Topic 15: Bipolar Junction Transistor Analysis

Preface:

This set of notes builds upon the intuitive understanding of BJT by developing equations expression the various currents and characteristics of the device. The mathematics will be quite involved, and reference heavily to the PN junction analysis. It is essential you are confident with the PN junction before reading this set of notes.

Notation & Device Properties:

In our analysis, it will be helpful to define several quantities to both simplify the analysis and to characterize the BJT. Again, our focus will be on the PNP BJT but applies to NPN devices with proper subscript changes.

Emitter Efficiency, γ: $\gamma = \frac{I_{Ep}}{I_{Ep}+I_{En}}$.

Base Transport Factor, α_T: $\alpha_T = \frac{I_{Cp}}{I_{Ep}}$.

Common-base Current Gain, α_o: $\alpha_o = \frac{I_C}{I_E}$.

Common-emitter Current Gain, β_o: $\beta_o = \frac{I_C}{I_B} = \frac{\alpha_o}{1-\alpha_o}$.

In the order of the above list, the **emitter efficiency** describes how efficient the emitter emits carriers. The emitter efficiency is bounded between $(0, 1)$ with 1 representing perfect efficiency. The only way to achieve an efficiency of 1 is by eliminating the minority component of the emitter, implying γ will typically be slightly less than 1. The **base transport factor** describes the ratio of carriers that are injected into the base and survive to the collector. Again, the transport factor will be bounded between $(0, 1)$. Lastly, the **common-base current gain** is the current gain in the common base configuration.

Common-base:

We may try some algebraic manipulations to attempt to relate these various quantities. Recall that the minority component collector current is negligible when compared to the majority component. The minority component is due to the random distribution of majority carriers in the base wandering into the base-collector depletion region; hence, $I_C \approx I_{Cp}$. Expanding the emitter current in the common-base current gain we have,

$$\begin{aligned} \alpha_o &= \frac{I_C}{I_E} \\ &= \frac{I_{Cp}}{I_{Ep} + I_{En}} \\ &= \frac{I_{Ep}}{I_{Ep} + I_{Ep}} \cdot \frac{I_{Cp}}{I_{Ep}} \\ \Rightarrow \alpha_o &= \gamma \alpha_T \end{aligned} \tag{1}$$

The transition from step 2 to step 3 involved multiplying by 1, where $\frac{I_{Ep}}{I_{Ep}} = 1$. Ideally, we want α_o to equal one or else power is lost. When $\alpha_o < 1$ current injected into the emitter is "lost" within the device, decreasing the signal strength at the collector. Since both γ and α_T are slightly below one it is necessary to maximize both of them to maximize their product. As stated, the emitter efficiency is maximized when the minority current component is driven toward zero. This can be accomplished by heavily doping the emitter above the base; $N_{AE} \ggg N_{DB}$ for a PNP device. To drive the base transport factor to 1, the neutral base width must be incredibly smaller than the diffusion length; $w_{NB} \lll L_p$.

These two conditions are possible to achieve a current gain of 0.999 but it will always be slightly less than 1.

Common-emitter:

We can repeat the process above to derive an expression for the common-emitter current gain. The base transport factor and emitter efficiency are independent of the device configuration.

$$
\begin{aligned}
I_C &\approx I_{Cp} \\
I_{Cp} &= \alpha_T I_{Ep} = \frac{I_{Cp}}{I_{Ep}} I_{Ep} \\
\Rightarrow I_C &= \alpha_T I_{Ep} \\
I_C &= \alpha_T \gamma \frac{I_{Ep}}{\gamma} \\
&= \alpha_o I_E \\
&= \alpha_o (I_B + I_C) \\
&= \frac{\alpha_o}{1 - \alpha_o} I_B \qquad (2) \\
\Rightarrow I_C &= \beta_o I_B \qquad (3)
\end{aligned}
$$

Before we step through the algebra, the goal is to relate the collector and base current. Under the common-emitter configuration, the base current is our input, and the collector current is the output. We currently have no way of relating the two. Stepping through the algebra above, we have stated that the collector current is due to the hole, or majority, component. By multiplying by $^{I_{Ep}}/_{I_{Ep}}$, or 1, we can introduce the base transport factor. Repeating with gamma allows the base transport factor term to be converted to the common-base current gain. However, gamma is introduced to convert the majority component of the emitter current into the total emitter current. Since the emitter current is composed of the total base and collector current the base current is now included in the expression. Distributing α_o and factoring results in the final equation, Eq. 2.

