FUN*da*MENTALS of Design

Topic 4
Linkages

Linkages

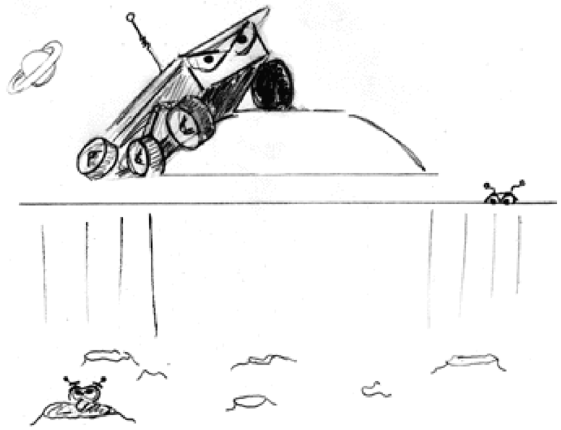
Linkages are perhaps the most fundamental class of machines that humans employ to turn thought into action. From the first lever and fulcrum, to the most complex shutter mechanism, linkages translate one type of motion into another. It is probably impossible to trace the true origin of linkages, for engineers have always been bad at documentation. Images of levers drawn in Egyptian tombs may themselves be documenting ancient (to them!) history. But given their usefulness, linkages will be with us always. They form a link to our past and extend an arm to our future. As long as we keep turning the technological crank, they will couple our efforts together so all followers of technology can move in sync.

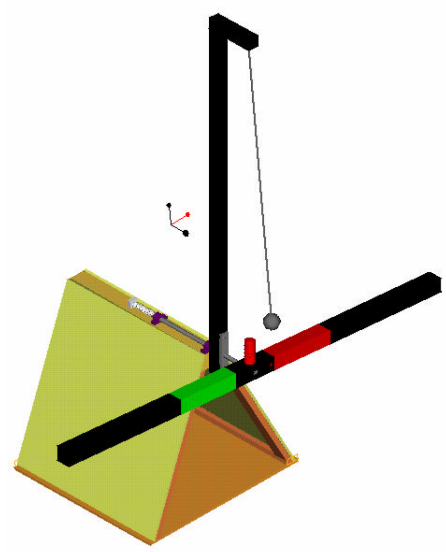
As you read this chapter on linkages, it is important to realize that history plays a vital role in the development of your own personal attitude towards becoming competent at creating and using linkages. As it was with many other areas of engineering, applied mathematicians and their curiosity for how their new analysis tools could be used to understand problems (opportunities!) catalyzed the discovery of linkages and analysis methods. The study of linkages is a very mature and rich subject area but it is by no means over. On the contrary, entire courses are dedicated to teaching students how to master what is and is not known about the design of linkages. Perhaps what is not known is just waiting for someone like you to make the next discovery! In particular, most of us are confined to using simple four or six bar linkages that move in a plane, but the world is three dimensional and waiting for you![1]

Fortunately, for us mere mortal linkage designers, there is powerful linkage design software that seemlessly links to many solid modelling programs. Just lkike snowboarding, you have to learn on the bunny slope before you ride extreme slopes, and you must learn the basics of linkage design before you attempt to zoom from the top! Accordingly, this chapter will focus on the fundamentals of linkage design: physics, synthesis and robust design & manufacturing.[2]

1. An awesome book containing many great mechanism ideas is N. Sclater and N. Chironis, Mechanisms and Mechanical Devices, McGraw-Hill, New York, 2001
2. If the design of machines is of real interest, you should take a course on the design of mechanisms where the entire focus of the course would be on the details of designing many different types of mechanisms from linkages to gear trains. An excellent reference is A. Erdman, G. Sandor, S. Kota, Mechanism Design, 2001 Prentice Hall Upper Saddle River, NJ USA

Topic 4
Linkages





Topics

- History
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- Links
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- 3-Bar Linkages
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- 6-Bar Linkages
- Extending Linkages
- Compliant Mechanisms
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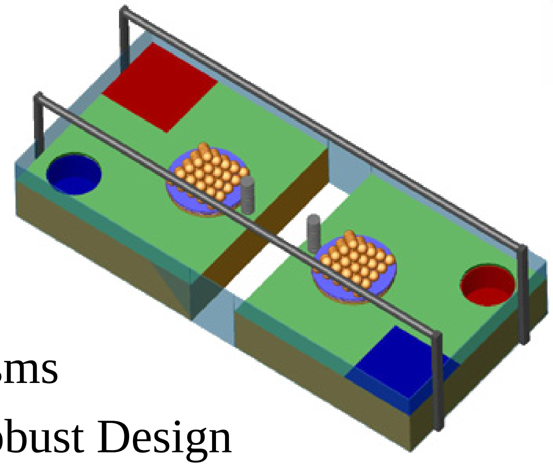
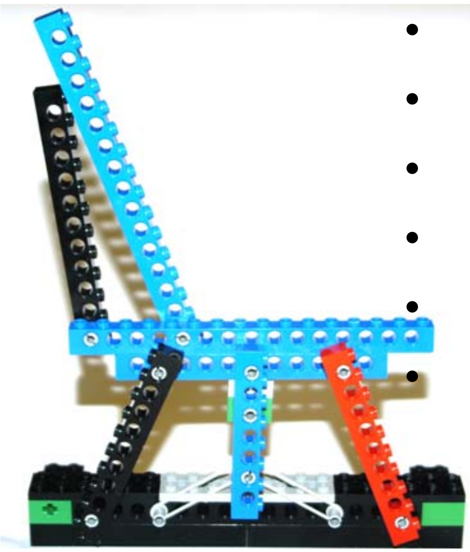
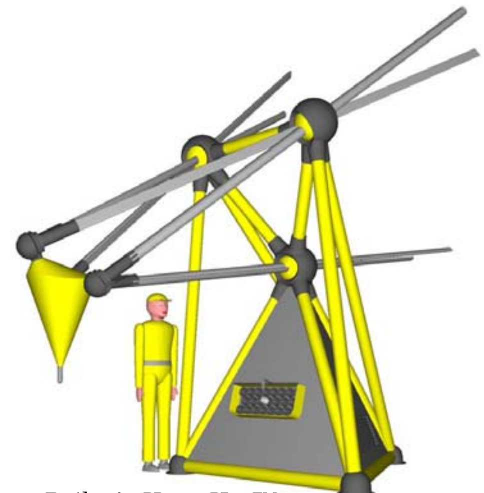
- *Mechanism Mania!*







Peter Bailey's *HyperHex*™
hexapod machining center

History

A *machine* is the combination of two or more machine elements that work together to transform power from one form to another. While the first *tools* used by humans are likely to have been rocks or sticks, the first *machine* was likely to have been a lever and fulcrum. More advanced machines also utilize control systems, which in the early days were also mechanical. This allowed machines to do work without humans attending to the their every function.

Could it be that the simple levers were mistakenly discovered when Ogette stepped on a fallen tree and she saw one end of the tree lift up another heavier tree that had fallen across it? Something was observed somewhere, and the lever was born as a means to amplify the force of a human. Simple cranes are also likely to have emerged, where the simplest crane merely used rope to extend the reach of the lever and the means of force application. From there, the idea that things could be combined to magnify and/or direct forces likely catalyzed the development of many new machines.

Was it watching a farmer turn over soil that gave Archimedes the idea for the screw? Who thought of using a screw to move an abject and thus created the first machine tool? Who first thought of toothed wheels and why? Leonardo da Vinci drew gears as wheels with protruding pegs, but these early ears wore quickly. Who observed the wear that accompanies simple peg-type gears might be done away with by using an involute tooth form so motion between the teeth could be made to be rolling like that of a wheel? Who put all these elements together to create machine tools to form metal faster so we could make more machines? Humans' curiosity and drive were amplified by religion as perhaps best described by Francis Bacon:

"The introduction of new inventions seems to be the very chief of all human actions. The benefits of new inventions may extend to all mankind universally; but the good of political achievements can respect but some particular cantons of men; these latter do not endure above a few ages, the former forever inventions make all men happy, without either injury or damage to any one single person. Furthermore, new inventions are, as it were, new erections and imitations of God's own works."

A consistent theme in the development of precision linkages has been time, although it was not until 1000 AD that the first Chinese water clocks appeared. In the 1300's mechanical clocks appeared in Europe and their value in navigation became a strategic technology that was mastered by one of the greatest precision mechanicians of all time John Harrison[1]. The more accurate the timepiece, the more accurate the navigation, and this trend continues to this day. This quest for precision in timepieces and the machines used to make them and other tools and instruments is well documented by Evans.[2] In addition, a review of the development of the most accurate machine tools which formed the foundation of our modern society is given by Moore[3]. Without precision mechanical machines, we would still be an agrarian society.

The birth of the modern history of linkages is often associated with James Watt who some say invented the steam engine; however, it was not Watt who invented the steam engine which perhaps had its origins in ancient Egypt as a means to open temple doors.[4] However, it was Watt who recognized the need for the application of thermodynamics, even though the subject was not yet invented, to increase efficiency of steam engines. He then give birth to the flyball governor to control the speed of an engine. Once steam was harnessed, the industrial revolution took off, and many other great minds linked together to create new machines and analytical tools to predict their performance in order to conserve scare resources.

Think about what people have done through the ages with observation and curiosity and the drive to understand! So it should be with you! With a few hours application of fundamental principles, catalyzed by simple experiments, countless days of frustration in the shop can be saved!

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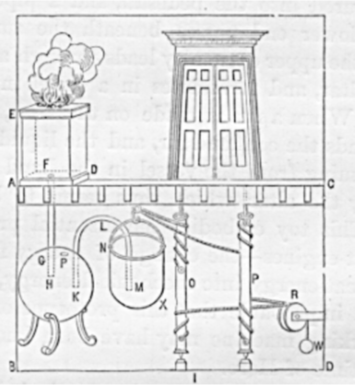
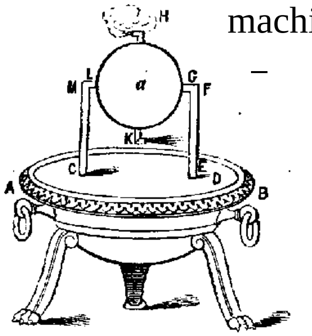
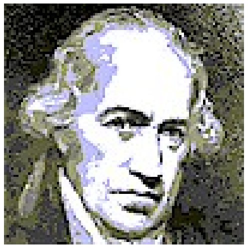
1. Dave Sobel, Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time
2. Chris Evans, Precision Engineering: An Evolutionary View, 1989 Cranfield Press, Cranfield, Bedford, England.
3. Wayne Moore, Foundations of Mechanical Accuracy, Moore Special Tool Co.
4. See for example http://www.history.rochester.edu/steam/thurston/1878/



History

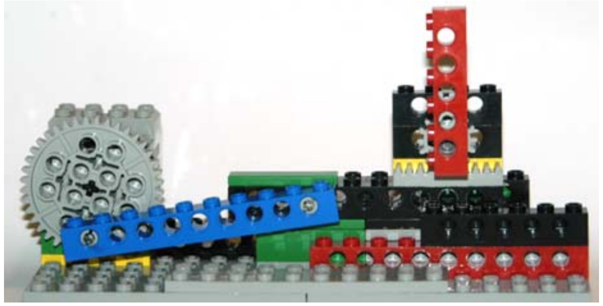
- The weaving of cloth gave rise to the need for more complex machines to convert waterwheels' rotary motion into complex motions
- The invention of the steam engine created a massive need for new mechanisms and machines
  - Long linear motion travel was required to harness steam power
    - *James Watt* (1736-1819) applied thermodynamics (though he did not know it) and rotary joints and long links to create efficient straight line motion
      - Watt also created the *flyball governor*, the first servomechanism, which made steam engines safe and far more useful
    - *Leonard Euler* (1707-1783) was one of the first mathematicians to study the mathematics of linkage design (synthesis)





- Most linkages are *planar*, their motion is confined to a plane
  - The generic study of linkage motions, *planar* and *spatial*, is called *screw theory*
    - *Sir Robert Stawell Ball* (1840-1913) is considered the father of screw theory
- There is a HUGE variety of linkages that can accomplish a HUGE variety of tasks
  - It takes an entire course just to begin to appreciate the finer points of linkage design
- History is a GREAT teacher: See http://kmoddl.library.cornell.edu/ for a fantastic collection of linkages created through the years, many of which are still very useful today!

http://visite.artsetmetiers.free.fr/watt.html



http://www.fcs-cs.com/motionsystems/productsandappl

The First Mechanism: *The Lever is a 2-bar Linkage*

The simplest mechanism, and perhaps the first, is a lever and a fulcrum. The lever is a *link*, the fulcrum a *joint*, and the ground is also a link. Together they form a *2-bar linkage*. These simple elements (a tree branch and a rock) with a force (Og) can create huge forces to do useful work. Once a person witnesses the mechanical advantage offered by a lever, they never seem to forget it, and often use it. From using a pry bar, or sometimes naughtily a screwdriver, to pry open a box, to a wine bottle opener, many of us use levers in our daily lives. Got pliers? A pair of pliers is essentially two levers that share a common fulcrum and hence are essentially levers placed back-to-back. Got scissors? Scissors shear paper (and rock smashes scissors) and the mechanism is again a pair of levers placed back-to-back with a common pivot.

Have you ever tried to cut thick wire or a bolt with a pair of wire cutters and just could not do it? Have you ever then taken the time to do the job right so you went and got a pair of bolt cutters and then found the job was easier? Thinking of the philosophy of physics and fundamental laws, why did the bolt cutters work so well and the wire cutters did not? You might have thought that the bolt cutters had longer handles and thus gave you more leverage, and that is partially correct. Energy is essentially conserved and the bolt cutters let you apply the force of your muscles over a much longer distance, so the cutting force acting over a small distance of travel becomes very high.

What differentiates bolt cutters from a simple giant size pair of wire cutters, is that the bolt cutters have a linkage that allows them to achieve in a much smaller space the amplification of force. Large bolt cutters use what is known as a *5-bar linkage*, and if you count the *links* and the *joints* in the picture, you see that there are 5 of each. You will soon see from *Gruebler's Equation* that there are 3*(5 - 1) - 2*5 = 2 *degrees of freedom*, which means that you need to control each handles' motion in order to control the motion of the linkage. This actually gives great versatility in their use as to how you grab and squeeze the handles, or place on of them on the floor and then lean your belly onto the other handle... Smaller cheaper bolt cutters have just a 4-bar linkage with 4 *links* and 4 joints and 3*(4 - 1) - 2*4 = 1 *degree of freedom*. This means they will not open as wide which makes them less ergonomic for monster cutting applications, but they will often do the job. Returning to the pliers, they have two links and one joint or 3*(2 - 1) - 2*1 = 1 *degree of freedom.*

The right linkage must be selected and engineered for the right job, *BUT* if you want higher performance with more action in less space, you often have to use a more complex linkage! Fortunately, even higher order linkages are essentially just cascaded series of levers. Regardless of the type of linkage, they are all based on simple elements, and the analysis of their motion is based on simple trigonometric relations. Likewise, an analysis of their force capabilities is based on simple vector cross products, which are also themselves based on simple trigonometry. In either case, the forces on bolt cutters are huge. Consider you might apply 100 Newtons of force over 500 mm of motion, but the jaws may only close over a range of 5 mm; hence the force on the cutting edges may be 10,000 Newtons! What about the links and joints?

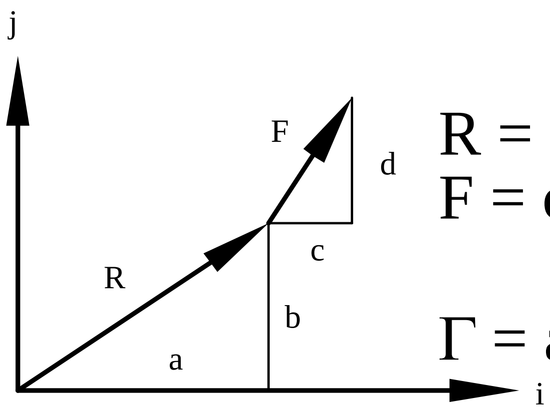
With this simple introduction, your curiosity should be piqued, but in order to move along the desired path of learning to design linkages, definitions must first be established, followed by an understanding of the different types of links and joints and how they operate together. Then different types of linkages, their mechanics, and the synthesis (creation) of their designs can be considered in detail. For example, starting with the idea of a simple 2D lever, the micro silicon Nanogate is essentially a circular plate whose outer circumference is bent down, causing it to pivot about an annular ring and open a small gap up between the center of the plate and a bottom plate.[1]

The pendulums in the robot design contest The MIT and the Pendulum represent significant scoring potential if you could clamp on to them, climb up to the supporting axle, and spin them like a propellor. How could you engage the round support shaft in order to cause the pendulum to spin? Again, how could you ensure that the clamping force remains sufficient and constant? Is some sort of suspension system in order? Might this suspension system use some sort of linkage? On the other hand, maybe you want to block pendulum motion and focus on scooping balls and pucks and dumping them in the scoring zone. Take a close look at construction equipment! In either case, remember, you have other duties and a vibrant social life, so you need tools to enable you to rapidly create and engineer awesome linkages. Taking the time to learn how to engineer linkages, as opposed to just blindly trying stuff will save you a LOT of fruitless failures! Read on and read carefully!

