Digital systems and basics of electronics

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Passive elements - lecture 2

Powers of 10 and prefixes

Quantities greater than 100 or less than 0.01 are usually expressed in the standard form of $A \times 10^n$, where A is a number, called the *mantissa* and n is a whole number called the *exponent*.

Prefix	Abbreviation	Power of ten	Multiplier
Giga	G	9	1 000 000 000
Mega	M	6	1 000 000
kilo	k	3	1000
milli	m	-3	$\frac{1}{1.000}$
micro	μ	-6	$\frac{1}{1.000.000}$
nano	n	-9	$\frac{1}{1.000.000.000}$
pico	p	-12	$\frac{1}{1.000.000.000.000}$

Conductors and Insulators

- A voltage applied between two points on a length of a metallic conductor produces the flow of an electric current, and an electric field is established around the *conductor*,
- In some nonmetallic materials, called *insulator*, the free electrons are so tightly bound by forces in the atom that, upon the application of an external voltage, they will not separate from their atom,
- *Semiconductors* are electronic conducting materials wherein the conductivity is dependent primarily upon impurities in the material.

DC and AC current

- Direct current **DC** is defined as a unidirectional current in which there are no significant changes in the current flow,
- Alternating current **AC** is defined as a current that reverses direction at a periodic rate. The average value of alternating current over a period of one cycle is equal to zero.

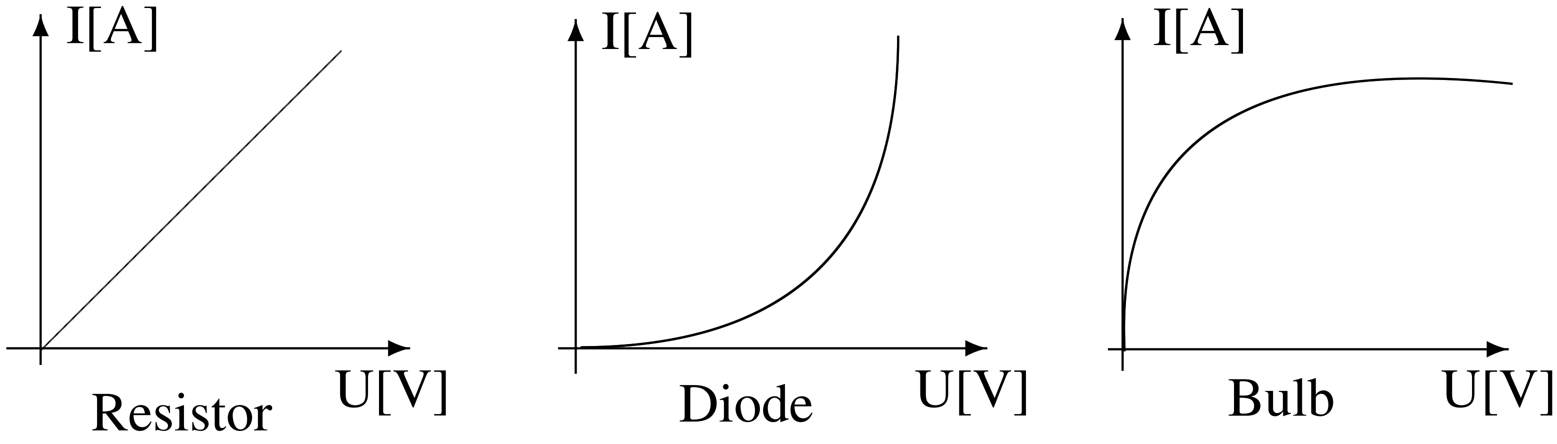
Passive and active components

- *Passive components* are those that need no power supply for their operation
- No purely passive component can have an output that supplies more power than is available at the input.
- *Active components* make use of a power supply, usually DC, so that the signal power output of an active component can be higher than the signal power at the input.

