Digital systems and basics of electronics

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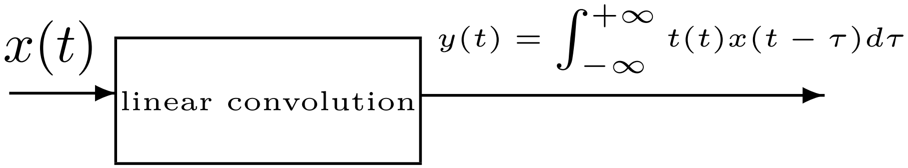
Passive Signal Processing - lecture 3

Repetition

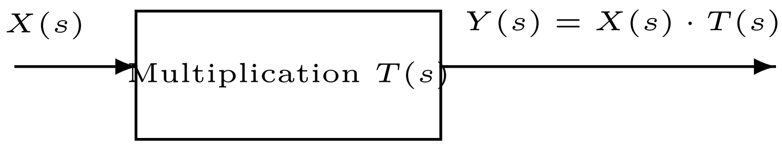
- Laplace Transform is a useful analytical tool for converting time-domain *(t)* signal descriptions into unctions of a complex variable *(s)*. This complex domain description of a signal provides new insight into the analysis of signals and systems.
- The Laplace transform often simplifies the calculations of differential and integral equation (integral "becomes"multiplying by $\frac{1}{s}$ and differential by s
- In addition, the Laplace transform method often simplifies the calculations involved in obtaining system response signals.

Transfer function, transmittance

Time domain



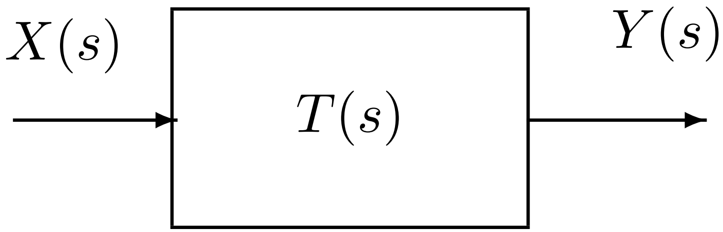
Laplace domain



Function $T(s)$ is also called **transfer function** or *transmittance* and can be expressed as

$$T(s) = \frac{Y(s)}{X(s)} \tag{1}$$

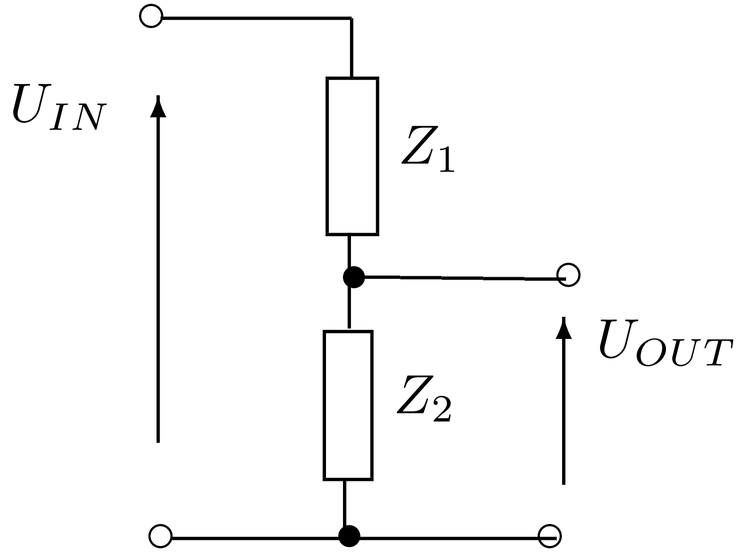
Transfer functions (*transmittance*)



$$T(s) = \frac{Y(s)}{X(s)}$$

A transfer function is a mathematical representation (in terms of frequency), of the relation between the input and output of a (linear time-invariant) system

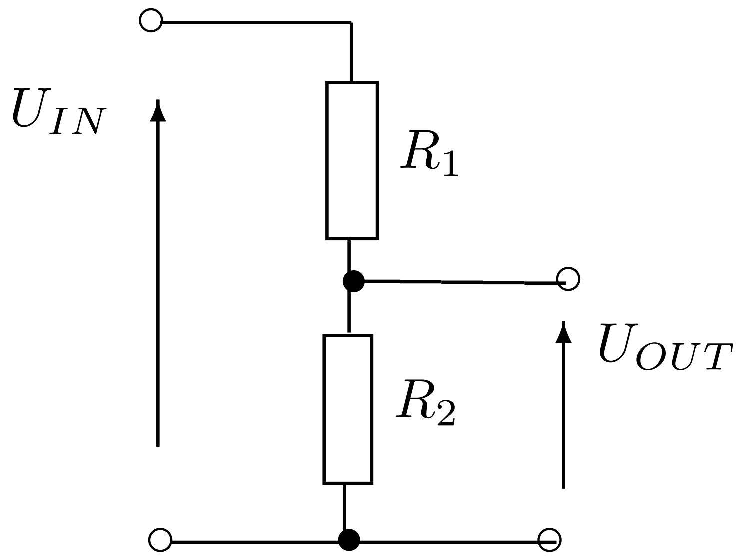
Impedance divider



$$U_{OUT} = \frac{Z_2}{Z_1+Z_2} \cdot U_{IN}$$

The ratio contains an imaginary number, and actually contains both the amplitude and phase shift information of the filter. To extract just the amplitude ratio, calculate the magnitude of the ratio.

