

Peter Willendrup, DTU Physics & ESS DMSC

Neutron optics

- And thanks to Ken Andersen ESS for
letting me use a lot of his slides on guides!

McStas



Neutron optics - subtopics

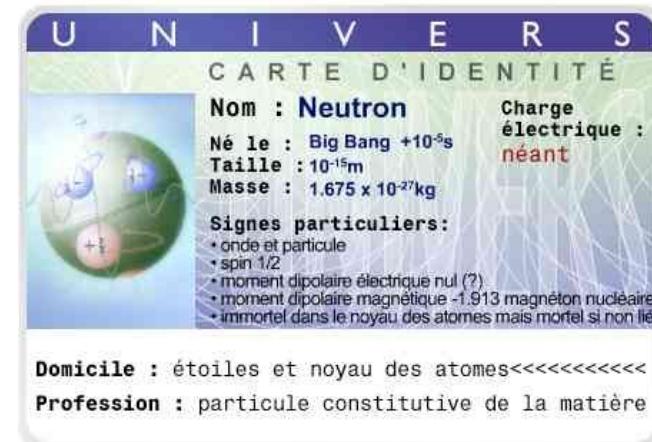
- Key properties of the neutron
- Transporting the beam
 - Neutron guides and how they work
- Tailoring the beam
 - Spatially
 - Directionally
 - Temporally
 - Energetically

The neutron - a reminder

Life time:	$\tau_{1/2} = 890s$
Mass:	$m = 1.675 \times 10^{-27} kg$
Charge:	$Q = 0$
Spin:	$s = \hbar/2$
Magnetic moment:	$\mu/\mu_n = -1.913$

$$E = \frac{1}{2}mv^2 = \frac{\hbar^2k^2}{2m}, \quad \lambda = 2\pi/k$$

$$E = 81.81 \cdot \lambda^{-2} = 2.07 \cdot k^2 = 5.23 \cdot v^2$$



Mr. Neutron

	Energy	Wavelength	n-Wavevector	Velocity	Frequency
cold neutrons:	$E = 1 \text{ meV}$	$1 = 9.0446 \text{ Å}$	$k = 0.6947 \text{ 1/Å}$	$v = 437 \text{ m/s}$	$v = 0.2418 \text{ THz}$
	$E = 5 \text{ meV}$	$1 = 4.0449 \text{ Å}$	$k = 1.5534 \text{ 1/Å}$	$v = 978 \text{ m/s}$	$v = 1.2090 \text{ THz}$
thermal neutrons:	$E = 25 \text{ meV}$	$1 = 1.8089 \text{ Å}$	$k = 3.4734 \text{ 1/Å}$	$v = 2187 \text{ m/s}$	$v = 6.045 \text{ THz}$
	$E = 50 \text{ meV}$	$1 = 1.2791 \text{ Å}$	$k = 4.9122 \text{ 1/Å}$	$v = 3093 \text{ m/s}$	$v = 12.090 \text{ THz}$

Cross section: coherent + incoherent + absorption

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Cross section: coherent + incoherent + absorption



Optical properties - slow neutrons vs. light

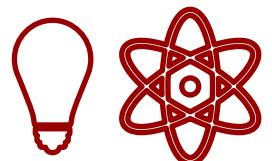
	light	neutrons
λ	$< \mu\text{m}$	$< \text{nm}$
E	$> \text{eV}$	$> \text{meV}$
n	$1 \rightarrow 4$	$0.9997 \rightarrow 1.0001$
θ_c	90°	1°
B	$10^{18} \text{ p/cm}^2/\text{ster/s}$ (60W lightbulb)	$10^{14} \text{ n/cm}^2/\text{ster/s}$ (60MW reactor)
spin	1	$\frac{1}{2}$
interaction	electromagnetic	strong force, magnetic
charge	0	0

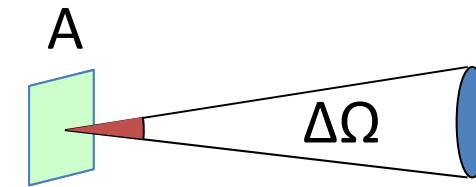
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Corresponding to distances and excitations within condensed matter

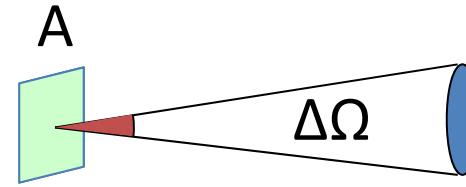
Neutrons are scarce, but we have a chance to transport them



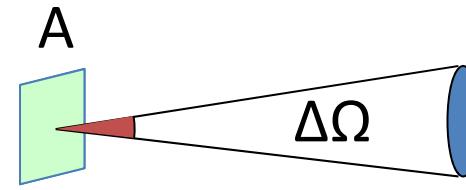


$B = N \text{ per time per } A \text{ per } \Delta\Omega$

units = $\text{n/s/cm}^2/\text{sr}$



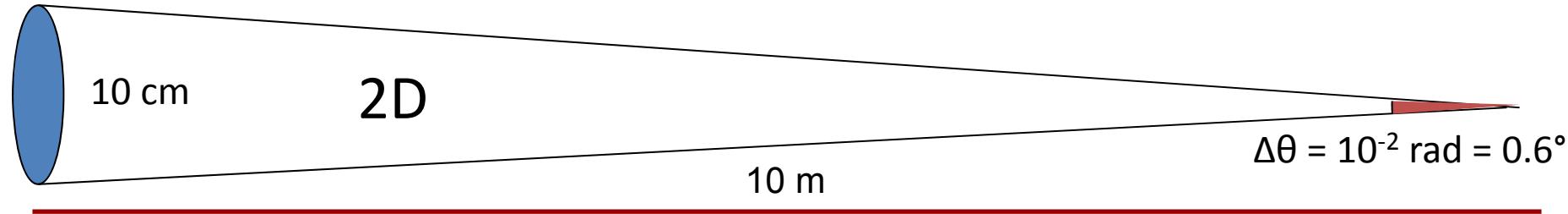
B is independent of distance
- property of the source



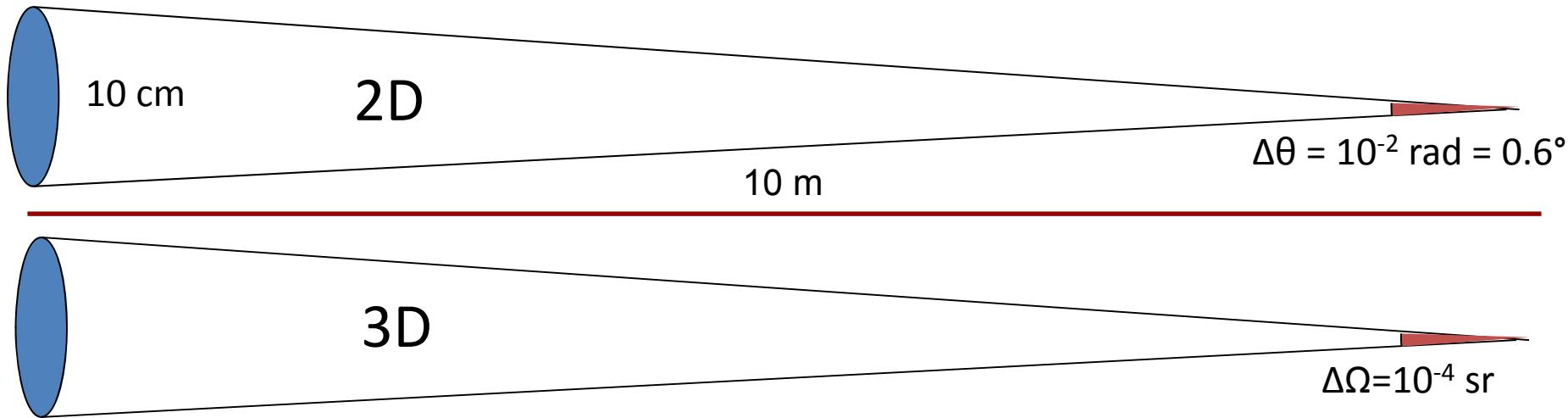
Brilliance/Brightness B $\text{n/s/cm}^2/\text{sr}$

Flux Ψ n/s/cm^2

Neutron flux - and brilliance / brightness



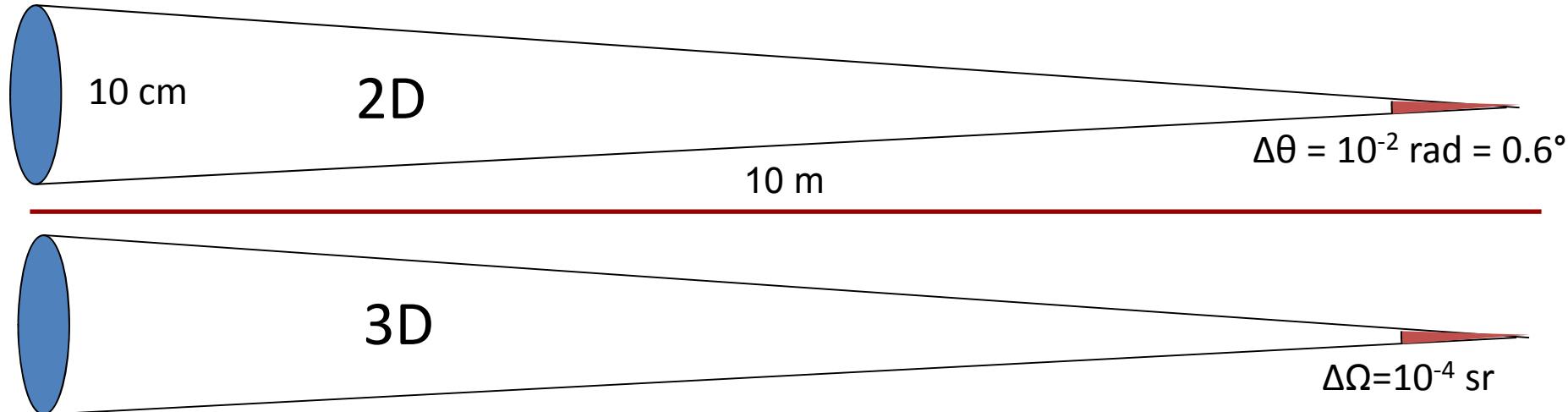
Neutron flux - and brilliance / brightness



$$\text{Flux} = \text{Source Brightness} \times \text{Solid Angle}$$

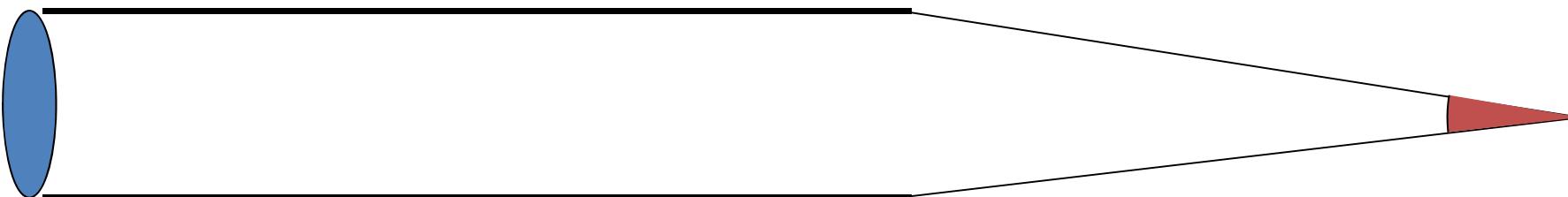
$$\Phi = B \times \Delta\Omega$$

Neutron flux - and brilliance / brightness



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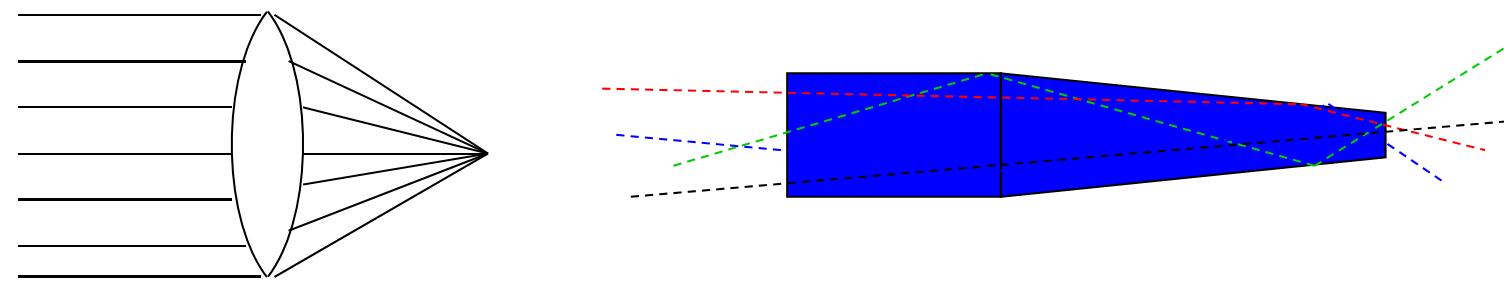
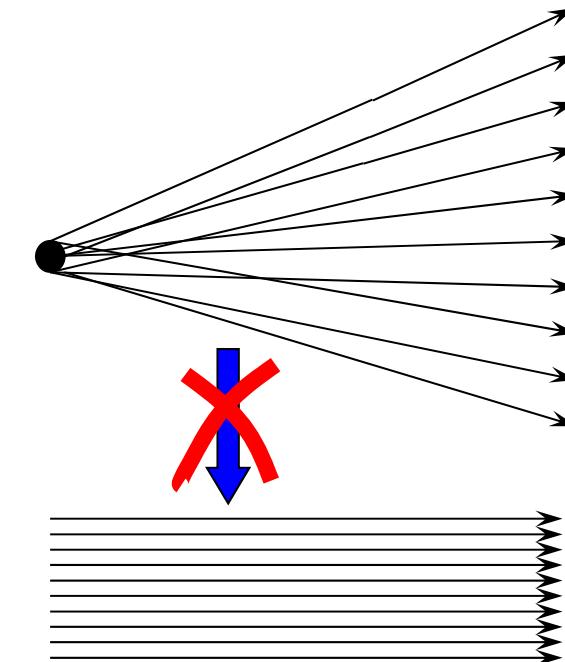
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Liouville theorem
 $\rightarrow \Delta x \Delta \phi = \text{const}$

Liouvilles Theorem

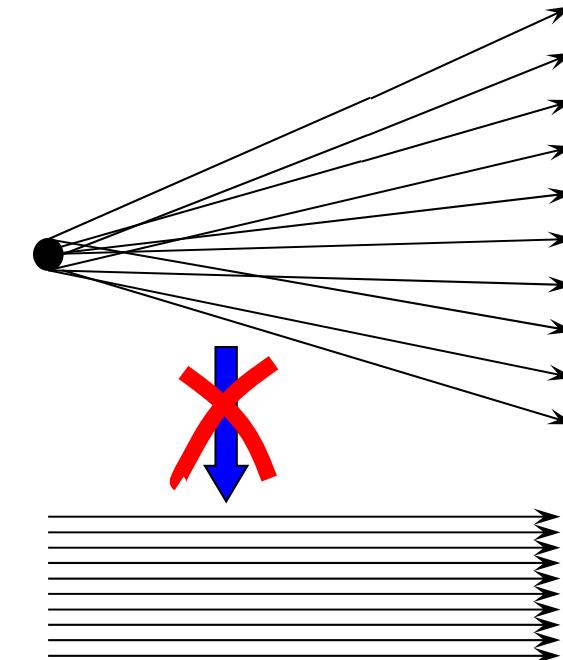
- Conservation laws:
 - neutrons can't be created from thin air
 - neither can “phase space density”
- There is no such thing as a free lunch
 - Beam manipulation transfers distribution between time, area, divergence, energy
- Most common application:
 - Focusing increases divergence
 - improve flux, lose angular resolution



