., (2012)

The chart above plots mobility versus temperature for several doping concentrations. At the top of the graph is the lightest doping at 10^{14} cm^{-3}and at the bottom the heaviest doping is at 10^{19} cm^{-3}. The top-right of the figure includes a sub-figure showing which scattering is more pertinent over temperature. Hence, for any doping concentration impurity scattering happens at low temperatures, and lattice scatting occurs at high temperatures. The reduction in mobility for both scatting types is most noticeable for large doping concentrations since 10^{14} is significantly less than 10^{22} in a mole of silicon. Regardless of doping concentration, each line decreases at larger temperatures due to lattice scattering.

Energy Band Diagrams:

In developing the expressions for mobility, drift velocity, and current it will be helpful to consider how we can relate these concepts to the energy band diagram. We want to find a bridge between the information encoded in the band diagram to how carriers move under drift. Recall that force is work per unit distance and that the difference in energy bands is in units of work. For an electron, let E_i represent an energy level, i, within the material. An increment in energy dE_i over a distance dx results in a force F.

$$
\begin{aligned}
F &= -q\mathscr{E} \\
F &= -\frac{dE_i}{dx} \\
\Rightarrow \mathscr{E} &= -\frac{1}{q}\frac{dE_i}{dx}
\end{aligned} \tag{4}
$$

The additional negative when taking the derivative is due to the relationship between electric field and potential. The energy between two bands is dependent on the potential between them; qV. Potential is the negative spatial derivative of the electric field, implying $E_i = qV$. I encourage you to verify this algebraically since this result is very significant to us.

We do not need to repeat this process for holes since energy band diagrams are with respect to electrons. The result above tells us that an electric field across a device will cause energy bands to bend. We call $\frac{dE_i}{dx} \neq 0$ **band bending** because the energy band will have a non-zero slope. Let's see if this concept aligns with our analysis to this point.

Let a piece of silicon sit on a table and experience zero electric fields. Drift current cannot occur within the device, implying the force acting on a carrier is zero. Note we are neglecting thermal effects on carrier motion. If the electric field is zero, the fundamental charge is a positive constant, implying $\frac{dE_i}{dx} = 0$. We have shown in the past that the energy bands are flat in this situation, so this aligns with our analysis. Say we have an n-type material with a non-zero electric field present. The process above forces $\frac{dE_i}{dx} = q\mathscr{E}$, resulting in the band diagram below.

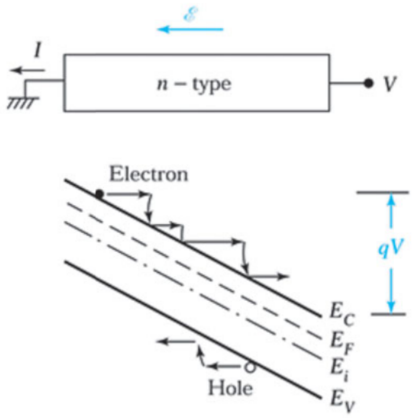


Figure 4: Drift Band Bending diagram. Ch. 2 Fig. 4 of (Sze S. M., (2012)

I particularly like the figure above from the Sze text due to it revealing not just band bending but also what it means to an individual carrier within the lattice. The electric field is pointing to the right, implying electrons will travel to the left. Using our formulas above, $\frac{dE_i}{dx} < 0$, with the negative being due to the direction of the electric field. **The distance between energy levels is unchanged since each band bends equally.**

In the figure, it may seem that an electron is rolling down the band. While this description is not fully accurate, it is a useful analogy when thinking of how carriers move within the band diagram. We represent the rolling as horizontal and vertical steps. The horizontal steps represent an electron moving with some energy through the lattice. The electron will scatter on something within the lattice, losing energy. This process repeats as the electron is forced to drift from the electric field. The inverse is true with holes moving from a low energy region to a high energy region.