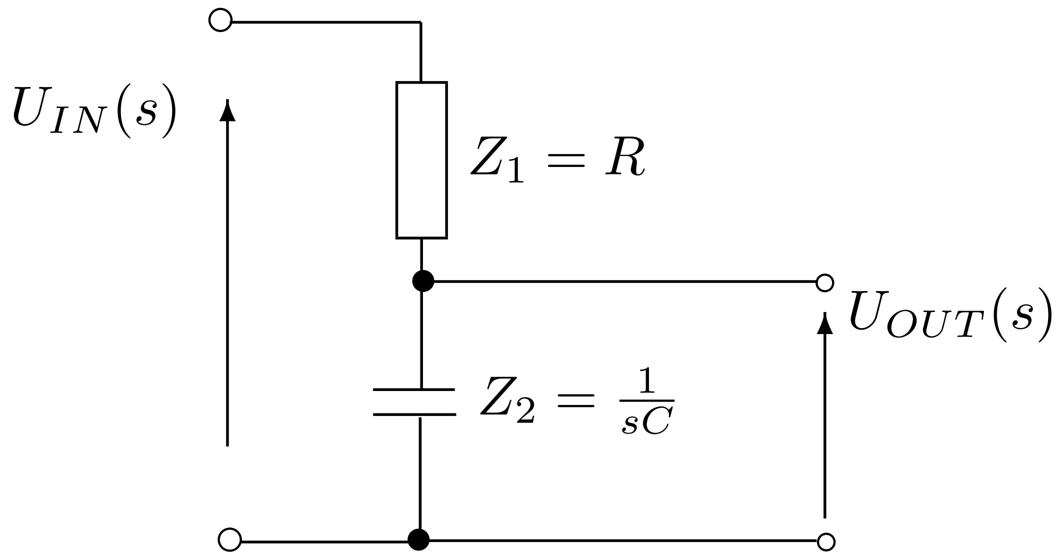
Voltage divider



$$U_{OUT} = \frac{R_2}{R_1+R_2} \cdot U_{IN}$$

Voltage divider is a simple device designed to create a voltage (V_{out}) which is proportional to another voltage (V_{in}). It is commonly used to create a reference voltage, and may also be used as a signal attenuator at low frequencies. Voltage dividers are also known by the terms *resistor divider*.

RC impedance divider - example



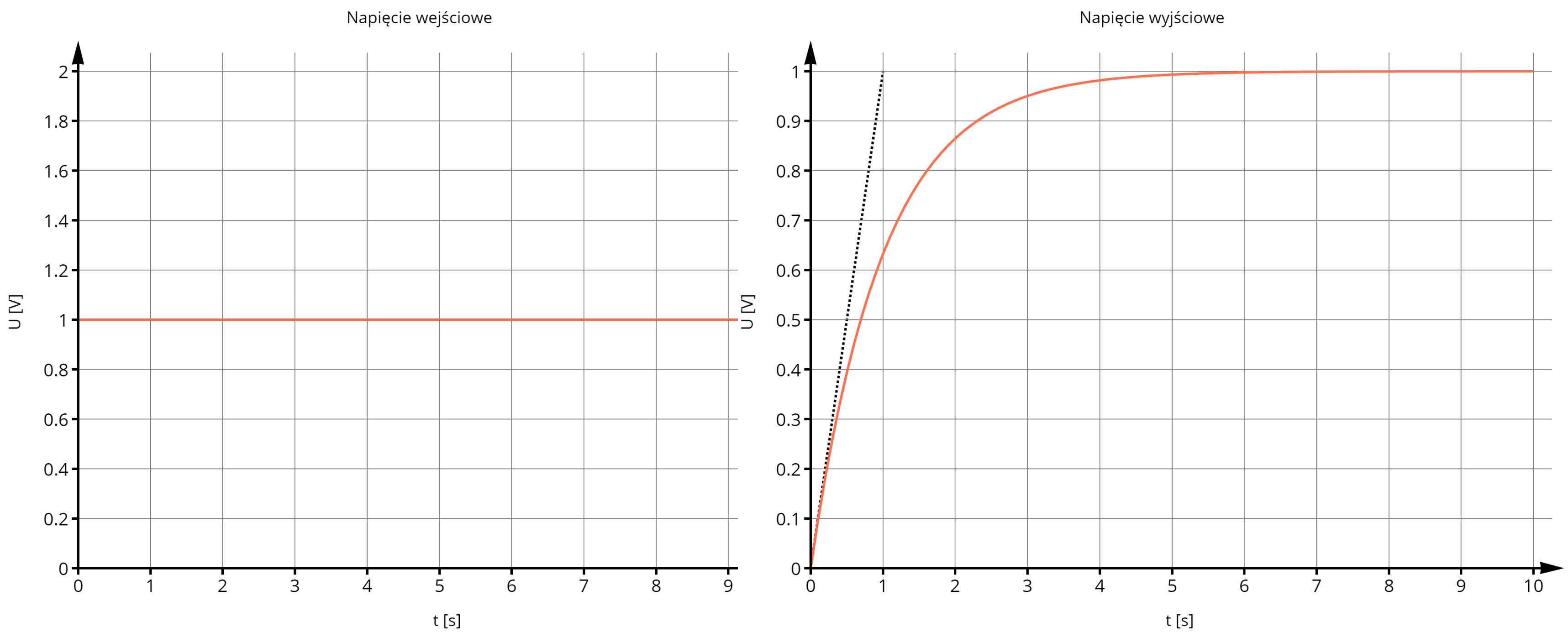
Transfer function can be calculated as follow:

$$T(s) = \frac{U_{out}(s)}{U_{in}(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + s \cdot RC} = \frac{1}{1 + s \cdot T} \tag{2}$$

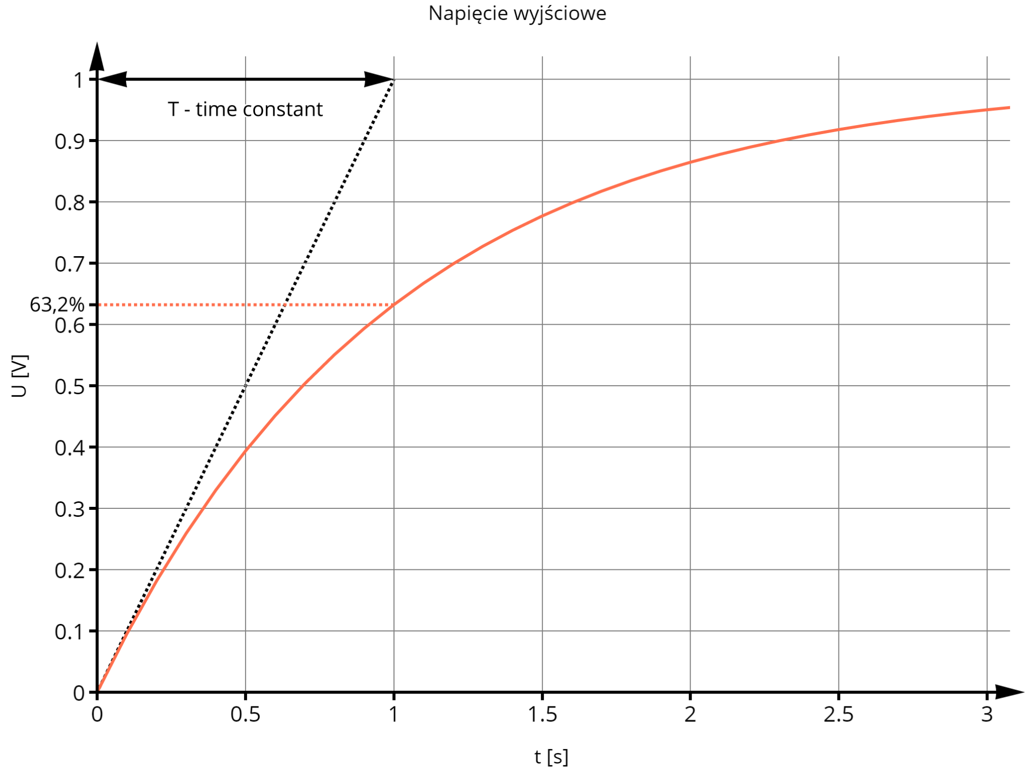
where $T = R \cdot C$ is called **time constant**

Step response of RC divider

Time constant is equal: $(T = RC = 1)$



Time constant



In a capacitor-resistor circuit, the number of seconds required for the capacitor to reach 63.2% of its full charge after a voltage is applied. The time constant of a capacitor with a capacitance (C) in farads in series with a resistance (R) in ohms is equal to $R \cdot C$ seconds.

How *transfer function* $T(j\omega)$ **depends on frequency** f **?**

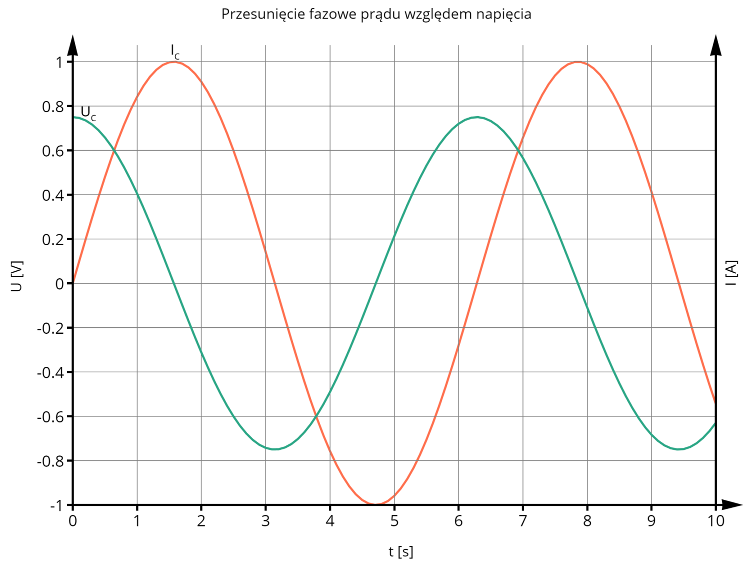
For any frequency *transfer function* is an complex number, and can be describe as

$$T(j\omega) = |Z|e^{j\cdot\varphi} \tag{3}$$

where $|Z|$ is an *magnitude* and φ is and angle (*phase* - in electronic terminology)