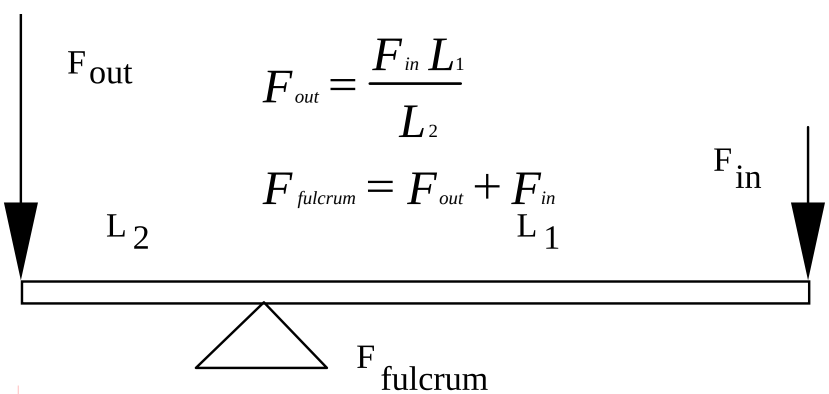
1. The Nanogate is a Micro Electro Mechanical System (MEMS), and it is the thesis topic of James White, who is one of Prof. Slocum's graduate students. See US Patent #5,964,242 and White, J., Ma. H., Lang, J. and Slocum, A. "An instrument to control parallel plate separation for nanoscale flow control." Rev. Sci. Inst. v. 74 no. 11, Nov. 2003.

The First Mechanism: *The Lever is a 2-bar Linkage*

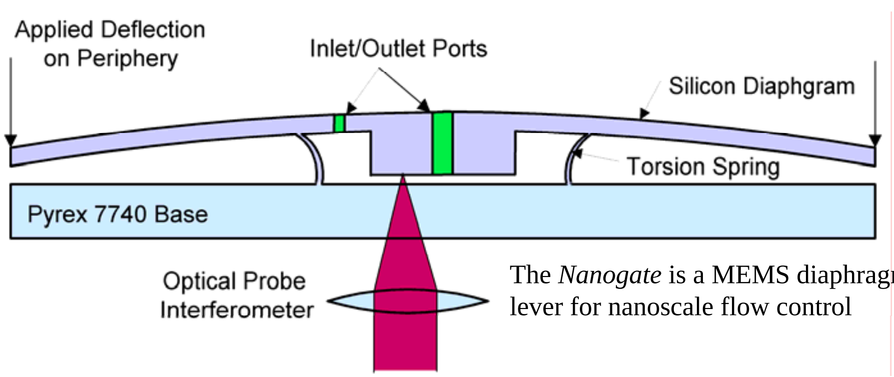
- A lever (link) can be used with a fulcrum (pivot) against the ground (link) to allow a small force moving over a large distance to create a large force moving over a short distance…
  - When one considers the means to input power, a lever technically becomes a 4-bar linkage
- The forces are applied through pivots, and thus they may not be perpendicular to the lever
  - Torques about the fulcrum are thus the best way to determine equilibrium, and torques are best calculated with vector cross product
  - Many 2.007 machines have used levers as flippers to assist other machines onto their backs…



$$R = ai + bj$$
$$F = ci + dJ$$
$$\Gamma = ad - bc$$



$$F_{out} = \frac{F_{in} L_1}{L_2}$$
$$F_{fulcrum} = F_{out} + F_{in}$$



The *Nanogate* is a MEMS diaphragm-type lever for nanoscale flow control

Definitions

A ***linkage*****,** or ***kinematic chain***, is an assembly of *links* and *joints* that provide a desired output motion in response to a specified input motion. A ***link*** is a nominally rigid body that possess at least 2 nodes. A ***node*** is an attachment point to other *links* via *joints*. The ***order*** of a *link* indicates the number of *joints* to which the *link* is connected (or the number of *nodes* per link). There are ***binary*** (2 nodes), ***ternary*** (3 nodes), and ***quaternary*** (4 nodes) links. A ***joint*** is a connection between two or more *links* at their nodes, which allows motion to occur between the *links.* A ***pivot*** is a joint that allows rotary motion, and a ***slider*** is a joint that allows linear motion. A ***mechanism*** is a *kinematic chain* in which at least one link is connected to a frame of reference (ground), where the ground is also counted as a link.

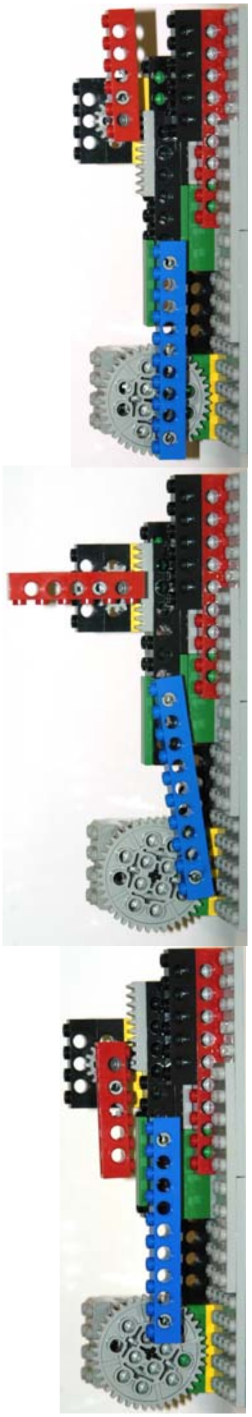
Even a lever with some sort of means to apply an input force is a linkage. One of the most common types of *linkages* is the ***4-bar linkage***, which is comprised of four links and four joints. A ***ground link*** acts as the reference for all motions of the other three links, and attached to it is the power input device, usually a motor, and another joint. The motor output shaft is connected to the link called the ***rocker***, in the case of oscillating input motion, but the same link is called the ***crank***, in the case of continuous input motion. The ***follower*** is connected to the *ground link* through a joint at one end. The ***coupler*** link couples the ends of the *crank* (or *rocker*) and the *follower* links. These four links are thus geometrically constrained to each other; however, their motion may not be deterministic, for there are link lengths and ground joint locations that can lead to instability in the linkage. Even though two points define a line, a straight line structure need not connect the region between the nodes of a link. A link may be curved or have a notch-shape to prevent interference with some other part of the structure or linkage as it moves.

Because each end of the *coupler* is connected to links which may not be of the same length or orientation, the *coupler* is a link not connected to ground that undergoes complex motion. It is often the “output” link for the mechanism (particularly in a 4-bar linkage) and its motion is often very nonlinear and of the highest interest. One very important and insightful means of describing the motion of the *coupler* at any instant in time, is the ***instant center****.* For very small motions, the *instant center* is the point about which a link appears to rotate. Because the coupler’s ends are constrained to move with the ends of the crank and follower links, whose ends themselves trace out circles, the instant center is the imaginary center of a circle which has radii that are coincident with the radii of the crank and follower links’ circles. Hence the *instant center* can be found by drawing lines through the link’s pivots, and the point at which they intersect is called the *instant center.* The *instant center* can also be used to help determine stability, but more on this later (see pages 4-16 to 4-18)

The number of ***degrees-of-freedom*** (DOF) of a linkage is equal to the number of input motions needed to define the motions of the linkage. When one looks at a 4-bar linkage and sees the coupler translating and rotating as it moves, the coupler does not have 3 degrees of freedom (x, y, θ) because the motions are all related. Indeed, the linkage has only 1 DOF. Is there a way to quickly look at a linkage and determine its degrees-of-freedom? ***Gruebler’s Equation*** as described on the facing page is perhaps the most commonly used equation for evaluating simple linkages. From *Gruebler’s Equation* we can see that a 2-bar linkage, an arm attached to a motor’s output shaft will have 1 DOF. A 3-bar linkage with 3 links and 3 joints will have 0 DOF, as expected, and hence triangles make stable structures! A 4-bar linkage has 4 links and 4 joints and 1 DOF. 5-bar linkages can be configured many different ways and thus may have more than 1 DOF. However, these are not generally stable unless multiple input power sources are used. 6-bar linkages can have 1 DOF and they can be extraordinarily useful.

There are many different processes for designing linkages. ***Synthesis*** is the process used to create a linkage. ***Number synthesis*** is the determination of the number and order of links needed to produce desired motion. ***Kinematic synthesis*** is the determination of the size and configuration of links needed to produce desired motion. In either method, ***precision points*** are the defined desired position and orientations of a link at a point in its motion.

What sort of motions may require you to create a linkage for your machine? Can a linkage enable your machine to meet the starting space constraints and then unfold into a bigger machine?

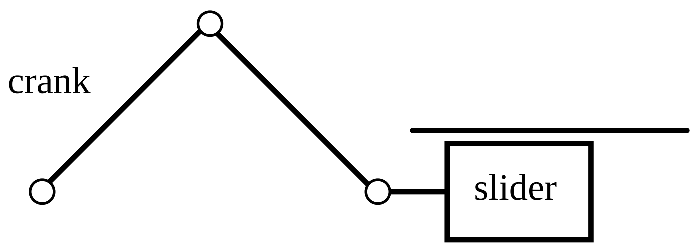


Definitions

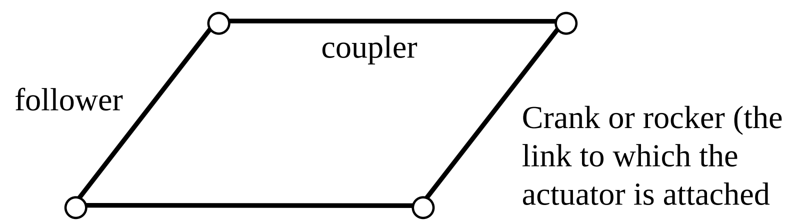
- *Linkage:* A system of *links* connected at *joints* with rotary or linear bearings
  - *Joint (kinematic pairs):* Connection between two or more links at their nodes, which allows motion to occur between the links
  - *Link:* A rigid body that possess at least 2 nodes, which are the attachment points to other links
- *Degrees of Freedom (DOF):*
  - The number of input motions that must be provided in order to provide the desired output, OR
  - The number of independent coordinates required to define the position & orientation of an object
  - For a *planar* mechanism, the degree of freedom (mobility) is given by *Gruebler's Equation*:

$$F = 3(n-1) - 2f_1$$

  - n = Total number of links (including a fixed or single ground link)
  - f_1 = Total number of joints (some joints count as f = ½, 1, 2, or 3)
    - Example: Slider-crank $n = 4$, $f_1 = 4$, $F = 1$
    - Example: 4-Bar linkage $n = 4$, $f_1 = 4$, $F = 1$
    - The simplest linkage with at least one degree of freedom (motion) is thus a 4-bar linkage!
    - A 3-bar linkage will be rigid, stable, not moving unless you bend it, break it, or throw it!



4 links (including ground), 4 joints



4 links, 4 joints

Links

The four most common links are known as ***binary***, ***ternary***, ***quaternary*** and ***pentanary*** links and they have two, three, four and five joints (nodes) respectively on their structures. Look closely at the picture of the excavator and try to identify each of these types of links. What types of links represent the hydraulic cylinders? The hydraulic cylinders have pivot joints at each end, and the rod slides inside the cylinder; thus a hydraulic cylinder is comprised of two binary links, each with a pivot joint and a slider in between. Note the first link, which has the name of the excavation company printed on it. What type of link is it? This link has a pivot at its base, which cannot be seen but obviously it must be present, a pivot at its end for the second link, and two other pivots to which hydraulic cylinders are attached; thus it is a *quaternary* link. How about the second major link? How many joints are on it and what type of link is it? Look closely and you can see it is a *pentanary* link.

Examine the bucket, which itself is a binary link, and see that is connected with several other links to form what type of linkage? Imagine that the hydraulic cylinder was taken off for repair. The bucket is connected to the boom link and to a *binary* link which is connected to another *binary* link that is also connected to the boom link. The bucket could be said to be a *follower*, and the *binary* link opposite it is a *rocker* link. Thus the bucket linkage is a *4-bar linkage*. The *rocker* is driven by the hydraulic cylinder which is connected to the boom link. Recall from above that the hydraulic cylinder is modeled as two binary links with pivots at their ends, but they happen to share a slider joint. Thus the bucket system is comprised of two 4-bar linkages that share a common link. The *follower* for one (the hydraulic cylinder side) and the rocker for the other (the bucket side). Together, they actually form a 6-bar linkage.

Gruebler's Equation was developed to enable a designer to quickly ascertain the mobility or degrees of freedom in a linkage. For the bucket linkage, there clearly are 4 links and 4 joints, and so 3*(4 - 1) - 2*4 = 1 degree of freedom. Physically, this means the bucket can only move in a single prescribed path and observation of an excavator will show this to be true.[1] Similarly, the hydraulic cylinder side of the linkage has 4 links and 4 joints so it is also a single degree of freedom linkage. If the bucket is removed, the small binary link that is attached to the end of the hydraulic cylinder rod will also move in a prescribed path. What would happen if we just counted all the links and joints at once? The boom forms one ternary link for consideration of the bucket motion linkage. The bucket is a binary link and there are two other binary links to which it is attached. The hydraulic cylinder is comprised of two binary links, and hence the total number of links is 6. There are 5 pivots and one slider joint which is the joint between the hydraulic cylinder the rod. Gruebler's Equation would then indicate that there are 3*(6 - 1) - 2*6 = 3 degrees of freedom! Something is wrong, because we indeed know that there is just one deterministic motion the bucket makes and there is just one actuator. Indeed the joint where the hydraulic cylinder rod and the two binary links are joined at a common node is called a second order pin joint, and it counts as 2 joints in Gruebler's Equation. Thus the bucket actuation systems has 3*(6 - 1) - 2*7 = 1 degree of freedom. As linkages get more complex, the use of Gruebler's Equation becomes more apparent, for we want mechanism to be exactly constrained to have the number of degrees of freedom desired.

Consider the two linkage systems shown. Although they appear similar, they are different in that the "coupler" link in one is a single ternary link, whereas the other has two binary links instead. In the latter system, which is similar to the bucket linkage in that it is two 4-bar linkages linked together (do not forget the second order pin joint!), Gruebler's Equation gives 3*(6 - 1) - 2*7 = 1! In the former system, Gruebler's Equation gives 3*(5 - 1) - 2*6 = 0! Indeed, unless all the dimensions of all the links were perfect, or the joints had enough clearance in them, the linkage would lock up or it would produce very high forces on the joints that would cause premature wear.

Links are indeed considered as rigid elements for the purpose of synthesis of a linkage, but of course they are subject to real loads; hence before a linkage is to be manufactured, a careful stress analysis must be performed. This may sometimes require the size of the links to be increased, which may interfere with the motion of the links; thus some design iteration may be required. In fact, out-of-plane motion and loading often requires links and joints to be substantially sized to also accommodate out-of-plane forces.

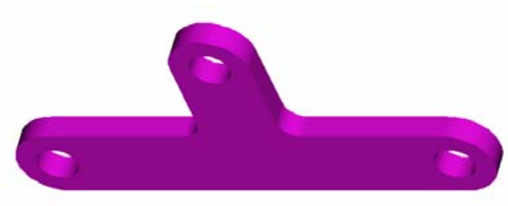
How would you resign the overconstrained linkage with 2 followers? What sort of links might your system need? Will your ideas for a linkage have enough room to accommodate structurally appropriate links?

1. If you have never watched an excavator work, you must rent one of those great construction videos little kids like to watch. Ask someone you are interested in to watch one with you as a date movie!

Links

- Binary Link: Two nodes:

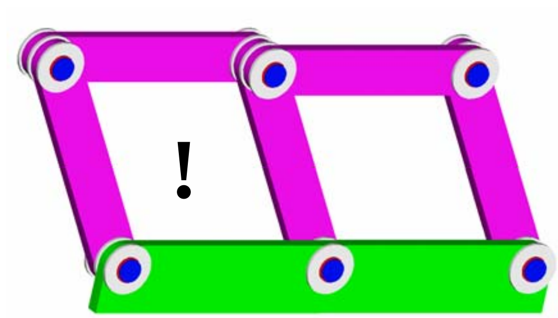
- Ternary Link: Three nodes:

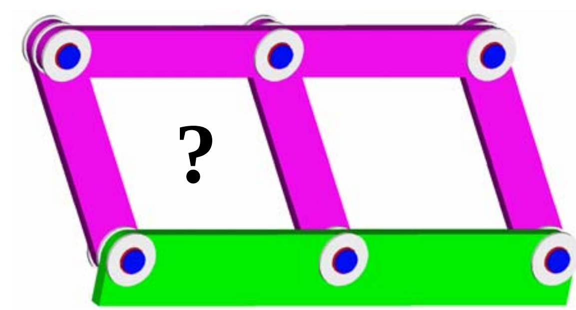


- Quaternary Link: Four nodes:

- Pentanary Link: Five nodes! *(Can you find it?!)*

Can you identify all the links?





Joints: *Single Degree-of-Freedom*

Recall that a ***pivot*** is a joint that allows rotary motion and a ***slider*** is a joint that allows linear motion. They are single degree of freedom joints for which $f = 1$ in Gruebler's Equation. They and others share the common characteristic that they must transmit loads from one member to another, and they must do so with a certain amount of precision lest the joint wobble too much and reduce mechanism quality and robustness. Thus they need bearings which must be carefully engineered as discussed in Chapters 10 and 11.