Liouville theorem
 $\rightarrow \Delta x \Delta \phi = \text{const}$

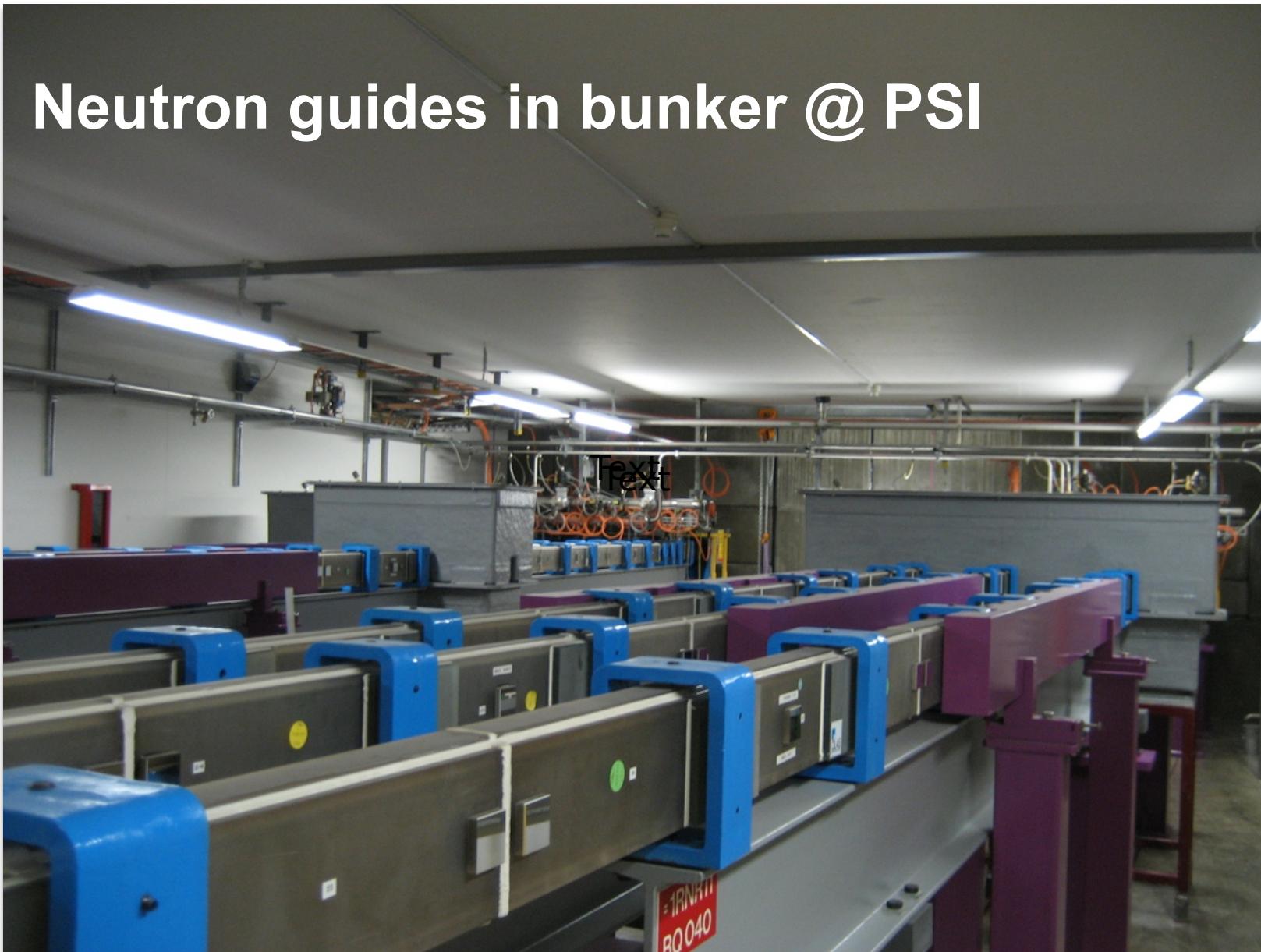
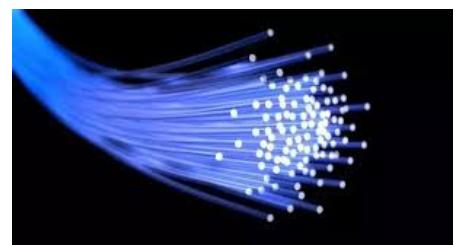
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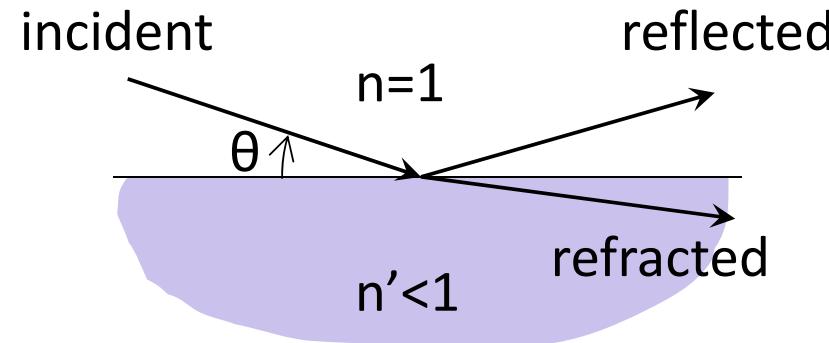


Integrated flux $\int \Psi dA d\Omega$ can never increase

- think optical fiber,
however not total
internal, rather total
external reflection

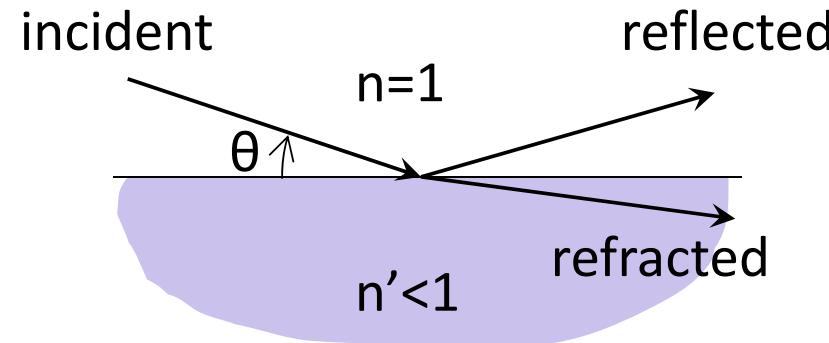


Reflection: Snell's Law



critical angle of total
reflection θ_c

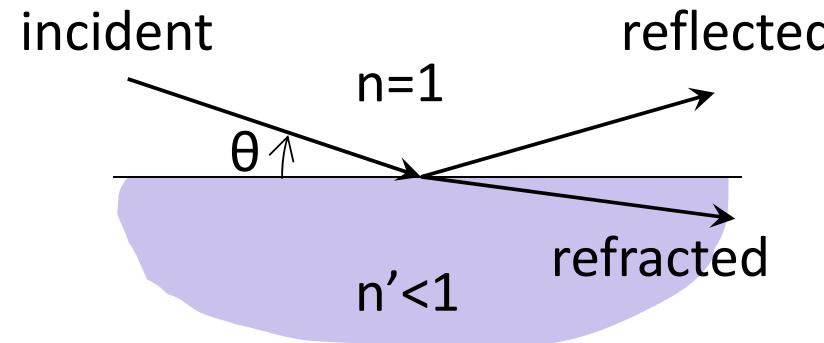
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$$\left. \begin{aligned} \cos\theta_c &= n'/n = n' \\ n' &= 1 - \frac{N\lambda^2 b}{2\pi} \\ \cos\theta_c &\approx 1 - \theta_c^2/2 \end{aligned} \right\} \Rightarrow \theta_c = \lambda \sqrt{Nb/\pi}$$

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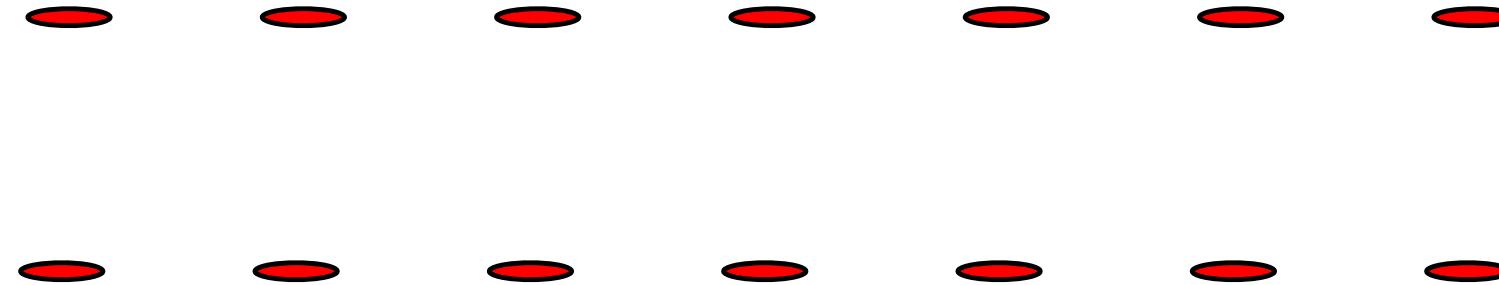
for natural Ni:
 $\theta_c = \lambda[\text{\AA}] \times 0.1^\circ$
 $Q_c = 0.0218 \text{ \AA}^{-1}$

Definition:
 $Q = 4\pi \sin \theta / \lambda$

Idea:

“Expand reflectivity by means of Bragg scattering?

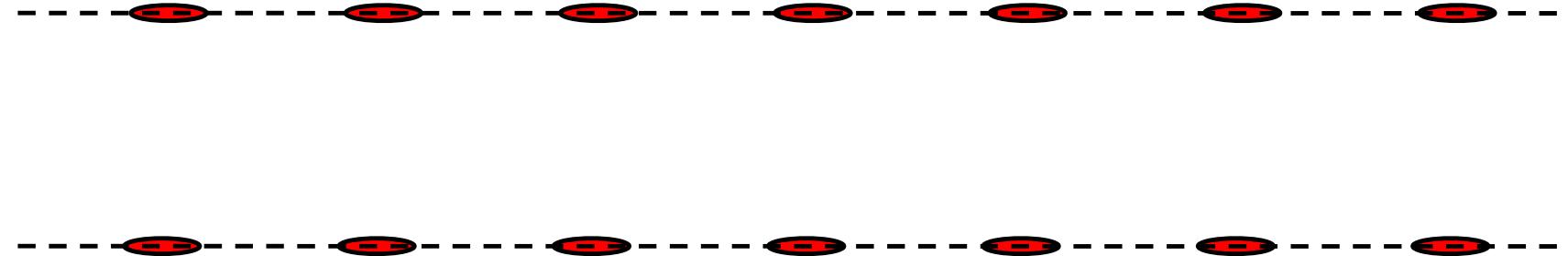
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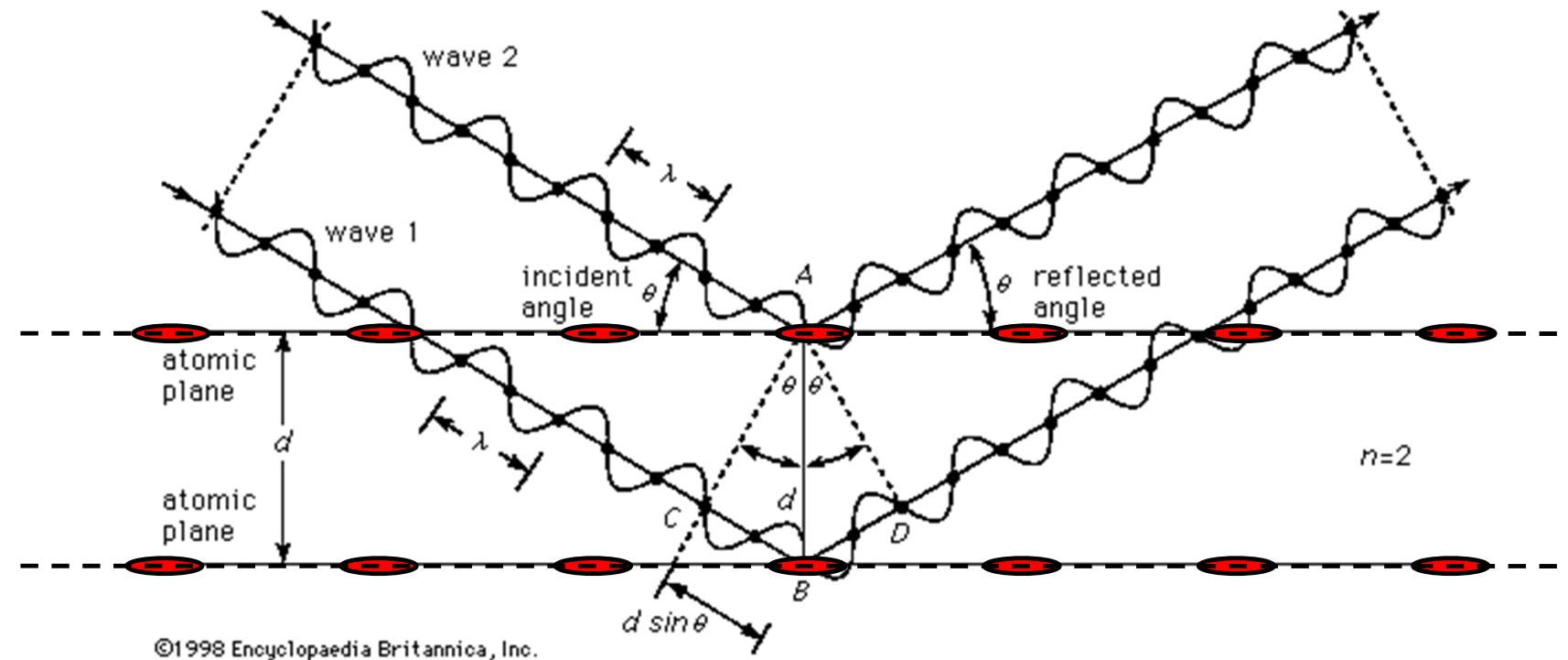
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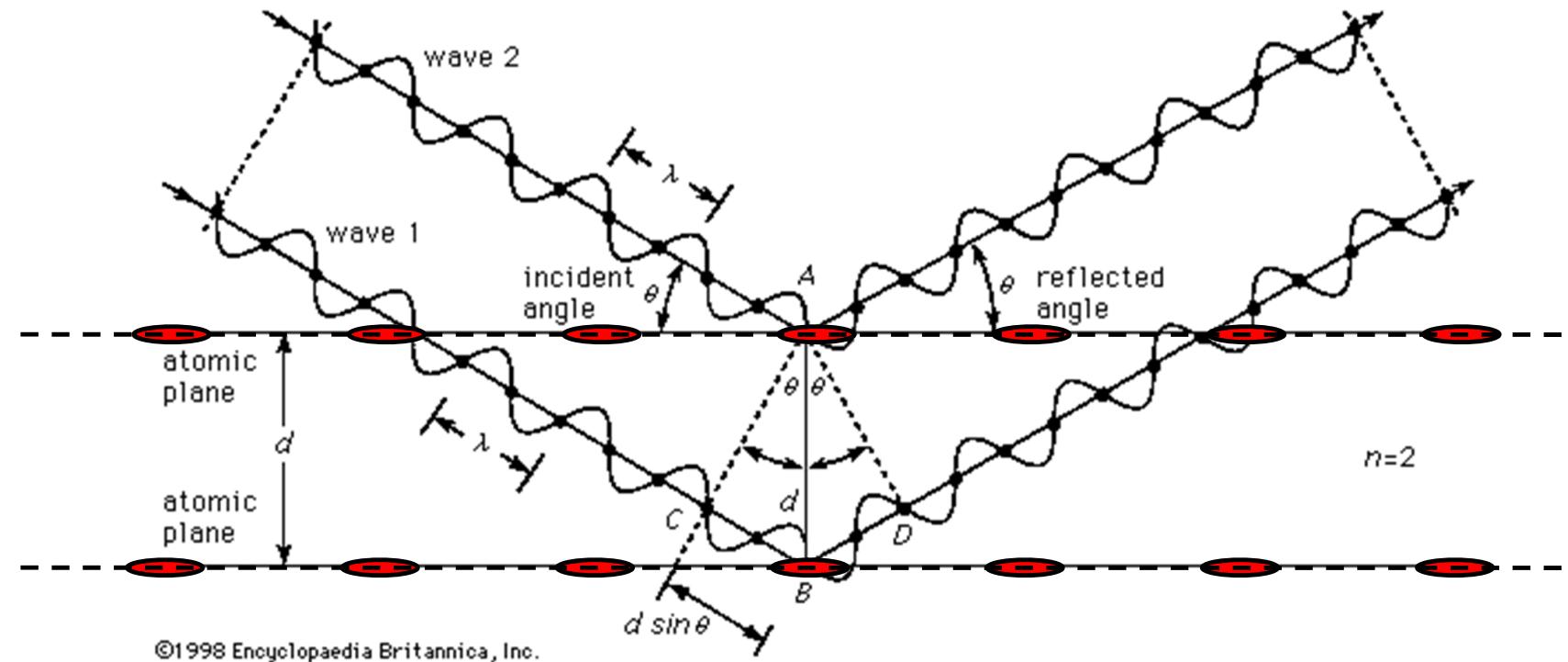
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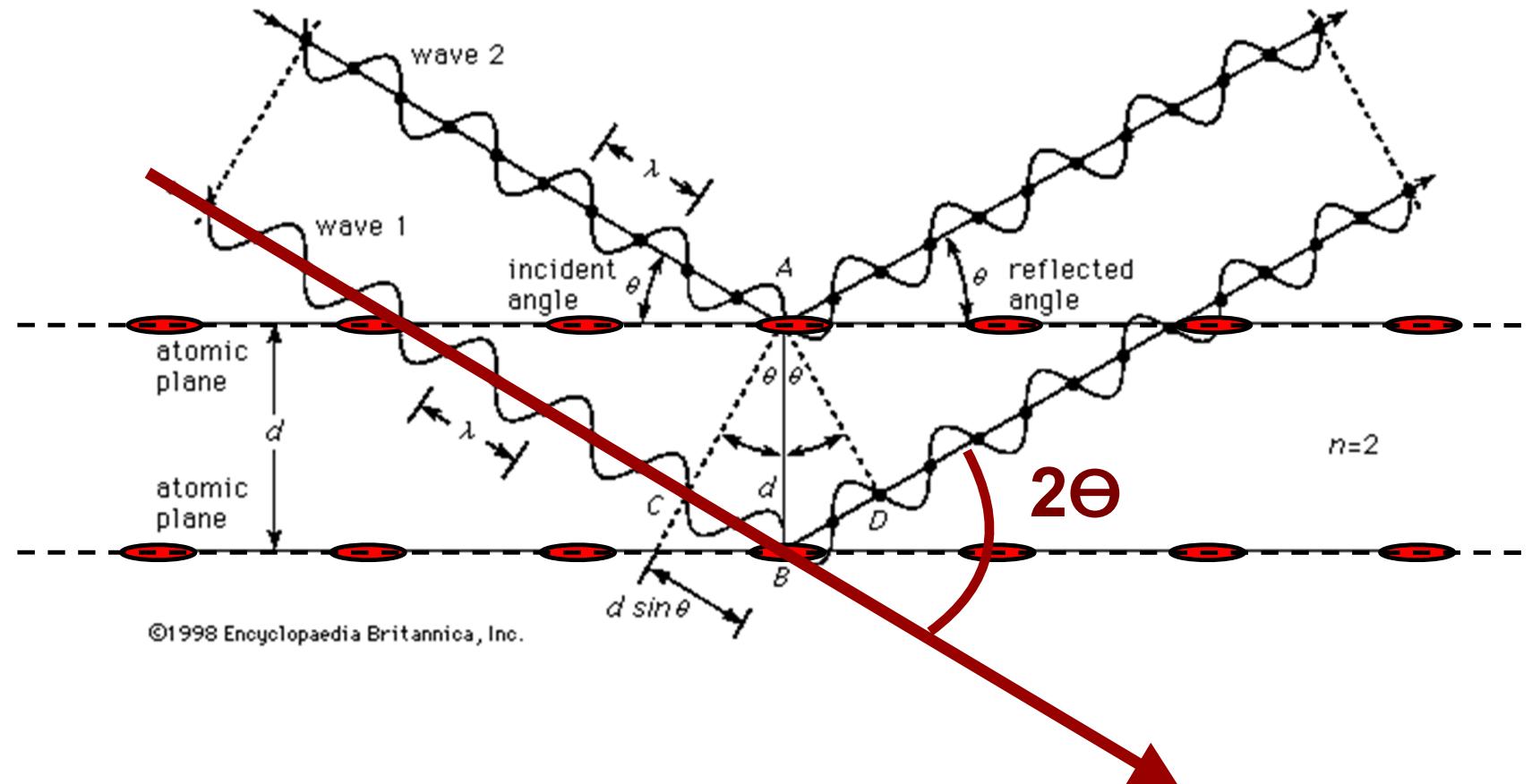
$$\lambda = 2d \sin \theta$$



