# Thermal Neutron Scattering Data 

 for the Moderator Materials $\mathrm{H}_{2} \mathrm{O}, \mathrm{D}_{2} \mathrm{O}$ and $\mathrm{ZrH}_{x}$
## in ENDF-6 Format and as ACE Library for MCNP(X) Codes

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#### Abstract

At thermal neutron energies, the binding of the scattering nucleus in a solid, liquid, or gas affects the cross section and the angular and energy distributions of the scattered neutrons. These effects are described in the thermal sub-library of evaluated files in File 7 of the ENDF6 format.

New and re-evaluations are described for the three thermal moderator materials: hydrogen bound in light water $\left(\mathrm{H}_{2} \mathrm{O}\right)$, deuterium bound in heavy water $\left(\mathrm{D}_{2} \mathrm{O}\right)$ and hydrogen in ZrH (zirconium hydride). The calculations for a variety of temperatures were made with the LEAPR module of NJOY to obtain new evaluated thermal neutron scattering files that are accurate over a wider range of energy and momentum transfer than existing files. The IKE physics models are described in detail, and the inputs to LEAPR are given.

Detailed comparisons with a significant number of measurements of differential and integral neutron cross sections and other relevant data are reported (for the validation of the generated Scattering Law data files $S(\alpha, \beta, T)$ ). Experimental data are reproduced reasonably well. In addition, thermal MCNP data sets for use in the continuous Monte Carlo codes MCNP and MCNPX were generated from these evaluations. Calculated neutron spectra agree rather well with measurements.

Details are also given of the updates to the NJOY modules LEAPR, THERMR, and ACER necessary in generating and processing the thermal neutron scattering data.


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## 1 Introduction

At thermal neutron energies, the binding of the scattering nucleus in a solid, liquid or gas moderator material affects the neutron cross section and the energy and angular distribution of secondary neutrons, as the neutron can give up energy to excitations in the material, or can gain energy. In the evaluated nuclear data files (ENDF/B-VI and JEF-2.2), these effects are described in the thermal sub-library using the File 7 format [1].

Status of Thermal Neutron Scattering Data

- The majority of the work on thermal neutron scattering was performed in the 1950s and 1960s
- Subsequently (1970s), data libraries were generated mainly using the GASKET methodology, which was developed at General Atomic (GA) [2, 3]
- New ENDF files that are accurate over a wider range of energy and momentum transfer than the existing files were calculated with the scattering law module LEAPR [4] (implemented in the early 1990s in the nuclear data processing system NJOY [5]) which uses methods based on the British code LEAP, together with the original GA physics models
- To date, this is the only set of data that is available as part of the ENDF/B-VI libraries
- In JEF-2.2 the Scattering Law data are carried over from JEF-1 (1984) [6]. The models for the neutron scattering of H in light water $\mathrm{H}_{2} \mathrm{O}$ and D in heavy water $\mathrm{D}_{2} \mathrm{O}$ were developed at IKE and the generation of $S(\alpha, \beta)$ was done using GASKET-2.
- The LANL report on LEAPR [4] shows that there is reasonable agreement between LEAPR and GASKET results. This is also the experience at IKE after modifications of the code GASKET-2.
- Work started at IKE in 2003 to re-evaluate and review the currently used thermal scattering law data $\mathrm{S}(\alpha, \beta)$ for $\mathrm{H}_{2} \mathrm{O}, \mathrm{D}_{2} \mathrm{O}$ and $\mathrm{ZrH}_{\mathrm{x}}$ as a function of temperatures equal to or greater than room temperature. This includes updating of models and model parameters, using LEAPR for the generation of $S(\alpha, \beta)$ with more details in the $\alpha$ and $\beta$ grids and higher energy transfer, the processing of the generated $S(\alpha, \beta)$ with NJOY and the comparison with measurements for differential and integral thermal neutron cross sections as well as neutron spectra.
- Documentation of the results was completed in 2004.

Newer experimental data for the verification of generated thermal neutron scattering data are very scarce. Measurements were mainly done in the 1960s, especially at GA and RPI in USA, in UK and in Germany in the research centres of Jülich and Karlsruhe.

## 2 Thermal Neutron Scattering

The thermal neutron scattering cross section is usually divided into three different parts:

- Inelastic: Important for all materials (both incoherent and coherent inelastic fall in this category) and described by the scattering law $\mathrm{S}(\alpha, \beta)$.
- Incoherent elastic: Important for hydrogenous solids like zirconium hydride and polyethylene or light water ice.
- Coherent elastic: Important for crystalline solids like graphite, beryllium or $\mathrm{UO}_{2}$.


### 2.1 Inelastic Scattering

In standard references or in the LEAPR documentation in [4] it is shown that the double differential scattering cross section for thermal neutrons for gases, liquids, or solids consisting of randomly ordered atoms or molecules (principle scatterer) can be written as

$$
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}\left(E \rightarrow E^{\prime}, \Omega \rightarrow \Omega^{\prime}\right)=\frac{\sigma_{b}}{4 \pi k T} \sqrt{\frac{E^{\prime}}{E}} e^{-\frac{\beta}{2}} S(\alpha, \beta)
$$

where $E$ and $E$ ' are the incident and secondary neutron energies in the laboratory system, $\Omega$ is the scattering angle in the laboratory system, $\sigma_{\mathrm{b}}$ is the bound scattering cross section for the material (sum of $\sigma_{\mathrm{b}}{ }^{\text {inc }}+\sigma_{\mathrm{b}}{ }^{\text {coh }}$ ), $k T$ is the temperature in eV , and $\mathrm{S}(\alpha, \beta)$ is the symmetric form of the thermal scattering law. The scattering law depends on only two variables:

- the momentum transfer $\kappa$

$$
\alpha=\frac{E^{\prime}+E-2 \sqrt{E^{\prime} E} \cos \theta}{A k T}=\frac{\hbar^{2} \kappa^{2}}{2 M k T}
$$

where A is the ratio of the mass of the scattering atom M to the neutron mass, and

- the energy transfer $\varepsilon$

$$
\beta=\frac{E^{\prime}-E}{k T}=\frac{\varepsilon}{k T}
$$

The asymmetric scattering law is the Fourier transform of the intermediate scattering function

$$
e^{-\frac{\beta}{2}} S(\alpha, \beta)=\frac{1}{2 \pi \hbar} \int_{-\infty}^{\infty} e^{\frac{i \varepsilon t}{\hbar}} \chi(\kappa, t) d t
$$

For a medium with isotropic neutron scattering this scattering function is given in the Gaussian approximation by

$$
\chi(\kappa, t)=\exp \left[\frac{\hbar^{2} \kappa^{2}}{2 M}\left(-\int_{0}^{\infty} \frac{\rho(\omega)}{\hbar \omega} \operatorname{coth} \frac{\hbar \omega}{2 k T} d \omega+\int_{0}^{\infty} \frac{\rho(\omega)}{\hbar \omega}\left(\operatorname{coth} \frac{\hbar \omega}{2 k T} \cos \omega t+i \sin \omega t\right) d \omega\right)\right]
$$

with $\rho(\omega)$ as the generalised frequency spectrum of excitations in the system expressed as a function of $\omega$, where the spectrum must be normalised as follows

$$
\int \rho(\omega) \mathrm{d} \omega=1
$$

The frequency spectrum is decomposed into a sum of simple excitation spectra

$$
\rho(\omega)=\sum_{i=1}^{K} \rho_{i}(\omega)
$$

where the following possibilities are allowed in the codes LEAPR and GASKET:

$$
\begin{array}{ll}
\rho_{\mathrm{i}}(\omega)=\rho_{\mathrm{t}}(\omega)=\mathrm{w}_{\mathrm{t}} \delta(\omega) & \text { translational spectrum } \\
\rho_{\mathrm{i}}(\omega)=\rho_{\mathrm{d}}(\omega) & \text { diffusion } \\
\rho_{\mathrm{i}}(\omega)=\rho_{\mathrm{s}}(\omega) & \text { solid-type spectrum (broad band of } \omega \text { ) } \\
\rho_{\mathrm{i}}(\omega)=\mathrm{w}_{\mathrm{j}} \delta\left(\omega-\omega_{\mathrm{j}}\right) & \text { discrete oscillator at } \omega_{\mathrm{j}}
\end{array}
$$

The solid-type spectrum must vary as $\omega^{2}$ as $\omega$ goes to zero, and it must integrate to $\mathrm{w}_{\mathrm{s}}$ the weight for the solid-type law. The translational spectrum must integrate to $\mathrm{w}_{\mathrm{t}}$, the translational weight.
In this formalism the inelastic scattering of thermal neutrons is treated in incoherent approximation as the bound scattering cross section is taken as the sum of $\sigma_{\mathrm{b}}{ }^{\text {inc }}+\sigma_{\mathrm{b}}{ }^{\text {coh }}$. To take into consideration the coherent inelastic neutron scattering in liquids see section 2.5.

### 2.2 Short-Collision-Time Approximation

For high incident energies, $\alpha$ and/or $\beta$ values may be required that are outside the ranges of tabulated $S(\alpha, \beta)$. In these cases, the short-collision-time (SCT) approximation should be used for the deep inelastic scattering of thermal neutrons as follows:

$$
S^{S C T}(\alpha, \beta, T)=\frac{e^{-\left[\frac{(\alpha-|\beta|)^{2} T}{4 \alpha T_{e f f}(T)}+\frac{|\beta|}{2}\right]}}{\sqrt{4 \pi \alpha \frac{T_{e f f}(T)}{T}}}
$$

where the effective temperature $\mathrm{T}_{\text {eff }}(\mathrm{T})$ is correlated to the generalised frequency distribution $\rho(\omega)$ according to

$$
T_{e f f}=\frac{1}{2} \int_{0}^{\omega_{\max }} \hbar \omega \operatorname{coth} \frac{\hbar \omega}{2 k T} \rho(\omega) d \omega
$$

It is important to mention that the principle of detailed balance is forced in the formula for the short-collision-time approximation (SCTA).

### 2.3 Incoherent Elastic Scattering

In hydrogenous solids there is an elastic (no energy change) component of scattering. In ENDF terminology this is called the "incoherent elastic" term.

The corresponding differential scattering cross section is

$$
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\frac{\sigma_{b}}{4 \pi} \exp \left(-2 E(1-\mu) \frac{\gamma(0)}{A}\right) \delta(\mu) \delta\left(E-E^{\prime}\right)
$$

and the integrated scattering cross section is

$$
\sigma(E)=\frac{\sigma_{b} A}{4 E \gamma(0)}\left[1-\exp \left(-4 E \frac{\gamma(0)}{A}\right)\right]
$$

where the Debye-Waller integral $\gamma(0)$ is computed from the frequency spectrum as

$$
\gamma(0)=\int_{0}^{\omega_{\max }} \frac{\rho(\omega)}{\hbar \omega} \operatorname{coth} \frac{\hbar \omega}{2 k T} d \omega
$$

The bound scattering cross section $\sigma_{\mathrm{b}}$ and the Debye-Waller integral divided by the reduced atomic mass $(=\gamma(0) / \mathrm{A})$ are given as a function of temperature in the section MF=7 and MT=2 in the ENDF-6 format.

### 2.4 Coherent Elastic Scattering

In solids consisting of coherent scatterers, interference scattering from the various planes of atoms of the crystals making up the solid occurs. The ENDF term for this process is "coherent elastic scattering" as there is no energy loss.
The differential coherent elastic scattering cross section for polycrystalline materials is given by

$$
\frac{d^{2} \sigma_{c o h}}{d \Omega d E^{\prime}}\left(E \rightarrow E^{\prime}, \Omega \rightarrow \Omega^{\prime}\right)=\frac{\sigma_{b}}{E} \sum_{E_{i}<E} f_{i} e^{-4 \gamma(0) E_{i} / A} \delta\left(\mu-\mu_{i}\right) \delta\left(E-E^{\prime}\right)
$$

where

$$
\mu_{i}=1-\frac{2 E_{i}}{E}
$$

and the integrated cross section is given by

$$
\sigma_{c o h}=\frac{\sigma_{b}}{E} \sum_{E_{i}<E} f_{i} e^{-4 \gamma(0) E_{i} / A}
$$

$\sigma_{\mathrm{b}}$ is the effective bound coherent scattering cross section for the material, $\mathrm{E}_{\mathrm{i}}$ are the so-called "Bragg edges", and the $f_{i}$ are related to the crystallographic structure factors.

The coherent elastic cross section is zero below the first Bragg edge $\mathrm{E}_{1}$ (typically 2 to 5 meV ). The quantity $\mathrm{E}_{\mathrm{i}} \sigma_{\mathrm{coh}}\left(\mathrm{E}_{\mathrm{i}}\right)$ as a function of energy and temperature is stored in the ENDF-6 format in $\mathrm{MF}=7$ and $\mathrm{MT}=2$. The knowledge of the crystal structure of the scattering material
is necessary to calculate the $\mathrm{E}_{\mathrm{i}}$ and $\mathrm{f}_{\mathrm{i}}$. The methods used in LEAPR are taken over from HEXSCAT. Crystal structures other than hexagonal are added to LEAPR:

### 2.5 Intermolecular Neutron Interference Scattering in Liquids

In general the thermal neutron scattering can be formulated as

$$
S(\alpha, \beta)=\frac{\sigma^{i n c}}{\sigma} S^{\text {inc }}(\alpha, \beta)+\frac{\sigma^{c o h}}{\sigma} S^{c o h}(\alpha, \beta)
$$

with $\sigma=\sigma^{\text {inc }}+\sigma^{\text {coh }}$
According to the formalism by Van Hove [7] the incoherent scattering is a part of the self term $\mathrm{S}_{s}$ and the distinct coherent part $\mathrm{S}_{d}$ is caused by intermolecular interference scattering. In calculating $\mathrm{S}^{\mathrm{coh}}$ for liquids various approximations are in use:

- Incoherent approximation ( $\mathrm{S}_{d} \equiv 0$ )

$$
S^{\mathrm{coh}}(\alpha, \beta)=\mathrm{S}^{\mathrm{inc}}(\alpha, \beta)
$$

- Vineyard approximation [8]

$$
\mathrm{S}^{\mathrm{coh}}(\alpha, \beta)=\mathrm{S}^{\mathrm{inc}}(\alpha, \beta) * \mathrm{~S}(\kappa)
$$

with the static structure factor $S(\kappa)$

- Sköld approximation [9]

$$
S^{\mathrm{coh}}(\alpha, \beta)=\mathrm{S}^{\mathrm{inc}}\left(\alpha^{\prime}, \beta\right) * S(\kappa)
$$

with $\alpha^{\prime}=\alpha / S(\kappa)$
Among these approximations the back-scattering of the interacting molecule is best described by the Sköld approximation.