Resistors

- The resistance of a sample of material, measured in units of ohms (Ω), is defined as the ratio of voltage (in units of volts) across the sample of material to the current (in units of amperes) through the material.

$$R = \frac{U}{I}$$



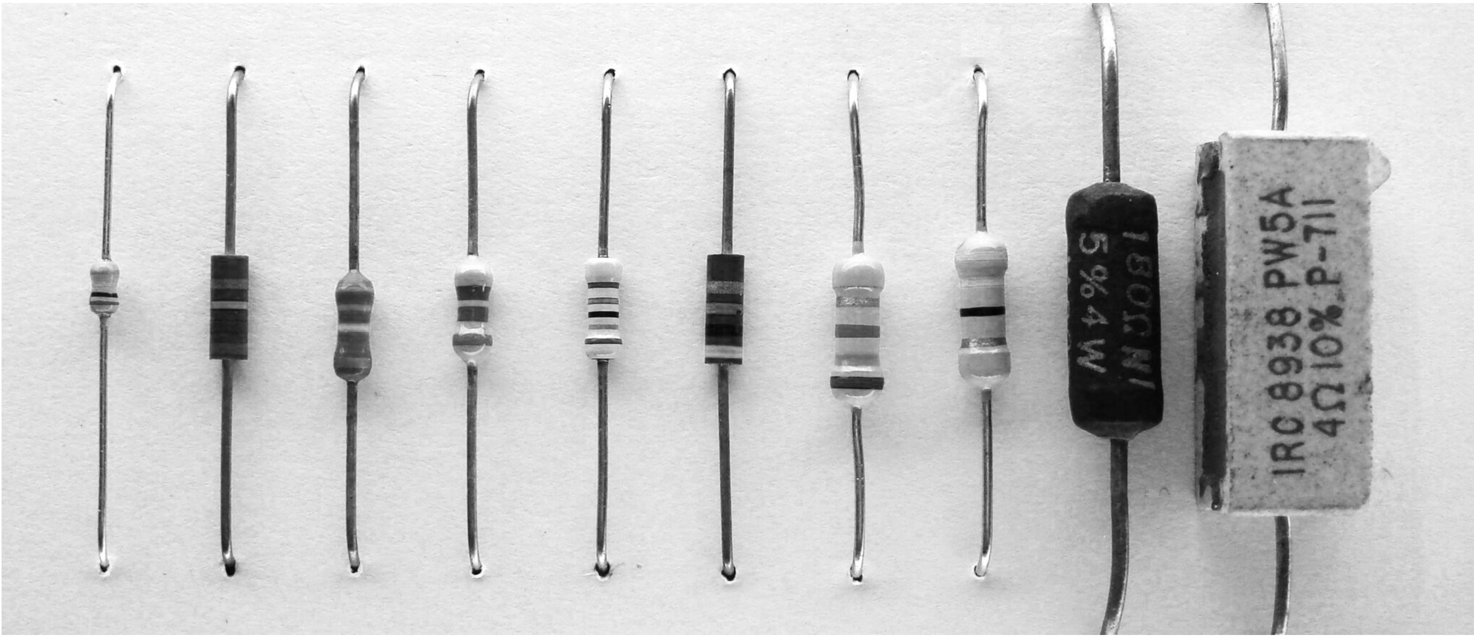
Resistivity and resistance

- The resistance of any sample of a material is determined by its dimensions and by the value of resistivity of the material,
- *Resistance* R can be calculated as follow:

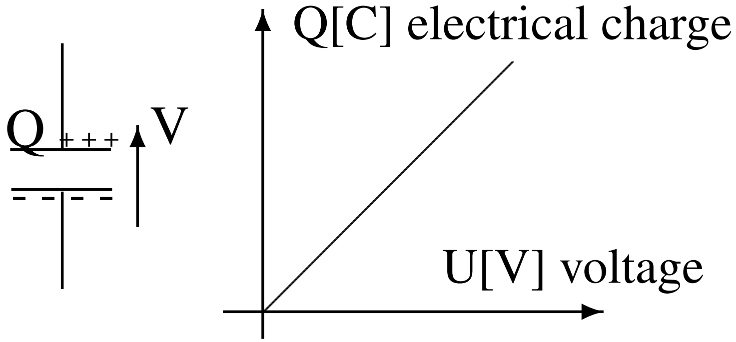
$$R = \frac{\rho L}{A}$$

where ρ is *resistivity*, A area of cross-section and L length.