The simplest joint that allows rotational motion to occur between two links is the ***revolute*** (R) joint. Also called a *pin joint* or a *pivot*, generally it is formed by a pin that passes through both links. One end of the pin has typically been formed and after the pin is placed in the joint, a snap-on clip is placed on the other end. Sometimes the other end is cold-formed in placed to create a permanent joint that is not likely to fail by means of a fastener coming off. The crudest form of a pin joint, often used in simple robot design contests, is made with a screw, but the motion of the joint acts on the threads which can cause a lot of wear and a lot of error. It can also literally screw itself apart. It is far better to use a shoulder bolt or a shaft with snap-on clips on the ends. Even better, it would be desirable to press-fit the pin in one of the links and to provide clearance between the pin and the other link.

Note that a *revolute* joint is referred to as a planar joint because the links are nominally confined to move in a plane; however, the links are actually offset from each other. Therefore loads are offset by the half-thicknesses of the links and a moment is transmitted across the joint. The moment can cause bending in the links and the pins in the joints, and the resulting stresses will have to be evaluated. The best pivot joint is symmetrical with the end of one link flared into a U shape and the other link between it, so there are no moment loads on the links. This is called a *clevis*. The pin is primarily in shear, and at worst, acts as a simply supported beam. This is the way many highly loaded joints on construction equipment are designed.

The next most common joint is the ***prismatic*** (P) joint, which is also called a slider or sliding joint, and it allows for linear motion to occur between two links. From drawers to windows, sliders are commonplace, but beware Saint-Venant when selecting proportions of the joint elements as discussed on page 3-3 in order to minimize the chances of the joint jamming. Crank mechanisms also often use sliders, and they have the same precision issues as revolute joints do as far as loads and errors are concerned.

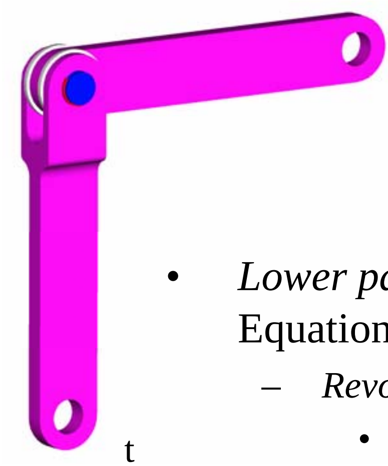
Helical (H) joints, also called *screws*, are another common joint which form the basis for a common means to transform rotary power into linear power. Beware of thread strength, friction and efficiency, all of which are discussed in detail in Chapter 6! Screws can be used in place of hydraulic cylinders to actuate linkages, where they can have the advantage of they are not backdriveable and thus fail-safe.

Return to the issue of clearances between joint components, which can be too large and create quality and robustness problems. Recall Abbe-type (sine) errors discussed beginning on page 3-8. Shown here are pictures of the gaps that must exist between Lego™ bricks and the cumulative effect allowing a long wall to be curved. In addition, a diagram of how these sine errors manifest themselves in a pivot joint are also shown. Note the large amplification δ of the angular error ε on the end of the link! For a pin to fit into a joint and allow easy motion, there must be some clearance between the pin and the joint. This allows the links to twist about their length, causing the planes of the links to no longer be parallel. How would you calculate the twist error that could occur? Drawing the system in the ideal and twisted cases shows that the tilt ε of the shaft in the hole and the amplified sine error δ are:

$$\varepsilon = \operatorname{atan}\left(\frac{D-d}{t}\right) \qquad \delta = L\varepsilon$$

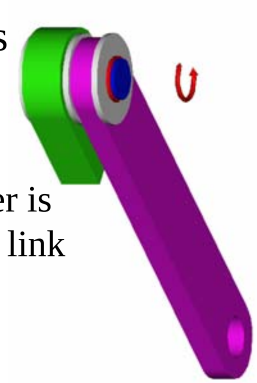
A design engineer must often develop a closed-form expression that can be used to select a clearance or a dimension before one details a mechanism. Solid modeling software generally does not allow a designer to design a machine with all the clearances required, and then enter "wiggle" to see how floppy the mechanism might be. The all too common method of "build it and see what happens, and if it's too floppy we can tighten it up" is costly and in the case of a design contest, you do not have such time to waste. When assessing the risk of a mechanism, you must ask yourself "what unwanted error motions can the clearance in the joints cause?"

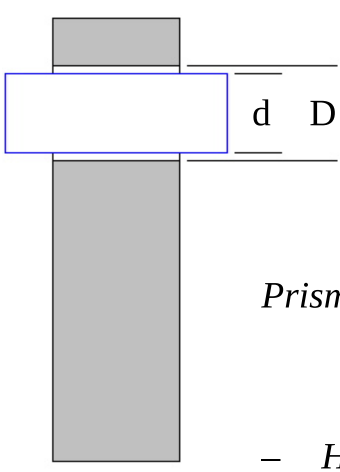
What is the effect on machine performance of clearances in joints on the accuracy or repeatability of mechanisms you are contemplating?



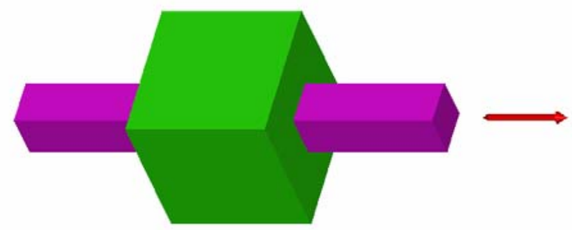
Joints: *Single Degree-of-Freedom*

- *Lower pairs (first order joints)* or *full-joints* (counts as $f = 1$ in Gruebler's Equation) have one degree of freedom (only one motion can occur):
  - *Revolute (R)*
    - Also called a pin joint or a pivot, take care to ensure that the axle member is firmly anchored in one link, and bearing clearance is present in the other link
    - Washers make great thrust bearings
    - Snap rings keep it all together
    - A *rolling contact* joint also counts as a one-degree-of-freedom revolute joint

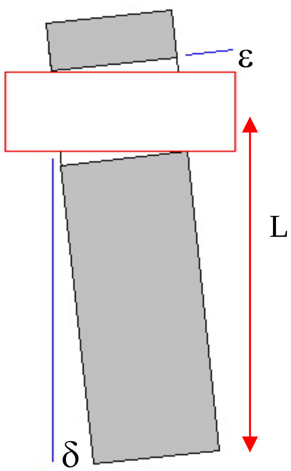


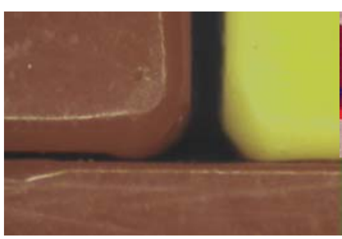


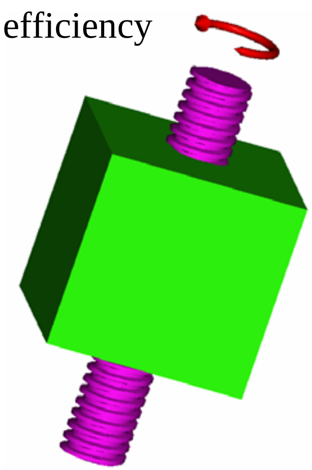
  - *Prismatic (P)*
    - Also called a slider or sliding joint, beware Saint-Venant!



  - *Helical (H)*
    - Also called a screw, beware of thread strength, friction and efficiency







Joints: *Multiple Degree-of-Freedom*

Some joints allow for multiple degrees of freedom, which can yield large space savings; however, this also means that much more care needs to be taken when considering joint clearance and the potential for error motions to cause problems. A common two degree of freedom joint is the ***Cylindrical*** (C) joint in which $f = 2$ in Gruebler's Equation. This joint is formed by a *bushing*, a round sliding bearing, that fits over a round rod, which allows the bushing to slide or rotate on the rod. It is a *superposition* of a *pivot* and a *slider.* Sometimes the motions are large, as would be required for some types of robot manipulators where an insertion and twist is required. In the earlier discussion of hydraulic cylinders, it was said that the piston rod and cylinder have a slider joint between them, which would count as 1 in Gruebler's Equation when analyzing linkages such as that in the excavator. This is true for the analysis of a planer linkage problem. However, the rod is actually free to rotate in the cylinder, so it would be possible to use this joint as a cylindrical joint if needed.

A ***Spherical*** (S) joint is a three degree of freedom joint in which $f = 3$ in Gruebler's Equation. This joint is commonly found in automotive and aircraft linkages where the primary degree of freedom is the revolute motion. The other two rotational degrees of freedom provide for small motions to accommodate deflections that usually occur in a suspension system. A common machine element that incorporates these features is called a ***rod-end***, and it is typically threaded onto the end of a link, and the threaded connection allows for an adjustment in length. Spherical bearings can use sliding contact bearing interfaces or spherical rollers to allow rolling motion to minimize friction. Such bearings allow for large shaft deflections without the shaft deflection causing moment loads on the bearings which could cause excessive loading. In addition, they accommodate manufacturing misalignment errors.

All linkages must accommodate error motions between components ranging from joint tolerance errors to deformations caused by heavy loads. In a machine like an excavator, for example, revolute joints must have some clearance between the pins and the bearings to allow for small angular motions (misalignments). This effectively gives them some very limited spherical motion capacity, but they should not be considered spherical joints. When reasonably large errors or deflections must be accommodated, an actual spherical joint must be used.

The generic spherical joint shown consists of a spherical socket with a mating ball, such as found in your hip! Unfortunately, the ball can never be made to exactly fit the socket, and friction will also always be present in a sliding contact joint. When greater accuracy and lower friction are required, small rolling balls can be used as the interface between the socket and ball. A common machine component with this design is a ***ball transfer***. Ball transfers are used in large arrays to allow heavy planar objects to roll across them. INA Corp. also manufactures a precision version of this concept as a spherical rolling element joint for precision parallel kinematic machine tools. An example is a *hexapod* which uses six extendable legs to support a moving platform.

A ***Planar*** (F) joint is a three degree of freedom joint that allows for two translational motions and a rotational motion in a plane (X, Y, and θ) so $f = 3$ in Gruebler's Equation. As mentioned above, ball transfers can be placed on a plane to allow for this type of motion. A more exotic, but increasingly common use of this type of joint is in planer stepper motor named a *Sawyer Moto*r after its inventor. The plane is comprised of raised square iron features where the gaps between them are filled in with epoxy. The platen containing the three motor coils floats above this surface using pressurized air (air bearings). Two of the motor coils are orthogonal to each other and provide the two translational motions. The third coil is parallel and offset from one of the other coils. Together, two coils form a force couple that can provide small rotational motion and rotational stiffness. This design forms a planar robot, and such machines have formed the basis for high speed high precision pick-and-place machines used in the semiconductor industry[1]. Their primary advantage is that as stepper motors, they do not require feedback measurements to control their position; however, their primary drawback is that they require a service loop (cable bundle) to deliver power to the coils. At high speed, with many robots on a single surface, entanglement can occur; thus typically only one or two such robots are used on a surface at a time. Because of their simplicity, the mean time between failure (MTBF) and the mean time between service (MTBS) can be in the thousands or tens of thousands of hours.

Can the use of a multiple degree of freedom joint be used to reduce complexity or increase design flexibility in your robot?

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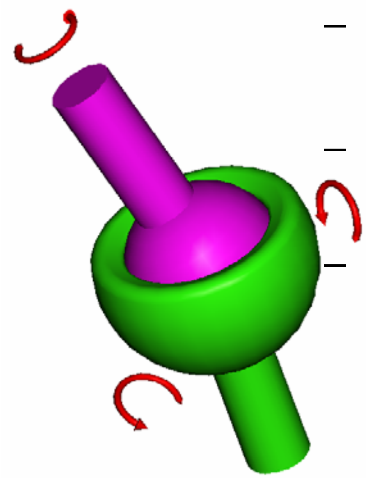
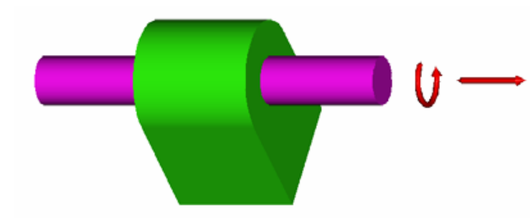
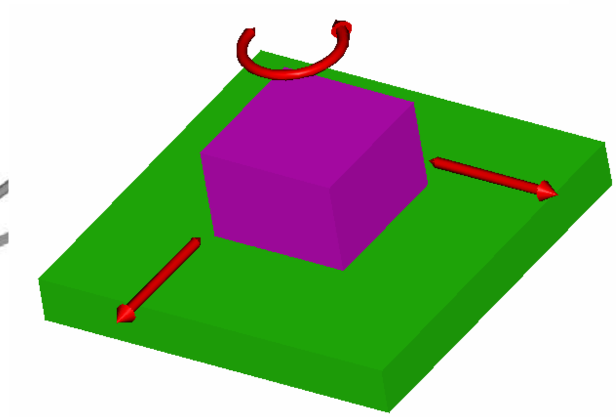
1. See for example W. J. Kim, D.L. Trumper, J.H. Lang, "Modelling and Vector Control of Planar Magnetic Levitator", IEEE Trans. Industry Applications, VOL. 34, NO. 6, 1998, pp 1254-1262

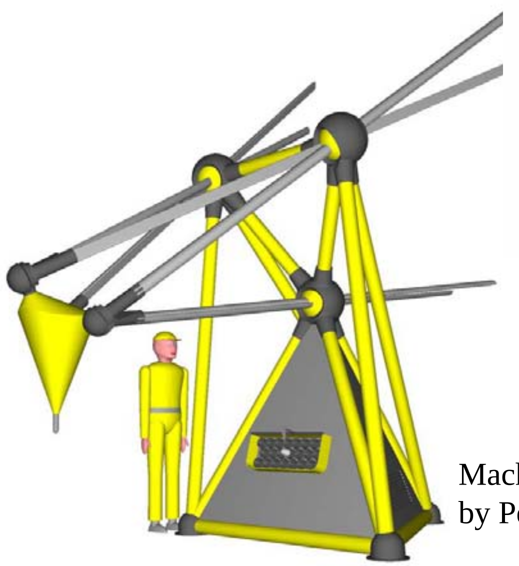
Joints: *Multiple Degree-of-Freedom*

- *Lower Pair* joints with multiple degrees of freedom:
  - Cylindrical (C) 2 DOF
    - A *multiple-joint* (f = 2)
  - Spherical (S) 3 DOF
    - » A *multiple-joint* not used in planar mechanisms (f = 3)
  - Planar (F) 3 DOF
    - A *multiple-joint* (f = 3)

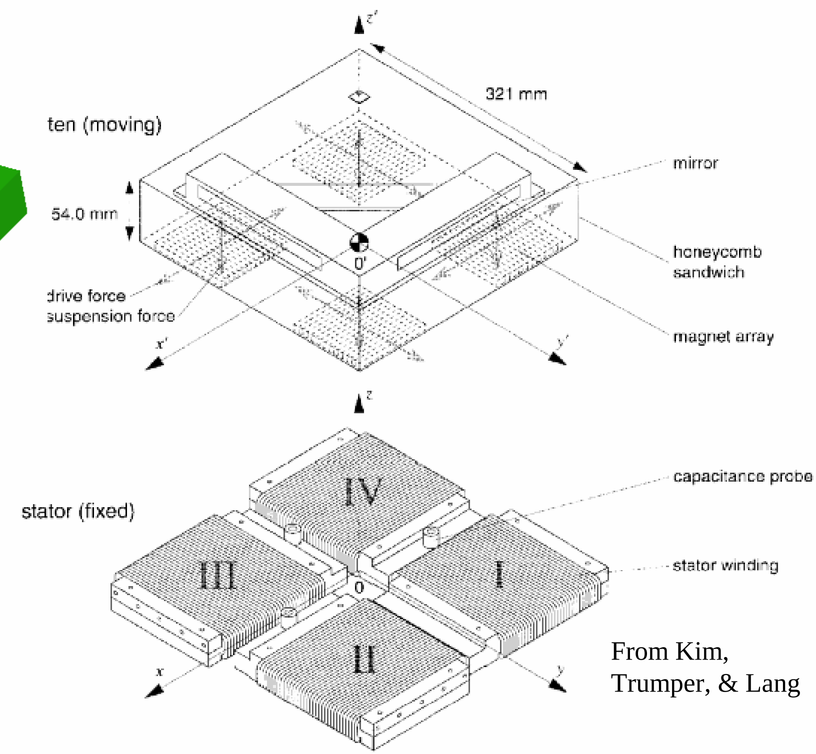








Machine concept
by Peter Bailey



From Kim,
Trumper, & Lang

Joints: *Higher Pair Multiple Degree-of-Freedom*

Higher pair joints are those comprised of multiple elements that can also allow for multiple degrees of freedom. A link acting against a plane is an example of a higher order pair that allows for one linear and one rotary degree of freedom. The link also requires a force to preload it (keep it in contact with the plane) and keep it a *form-closed* joint, and $f = 2$ in Gruebler's Equation. Such a link may be used in a walking mechanism, but it is not very common.

