Topic 6: Hall Effect & Hall Effect Sensors:

Introduction:

Edwin Hall is a name you likely do not recognize, but the Hall Effect and the subsequent Hall effect sensor is an incredibly powerful phenomenon and tool, respectively. The Hall effect sensor is one of the most common solid-state magnetometers, or devices to measure magnetic fields. The effect was discovered in 1879 and was one of the first vector magnetometers invented. The accuracy associated with the sensor is very impressive due to most magnetometers measuring the strength at a particular point, a scalar, whereas a Hall effect sensor can measure both the strength and the direction, a vector. Hall's Ph.D. thesis is public if you want to learn more about his discovery process.

Magnetic Deflection Simulation:

Provided in the following link is a Hall effect simulation created by National MagLab; https://nationalmaglab.org/education/magnet-academy/watch-play/interactive/hall-effect. The simulation is not identical to our experiment, but it emphasizes how a magnetic field will deflect, or bend, a beam of electrons. Provided below is a figure of the simulation.

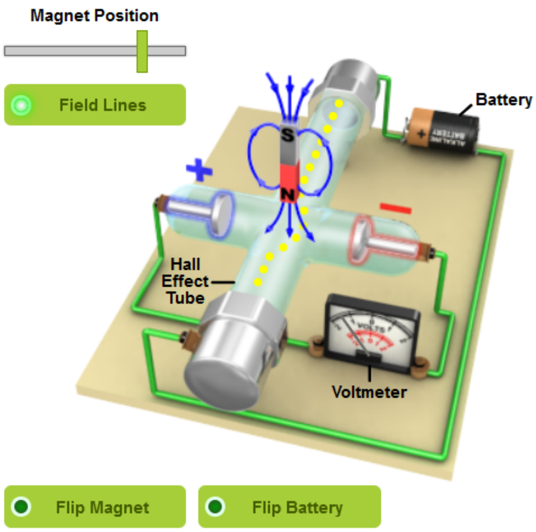

Figure 1: Magnetic deflection simulation by (Collaborators, 2019)

Experiment Setup:

Imagine we have a chunk of p-type silicon, abbreviated p-Si, sitting on a table. The material has a length, l, width, w, and thickness, t. The cross-sectional area of the material, A, equals wt; $A = wt$. A DC voltage source is attached to two ends of the material, resulting in an electric field across the length of the device in the $\hat{x}$ direction. This setup is no different from our study of drift current and is visualized in the figure below. Let empty circles represent holes and filled circles electrons.

We know that holes drift in the direction of the electric field, implying their velocity is in the $\hat{x}$ direction. There is only so much motion you can express in a single image, so imagine that these holes are slowly moving in the $\hat{x}$ direction. We will now activate an external magnetic field in the $\hat{z}$ direction.

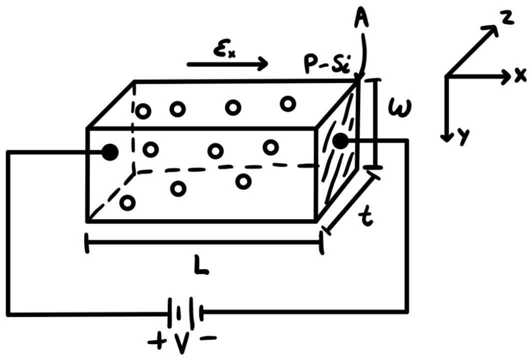

Figure 2: Hall effect initial setup.

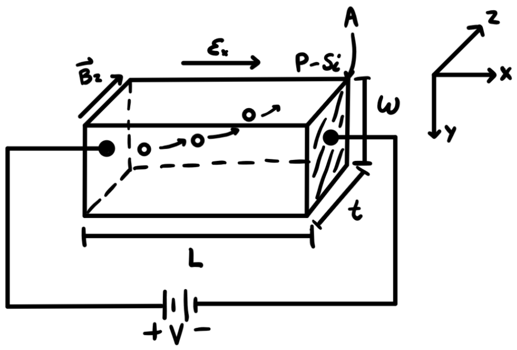

Figure 3: Experiment setup after the magnetic field is activated.