### 2.6 Free Gas

For a free gas of scatterers with no internal structure the frequency distribution degenerates into $\rho(\omega)=1 / \mathrm{A} * \delta(0)$ and the scattering law is then given by

$$
S(\alpha, \beta)=\frac{e^{-\frac{\alpha^{2}+\beta^{2}}{4 \alpha}}}{\sqrt{4 \pi \alpha}}
$$

## 3 Generation of $\mathbf{S}(\alpha, \beta)$ - Processing of Thermal Neutron Scattering Data

The LEAPR module of NJOY is used to prepare the scattering law $S(\alpha, \beta)$ and related quantities, which describe thermal neutron scattering from bound moderators, in the ENDF-6 format used by the THERMR module of NJOY.

| 8 <br> 9 <br> 9 <br> 0 <br> 1 <br>  <br> 2 <br> 2 | LEAPR | Generates thermal scattering data in ENDF format File 7 for various temperatures: <br> MT=2 coherent elastic or incoherent elastic MT=4 (incoherent inelastic): tabulated $S(\alpha, \beta)$ and $T_{\text {eff }}$ |
| :---: | :---: | :---: |
|  | THERMR | Calculates pointwise thermal scattering cross sections in PENDF format, File 3 (cross sections) and File 6 (double differential data): <br> $\mathrm{MT}=2+221$ free gas <br> for solid, liquid and gaseous moderators <br> MT>221 incoherent scattering XS <br> $\mathrm{MT}=\mathrm{MT}+1$ elastic scattering XS |
|  | ACER | Generates thermal scattering data for MCNP(X) code in ACE format |

Figure 3.1 NJOY modules for the generation of thermal neutron scattering data for application

For binding in solids and liquids, the scattering law $S(\alpha, \beta)$ is given in ENDF format File 7 as tables of $S$ versus $\alpha$ for various values of $\beta$. Values of $S$ for other values of $\alpha$ and $\beta$ can be obtained by interpolation. The scattering law is normally symmetric in $\beta$ and only has to be tabulated for positive values, but for materials like ortho-hydrogen and para-hydrogen of interest for cold moderators, this is not true as the principle of detailed balance is not fulfilled.

If the $\alpha$ or $\beta$ required is outside the range of the table in File 7 , the differential scattering cross section is computed in THERMR using the short-collision-time approximation (SCTA). The effective temperatures $T_{\text {eff }}$ needed are included in the data file.

THERMR expects the requested temperature T to be one of the temperatures included on the ENDF thermal file, or within a few degrees of that value. Intermediate temperatures should be obtained

- by interpolating between the resulting cross sections and not by interpolating $S(\alpha, \beta)$ or
- by running LEAPR for the wanted temperature $T$.


### 3.1 General Remarks on the Choice of the $\alpha, \beta$-grid for the Scattering Law Tables

The choice of the $\alpha, \beta$-grid has to be oriented towards the peak structure of the generalised frequency distribution $\rho(\omega)$ of the scattering nucleus to assure resonance scattering of the thermal neutrons in the system of bound moderator atoms. The $\alpha, \beta$-grid must consider multiphonon excitations of the individual peaks, the minima in $\rho(\omega)$ as well as combinations of the peaks of different modes because these are independent motions (known as sum of excitation levels).
In the past (JEF-1 or ENDF/B-III) the strong correlation of neutron energy transfer with excitation of the dynamical modes of individual scattering nucleus was not always considered.

The maximum values of the $\alpha, \beta$-grid were often treated somehow arbitrarily. In the theoretical formulation for the calculation of thermal neutron scattering cross sections the frequency distribution $\rho(\omega)$ is treated in the harmonic approximation. This implies that a parabolic potential is taken for the harmonic oscillators allowing excitations up to infinity. The realistic potential is unharmonic. Therefore possible excitations are limited by the binding energy $\mathrm{E}_{\mathrm{b}}$ of the scattering nucleus in the molecule or lattice structure. Therefore the maximum values in the $\alpha, \beta$-grid should be taken as

$$
\begin{aligned}
& \beta_{\max }=E_{b} / k T \\
& \alpha_{\text {max }}=4 \beta_{\text {max }} / A
\end{aligned}
$$

in a consistent set and the maximum energy transfer $\mathrm{E}_{\text {max }}$ is set equal to $\mathrm{E}_{\mathrm{b}}$ in the Scattering Law data tables. For greater energy transfers up to the thermal cut-off energy ( $\leq 5 \mathrm{eV}$ ) the neutron interaction is treated within the short collision time approximation (SCTA) as with a free gas of the scattering nuclei with the effective temperature $T_{\text {eff }}$. This extension of the $S(\alpha, \beta)$ tables is done in the THERMR module in NJOY in the calculation of the double differential neutron scattering cross sections.

Considering this procedure with $\mathrm{E}_{\mathrm{b}}$ compared to the older arbitrary treatment, one has the advantage of producing much smaller $S(\alpha, \beta)$ tables without any loss of accuracy. In addition we do not have to provide the $\alpha, \beta$-grid dense enough for large values of $\alpha$ and $\beta$ to avoid interpolation errors as is demonstrated for $\mathrm{d} \sigma / \mathrm{dE}$ for H bound in $\mathrm{H}_{2} \mathrm{O}$ in section 4.2 in Figure 4.5.

## 4 Light Water, H in $\mathrm{H}_{2} \mathrm{O}$

### 4.1 Model Description

At low energies ( $\mathrm{E} \leq 4 \mathrm{eV}$ ) the interaction of the scattered neutron with the scattering nucleus is pronounced by the individual dynamical excitations of the scattering nucleus.

The IKE model for water assumes that the scattering law for the primary scatterer $(\mathrm{H})$ is well represented by a set of hindered rotations given as a solid-type frequency distribution (temperature dependent), two discrete oscillators (.205 and . 436 eV ) to represent the molecular vibrations, and a hindered translational mode with effective temperature dependent masses.

Based on work by Eucken [10] and experimental data by Haywood and Page [12], the modes of the scattering dynamics of H bound in $\mathrm{H}_{2} \mathrm{O}$ in the liquid phase are treated as

- hindered translations with effective temperature dependent masses. These values are listed for a grid of temperatures in Table 4.1 and are derived from the work of Eucken [10]. The moving translational units can be thought to be clusters of single molecules as well as of two, four, and eight complexes of $\mathrm{H}_{2} \mathrm{O}$ molecules with varying fractions depending on temperature as given in Figure 4.1. This assumption was corroborated later by Bertagnolli [11].
- hindered rotations with a broad band of frequencies, which are temperature-dependent. For a specific temperature, interpolation for the continuous part $\rho(\omega)$ is done between the two limiting curves at 294 and 624 K given in Figure 4.2, based on measurements by Haywood and Page [12]. Also shown in this figure is the temperature independent frequency distribution used in the ENDF model.
- two Einstein $\delta$-oscillators describing the bending and stretching vibrations within the water molecule (see Figure 4.3). Compared to our model for the JEF data the frequency of the degenerated stretching vibrations are slightly reduced from .48 meV to .436 meV taking into account the liquid state (Springer [13]). The numerous values for the frequencies of the intra-molecular oscillations for the three phases (solid, liquid, vapour) are listed in Table 4.2.

For the optical modes it is assumed that each of the six degrees of freedom are equally weighted contrary to the ENDF model [2] [4] and the previous model for JEF [6].

The scattering on oxygen is represented using a free-gas law for mass 16 .


Figure 4.1 Molecular cluster structure of liquid water as a function of temperature (derived from Eucken [10])

Table 4.1 Effective masses $\mathrm{M}_{\text {trans }}$ for the translational mode, effective temperature $\mathrm{T}_{\text {eff }}$, Debye-Waller integral $\gamma(0)$ and the average scattering amplitude $<\mathrm{u}>$ of H bound in $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature T

| T in K | $\mathbf{T}_{\text {eff }}$ in K |  | $\gamma(\mathbf{0})$ |  | in $\mathrm{eV}^{-1}$ | $<\mathbf{u}>$ |  | in $\AA$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{M}_{\text {trans }}$ in amu |  |  |  |  |  |  |  |  |
|  | IKE | GA | IKE | GA | IKE | GA | IKE | GA |
| 293.6 | 1297.1 | 1398.5 | 11.66 | 10.19 | 0.269 | .252 | 46.00 | 18 |
| 323.6 | 1300.7 | 1406.3 | 12.16 | 10.66 | 0.275 | .258 | 39.00 | 18 |
| 373.6 | 1308.9 | 1420.9 | 13.01 | 11.48 | 0.284 | .267 | 31.00 | 18 |
| 423.6 | 1321.2 | 1437.4 | 13.91 | 12.34 | 0.294 | .277 | 27.00 | 18 |
| 473.6 | 1337.2 | 1455.4 | 14.85 | 13.24 | 0.304 | .287 | 25.00 | 18 |
| 523.6 | 1354.6 | 1475.4 | 15.79 | 14.16 | 0.313 | .297 | 23.00 | 18 |
| 573.6 | 1374.9 | 1496.6 | 16.76 | 15.11 | 0.323 | .307 | 22.00 | 18 |
| 623.6 | 1396.5 | 1519.1 | 17.73 | 16.07 | 0.332 | .316 | 21.00 | 18 |
| 647.2 | 1407.0 | 1530.1 | 18.22 | 16.53 | 0.337 | .321 | 20.40 | 18 |
| $800^{*}$ | 1488.4 | 1607.1 | 21.63 | 19.57 | 0.367 | .349 | 20.40 | 18 |
| $1000^{*}$ | 1608.5 | 1720.9 | 26.31 | 23.68 | 0.404 | .384 | 20.40 | 18 |

In Table 4.1 the effective scattering temperature, the Debye-Waller integral and the average scattering amplitude $<\mathrm{u}\rangle$ of the bound H are given. $\left\langle\mathrm{u}^{2}\right\rangle$ is proportional to the Debye Waller integral $\gamma(0)$. All these parameters are simply correlated to the chosen $\rho(\omega)$.

[^0]

Figure 4.2 Continuous frequency spectra $\rho_{\mathrm{s}}(\omega)$ for two temperatures used for H in $\mathrm{H}_{2} \mathrm{O}$, based on experimental work of Haywood and Page.

Table 4.2 Frequencies $\omega_{\mathrm{j}}$ in meV of intra-molecular oscillations of H in $\mathrm{H}_{2} \mathrm{O}$

| Oscillations |  | Ice | Liquid | Vapour |
| :---: | :---: | :---: | :---: | :---: |
| stretching | $\omega_{1,3}$ | 409 | 436 | 460 |
| bending | $\omega_{2}$ | 203 | 205 | 198 |


$\omega_{1}$

$\omega_{2}$


$\omega_{3}$

symmetric stretching
asymmetric stretching

Figure 4.3 Vibrations within a single water molecule $\mathrm{H}_{2} \mathrm{O}$

### 4.2 LEAPR - Input and Results

The LEAPR module is used to prepare the scattering law $S(\alpha, \beta)$ and related quantities, which describe thermal scattering from bound moderators, in the ENDF-6 format used by the THERMR module of NJOY.

LEAPR requires a uniform grid for the continuous frequency distribution $\rho(\omega)$ for every temperature. The final frequency spectra for the range of temperatures from 293.6 K up to 647.2 K (critical temperature, triple point in the phase diagram) are given in the listed input for LEAPR in the Appendix.

The specific points in the $\alpha$ and $\beta$ grid were carefully chosen for all dynamical modes and resulted in 182 values for $\alpha$ and 259 values for $\beta$. The $\beta$ grid reflects all the structure in the frequency distribution. As translational modes are to be included, a fine $\beta$ grid for small $\beta$ is required to get good results at small $\alpha$. For the discrete oscillators additional $\beta$ values are needed at the $\mathrm{n}^{*} \omega_{\mathrm{i}}$ (multi-phonon excitations) and their various sums.

The range of $\beta$ are from zero up to $\beta_{\max }=158.1$ (equal maximum energy transfer of 4 eV ) and that of $\alpha$ from . 0005 up to 632.9. In LEAPR the grids are currently limited to 200 elements each, so all arrays for the beta and alpha values have to be increased.

With this choice for the $\alpha$ and $\beta$ grid, details of the scattering dynamics are well represented in the differential neutron scattering cross section as is given in Figure 4.4.


Figure 4.4 Spectra of secondary energies for double differential neutron scattering cross section of H in $\mathrm{H}_{2} \mathrm{O}\left(\mathrm{E}_{\mathrm{i}}=625 \mathrm{meV}\right.$ at an angle of 15 degree $)$

In Figure 4.4 the peak structure around the initial neutron energy describes the quasi-elastic domain where hindered or free translations of the hydrogen can be excited. The peak around 560 meV corresponds to the interaction with the hindered rotations of the hydrogen, whereas the peaks around 420 meV and 190 meV are clearly correlated to the bending and stretching vibrations of the bound hydrogen.