Time domain interpretation of *magnitude* and *phase* of transmittance $T(j\omega)$

Time constant $T = RC = 1$, frequency $f = 1\frac{rad}{s}$



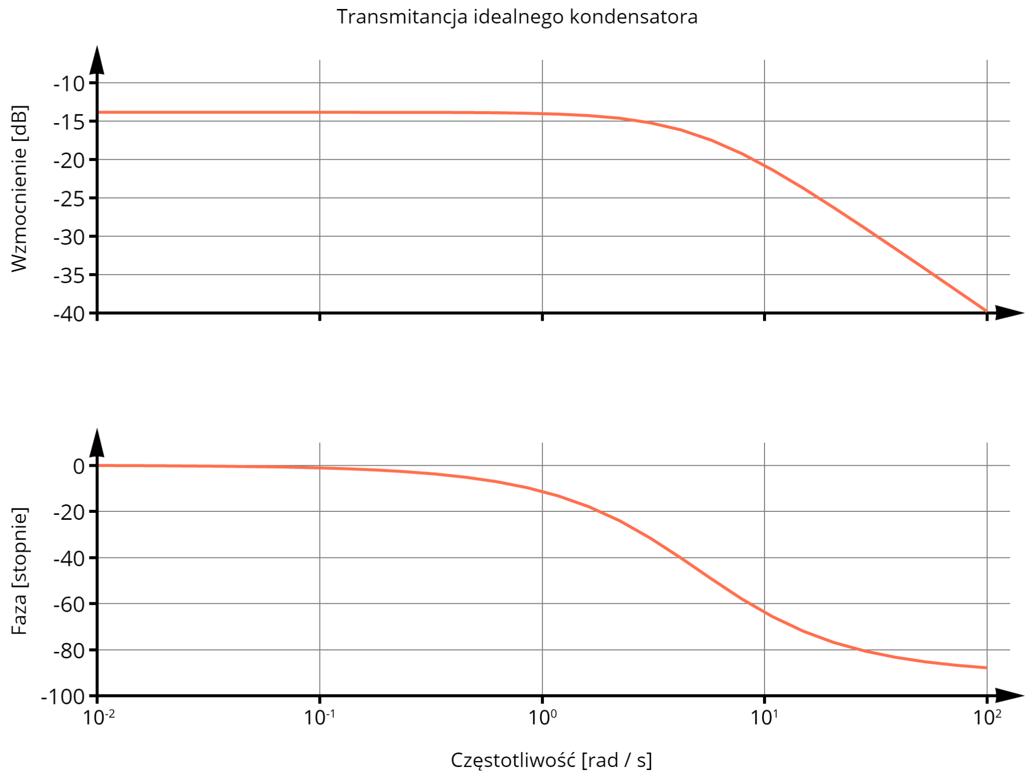
- transmittance amplitude: $|Z| = \frac{AMPLITUDE_{OUT}}{AMPPLITUDE_{IN}}$
- transmittance phase: time shift of output signal expressed in radians (or degree)

Important remarks on linear systems

- Transmittance describes linear object (or linear approximation of object)
- INPUT Sinusoidal signal with frequency f ”generates“ OUTPUT sinusoidal signal with the same frequency f.
- Superposition: If input signal is combination of frequency f_1 and f_2 output signal of *linear system* is superposition (sum) of response for sinusoidal f_1 and f_2.

Magnitude and phase diagrams

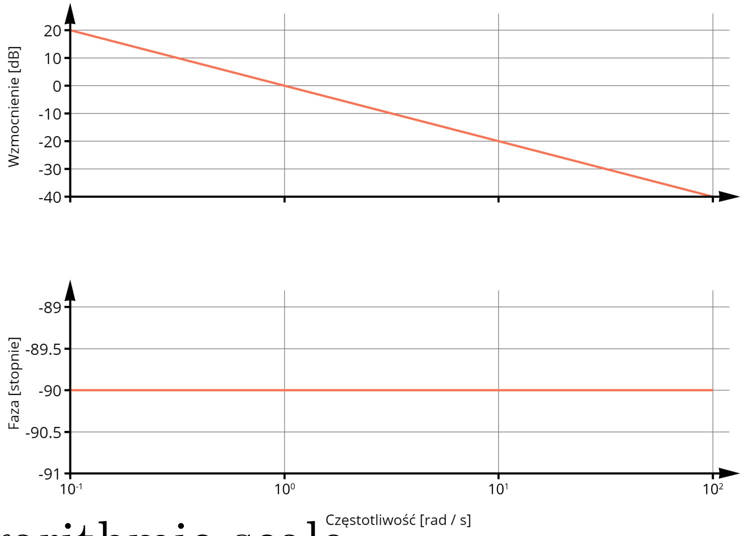
Time constant: $T = RC = 1$



Logarithmic scale of Magnitude

Transmittance of capacitor

$T(S) = \frac{U(S)}{I(s)}$



- Module $|Z|$ can be expressed in logarithmic scale

$$|Z| = 20 \cdot \log_{10}(\frac{AMPLITUDE_{OUT}}{AMPPLITUDE_{IN}}) \tag{4}$$

unit decybele [dB]