Resistors overview



Capacitors



- Two conductors that are not connected and are separated by an insulator constitute a *capacitor*,
- When electrical charge Q (measured in units of coulombs) has been transferred, the voltage across the plates equals the voltage V across the voltage source *capacity* C (measured in units of farads) is equal

$$C = \frac{Q}{V}$$

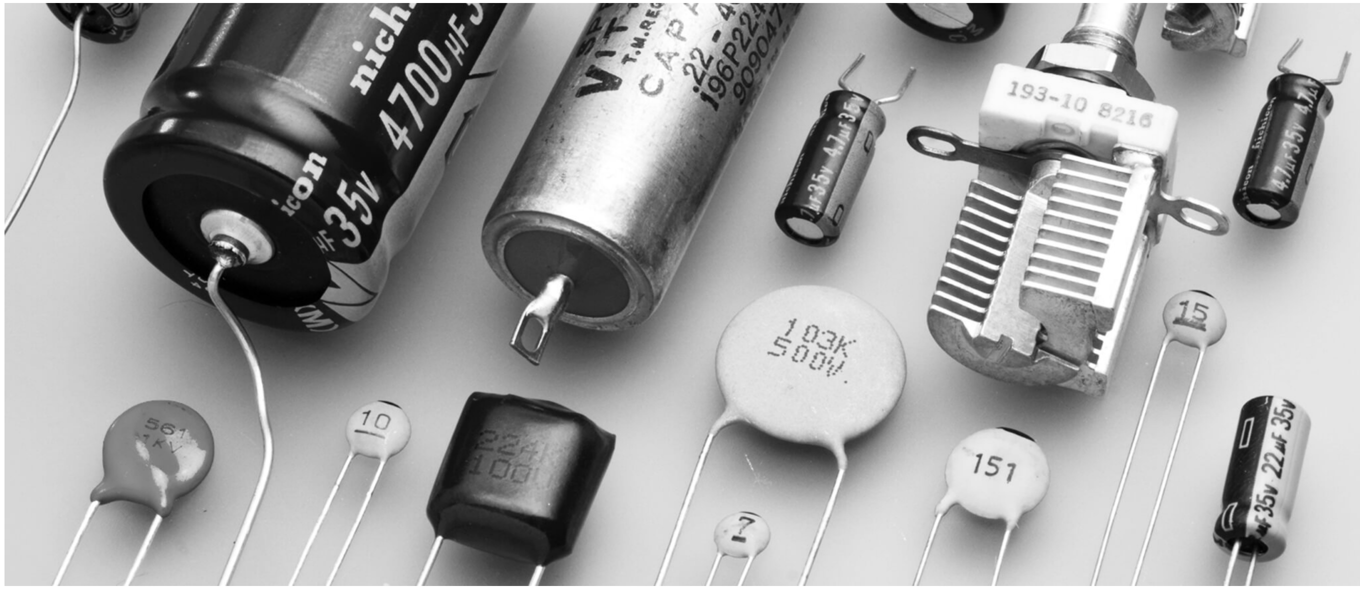
Capacitor and capacity

- The parallel-plate *capacitor* is the simplest (theoretical) arrangement and its capacity value C is, for ideal conditions, easy to calculate. For a pair of parallel plates of equal area A, separation d, the *capacity* is given by

$$C = \frac{\xi_r \xi_0 A}{d}$$

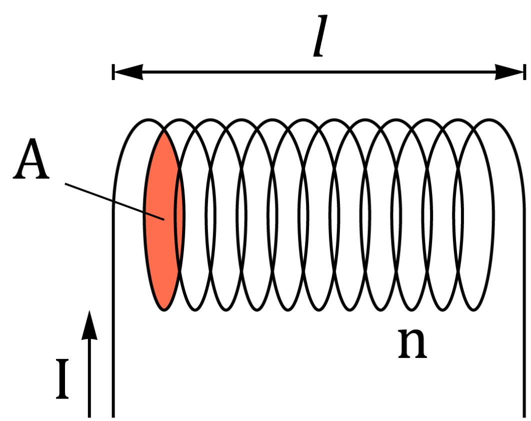
where the quantity ξ_0 is a universal constant called the *permittivity of free space*, and it has the fixed value of 8.84×10^{-12} farads per metre.