The domain of the stretching vibrations are not so pronounced for ENDF/B-VI data. The reason may be in the $\alpha$ and $\beta$ grid and a higher frequency for these vibrations.

A reduction of about $50 \%$ in the number of the $\alpha$ and $\beta$ values can be achieved without any loss of accuracy by using a lower value for $\mathrm{E}_{\text {max }}$ as given in the following table and shown in Figure 4.5. The energy $\mathrm{E}_{\mathrm{b}}$ of 1.48 eV corresponds to the binding energy of the single proton in a water molecule [14] [15].

Table 4.3 Maximum energy transfer $\mathrm{E}_{\text {max }}$ and the numbers of $\alpha$ and $\beta$ values used in different evaluated files for $\mathrm{S}(\alpha, \beta)$ of H in H 2 O

| Data | $\mathrm{E}_{\max }$ <br> eV | Number of <br> $\alpha$ values | Number of <br> $\beta$ values |
| :---: | :--- | :---: | :---: |
| IKE-IAEA | 4 | 259 | 182 |
| ENDF/B-VI | 4.048 | 95 | 97 |
| IKE with $\mathrm{E}_{\max }=\mathrm{E}_{\mathrm{b}}$ | 1.48 | 178 | 137 |

The interrelation between $\mathrm{E}_{\text {max }}$ and the short collision time approximation SCTA as calculated by THERMR is demonstrated in Figure 4.5 as an example for the differential neutron scattering probability for H bound in $\mathrm{H}_{2} \mathrm{O}$ at an incident neutron energy of 4.46 eV at room temperature (RT).


Figure 4.5 Differential neutron scattering probability of H in light water from different evaluated $S(\alpha, \beta)$ files $\left(E_{i}=4.46 \mathrm{eV}, R T\right)$

For an incident energy of 4.46 eV the neutron scattering cross section is 20.77 barn for both IKE evaluations whereas the ENDF/B-VI file yields a value of 20.36 barn which seems to be too low. The free atomic neutron scattering cross section is 20.478 barn. This limit is however reached asymptotically at higher incident energies ( $\mathrm{E}>10 \mathrm{eV}$ ) from cross section values higher than the values of the free gas assumption.

Using the GA/ENDF model and a more adequate choice in the $\alpha$ and $\beta$ grid yields a value of 20.76 barn for the scattering cross section.

The effective temperatures and Debye-Waller integrals for H in $\mathrm{H}_{2} \mathrm{O}$ are given Table 4.1 in the section before. The IKE model results in lower values for $\mathrm{T}_{\text {eff }}$ compared to the GA model used in ENDF/B. This can be seen also for the average kinetic energy for H in $\mathrm{H}_{2} \mathrm{O}$ shown in Figure 4.6 together with experimental data [16] [17].


Figure 4.6 Average kinetic energy of H bound in liquid water

As in the coding of the LEAPR module one component for $\mathrm{T}_{\text {eff }}$ is missing (translational part not included in the output to the ENDF file), the results differ for the ENDF/B-VI. 3 data compared to the original GA values. LEAPR was updated at IKE for the correct weighting of the modes contributing to $\mathrm{T}_{\text {eff }}$.

### 4.3 Neutron Scattering Cross Sections

Using the THERMR module of NJOY the total scattering cross section can be calculated from the evaluated $S(\alpha, \beta)$ as well as secondary neutron distributions for a fix grid of incident energies (given in a data statement in THERMR). Examples of secondary neutron spectra are given in Figure 4.7.


Figure 4.7 Secondary neutron spectra for H in $\mathrm{H}_{2} \mathrm{O}$ for several incident neutron energies at a temperature of 293.6 K (red curve IKE) compared to ENDF/B-VI data (black curve)

In THERMR it is not possible to have as input discrete incident or secondary neutron energies or scattering angles nor to calculate directly values for the $\mathrm{H}_{2} \mathrm{O}$ molecule, that means to add directly the free gas value for oxygen.

For the validation and verification of the generated thermal scattering data calculations of differential and double-differential cross sections at specific incident and secondary energies as well as at specific scattering angles of the experiments are necessary. For these purposes codes have been written specifically.

For energy transfer higher than $\beta_{\max }$ the short collision time approximation is used in THERMR.

### 4.4 Validation of Differential Neutron Scattering Cross Sections

The following figures show measured double-differential neutron scattering cross sections of light water $\mathrm{H}_{2} \mathrm{O}$ [18] for three different incident neutron energies ( 154,231 and 631 meV ) at an scattering angle of $60^{\circ}$ together with calculated ones based on $\mathrm{S}(\alpha, \beta)$ from the IKE model as well as those derived from the ENDF/B-VI. 3 data.
Good agreement and an improvement over the ENDF/B-VI data can be seen at least for the two lowest incident energies.


Figure 4.8 Double differential neutron scattering cross section of water around room temperature $\left(\mathrm{E}_{\mathrm{i}}=154 \mathrm{meV}, \theta=60^{\circ}\right)$


Figure 4.9 Double differential neutron scattering cross section of water around room temperature $\left(\mathrm{E}_{\mathrm{i}}=231 \mathrm{meV}, \theta=60^{\circ}\right)$


Figure 4.10 Double differential neutron scattering cross section of water around room temperature $\left(\mathrm{E}_{\mathrm{i}}=631 \mathrm{meV}, \theta=60^{\circ}\right)$

In Figure 4.11 up to Figure 4.14 angular distributions of neutrons scattered in $\mathrm{H}_{2} \mathrm{O}$ are represented for several incident neutron energies (9.1, 25.3, 56.9 and 114 meV ) near room temperature compared with experimental data of Beyster [20].


Figure 4.11 Differential neutron scattering cross section of water around room temperature ( $\mathrm{E}_{\mathrm{i}}=9.1 \mathrm{meV}$ )


Figure 4.12 Differential neutron scattering cross section of water around room temperature ( $\mathrm{E}_{\mathrm{i}}=25.3 \mathrm{meV}$ )


Figure 4.13 Differential neutron scattering cross section of water around room temperature ( $\mathrm{E}_{\mathrm{i}}=56.9 \mathrm{meV}$ )


Figure 4.14 Differential neutron scattering cross section of water around room temperature ( $\mathrm{E}_{\mathrm{i}}=114 \mathrm{meV}$ )

### 4.5 Total Neutron Cross Sections for $\mathbf{H}_{2} \mathrm{O}$

In the following figures the total neutron cross sections for $\mathrm{H}_{2} \mathrm{O}$ are given for room temperature and 473 K compared to measurements found in EXFOR [21], [22], [23], [24], [25]. The neutron cross sections for oxygen in free gas approximation are taken from ENDF/B-VI.


Figure 4.15 Total cross section for light water around room temperature for ultracold, cold and thermal neutrons ( 0.000001 up to 10 eV )


Figure 4.16 Total neutron cross section for light water around room temperature (linear scale)


Figure 4.17 Total neutron cross section for light water at 473 K

The temperature dependence of the scattering cross section of H bound in $\mathrm{H}_{2} \mathrm{O}$ is shown in Figure 4.18.


Figure 4.18 Neutron scattering cross section for H bound in $\mathrm{H}_{2} \mathrm{O}$ at several temperatures together with the free-gas approximation for H at RT .

### 4.6 Average Cosine of the Neutron Scattering Angle in Water

Calculated results for the water molecule are compared with measurements from [26], [27], [20] in Figure 4.19 and Figure 4.20.


Figure 4.19 Average cosine of the neutron scattering angle for H 2 O around room temperature


Figure 4.20 Average cosine of the neutron scattering angle in light water at 473 K

### 4.7 Neutron Flux Density Spectrum in $\mathbf{H}_{\mathbf{2}} \mathrm{O}$

MCNP data sets in ACE format based on the IKE model were generated with NJOY using the modules THERMR and ACER for 16 angles and 64 equally energy bins for the secondary energies. For the verification of the data neutron flux density spectra in $\mathrm{H}_{2} \mathrm{O}$ were calculated by MCNP-4C3 and MCNPX and compared with experiments. As an example Figure 4.21 shows the results for pure light water at room temperature. The measured values are taken from [28]


Figure 4.21 Neutron flux density spectrum of light water at room temperature

## 5 Zirconium Hydride

### 5.1 General Remarks

This evaluation for scattering law data for H in $\mathrm{ZrH}_{\mathrm{x}}$ (variable stoichiometry with $\mathrm{x} \leq 2$ ) is based on former work on metal hydrides in fcc lattice structure done at IKE [29] [30] using the code PHONON (developed at IKE) in addition to the code GASKET. The basis for PHONON was the so-called phonon expansion method as used in LEAPR to generate Scattering Law data.

No changes have been made in the physics for the generalised frequency distribution $\rho(\omega)$. For the input to LEAPR an equidistant grid for $\rho(\omega)$ was built and the $\alpha$ and $\beta$ grid is strongly associated to the maximal and minima of the frequency spectra and its multi-phonon excitations. The $\alpha$ range is extended to small values.

Compared to the ENDF model from Slaggie [31] for H in $\mathrm{ZrH}_{\mathrm{x}}$ and Zr in $\mathrm{ZrH}_{\mathrm{x}}$ the overall agreement of measured data and calculated cross sections based on the IKE model for H in $\mathrm{ZrH}_{\mathrm{x}}$ and treating Zr as free gas is as good or better.

### 5.2 Physical basis of thermal neutron interaction in zirconium hydride

The thermal neutron interaction in zirconium hydride is strongly influenced by the bound hydrogen in the lattice. As is known from the phase diagram under normal conditions zirconium hydride has several lattice structures dependent on the hydrogen content. At low hydrogen concentrations the lattice structure is a mix of hexagonal lattice ( $\alpha$-phase) and face centred cubic (fcc) lattice structure ( $\delta$-phase). At medium concentrations as for $\mathrm{ZrH}_{\mathrm{x}}$ with x around 1.5 we have a pure fcc lattice for the hydride. For greater values of x a mix of fcc and face centred tetragonal (fct) hydride exists. For x around 2 we have the pure fct $\varepsilon$-phase. X equal 2 is the maximal possible value for zirconium hydride.

For the $\delta$ - and $\varepsilon$-phases the zirconium hydride lattice can be approximately represented by the main lattice of fcc metal atoms. The hydrogen atoms form a primitive cubic sub-lattice, so that the hydrogen atom is centred in a tetrahedron of surrounding metal atoms. At room temperature the distance between Zr and H is $2.07 \AA$, whereas the distance of neighboured H is $2.39 \AA$. From this structure the bound proton has an isotropic harmonic potential. Therefore the optical mode excitation can be well be described by a harmonic Einstein $\delta$-oscillator. From known force constants an optical peak around 137 meV can be derived. As the $\mathrm{H}-\mathrm{H}$ interacting forces are neglected, no splitting of the peak structure will be seen.

The generalized frequency spectra for H bound in zirconium hydride is composed of

- acoustical modes with a Debye Ansatz and a Debye temperature of 20 meV
- an optical branch approximated by a Gaussian Peak with a FWHM of 28 meV (full width of half maximum).
The ratio of the optical branch to the acoustical one is $240: 1$ which is taken over from the study of Slaggie [31] based on a central force dynamical calculation of the fcc zirconium hydride lattice. From the derived dispersion relations frequency distributions are calculated for the molecule as well as for the metal and hydrogen components. The consideration of the $\mathrm{H}-\mathrm{H}$ interaction yields a splitting up of the optical peak for hydrogen. As shown in
comparisons with measured double differential neutron scattering cross sections, this splitting up is overestimated (see Figure 5.3 up to Figure 5.6).

The frequency distribution of the IKE model is presented in Figure 5.1 in comparison to the model used for the ENDF/B-VI data.


Figure 5.1 Frequency distribution of H bound in zirconium hydride

In all known models the frequency distributions $\rho(\omega)$ are handled in harmonic approximation. This means that the effects of temperature changes on the lattice structure and the force constants are not taken into account and $\rho(\omega)$ is temperature independent.

For the optical part a phonon spectrum of H in $\mathrm{ZrH}_{2}$ published by Evans et al. [32] and based on experimental data was used to derive a complete frequency distribution with an acoustical part and weights as in the IKE model (green curve in the figure aside). For room temperature the calculated differential neutron cross sections compared to experimental data show a similar behaviour as the results based on the Slaggie model as can be seen in Figure 5.5 and Figure 5.8. The agreement with experiments and the results based on the IKE model is often better.


### 5.3 Notes to LEAPR input

The complete input to LEAPR is given in the Appendix (section10.2.2) for H in ZrH . For 8 temperatures as given in Table 5.1 scattering law data are generated for the incoherent inelastic part and the Debye-Waller integrals needed for the incoherent elastic part of thermal neutron scattering data sets in ENDF-6 format.
As an adequate choice to represent the structure of the frequency distribution as well as multiple phonon excitations $147 \alpha$ values and $185 \beta$ values have been chosen for the $\alpha, \beta$ grid. The upper limit for $\beta$ corresponds to $\mathrm{E}_{\max }$ of 1.8554 eV , which is higher than the binding energy [33] of 1.04 eV for H bound in $\mathrm{ZrH}_{2}$. This energy value decreases to 0.90 eV for $\mathrm{ZrH}_{1.5}$. The number of principal scattering atoms in compound (npr) is set to one in LEAPR input.
Table 5.1 Effective Temperatures and Debye-Waller integrals for H in ZrH

| Temperature <br> K | Effective Temperature <br> K | Debye-Waller Integral <br> $\mathrm{eV}^{-1}$ |
| :---: | :---: | :---: |
| 293.6 | 800.0 | 9.031 |
| 400 | 824.2 | 9.835 |
| 500 | 863.4 | 10.75 |
| 600 | 915.6 | 11.79 |
| 700 | 977.8 | 12.92 |
| 800 | 1047.5 | 14.12 |
| 1000 | 1202.5 | 16.67 |
| 1200 | 1370.8 | 19.34 |

The new evaluation was run through the THERMR module of NJOY to obtain integrated cross sections. Figure 5.2 shows the total and the partial neutron scattering cross sections for H in ZrH at room temperature and 1000 K .