- slope of amplitude curve $-20\frac{dB}{dec}$

Cut-off frequency

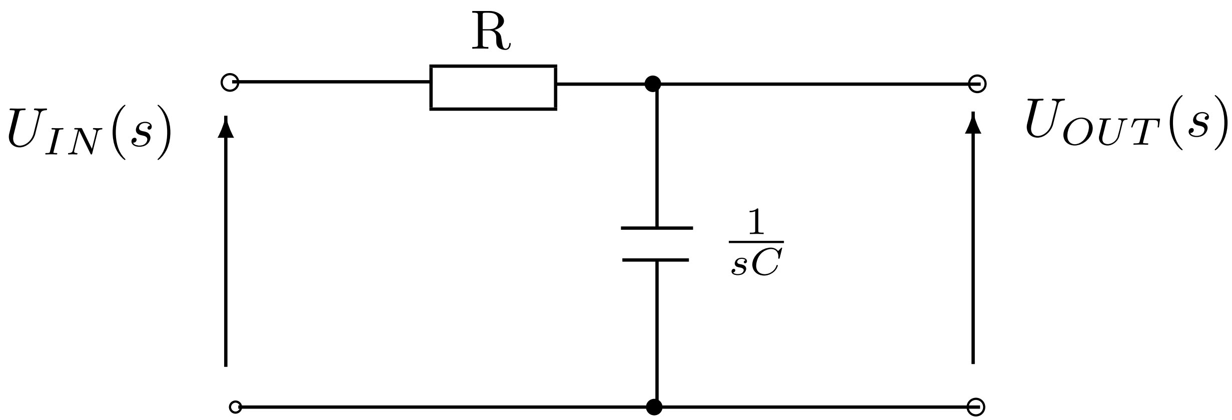
The cut-off frequency is the point at which the magnitude is $\frac{1}{\sqrt{2}} \approx 0.707$. (–3 dB in decybel scale)

$$20 \cdot \log_{10}(\frac{1}{\sqrt{2}}) \approx -3dB \tag{5}$$

when the amplitude decrees $\frac{1}{\sqrt{2}}$ times power decrees $\frac{1}{2}$ times $(P = \frac{U^2}{R})$

$$10 \cdot \log_{10}(\frac{1}{2}) \approx -3dB \tag{6}$$

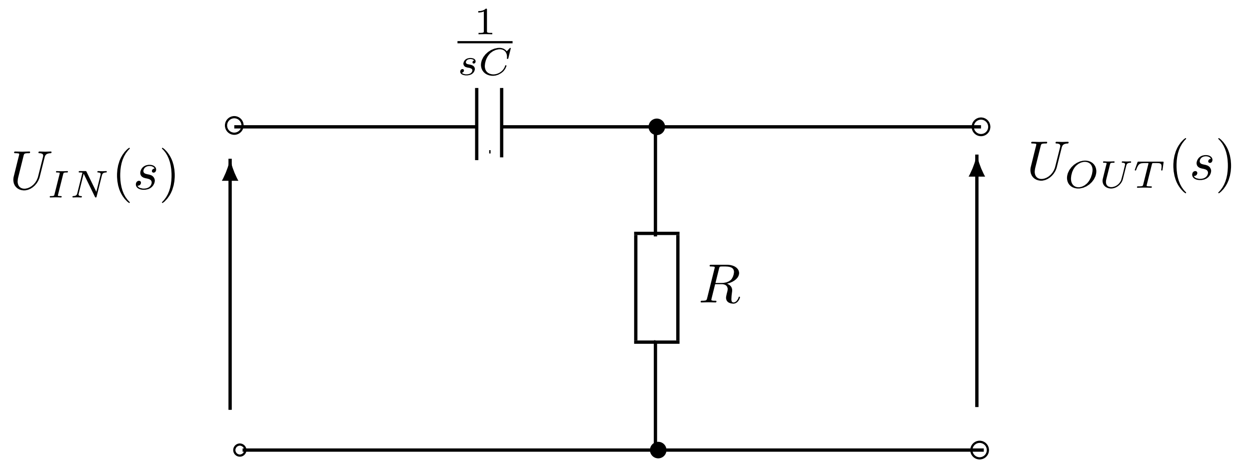
Low-Pass Filters



- *Low-pass filter* is a filter whose passband extends from dc to some finite cut-off frequency.
- RC - divider is low-pass filter
- Cut-off frequency is equal