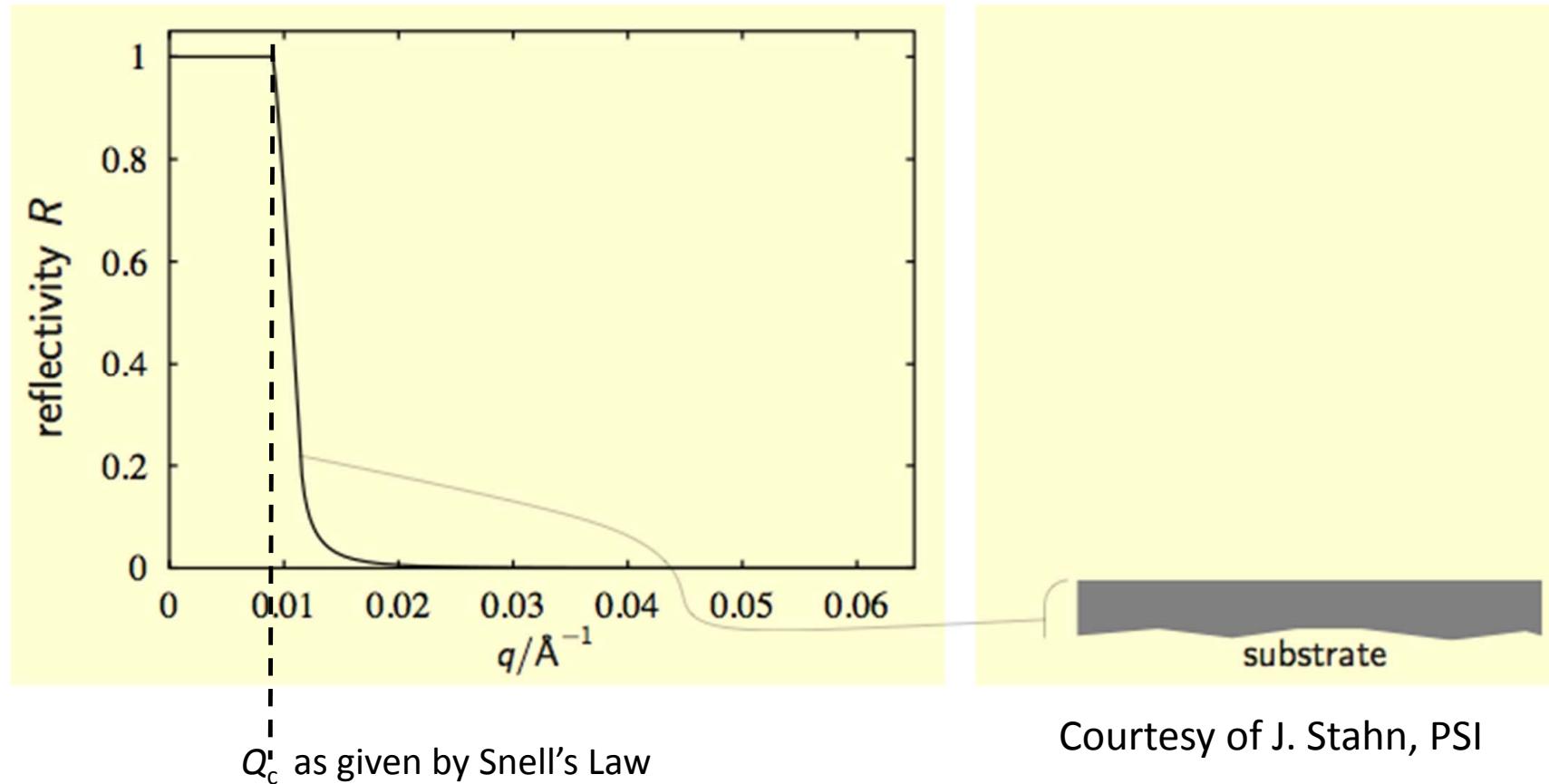
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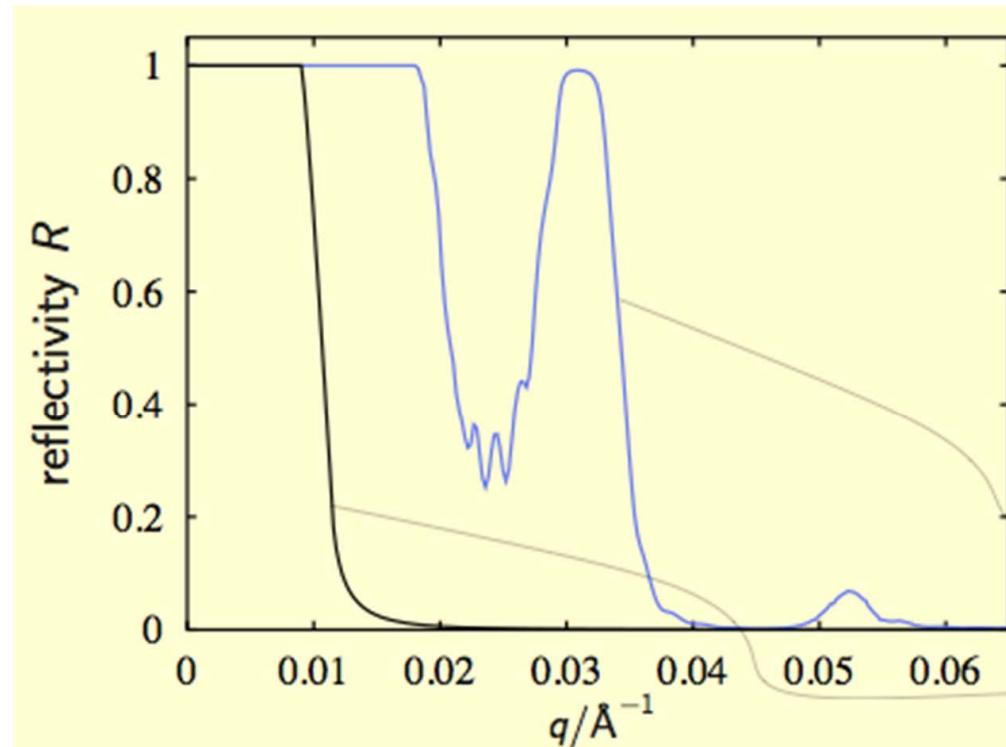
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Neutron Supermirrors