Figure 5.2 Neutron scattering cross sections for H in ZrH at 293.6 K and 1000 K

### 5.4 Validation of the Generated Scattering Law Data Files for $\mathbf{H}$ in $\mathbf{Z r H}$

From the generated neutron scattering data files in $\mathrm{MF}=7$ for the inelastic incoherent part (MT=4 with $\mathrm{S}(\alpha, \beta)$ ) and the incoherent elastic part (MT=2) differential and integral neutron cross sections are calculated and compared with experiments. Zr was considered in free gas approximation. Treating Zr as in the GA model $\mathrm{Zr}(\mathrm{ZrH})$ showed no differences.

### 5.4.1 Double Differential Neutron Cross Sections

The structure predicted in the optical peak by the central force model of Slaggie is not confirmed by experimental data for $\mathrm{ZrH}_{2}$ [34] as shown in the following figures for an incident energy of 238 meV and several scattering angles.


Figure 5.3 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$

$$
\left(\mathrm{E}_{\mathrm{i}}=238 \mathrm{meV}, \theta=25^{\circ}, \mathrm{RT}\right)
$$



Figure 5.4 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$ ( $\mathrm{E}_{\mathrm{i}}=238 \mathrm{meV}, \theta=40^{\circ}$, RT)


Figure 5.5 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$ ( $\mathrm{E}_{\mathrm{i}}=238 \mathrm{meV}, \theta=60^{\circ}$, RT)


Figure 5.6 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$ ( $\mathrm{E}_{\mathrm{i}}=238 \mathrm{meV}, \theta=90^{\circ}$, RT)

Double-differential neutron scattering cross of $\mathrm{ZrH}_{2}$ at an incident energy of 572 meV for several scattering angles are given in the following figures based on the IKE model as well as generated from the ENDF/B-VI. 3 data in comparison with measurements done at RPI [35] around room temperature RT.


Figure 5.8 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$ $\left(\mathrm{E}_{\mathrm{i}}=572 \mathrm{meV}, \theta=60^{\circ}, \mathrm{RT}\right)$


Figure 5.9 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$ $\left(\mathrm{E}_{\mathrm{i}}=572 \mathrm{meV}, \theta=90^{\circ}, \mathrm{RT}\right)$


Figure 5.10 Double-differential neutron scattering cross section of $\mathrm{ZrH}_{2}$ $\left(\mathrm{E}_{\mathrm{i}}=572 \mathrm{meV}, \theta=120^{\circ}, \mathrm{RT}\right)$

### 5.4.2 Comparison with Measured Scattering Law Data at 483 K

For $\mathrm{ZrH}_{1.08}$ there are available measured scattering law data $\mathrm{S}(\alpha, \beta)$ at a temperature of 483 K
[36] for several values of $\beta$ i.e. energy transfers. In Figure 5.11 these $\mathbf{S}(\alpha, \beta)$ data are compared with the generated ones from IKE and ENDF/B at four energy transfers from 16.6 up to 158.2 meV .


Figure 5.11 Scattering law for $\mathrm{ZrH}_{1.08}$ at a temperature of 483 K for several energy transfers ( $16.6,133.2,141.5$ and 158.2 meV )

### 5.4.3 Differential Neutron Scattering for $\mathbf{Z r H}_{1.85}$ and $\mathbf{Z r H}_{1.92}$

For zirconium hydride $\mathrm{ZrH}_{1.85}$ and $\mathrm{ZrH}_{1.92}$ there are measured data by Carriveau [37] and Kornbichler [38] for differential neutron scattering cross sections as a function of the scattering angles for several incident energies. As there is an agreement between the new data (IKE) and ENDF/B compared to the experimental data only the following two figures are shown.


Figure 5.12 Differential neutron scattering cross sections around room temperature of $\mathrm{ZrH}_{1.85}$ for $\mathrm{E}_{\mathrm{i}}=0.8824 \mathrm{eV}$


Figure 5.13 Differential neutron scattering cross sections around room temperature of $\mathrm{ZrH}_{1.92}$ for $\mathrm{E}_{\mathrm{i}}=22.5 \mathrm{meV}$

The experimental data given in Figure 5.13 show some interference structure which may be assigned to the metal component in the lattice. As the neutron scattering on Zr is treated in free gas approximation, no coherent elastic scattering is considered in the IKE model. In the ENDF model only incoherent elastic scattering is included for Zr.

### 5.4.4 Neutron Scattering Cross Section for $\mathbf{Z r H}$

The neutron scattering cross section for H in ZrH is calculated as the sum of the incoherent inelastic cross section from $\mathrm{S}(\alpha, \beta)(\mathrm{MT}=4)$ and the incoherent elastic cross section (MT=2). For the scattering cross section of ZrH (shown in Figure 5.14) the contribution for Zr is treated in free gas approximation. Experimental data are from [39].


Figure 5.14 Total neutron scattering cross section for ZrH at room temperature

However for ZrH as a solid there should be a coherent elastic part. From the structure of the experimental data in the low energy range this could be expected. Therefore the coherent


Figure 5.15 Coherent elastic scattering included in the total neutron scattering cross section in ZrH
elastic scattering was calculated for an assumed fcc lattice structure of $\mathrm{ZrH}_{2}$ and stored in $\mathrm{MF}=7$ and $\mathrm{MT}=2$ instead of the original data in the file for Zr in $\mathrm{ZrH}_{\mathrm{x}}$ of ENDF/B. The effect is clearly seen in Figure 5.15.
Due to the different stoichiometry and lattice structures of ZrH and $\mathrm{ZrH}_{2}$ this is a first approximation to consider the coherent elastic cross section and is not universally valid. The effect is seen only for low energies less 5 meV .

Figure 5.16 represents the total scattering cross section for the n -p interaction in $\mathrm{ZrH}_{\mathrm{x}}$ at room temperature in the energy range from 0.1 eV up to 1 eV where two measurements are available [40], [41]. A better agreement with the experimental data and the calculated ones (IKE or ENDF/B) could be achieved when additional energy points were introduced into the energy grid built into THERMR. In Figure 5.17 the calculated average cosine of the scattering angle is compared with measurements.


Figure 5.16 Total neutron scattering cross section of n -p interaction in $\mathrm{ZrH}_{\mathrm{x}}$ at RT


Figure 5.17 Average cosine of the scattering angle in $\mathrm{ZrH}_{\mathrm{x}}(\mathrm{x} \approx 1.84)$ at RT

### 5.5 Neutron flux density spectra

Measured neutron flux density spectra [42] are compared with calculations done by Bernnat/IKE [43] using the Monte Carlo code MCNPX and the generated data sets in ACE format based on the IKE evaluation for $\mathrm{H}(\mathrm{ZrH})$ (see section 7).


Figure 5.18 Infinite medium spectrum in borated ZrH at $295^{\circ} \mathrm{C}, 3.4 \mathrm{~b} / \mathrm{H}$ atom


Figure 5.19 Infinite medium spectrum in borated ZrH at $468{ }^{\circ} \mathrm{C}, 3.4 \mathrm{~b} / \mathrm{H}$ atom

## 6 Heavy Water, D in $\mathrm{D}_{2} \mathrm{O}$

The dominant coherent neutron scattering of D was never considered in the ENDF and JEF model. For the correct description of neutron scattering in $\mathrm{D}_{2} \mathrm{O}$ the intermolecular interaction for D-D, O-D and O-O interference as well as the intra-molecular interference should be taken into account. For room temperature a study at IKE in 1992 has shown that the different intermolecular and intra-molecular contributions almost cancel out each other excluding the D-D intermolecular part.

The scattering law data for D in $\mathrm{D}_{2} \mathrm{O}$ in incoherent approximation can be improved by consideration of the D-D intermolecular interference scattering according to the Sköld approximation. The neutron scattering on oxygen is treated as free gas.

Compared to the JEF model [6] for D in $\mathrm{D}_{2} \mathrm{O}$ the treatment of the translational mode was modified. The $\alpha$ and $\beta$ grid (input to LEAPR) is strongly correlated to the maxima and minima of the frequency distribution and, in addition, the $\alpha$ range is extended to smaller values.

### 6.1 The Scattering Dynamics of Deuterium bound in Heavy Water

For the dynamics of deuterium in liquid heavy water three types of motions according to the different degrees of freedom are characterising the generalised frequency distribution:

- the three acoustical modes are split into a part of hindered translations and of translational vibrations. As in our model for liquid deuterium [44] this may be discussed as a model for jump and hindered translations. The deuterium is assumed to diffuse for a certain time in the liquid and then it is localised for a certain time at a fixed place and vibrates in the condensed matter. The weight of the hindered translational mode is taken to be 0.05 , which corresponds to an effective translational mass of 20. The fixed vibrational part also has a weight of 0.05 but with a Debye type distribution for a Debye temperature of 20.2 meV .
- for the three rotational degrees of freedom a broad band of frequencies is assumed according to the results of Haywood and Page [12] being dependent on temperature. For a given temperature the distribution is derived by linear interpolation. In contrary to the previous JEF model the weight $\mathrm{W}_{\mathrm{s}}$ is slightly increased by the Debye part (see Figure 6.1).
- the three vibrational degrees of freedom are represented by three discrete Einstein $\delta$ oscillators from which the two stretching vibrations are handled as degenerated with doubled weighting. As in the JEF model the frequencies are taken as 145 and 338 meV . The weight of these modes is assumed to be 0.5 .

This generalised frequency distribution was taken to generate in a first step $S(\alpha, \beta)$ data in incoherent approximation by the LEAPR module. As deuterium is a dominant coherent neutron scatterer, an improvement of the scattering law data for D in $\mathrm{D}_{2} \mathrm{O}$ could be achieved


Figure 6.1 Continuous part $\rho_{s}(\omega)$ for the frequency spectrum for $D$ bound in $D_{2} O$
by considering the intermolecular $\mathrm{D}-\mathrm{D}$ interference. With a static structure factor $\mathrm{S}(\kappa)$ a correction of the incoherent data file according to the Sköld approximation [9]

$$
S(\alpha, \beta)=\frac{\sigma^{i n c}}{\sigma} S^{i n c}(\alpha, \beta)+\frac{\sigma^{c o h}}{\sigma} S^{i n c}\left(\alpha^{\prime}, \beta\right) S(\kappa)
$$

with

$$
\alpha^{\prime}=\frac{\alpha}{S(\kappa)} \quad \text { and } \quad \sigma=\sigma^{\mathrm{inc}}+\sigma^{\mathrm{coh}}
$$

was done. The static structure factor was derived by assuming for the interacting potential for small distances a hard core model and for greater distances a Lennard-Jones potential. In Figure 6.2 the structure factors $\mathrm{S}(\kappa)$ for the two limiting temperatures 293.6 and 643.2 K are represented.


Figure 6.2 Static structure factors for intermolecular D-D interaction in liquid $\mathrm{D}_{2} \mathrm{O}$ These modified scattering law data with coherent contributions are stored in consistent manner in MF=7 and MT=4 of the ENDF-6 format. The further processing with THERMR is as usual. The oxygen part has to be handled in free gas approximation.

### 6.2 Notes to LEAPR input

The complete input to LEAPR is given in the Appendix (section 10.2.3) for D in $\mathrm{D}_{2} \mathrm{O}$. For 8 temperatures up to the critical temperature as given in Table 6.1 scattering law data are generated for the incoherent part of the neutron scattering. This is needed as base for the Sköld approximation to consider inter-atomic interference scattering.

As an adequate choice to represent the structure of the frequency distribution as well as all multiple phonon excitations $277 \alpha$ values and $159 \beta$ values have been chosen for the $\alpha, \beta$ grid. The upper limit for $\beta$ corresponds to $\mathrm{E}_{\max }$ of 1.8554 eV , which is higher than the binding energy of 1.525 eV as given in [14]. The range for the $\alpha$ values is from .0001 to 146.9 and for the $\beta$ values from zero to 73.334 .

For each temperature of the new evaluation of $\mathrm{S}(\alpha, \beta)$ for $\mathrm{D}\left(\mathrm{D}_{2} \mathrm{O}\right)$ the results from LEAPR are modified as specified in Section 2.5 with the appropriate structure factor which is given in Figure 6.2 for two temperatures.

Table 6.1 Effective temperature for D in $\mathrm{D}_{2} \mathrm{O}$

| Temperature <br> K | Effective Temperature <br> K |
| :---: | :---: |
| 293.6 | 1010.1 |
| 323.6 | 1021.6 |
| 373.6 | 1042.3 |
| 423.6 | 1064.6 |
| 473.6 | 1088.4 |
| 523.6 | 1113.6 |
| 573.6 | 1140.1 |
| 643.9 | 1179.4 |

### 6.3 Validation

### 6.3.1 Differential Neutron Cross sections

From the generated scattering law data differential cross sections are calculated and compared with experimental results [45] [46] for several incident neutron energies as given in Figure 6.3 up to Figure 6.6. Cross sections based on $S(\alpha, \beta)$ in incoherent approximation (LEAPR) as well as cross section based on the new $S(\alpha, \beta)$ with intermolecular D-D interference are shown. The agreement of the "coherent" model is excellent.