$$f_{cut} = \frac{1}{2\pi RC} \tag{7}$$

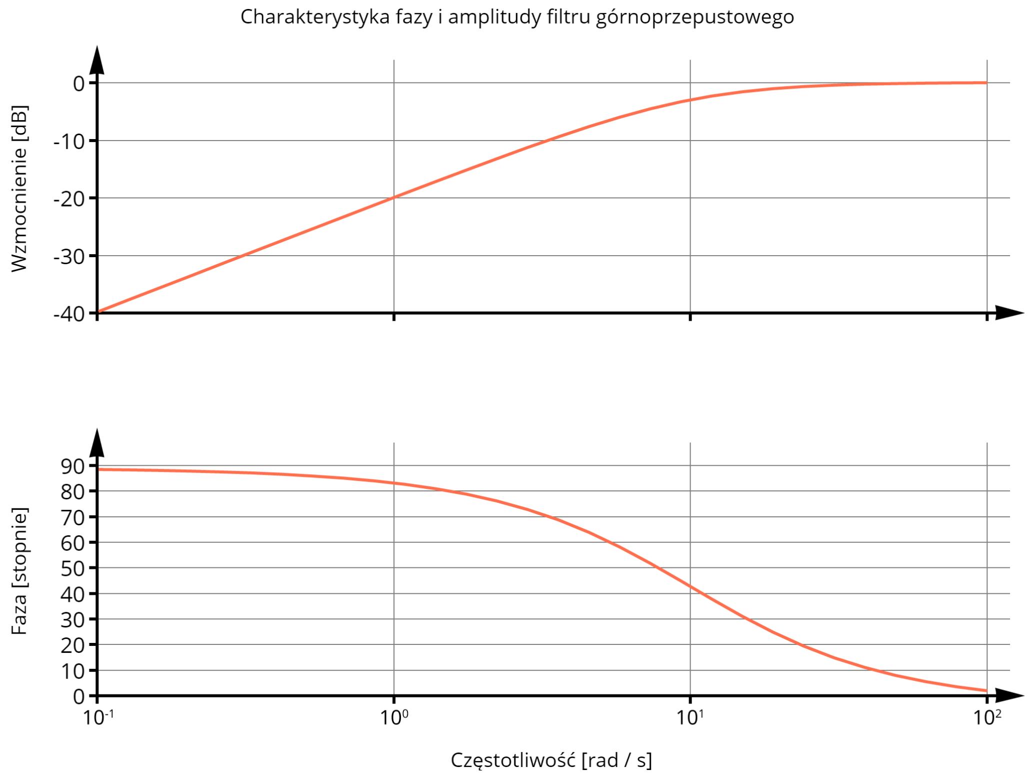
High-Pass Filters



- *High-pass filter* is a filter whose band extends from some finite cut-off frequency to infinity.
- Transfer function of high-pass filter is equal $T(s) = \frac{R}{R+\frac{1}{sC}} = \frac{sRC}{1+sRC}$
- CR - divider is low-pass filter
- Cut-off frequency is equal $f_{cut} = \frac{1}{2\pi RC}$

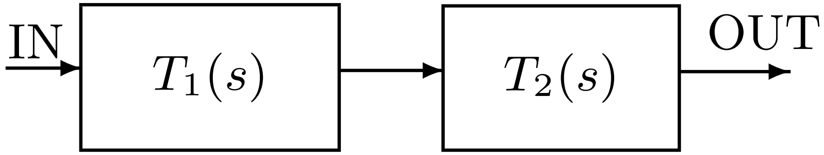
High-pass filter magnitude and phase characteristic

Time constant: $T = RC = 1$



Bandpass filter

- *Bandpass filter* is a filter whose passband extends from a finite lower cut-off frequency to a finite upper cutoff frequency.
- Bandpass filter is an combination of low-pass and high-pass filters