Example:

Now that we know band bending results in an electric field **proportional to the electric field** there is a new set of questions we may answer. Say we have the band diagram below,

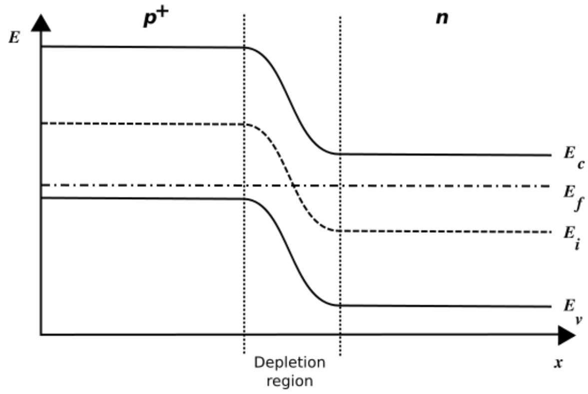
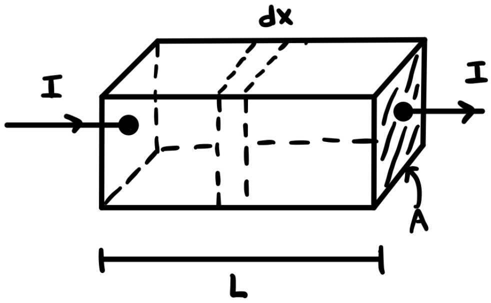


Figure 5: PN junction energy band diagram. Provided by (User, 2020)

For now, ignore everything except for the bands in the depletion region. I include this figure to show that our theory now will become important in future topics. In the center of the depletion region the band bends downward, $\frac{dE_i}{dx} < 0$, implying an electric field is pointing to the left. As we follow a given energy level into the depletion region the slope does not instantaneously change to negative. There is a smooth transition from a flat band to an approximately linear band, back to a flat band. From the simple-looking expression, $q\mathscr{E} = \frac{dE_i}{dx}$, we know that there exists a maximum electric field strength in the center of the depletion region. The electric field of a flat energy band is zero, implying the electric field in the p^+ and n region is zero. This process applies to any energy band diagram, the value of the slope encodes information about the magnitude and direction of an electric field.

Drift Current:

With an understanding of the mechanisms behind drift, we have the prerequisites to explicitly define drift current. Beginning with a piece of semiconductor material, the type does not matter, so let's assume it is an n-type.



Current is defined as the amount of charge per unit time; $I = \frac{dq}{dt}$. If we have some volume of semiconductor material, V, the number of charges within the volume is nV. Unit wise, the cm^3 cancels with the $\text{cm}^{-3}$of the carrier concentration. To convert the number of charges into a net charge, we multiply by the number of charges by q.

$$\frac{dq}{dt} = \frac{qnV}{dt}$$

It is possible to determine the volume of the material, but this expression is not as accurate as it could be. Current is defined with respect to some spatial point. Let us look at a small cross-section of the semiconductor with width dx and determine the current passing through it. We can represent the volume of this chunk as Adx. Plugging this into the formula above we get,

$$\begin{aligned}
\frac{dq}{dt} &= \frac{-qnAdx}{dt} \\
\frac{dx}{dt} &= v_{drift} = v_n \\
\Rightarrow \frac{dx}{dt} &= -qnAv_n
\end{aligned}$$

The negative is purely due to the electron charge. This derivation is identical for holes, excluding the negative associated with the charge. We have derived an expression for the drift velocity which we can substitute into the equation above, $v_n = \mu_n\mathscr{E}$.

$$\begin{aligned}
\frac{dq}{dt} &= -qnA * -\mu_n\mathscr{E} \\
I_n &= qnA\mu_n\mathscr{E} \qquad (5)
\end{aligned}$$

Each term in the above equation **must be positive**, excluding the electric field. This implies that the electron current is in the same direction as the electric field. Using the information from the energy band diagram, we can add the current direction to the list of useful information encoded within it. The current is proportional to each term because increasing any quantity allows more charge to pass through a given area. The last step we need to do is divide by the area, resulting in a current density as opposed to the current.

$$J_n = qn\mu_n\mathscr{E} \qquad (6)$$

$$J_p = qp\mu_p\mathscr{E} \tag{7}$$

These two equations tell us that there is more than one component to current within the semiconductor. We already knew this in our study of the energy band diagram, but interestingly both the electron and hole current flow in the same direction. Hence, the total current through the device is the addition of both terms.

$$J_d = J_{drift} = q(n\mu_n + p\mu_p)\mathscr{E} \tag{8}$$

$$\Rightarrow J_d = \sigma\mathscr{E} \tag{9}$$

$$\sigma = q(n\mu_n + p\mu_p)$$

The $q(n\mu_n + p\mu_p)$ term occurs so often throughout the study of carrier transport that it has been abstracted to the symbol sigma, which represents the conductivity of the device. The conductivity expression should look similar to those who have taken a course in Electromagnetic Fields. Typically we can represent a material with a conductivity, σ, to abstract the material parameters that influence how much current an electric field may induce. Conductivity also matters due to it being the inverse of resistivity. Recall from Electrostatics the equation $R = \frac{\rho L}{A}$. The resistivity of a material allows a material to be characterized regardless of the shape of the material.

