note: the tex file is here: /users/dor/armn/jfl/1D_SIM_storage/1Dequations

1D momentum VP equation

We start from eq. 3 in [1]. The stresses in 2D are given by

$$\sigma_{ij} = 2\eta\dot{\epsilon}_{ij} + (\zeta - \eta)(\dot{\epsilon}_{11} + \dot{\epsilon}_{22})\delta_{ij} - \frac{p}{2}\delta_{ij} \tag{1}$$

where ζ and η are respectively the bulk viscosity and the shear viscosities and is a pressure-like term (which depends on the compressive P_p and tensile strength T_p). In 1D, v=0 and $\frac{\partial}{\partial y}$ such that

$$\sigma = \sigma_{11} = (\zeta + \eta)\frac{\partial u}{\partial x} - \frac{p}{2} \tag{2}$$

where $\frac{\partial u}{\partial x} = \dot{\epsilon}_{11}$. With the ellipse, $\eta = e^{-2}\zeta$. We can then write the stress as

$$\sigma = \alpha^2\zeta\frac{\partial u}{\partial x} - \frac{p}{2} \tag{3}$$

with $\alpha = \sqrt{1 + e^{-2}}$. The viscous coefficient is given by

$$\zeta = \frac{P_p + T_p}{2\Delta}, \tag{4}$$

where $\Delta = \alpha|\frac{\partial u}{\partial x}|$ [1]. P_p and T_p are respectively the compressive and tensile strengths. P_p follows the parameterization of [2] and is given by

$$P_p = P^* h \exp(-C(1 - A)). \tag{5}$$

The tensile is defined based on the compressive strength. It is a fraction of P_p such that

$$T_p = T_{frac}P_p. \tag{6}$$

In the code, ζ is usually regularized by a tanh [3] such that

$$\zeta = \frac{(P_p + T_p)}{2\Delta_{min}}\tanh\frac{\Delta_{min}}{\Delta}, \tag{7}$$

with $\Delta_{min} = 2{\times}10^{-9}s^{-1}$. For $\Delta \to 0$, $\zeta \to \frac{P_p+T_p}{2\Delta_{min}}$ while for $\Delta \gg 0$, $\zeta \to \frac{P_p+T_p}{2\Delta}$.

The pressure-like term in eq. 2 is just equal to $P_p - T_p$ unless the replacement closure is used. In this case,

$$p = (P_p - T_p)\frac{\Delta}{\Delta_{min}} \tanh\left(\frac{\Delta_{min}}{\Delta}\right), \tag{8}$$

which means that $p \to 0$ for $\Delta \to 0$ while $p \to P_p - T_p$ for $\Delta \gg 0$.

Note that in the code, $\frac{P_p}{2} = P_{p\_half}$, $\frac{T_p}{2} = T_{p\_half}$, $\frac{p}{2} = P_{\_half}$, $\Delta =$deno.

1. The EVP solver in 1D (with tensile strength)

Neglecting the advection of momentum and assuming an ocean at rest, the sea ice momentum equation in 1D is

$$\rho h \frac{\partial u}{\partial t} + C_w(u)u = \tau_a + \frac{\partial \sigma(u)}{\partial x} \tag{9}$$

where ρ is the ice density, h the sea ice thickness, u the velocity, $C_w(u)u$ is the water drag, τ_a is the wind stress and $\sigma(u)$ is the internal stress given by eq. 3. See [1] and [4] for details on 1D equations. Here is how u is calculated with the EVP solver [5].

We first write equation (3) as

$$\frac{\sigma}{\alpha^2\zeta} + \frac{p}{2\alpha^2\zeta} = \frac{\partial u}{\partial x} \tag{10}$$

Adding an elastic term [6] we get

$$\frac{1}{E}\frac{\partial \sigma}{\partial t} + \frac{\sigma}{\alpha^2\zeta} + \frac{p}{2\alpha^2\zeta} = \frac{\partial u}{\partial x} \tag{11}$$

where E is similar to Young's modulus

Introducing $T = \zeta/E$ [5], we can write equation (11) as

$$\frac{\partial \sigma}{\partial t} + \frac{\sigma}{\alpha^2 T} + \frac{p}{2\alpha^2 T} = \frac{\zeta}{T}\frac{\partial u}{\partial x} \tag{12}$$

The EVP time-stepping scheme is based on the following two equations (the stresses are time-stepped first)

$$\frac{(\sigma^s - \sigma^{s-1})}{\Delta t_e} + \frac{\sigma^s}{\alpha^2 T} + \frac{p^{s-1}}{2\alpha^2 T} = \frac{\zeta(u^{s-1})}{T}\frac{\partial u^{s-1}}{\partial x} \tag{13}$$

$$\rho h \frac{(u^s - u^{s-1})}{\Delta t_e} + C_w(u^{s-1})u^s = \tau_a + \frac{\partial \sigma^s}{\partial x} \tag{14}$$

where Δt_e is the EVP time step and $N_{sub}\Delta t_e = \Delta t$ (N_{sub} is the number of subcycles). Δt is the same as the implicit time step and corresponds to the time step at which the forcing and the thickness are updated. Δt is typically on the order of 30 minutes and Δt_e on the order of 10 s. $u^s \approx u^n$ after N subcycles. p varies during the subcycling if the replacement closure is used.

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