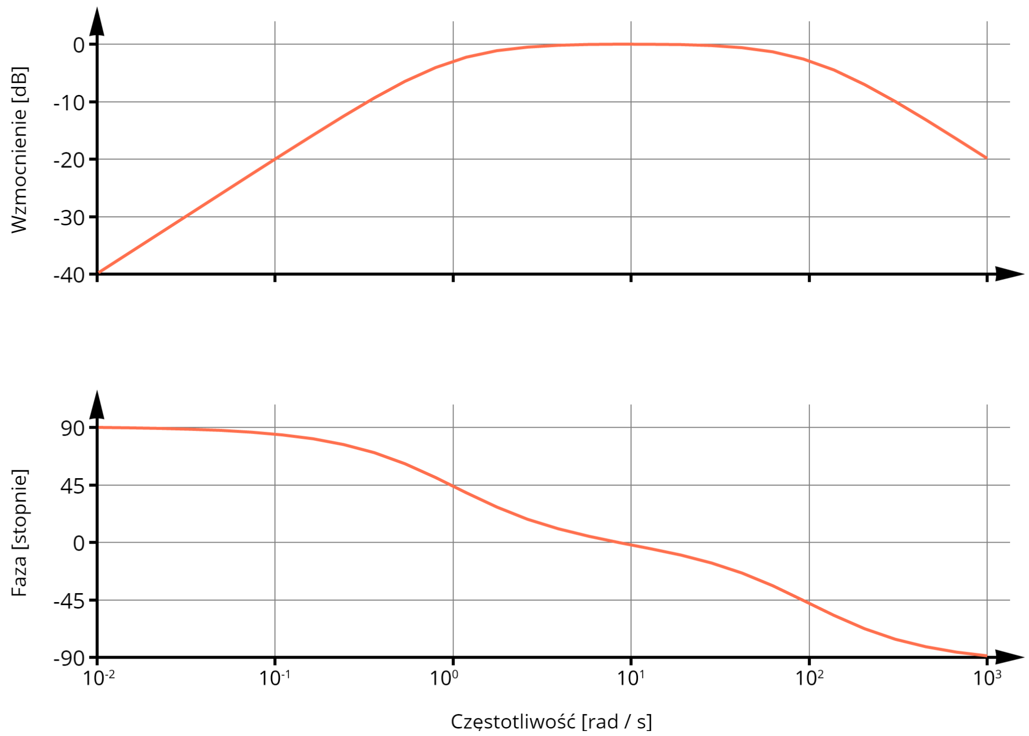


- transfer function

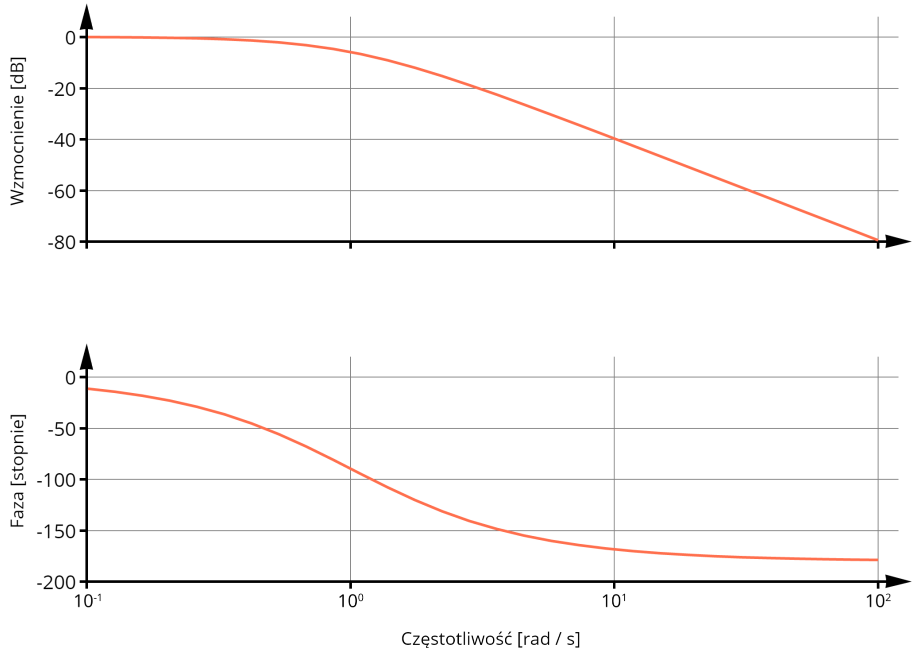
$$T(s) = \frac{sR_1C_1}{1 + R_1C_1} \cdot \frac{1}{1 + R_2C_2}$$

Bandpass filter magnitude and phase characteristic

Time constants: $T_1 = R_1C_1 = 1$, $T_2 = R_2C_2 = 0.01$

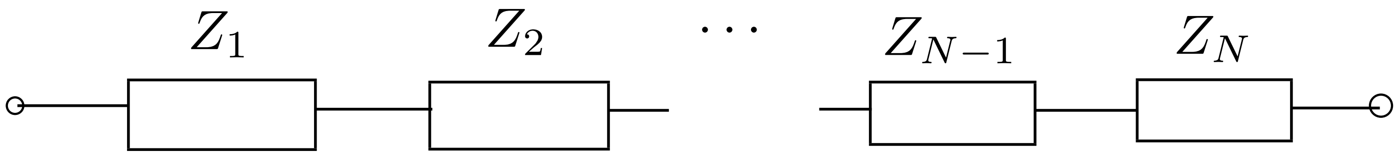


Higher-order filters



- Slope of amplitude characteristic can be change by connection in serial two or three filters. In that case $-40\frac{dB}{dec}$
- Number of filters is limited to 3 (stability problem - will be discuss later.)

