1 Polynomial Approximation

Polynomial approximation is a mathematical technique used to approximate a given function with a polynomial. A polynomial is a mathematical expression composed of one or more terms, each term representing a power of the variable. The general form of a polynomial is:

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

Here, $P_n(x)$ is the polynomial of degree n, and $a_0, a_1, \ldots, a_n$ are the coefficients. The goal of polynomial approximation is to find the coefficients that best fit the given function within a specified range. This technique is often used for curve fitting and interpolation.

2 Chebyshev Polynomial Approximation

Chebyshev polynomial approximation is a method for approximating a function using Chebyshev polynomials. Chebyshev polynomials are a family of orthogonal polynomials defined on a specific interval, typically $[-1, 1]$. The nth Chebyshev polynomial, denoted as $T_n(x)$, is defined as:

$$T_n(x) = \cos(n \cos^{-1}(x))$$

Chebyshev polynomial approximation is particularly useful for approximating functions on a bounded interval, as it minimizes the "Runge's phenomenon" that can occur with other polynomial approximations.

3 Recursion Formulas

3.1 Laguerre Polynomials

Laguerre polynomials, denoted as $L_n(x)$, satisfy the Laguerre's differential equation:

$$x \frac{d^2 L_n(x)}{dx^2} + (1 - x) \frac{dL_n(x)}{dx} + nL_n(x) = 0$$

The recursion formula for Laguerre polynomials is:

$$(n + 1)L_{n+1}(x) = (2n + 1 - x)L_n(x) - nL_{n-1}(x)$$

3.2 Legendre Polynomials

Legendre polynomials, denoted as $P_n(x)$, satisfy the Legendre's differential equation:

$$(1 - x^2) \frac{d^2 P_n(x)}{dx^2} - 2x \frac{dP_n(x)}{dx} + n(n + 1)P_n(x) = 0$$

The recursion formula for Legendre polynomials is:

$$(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x)$$

3.3 Hermite Polynomials

Hermite polynomials, denoted as $H_n(x)$, satisfy the Hermite's differential equation:

$$\frac{d^2H_n(x)}{dx^2} - 2x\frac{dH_n(x)}{dx} + 2nH_n(x) = 0$$

The recursion formula for Hermite polynomials is:

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$