Neutron Supermirrors

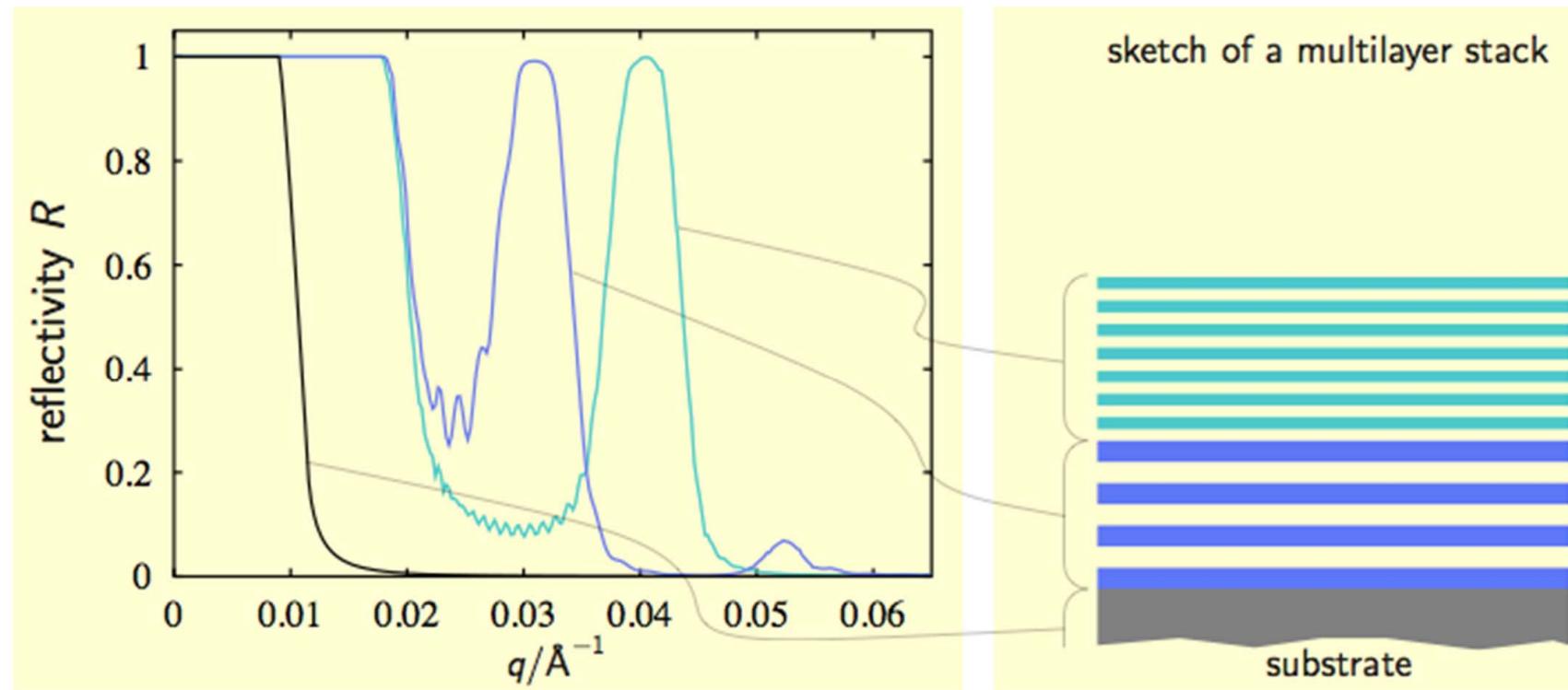


sketch of a multilayer



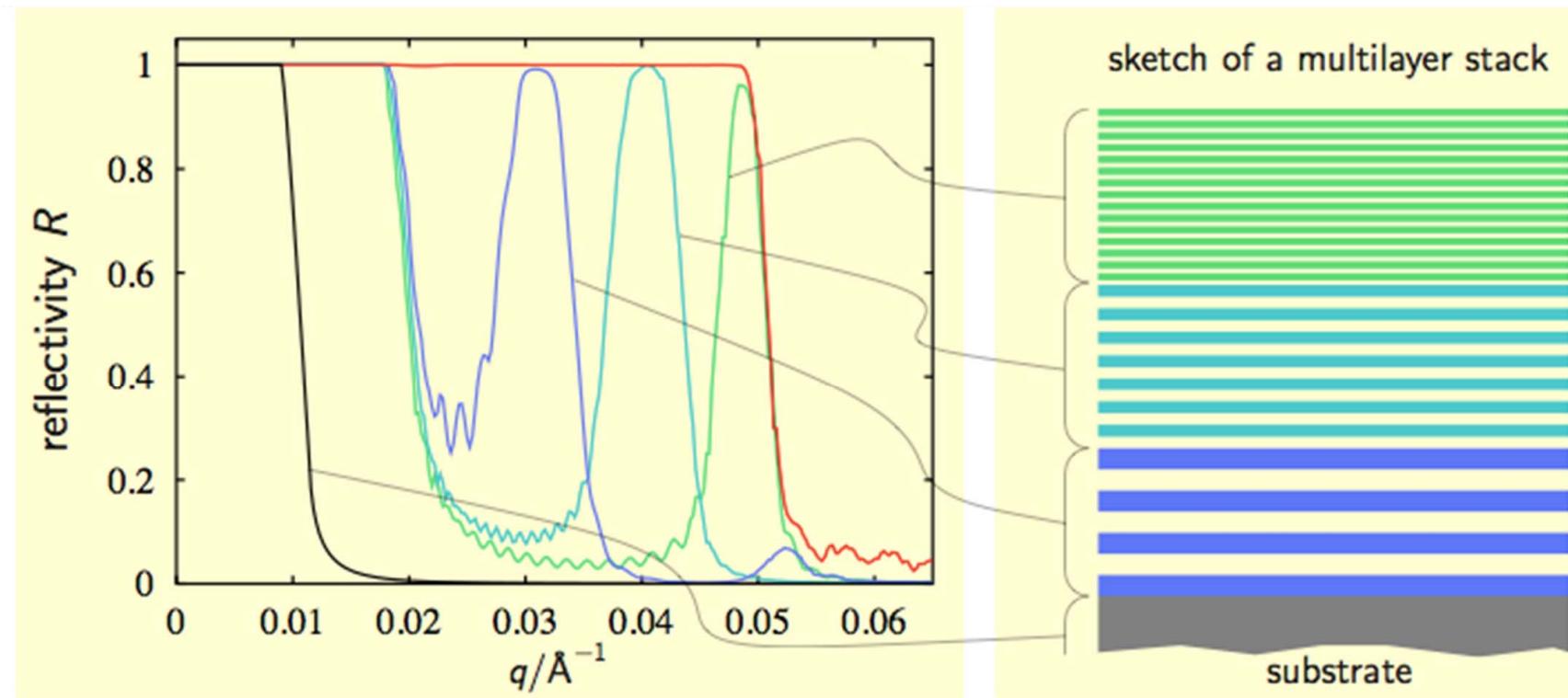
Courtesy of J. Stahn, PSI

Neutron Supermirrors



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Neutron Supermirrors

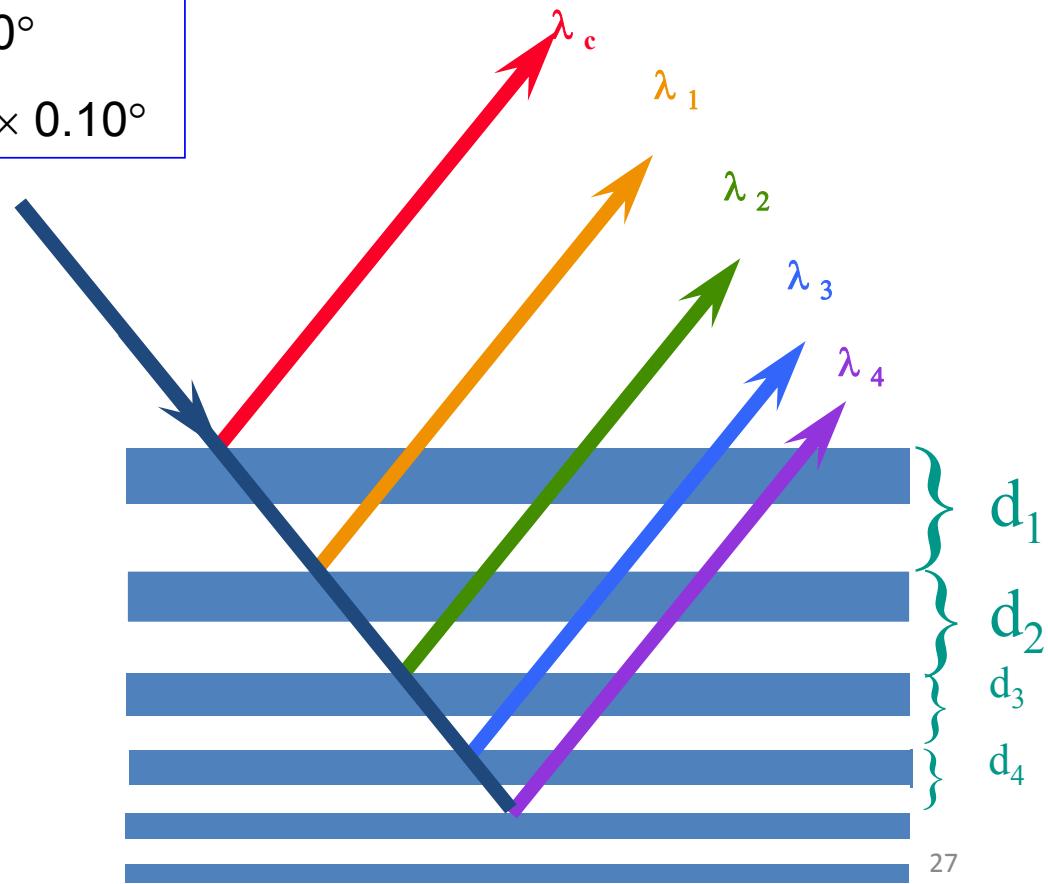


Courtesy of J. Stahn, PSI

Neutron Supermirrors

Reflection: $\theta_c(\text{Ni}) = \lambda[\text{\AA}] \times 0.10^\circ$

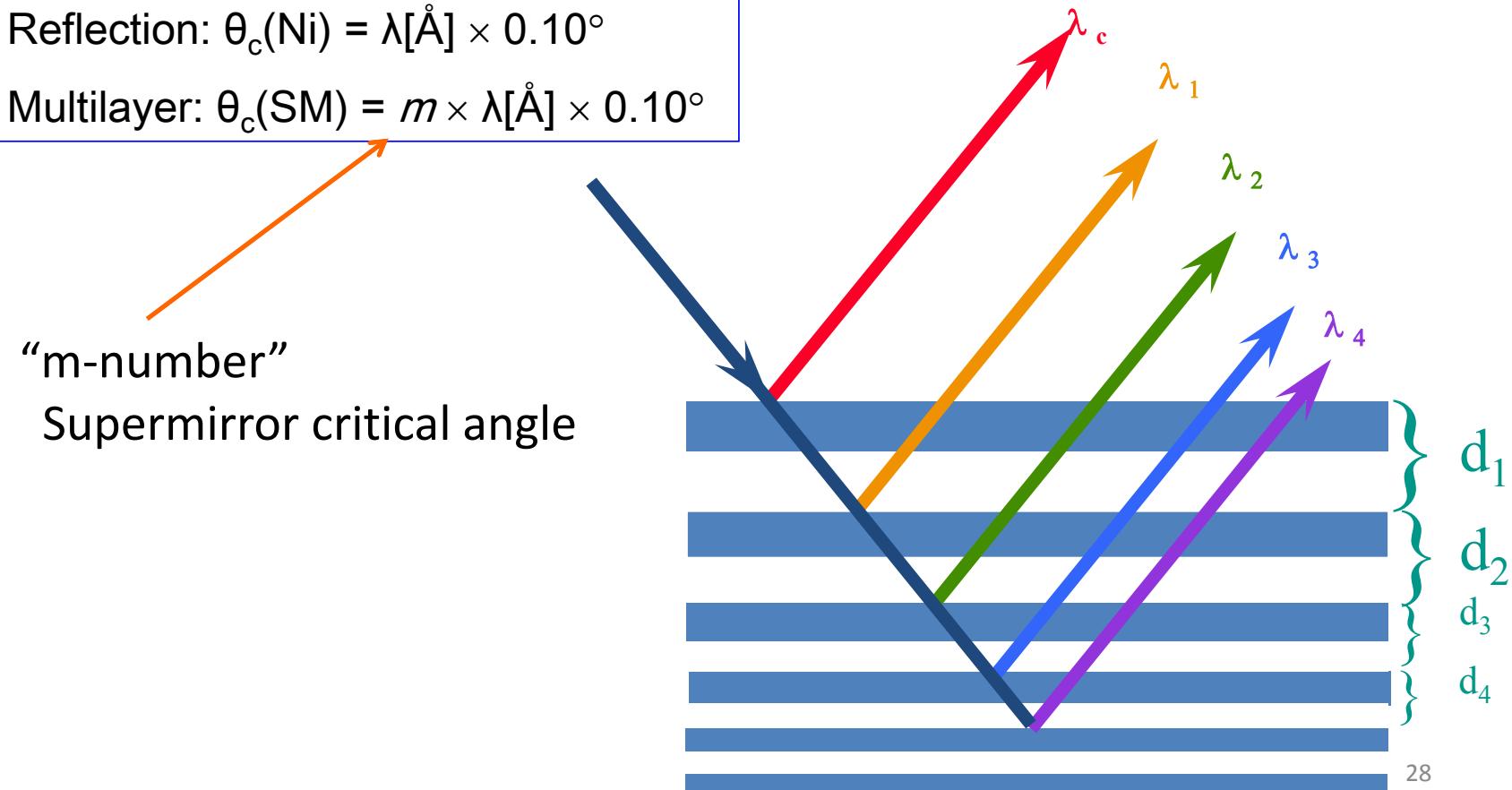
Multilayer: $\theta_c(\text{SM}) = m \times \lambda[\text{\AA}] \times 0.10^\circ$



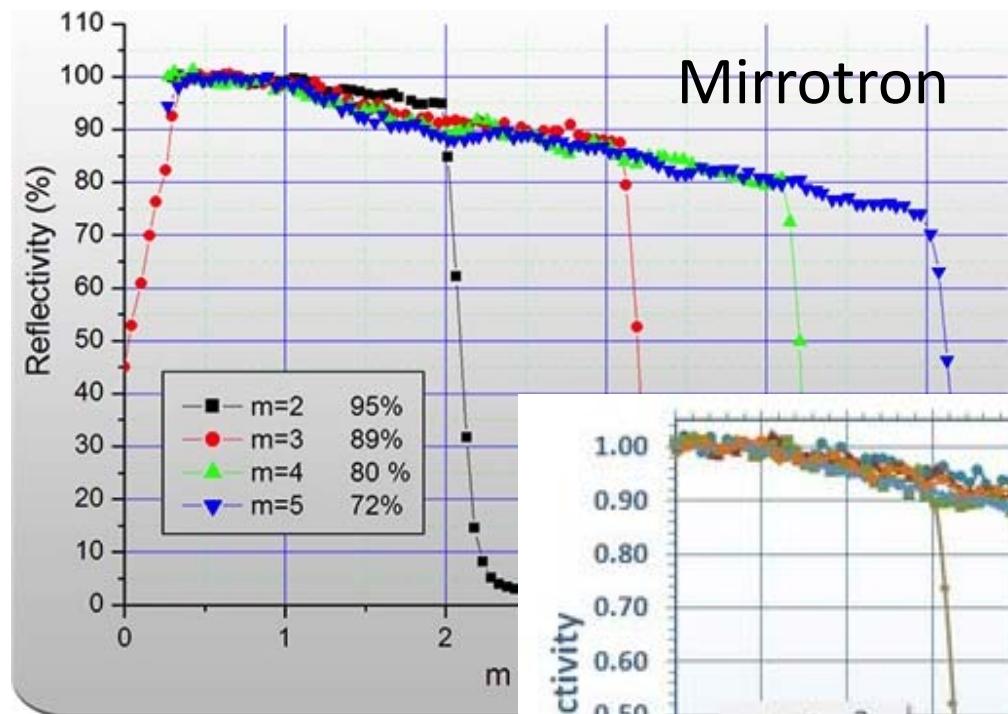
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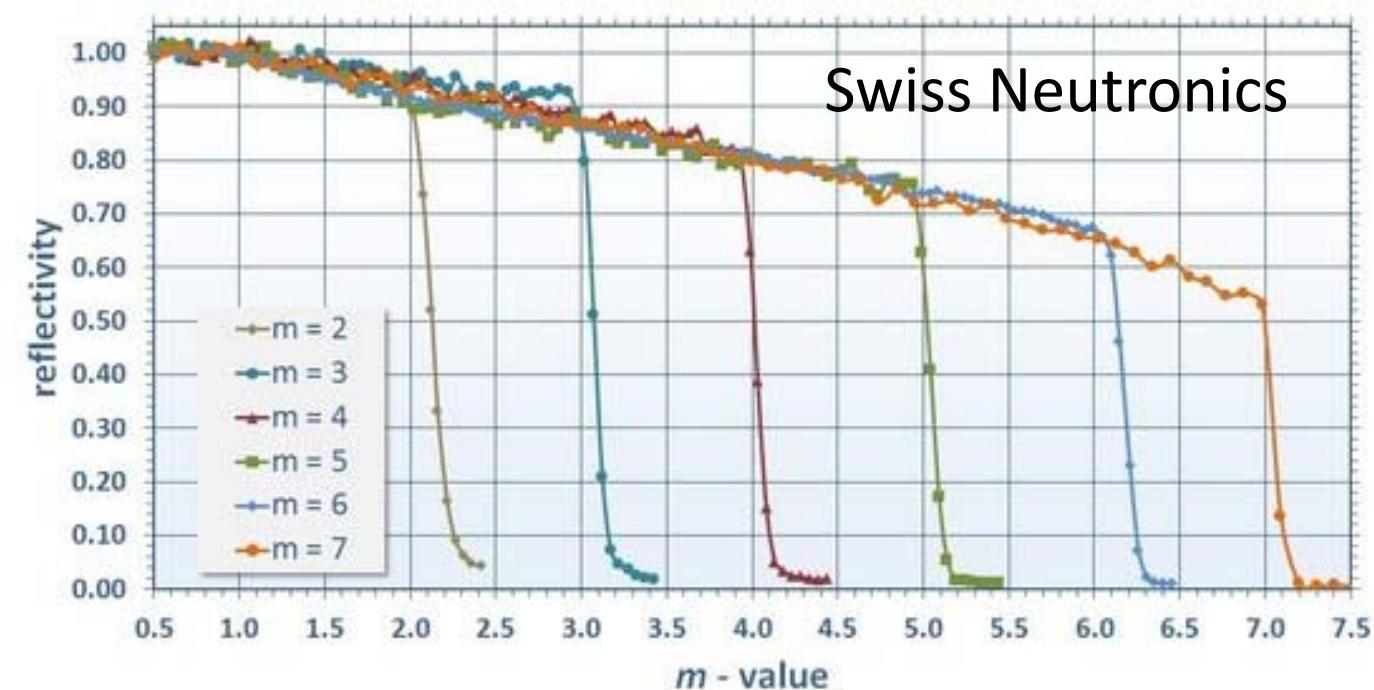
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State-of-the-art Supermirrors (2015?)



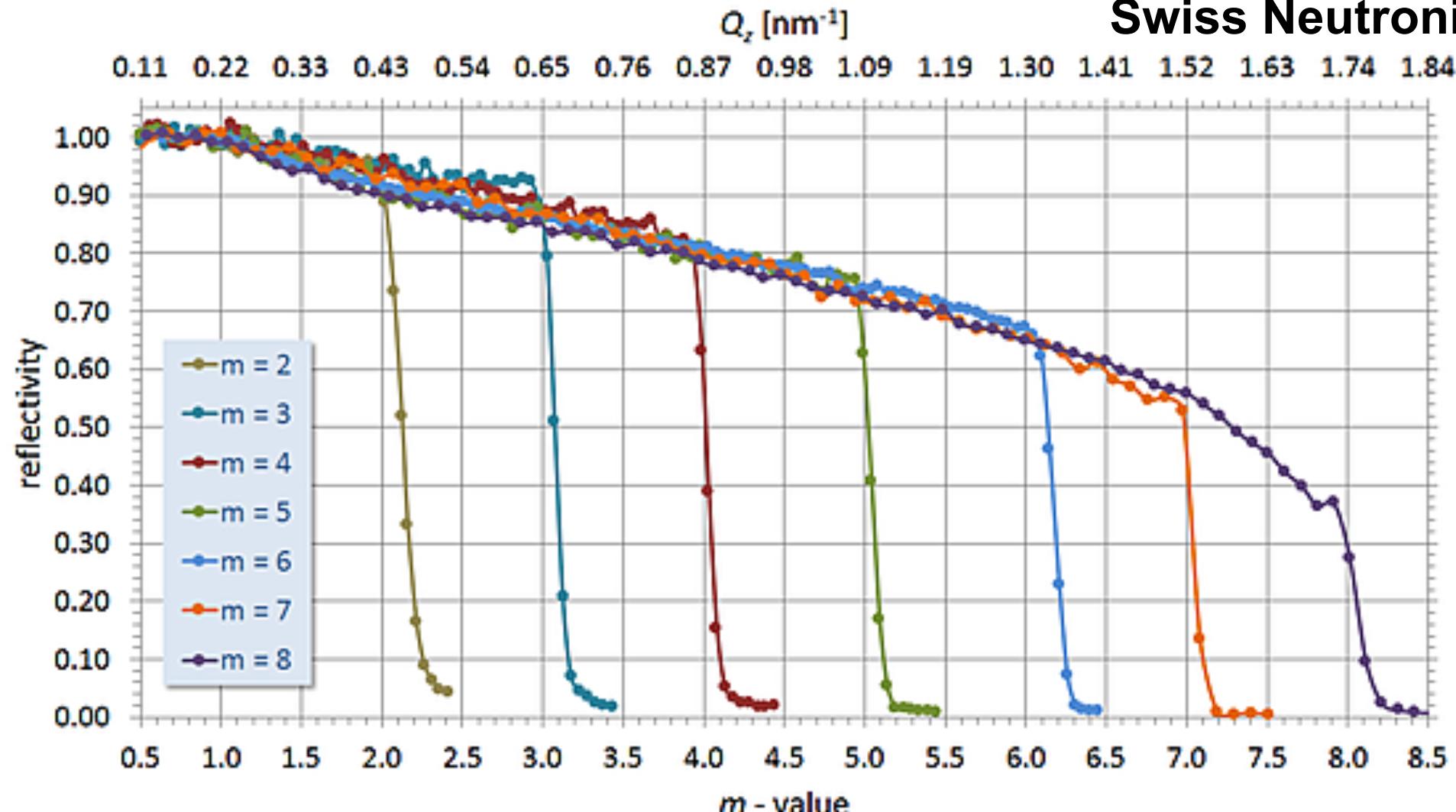
Mirrotron



Swiss Neutronics

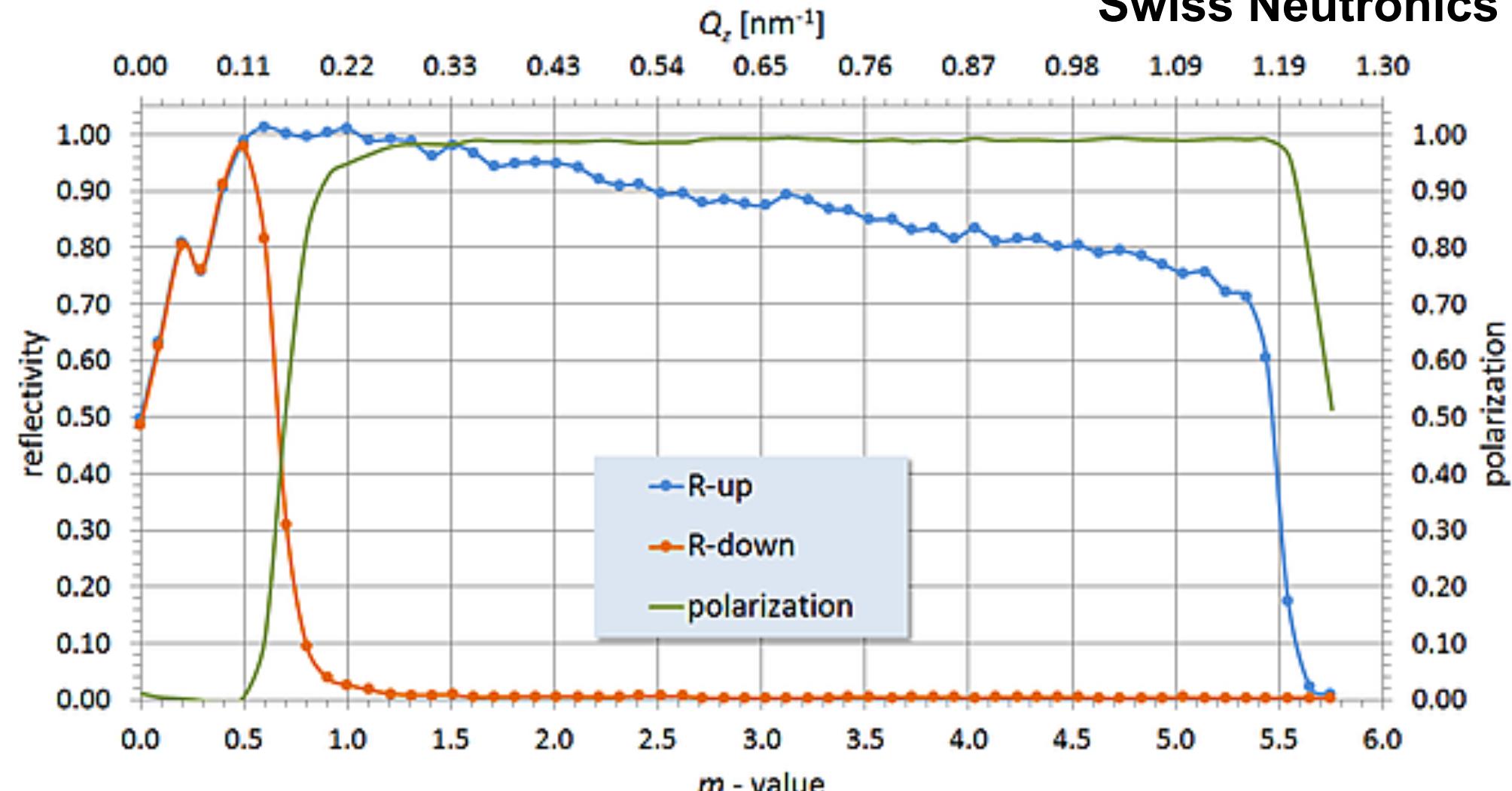
State-of-the-art Supermirrors (2019)

Swiss Neutronics



State-of-the-art Polarising Supermirrors (2019)

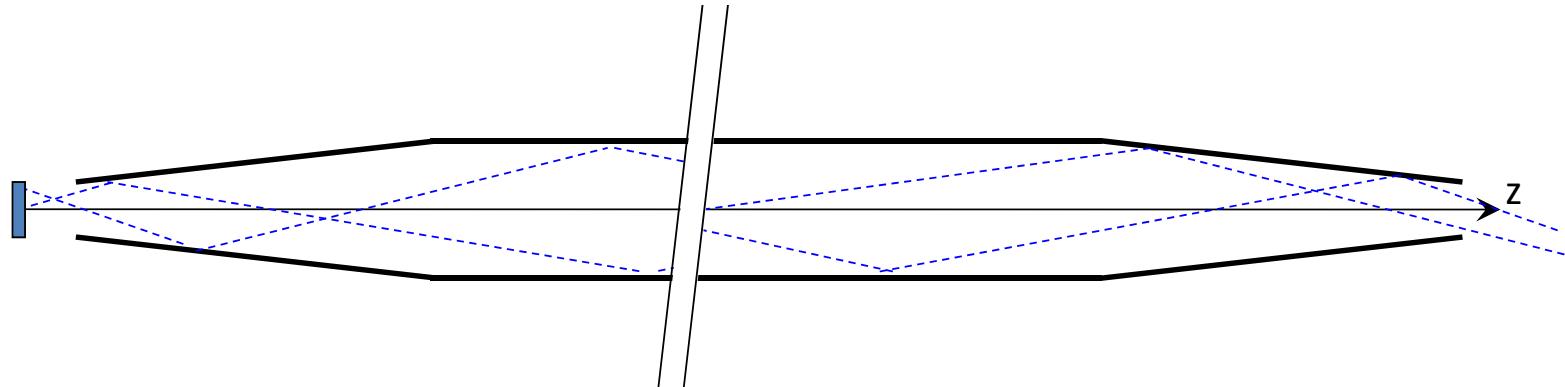
Swiss Neutronics



What are guides used for?