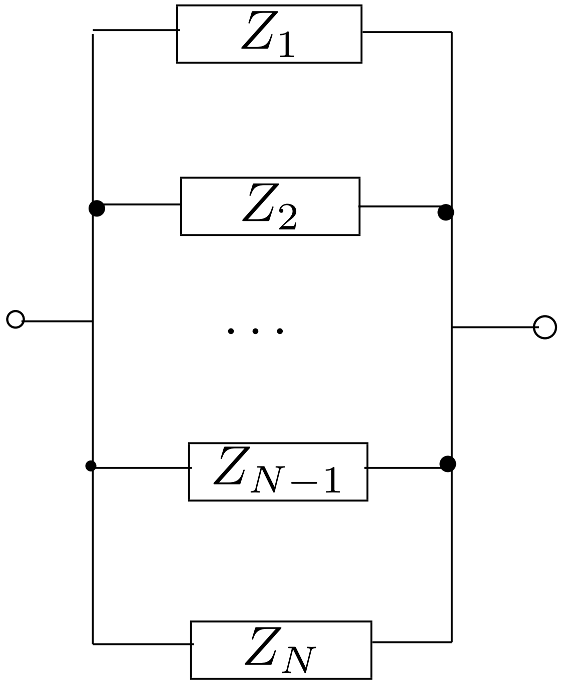
Impedance Networks - impedances connected in series



$$Z = Z_1 + Z_2 + \ldots Z_{n-1} + Z_n = \sum_{i=1}^{n} Z_i \tag{8}$$

- resistance: $R = R_1 + R_2 + \ldots R_{n-1} + R_n$
- capacity: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_{n-1}} + \frac{1}{Cn}$
- inductance: $L = L_1 + L_2 + \ldots L_{n-1} + L_n$

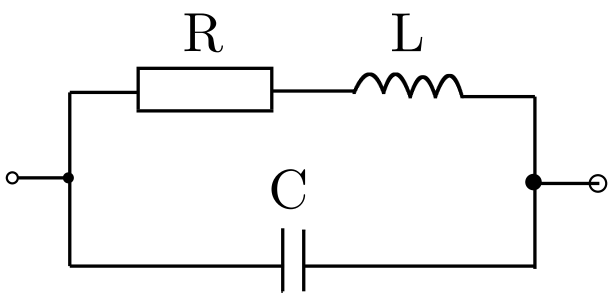
Impedances connected in parallel.



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n} = \sum_{i=1}^{n} \frac{1}{Z_i}$$

- resistance: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{Rn}$
- capacity: $C = C_1 + C_2 + \ldots C_{n-1} + C_n$
- inductance: $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{Ln}$

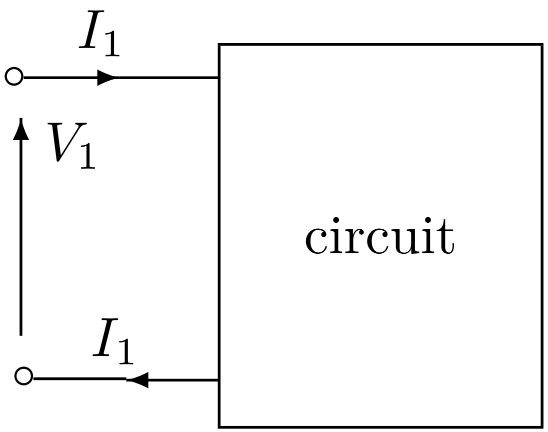
Impedance Networks - other example



$$\frac{1}{Z} = \frac{1}{sC} + \frac{1}{R + sL} \tag{9}$$

$$Z = \frac{\frac{1}{sC} \cdot (R + sL)}{\frac{1}{sC} + R + sL} = \frac{R + sL}{1 + sRC + s^2LC} \tag{10}$$

One-Port

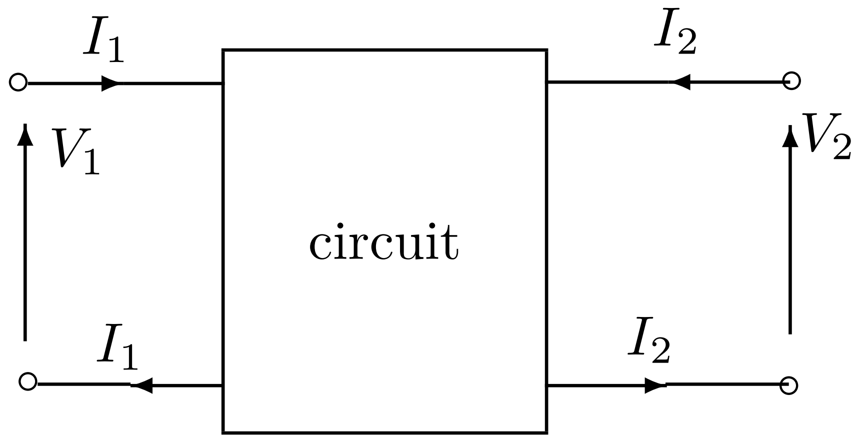


Many times we want to model the behavior of an electric network at only two terminals as shown in Figure. Here, only V_1 and I_1, not voltages and currents internal to the circuit, need to be described. We define the pair of terminals shown as a port, where the current, I_1, entering one terminal equals the current leaving the other terminal.

We can mathematically model the network at the port as

$$V_1 = Z \cdot I_1$$

Two-Port Networks



We can model such circuits as two-port networks as shown in Figure. Here we see the input port, represented by V_1 and I_1, and the output port, represented by V_2 and I_2. Currents are assumed positive if they flow as shown in Figure.