We mentioned that the common-base current gain must be less than one. This implies the denominator of Eq. 2 will be small. Dividing 1 by a small number results in a fairly large number. Beta is generally in the range of 100-300 for practical transistors like the 2n2222.

Device Notation:

Before we can derive the various equations for the currents within the BJT we need to define our notation. The two PN junctions give us a choice in defining the zero on our x-axis. We will use the notation in the following figure for each configuration.

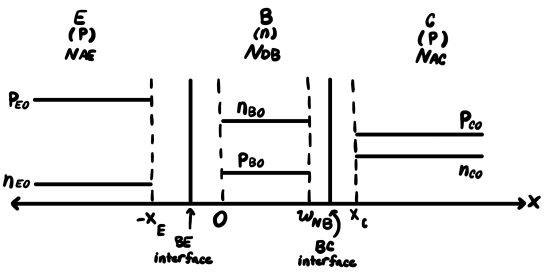

Figure 1: PNP BJT thermal equilibrium concentrations ignoring bias voltage.

We assign $x = 0$ at left-most point of the neutral base. The base-emitter depletion has been included because diffusion may occur for V_{BE} slightly below the junction's built-in voltage. Implying a small depletion region may be present. The neutral base extends to some point which we will call w_{NB}. Since we start the neutral base at $x = 0$, W_{NB} accurately represents the total width of the neutral base. The base-emitter depletion extends into the emitter until $-x_E$. Similarly, the collector depletion extends to the point x_c. The physical base-emitter interface will lie between $x = -x_E$ and $x = 0$. Likewise, the base-collector interface will lie between $x = W_{NB}$ and $x = x_c$.

The device is doped such that $N_{AE} \gg N_{DB} \gg N_{AC}$. Using our notation from PN junctions, the majority concentration for each region is: $p_{Eo} = N_{AE}$, $n_{Bo} = N_{DB}$, and $p_{Co} = N_{AC}$. The first subscript denotes the region of the device and the subscript, o represents thermal equilibrium. We cannot iterate how important it is to understand the notation because it will lead to many mistakes throughout the derivation. Utilizing the mass-action law, we can find the thermal equilibrium minority carrier concentration.

$$n_{Eo} = \frac{{n_i}^2}{p_{Eo}} \tag{4}$$

$$p_{Bo} = \frac{{n_i}^2}{n_{Bo}} \tag{5}$$

$$n_{Co} = \frac{{n_i}^2}{p_{Co}} \tag{6}$$

From the doping concentration we know that $p_{Eo} \gg n_{Bo} \gg p_{Co}$. Since p_{Eo} is the largest majority concentration, it must have the smallest minority concentration. Hence, we will see a decreasing staircase in the majority concentration and an increasing staircase in the minority concentration.

Forward Active:

Under forward active we know that the base-emitter junction is forward biased and the base-collector reversed biased. We can use our knowledge of PN junction minority curves for forward and reverse bias to avoid re-deriving the equations. Therefore, the n_{Eo} curve will bend upward towards $x = -x_E$ and n_{Co} curve downwards toward zero at $x = x_c$. We must ask ourselves what will happen in the neutral base?

Collector & Emitter:

Since the base-emitter junction is a PN junction we can rely on the following equation from the PN junction notes and modify it for the emitter notation,

$$n_p(x) = n_{po} + n_{po}\left(e^{qV_{bias}/kT} - 1\right)e^{(x\,+\,x_p)/L_n} \text{ , for } x \leq -x_p \tag{7}$$

Within the emitter, n_{po} becomes n_{Eo}, V_{bias} becomes V_{EB}, x_p becomes x_E, and L_n becomes L_E. Be aware of the diffusion constant and carrier lifetime. Using the variable name L_E abstracts the region type. We know we have a PNP device, implying the emitter is a p-type with electrons as the minority. Hence, L_E will depend on the electron diffusion constant and electron lifetime. Substituting into the above equation, we have our minority concentration curve for the emitter. The collector equation has been included as well using the same process.