A more common higher pair is a ***pin-in-slot*** joint where a pin allows a link to rotate and the pin itself can slide in a slot. The geometry keeps the joint constrained or closed (form closed). This joint can be considered the combination of a pivot joint and a slider joint into one compact unit. It is commonly used in mechanisms such as those used to open and close casement windows. It it is a multiple-joint for which $f = 2$ in Gruebler's Equation.

Another common joint is a ***second order pin joint***, in which 3 links are joined at a single node. Since the links can move in different directions, depending on how their ends are constrained, it is considered a multiple-joint and so $f = 2$ in Gruebler's Equation. As shown in the picture, this joint is what enables the hydraulic piston to produce a very large range of motion in the excavator bucket. Indeed, this type of linkage is very commonly used in construction equipment to allow a linear actuator to actuate a link through a very large angular range of motion with a much more even torque capability than would be possible if the cylinder pushed directly on the load.

Part of the fun of designing linkages is the geometry problem that one encounters when trying to evaluate ranges of motions and the relationship between actuator force and joint torques. No matter how complex the linkage, imaginary lines can be drawn between nodes to form triangles. Then its just a matter of using trigonometry, especially the laws of sines and cosines, to solve for the unknowns. Analysis is often used during the concept phase to determine the best type of linkage to use. For example, compare two linkages for moving an arm (boom): a simple piston attached to a pivoting arm (a 4-bar linkage with pin joints at points *A*, *B*, and *D*) and a more complex 6-bar linkage, such as used for an excavator bucket, with pin joints at points *A*, *B*, *D*, *E*, and *H*. The lengths of the segments and the angles defined are coded by color, where the black letters are known dimensions and the red and blue dimensions are intermediate calculations. This is helpful for documenting one's analysis so other engineers can follow your work. The solutions for the 4-bar linkage are:

$$e = \sqrt{d^2 + c^2} \qquad f = \sqrt{a^2 + b^2}$$

$$\alpha = \sin^{-1}\left(\frac{d}{e}\right) \qquad \gamma = \sin^{-1}\left(\frac{b}{f}\right)$$

$$\beta = \cos^{-1}\left(\frac{e^2 + f^2 - L^2}{2ef}\right) \qquad \phi = \cos^{-1}\left(\frac{f^2 + L^2 - e^2}{2fL}\right)$$

$$\theta = \pi - \alpha - \beta - \gamma \qquad R = f\sin\phi$$

The solutions for the 6-bar linkage are a bit more involved:

$$i = \sqrt{e^2 + f^2} \qquad j = \sqrt{a^2 + b^2} \qquad k = \sqrt{c^2 + (d - f)^2}$$

$$\beta_1 = \tan^{-1}\left(\frac{d - f}{c}\right) \qquad \beta_2 = \cos^{-1}\left(\frac{k^2 + g^2 - L^2}{2kg}\right) \qquad \alpha_1 = \tan^{-1}\left(\frac{f}{e}\right)$$

$$\beta_3 = 2\pi - \pi/2 - \beta_1 - \beta_2 - \left(\pi - \pi/2 - \alpha_1\right) = \pi - \beta_1 - \beta_2 + \alpha_1$$

$$m = \sqrt{i^2 + g^2 - 2ig\cos\beta_3} \qquad \alpha_2 = \cos^{-1}\left(\frac{i^2 + m^2 - g^2}{2im}\right) \qquad \alpha_3 = \cos^{-1}\left(\frac{m^2 + j^2 - h^2}{2mj}\right)$$

$$\alpha_4 = \tan^{-1}\left(\frac{b}{a}\right) \qquad \phi_1 = \cos^{-1}\left(\frac{g^2 + m^2 - i^2}{2gm}\right) \qquad \phi_2 = \cos^{-1}\left(\frac{g^2 + L^2 - k^2}{2gL}\right)$$

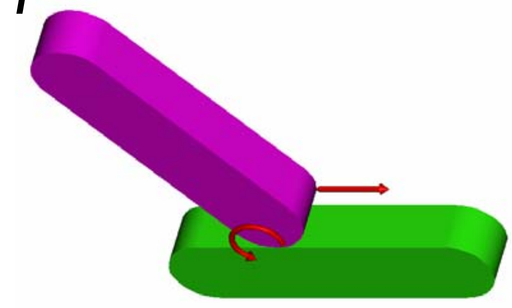
$$\theta = \pi - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 \qquad R = m\sin\left(\phi_1 + \phi_2\right)$$

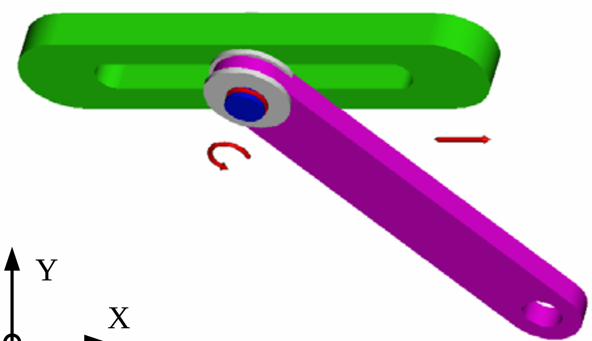
In both cases, the angle θ and the radius R on which the piston acts to create a moment on the output link would be determined for the piston length L as it increases from its contracted to extended states. Plots of θ and R for the 4 and 6-bar linkages can then be done to determine which is the most appropriate for the system being designed. When a large range of motion is required, the 6-bar linkage is well-worth the design and manufacturing effort!

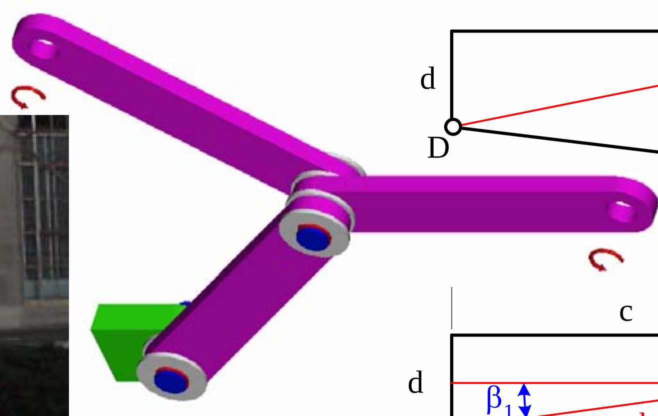
Study the figures carefully and derive the above equations independently. Where in you machine might you want to use a more complex, but larger range of motion 6-bar linkage? Check out the spreadsheets!

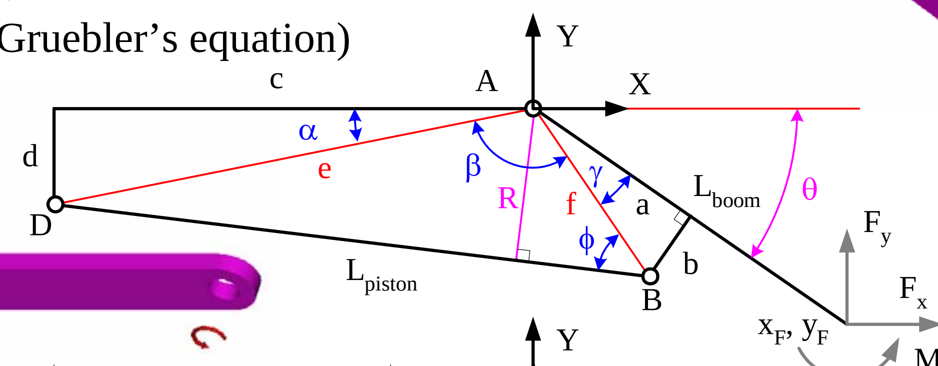
Joints: *Higher Pair Multiple Degree-of-Freedom*

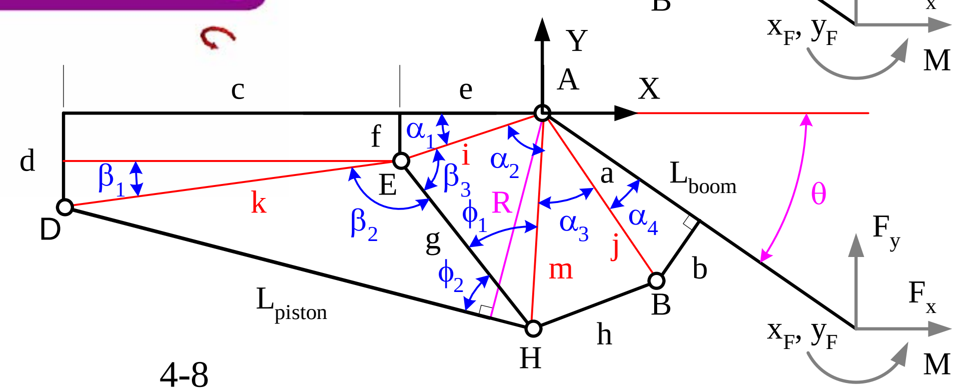
- *Higher Pair* joints with multiple degrees of freedom:
  - Link against a plane
    - A force is required to keep the joint closed (force closed)
      - A *half-joint* (*f* = 2 in Gruebler's equation)
    - The link may also be pressed against a rotating cam to create oscillating motion
  - Pin-in-slot
    - Geometry keeps the joint closed (form closed)
      - A *multiple-joint* (*f* = 2 in Gruebler's equation)
  - Second order pin joint, 3 links joined, 2-DOF
    - A *multiple-joint* (*f* = 2 in Gruebler's equation)











2-Bar Linkages: *Triggers*

A lever and fulcrum is a simple two-bar linkage that has many different uses. Recall that the lever itself is a link to which the input and output forces are both applied. The fulcrum acts as the pivot, and the structure to which the fulcrum is attached is the ground link. Gruebler's Equation gives 3*(2 - 1) - 2*1 = 1 degree of freedom. Pliers allow a small grip force to apply a large grip force. Another particularly useful class of 2-bar linkages are *triggers*. Triggers are used to hold back large forces, such as those from constant force springs, and release them with a small force.

A lever-type (latch) trigger is a simple 2-bar linkage, where the location of the pivot point with respect to the force being resisted (the latch force) determines if the trigger is *hard*, *neutral*, or *hair.* A *hard* trigger is when the dimension y_s is positive so the force acts to keep the trigger from misfiring; however, it requires more force to trigger. A *neutral* trigger is when $y_s = 0$, and it is easy to release. A hair trigger is where y_s is negative and the only thing that keeps it from firing is friction. The equilibrium equation is:

$$y_s F_s - y_t F_t + L_t F_s \left(IF(y_s < 0, -1, 1) * MIN(\mu_o, \frac{\mu_i d_i}{d_o} \right) = 0$$

Friction is dealt with by using a roller, or a curved surface as shown in the figure. If a hard surface is used (no roller), then μ_i is set to a very large number in the above equation. The spreadsheet trigger.xls can be a useful design tool to determine if a roller should be used. it can also be used to ensure that the actuation method used to release the trigger has enough force.

A variation on this type of design is the bent-wire trigger. The wire is shown in blue and is released by pulling up on the purple string. The red string is shown tied, so when it releases its total stroke is limited, but a hook that releases can be used if needed. Be careful of flying parts! Why is the blue wire shown with the wavy bends? Are they really needed?

A simple pin-type trigger uses a pin in a bore. One end of the pin sticks out of the bore and resists a shear force. An axial force applied to the other end of the pin will pull the pin into the bore and release the applied force. Although conceptually simple, the existence of friction can cause the pull force to be large. How should L_1 be determined?

$$\sum M = 0 = F_1 L_2 + F(L_1 + L_2) \qquad \sum F = 0 = F_1 - F_2 - F$$

$$F_1 = \frac{F(L_1 + L_2)}{L_2} \qquad F_2 = \frac{FL_1}{L_2}$$

$$F_{Trigger} = \mu(F_1 + F_2 + F) \Rightarrow 2\mu F \left(\frac{L_1 + L_2}{L_2} \right)$$

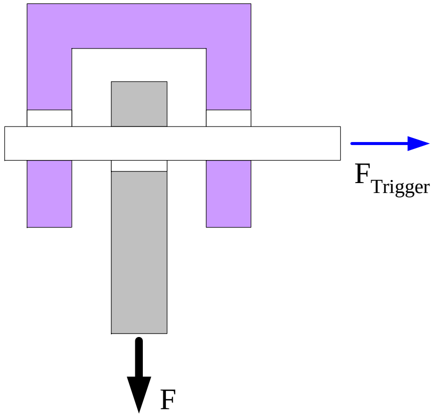
Despite the simplicity of triggers, it is amazing the number of novice designers who do not use these simple equations to optimize their trigger designs. Often they are stuck with triggers that do not release, or release too easily. *Use the equations to design your trigger before you build it!*

Often a machine designed for a contest will want to launch a projectile the moment the contest starts and the machine starts moving. The use of one channel on the control system and one actuator can be saved by using the motion of the machine's wheels as the trigger. To do this, use a *pawl*[1] *trigger* as shown where the pawl would be attached to the same shaft that supports one of the machine's drive wheels. A string can be held in the root of the pawl tooth, and when the wheel starts turning the string is either let go or drawn in to release a trigger. Just make sure that with continued motion of the wheel the string falls clear and does not wind up around the axle.

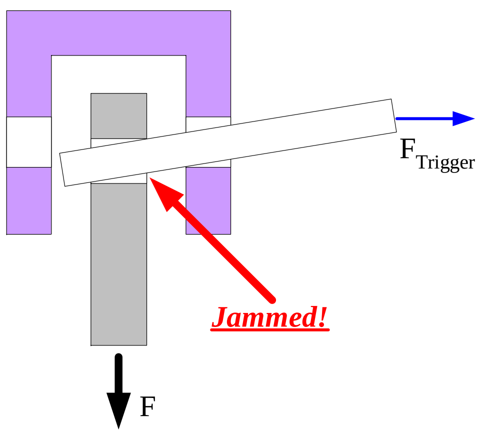
Look for triggers on common objects in your home. Have you examined a classic mousetrap lately? If not, go buy one and examine it (carefully) and take it apart. Sketch a free body diagram of the parts and see if you can determine with what force the mouse steps to trigger the trap. Given the strength of the spring and inertia properties of the moving member, can you determine how long it takes the trap to close? How fast does the mouse have to be? Does it even have a chance to accelerate out of the way? Do you need a trigger in your machine? Can you scale one of the triggers you have seen? How will you analyze your trigger <u>before</u> you build it to make sure it will work?

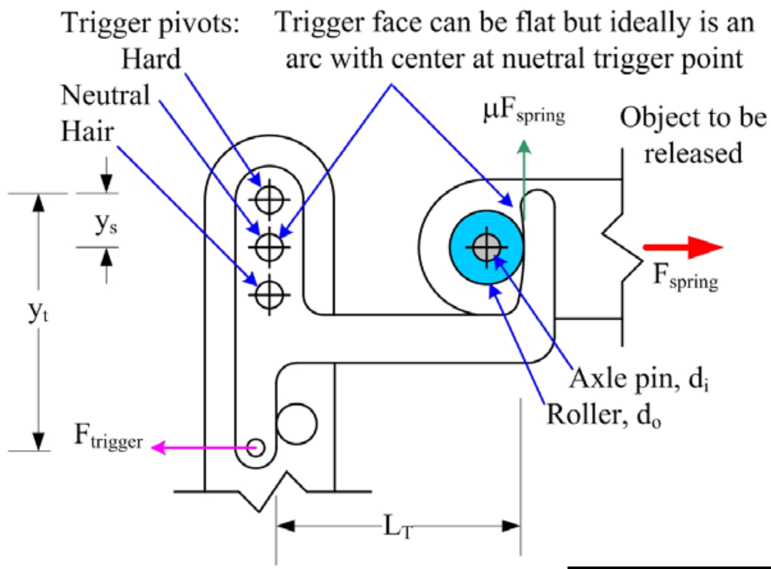
1. A pawl is a toothed wheel where the teeth are angled so in one direction of motion they grab, but slide in the other direction.

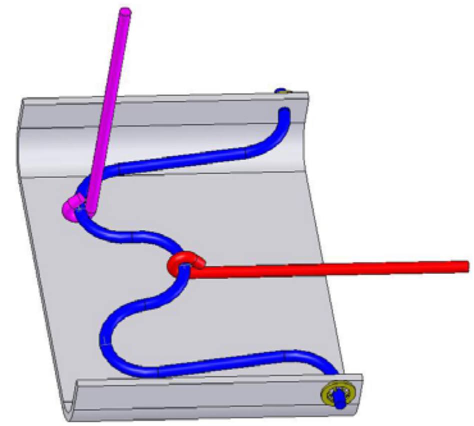
2-Bar Linkages: *Triggers*

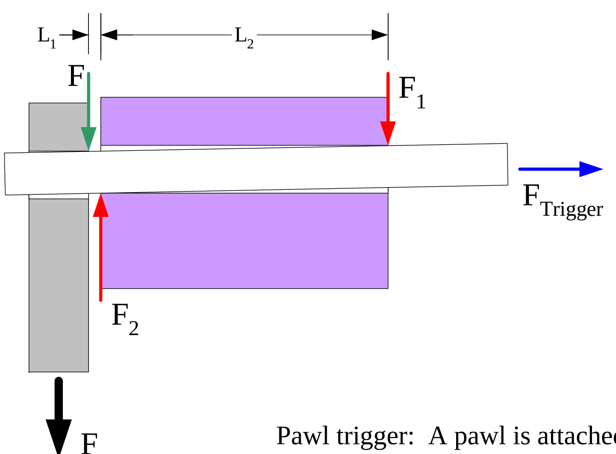


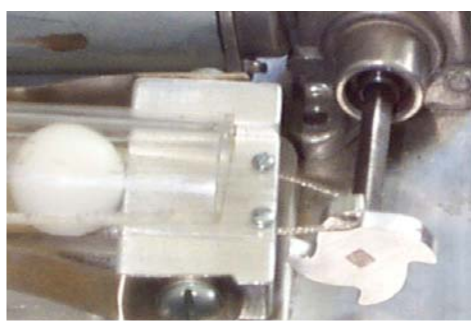
- A trigger is a mechanism that uses a small input to release a big output
  - Stable (hard trigger), neutrally stable, or marginally stable (hair trigger)
  - Beware of fundamentals, e.g., Saint Venant, and stress reliability!
    - Leverage is often the key!