Figure 6.3 Differential neutron scattering cross section of heavy water at room temperature $\left(\mathrm{E}_{\mathrm{i}}=22.5 \mathrm{meV}\right)$


Figure 6.4 Differential neutron scattering cross section of heavy water at room temperature $\left(\mathrm{E}_{\mathrm{i}}=44 \mathrm{meV}\right)$


Figure 6.5 Differential neutron scattering cross section of heavy water at room temperature $\left(\mathrm{E}_{\mathrm{i}}=71 \mathrm{meV}\right)$


Figure 6.6 Differential neutron scattering cross section of heavy water at room temperature $\left(\mathrm{E}_{\mathrm{i}}=105 \mathrm{meV}\right)$

The improvement due to consideration of interatomic D-D interference at least at smaller angles is quite evident.

In Figure 6.7 an experimental scattering law [47] is compared with the IKE and ENDF/B-VI. 3 data sets. There is nearly no difference between the calculated data as the energy transfer is outside of the influence of the coherent effects.
This is valid for all greater energy transfers we have compared and therefore not given here.


Figure 6.7 $\mathrm{S}(\alpha, \beta)$ for heavy water with an energy transfer of 38.6 meV around room temperature

### 6.3.2 Total Neutron Cross Sections for $\mathbf{D}_{2} \mathbf{O}$

In Figure 6.8 up to Figure 6.10 the calculated total neutron cross sections for heavy water are compared with measurements given in EXFOR and data taken from [48], [23] for room temperature and at $200^{\circ} \mathrm{C}$


Figure 6.8 Total neutron cross section for heavy water at room temperature


Figure 6.9 Total Neutron cross section for heavy water at room temperature


Figure 6.10 Total neutron cross section for heavy water at $200^{\circ} \mathrm{C}$

### 6.3.3 Average Cosine of the neutron scattering angle

Calculated results for the heavy water molecule are compared with measurements from [45] in Figure 6.11 and Figure 6.12.


Figure 6.11 Average cosine of the neutron scattering angle in heavy water at RT


Figure 6.12 Average cosine of the neutron scattering angle in heavy water at 473 K

## 7 MCNP Data Sets

The NJOY processing code system was used to create $S(\alpha, \beta)$ tables for MCNP from the IAEA/IKE evaluations at a variety of temperatures. NJOY version 99.90-with additional updates particular in the modules THERMR and ACER - was used.
The maximum incident neutron energy is 4.46 eV for H in $\mathrm{H}_{2} \mathrm{O}$, D in $\mathrm{D}_{2} \mathrm{O}$, and H in ZrH . In THERMR extra points to the energy grid for ZrH have been added to get agreement with measured neutron scattering cross sections.
A total of 27 MCNP data sets have been created for 3 moderator materials: H bound in $\mathrm{H}_{2} \mathrm{O}$, H in ZrH and D in $\mathrm{D}_{2} \mathrm{O}$. Information about these 27 data sets similar to that contained in Appendix G of the MCNP manual is provided here in the following Table 7.1. Note that these data sets may be used with any version of MCNP(X).
Given in parenthesis are the nuclides for which the $S(\alpha, \beta)$ data are valid. For example, $h 20.01 t$ provides scattering data only for ${ }^{1} \mathrm{H} ;{ }^{16} \mathrm{O}$ would still be represented by the default free-gas treatment.

Table 7.1 MCNP Data Sets in the SAB-IKE-2004 Library

| Hydrogen in Light Water $\mathrm{H}_{2} \mathrm{O}$ (1001) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| h20.01t | IAEA/IKE | 02/04/04 | 293.6 | 16 | 64 | none |
| h20.02t | IAEA/IKE | 02/04/04 | 323.6 | 16 | 64 | none |
| h20.03t | IAEA/IKE | 02/04/04 | 373.6 | 16 | 64 | none |
| h20.04t | IAEA/IKE | 02/04/04 | 423.6 | 16 | 64 | none |
| h20.05t | IAEA/IKE | 02/04/04 | 473.6 | 16 | 64 | none |
| h20.06t | IAEA/IKE | 02/04/04 | 523.6 | 16 | 64 | none |
| h20.07t | IAEA/IKE | 02/04/04 | 573.6 | 16 | 64 | none |
| h20.08t | IAEA/IKE | 02/04/04 | 623.6 | 16 | 64 | none |
| h20.09t | IAEA/IKE | 02/04/04 | 647.2 | 16 | 64 | none |
| h20.10t | IAEA/IKE | 11/12/04 | 800 | 16 | 64 | none |
| h2o.11t | IAEA/IKE | 11/14/04 | 1000 | 16 | 64 | none |
| Deuterium in Heavy Water $\mathrm{D}_{2} \mathrm{O}$ (1002) |  |  |  |  |  |  |
| d20.01t | IAEA/IKE | 11/09/04 | 293.6 | 16 | 64 | none |
| d20.02t | IAEA/IKE | 11/09/04 | 323.6 | 16 | 64 | none |
| d2o.03t | IAEA/IKE | 11/09/04 | 373.6 | 16 | 64 | none |
| d20.04t | IAEA/IKE | 11/09/04 | 423.6 | 16 | 64 | none |
| d20.05t | IAEA/IKE | 11/09/04 | 473.6 | 16 | 64 | none |
| d20.06t | IAEA/IKE | 11/09/04 | 523.6 | 16 | 64 | none |
| d20.07t | IAEA/IKE | 11/09/04 | 573.6 | 16 | 64 | none |
| d20.08t | IAEA/IKE | 11/09/04 | 643.9 | 16 | 64 | none |
| Hydrogen in Zirconium Hydride ZrH (1001) |  |  |  |  |  |  |
| h_zrh.01t | IAEA/IKE | 11/09/04 | 293.6 | 16 | 64 | incoh |
| h zrh.02t | IAEA/IKE | 11/10/04 | 400 | 16 | 64 | incoh |
| h zrh.03t | IAEA/IKE | 11/10/04 | 500 | 16 | 64 | incoh |
| h zrh.04t | IAEA/IKE | 11/10/04 | 600 | 16 | 64 | incoh |
| h zrh.05t | IAEA/IKE | 11/10/04 | 700 | 16 | 64 | incoh |
| h zrh.06t | IAEA/IKE | 11/10/04 | 800 | 16 | 64 | incoh |
| h_zrh.07t | IAEA/IKE | 11/10/04 | 1000 | 16 | 64 | incoh |
| h_zrh.08t | IAEA/IKE | 11/10/04 | 1200 | 16 | 64 | incoh |

The first column of the Table contains the ZAID, which is the data set identification to be specified on $\operatorname{MCNP}(\mathrm{X})$ MTn cards. The portion of the ZAID before the decimal point provides a shorthand alphanumeric description of the material. The two digits after the decimal point differentiate among different data sets (different temperatures) for the same material. The final character in the ZAID is a " t " which indicates thermal $\mathrm{S}(\alpha, \beta)$ table.
The second column of the Table is the evaluated source file. For this $\operatorname{MCNP}(\mathrm{X})$ library, all data are from the IAEA/IKE evaluations.
The thirth column provides the date that the data table was processed by the NJOY code.
The fourth column is the temperature of the data (in degrees Kelvin).
The fifth column contains the number of equally-likely discrete secondary cosines provided at each combination of incident and secondary energy for inelastic scattering, and for each incident energy for incoherent elastic scattering.
The sixth column gives the number of secondary energies provided for each incident energy for inelastic scattering.
There are three options for the elastic scattering entry in the seventh column:
none - no elastic scattering data for this material.
coh -- coherent elastic scattering data provided for this material (Bragg scattering).
incoh - incoherent elastic scattering data provided for this material.

For each moderator an ACE-library was generated named as follows

| H_H2O.ace | for H in $\mathrm{H}_{2} \mathrm{O}$ |
| :--- | :--- |
| D_D2O.iaea | for D in $\mathrm{D}_{2} \mathrm{O}$ |
| H_ZrH.iaea | for H in ZrH |

The associated xsdir file is given below.
xsdir

```
h2o.01t 0.999170 H_H2O.ace 0 1 1 63221 0 0 2.5301E-08
h2o.02t 0.999170 H_H2O.ace 0 1 15819 63221 0 0 2.7886E-08
h2o.03t 0.999170 H_H2O.ace 0 1 31637 63221 0 0 3.2195E-08
h2o.04t 0.999170 H_H2O.ace 0 1 47455 63221 0 0 3.6503E-08
h20.05t 0.999170 H_H20.ace 0 1 63273 63221 0 0 4.0812E-08
h20.06t 0.999170 H_H20.ace 0 1 79091 63221 0 0 4.5121E-08
h20.07t 0.999170 H_H20.ace 0 1 94909 63221 0 0 4.9429E-08
h2o.08t 0.999170 H_H2O.ace 0 1 110727 63221 0 0 5.3738E-08
h2o.09t 0.999170 H_H2O.ace 0 1 126545 63221 0 0 5.5772E-08
h2o.10t 0.999170 H_H2O.ace 0 1 142363 63221 0 0 6.8939E-08
h2o.11t 0.999170 H_H20.ace 0 1 158181 63221 0 0 8.6174E-08
d20.01t 1.996800 D_D20.iaea 0 1 1 63221 0 0 2.5301E-08
d2o.02t 1.996800 D_D20.iaea 0 1 15819 63221 0 0 2.7886E-08
d2o.03t 1.996800 D_D20.iaea 0 1 31637 63221 0 0 3.2195E-08
d20.04t 1.996800 D_D20.iaea 0 1 47455 63221 0 0 3.6503E-08
d20.05t 1.996800 D D20.iaea 0 1 63273 63221 0 0 4.0812E-08
d20.06t 1.996800 D_D20.iaea 0 1 79091 63221 0 0 4.5121E-08
d20.07t 1.996800 D_D20.iaea 0 1 94909 63221 0 0 4.9429E-08
d20.08t 1.996800 D_D20.iaea 0 1 110727 63221 0 0 5.5487E-08
h_zrh.01t 0.999170 H_ZrH.iaea 0 1 1 91592 0 0 2.5301E-08
h_zrh.02t 0.999170 H_ZrH.iaea 0 1 22911 91592 0 0 3.4470E-08
h_zrh.03t 0.999170 H_ZrH.iaea 0 1 45821 91592 0 0 4.3087E-08
h_zrh.04t 0.999170 H_ZrH.iaea 0 1 68731 91592 0 0 5.1704E-08
h_zrh.05t 0.999170 H_Z_ZrH.iaea 0 1 91641 91592 0 0 6.0322E-08
h_zrh.06t 0.999170 H_ZrH.iaea 0 1 114551 91592 0 0 6.8939E-08
h_zrh.07t 0.999170 H_ZrH.iaea 0 1 137461 91592 0 0 8.6174E-08
h_zrh.08t 0.999170 H_ZrH.iaea 0 1 160371 91592 0 0 1.0341E-07
```


## 8 Conclusions

New thermal neutron scattering law files in ENDF-6 format for hydrogen bound in water, hydrogen bound in zirconium hydride and deuterium bound in heavy water are available as well as thermal scattering data sets for use in the Monte Carlo codes MCNP and MCNPX. The data are given at a variety of temperatures.
The adequacy of thermal scattering law data was re-considered in view of enhanced computational capabilities and availability of modern computational tools. The following conclusions can be drawn:

- There were hardly any new experimental measurements performed since the time the thermal scattering methods were developed.
- Existing models are able to describe available experimental data reasonably well.
- The generated data for the three moderator materials have been tested quite extensively and in addition compared with the files given in ENDF/B-VI. 3 or ENDF/B-VI.8.
- The generating and processing chain for the thermal neutron scattering files with NJOY-99.90 was carefully investigated and necessary updates are given.
- New thermal neutron scattering libraries are available from the IAEA-NDS web site.