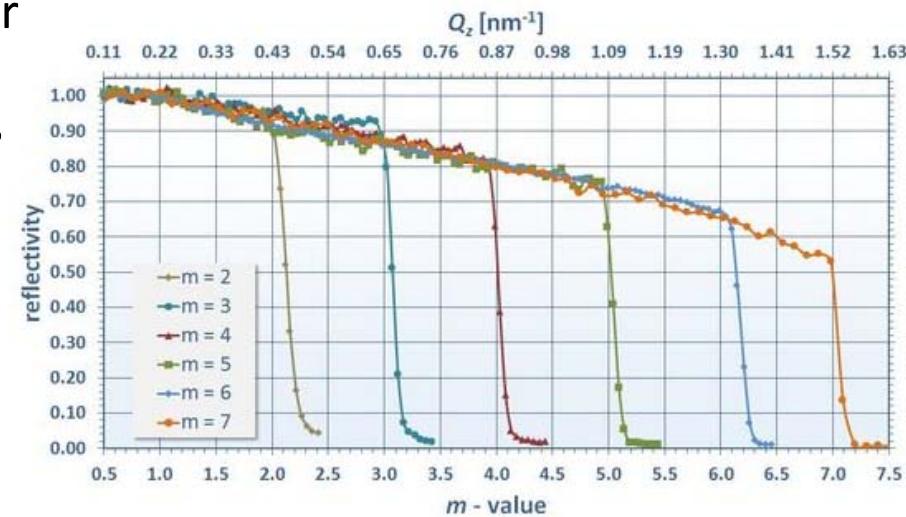
- Transport divergence
 - large m-numbers needed for short wavelengths
 - ballistic geometry required for supermirror guides
- Create space
 - build instruments far from neutron source
- Improve TOF resolution
- Reduce background
 - transport only “good” neutrons
- Focusing
 - increased divergence: increased flux

Ballistic guides

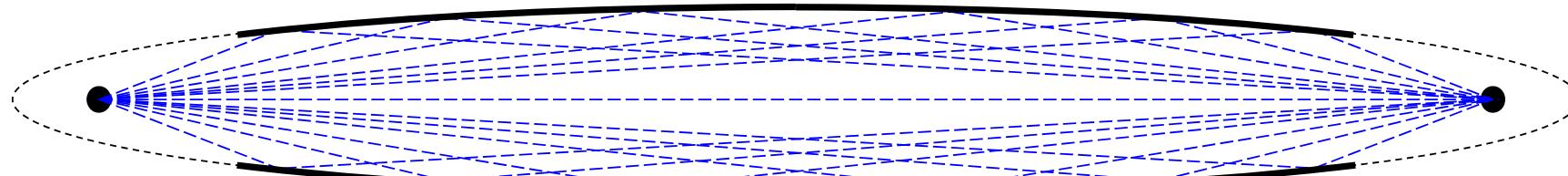


- Used to transport neutrons over long distances
- Minimise number of reflections
- Minimise reflection angles
- Increase width slowly to decrease divergence
adibaticity - reversible

Central part has few reflections: reduce m here to reduce cost.

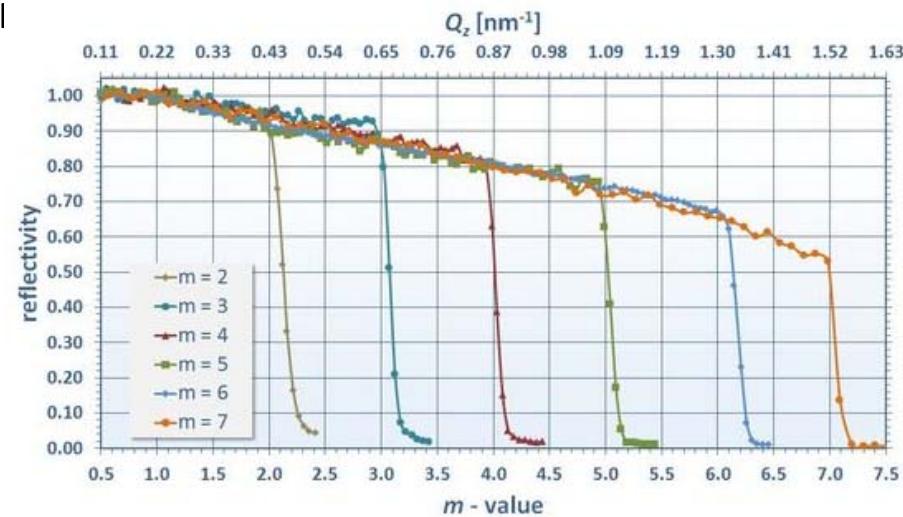


Ballistic guides



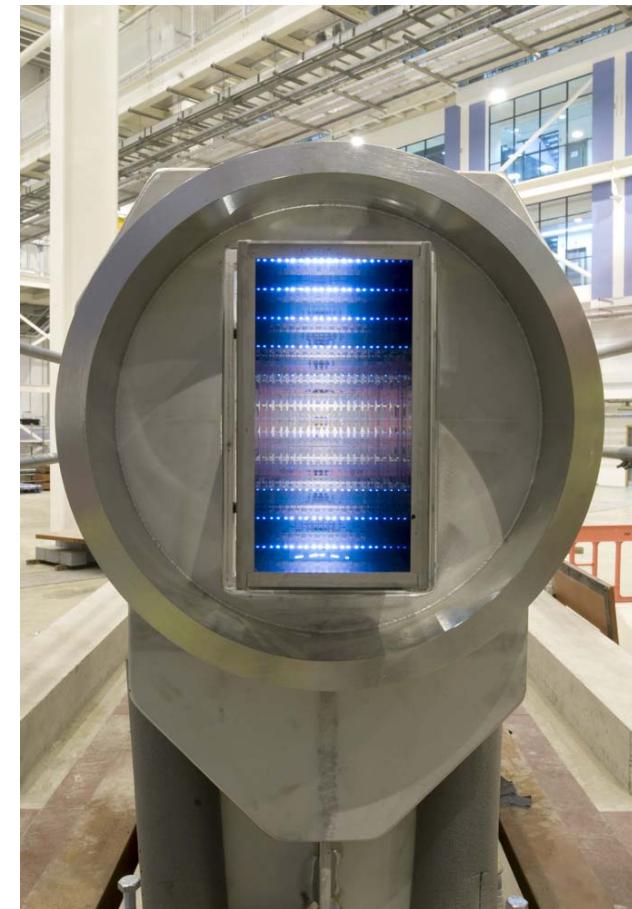
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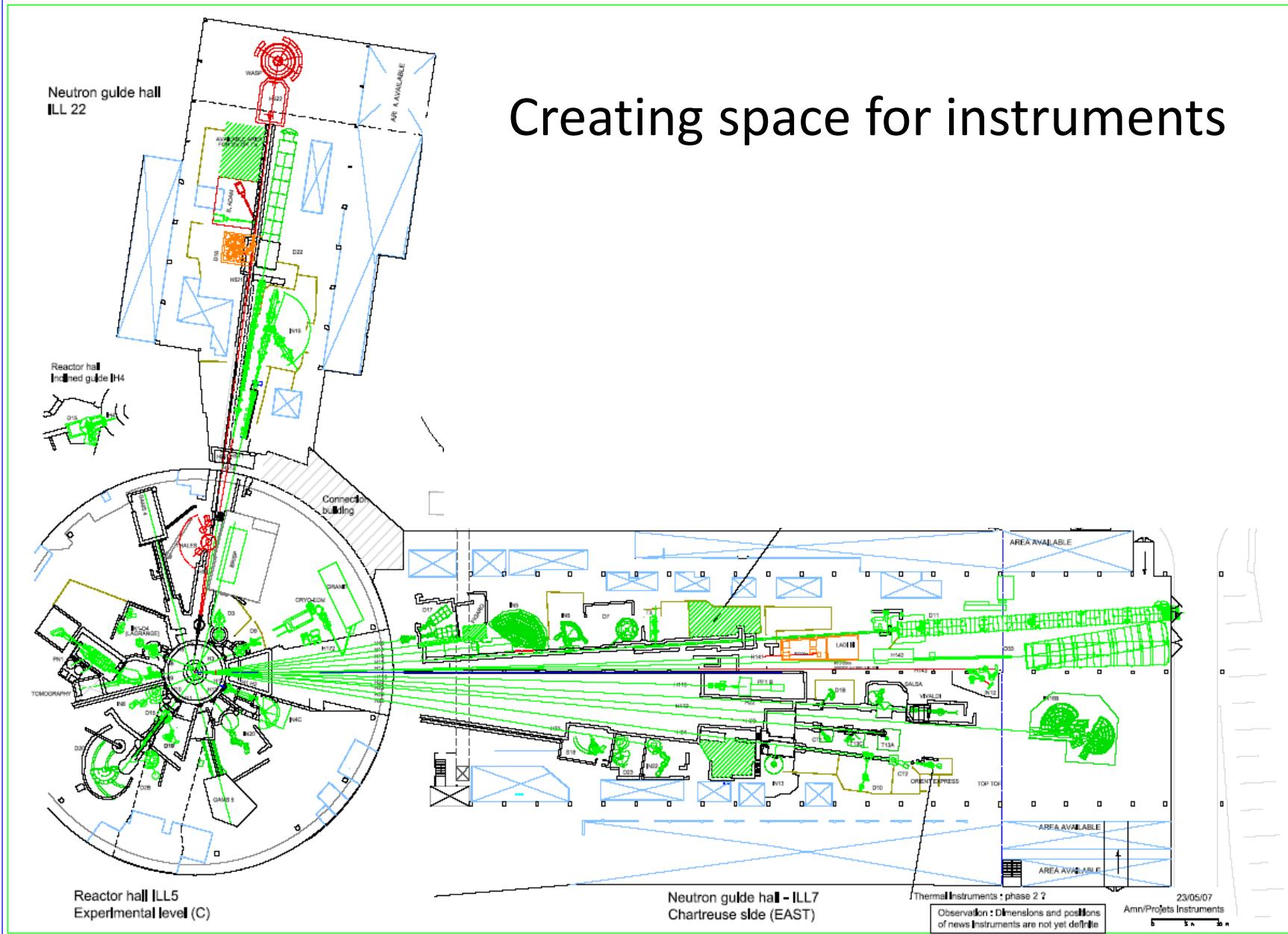
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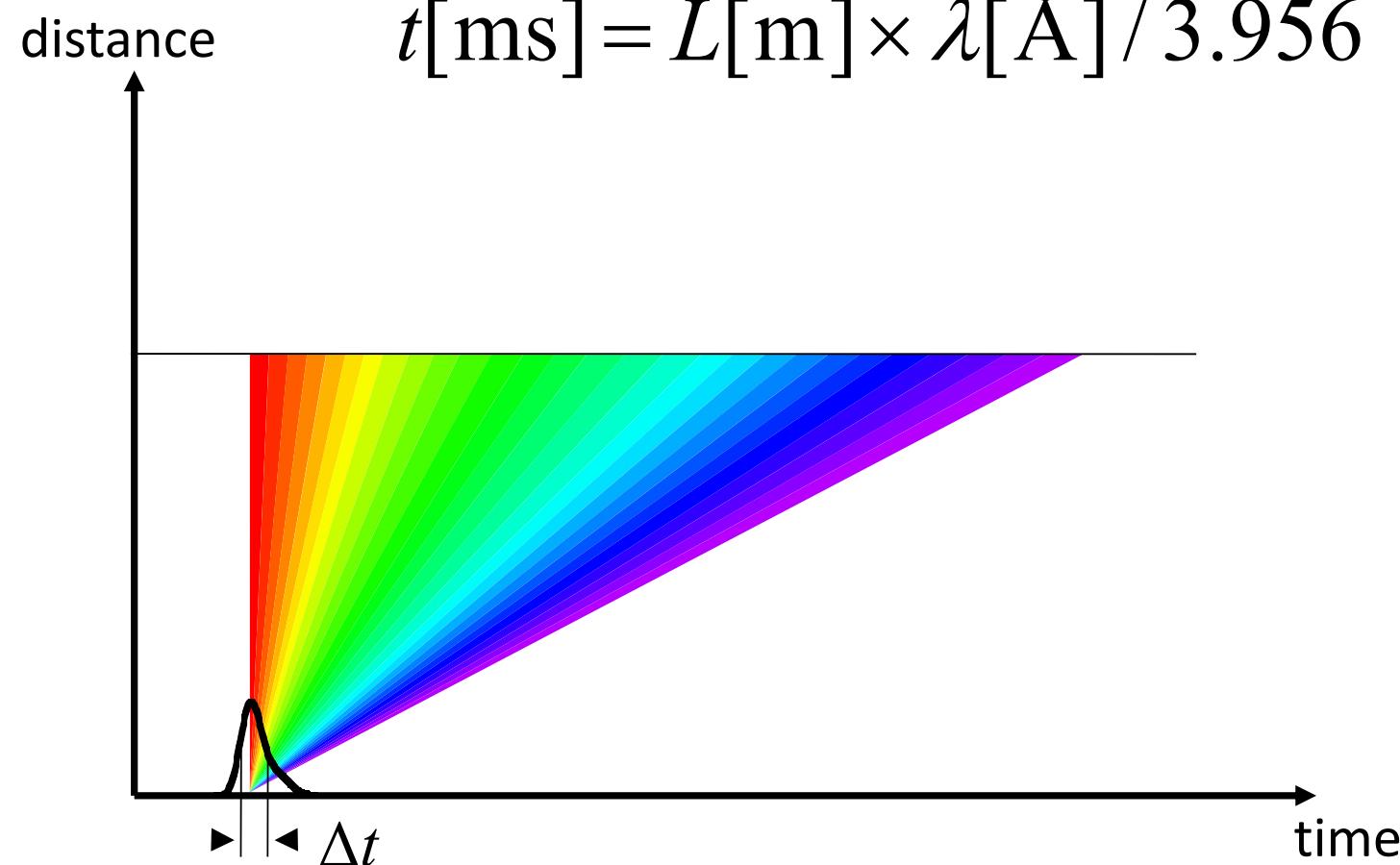
Ballistic guides