The magnetic field **does not** affect the electric field produced by the DC voltage source. However, the magnetic field does affect the motion of the carriers within the semiconductor. Recall that moving electrons within a magnetic field will begin to curve.

$$F_B = q(\vec{v_h} \times \vec{B_z}) \tag{1}$$

We do not need to be strict with our vector notation due to motion along the coordinate axes, it is primarily aimed at keeping track of directions more than for rigor. We can simplify the equation by extracting the magnitudes of each vector and substituting the unit vectors into the cross product. To be more precise, the Lorentz force equation should be used to view the motion due to both the electric and magnetic fields:

$$F = \pm q(\vec{\mathcal{E}} + (\vec{v_h} \times \vec{B_z}) \tag{2}$$

Since the external electric and magnetic fields are independent of one another all the constants can be extracted resulting in an addition of two unit vectors composing the net force. Looking at just the influence of the magnetic field, we can determine how the carriers will curve through the semiconductor.

$$\begin{aligned} F_B &= qv_hB_z(\hat{x} \times \hat{z}) \\ F_B &= -qv_hB_z\hat{y} \end{aligned} \tag{3}$$

The right-hand rule states that $\hat{x} \times \hat{z} = -\hat{y}$. The presence of the magnetic field in the $\hat{z}$ direction results in a force applied to the holes in the $-\hat{y}$ direction. This force will result in holes being pushed to the top face of the semiconductor. Inversely, electrons will experience a force pushing them to the bottom face. The charge on the electron is negative, which inverts the sign of Eq. 3; $F_B = qv_hB_z\hat{y}$. With time charge will accumulate on the top and bottom faces as shown in Fig. 3.

The holes have accumulated on the top face and the electrons on the bottom face. Note that the concentration of holes is much greater than the electrons, so we might ask how is it possible for this charge imbalance to form? The material is p-type implying boron is the dopant. The holes are forced

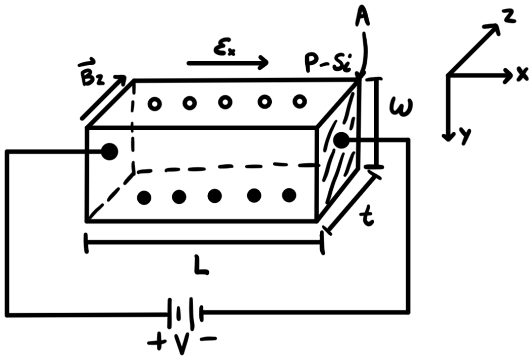

Figure 4: Carrier Steady-state.

by the electic field to leave the host atom and the magnetic field forces them to the top face. A hole cannot just leave an atom the same way an electron can, it requires an electron to combine with the hole. When an electron combines with a hole it leaves a negatively charged boron ion on the bottom face of the semiconductor. Now hopefully this situation looks familiar, a charge accumulation separated by some distance? A capacitor is formed within the material in the y-direction. Charged capacitors imply a potential difference across the faces and a net electric field in the y-direction, denoted $\vec{\mathscr{E}}_y$.

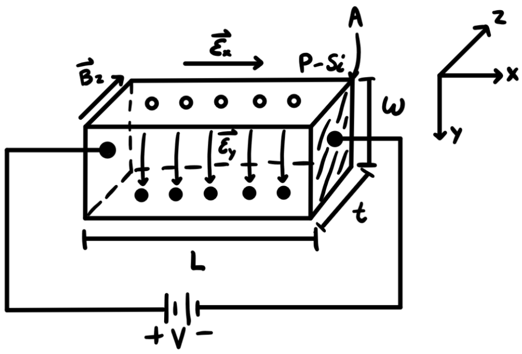

Figure 5: Perpendicular electric field formation.

The formation of the capacitor is not significant to us, but the incident voltage and electric field are what composes the Hall effect. Say we attached a sensitive voltmeter to the top and bottom faces of the semiconductor. I mention sensitivity because some Hall effect sensors measure on the order of several mV.

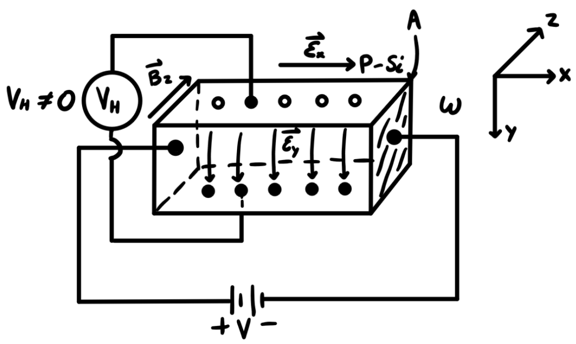

Figure 6: Hall effect measurement by Hall voltage.

With a sensitive voltmeter attached to the faces, it is possible to detect the strength and direction of a magnetic field. Imagine if B_z were immediately removed, $B_z = 0$. The $\hat{y}$ component of the Lorentz force would disappear, resulting in carrier motion only in the $\hat{x}$ direction. In short, $\vec{\mathscr{E}}_y = 0 \Rightarrow V_{hall} = 0$. Hence, no magnetic field produces no hall voltage. If the magnetic field were in another direction, only the $\hat{z}$ component would be detected.