$$n_E(x) = n_{Eo} + n_{Eo}\left(e^{qV_{EB}/kT} - 1\right)e^{(x\,+\,x_E)/L_E} \text{ , for } x \leq -x_E \tag{8}$$

$$n_C(x) = n_{Co} + n_{Co}\left(e^{qV_{CB}/kT} - 1\right)e^{(x\,-\,x_C)/L_C} \text{ , for } x \geq x_C \tag{9}$$

Equations 8 and 9 will be useful later in our discussion. Our focus at the moment is the boundary condition at $x = x_E$ and $x = x_C$. Evaluating Eq. 8 at $x = -x_E$ reduces the exponential term to one, and one term of n_{Eo} is canceled. Therefore,

$$n_E(-x_E) = n_{Eo}e^{\frac{qV_{EB}}{kT}} \tag{10}$$

$$n_C(x_c) = n_{Co}e^{\frac{qV_{CB}}{kT}} \tag{11}$$

$$\tag{12}$$

Note that V_{CB} is used in the above equations instead of V_{BC}. When we derived our equations for the PN junction we assumed the voltage source is placed with the positive terminal on the p-type bulk and the negative on the n-type bulk. We need to remember that the polarity is opposite the common-base configuration and if we use the equation to calculate the concentration a negative value must be included; $V_{CB} = -V_{BC}$.

Moving onto the neutral base, we must determine the hole concentration at $x = 0$ and $x = W_{NB}$. Luckily the equation for the minority concentration is of the same form. The following equation was derived in the diode IV characteristic lecture notes and is provided for reference.

$$p_n = p_{no} + p_{no}\left(e^{qV_{bias}/kT} - 1\right)e^{-\frac{(x-x_n)}{L_p}} \ , \text{for } x \geq x_n \tag{13}$$

Neutral Base:

We may be tempted to take the PN junction equations and apply them to our new notation, however, **this is wrong!** With our new notation, p_{no} becomes p_{Bo}, V_{bias} becomes V_{EB}, x_n becomes 0 and L_p becomes L_C. The horizontal shift becomes zero since we conveniently placed the base depletion region boundary at $x = 0$.

$$p_B(x) = p_{Bo} + p_{Bo}\left(e^{qV_{EB}/kT} - 1\right)e^{-\frac{x}{L_B}} \tag{14}$$

$$\Rightarrow p_B(0) = p_{Bo}e^{q/V_{EB}kT} \tag{15}$$

At the $x = W_{NB}$ boundary, we utilize the same replacement strategy.

$$p_B(x) = p_{Bo} + p_{Bo}\left(e^{qV_{CB}/kT} - 1\right)e^{-\frac{x-W_{NB}}{L_B}} \tag{16}$$

$$\Rightarrow p_B(W_{NB}) = p_{Bo}e^{q/V_{CB}kT} = 0 \tag{17}$$

The last equation claims that the hole concentration at $x = W_{NB}$ equals zero. Remember the depletion region is present at the base-collector junction. The built-in electric field will sweep away any carriers. Hence, any holes which successfully traverse the base will be swept away producing a region with zero concentration.

From the continuity equation, several simplifications are present in steady state. The partial derivative with respect to time, $\frac{\partial p_n}{\partial t}$, becomes zero. We assume zero electric field within the neutral base, implying the drift terms become negligible and we are left with only diffusion and recombination terms. For the electron concentration in a p-type region, the continuity equation simplifies to,

$$D_p\frac{d^2p_n}{dx^2} = R_p \tag{18}$$

However, there are further simplifications we can make. Recall the base transport factor from the beginning of our BJT discussion. If the base transport factor is approximately one, then a small portion of carriers are being lost in the base. Small loss implies the recombination rate is small, even negligible. The underlying assumption which minimized the base recombination was an extremely narrow neutral base width. This is why applying the general PN junction formula is incorrect, the base is not infinitely long; **long base assumption**. We need to derive the equation for a **short base**.