Capacitors overview



Capacitor types: ceramic, polyester stacked film, electrolytic, tantalum.
electrolytic.

Inductor



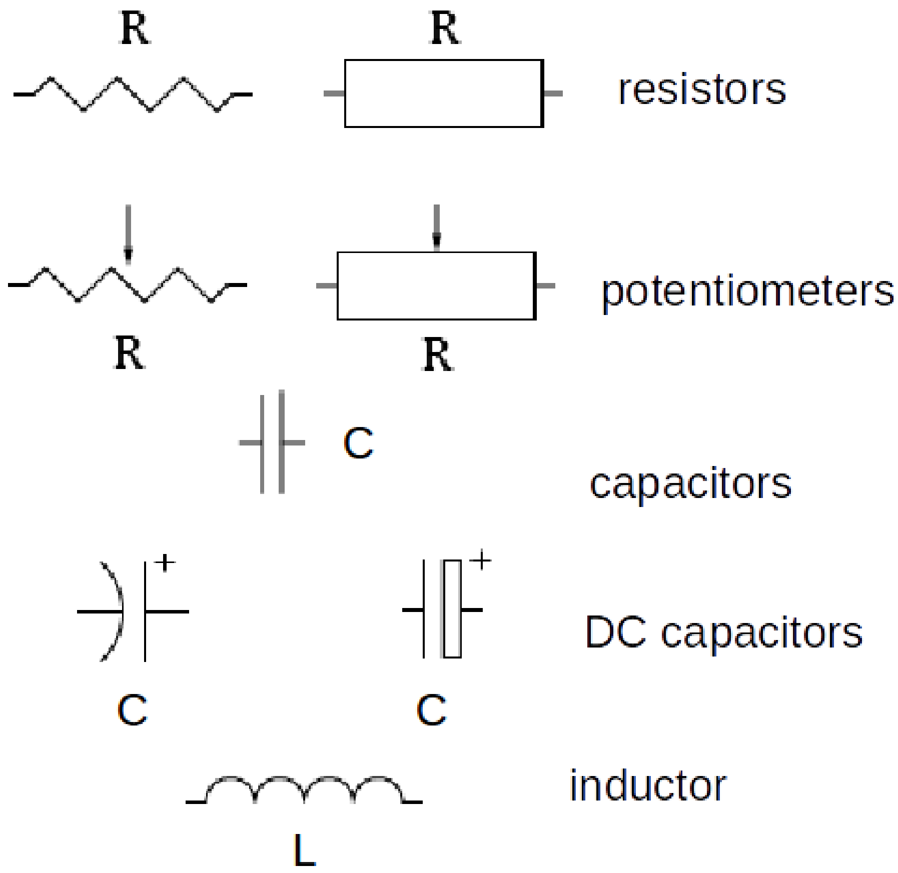
- An *Inductor* is a component whose action depends on the magnetic field that exists around any conductor when a current flows through that conductor.
- *Inductance* is proportional to the square of turns n^2, area of cross-section A, *magnetic permeability* μ and reverse proportional to the length of coil L:

$$L = \frac{\mu \cdot n^2 \cdot A}{L}$$

- Voltage E is induced between the ends of the conductor by magnetic field (or magnetic flux Φ) changes. This voltage is termed *electromotive force*

$$E = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}.$$

Electrical symbols of resistor, capacitor and inductor



The Laplace transform

- is a useful analytical tool for converting time-domain *(t)* signal descriptions into functions of a complex variable *(s)*. This complex domain description of a signal provides new insight into the analysis of signals and systems.
- In addition, the Laplace transform method often simplifies the calculations involved in obtaining system response signals.