Pawl trigger: A pawl is attached to a shaft (which may also hold a wheel), that releases when the shaft turns

	Trigger.xls To design a trigger By Alex Slocum 8/28/2005 Enter numbers in **BOLD,** results are in **RED** Be consistent with units (e.g., mm, N)	
LT	**50**	Horizontal distance between trigger pivot and trigger latch
ys	**0**	Vertical distance between trigger pivot and trigger latch
yt	**35**	Vertical distace between trigger pull and pivot
di	**6**	Trigger latch pin diameter
mi	**0.05**	Trigger latch pin friction coefficient
do	**12**	Trigger latch roller diameter
mo	**0.1**	Trigger latch-to-roller friction coefficient
Fs	**100**	Force to be held by trigger
Ft	**3.6**	Force to release load
	Stable	Trigger stability

3-Bar Linkages (?!)

A 3-bar linkage has three links and 3 pivots, and Gruebler's Equation gives 3*(3 - 1) - 2*3 = 0 degrees of freedom. However, being a triangle, it is stable even if the links inadvertently change length! Consider the development of a concept for a large low-cost precision gantry machine. In order to achieve precision linear motion, bearings must be spaced apart so they act as a force couples to resist moments. This generally means that the surfaces on which they move are also spaced apart; however, it is not possible for two elements to be exactly parallel, so the ground link's length is not always constant.

Misalignment between bearing rails can be accommodated in many different ways. The simplest way in which misalignment is accommodated is by allowing for clearance between the bearing and the rail. If the loading of the system is always from the same direction, this configuration can still provide acceptable accuracy. The clearance provided can accommodate misalignment, but then this places a limit on the accuracy of the system being supported. Another method that allows for rail misalignment is to mount one of the bearing assemblies rigidly to the moving structure, and compliantly mount the other bearing. This can be achieved with metal flexures or even resilient mounts, such as rubber. However, the product of the misalignment and the flexure stiffness is a force that must be subtracted from the load capacity of the bearing. The use of clearance or compliance in a machine with reasonable precision can typically accommodate 0.1 mm of rail misalignment over the length of the rail.

In order to accommodate misalignment without sacrificing as much performance, the principle of reciprocity can be used. Misalignment is fundamentally an angular error motion that is amplified by distance into a larger displacement between the bearing rails. There must be a way to use angular motion to counter these effects. A sine error, as discussed starting on page 3-8, is a linear distance that results from an angular error being amplified by the length of a machine component on which it acts. It thus makes sense that there should be a way to properly constrain the bearings on two misaligned rails, such that the misaligned rail's errors are accommodated by sine errors.

As shown in the figure, this can be achieved by having one side of a machine's bridge rigidly mounted to a bearing on a rail and the other side mounted to a pivot located atop a link whose base pivot is mounted to a bearing on the misaligned bearing rail. As long as the bearings can accommodate linear as well as rotary motion, they can be preloaded to move with zero clearance. As one bearing rail starts to diverge from the other, the connection via the link with the pivot to the bridge rotates about its bearing mounted on the rail. This also results in some small vertical motion of the bridge, a cosine error, but it can be predicted and in most cases, it is negligible.

Hence the system is stable and rigid as required for a machine tool. It is a 3 bar linkage with 3 pivots. When the spacing between the bearing rails changes, what was the ground link is actually a slider, and the system essentially becomes a 4 bar linkage. The pivots accommodate motion, but for any instant in time, it is a stable 0 degree of freedom 3 bar linkage!

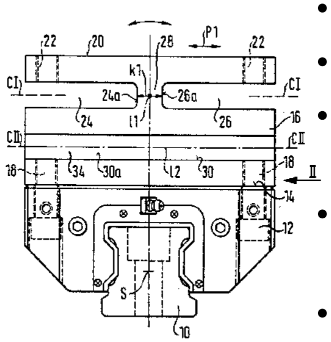
This clever design[1] is an *exact constraint design*, as discussed in principle on page 3-24. If a flexure, or spherical pivot, was not used between the riser and the bridge, then bearing rail misalignment must to be allowed for by bearing clearance or by elastic deformation. This common issue can result in the bearings failing early unless the product of the misalignment and the bearing stiffness is accounted for in the assessment of the load/life analysis for the bearing (see page 10-32). This same lesson can be applied to machines and to linkages. As you read this book, keep thinking of how the links and joints would be designed to have the exactly proper constraints so that they can move just the way they are supposed to be, without overloading and prematurely wearing out the bearings!

Think of your machine as a series of links, some of which are pinned and cannot move, and some that change shape and cause the machine to move. Whatever your machine does, make sure it does only what you want it to!

1. This great patentable idea seemed too simple to the author, so he did a patent search and found US patent 4,637,738. The patent claims the use of angular motion about a round rail and a angularly compliant connection between the bearing and the carriage to compensate for a varying center distance between round rails. This patent was issued January 20, 1987, and a company was worried about using this principle. Since there were no products on the market that appeared to have used this principle, the company was encouraged to check to see if perhaps the independent inventor got tired of paying the maintenance fees and just abandoned the patent. It turns out they did, and so the patent was then in the public domain. The company did the right thing. Of course this did not address US patent 5,176,454 which was essentially the same patent but with a double flexure (X and Y), but its claims were very narrow.

3-Bar Linkages (?!)

- A 3-Bar linkage (is there really a "3-bar" linkage?!) system can minimize the need for precision alignment of bearing ways
  - Accommodates change in way parallelism if machine foundation changes
  - US Patent (4,637,738) now available for royalty-free public use
    - Round shafts are mounted to the structure with reasonable parallelism
    - One bearing carriage rides on the first shaft, and it is bolted to the bridge structure risers
    - One bearing carriage rides on the second shaft, and it is connected to the bridge structure risers by a spherical bearing or a flexure
    - Alignment errors (pitch and yaw) between the round shafts are accommodated by the spherical or flexural bearing
    - Alignment errors (δ) between the shafts are accommodated by roll (θ) of the bearing carriage
    - Vertical error motion (Δ) of the hemisphere is a second order effect
    - Example: $\delta = 0.1''$, H = 4", $\theta = 1.4$ degrees, and $\Delta = 0.0012''$
    - Abbe's Principle is used to the advantage of the designer!



US patent 5,176,454

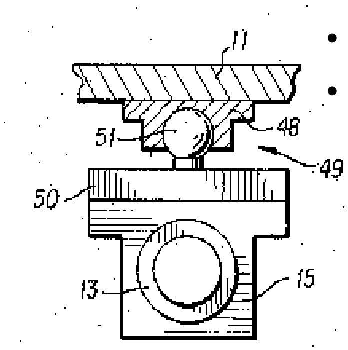
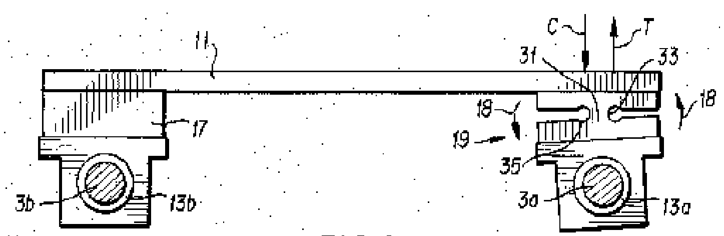
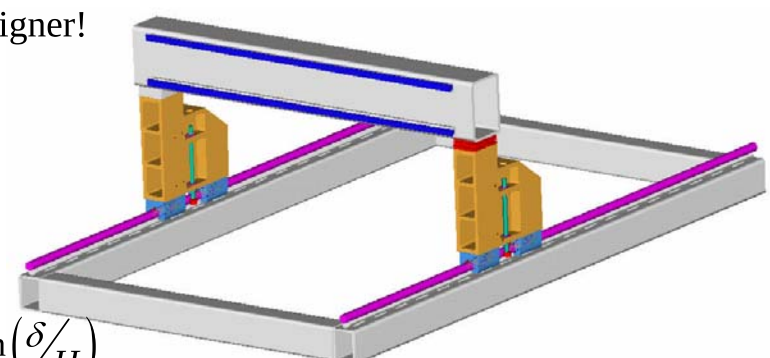


FIG. 9



US patent 4,637,738

FIG. 3



$$\theta = \arcsin\left(\delta/H\right)$$

$$\Delta = H\left(1-\cos\theta\right) \approx \frac{\delta^2}{2H}$$

4-Bar Linkages

A 4-bar linkage has four binary links and 4 revolute joints; hence from *Gruebler's Equation* there are 3*(4 - 1) - 2*4 = 1 *degree of freedom*. This means that only one input is required to make the linkage move. If designed properly, the instant center never becomes coincident with a joint and it will move in a deterministic manner. Because of its simplicity, and perhaps also because of the rapid increase in design complexity suffered by linkages with more than 4 bars, the 4-bar linkage is one of the most commonly used linkages. Thus considerable attention will be paid to its operation and its creation or *synthesis*. In its simplest manifestation, a 4-bar linkage is a parallelogram so the rocker and follower links are parallel and of equal length so the coupler moves without rotation. In this case, the velocities of the coupler in the X and Y directions are respectively:

$$V_x = a\omega \sin\Omega \qquad V_y = a\omega\cos\Omega$$

If the crank is driven by a motor with maximum rated torque Γ, then what is the maximum force F_y that the coupler can support? The easiest way to determine the maximum force is to equate the work-in with the work-out. In addition, we will consider the effect of friction μ in the pin joints of diameter d_{pin} (we know the pin rotation equals the rocker rotation for this configuration):

$$\left(\Gamma - \mu F_y d_{pin}/2\right) d\Omega = F_y d_y \qquad y = a\sin\Omega \qquad dy = a\cos\Omega d\Omega$$

$$F_y = \frac{\Gamma}{\mu d_{pin}/2 + a\cos\Omega}$$

For the generic 4-bar linkage with different length links, as shown on the previous page in the context of instant centers, the same method of equating the work-in to the work-out can be applied. As shown, a force F acting at a radius from a pivot and moving through an angle increment $d\theta$ moves a distance *ds* and does work *Fds*. This is a very important principle that greatly simplifies finding linkage output forces given input forces. It allows the engineer to create a spreadsheet or program to determine the position of the linkage given an input parameter, such as crank angle, and then numerically determine *ds* by incrementing the crank angle by say 0.001 radians. When the forces are significant, or friction high, as is the case for sliding contact bearings, the energy dissipated by friction can be accounted for in the analysis:

$$\Gamma_{\text{rocker\_torque}} d\Omega = F_x dx + F_y dy + \mu\sqrt{F_x^2 + F_y^2}\, d_{pin}/2 \qquad F_{out} = \frac{F_{in}\sqrt{dx_{in}^2 + dy_{in}^2}}{\sqrt{dx_{out}^2 + dy_{out}^2}}$$

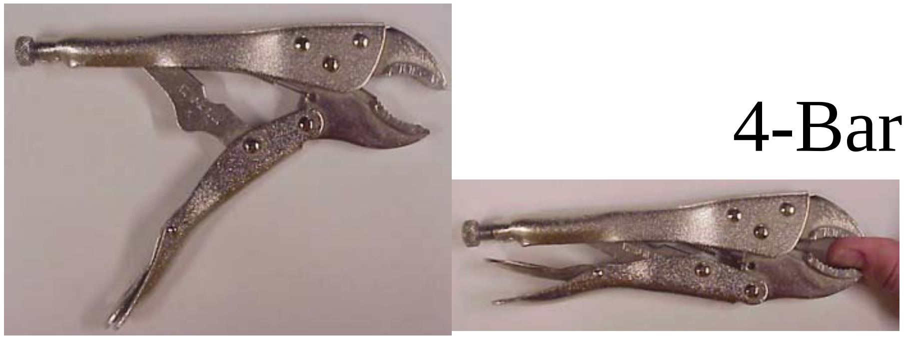
One may design a 4-bar linkage as a parallelogram to provide horizontal motion of the coupler; however, the horizontal X motion will also be accompanied by vertical Y motion. Unwanted deflections in the Y direction are known as *parasitic error motions*. Parasitic error motions also plague structural linkage systems and can lead to a reduction in quality and decreased robustness. For small horizontal motions, the parasitic error motion is determined using small angle approximations to be:

$$\delta_y = \frac{\delta_x^2}{2L}$$

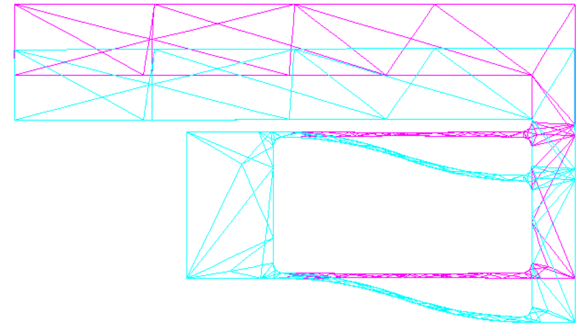
Must pinned joints always be used? No, and in fact, flexural members can be used which are constrained at each end by a zero-slope condition. However, the actuation force must overcome the spring force of the flexures. To avoid pitching motions on flexural element supported platforms that are not subject to external loads, the actuator force should be applied at a point midway between the moving and fixed platforms.[1] Can the error motions and sensitivities to actuation force placement be reduced? The fundamental principle of *Reciprocity*, as discussed on page 3-14, comes to the rescue! The error motion of one set of flexures can accommodate the error motion of the second set by placing both sets back-to-back to create a *folded flexure stage* as shown in the solid model image. These flexures are discussed in detail on page 4-24.

Given the simplicity of a 4-bar flexure, can you think of applications in your machine? How about for a module to scoop up balls or hockey pucks? Or maybe you want to create a linkage that can help your opponent to turn over so they can show the crowds what a nice paint job they did on their machine's belly?!