Substituting a negligible recombination rate,

$$\begin{aligned} D_p\frac{d^2p_n}{dx^2} &= 0 \\ \Rightarrow \frac{dp_n}{dx} &= C \\ \Rightarrow p_n(x) &= Cx + D \ , \text{for } 0 \leq x \leq W_{NB} \end{aligned} \tag{19}$$

From this, the minority concentration curve is linear within the neutral base if a short base is present. Switching to the base notation, we have one boundary condition of $p_b(W_{NB}) = 0$ but not at $x = 0$. Luckily we can utilize our former equations to determine the boundary condition at $x = 0$. Our issue with Eq. 13 is the inaccuracy for large x when compared to the short base junction. Imagine a charge which has just entered the neutral base. The charge is unaware of how long the base is. From a physical

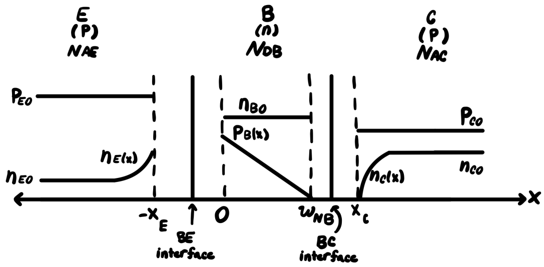

Figure 2

perspective, both equations must have the same boundary conditions at $x = 0$. It is only when the carrier has traveled sufficiently that the diffusion length will influence the minority carrier concentration. Therefore, our two boundary conditions are,

$$p_B(0) = p_{Bo}e^{qV_{EB}/kT} \tag{20}$$
$$p_B(W_{NB}) = 0 \tag{21}$$

Substituting these two conditions into the linear into the linear equation, Eq. 19, we can solve for the constants C and D.

$$\begin{aligned} p_B(0) &= D = p_{Bo}e^{qV_{EB}/kT} \\ p_B(W_{NB}) &= 0 = CW_{NB} + D \Rightarrow C = -\frac{D}{W_{NB}} \\ \Rightarrow p_B(x) &= D\left(1 - \frac{x}{W_{NB}}\right) \\ p_B(x) &= p_{Bo}e^{q/V_{EB}kT}\left(1 - \frac{x}{W_{NB}}\right) \end{aligned} \tag{22}$$

Current Components:

We do not have equations for the minority carrier concentrations in the emitter, base, and collector. As with the PN junction, we may use the diffusion current formula to derive the expressions and evaluate the current at a given point. Before we substitute the expressions, it will be helpful to determine which current components these curves produce. Due to their nonzero slope, the three curves are diffusion currents. We have the electron concentration in the emitter, implying it composes I_{En}. Similarly, the collector will compose I_{Cn} since the minority curve is the electron. Lastly, the hole concentration in the base composes the hole current in the base. We did not explicitly provide the notation for the base current but it represents I_{Bp}.

To calculate the emitter hole current, I_{Ep}, we apply Eq 22 to the diffusion current equation.

$$J_p = -qD_p \frac{dp}{dx}\bigg|_{x=0} \tag{23}$$

We choose $x = 0$ because this point represents the current of holes that have passed the base-emitter depletion region. If we were to assume recombination in the base-emitter SCR the number of holes injected into the junction would be less than the number seen at the boundary of the neutral base. However, we assume that any carrier entering the SCR will pass without issue.

$$\begin{aligned} J_{Ep} &= -qD_B \frac{dp_B(x)}{dx}\bigg|_{x=0} \\ p_B(x) &= p_{Bo}e^{q/V_{EB}kT}\left(1 - \frac{x}{W_{NB}}\right) \\ &= -qD_B\left(-\frac{p_{no}e^{qV_{EB}/kT}}{W_{NB}}\right) \end{aligned}$$

$$
\begin{aligned}
&= \frac{qD_B p_{no}}{W_{NB}} e^{qV_{EB}/kT} \\
\Rightarrow I_{Ep} &= \frac{qAD_B p_{no}}{W_{NB}} e^{qV_{EB}/kT} \approx I_{Cp} \qquad (24)
\end{aligned}
$$

Since we have shown that $I_{Ep} \approx I_{Cp}$ we do not need to explicitly solve for I_{Cp}. The emitter current is not composed purely of holes, we must also find the minority carrier component.

$$
\begin{aligned}
J_n &= qD_n \frac{dn}{dx}\bigg|_{x=-x_E} \\
n_E(x) &= n_{Eo} + n_{Eo}\left(e^{qV_{EB}/kT} - 1\right) e^{(x\,+\,x_E)/L_E} \text{ , for } x \leq -x_E \\
J_{En} &= qD_E \frac{dn_E(x)}{dx}\bigg|_{x=-x_E} \\
&= \frac{qD_E n_{Eo}}{L_E}\left(e^{qV_{EB}/kT} - 1\right) \\
\Rightarrow I_{En} &= \frac{qAD_E n_{Eo}}{L_E}\left(e^{qV_{EB}/kT} - 1\right) \qquad (25)
\end{aligned}
$$