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## 10 APPENDIX

### 10.1 Updates to NJOY-99.90 for the modules THERMR, LEAPR, ACER

In generating and processing the new thermal neutron scattering data in ENDF-6 and ACE format updates to NJOY- 99.90 were necessary for a correct representation of the evaluated data as well as the processed data. Some corrections concern also other moderator materials. The updates listed below are limit to the modules LEAPR, THERMR and ACER.

```
*ident up_thermr
*/ thermr - M. Mattes/IKE ----------------------------------------------
*/ increase array size for IKE evaluations
*d thermr.101
            dimension a(800000)
*d thermr. }13
    namax=800000
*/
*/ fix reading long TAB1 records
*i thermr.1543
        ll=loc
        do while (nb.ne.0)
            ll=ll+nw
            call moreio(nendf,0,0,a(ll),nb,nw)
        enddo
*i thermr.1602
    ll=loc
    do while (nb.ne.0)
            ll=ll+nw
            call moreio(nendf,0,0,a(ll),nb,nw)
        enddo
*/
*/ -------------------------------------------------------------------mm
*/ident up2 thermr
*/ maximum.energy transfer at higher temperatures (print only)
*i thermr.1622
    if (lat.eq.1) tmax=tmax*tevz/(bk*temp)
*/ -------------------------------------------------------nd up2_thermr
*ident up3_thermr
*/ more incident energies necessary for ZrH for an adequate
*/ represention of the neutron scattering xs in comparison
*/ with experimental results (.1 < E_i < 1 eV)
*d thermr.1422,thermr.1423
            dimension egrid(79)
            dimension ubar(79)
*d thermr.1427
            data ngrid/79/
*d thermr.1433,thermr.1436
    & .030613d0,.042757d0,.056925d0,.081972d0,
    * .1d0, .111573d0, .12d0, .13d0, .145728d0, .16d0, .17d0,
    & .184437d0,
        .195d0, .2277d0, .24d0, .2510392d0,.2705304d0,.2907501d0,
        .3011332d0,.3206421d0,.3576813d0,
            .38d0, .4d0, .4170351d0, .44d0, .48d0, .5032575d0, .53d0,
            .56d0, .59d0, .6249328d0, .65d0, .69d0, . 72d0, . 75d0,
            .7821141d0, .82d0, .89d0, .9506956d0,1.0137432d0,1.1664337d0,
*d thermr.1455,thermr.1458
    & .030613e0,.042757e0,.056925e0,.081972e0,
    * .1e0, .111573e0, .12e0, .13e0, .145728e0, .16e0, .17e0,
```

```
        .184437e0,
        .195e0, .2277e0, .24e0, .2510392e0,.2705304e0,.2907501e0,
        .3011332e0,.3206421e0,.3576813e0,
        .38e0, .4e0, .4170351e0, .44e0, .48e0, .5032575e0, .53e0,
        .56e0, .59e0, .6249328e0, .65e0, .69e0, . 72e0, . 75e0,
        .7821141e0, .82e0, .89e0, .9506956e0,1.0137432e0,1.1664337e0,
*/
```



```
*ident up1_leapr
*/
*/ if number(>200) for alpha and/or beta values then
*/ increase array sizes plus working buffers
*/ e.g. beta(200) -> beta(400) for H2O
*d leapr.164
    common/ab/nalpha,nbeta,naint,nbint,alpha(200),beta(400)
*d leapr.173
    common/lstore/a(7500000)
*d leapr.178
    data nbmax,namax/400,200/
*d leapr.189
    maxa=7500000
*d leapr. }39
    mscr=4000
*d leapr.420
    common/ab/nalph1,nbetal,naint,nbint,alph1(200),beta1(400)
*d leapr.426
    dimension maxt(400)
*d leapr.794
    common/ab/nalpha,nbeta,naint,nbint,alpha(200),betal(400)
*d leapr.1252
    common/ab/nalpha1,nbeta1,naint,nbint,alpha(200),betal(400)
*d leapr.1852
    common/ab/nalph1,nbeta1,naint,nbint,alpha(200),beta1(400)
*/
*/ IKE evaluation of H(H2O) creates more pairs of alpha
*/ and S(alpha,beta) than npage
*i leapr.3122
l_mm=1+nw
d\overline{O} while (nb.ne.0)
                                    call moreio(0,nout,nprnt,scr(l_mm),nb,nw)
                    l_mm=1_mm+nw
enddo
*/ ----------------------------------------------------------
*ident up2_leapr
*/ for }\mp@subsup{}{}{-H(H2O), D(D2O) NS should be equal 1
*d leapr.2976,2977
    if(nss.gt.0) scr(5)=6*(nss+1)
    scr(6)=nss
*/ -----------------------------------------------------nd up2_leapr
*ident up3_leapr
*/ T-eff calculation
*d leapr. }155
    tempf(itemp)=(tbeta+twt) *tempf(itemp) +tsave
*/ ------------------------------------------------------nd up3_leapr
*ident up4_leapr
*/ generation of more than 10 temperatures
*d leapr.166,167
    common/te/tempr(20),tempf(20),tempf1(20)
    common/dw/dwpix(20),dwp1(20)
*d leapr. }17
    data ntmax/20/
*d leapr.600,601
    common/te/tempr (20),tempf(20),tempf1 (20)
```

common/dw/dwpix(20), dwp1 (20)
*d leapr. 797
common/te/tempr (20), tempf(20), tempf1 (20)
*d leapr. 1256
common/te/tempr (20), tempf(20), tempf1 (20)
*d leapr. 1258
common/dw/dwpix(20),dwp1(20)
*d leapr. 1856
common/te/tempr (20), tempf (20), tempf1 (20)
*d leapr. 2651.2652
common/te/tempr (20), tempf(20), tempf1 (20)
common/dw/dbw (20) , dbw1 (20)
*/ ---------------------------------------------------end up4_leapr
*ident up5_leapr
*/ correct directory
*i leapr. 2797 $\operatorname{scr}(5)=\operatorname{scr}(5)+1$
if(iel.ne.0) $\operatorname{scr}(5)=\operatorname{scr}(5)+1$
*/ --------------------------------------------------end up5_leapr
*
*ident up_acer
*/ id-name of thermal data set with 6 characters else blank
*i acer. 358
if (nch.eq.0) nch=6
*/
*/ more than 64 bins wt(65)
*d acer. 13079
dimension wt(401)
*/
*/ increase of scratch buffer (?)
*d acer. 13098
ninmax=8000
*/


### 10.2 Inputs to LEAPR

For the three moderator materials light water, heavy water and zirconium hydride the inputs to the module LEAPR of NJOY-90.99 are listed.

### 10.2.1 H in $\mathrm{H}_{2} \mathrm{O}$

In the following the input to LEAPR for H bound in $\mathrm{H}_{2} \mathrm{O}$ is given for the calculation of $\mathrm{S}(\alpha, \beta)$ in incoherent approximation for 8 temperatures from 293.6 K up to 647.2 K according to the IKE model.




```
\begin{tabular}{lcccccc}
0.009241 & 0.036971 & 0.083191 & 0.147901 & 0.231083 & 0.332760 & 0.452849 \\
0.591487 & 0.748599 & 0.924198 & 1.118324 & 1.330876 & 1.561874 & 1.811430 \\
2.079492 & 2.366205 & 2.671149 & 3.004446 & 3.377992 & 3.807549 & 4.210421 \\
4.601901 & 4.905032 & 5.137760 & 5.343403 & 5.522043 & 5.528789 & 5.496279 \\
5.470710 & 5.443103 & 5.305704 & 5.161395 & 4.980206 & 4.809356 & 4.660707 \\
4.515263 & 4.372950 & 4.219132 & 4.068416 & 3.907689 & 3.690725 & 3.576285 \\
3.408925 & 3.244872 & 3.068391 & 2.910673 & 2.747111 & 2.613069 & 2.470184 \\
2.347965 & 2.202827 & 2.064036 & 1.946403 & 1.820161 & 1.699877 & 1.588447 \\
1.528279 & 1.458090 & 1.392651 & 1.331715 & 1.275806 & 1.239591 & 1.193201 \\
1.151560 & 1.114547 & 0.529185 & 0.000000 & \(/\) & & \\
0.047620 & 0. & 0.47619 & & & &
\end{tabular}
2 /
205 . 436
0.15873 0.31746 /
    647.2 /
0.002542 68 /
    0.0
        589657 . }74628
    2.073057
    4.58766
    5.45378
    4.501290
    3.398377-3.234830
    2.340699 2.196011 2.057649 1.940380
    1.523550 1.453578 1.388341 1.327594 1.271858 1.235755 1.189508
    1.147997 1.111098 . 527548 .000000 /
    0.049020 0. 0.47549 /
    2 /
    .205 . 436 /
    0.158496667 0.316993333 /
' H(H2O) IKE EVAL-jan04 Keinert,Mattes '
' IKE 6-201 DIST-feb04 040202 ',
'----IKE/IAEA MATERIAL 1
'-----THERMAL NEUTRON SCATTERING DATA
'------ENDF-6 FORMAT
'*
'*---------------------------------------------------------------------------
'* Temperatures (K)
'* 293.6 323.6 373.6 423.6 473.6 523.6 573.6 647.2 *'
'* **'0 *',
'* Modifications of the IKE model compared to the JEF-2.2 thermal *'
'* data file:
'* - alpha,beta-grid strictly correlated to the phonon spectra *'
'* - modification of hindered rotational mode spectrum at high *'
'* frequencies
'* _ oscillator frequency omegal=omega3 changed to 0.436 eV *'
'* - optical modes equally weighted *'
* optical modes equally weighted
'* generated with LEAPR/NJOY-99.90++ IKE jan2004 *'
'*-----------------------------------------------------------------------------
/ end leapr
stop
```


### 10.2.2 H in ZrH

In the following the input for LEAPR for H bound in ZrH is tabulated for the calculation of $\mathrm{S}(\alpha, \beta)$ in incoherent approximation for 8 temperatures from 293.6 K up to 1200 K according to the IKE physics model.

| leapr$40$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H in ZrH IKE model'/ |  |  |  |  |  |  |
| $81200 /$ |  | 8 tempe | eratures |  |  |  |
| 7 107./ |  |  |  |  |  |  |
| . 99917 20.478 1-1/ |  |  |  |  |  |  |
| 1190.436 6.37 1/ Zr as free gas / |  |  |  |  |  |  |
| 1471851 / ---- alpha grid |  |  |  |  |  |  |
| .0001 .0 |  |  |  |  |  |  |
| . 0006 | . 001 | . 005 | . 01 | . 025 | . 05 | . 075 |
| . 1 | . 125 | . 15 | . 2 | . 25 | . 3 | . 325 |
| . 35 | . 375 | . 4 | . 425 | . 45 | . 475 | . 5 |
| . 525 | . 55 | . 58 | . 61 | . 65 | . 69 | . 73 |
| . 7905 | . 83 | . 88 | . 94 | 1. | 1.08 | 1.16 |
| 1.24 | 1.33 | 1.45 | 1.581 | 1.7 | 1.8 | 1.95 |
| 2.1 | 2.26 | 2.3715 | 2.5 | 2.65 | 2.83 |  |
| 2.96 | 3.162 | 3.38 |  |  |  |  |
| 3.67 | 3.9525 | 4.32 | 4.52 | 4.743 | 5.1 | 5.415 |
| 5.97 | 6.324 |  |  |  |  |  |
| 6.59 | 7.114 | 7.54 | 7.905 | 8.695 | 9.486 | 10.276 |
| 10.830 | 11.857 | 12.648 | 13.438 | 14.229 | 15.019 | 15.810 |
| 16.245 | 17.2 |  |  |  |  |  |
| 18.245 | 19.4 | 20.3 | 21.659 | 22.9 | 24.8 |  |
| 26.2 | 27.074 | 28.9 | 30.2 | 31.5 | 32.489 | 34. |
| 36.6 |  | 37.904 | 39. | 41.1 | 43.319 |  |
| 44.4 | 45.5 | 47.7 | 48.734 | 50. | 51.2 | 52.5 |
| 53.9 | 55.149 | 56.3 | 58.4 | 59.564 | 61.2 |  |
| 63.8 | 64.978 | 66.5 | 68.4 | 69.4 | 70.393 | 71.6 |
| 73.334 | 75.808 | 80. | 84. | 89. | 94. | 100. |
| 105. | 113. | 120.63 | 126. | 132. | 140. | 147. |
| 154. | 162. | 170. | 177. | 184. | 191. | 199. |
| 208. | 218. | 227. | 237. | 246. | 255. | 265. |
| 275.72 | 284. | 293.58 | / --- beta | grid |  |  |
| 0.00.15 | 0.005 | 0.01 | 0.025 | 0.05 | 0.075 | 0.1 |
|  | 0.20 | 0.250 | 0.30 | 0.35 | 0.40 | 0.45 |
| 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.7905 |
| 0.85 | 0.90 | 0.95 | 1.0 | 1.05 | 1.10 | 1.15 |
| 1.20 | 1.25 | 1.30 | 1.35 | 1.4 | 1.45 | 1.5 |
| 1.55 |  |  |  |  |  |  |
| 1.581 | 1.6 | 1.65 | 1.7 | 1.75 | 1.8 | 1.85 |
| 1.9 | 1.95 | 2.0 | 2.05 | 2.1 | 2.15 | 2.2 |
| 2.25 | 2.3 | 2.35 | 2.3715 |  |  |  |
| 2.4 | 2.45 | 2.5 | 2.55 |  |  |  |
| 2.6 | 2.65 |  | 2.77 | 2.83 | 2.90 | 2.96 |
| 3.03 | 3.11 | 3.162 |  |  |  |  |
| 3.20 | 3.26 | 3.34 | 3.43 | 3.52 |  |  |
| 3.61 | 3.71 | 3.81 | 3.89 | 3.9525 |  |  |
| 4.03 | 4.14 | 4.26 |  |  |  |  |
| 4.39 | 4.52 | 4.65 | 4.743 |  |  |  |
| 4.84 | 4.94 | 5.10 | 5.26 | 5.415 |  |  |
| 5.533 | 5.60 | 5.78 | 5.97 | 6.17 | 6.324 |  |
| 6.48 | 6.59 |  |  |  |  |  |
| 6.81 | 6.96 | 7.114 |  |  |  |  |
| 7.29 | 7.54 | 7.72 | 7.905 | 8.15 | 8.37 |  |
| 8.695 | 8.98 | 9.30 | 9.486 |  |  |  |
| 9.64 | 10. | 10.276 | 10.6 | 10.830 | 10.95 |  |
| 11.067 |  |  |  |  |  |  |
| 11.31 | 11.57 | 11.857 |  |  |  |  |
| 12.15 | 12.46 | 12.648 |  |  |  |  |
| 12.98 | 13.438 | 13.94 | 14.229 |  |  |  |
| 14.48 | 15.019 | 15.42 | 15.810 | 16. | 16.245 |  |
| 16.6 | 17.4 | 18.2 |  |  |  |  |
| 18.8 | 19.4 | 20. | 20.7 | 21.659 | 22.6 |  |
| 23.5 |  | 24.8 |  | 26.2 | 27.074 | 27.9 |
|  | 28.9 |  | 30.2 |  | 31.5 | 32.489 |
| 33.2 |  | 34. |  | 35.3 |  | 36.6 |