HRPD @ ISIS: ballistic (elliptical) guide





Time-of-flight (TOF) resolution

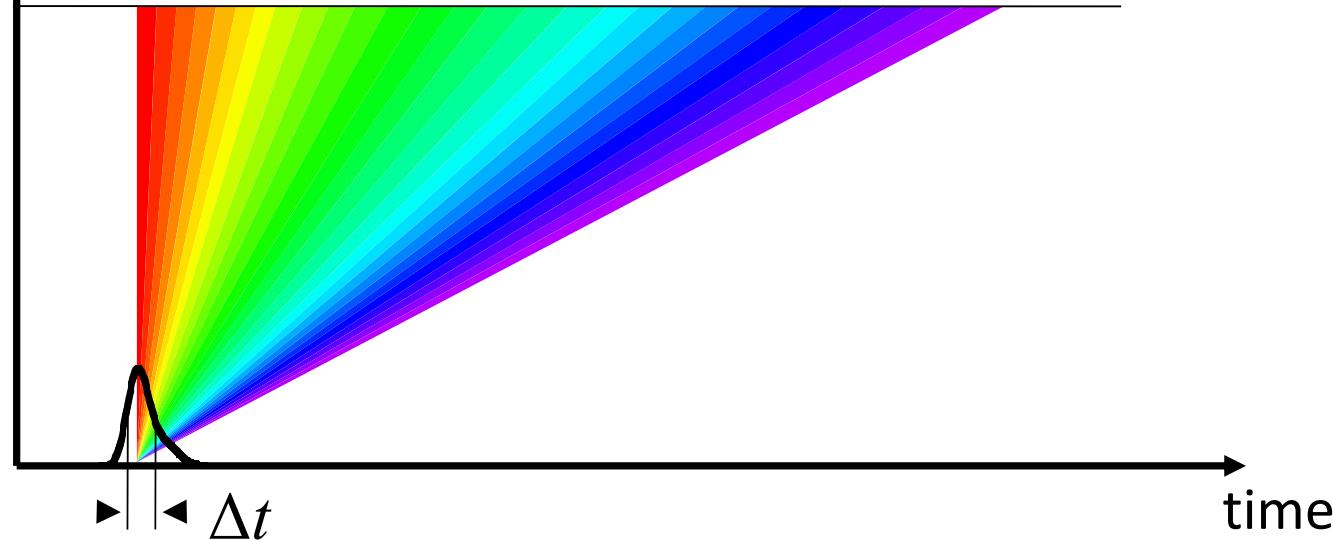


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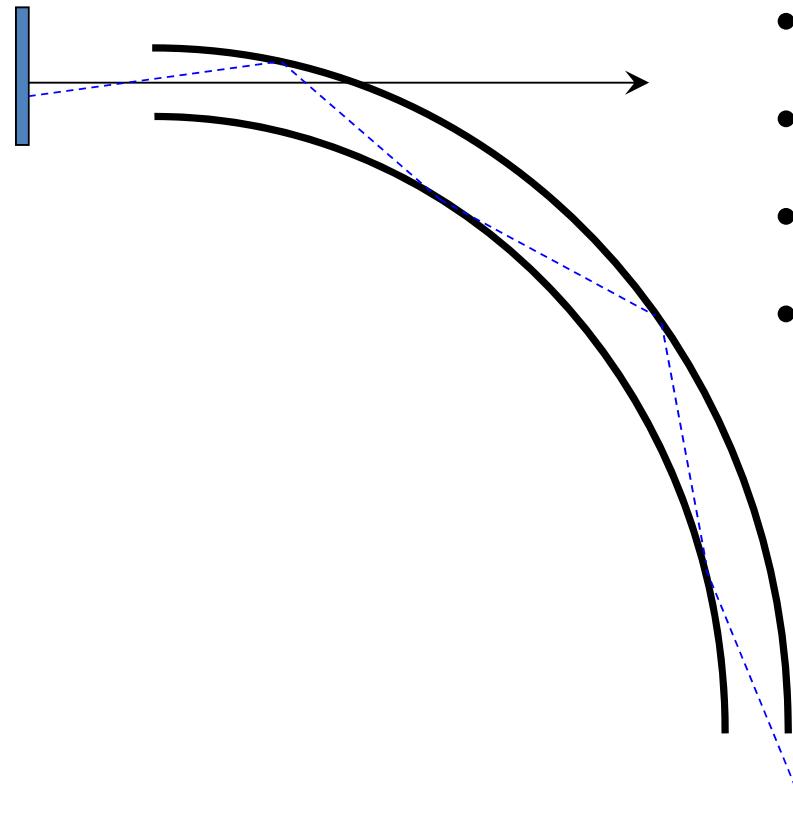
distance

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

$$\Rightarrow \Delta\lambda[\text{\AA}] = \Delta t[\text{ms}] \times 3.956 / L[\text{m}]$$

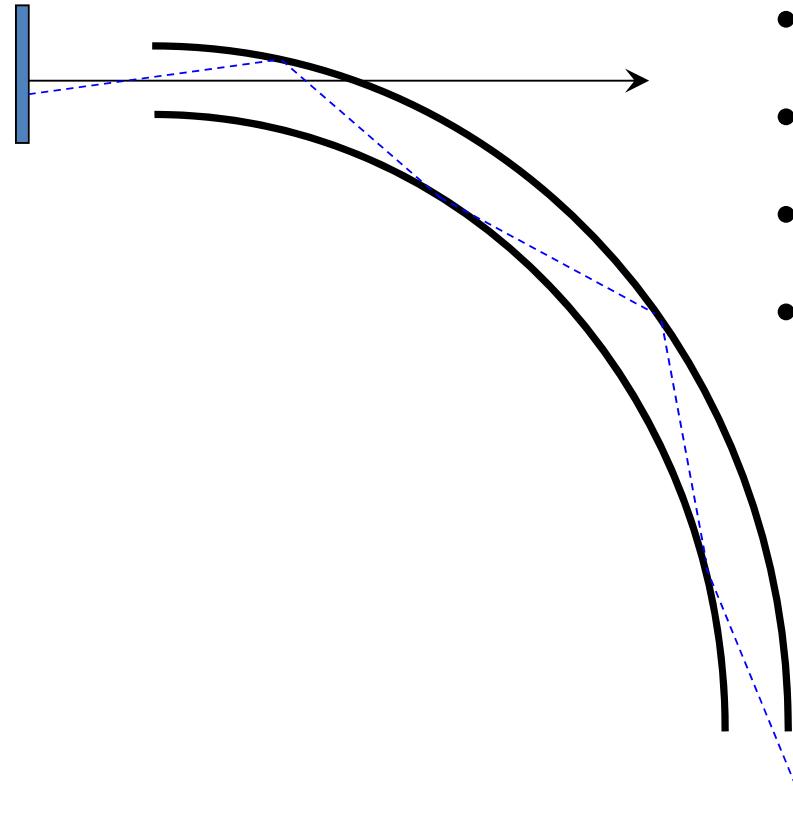


Curved guides



- Avoid direct line-of-sight
- Avoid gammas
- Avoid fast neutrons
- Reduce background

Curved guides

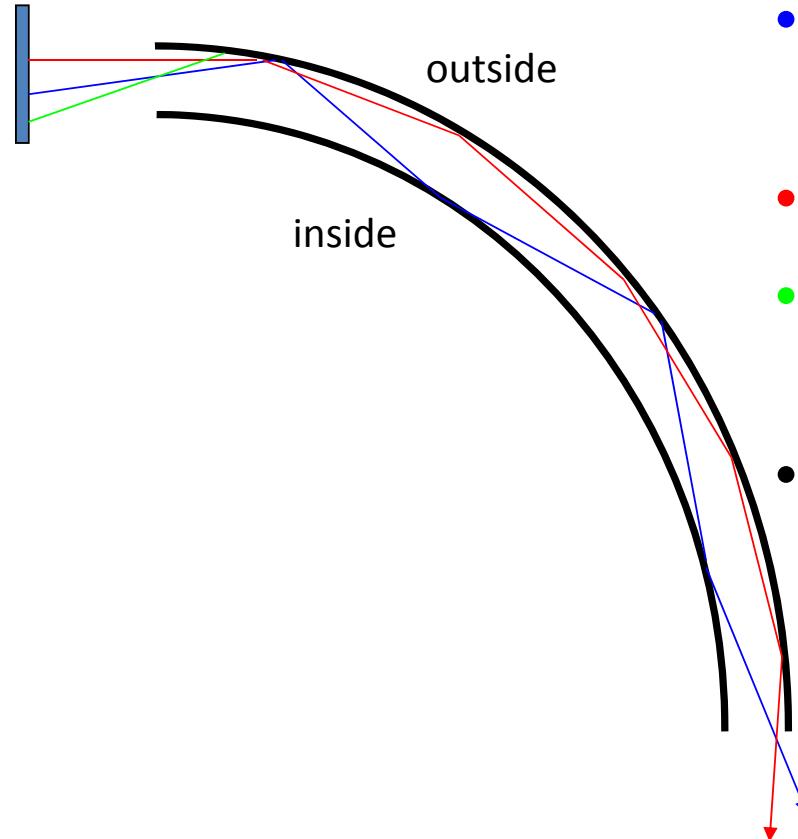


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Shorter, with multiple channels typically denoted “bender” - also used in polarisation-settings

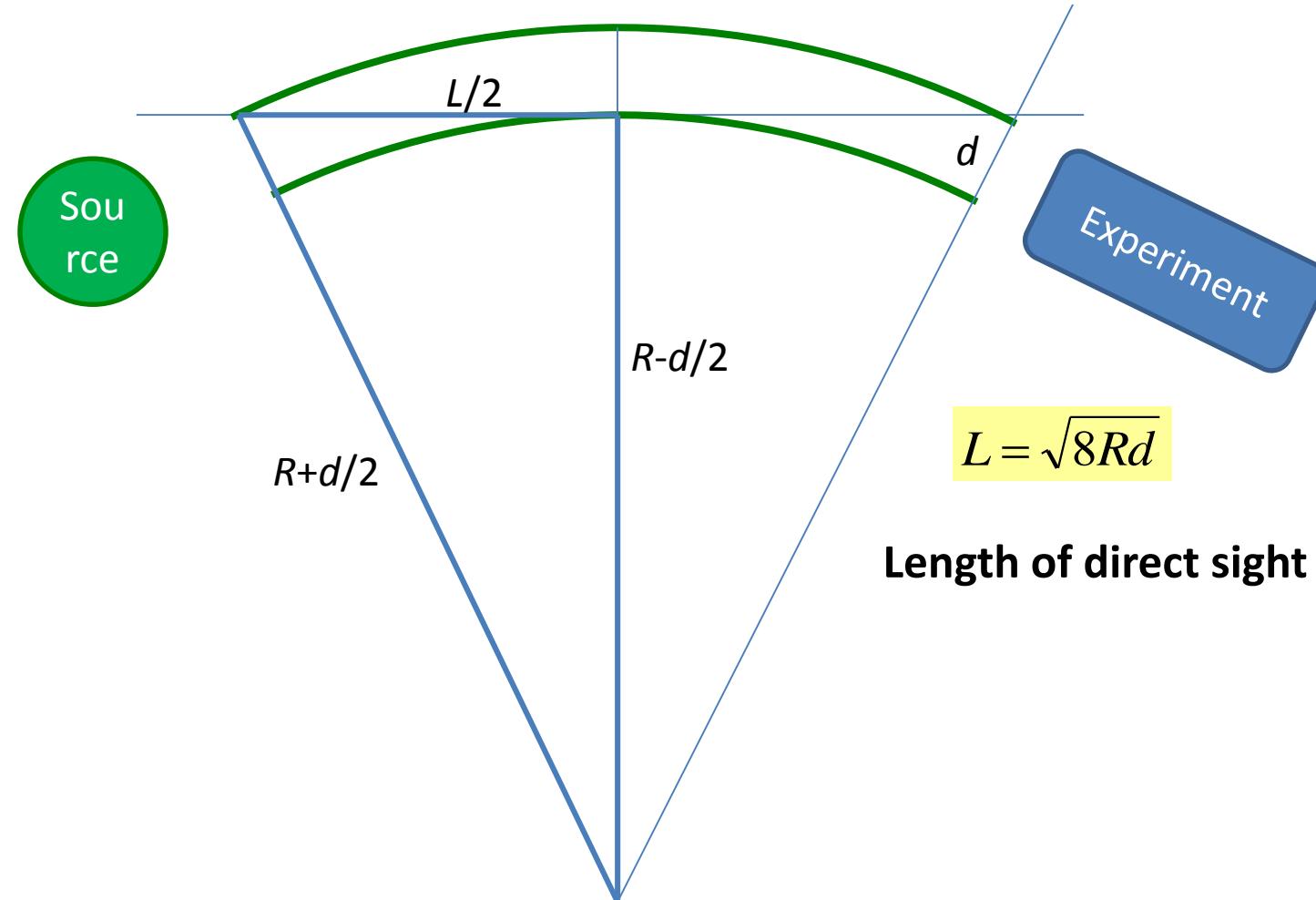


Curved guides



- **Blue** – reflecting from both sides
- **Red** – garland reflections
- **Green** – exceeds critical angle
- Fewer neutrons along inside face - quantify

Curved guides - 1



Length of direct sight

Curved guides - 2

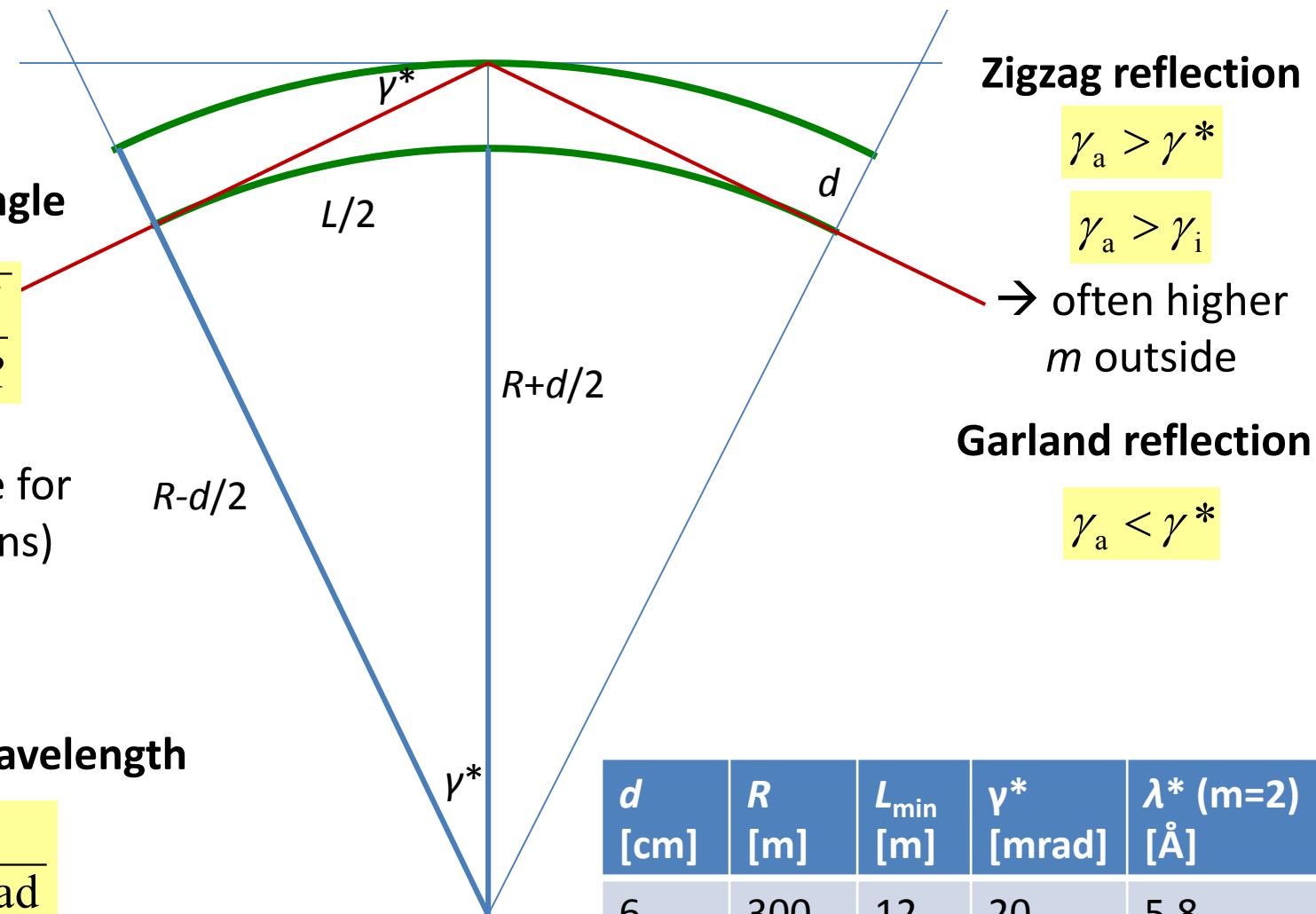
Characteristic angle

$$\gamma^* = \frac{L}{2R} = \sqrt{2 \frac{d}{R}}$$

(Maximum angle for
garland reflections)

Characteristic wavelength

$$\lambda^* = \frac{\gamma^*}{m \cdot 1.73 \frac{\text{mrad}}{\text{\AA}}}$$



d [cm]	R [m]	L_{\min} [m]	γ^* [mrad]	$\lambda^* (m=2)$ [\text{\AA}]
6	300	12	20	5.8
6	3000	38	6	1.8
9	4000	54	7	1.9