The first two steps above included the electron diffusion equation and the expression for the emitter minority carriers for reference. The difficulty is in developing expressions for the minority carrier concentrations, whereas only a step or two of calculus is needed to convert the concentration expression into a current. The last component we must find is the electron current in the collect, I_{Cn}.

$$
\begin{aligned}
J_n &= qD_n \frac{dn}{dx}\bigg|_{x=x_C} \\
n_C(x) &= n_{Co} + n_{Co}\left(e^{qV_{CB}/kT} - 1\right) e^{(x\,-\,x_C)/L_C} \text{ , for } x \geq x_C \\
J_{Cn} &= qD_C \frac{dn_c(x)}{dx}\bigg|_{x=x_C} \\
&= -\frac{qD_C n_{Co}}{L_C}\left(e^{qV_{CB}/kT} - 1\right) \\
\Rightarrow I_{Cn} &= -\frac{qAD_E n_{Eo}}{L_C}\left(e^{qV_{CB}/kT} - 1\right) \qquad (26) \\
\Rightarrow I_{Cn} &\approx \frac{qAD_E n_{Eo}}{L_C} \qquad (27)
\end{aligned}
$$

Note that an additional negative was necessary for the collector current derivation. We know that the hole and electron currents in the collector are aligned in the same direction. The negative is due to the way we derived our PN junction equations. We assumed that the p-type region was to the left and the n-type to the right, however, the base-collector junction is inverted. Hence, we need to multiply the argument of the exponential by -1 to reflect it about the junction. The equations above also include a strong base-collector reverse bias, which drives the exponential term toward zero. In general, we need to be aware of the directions of the various currents. Sometimes this means reflecting the minority concentration curve, but it is easier to consider the absolute value, or magnitude, of each current component. In the end, we can account for any missing negatives by placing the current components in the proper direction, granted it is not rigorous.

We have now calculated 4 of the 5 current components. We will not calculate the majority current in the base because it is not the focus. The BJT works on the fundamental principle of minority carrier diffusion. The current through the device is encoded in the minority carriers. For example, we used the minority base concentration to infer the majority carrier current in the base and collector. We gain no information by attempting to express the hole concentration current in the emitter based on the hole concentration. It is not until the holes from the emitter diffuse into the base that a significant current is formed.

With the emphasis on minority current components reigning dominant in the BJT we can rank the strength of each component from our equations above. In order of decreasing strength, the list of current component strength is I_{Ep}, I_{Cp}, I_{En}, and I_{Cn},

I_{Cn} is the smallest of the components due to the strong reverse bias. The strong reverse bias removes the exponential influence leaving an incredibly small constant for $\frac{qAD_C n_{Co}}{L_C}$ on the order of nano or picoamps. I_{En} is the next largest component due to the forward-biased base-emitter junction. As holes diffuse from the emitter electrons diffuse from the base and the current is not completely negligible. From our discussion on emitter efficiency, we know that I_{En} must be less than I_{Ep}. If the emitter efficiency is 0.99, or 99%, electron component is 100x less than the hole current. As for I_{Ep} and I_{Cp} it has been stated several times that we approximate them as equal but I_{Ep} is slightly larger. **Note that this chart only applies to forward active.**

Saturation:

This section will provide a short overview of the saturation derivation. Much of the work has been done in the forward active derivation, hence, we will utilize some of the pre-derived equations. We start by copying the $n_E(x)$ and $n_C(x)$ equations.

$$n_E(x) = n_{Eo} + n_{Eo}\left(e^{qV_{EB}/kT} - 1\right)e^{(x\,+\,x_E)/L_E}\ \text{, for } x \le -x_E \tag{28}$$

$$n_C(x) = n_{Co} + n_{Co}\left(e^{qV_{CB}/kT} - 1\right)e^{(x\,-\,x_C)/L_C}\ \text{, for } x \ge x_C \tag{29}$$

These two equations above are independent of the bias voltage. To apply the equations to saturation we know that V_{EB} and V_{CB} are much greater than the thermal voltage. The large bias voltage implies the -1 term is negligible.