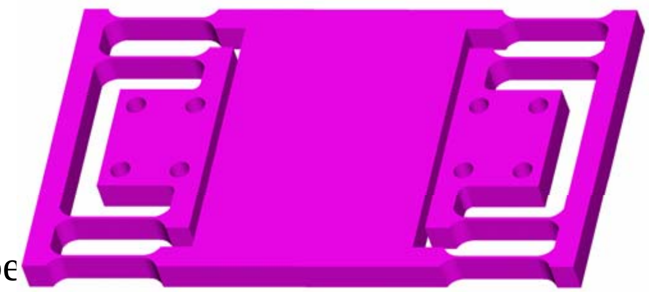
1. Section 8.6, A. Slocum, Precision Machine Design, 1995, Society of Manufacturing Engineers, Dearborn, MI

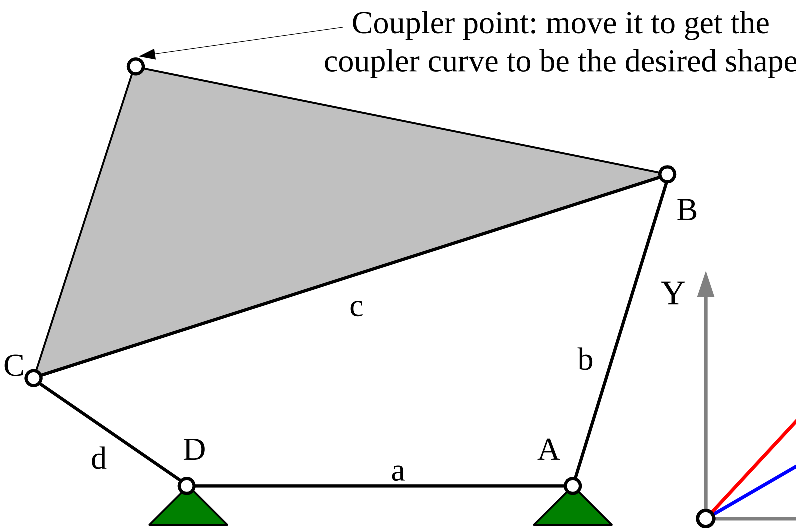


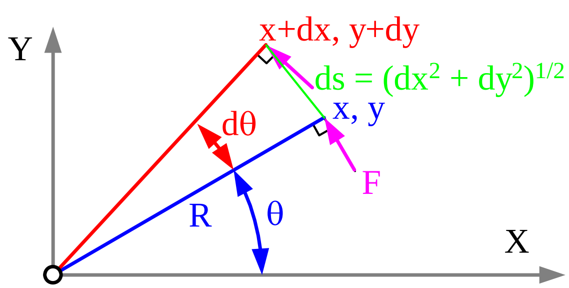
4-Bar Linkages

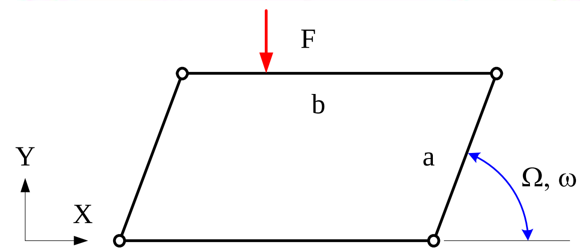


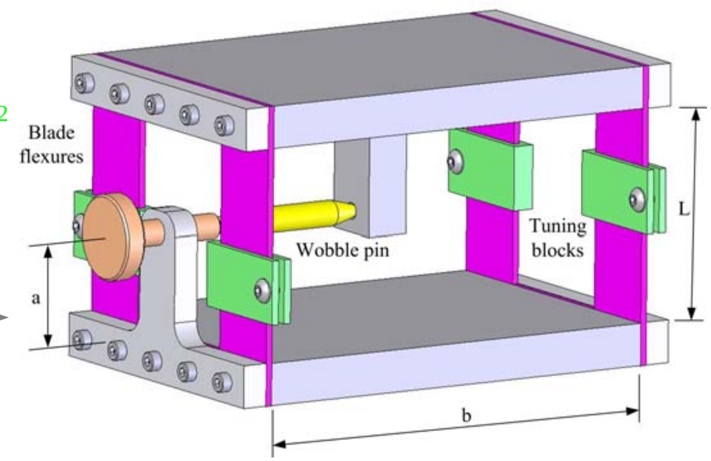
- 4-Bar linkages are commonly used for moving platforms, clamping, and for actuating buckets on construction equipment
- They are perhaps the most common linkage
  - They are relatively easy to create
  - One cannot always get the motion and force one wants
    - In that case, a 5-Bar or 6-bar linkage may be the next be











4-Bar Linkages: *Booms*

4-bar linkages are often used to actuate booms or robot arms. Page 4-8 gave us our first glimpse of a piston actuated 4-bar linkage boom, where equations were presented for the determination of the perpendicular distance from the piston to the pivot point. The analysis showed that if we know the loads applied to the end of the boom, we can find the moment on the pivot *A* and the required piston force F_{piston}. Although the term *piston* is used here, it could just as easily be a leadscrew actuator that is used. Furthermore, note the inclusion of elements of length *b* and *d* which represent offsets for the piston attachment points from boom and link c respectively. These offsets represent a more real-world design than if the pivots were located on the link lines and then the designer would have to do small rotations to align these virtual links up with the reference planes in an actual structure. This small increase in complexity for analysis makes actual dimensioning of mechanism much more realistic and hence faster and less prone to errors.[1]

The spreadsheet *4barpistonlinkage.xls* shows that as a piston extends, the effective radius upon which it acts to create a moment about the boom pivot point *A* decreases substantially. As a result, the required piston force to support the load increases. In some situations, this may mean that the boom also becomes more vertical and the load would be creating less of a moment on the boom. Because this is not always the case, this type of analysis is very valuable. Note that the effect of a moment on the end of the boom is included. This moment could be created by another boom cantilevered off the first boom. One can see this type of arrangement in some types of cranes and in concrete pump trucks' booms.

4barpistonlinkage.xls shows the ground link in a horizontal plane. When the piston retracts, the boom is angled down almost 56 degrees, and then when the piston is fully extended, the boom is nearly horizontal. The ground link *c* could just as well be in the vertical plane, and the spreadsheet is equally valid and useful. All that must be done is to be careful with the magnitude and direction of the input forces. It is also useful to note that the total length of the piston in the extended state is about 50% longer than the contracted length. This reflects the *overhead* associated with the space required for the end pivots and the structure of the piston. If one needed more stroke from a piston, one would use a *telescoping* cylinder. Telescoping leadscrews have also been used in applications such as aircraft control surfaces.

Enter numbers in bold	Results in red				
a (mm)	**75**			Fx (N)	**0**
b (mm)	**50**			Fy (N)	**-20**
cc (mm)	**100**			M (N-mm)	**-1000**
d (mm)	**25**		**Piston (actuator) length**		
Lboom (mm)	**200**			Contracted (mm)	**120**
				Extended (mm)	**180**
Results					
Lpiston (mm)	120	135	150	165	180
R	67.3	59.7	50.0	36.3	5.0
theta (deg)	55.8	43.9	30.6	15.1	-5.0
Fpiston (N)	-48	-65	-89	-134	-997

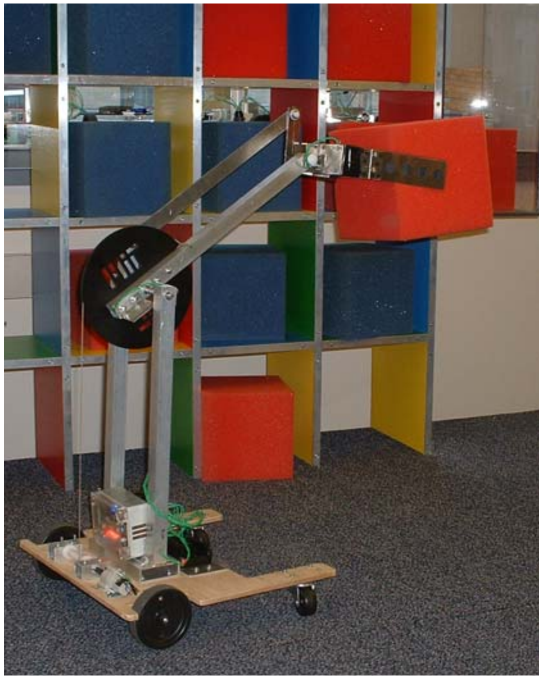
The above analysis only considers the kinematics and overall loading. It does not consider the effect of the loads on the stress in the links. Given the forces from the applied loads and the piston and the angles between links, it is a straightforward exercise to determine the bending moments and hence the required link cross sections to support them. The spreadsheet provided is just a starting point and can easily be modified for your application.

Have you any 4-bar linkages that could be actuated by an extending actuator such as a piston or leadscrew? Would a 4-bar linkage be useful for preloading your vehicle to the square plastic tube so you can drive up to the support tube, engage it and rapidly spin the tube for a large score multiplier? Could you design a 4-bar linkage that lifts up your opponent and perhaps help them turn over onto their back so they could have a nice gentle rest, but keeps the lifting force close to your vehicle so you do not tip over? Synthesize and analyze these linkages and determine what geometries could minimize the forces required to actuate them.

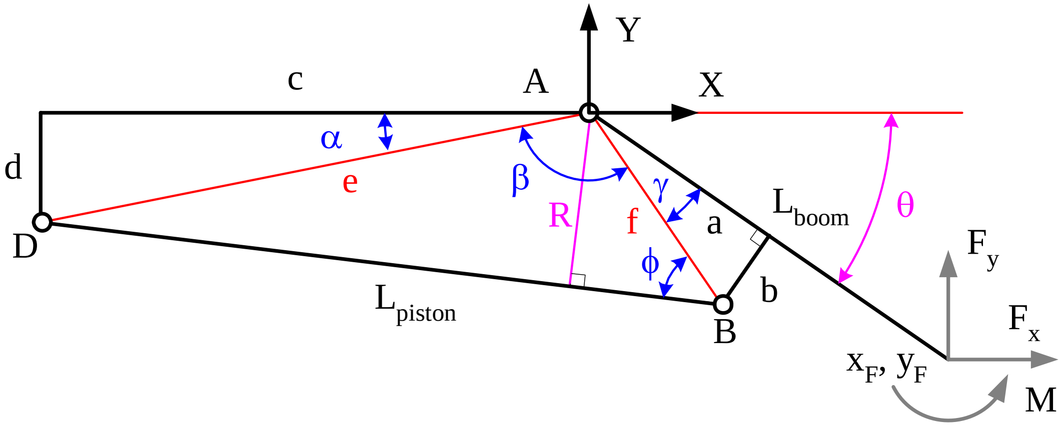
1. The author's first boss and dear friend Donald Blomquist used to say "Silicon is cheaper than cast iron, and it does not rust" to mean use computers in analysis and control to help you minimize mechanical complexity. Don was the Chief of the Automated Production Technology Division at the National Institute of Standards and Technology. He was one of those rare people who understood mechanical and electrical and digital hardware AND software. He died in a boating accident, but he has never left my thoughts. I know that in the future I will join him to ride (although he will be on his skis, but maybe he has had time to learn to snowboard) the deep powder formed by the galaxies that make up our universe.

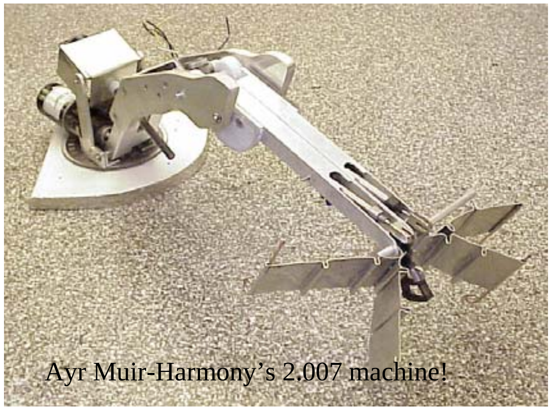
4-Bar Linkages: *Booms*

Mark Cote's winning 2005 2.007 "Tic-Tech-Toe" machine



- Linkages for cranes and booms are 4-bar linkages that replace one of the pivot joints with a slider
  - The *boom* is the *follower* even though it is used as the output link
  - The *piston rod* is the "coupler"
  - The *piston cylinder* is the "rocker", and the connection between the "rocker" and the "coupler" is a *slider joint*
- Link configurations can be determined using parametric sketches, sketch models, or spreadsheets
  - Their simple nature makes them particularly well-suited for development by a spreadsheet





Ayr Muir-Harmony's 2.007 machine!

4-Bar Linkages: *Kinematic Synthesis*

If you are given all the dimensions of a linkage and the input angle of the crank, you can easily determine the position of the coupler. The problem of determining the position of a linkage's elements given their dimensions and constraints, either relative to each other or to the positions of the actuators, is called the *forward (or direct) kinematics problem*. What if you were given desired positions of a coupler and had to find the link parameters that would enable the linkage to move the coupler through the desired positions? This is called the *inverse kinematics problem* when determining the position, such as crank angle, of the actuator(s). *Linkage synthesis* is when the lengths and positions of the links themselves must also be determined.

Image a coupler in three different required positions. The pivots at each end of the coupler in each of these three positions are called *precision points*. The crank and follower must each be attached to the coupler at its ends respectively, and since the crank and follower are also fixed to the ground link by pivots, the task is simply to find the location of the ground pivots. The key skill required for synthesizing 4-bar linkages is to be able to find the center of a circle that passes through three points.

As shown in the figure, to find the center of a circle that passes through three points, **first connect the points with lines. Next, find the perpendicular bisector of each red line by drawing equal radii arcs with their centers at each end the line. Connect the arcs' intersections with a line, which will be the perpendicular bisector for that line. The center of the circle (arc) that contains all three points will lie at the intersection of the perpendicular bisectors.** If this process is done for each end of a coupler, then you will have located the ground pivot locations for both the crank and the follower! This method is called the *three precision point linkage synthesis method*. Finding the center of a circle that contains the three precision points can also be done with the 3-point-circle icon on many CAD systems.

The next step is to find the curve that plots the locations of the coupler's *instant center* as the linkage moves through its desired range of motion. If the instant center ever passes through one of the linkage's joints, then at that point an instability can occur, and the linkage can move in one of two different directions. This generally is not a desirable situation, and thus different precision points might have to be selected, or the follower might have to become the rocker and vice versa!

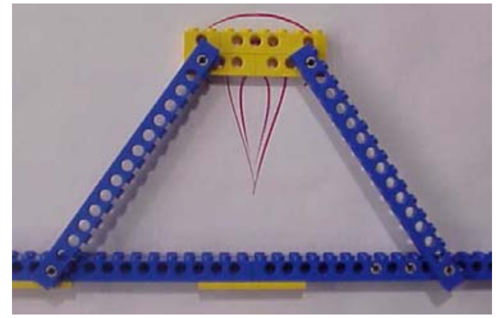
When a 4-bar linkage is a parallelogram, the instability will never occur; so then why would anyone want to use anything else? When designing a bucket for a scoop, for example, it is desired for the coupler to also rotate as it translates. In addition, when the actuation method is a hydraulic or pneumatic piston that causes the crank's length to change, rotation will occur! The mechanism shown modeled with Legos^{TM} uses a 4-bar linkage to raise a scoop and dump it behind itself. This system might be used, for example, to scoop balls or pucks and dump them into a collection bin for later dumping into a scoring bin. This linkage would be actuated by a motor/gearbox driving the rocker. Here it is a rocker because it does not keep revolving, but rather its motion will be oscillatory. How might a crank be used instead?

One of the advantages of physically modelling a linkage is that you can move it and experience whether it will lock up, and discover the mechanical advantages/disadvantages with respect to the force inputs and outputs. Even though a linkage may have some unstable points, some regions may produce highly desirable motion. James Watt invented such a linkage to create near straight-line motion to guide the connecting rod of one of his steam engines! As shown, his 4-bar linkage creates nearly straight-line motion for a limited range of motion of the rocker.

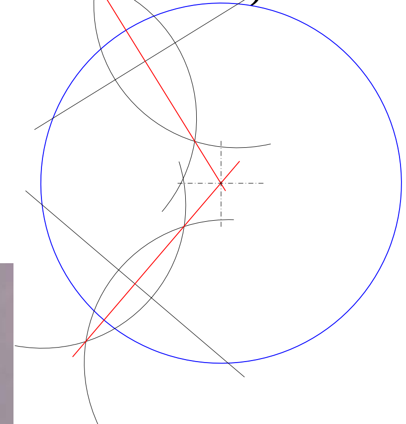
Creating linkage sketch models from Legos^{TM} or other construction toys is a great way to rapidly experiment with potential linkage designs. Even though the spacing between possible pivot points is relatively coarse, they can enable you to converge on an overall linkage configuration that can then be optimized using the equations discussed earlier (or write your own!). This will help you develop a physical instinct for the design of linkages. The next step would be to learn to use one of the many CAD programs specifically developed to help synthesize and analyze linkages.

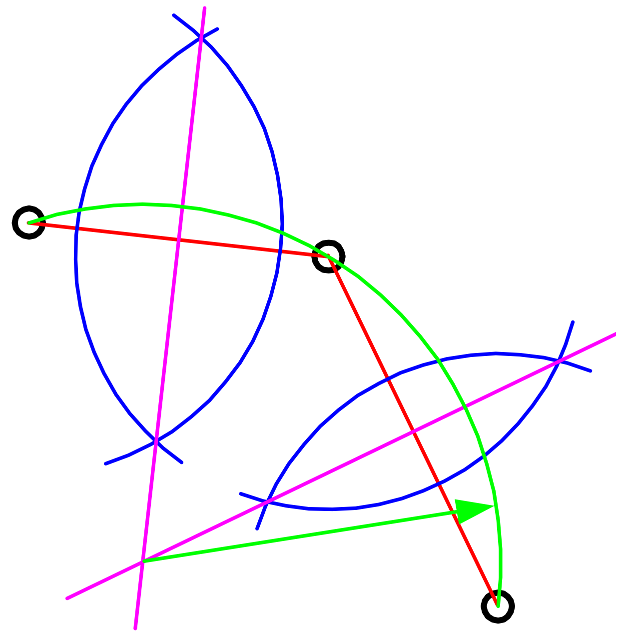
Do you need linkages for suspensions or preload mechanisms? Do you need linkages for large motions for buckets to scoop up stuff? Can you connect a motor up to a crank or rocker, or should your motor power a screw? Generate ideas by visiting construction equipment (web) sites and look at how machines move and work.