### 10.2.3 $D$ in $\mathrm{D}_{2} \mathrm{O}$, incoherent part only.

In the following the input to LEAPR for D bound in $\mathrm{D}_{2} \mathrm{O}$ is tabulated for the calculation of $\mathrm{S}(\alpha, \beta)$ in incoherent approximation for 293.6 K . D-D interference scattering is not included as this is not treated in LEAPR.

| $\begin{gathered} \text { leapr } \\ 40 \end{gathered}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ' D in D20 T=293.6 K only incoherent part IKE njoy99.90+ feb2004' |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 1111 / |  |  |  |  |  |  |
| 1.9968 | 3.3952 |  |  |  |  |  |
| 11 | $1.585316 \mathrm{e}+1$ | 3.761 | / |  |  |  |
| 159277 | 71 / |  | lat $=1$ |  |  |  |
| 0.0001 | 0.0002 | 0.0004 | 0.0006 | 0.0008 | 0.0010 | 0.0015 |
| 0.0020 | 0.0025 | 0.0030 | 0.0040 | 0.0050 | 0.0060 | 0.0080 |
| 0.0100 | 0.0150 | 0.0200 | 0.0250 | 0.0350 | 0.0500 | 0.0600 |
| 0.0750 | 0.0850 | 0.1000 | 0.1250 | 0.1500 | 0.1750 | 0.2000 |
| 0.2250 | 0.2500 | 0.2750 | 0.3000 | 0.3250 | 0.3500 | 0.3750 |
| 0.4000 | 0.4250 | 0.4500 | 0.4750 | 0.5000 | 0.5300 | 0.5600 |
| 0.5900 | 0.6200 | 0.6600 | 0.7000 | 0.7500 | 0.8000 | 0.8500 |
| 0.9000 | 0.9600 | 1.0200 | 1.1000 | 1.2000 | 1.2800 | 1.3500 |
| 1.4250 | 1.5000 | 1.5700 | 1.6500 | 1.7500 | 1.8500 | 1.9500 |
| 2.0500 | 2.2000 | 2.3500 | 2.5500 | 2.7500 | 3.0300 | 3.2600 |
| 3.5300 | 3.8600 | 4.1900 | 4.5000 | 4.9500 | 5.3400 | 5.7800 |
| 6.1800 | 6.5200 | 6.9000 | 7.4000 | 7.9500 | 8.2500 | 8.7000 |
| 9.6000 | 10.4000 | 11.4000 | 12.4400 | 13.5000 | 14.7000 | 16.0000 |
| 17.2000 | 18.7000 | 20.0000 | 21.1000 | 22.5000 | 23.4000 | 24.5000 |
| 25.5000 | 27.0000 | 29.2000 | 31.7000 | 33.2000 | 34.6000 | 36.0000 |
| 37.2000 | 39.1000 | 41.0000 | 42.5000 | 44.2000 | 46.0000 | 48.5000 |
| 50.6000 | 53.4000 | 54.1000 | 55.2000 | 57.6000 | 60.0000 | 62.2000 |
| 63.9000 | 65.7000 | 67.1000 | 68.5000 | 69.7000 | 71.0000 | 72.0000 |
| 73.3000 | 74.7000 | 76.4000 | 78.0000 | 79.5000 | 81.4000 | 83.2000 |
| 84.9000 | 86.7000 | 88.5000 | 90.5000 | 92.5000 | 94.5000 | 96.6000 |
| 98.7000 | 100.9000 | 103.0000 | 105.2000 | 108.0000 | 112.4000 | 114.7000 |
| 117.0000 | 119.4000 | 122.2000 | 125.0000 | 128.0000 | 132.0000 | 136.2000 |
| 138.0000 | 140.0000 | 142.0000 | 144.0000 | 146.9000 | / end of | pha grid |
| 0.0000 | 0.0025 | 0.0050 | 0.0075 | 0.0100 | 0.0150 | 0.0200 |
| 0.0250 | 0.0300 | 0.0350 | 0.0500 | 0.0600 | 0.0750 | 0.0850 |
| 0.1000 | 0.1250 | 0.1500 | 0.1750 | 0.2000 | 0.2250 | 0.2500 |
| 0.2750 | 0.3000 | 0.3250 | 0.3500 | 0.3750 | 0.4000 | 0.4250 |
| 0.4500 | 0.4750 | 0.5000 | 0.5250 | 0.5500 | 0.5750 | 0.6000 |
| 0.6250 | 0.6500 | 0.6750 | 0.7000 | 0.7250 | 0.7500 | 0.7750 |
| 0.7984 | 0.8250 | 0.8500 | 0.8750 | 0.9000 | 0.9250 | 0.9500 |
| 0.9750 | 1.0000 | 1.0200 | 1.0500 | 1.0750 | 1.1000 | 1.1250 |
| 1.1500 | 1.1750 | 1.2000 | 1.2250 | 1.2500 | 1.2750 | 1.3000 |
| 1.3500 | 1.4000 | 1.4500 | 1.5000 | 1.5500 | 1.5968 | 1.6500 |
| 1.7000 | 1.7500 | 1.8000 | 1.8500 | 1.9104 | 1.9500 | 2.0000 |
| 2.0500 | 2.1000 | 2.1500 | 2.2000 | 2.2500 | 2.3000 | 2.3500 |
| 2.3952 | 2.4500 | 2.5000 | 2.5500 | 2.5911 | 2.6500 | 2.7000 |
| 2.7500 | 2.8000 | 2.8400 | 2.8800 | 2.9300 | 2.9800 | 3.0300 |
| 3.0800 | 3.1400 | 3.1936 | 3.2600 | 3.3500 | 3.4400 | 3.5300 |
| 3.6200 | 3.7100 | 3.8208 | 3.8600 | 3.9200 | 3.9920 | 4.0300 |
| 4.0800 | 4.1400 | 4.1900 | 4.2500 | 4.3100 | 4.3700 | 4.4300 |
| 4.5000 | 4.5700 | 4.6500 | 4.7200 | 4.7904 | 4.8700 | 4.9500 |
| 5.0200 | 5.1000 | 5.1822 | 5.2500 | 5.3400 | 5.5000 | 5.5888 |
| 5.6262 | 5.6600 | 5.7311 | 5.7800 | 5.8800 | 5.9700 | 6.0700 |
| 6.1700 | 6.2800 | 6.3872 | 6.5200 | 6.6500 | 6.7700 | 6.9000 |
| 7.0200 | 7.1856 | 7.4000 | 7.6416 | 7.7733 | 7.9840 | 8.1000 |
| 8.2500 | 8.4500 | 8.7000 | 9.0000 | 9.3000 | 9.5520 | 9.9500 |
| 10.3644 | 10.8000 | 11.0000 | 11.2000 | 11.4622 | 11.6500 | 12.1000 |
| 12.6000 | 13.1000 | 13.3593 | 13.6000 | 13.8000 | 14.0000 | 14.2000 |
| 14.5500 | 15.1000 | 15.2832 | 15.5466 | 15.7000 | 16.0000 | 16.1700 |
| 16.3500 | 16.5000 | 16.7000 | 16.9000 | 17.1933 | 17.5000 | 18.1377 |
| 18.7000 | 18.9000 | 19.1040 | 19.3000 | 19.5000 | 19.7500 | 20.0000 |
| 20.3500 | 20.7288 | 21.1000 | 21.8000 | 22.2000 | 22.5500 | 22.9244 |
| 23.4000 | 23.9000 | 24.3000 | 24.5000 | 24.8000 | 25.1000 | 25.3000 |
| 25.5000 | 26.0000 | 26.4000 | 26.7186 | 27.0000 | 27.6000 | 28.2000 |
| 28.6555 | 29.7000 | 30.3000 | 31.0000 | 31.7000 | 32.4000 | 33.2000 |
| 34.3866 | 34.8000 | 35.2000 | 36.0000 | 36.6000 | 37.2000 | 37.9000 |
| 38.5000 | 39.1000 | 39.8000 | 40.0900 | 40.5000 | 41.0000 | 41.5000 |
| 42.0000 | 42.5000 | 43.0000 | 43.6000 | 44.2000 | 44.8000 | 45.4000 |
| 45.8500 | 46.0000 | 46.7000 | 47.5000 | 48.5000 | 49.5000 | 50.6000 |


| 51.5800 | 52.5000 | 53.4400 | 54.1000 | 55.2000 | 56.5000 | 57.6000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 58.7000 | 60.0000 | 61.1000 | 62.2000 | 63.5000 | 64.6000 | 65.7000 |
| 67.0000 | 67.5000 | 68.0000 | 68.5000 | 70.5000 | 71.0000 | 71.5000 |
| 72.0000 | 72.5000 | 72.9000 | 73.3340 / end of beta grid |  |  |  |