$$n_E(x) = n_{Eo} + n_{Eo}e^{qV_{EB}/kT}e^{(x\,+\,x_E)/L_E}\ \text{, for } x \le -x_E \tag{30}$$

$$n_C(x) = n_{Co} + n_{Co}e^{qV_{CB}/kT}e^{(x\,-\,x_C)/L_C}\ \text{, for } x \ge x_C \tag{31}$$

The two minority carrier equations may be differentiated via the diffusion equation to determine the electron current components in the emitter and collector.

$$J_n = qD_n\frac{dn}{dx} \tag{32}$$

$$J_{En}(x) = \frac{qD_E n_{Eo}}{L_E}e^{qV_{BE}/kT}e^{(x\,+\,x_E)/L_E} \tag{33}$$

$$J_{Cn}(x) = \frac{qD_C n_{Co}}{L_C}e^{qV_{CB}/kT}e^{(x\,-\,x_C)/L_C} \tag{34}$$

$$\tag{35}$$

The two equations above may be evaluated at the boundaries $x = 0$ and $x = x_C$ to determine the net current to and from each depletion region. The next step involves determining the minority concentration within the neutral base. Unlike cutoff or forward active, where we know a boundary condition is zero, we are unsure of the concentration at the $x = 0$ and $x = W_{NB}$ boundaries. The minority concentration in the base is assumed to vary linearly. We justify the linearization for reasons identical to the forward active case.

$$p_n = p_{no} + p_{no}\left(e^{qV_{bias}/kT} - 1\right)e^{-\frac{(x-x_n)}{L_p}}\ \text{, for } x \ge x_n \tag{36}$$

$$p_B(0) = p_{Bo}e^{qV_{EB}/kT}$$

$$p_B(W_{NB}) = p_{Bo}e^{qV_{CB}/kT}$$

The minority carrier concentration in the base is of the form $p_n(x) = Cx + D$. We two boundary conditions so we can solve for C and D.

$$p_B(0) = D = p_{Bo}e^{qV_{EB}/kT} \tag{37}$$

$$p_B(W_{NB}) = Cx + D = p_{Bo}e^{qV_{CB}/kT} \tag{38}$$

$$\Rightarrow C = \frac{p_{Bo}}{W_{NB}}e^{qV_{CB}/kT} - \frac{D}{W_{NB}} \tag{39}$$

$$= \frac{p_{Bo}}{W_{NB}} \left(e^{qV_{CB}/kT} - e^{qV_{EB}/kT} \right) \tag{40}$$

$$\Rightarrow p_B(x) = \frac{p_{Bo}}{W_{NB}} \left(e^{qV_{CB}/kT} - e^{qV_{EB}/kT} \right) x + p_{Bo} e^{qV_{EB}/kT} \tag{41}$$

The final step of the derivation is to covert the minority hole concentration in the base to a current density. The total collector and emitter current will not be derived. The last steps involve adding the components. You may also verify the equations correctness via the two-port representation on the following pages.

$$J_p = -qD_p \frac{dp}{dx} \tag{42}$$

$$= -qD_P \frac{p_{Bo}}{W_{NB}} \left(e^{qV_{CB}/kT} - e^{qV_{EB}/kT} \right) \tag{43}$$

Two-port Representation:

The analysis above is helpful in determining the components and total current for the four modes of operation but it lacks convenience. We would need to derive the equations again for an NPN device, and even a PNP in a different mode of operation. It would be incredibly convenient if we could model the BJT in terms of the base-emitter and base-collector voltages.

Coincidentally the exists a set of coefficients that accurately describes the 4 modes of BJT. The equations for I_E and I_C apply to forward active, but the coefficients apply to any mode of operation.

$$I_E = a_{11} \left(e^{qV_{EB}/kT} - 1 \right) + a_{12} \tag{44}$$

$$I_C = a_{21} \left(e^{qV_{CB}/kT} - 1 \right) + a_{22} \tag{45}$$

$$a_{11} = qA \left(\frac{D_B p_{no}}{W_{NB}} + \frac{D_E n_{Eo}}{L_E} \right) \tag{46}$$

$$a_{12} = a_{21} = \frac{qAD_B p_{Bo}}{W_{NB}} \tag{47}$$

$$a_{22} = qA \left(\frac{D_B p_{Bo}}{W_{NB}} + \frac{D_C n_{Co}}{L_C} \right) \tag{48}$$