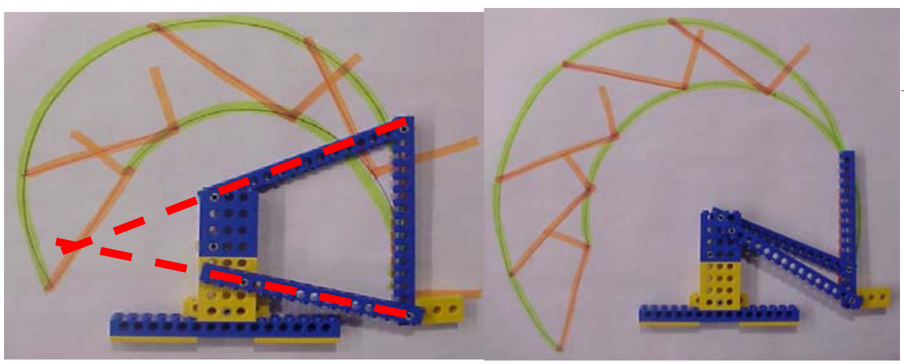
4-Bar Linkages: *Kinematic Synthesis*



- 4-Bar linkage motion can be developed using *kinematic synthesis:*
  - *3 Point Circle Construction (Precision Point Method)*
    - *3 Precision Point Example*
    - *Loader Example*
  - *Experimentation*

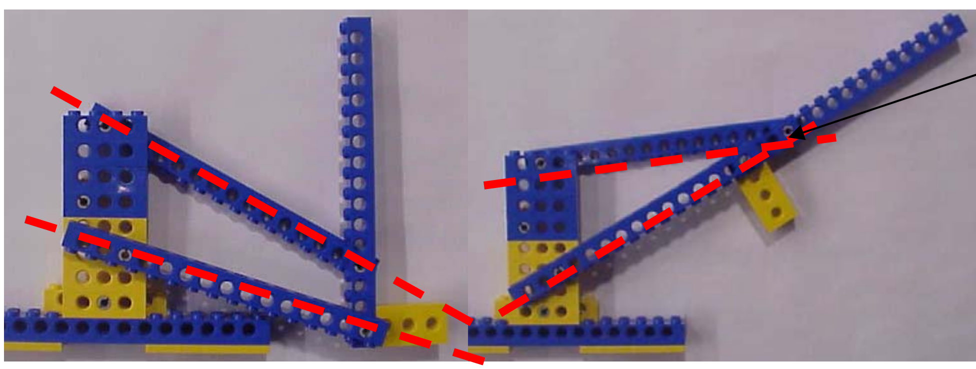


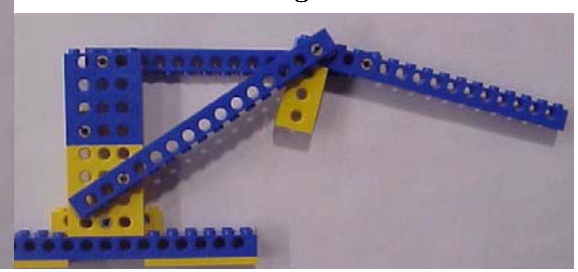




Apply reversal to the geometry and unstable becomes stable!

Instant Center and pivot point become coincident and linkage becomes unstable





Kinematic Synthesis: *3 Precision Point Example*

A good way to synthesize a design is to start with a search to see what exists. Ideally you can scale or evolve an existing design. There are so many different linkage designs for so many different pieces of equipment, that chances are what you need already exists, and you merely have to scale it. There is no loss of design genius in scaling an existing design, as long as you do not infringe a patent. Once you have identified a linkage to scale, or even if a new linkage is needed, its development can proceed either graphically or analytically. Often the former is used to generate the overall shape, and then equations can be created to optimize it or to understand its mechanics so links and joints and the actuator can be properly sized.

Consider a linkage for a single degree of freedom scoop to collect objects and then dump them into a bin. This would allow a machine to zoom around gathering balls from all sides of a contest table. In addition to the steps described above, the concept of bracketing the solution will be used. This means that one of the pivots on the coupler will be assumed fixed, and the other point will be assumed to be in one of two extremes. Whichever extreme yields the better linkage can then be further optimized. This means that we are using the fundamental principle of reciprocity from the start to investigate two very different ideas that will then be compared.

In the first case shown, the coupler pivots lie on a line parallel with the bottom of the scoop. The sequence of sketches shows the rocker and follower base pivot point locations. In the second case, the pivots lie on a line that is perpendicular with the bottom of the scoop. The sequence of sketches shows the rocker and follower base pivot point locations. A solution appears to have been found for the first case, where the pivots on the coupler are parallel to the bottom of the scoop, but the system is very long and takes up a lot of space. A long rocker would mean that for a given power source high speed could be obtained, at the expense of torque, or in this case, lifting capacity.

Conceptually, one can also see that the instant center stability criteria is met, but the pivot on the couplers exchange position. What was in front is now in back, so the rocker and follower links will have to cross each other. If one is offset from the other, than this can be made to happen, but what are the implications for stability and robustness? Does this create a point where the instant center moves near a pivot point? Here again is where a physical model can aid in the synthesis process, and it turns out, that crossing the links is not necessarily a bad thing with respect to stability. However, it may sometimes cause some difficulty in manufacturing.

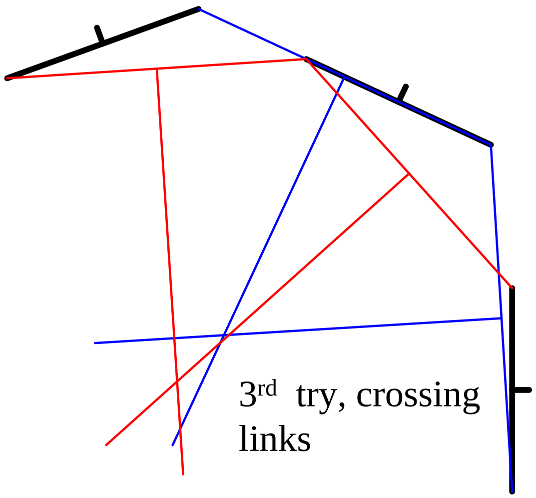
In the second case, the pivots lie on a line perpendicular to the bottom of the scoop. In general, it will be easier to manufacture the linkage when the rocker and follower base pivots are further apart. In addition, it is also desirable to not have the links cross so they can both reside on the same side of the base structure and are less likely to collide with other associated mechanism. By translating and rotating the coupler in the neighborhood of the desired precision points, the bottom sketch emerges which mostly meets the above criteria. As the center drawing shows, a kink needs to be added in the follower to clear the rocker base pivot. This is a simple and common thing to do, particularly if you are cutting your links out using a programmable torch or abrasive waterjet cutter.

These two cases illustrate the concept of bracketing a design. The optimal probably lies somewhere between. It is analogous to limit analysis, trying the extremes, or bracketing exposures in photography. If you try the extremes and observe the effects, you can converge on the best middle position. So what is better for synthesis by bracketing: sketch models or CAD systems? The former has more of a feel to it, but the resolution of the part size limits your creativity. On the other hand, it does help develop insight and physical feel, which are very important for developing your bio neural linkage net! The CAD system allows you to explore variations far more rapidly, and it is not resolution limited. The 3 point precision method is still where the points are defined using the sketching feature. Solid elements can then be added, and the system moved through its motions to check for interferences. The next step would be to size members and actuators and again check to make sure everything still fits.

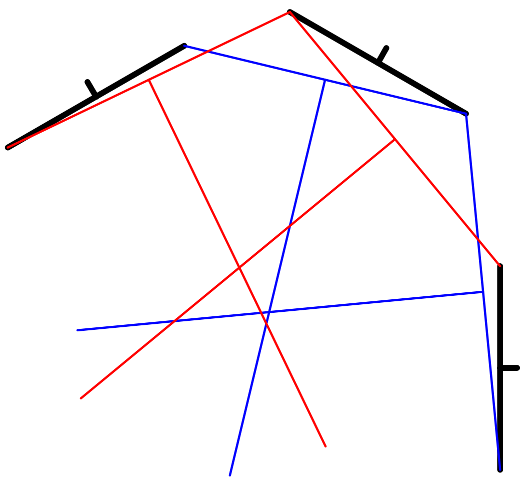
You must now have a good idea of what sort of 4-bar linkages might be useful for your machine. Use the 3 precision point method to synthesize potential linkage designs and build sketch models to verify the designs. Now is a good time to turn on the CAD system and try to create some linkages.

Kinematic Synthesis: 3 *Precision Point Example*

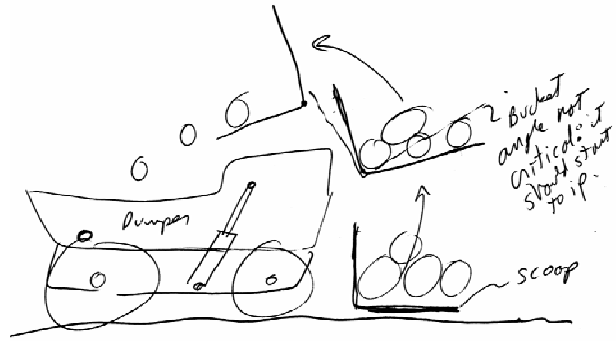
Use the 3 precision point method to find the ground pivot point for the crank and follower links



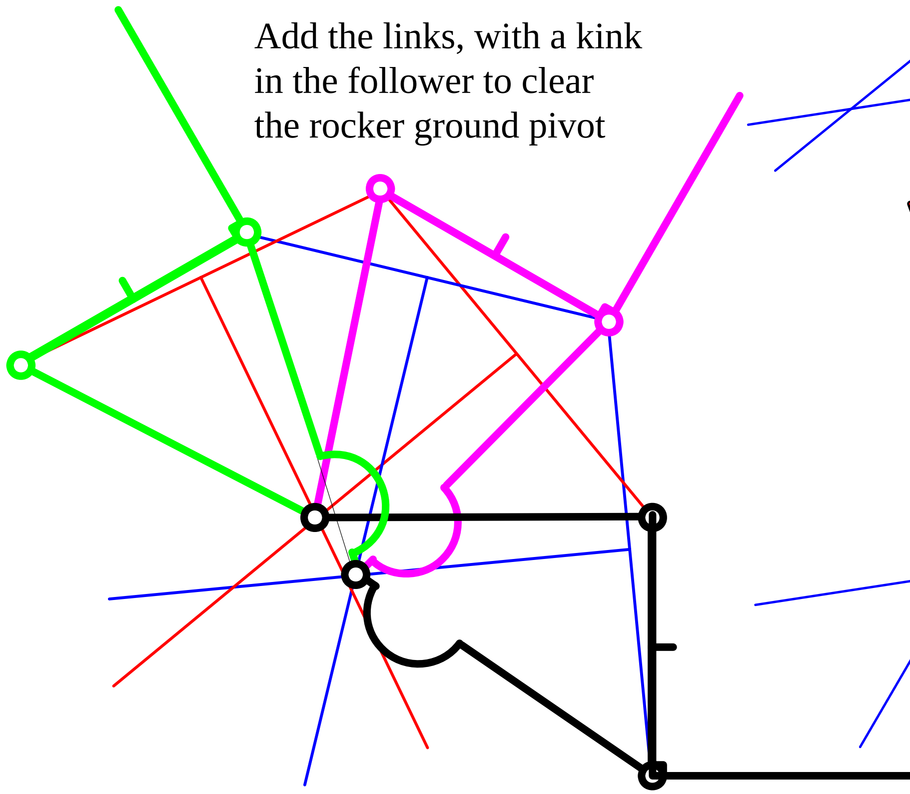
3rd try, crossing links

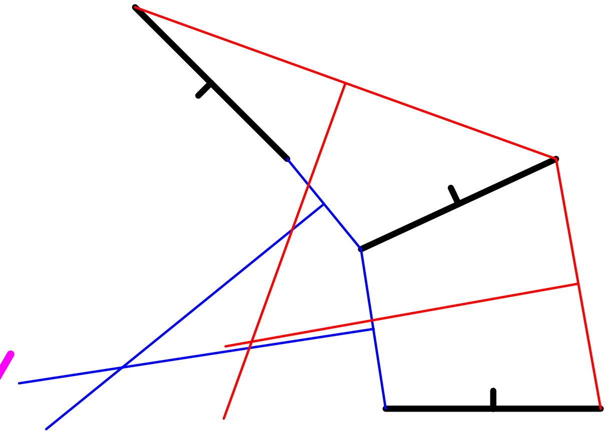


4th try, good!

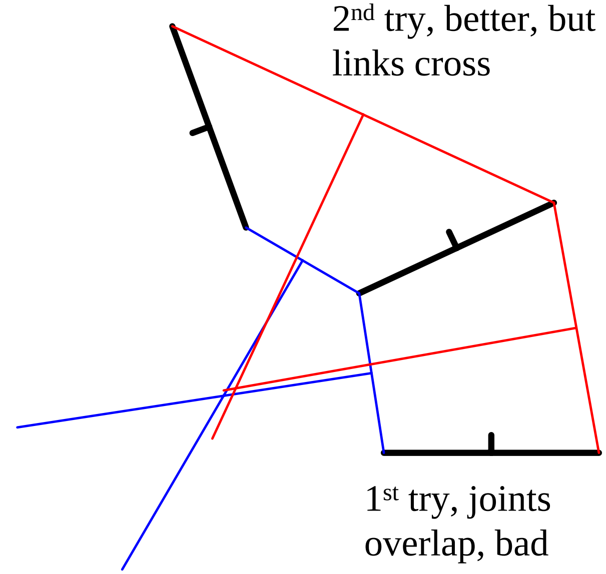


Add the links, with a kink in the follower to clear the rocker ground pivot





2nd try, better, but links cross



1st try, joints overlap, bad

Kinematic Synthesis: *Analysis*

Once a linkage design has been synthesized, for example by the 3 point method just shown, the next step is to perform the analysis needed to determine the velocities, accelerations, and loads in the system. This will enable you to size the links and the actuator to make sure that they are strong enough throughout their range of motion. Given the analysis tools and formulas available, it is rare and unacceptable to build a serious linkage by trial and error, particularly to build it and then find out that it is not strong enough to do the job. Perhaps when designing a machine totally from construction toy components, one could more rapidly build and test a system; however, where you are cutting and assembling components, synthesis and analysis will save you a lot of time in the shop. Once synthesized, the linkage should be sketch modeled, even by printing the CAD synthesis drawing and then cut out the links and pin them together with push-pins and then carefully move it for a geometry check. You may even wish to make a full-scale foam core sketch model and use it in a sketch model derby. If you are lucky, Lego^{TM} pieces will be of close-enough size that you could make a Lego sketch model.

In order to determine the motor torque to move the rocker which moves the load acting on the coupler, we can build on the instant center analysis from page 4-9. The drawing shows the added geometry in green. The goal is to determine the *x*, *y* global position of the loads F_x and F_y applied to the coupler at points *r* and s in the coupler reference frame. A spreadsheet can be used to numerically differentiate the closed form non-linear equations for the *x* and *y* coordinates of the loads to find $dx/d\Omega$ and $dy/d\Omega$ for the energy calculations needed to determine the required motor torque:

$$t = \sqrt{r^2 + s^2} \qquad \alpha_8 = \sin^{-1}\left(\frac{s}{t}\right)$$

$$u = \sqrt{t^2 + b^2 - 2tb\cos(\alpha_8 + \alpha_7 + \alpha_6)}$$

$$\alpha_9 = \cos^{-1}\left(\frac{b^2 + u^2 - t^2}{2bu}\right)$$

$$x = u\cos(\Omega + \alpha_9) \qquad y = u\sin(\Omega + \alpha_9)$$

$$\Gamma_{rocker\_torque} = \frac{F_x dx + F_y dy}{d\Omega} + \mu(F_x + F_y)d_{pin}/2$$

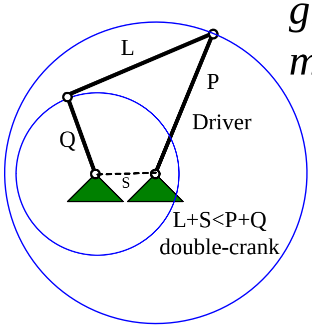
From the spreadsheet *4baranalysis.xls*, the motor torque as a function of loads applied to the coupler can be determined. This spreadsheet uses a numerical differential method to determine motor torque to move the applied load as a function of rocker angle. It is also possible to add rows to input link dimensions and calculate inertias and stresses and accelerations. In addition, note the *Grashof criteria* for initially selecting link lengths to obtain the general type of motion desired. Have a look at a portion of the spreadsheet:.

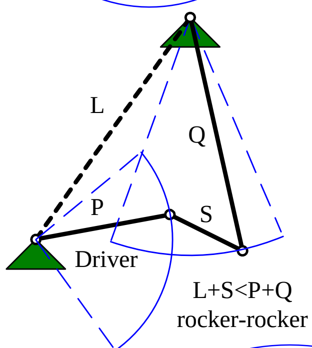
Enter numbers in bold, Results will be in red					
rr (mm)	**50**		a	**100**	
s (mm)	**0**		b	**25**	
Fx (N)	**0**		cc	**100**	
Fy (N)	**10**		d	**50**	
dpin (mm)	**4**		mu	**0.3**	
Linkage motions	**start**				**finish**
omega (degrees)	**0**	86	176	266	**360**
domega	1.00E-06				
x	-21.250	-48.425	-68.750	48.425	-21.250
y	18.998	37.450	-24.206	-37.450	18.998
dx	-3.81E-06	-2.48E-05	-8.07E-06	5.01E-05	-3.81E-06
dy	1.57E-05	7.76E-07	-1.04E-05	9.76E-05	1.57E-05
Static Motor Torque (+CCW) (N-mm)					
To overcome pin friction	6.0	6.0	6.0	6.0	6.0
To overcome load	157	8	-104	976	157
Total motor torque	163	14	-98	982	163
Velocities					
Motor speed ωb (rpm, rad/s)	**20**	2.09			
Link_c ωc = ωic (rad/s)	0.42	-0.03	-0.70	0.03	0.42
Link_d ωd (rad/sec)	0.42	1.03	-0.70	-1.00	0.42
VL at load point x, y (mm/s)	34	-52	-28	53	34
Vx at load point x, y (mm/s)	-7.97	-51.94	-16.91	104.93	-7.97
Vy at load point x, y (mm/s)	32.96	1.63	-21.80	204.47	32.96

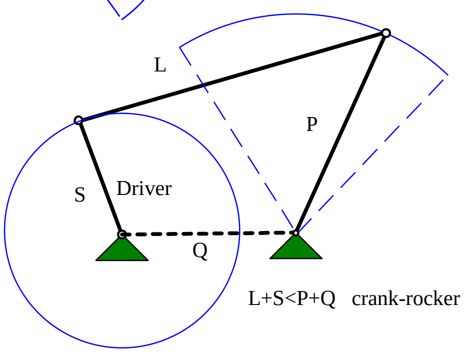
You now have the tools and methods to synthesize and analyze a 4-bar linkage for your machine. Do so for your most critical linkage. If you can achieve good motions, excellent. If not, you may need a higher order linkage as will soon be discussed.