### 0.00028879 572 / <br> 0.0

$2.23783 \mathrm{E}-036.65181 \mathrm{E}-031.45127 \mathrm{E}-02 \quad 2.68802 \mathrm{E}-02 \quad 4.31287 \mathrm{E}-02 \quad 5.93772 \mathrm{E}-02$ $7.56256 \mathrm{E}-02 \mathrm{9} .94981 \mathrm{E}-021.26215 \mathrm{E}-011.52931 \mathrm{E}-011.86715 \mathrm{E}-01 \quad 2.24982 \mathrm{E}-01$ $2.63249 \mathrm{E}-013.01516 \mathrm{E}-013.41114 \mathrm{E}-01 \quad 3.91436 \mathrm{E}-01 \quad 4.41758 \mathrm{E}-01 \quad 4.92080 \mathrm{E}-01$ $5.49531 \mathrm{E}-01 \quad 6.10859 \mathrm{E}-01 \quad 6.72188 \mathrm{E}-01 \quad 7.33517 \mathrm{E}-01 \quad 8.08931 \mathrm{E}-01 \quad 8.87053 \mathrm{E}-01$ $9.65175 \mathrm{E}-011.04330 \mathrm{E}+001.12142 \mathrm{E}+001.19954 \mathrm{E}+001.27766 \mathrm{E}+00 \quad 1.36823 \mathrm{E}+00$ $1.46831 \mathrm{E}+001.56839 \mathrm{E}+00 \quad 1.66847 \mathrm{E}+00 \quad 1.76854 \mathrm{E}+00 \quad 1.86862 \mathrm{E}+00 \quad 1.96870 \mathrm{E}+00$ $2.07546 \mathrm{E}+002.19817 \mathrm{E}+00 \quad 2.32088 \mathrm{E}+00 \quad 2.44360 \mathrm{E}+00 \quad 2.56631 \mathrm{E}+00 \quad 2.68902 \mathrm{E}+00$ $2.81173 \mathrm{E}+002.93444 \mathrm{E}+00 \quad 3.06836 \mathrm{E}+00 \quad 3.20761 \mathrm{E}+00 \quad 3.34687 \mathrm{E}+00 \quad 3.48852 \mathrm{E}+00$ $3.63893 \mathrm{E}+003.78934 \mathrm{E}+00 \quad 3.93975 \mathrm{E}+00 \quad 4.09489 \mathrm{E}+00 \quad 4.25695 \mathrm{E}+00 \quad 4.41901 \mathrm{E}+00$ $4.58107 \mathrm{E}+004.74958 \mathrm{E}+00 \quad 4.92244 \mathrm{E}+00 \quad 5.09530 \mathrm{E}+00 \quad 5.26966 \mathrm{E}+00 \quad 5.45372 \mathrm{E}+00$ $5.63779 \mathrm{E}+00 \quad 5.82185 \mathrm{E}+006.00950 \mathrm{E}+00 \quad 6.20457 \mathrm{E}+00 \quad 6.39964 \mathrm{E}+00 \quad 6.59471 \mathrm{E}+00$ $6.79904 \mathrm{E}+00 \quad 7.00486 \mathrm{E}+00 \quad 7.21067 \mathrm{E}+00 \quad 2.11225 \mathrm{E}+00 \quad 2.17407 \mathrm{E}+00 \quad 2.23589 \mathrm{E}+00$ $2.29772 \mathrm{E}+002.36082 \mathrm{E}+00 \quad 2.42789 \mathrm{E}+00 \quad 2.49496 \mathrm{E}+00 \quad 2.56204 \mathrm{E}+00 \quad 2.62911 \mathrm{E}+00$ $2.69618 \mathrm{E}+00 \quad 2.76325 \mathrm{E}+00 \quad 2.83032 \mathrm{E}+00 \quad 2.90194 \mathrm{E}+00 \quad 2.97370 \mathrm{E}+00 \quad 3.04545 \mathrm{E}+00$ $3.11720 \mathrm{E}+003.18895 \mathrm{E}+003.26071 \mathrm{E}+00 \quad 3.33246 \mathrm{E}+00 \quad 3.40460 \mathrm{E}+00 \quad 3.47690 \mathrm{E}+00$ $3.54921 \mathrm{E}+003.62151 \mathrm{E}+00 \quad 3.69381 \mathrm{E}+00 \quad 3.76612 \mathrm{E}+00 \quad 3.83842 \mathrm{E}+00 \quad 3.91072 \mathrm{E}+00$ $3.98288 \mathrm{E}+00 \quad 4.05505 \mathrm{E}+00 \quad 4.12721 \mathrm{E}+00 \quad 4.19938 \mathrm{E}+00 \quad 4.27155 \mathrm{E}+00 \quad 4.34371 \mathrm{E}+00$ $4.41588 \mathrm{E}+004.48816 \mathrm{E}+004.56046 \mathrm{E}+00 \quad 4.63277 \mathrm{E}+00 \quad 4.70507 \mathrm{E}+00 \quad 4.77737 \mathrm{E}+00$ $4.84968 \mathrm{E}+004.92198 \mathrm{E}+00 \quad 4.98916 \mathrm{E}+00 \quad 5.05196 \mathrm{E}+00 \quad 5.11476 \mathrm{E}+00 \quad 5.17756 \mathrm{E}+00$ $5.24036 \mathrm{E}+00 \quad 5.30316 \mathrm{E}+00 \quad 5.36596 \mathrm{E}+00 \quad 5.42543 \mathrm{E}+00 \quad 5.47578 \mathrm{E}+00 \quad 5.52612 \mathrm{E}+00$ $5.57647 \mathrm{E}+005.62682 \mathrm{E}+00 \quad 5.67717 \mathrm{E}+00 \quad 5.72752 \mathrm{E}+00 \quad 5.77787 \mathrm{E}+00 \quad 5.82674 \mathrm{E}+00$ $5.87480 \mathrm{E}+00 \quad 5.92287 \mathrm{E}+00 \quad 5.97093 \mathrm{E}+00 \quad 6.01899 \mathrm{E}+00 \quad 6.06706 \mathrm{E}+00 \quad 6.11512 \mathrm{E}+00$ $6.16434 \mathrm{E}+006.21543 \mathrm{E}+006.26652 \mathrm{E}+00 \quad 6.31762 \mathrm{E}+00 \quad 6.36871 \mathrm{E}+00 \quad 6.41981 \mathrm{E}+00$ $6.47090 \mathrm{E}+006.52065 \mathrm{E}+00 \quad 6.55917 \mathrm{E}+00 \quad 6.59769 \mathrm{E}+00 \quad 6.63621 \mathrm{E}+00 \quad 6.67472 \mathrm{E}+00$ $6.71324 \mathrm{E}+006.75176 \mathrm{E}+00 \quad 6.79028 \mathrm{E}+00 \quad 6.82714 \mathrm{E}+00 \quad 6.86226 \mathrm{E}+00 \quad 6.89738 \mathrm{E}+00$ $6.93250 \mathrm{E}+006.96762 \mathrm{E}+007.00274 \mathrm{E}+00 \quad 7.03786 \mathrm{E}+00 \quad 7.07276 \mathrm{E}+00 \quad 7.10689 \mathrm{E}+00$ $7.14101 \mathrm{E}+007.17514 \mathrm{E}+00 \quad 7.19185 \mathrm{E}+00 \quad 7.20289 \mathrm{E}+00 \quad 7.21394 \mathrm{E}+00 \quad 7.22498 \mathrm{E}+00$ $7.23254 \mathrm{E}+007.23990 \mathrm{E}+00 \quad 7.24726 \mathrm{E}+00 \quad 7.25334 \mathrm{E}+00 \quad 7.25132 \mathrm{E}+00 \quad 7.24930 \mathrm{E}+00$ $7.24727 \mathrm{E}+007.23793 \mathrm{E}+007.22505 \mathrm{E}+007.21216 \mathrm{E}+007.19928 \mathrm{E}+00 \quad 7.18822 \mathrm{E}+00$ $7.17744 \mathrm{E}+007.16666 \mathrm{E}+00 \quad 7.15412 \mathrm{E}+00 \quad 7.11190 \mathrm{E}+00 \quad 7.06968 \mathrm{E}+00 \quad 7.02745 \mathrm{E}+00$ $6.98064 \mathrm{E}+006.93069 \mathrm{E}+00 \quad 6.88073 \mathrm{E}+00 \quad 6.83077 \mathrm{E}+00 \quad 6.77766 \mathrm{E}+00 \quad 6.72367 \mathrm{E}+00$ $6.66968 \mathrm{E}+006.61569 \mathrm{E}+006.56171 \mathrm{E}+00 \quad 6.50772 \mathrm{E}+00 \quad 6.45373 \mathrm{E}+00 \quad 6.39848 \mathrm{E}+00$ $6.34201 \mathrm{E}+006.28554 \mathrm{E}+00 \quad 6.22908 \mathrm{E}+00 \quad 6.17261 \mathrm{E}+00 \quad 6.11615 \mathrm{E}+00 \quad 6.05968 \mathrm{E}+00$ $5.99975 \mathrm{E}+005.92889 \mathrm{E}+00 \quad 5.85804 \mathrm{E}+00 \quad 5.78718 \mathrm{E}+00 \quad 5.71632 \mathrm{E}+00 \quad 5.64546 \mathrm{E}+00$ $5.57460 \mathrm{E}+00 \quad 5.50375 \mathrm{E}+00 \quad 5.43808 \mathrm{E}+00 \quad 5.37555 \mathrm{E}+00 \quad 5.31303 \mathrm{E}+00 \quad 5.25050 \mathrm{E}+00$ $5.18798 \mathrm{E}+00 \quad 5.12546 \mathrm{E}+00 \quad 5.06293 \mathrm{E}+00 \quad 5.00011 \mathrm{E}+00 \quad 4.93676 \mathrm{E}+00 \quad 4.87341 \mathrm{E}+00$ $4.81006 \mathrm{E}+004.74670 \mathrm{E}+004.68335 \mathrm{E}+00 \quad 4.62000 \mathrm{E}+00 \quad 4.55785 \mathrm{E}+00 \quad 4.50960 \mathrm{E}+00$ $4.46135 \mathrm{E}+004.41311 \mathrm{E}+00 \quad 4.36486 \mathrm{E}+00 \quad 4.31661 \mathrm{E}+00 \quad 4.26837 \mathrm{E}+00 \quad 4.22012 \mathrm{E}+00$ $4.17259 \mathrm{E}+004.12591 \mathrm{E}+00 \quad 4.07922 \mathrm{E}+00 \quad 4.03253 \mathrm{E}+00 \quad 3.98585 \mathrm{E}+00 \quad 3.93916 \mathrm{E}+00$ $3.89247 \mathrm{E}+003.84715 \mathrm{E}+00 \quad 3.80762 \mathrm{E}+00 \quad 3.76809 \mathrm{E}+00 \quad 3.72856 \mathrm{E}+00 \quad 3.68904 \mathrm{E}+00$ $3.64951 \mathrm{E}+00 \quad 3.60998 \mathrm{E}+00 \quad 3.57046 \mathrm{E}+00 \quad 3.53257 \mathrm{E}+00 \quad 3.49484 \mathrm{E}+00 \quad 3.45710 \mathrm{E}+00$ $3.41937 \mathrm{E}+003.38163 \mathrm{E}+00 \quad 3.34390 \mathrm{E}+00 \quad 3.30616 \mathrm{E}+00 \quad 3.27013 \mathrm{E}+00 \quad 3.23503 \mathrm{E}+00$ $3.19993 \mathrm{E}+003.16483 \mathrm{E}+00 \quad 3.12973 \mathrm{E}+00 \quad 3.09463 \mathrm{E}+00 \quad 3.05953 \mathrm{E}+00 \quad 3.02453 \mathrm{E}+00$ $2.99285 \mathrm{E}+002.96117 \mathrm{E}+00 \quad 2.92950 \mathrm{E}+00 \quad 2.89782 \mathrm{E}+00 \quad 2.86615 \mathrm{E}+00 \quad 2.83447 \mathrm{E}+00$ $2.80279 \mathrm{E}+002.77404 \mathrm{E}+002.74622 \mathrm{E}+00 \quad 2.71840 \mathrm{E}+00 \quad 2.69058 \mathrm{E}+00 \quad 2.66276 \mathrm{E}+00$ $2.63494 \mathrm{E}+002.60712 \mathrm{E}+00 \quad 2.58266 \mathrm{E}+00 \quad 2.56176 \mathrm{E}+00 \quad 2.54085 \mathrm{E}+00 \quad 2.51995 \mathrm{E}+00$ $2.49905 \mathrm{E}+002.47814 \mathrm{E}+00 \quad 2.45724 \mathrm{E}+00 \quad 2.43634 \mathrm{E}+00 \quad 2.41290 \mathrm{E}+00 \quad 2.38907 \mathrm{E}+00$ $2.36525 \mathrm{E}+002.34142 \mathrm{E}+00 \quad 2.31760 \mathrm{E}+00 \quad 2.29377 \mathrm{E}+00 \quad 2.26995 \mathrm{E}+00 \quad 2.24957 \mathrm{E}+00$ $2.23152 \mathrm{E}+002.21348 \mathrm{E}+00 \quad 2.19544 \mathrm{E}+00 \quad 2.17740 \mathrm{E}+00 \quad 2.15936 \mathrm{E}+00 \quad 2.14132 \mathrm{E}+00$ $2.12363 \mathrm{E}+002.10669 \mathrm{E}+002.08976 \mathrm{E}+002.07282 \mathrm{E}+002.05588 \mathrm{E}+00 \quad 2.03894 \mathrm{E}+00$ $2.02200 \mathrm{E}+002.00510 \mathrm{E}+00 \quad 1.98906 \mathrm{E}+001.97303 \mathrm{E}+001.95699 \mathrm{E}+00 \quad 1.94095 \mathrm{E}+00$ $1.92491 \mathrm{E}+001.90887 \mathrm{E}+001.89283 \mathrm{E}+001.87760 \mathrm{E}+001.86342 \mathrm{E}+00 \quad 1.84923 \mathrm{E}+00$ $1.83505 \mathrm{E}+001.82086 \mathrm{E}+001.80667 \mathrm{E}+001.79249 \mathrm{E}+001.77882 \mathrm{E}+00 \quad 1.76780 \mathrm{E}+00$ $1.75678 \mathrm{E}+001.74576 \mathrm{E}+001.73475 \mathrm{E}+001.72373 \mathrm{E}+00 \quad 1.71271 \mathrm{E}+00 \quad 1.70169 \mathrm{E}+00$ $1.69227 \mathrm{E}+001.68304 \mathrm{E}+001.67381 \mathrm{E}+001.66459 \mathrm{E}+001.65536 \mathrm{E}+00 \quad 1.64613 \mathrm{E}+00$ $1.63690 \mathrm{E}+001.62623 \mathrm{E}+001.61466 \mathrm{E}+001.60309 \mathrm{E}+001.59152 \mathrm{E}+00 \quad 1.57995 \mathrm{E}+00$ $1.56838 \mathrm{E}+001.55681 \mathrm{E}+00 \quad 1.54525 \mathrm{E}+00 \quad 1.53368 \mathrm{E}+001.52211 \mathrm{E}+00 \quad 1.51054 \mathrm{E}+00$ $1.50101 \mathrm{E}+001.49324 \mathrm{E}+001.48547 \mathrm{E}+001.47770 \mathrm{E}+001.46993 \mathrm{E}+00 \quad 1.46215 \mathrm{E}+00$ $1.45438 \mathrm{E}+001.44661 \mathrm{E}+00 \quad 1.43884 \mathrm{E}+00 \quad 1.43107 \mathrm{E}+00 \quad 1.42329 \mathrm{E}+00 \quad 1.41563 \mathrm{E}+00$ $1.40810 \mathrm{E}+001.40056 \mathrm{E}+001.39303 \mathrm{E}+001.38550 \mathrm{E}+001.37796 \mathrm{E}+00 \quad 1.37043 \mathrm{E}+00$ $1.36290 \mathrm{E}+001.35536 \mathrm{E}+001.34783 \mathrm{E}+001.34030 \mathrm{E}+001.33276 \mathrm{E}+00 \quad 1.32523 \mathrm{E}+00$ $1.31770 \mathrm{E}+001.31016 \mathrm{E}+00 \quad 1.30372 \mathrm{E}+00 \quad 1.29810 \mathrm{E}+00 \quad 1.29248 \mathrm{E}+00 \quad 1.28685 \mathrm{E}+00$ $1.28123 \mathrm{E}+001.27561 \mathrm{E}+001.26999 \mathrm{E}+001.26437 \mathrm{E}+001.25875 \mathrm{E}+00 \quad 1.25313 \mathrm{E}+00$ $1.24751 \mathrm{E}+001.24189 \mathrm{E}+001.23627 \mathrm{E}+001.23065 \mathrm{E}+001.22503 \mathrm{E}+00 \quad 1.21941 \mathrm{E}+00$ $1.21379 \mathrm{E}+001.20817 \mathrm{E}+001.20286 \mathrm{E}+001.19872 \mathrm{E}+001.19457 \mathrm{E}+00 \quad 1.19042 \mathrm{E}+00$ $1.18628 \mathrm{E}+001.18213 \mathrm{E}+001.17798 \mathrm{E}+001.17383 \mathrm{E}+001.16969 \mathrm{E}+001.16554 \mathrm{E}+00$ $1.16139 \mathrm{E}+001.15725 \mathrm{E}+001.15310 \mathrm{E}+001.14895 \mathrm{E}+001.14480 \mathrm{E}+00 \quad 1.14066 \mathrm{E}+00$ $1.13651 \mathrm{E}+001.13236 \mathrm{E}+001.12822 \mathrm{E}+001.12525 \mathrm{E}+001.12246 \mathrm{E}+00 \quad 1.11968 \mathrm{E}+00$ $1.11690 \mathrm{E}+001.11412 \mathrm{E}+001.11133 \mathrm{E}+001.10855 \mathrm{E}+001.10577 \mathrm{E}+00 \quad 1.10298 \mathrm{E}+00$ $1.10020 \mathrm{E}+001.09742 \mathrm{E}+001.09464 \mathrm{E}+00 \quad 1.09185 \mathrm{E}+00 \quad 1.08907 \mathrm{E}+00 \quad 1.08629 \mathrm{E}+00$ $1.08351 \mathrm{E}+001.08072 \mathrm{E}+00 \quad 1.07794 \mathrm{E}+001.07532 \mathrm{E}+001.07286 \mathrm{E}+00 \quad 1.07039 \mathrm{E}+00$



[^0]:    * This data set is generated with the frequency distribution at the critical temperature of $647,2 \mathrm{~K}$.