Definition

The Laplace transform of the continuous-time signal it x (t) is

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} d(t) \tag{1}$$

The variable *s* that appears in this integrand exponential is generally complex valued and is therefore often expressed in terms of its rectangular coordinates

$$s = \sigma + j\omega, \tag{2}$$

where $\sigma = Re(s)$ and $\omega = Im(s)$ are referred to as the real and imaginary components of s, respectively.

The signal $x(t)$ and its associated Laplace transform $X(s)$ are said to form a Laplace *transform pair*. This reflects a form of equivalency between the two apparently different entities $x(t)$ and $X(s)$. We may symbolize this interrelationship in the following suggestive manner:

$$X(s) = \pounds[x(t)] \tag{3}$$

where the operator notation $\pounds$ means to multiply the signal $x(t)$ being operated upon by the complex exponential e^{-st} and then to integrate that product over the time interval $(-\infty, +\infty)$.

Examples of Laplace'a transform

dziedzina czasu	dziedzina Laplace'a
$f(t)$	$\mathcal{L}[f(t)] = F(s)$
1	$\frac{1}{s}$
$e^{at}f(t)$	$F(s-a)$
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$
$\delta(t)$	1
$\delta(t-t_0)$	e^{-st_0}
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$f'(t)$	$sF(s) - f(0)$
$f^n(t)$	$s^n F(s) - s^{(n-1)} f(0) - \cdots - f^{(n-1)}(0)$
$t^n \; (n = 0, 1, 2, \ldots)$	$\frac{n!}{s^{n+1}}$

$\sin kt$	$\dfrac{k}{s^2 + k^2}$
$\cos kt$	$\dfrac{s}{s^2 + k^2}$
e^{at}	$\dfrac{1}{s - a}$
$f(t)$	$\mathcal{L}[f(t)] = F(s)$
$\dfrac{ae^{at} - be^{bt}}{a - b}$	$\dfrac{s}{(s - a)(s - b)}$
te^{at}	$\dfrac{1}{(s - a)^2}$
$t^n e^{at}$	$\dfrac{n!}{(s - a)^{n+1}}$

Properties of Laplace Transform

- Let us obtain the Laplace transform of a signal, $x(t)$, that is composed of a linear combination of two other signals,

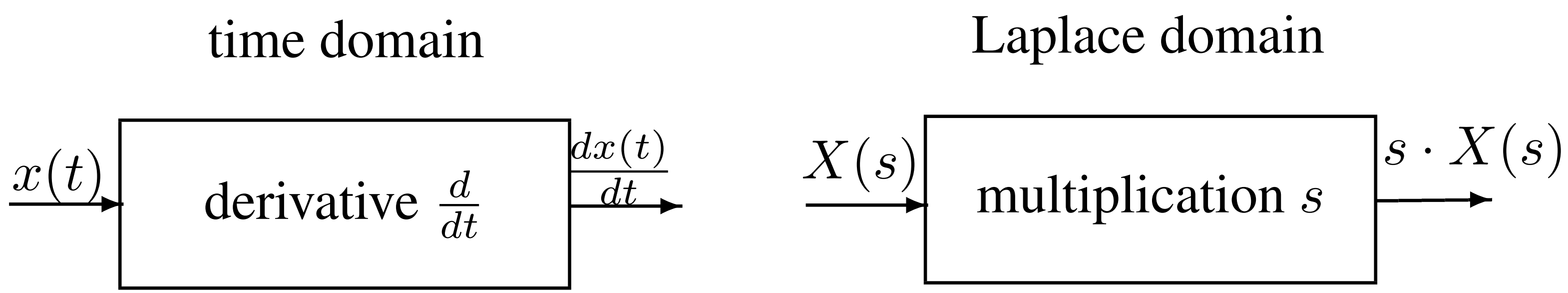
$$x(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t) \tag{4}$$

where α_1 and a_2 are constants.