Curved guides - 3

Characteristic angle

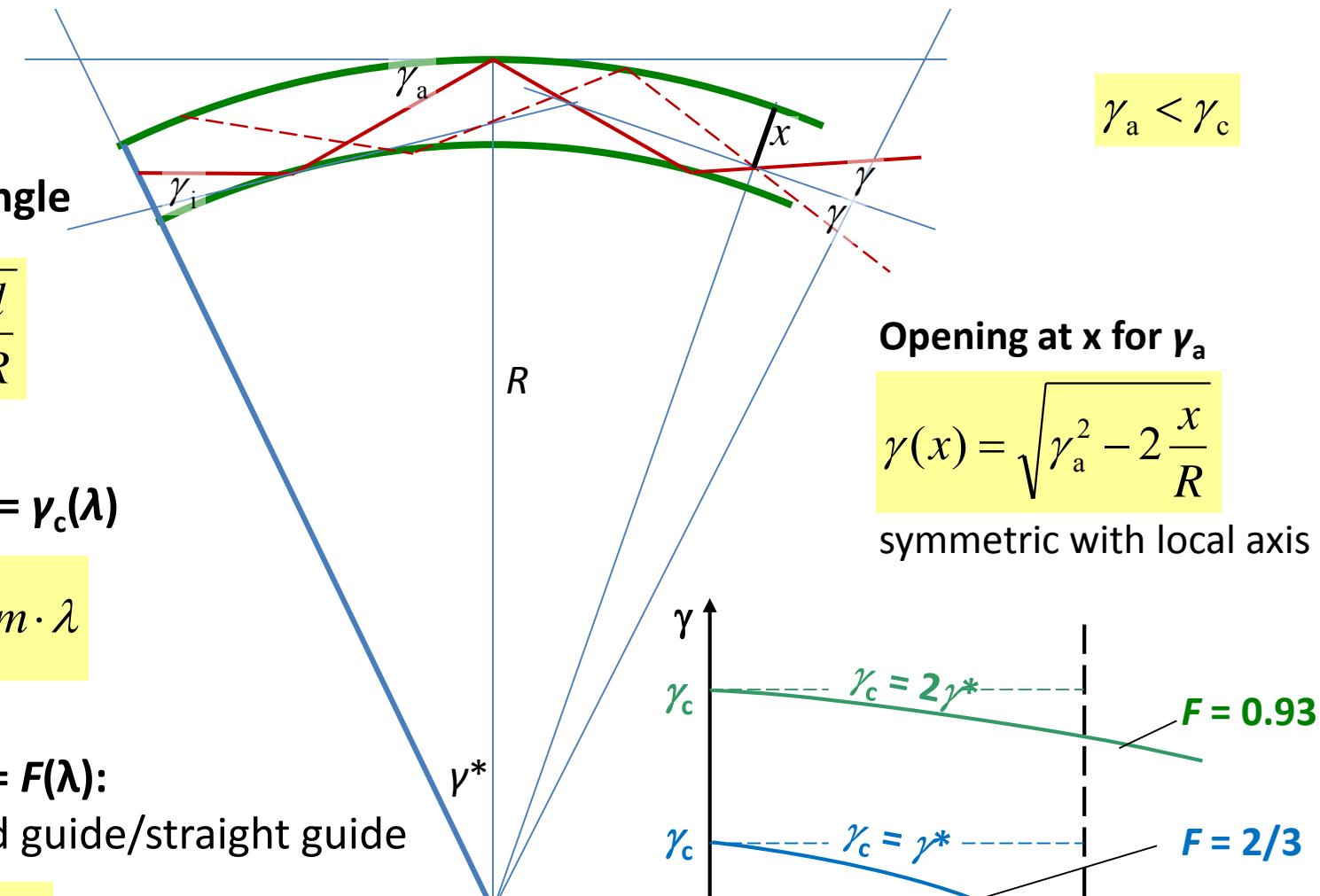
$$\gamma^* = \frac{L}{2R} = \sqrt{2 \frac{d}{R}}$$

Critical angle $\gamma_c = \gamma_c(\lambda)$

$$\gamma_c = 1.73 \frac{\text{mrad}}{\text{\AA}} m \cdot \lambda$$

Filling factor $F = F(\lambda)$:
Intensity curved guide/straight guide

$$F = \frac{1}{d\gamma_c} \int_0^{d_c} \gamma(x) dx$$



Curved guides - 3

Characteristic angle

$$\gamma^* = \frac{L}{2R} = \sqrt{2 \frac{d}{R}}$$

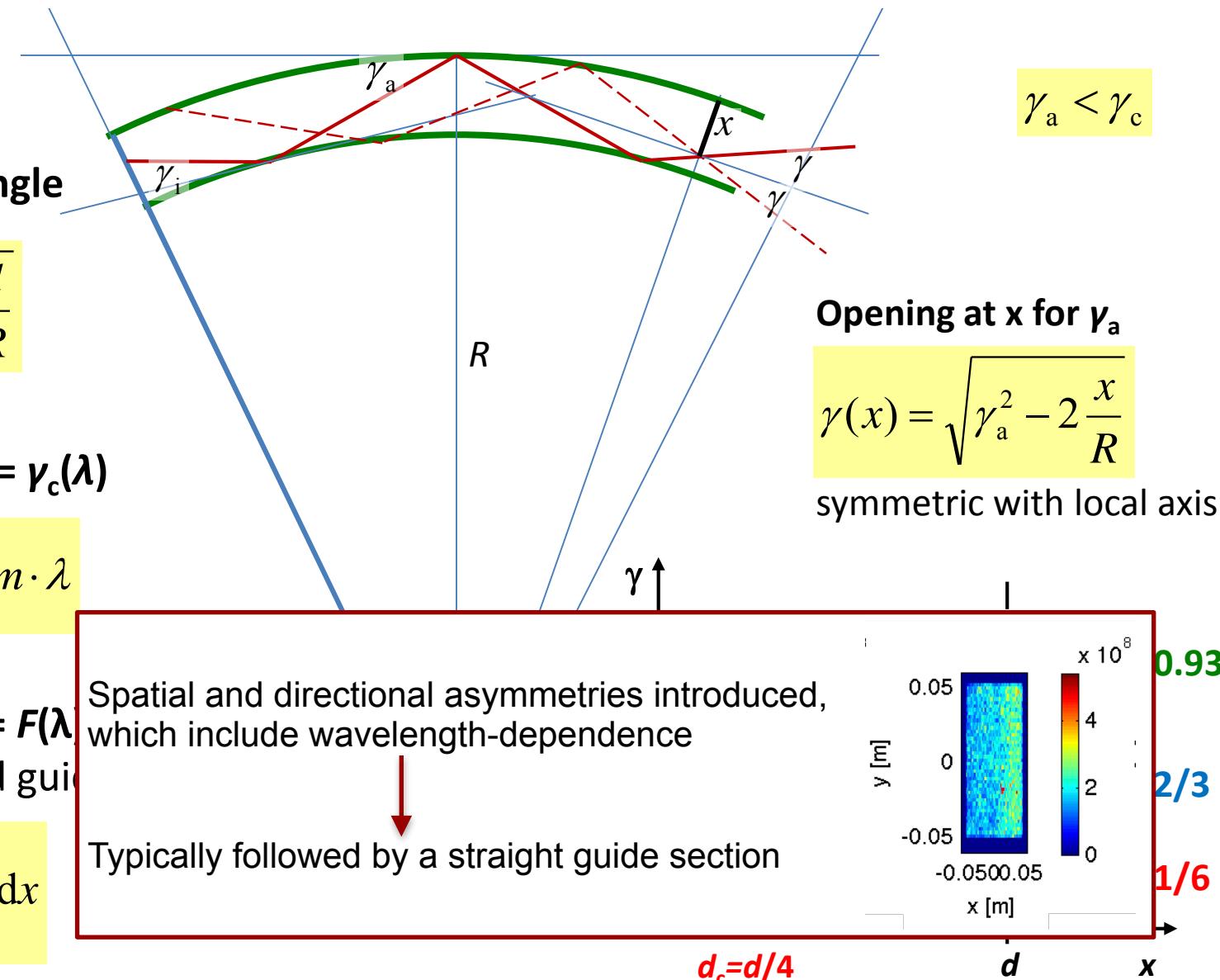
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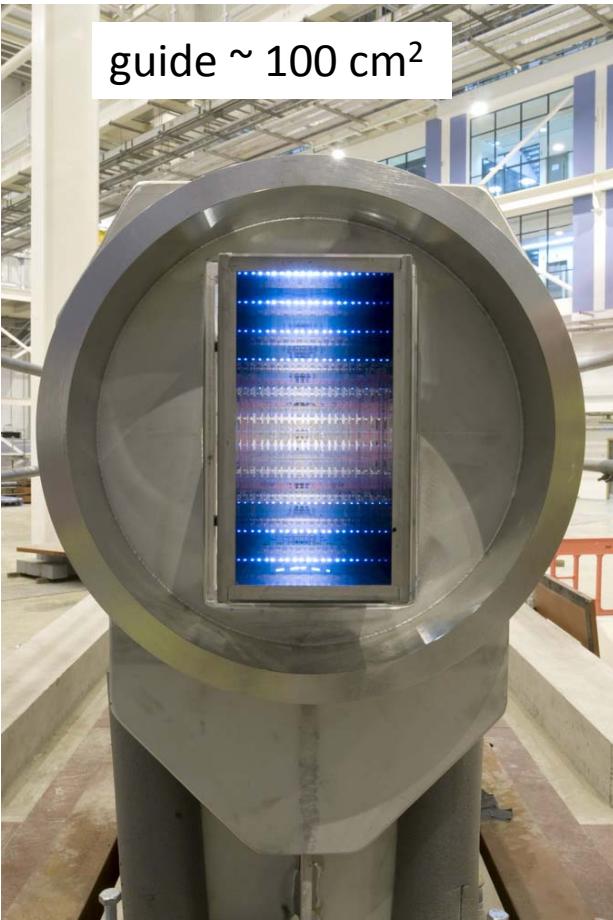
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Focusing

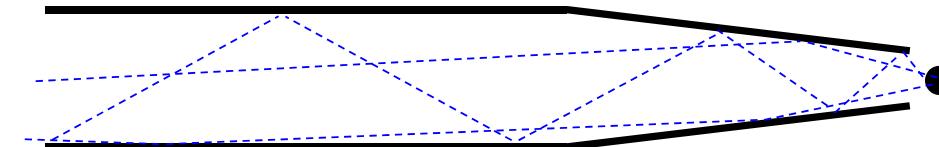


samples < 1 cm²

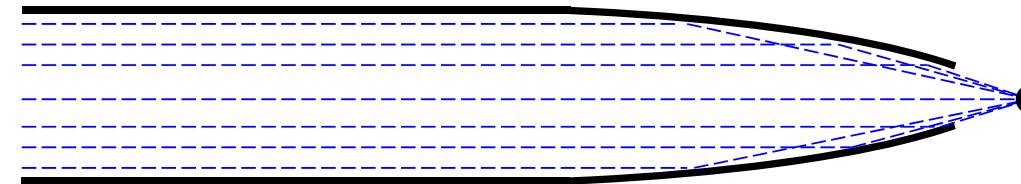


Focusing

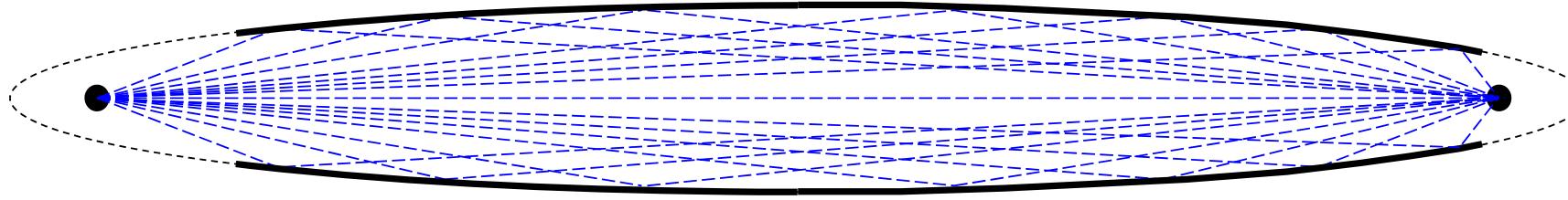
Linear tapering



Parabolic

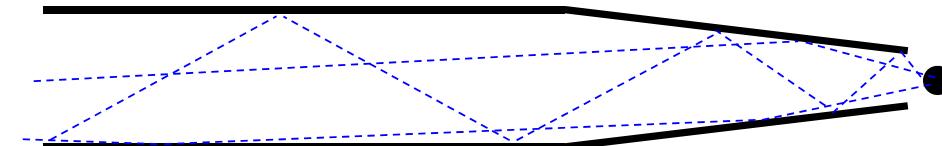


Elliptic

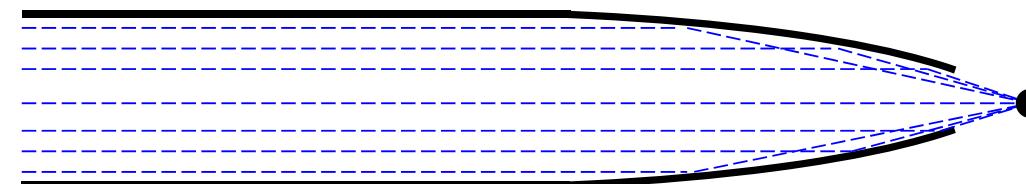


Focusing

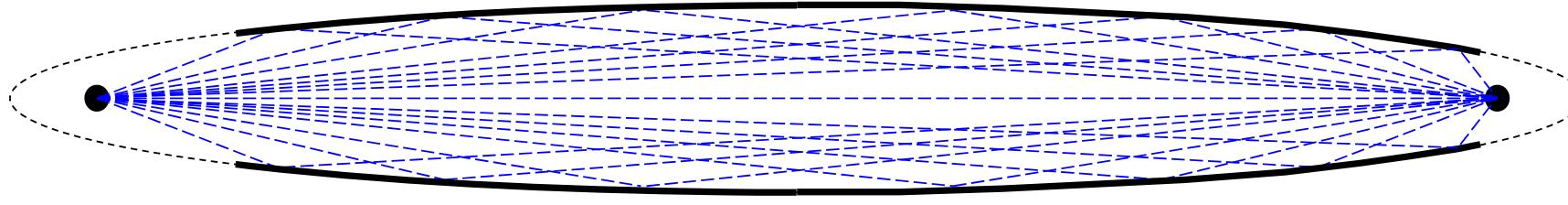
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Parabolic



Elliptic

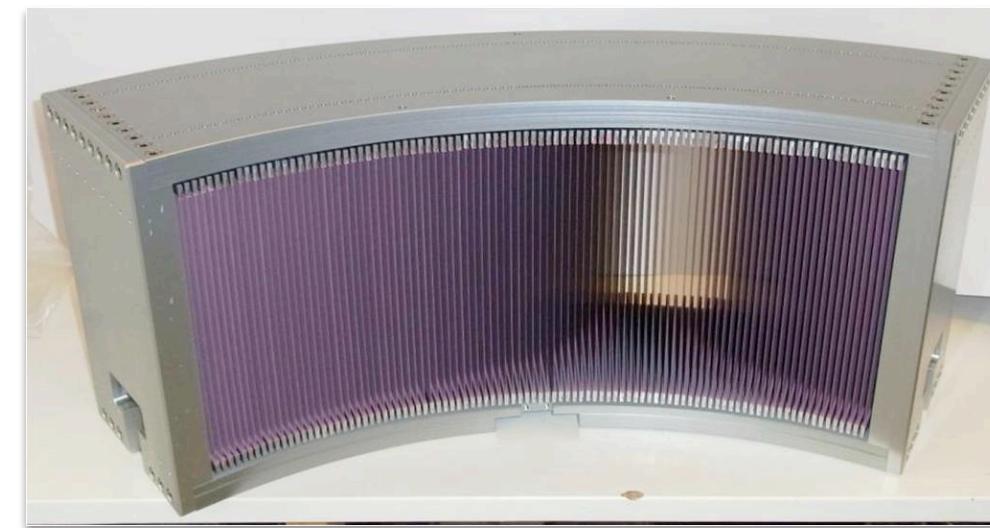


Liouville's theorem: more intensity => more divergence

Spatial, directional beam tailoring



Slit: (configurable) opening in absorbing material

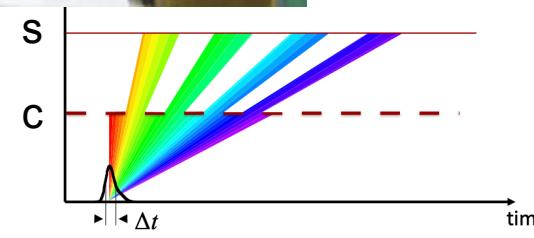
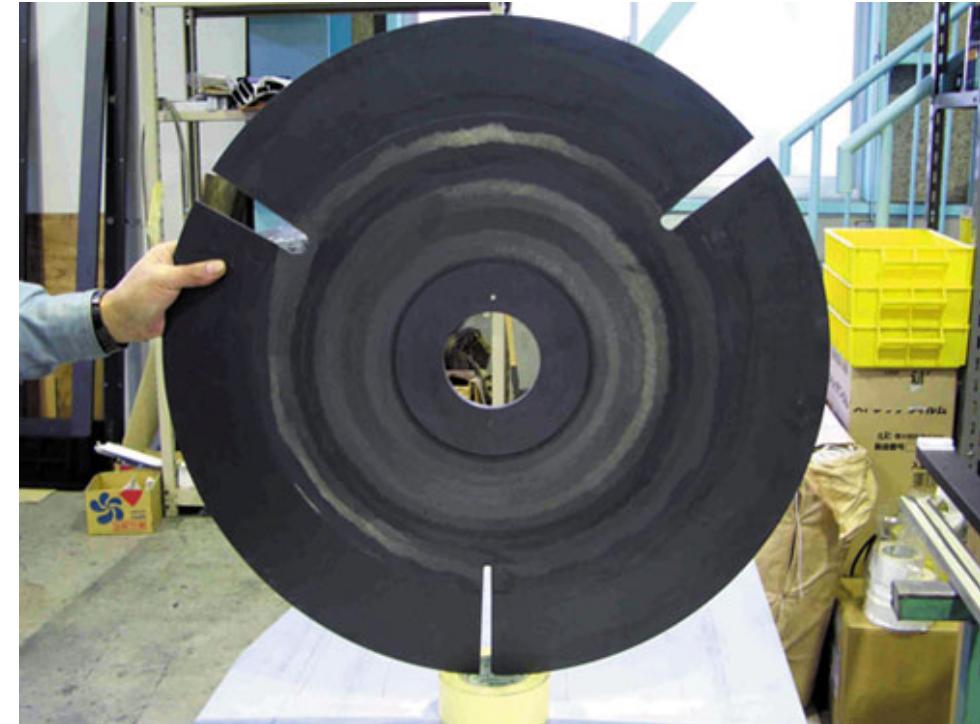
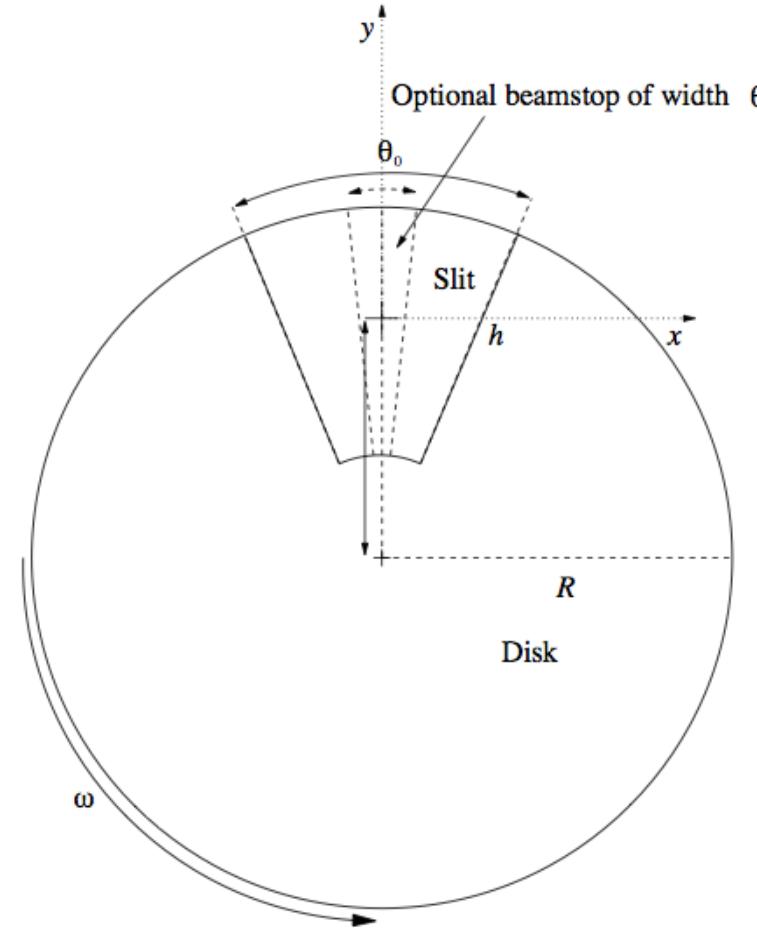