This matrix of a coefficients is a bold claim. Let's see if it provides the same results as our forward active derivation.

$$\begin{aligned} I_E &= qA \left(\frac{D_B p_{no}}{W_{NB}} + \frac{D_E n_{Eo}}{L_E} \right) \left(e^{qV_{EB}/kT} - 1 \right) + \frac{qAD_B p_{Bo}}{W_{NB}} \\ &= qA \left(\frac{D_B p_{no}}{W_{NB}} + \frac{D_E n_{Eo}}{L_E} \right) e^{qV_{EB}/kT} - qA \frac{D_E n_{Eo}}{L_E} \\ &= qA \frac{D_B p_{no}}{W_{NB}} e^{qV_{EB}/kT} + \frac{qAD_E n_{Eo}}{L_E} \left(e^{qV_{EB}/kT} - 1 \right) \end{aligned}$$

Eq. 49 has been split up into the hole and electron components. The first term is the hole component, given by Eq. 25, and the second term is the electron component, given by Eq. 24. The equations match identically, and only a few steps of algebra were necessary. We can repeat the process for the collector current,

$$\begin{aligned} I_C &= a_{21} \left(e^{qV_{EB}/kT} - 1 \right) + a_{22} \\ &= \frac{qAD_B p_{Bo}}{W_{NB}} \left(e^{qV_{EB}/kT} - 1 \right) + qA \left(\frac{D_B p_{Bo}}{W_{NB}} + \frac{D_C n_{Co}}{L_C} \right) \\ &= \frac{qAD_B p_{Bo}}{W_{NB}} e^{qV_{EB}/kT} + \frac{qAD_C n_{Co}}{L_C} \end{aligned}$$

To no surprise, the equation matches identically. However, the above equations only apply to forward active. The expressions below describe how these coefficients may be used for any mode of operation.

$$I_E = a_{11}\left(e^{qV_{EB}/kT} - 1\right) - a_{12}\left(e^{qV_{CB}/kT} - 1\right) \tag{49}$$

$$I_C = a_{21}\left(e^{qV_{EB}/kT} - 1\right) - a_{22}\left(e^{qV_{CB}/kT} - 1\right) \tag{50}$$

$$\tag{51}$$

It is a little bit more convenient to write the above equations in matrix notation,

$$\begin{bmatrix} I_E \\ I_C \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} \left(e^{qV_{EB}/kT} - 1\right) \\ \left(e^{qV_{CB}/kT} - 1\right) \end{bmatrix} \tag{52}$$

Note that the voltage subscripts are referenced to a PNP device. Simply swap the subscripts for different devices and common configurations but ensure that the argument of each exponential is purely the voltage across the junction.

With this general description of the BJT we can plug in values for V_{EB} and V_{CB} to see how each of the other modes work. In cutoff, both V_{EB} and V_{CB} are negative, implying the exponential terms approach zero. Removing the exponential terms results in $I_E = -a_{11} + a_{12}$ and $I_C = -a_{21} + a_{22}$. Each of these coefficients is a small number, implying little current is passing through the device. Again, little current is synonymous with the reverse saturation current since both junctions are reverse biased.

Derivation Aside:

If it is not apparent, this set of notes relies heavily on the PN junction derivations. For the sake of time and space in this set of notes, the full derivation has not been included. However, that does not mean we will not expect you to be able to derive the minority carrier functions from scratch. It is paramount that you understand the process beyond just the PN junction. Yes, the only difference between the PN junction and the BJT derivation would be notation. Regardless, take the time to step through the derivation without any notes, solutions, or guides. We are deriving the forward active equations, but cutoff, saturation, and reverse active are all fair questions that could be asked on an exam. I would recommend deriving for saturation since reverse active is a reflection of forward active, and cutoff is a relatively short derivation.

Conclusion:

This marks the end of our discussion on the BJT. The next, and last topic, of the course is the MOS system and MOSFET. Within this set of notes we built on the theory provided in the last set of notes. We derived the minority carrier concentrations and current expressions for a PNP device in forward active and saturation. The two derivations are relatively similar, algorithmically. However, we noted that the collector and emitters currents can easily be described by a matrix of two-port-like parameters. The parameters of which all rely on material properties of the device. The use of the two-port model provides confirmation our derivation was correct and a much easier way to express the current through the collector and emitter.