- The <u>**linearity property**</u> indicates that:

$$\pounds[\alpha_1 x_1(t)+\alpha_2 x_2(t)] = \alpha_1 \pounds[x_1(t)]+\alpha_2 \pounds[x_2(t)] = \alpha_1 X_1(s)+\alpha_2 X_2(s) \tag{5}$$

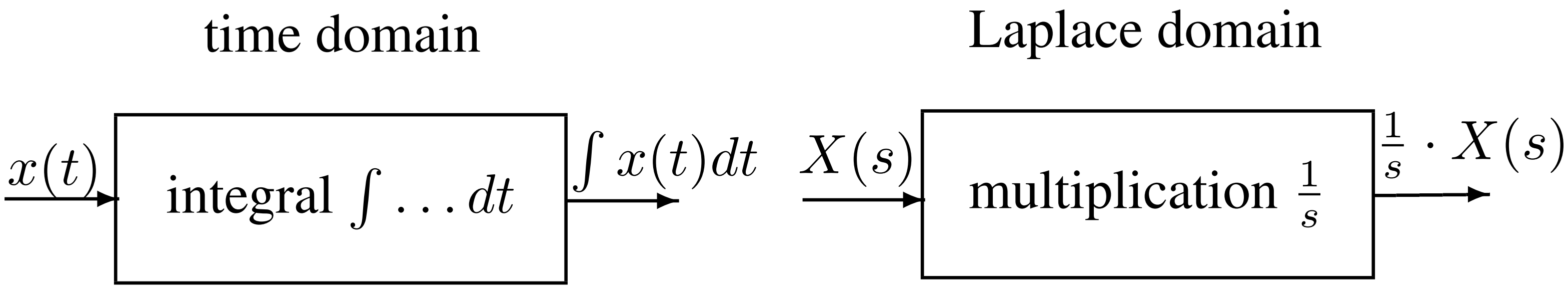
Time-Domain Differentiation



The operation of time-domain differentiation has then been found to correspond to a multiplication by s in the Laplace variable s domain. The Laplace transform of differentiated signal $\frac{dx(t)}{dt}$ is

$$\pounds[\frac{dx(t)}{dt}] = s \cdot X(s) \tag{6}$$

Time-Domain Integration



The operation of time-domain Integration has then been found to correspond to a multiplication by $\frac{1}{s}$ in the Laplace variable s domain. The Laplace transform of integration signal $\int x(t)dt$ is

$$\pounds[\int x(t)dt] = \frac{1}{s} \cdot X(s) \tag{7}$$

Inverse Laplace Transform

Given a transform function $X(s)$ and its region of convergence, the procedure for finding the signal $x(t)$ that generated that transform is called finding the inverse Laplace transform and is symbolically denoted as

$$x(t) = \pounds^{-1}[X(s)] \tag{8}$$

The signal $x(t)$ can be recovered by means of the relationship

$$x(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} X(s) \cdot e^{st} ds \tag{9}$$

Fourier transform - Frequency-Domain

- Laplace transform - s-domain

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} d(t) \tag{10}$$

- Fourier transform - Frequency-Domain (by replacing s with $j\omega$)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} d(t) \tag{11}$$

where $w = 2\pi f$, f - frequency, $j = \sqrt{-1}$ - imaginary unit.

Inductor Transformation

For an inductance L, the voltage is

$$v_L(t) = L \cdot \frac{di_L(t)}{dt} \tag{12}$$

Transforming, we have

$$V_L(s) = L \cdot s \cdot I_L(s) \tag{13}$$

Ratio $\frac{V(s)}{I(s)}$ is an **impedance**. For inductor impedance is equal $X_l(s) = s \cdot L$
(In frequency domain $X_l(j\omega) = j\omega \cdot L$)

Capacitor Transformation

In the case of a capacitance C we have

$$v_C(t) = \frac{1}{C}\int i_C(t)dt \tag{14}$$

which transforms to

$$V_C(s) = \frac{1}{C}\cdot\frac{1}{s}I_C(s) \tag{15}$$

For capacitor impedance is equal $X_c(s) = \frac{1}{s\cdot C}$ (In frequency domain $X_c(j\omega) = \frac{1}{j\omega\cdot C}$)

Impedance = Resistance + j*Reactance

- For alternating signals electrical resistance can be describe by complex number and denoted as impedance Z

$$Z = R + j \cdot X \tag{16}$$

where real part of Z is an resistance R and imaginary part of Z $j \cdot X$ is called reactance. Reactance represents żesistance of real capacitorór żesistance of real inductorąnd depends of frequency.