Assumptions:

The study of semiconductor devices is ripe with assumptions. Many of our assumptions will involve neglecting a term that is much smaller than other surrounding terms. We know that for n-type semiconductors $n \ggg p$, implying it is perfectly valid to ignore the current contribution due to holes; $J_{drift} \approx qn\mu_n\mathscr{E}$. Similarly, if we have a p-type material $p \ggg n$, implying $J_{drift} \approx qp\mu_p\mathscr{E}$. To prove this assumption is valid, let's try and calculate the conductivity of silicon doped with 10^{16} donor ions. $(10^{16})^2 = 10^{32}$ and $\sqrt{10^{32} - 10^{21}} \approx 10^{16}$. The 10^{21} term is the expanded form of $4n_i^2$ The donor concentration is much larger than the intrinsic, implying that $n = N_D = 10^{16}$ cm^{-3}. $p = \frac{n_i^2}{n} = \frac{2.25*10^{20}}{10^{16}} = 22,500$ cm^{-3}. Plugging these concentrations into our equation for conductivity gives $\sigma = q(10^{16} * 1400 + 22500 * 450) = 1.4 * 10^{19}$. Hole conduction does produce a current, but compared to electron conduction is negligible.

Drift Simulation:

The following link leads to a Geogebra animation visualizing drift current and band bending; https://www.geogebra.org/m/g5j4szgt. Information on how to use the simulation is provided on the Geogebra page. I highly encourage you to play around with the simulation to reinforce your intuition of drift current.

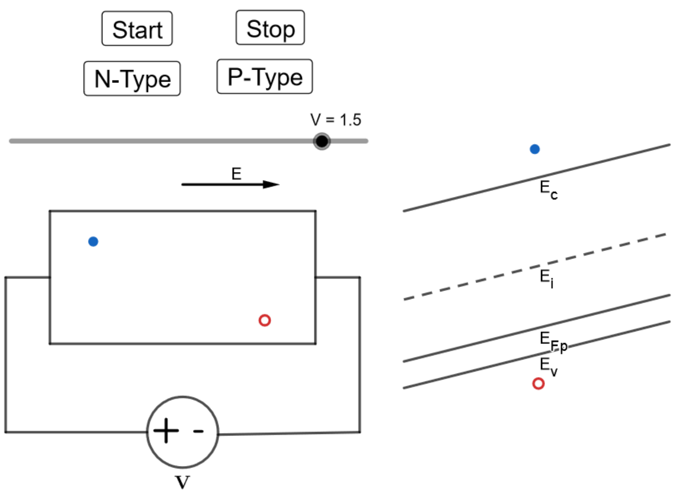


Figure 6: P-type drift current Geogebra simulation.

Conclusion

To keep this set of notes from becoming too lengthy we will cut it off here. This set of notes looked at the first of many carrier transport mechanisms, carrier drift. Drift is entirely due to an external electric field applying a force on free carriers within the lattice. The force experienced results in an increase of kinetic energy and a current is produced. We also discussed how electric field results in bending of the energy band diagram, how carriers "roll" up and down the bands to higher or lower energy states, and developed a quantitative expression for current density in terms of carrier concentrations and field strength. The next topic will be an interesting application of electron drift, and will likely be a product you will see in Senior Design, the Hall Effect.

References

Lee M. K. Sze S. M. Semiconductor devices: Physics and technology 3rd edition. Wiley, (2012).

Learning Lumen. Electric current. https://courses.lumenlearning.com/boundless-physics/chapter/electric-current/, Unknown.

BigBear User. Electronics stack exchange question. https://electronics.stackexchange.com/questions/529194/why-do-n-type-materials-have-a-smaller-conduction-energy-band, 2020.