Collimators: Limiting divergence by sheets of absorbing material

Disk Chopper - a “rotating slit”

1ST
2019
BILBAO
NEUTRON
SCHOOL
SCIENCE AND INSTRUMENTATION FOR
COMPACT ACCELERATOR-DRIVEN
NEUTRON SOURCES (CANS)
JUNE, 17, 18, 19

More information and registration:
www.BNS2019.org



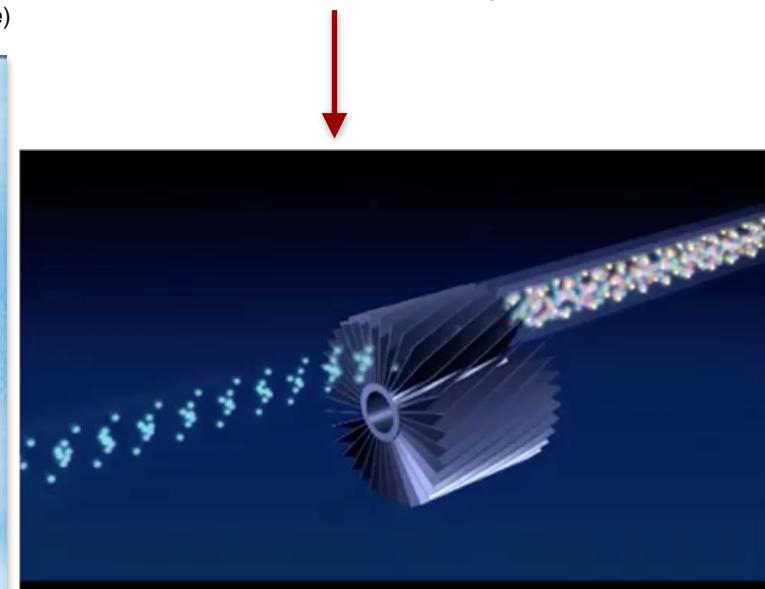
- • Spatio-temporal tailoring of the beam
- A cascade of these will introduce spectral filtering

Velocity selector

- A sort of “rotating collimator”



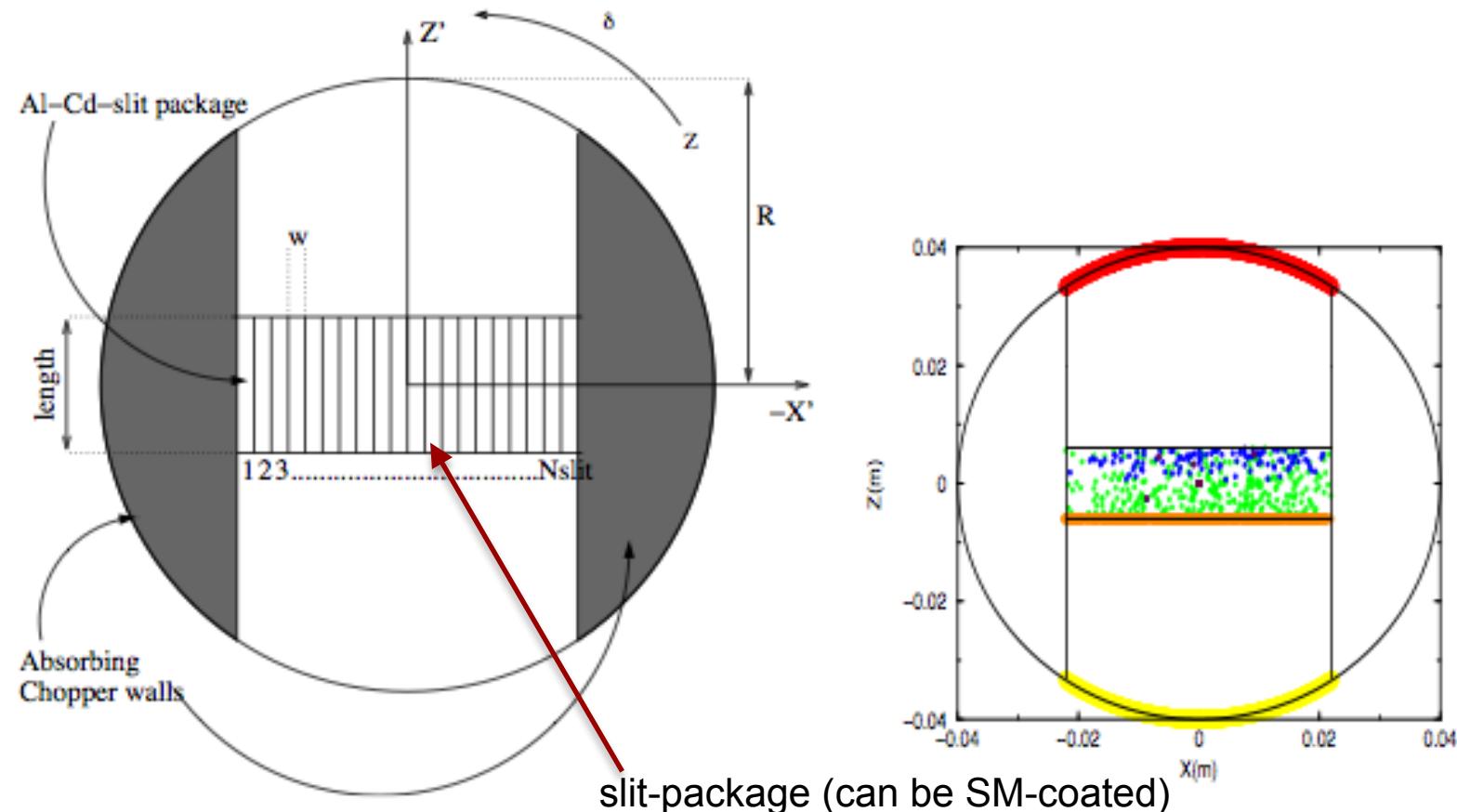
ILL - SANS instrument D11: a "microscope" for exploring the nanoworld
(YouTube)



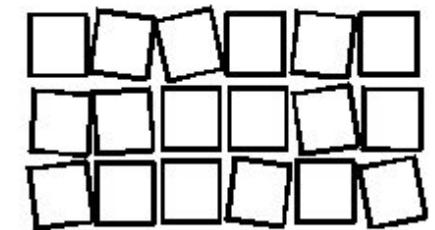
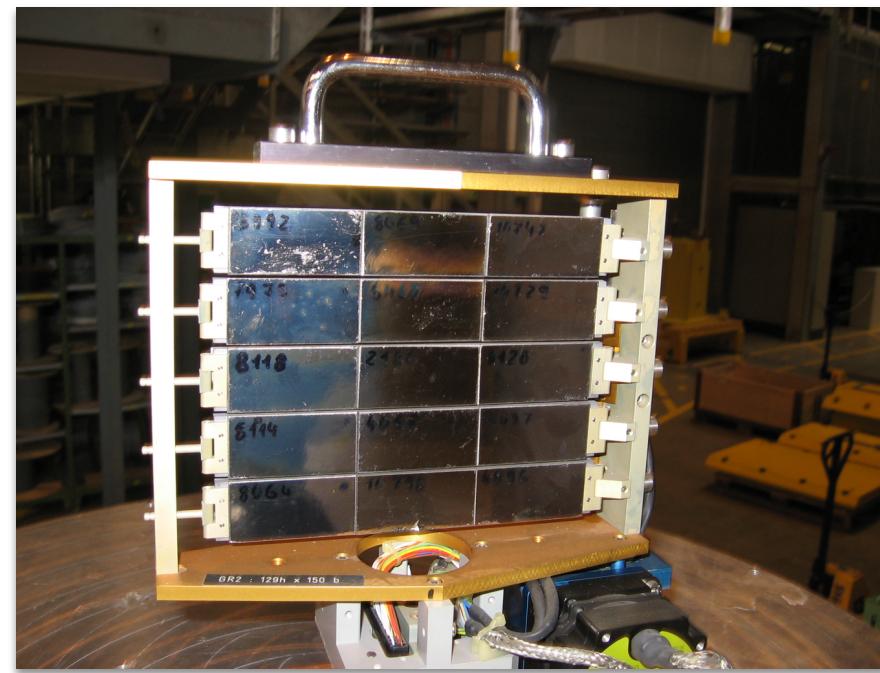
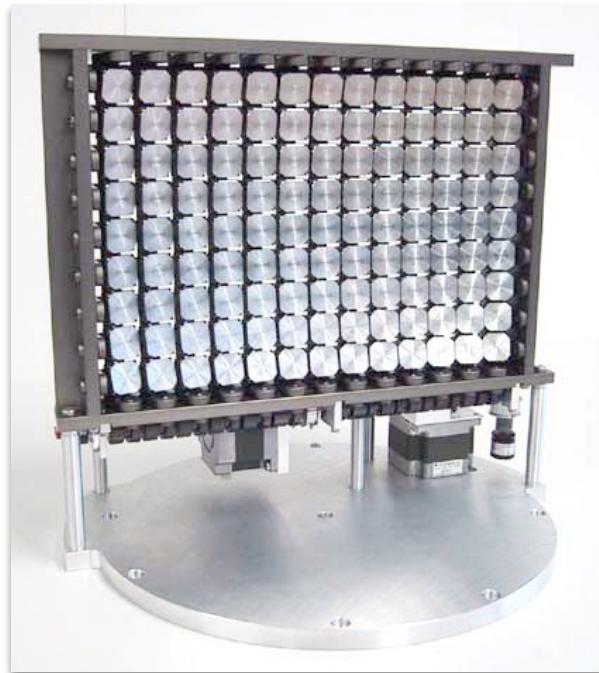
- Spectral tailoring of the beam

$$\frac{\delta\lambda}{\lambda} \approx 10\%$$

Fermi Choppers



- Spectro-temporal tailoring of the beam, quite monochromatic $\frac{\delta\lambda}{\lambda} \approx 1\%$



Often far from being perfect single crystals

Various means of increasing crystal **mosaicity** are used

- impacts divergence
- focusing geometries

- Spectral tailoring via Bragg's law $n\lambda = 2d \sin(\theta)$

$$\frac{\delta\lambda}{\lambda} \stackrel{PG}{\approx} 3\%$$

Filters...

- Removal of **fast neutrons or gammas**
- Suppression of **higher-order contributions λ/n** of monochromator

$$\sigma = \sigma_{\text{abs}} + \sigma_{\text{tds}} + \sigma_{\text{bragg}}$$

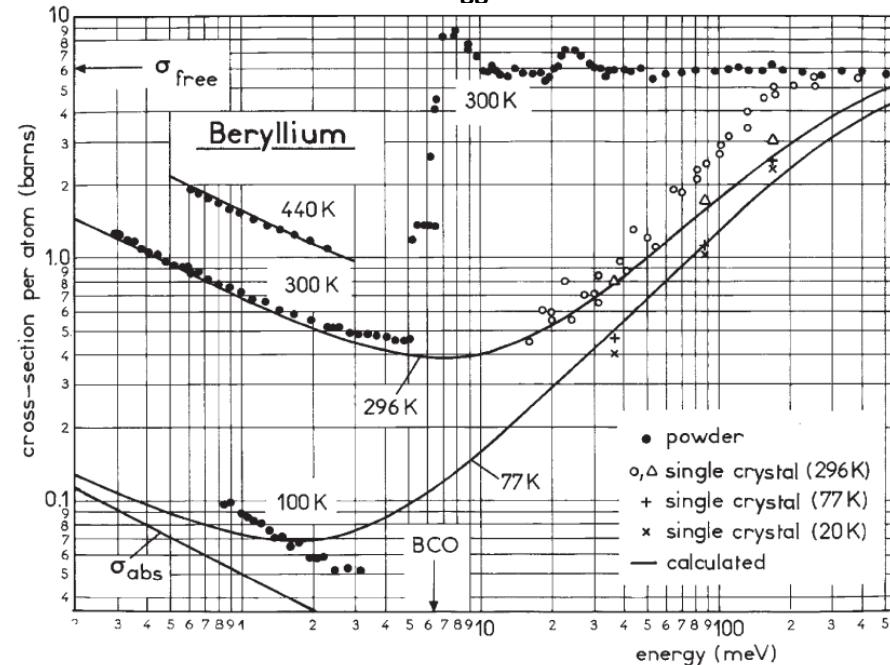
Freund, NIM 213 495 (1983)

σ_{abs} Absorption cross section ($1/v$ at low energy)

σ_{tds} Thermal diffusive cross section (single phonon at low, multi phonon at high energy)

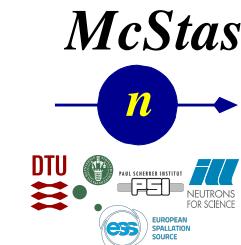
σ_{bragg} Bragg scattering in single or polycrystals

$$\sigma_{\text{bragg}} = 0 \text{ for } \lambda > 2d_{\text{max}}$$

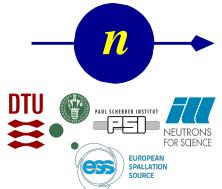


Techniques for neutron-optical design

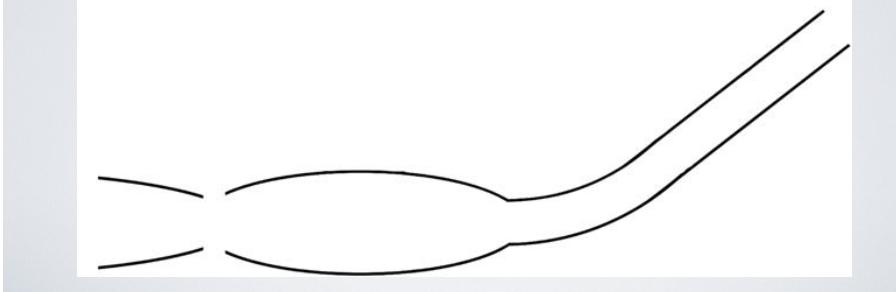
- Back of the envelope
- Acceptance diagrams
- Monte Carlo (ray-tracing) simulations
 - McStas
 - Vitess
 - SimRes
- Always apply sanity checks
 - Liouville's theorem
 - check Brilliance Transfer: variation of calculated flux: $\int \Psi dA d\Omega$
 - check divergence and beam profiles
- Optimisation package
 - Guide_bot



McStas



"I want a feeder system consisting of a parabola and a gap for the chopper followed by an elliptic guide"



Mads Bertelsen, Copenhagen Univ.

https://github.com/mads-bertelsen/guide_bot

CAMEA P G E S K S E

```
P
G(start=6.5,length=0.1)
E(maxStartWidth=0.030)
S(minlength=1)
K(minstart=66,maxstart=96,maxlength=2.5)
S(minlength=1,maxlength=12)
E
```

Demands

Sample size H	15mm
Sample size V	15mm
Divergence H	$\pm 0.75^\circ$
Divergence V	$\pm 1.00^\circ$
Wavelength	1.65 - 6.40 Å
Length	170m
Sample - guide	60cm

Literature

- G. L. Squires: Introduction to the Theory of Thermal Neutron Scattering, Cambridge
Great all-round resource for understanding neutron-matter interaction
- V. F. Sears: Neutron Optics, Oxford University Press 1989
Theoretically rigorous, not so many applications
- M Utsuro & V. K. Ignatovich, Handbook of Neutron Optics, Wiley-VCH 2010
Very complete with many applications
- R. Golub et al, Ultra-Cold Neutrons, IOP Publishing 1991
Introduction interactions neutrons – matter
- A.-J. Dianoux & G. Lander (eds.), Neutron Data Booklet, Institut Laue Langevin 2003
Compact overview and formulas
Download:
https://www.ill.eu/fileadmin/user_upload/ILL/1_About_ILL/Documentation/NeutronDataBooklet.pdf

Thank you!

- Questions?
- McStas talk and demo after the coffee break...