- Impedance can be expressed in the notion of Ohm's law :

$$Z = \frac{U}{I} \tag{17}$$

Ideal Capacitor Reactance

In the case of a capacitance C we have $v_C(t) = \frac{1}{C}\int i_C(t)dt$ which transforms to

$$V_C(s) = \frac{1}{C} \cdot \frac{1}{s} I_C(s) \tag{18}$$

For capacitor impedance (for ideal capacitor reactance=impedance) is equal

$$X_c(s) = \frac{V_C(s)}{I_C(s)} = \frac{1}{s \cdot C} \tag{19}$$

In frequency domain

$$X_c(j\omega) = \frac{1}{j\omega \cdot C} \tag{20}$$

Ideal Inductor Reactance

For an inductance L, the voltage is $v_L(t) = L \cdot \frac{di_L(t)}{dt}$. Transforming, we have

$$V_L(s) = L \cdot s \cdot I_L(s) \tag{21}$$

For inductor impedance (for ideal inductor reactance=impedance) is equal

$$X_l(s) = \frac{V_L}{I_L} = s \cdot L \tag{22}$$

In frequency domain

$$X_l(j\omega) = j\omega \cdot L \tag{23}$$

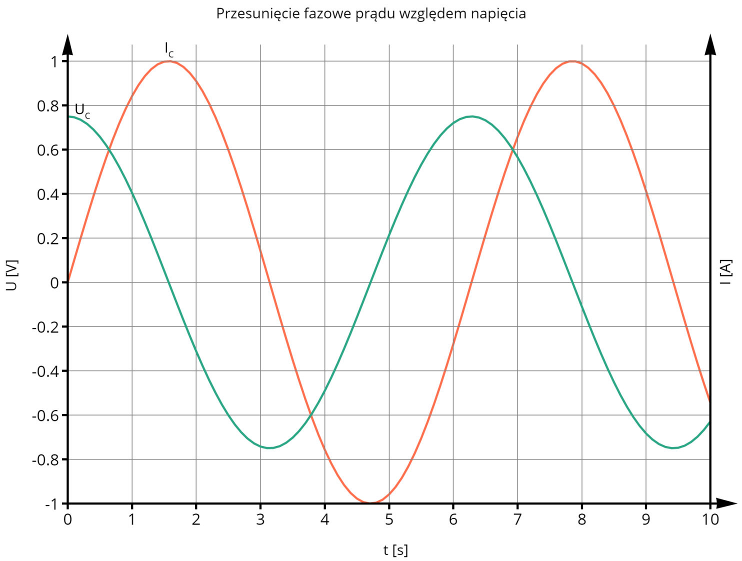
Properties of capacitor and inductor reactance

- observe, that for capacitor:
  $X_C = \frac{1}{j\omega \cdot C} = -j\frac{1}{\omega \cdot C}$ hence $X_C < 0$
- similar for inductor:
  $X_L = j\omega \cdot L$ hence $X_L > 0$
- If *reactance* is negative circuit has an *capacitive character*
- If *reactance* is positive circuit has an *inductive character*

How capacitor and inductor reactance depends on frequency ?

$\omega = 0$	$X_C = \frac{1}{j\omega \cdot C} = \infty$	$X_L = j\omega \cdot L = 0$
$\omega \uparrow$	$X_C \downarrow$	$X_L \uparrow$
$\omega \downarrow$	$X_C \uparrow$	$X_L \downarrow$
$\omega = \infty$	$X_C = \frac{1}{j\omega \cdot C} = 0$	$X_L = j\omega \cdot L = \infty$

Time dependencies between voltage and current



- Impedance amplitude: $|Z| = \frac{AMPLITUDE_{voltage}}{AMPPLITUDE_{current}}$
- Impedance phase: time shift between voltage and current expressed in radians (or degree)