Definition 1. *Short modular notation: These are all equivalent, let* $a, b, n \in \mathbb{Z}$

$$a \equiv b \mod n \tag{1a}$$
$$a \equiv_n b \tag{1b}$$

Definition 2. *Set of all primes: These are all equivalent, let* $n, m \in \mathbb{Z}$

$$Primes = \{\, n \mid \nexists m \in Primes, n \equiv_m 0 \,\} \tag{2a}$$
$$Primes = \{\, n \mid \forall m \in Primes, n \not\equiv_m 0 \,\} \tag{2b}$$
$$Primes = \{\, n \mid \forall m \in Primes, gcd(m, n) = 1 \,\} \tag{2c}$$
$$Primes = \{\, n \mid \nexists m \in (\mathbb{N} - Primes - \{1\}) \quad n \equiv_m 0 \,\} \tag{2d}$$

Statement 1. *The set of all primes is a complete order as a subset of all the orders on* $\mathbb{Z}$

Definition 3. *Root of the Prime Space of order* n*,* R_{PS_n}*: let* $P \subseteq Primes$ *containing the first* n *members of* $Primes$ *under the* $<$ *relation*

$$R_{PS_n} = \{-1\} \cup P \tag{3}$$

Statement 2. R_{PS_n} *contains exactly* $n + 1$ *members*

$$|R_{PS_n}| = n + 1 \tag{4}$$

Statement 3. *Closing* R_{PS_n} *under* $*$ *contains all the* $m \in \mathbb{Z}$ *such that* $\forall p \in R_{PS_n}, m \equiv_p 0$

$$cl(\langle R_{PS_n}, * \rangle) = \{\, m \in \mathbb{Z} \mid \forall p \in R_{PS_n}, m \equiv_p 0 \,\} \tag{5}$$

But not those $m \in \mathbb{Z}$ *that* $\forall p \in (Primes \oplus R_{PS_n}), m \equiv_p 0$

$$cl(\langle R_{PS_n}, * \rangle) \cap cl(\langle Primes \oplus R_{PS_n}, * \rangle) = \emptyset \tag{6}$$

Definition 4. *Prime Space,* PS_n*:*

$$PS_n = cl(\langle R_{PS_n}, * \rangle) \tag{7}$$