Steady-State:

The description above qualitatively described the Hall effect, but there are many questions we need to ask ourselves. The first of which being, How strong can $\vec{\mathscr{E}_y}$ become? I call this point steady-state because $\vec{\mathscr{E}_y}$ must be bounded, the field cannot become infinitely large. The field is caused by the external magnetic field, hence the force due to $\mathscr{E}_y$ must be less than or equal to the magnetic force in the $\hat{y}$ direction; $F_y = F_B$.

$$\begin{aligned} F_y &= F_B \\ q\mathscr{E}_y &= qv_hB_z \\ \mathscr{E}_y &= v_hB_z \qquad (4) \\ v_h &= \frac{\mathscr{E}_y}{B_z} \qquad (5) \end{aligned}$$

The vector notation above was was dropped since both terms are in the $\hat{y}$ direction. Interestingly, the velocity of the carrier is dependent on the strength of the magnetic field. If we apply a known magnetic field strength and measure the Hall voltage we can determine the velocity of carriers in the material. At this point, it is difficult to see what other information we can extract. However, the last set of notes saw the derivation of expressions for drift current.

$$\begin{aligned} J_p &= qp\mu_p\mathscr{E} \\ &= qpv_h \\ \Rightarrow v_h &= \frac{J_p}{qp} = R_HJ_p \qquad (6) \end{aligned}$$

Where R_H is called the Hall coefficient; $R_H = \frac{1}{qp} = \frac{1}{qn}$ for p and n-type, respectively. Feel free to use the Hall coefficient notation, just know that it does not add much to the analysis. The terms qp and qn represent the total charge from doping and are typically constant. However, not every device utilizing the Hall effect has a constant concentration, so we prefer to keep the constants explicit. Substituting into Eq. 5, we have the expressions:

$$\mathscr{E}_y = \frac{I_pB_z}{qpA} p = \frac{I_pB_z}{qA\mathscr{E}_y} \qquad (7)$$

In the last paragraph I mentioned measuring the Hall voltage to determine $\mathscr{E}_y$, why? The top and bottom faces are separated by a distance, w. An electric field is the spatial derivative of potential, $\frac{d\phi}{dx} = -\mathscr{E}_y$. The electric field is the potential over some distance, implying the Hall voltage, $\mathscr{E}_y = \frac{V_h}{w}$. Therefore,

$$p = \frac{I_pB_zw}{qAV_H} \qquad (8)$$

Let us take a moment to examine what this equation is telling us. If there is a non-zero magnetic field nearby, a current induced by an external voltage source, a known or easily measured cross-sectional area, and a voltmeter to measure the Hall voltage, we can determine the carrier concentration of the material. This may not seem important at first, but a great deal of work has gone into developing techniques to measure the doping concentration in a semiconductor. Not every device is uniformly doped implying there will be small variations within the material that can cause unexpected behaviors. These techniques are beyond this course, but I mention this to emphasize that the Hall effect is not just a one-off topic in this course.

N-Type Derivation:

Each equation derived so far has been for p-type materials. The Hall effect will occur regardless if a material is an n or p-type. There are some tricks that make redoing the derivation worthwhile. To keep these notes from becoming a textbooks, I will include a list of differences between the derivations below. The voltage source, magnetic field, and polarity of the voltmeter are unchanged from above.

- The magnetic field will deflect electrons to the top face. The Lorentz force gains a negative from the negative charge of the electron.
- The removal of electrons from phosphorous dopant atoms result in positive ions on the bottom face.
- The resultant electric field is in the $-\hat{y}$ direction.
- Steady-state occurs when $\mathscr{E}_y = v_h B_z$
- The voltmeter measures a negative voltage.
- The current direction remains the same.

By going through the n and p-type derivation, we gained a deeper understanding that a standard Hall effect setup may determine doping type and doping concentration. If we know the concentration, we may rearrange for the current. This expression forms the basis of contactless ammeters where the magnetic field given off by a device under test is measured.

Conclusion

For a seemingly small concept, this file has become fairly lengthy. If it has not become apparent, a Hall effect derivation would make for an excellent exam question. Parameters such as the direction of the magnetic field, which set of faces the voltage source is attached to, whether the material is n or p-type will change the derivation. I cannot encourage highly enough to try out at least one other configuration on your own to see if you thoroughly understand the physics.

References

Collaborators, M.