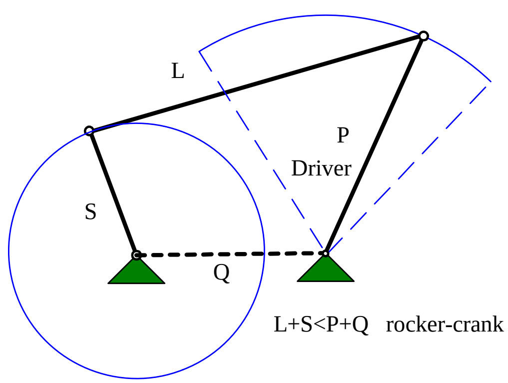
Kinematic Synthesis: *Analysis*

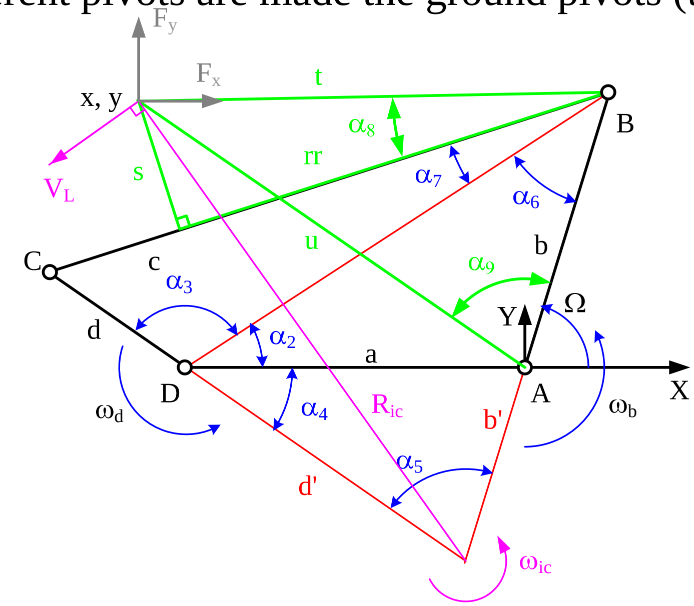
- Code or a spreadsheet can be written to analyze the a general 4-bar linkage, but types of motion can be anticipated using the Grashof criteria:
  - *The sum of the shortest (S) and longest (L) links of a planar four-bar linkage cannot be greater than the sum of the remaining two links (P, Q) if there is to be continuous relative motion between two links*
    - If $L + S < P + Q$, four *Grashof* mechanisms exist: crank-rocker, double-crank, rocker-crank, double-rocker
    - If $L + S = P + Q$, the same four mechanisms exist, but, change-point condition occurs where the centerlines of all links become collinear and the mechanism can toggle
    - If L + S > P + Q, non-Grashof triple-rocker mechanisms exist, depending on which is the ground link, but continuous rotation is not possible
    - *Geometric inversions* occur when different pivots are made the ground pivots (this is simply an application of reciprocity)











Kinematic Synthesis: *Coupler Curves*

Linkages are often drawn with more than just binary links connecting pivots. Sometimes ternary or even quaternary links are shown connecting pivots. The other nodes on the link represent attachment points for other objects such as a bucket on a loader on a robot for a design contest! By providing points on the coupler link away from the line connecting its pivots, different types of motions can be obtained. The paths that these points trace are called *coupler curves*. Coupler curves are the business end of a linkage, so it is extremely important to be able to select link lengths and a crank[1] to obtain the desired motions. *The best way to rapidly synthesize linkages with desired coupler curves is to use an appropriate combination of bio-neural-net application of fundamental principles, curve outputs from analysis programs, and playing with sketch models.* Optimization or fine tuning, can often be accomplished by calculating a penalty function and using the analysis program to search for, or tweak, link lengths to minimize the penalty function.

Fortunately, the analysis we have just completed forms the foundation for selecting linkage lengths to obtain desired coupler curves. Unfortunately, spreadsheets have limited numerical precision, typically 32 bits, so roundoff errors in trigonometric functions can create errors near singular points in linkage motion. However, a simple Internet search for “four bar linkage synthesis” yields many excellent web sites that people have created to assist in the synthesis and analysis of 4-bar linkages. Still, having your own code is useful to allow you to customize and calculate exactly what you want. When you create linkage analysis spreadsheets, and for example enter in equal link lengths, you may see that some cells say #DIV/0! Why is this? The error can be traced to the calculation of angle *alpha5* between the crank and follower link line extensions used to find the instant center. Use IF statements to force the value to be a small distance so you do not divide by zero. For example, in the spreadsheet use: alpha5 =IF(ABS(PI()-L51-B51)<1e-6,1e-6,PI()-L51-B51)

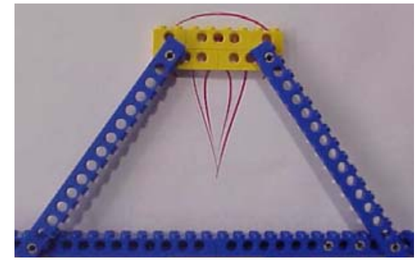
The spreadsheet *4baranalysis.xls* also calculates and plots coupler curves. Consider the two plots shown. The first plot shows a coupler curve for a parallelogram linkage, and it can give happiness. However, if you make a sketch model of a parallelogram linkage, for example using Legos™, and play with it by grabbing the coupler and moving it, you can make the coupler move so it is always parallel to the ground link, hence tracing out an elliptical coupler curve. Why then does the spreadsheet, or any other program, yield a kinked plot? Move the sketch model’s crank link and see that when the instant center moves from being at infinity to being at one of the pivots, which is when all the links are colinear. A singularity nearly occurs and the next motion can have the coupler either remaining parallel to the ground link, or starting to become inclined to the ground link. In fact, in a vertical plane, you will find that when the coupler link is below the plane of the ground link, it is parallel to the ground link, and when it is above the plane of the ground link, it tilts and is inclined to it. Joint clearance, friction, gravity and inexact link lengths create just enough of a bias to make the linkage stay far enough away from the singularity, making the linkage produce predictable motion. With a load attached to the coupler, one should make sure the load is attached to give the desired path a definite bias to maximize linkage determinism.

The other plot, on the other hand, shows a linkage where the **rocker** is 50 mm long, and the **follower** is 25 mm long. This linkage has a very limited range of motion, because the **rocker** reaches a position where its motion cause the **follower** and **coupler** to be colinear; and continued motion of the **rocker** places the **follower** and **coupler** in tension, so the linkage locks up. If the motion of the **rocker** is reversed, then the **follower** and **coupler** are placed in compression and they “buckle”. The linkage motion continues until they are again colinear, but overlapping, and the linkage again locks up. On the other hand, what if the **follower** is now the **crank** because it can move through 360 degrees. When it is colinear with the **coupler**, its input torque cannot create tensile forces along its length so the **follower** can never be colinear with the **coupler**. In fact, it will cause the **follower**, the previous rocker, to oscillate back and forth between what were its singularity points! We now have a very useful linkage where a continuous rotary motion from a motor produces oscillating output motion. When continuous rotary motion is the input, the **crank** should be shorter than the **follower**, and one linkage can drive another linkage to get even more interesting coupler curves.

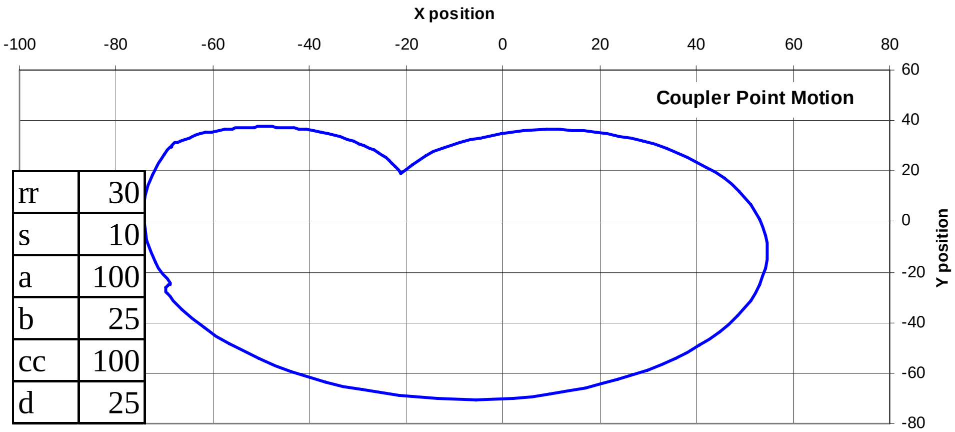
Make some sketch models of linkages and play with them. Compare their coupler curve motions to those that the spreadsheet generates. What modules might benefit from oscillating motion and what modules may benefit from continuous quasi-elliptical motions? Are neither appropriate and you require something more like a crane boom?

1. Remember, a *rocker* is an input link that has oscillating motion, and a *crank* is an input link that rotates continuously.

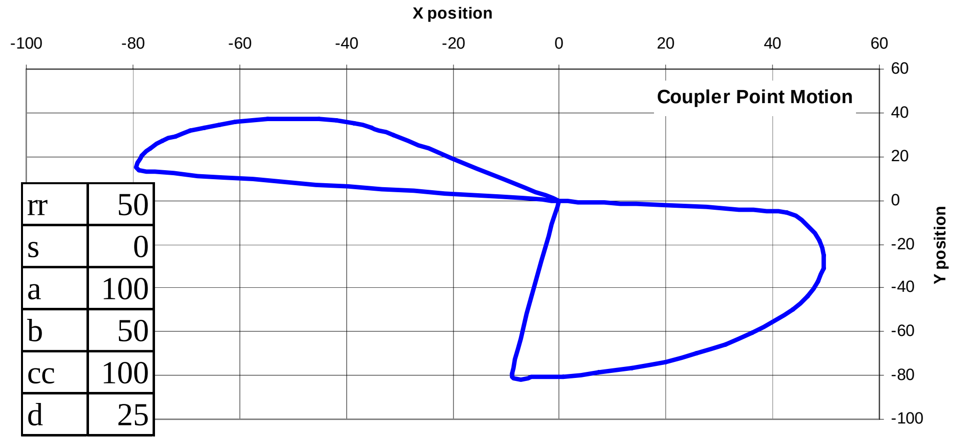
Kinematic Synthesis: *Coupler Curves*



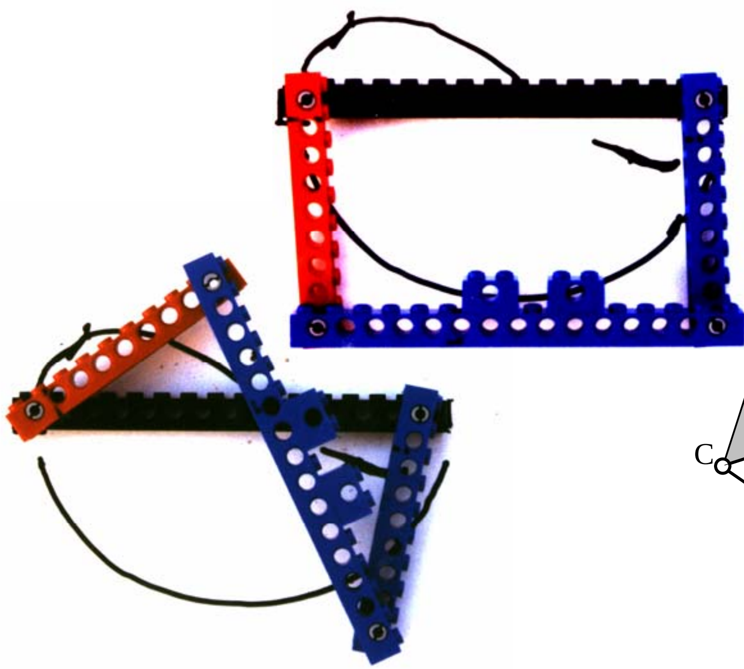
- From the same analysis, the motions of the coupler point can be plotted:
  - See *Linkage_4_bar_Analysis.xls*

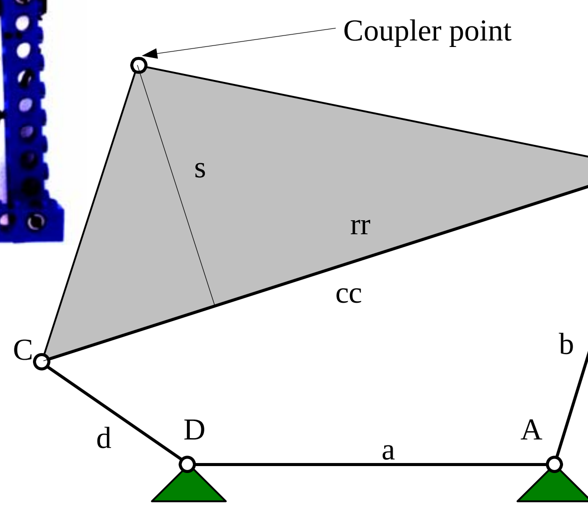


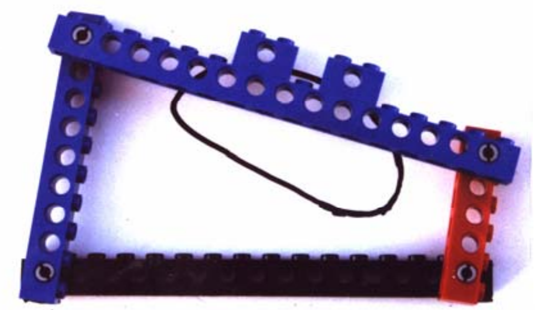
rr	30
s	10
a	100
b	25
cc	100
d	25

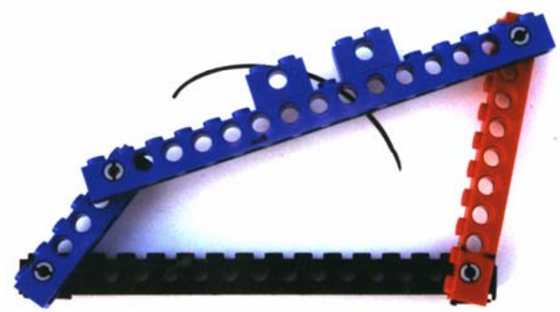


rr	50
s	0
a	100
b	50
cc	100
d	25









Instant Centers

A link's motion is a function of the motions of its endpoints, and the pivot points must follow the contour of curves traced out by the endpoints of other links. All these pivots following curves can become complex and make it nearly impossible to visualize what is going on. Fortunately, for an instant in time, a link's velocity vector at any point can be determined using the concept of the *instantaneous center of rotation*: The ***instant center*** is the point about which every point on a link acts as if it is rotating about for an instant in time. Furthermore, the velocity vector of any point on the link is perpendicular to the imaginary radius from the instant center to the point of interest. The product of the instant center of rotation's angular velocity (or acceleration) and the radius from the link's center of mass to the instant center is used to determine the link's center of mass, velocity, and acceleration.

The instant center allows for the calculation of linear and angular velocities of points and links, which when combined with the fact that power = force (torque) x velocity (angular velocity) allows us to determine, for example, the mechanical advantage of a linkage. If we know the input torque and angular velocity for the crank, and we can compute the angular velocity of the coupler or the follower, we can determine the potential output torque at the coupler or the follower.

Consider the simple case of a wheel rolling on the ground. The wheel may be rotating about its center, but for an instant in time, the point of the wheel that contacts the ground is NOT moving. In fact, the center of the wheel is rotating about the ground contact point, as is the top of the wheel! What is the forward velocity of the center of the wheel (the axle)? What is the forward velocity of the top of the wheel? Can you see that the top of the wheel is moving forward with twice the velocity of the center of the wheel (the car's velocity)?

Study the circle with the chord and the two radii connecting the ends of the chord to the center of the circle. If the radii are joined by pivots to the chord and by a second-order pin joint to the center, then the chord can spin around the circle. The instant center is at the center of the circle, and each end of the chord would be moving tangent to the circle if the radii starting rotating at a velocity ω. In addition to translating, the chord is also rotating.

For a linkage, one can actually identify many instant centers associated with the different links. Each pivot joint itself is an instant center, and there are instantaneous (they move in time) centers associated with every pair of links. *The instant center of velocity (rotation) for two bodies in plane motion is a point, common to the two bodies, which has the same instantaneous velocity in each body.* This point may be a virtual point physically located off of the two bodies. Some of these points are not very interesting with respect to analysis of the linkage, and some, as we shall see, are extremely interesting from both a graphical and analytical perspective. How many instant centers are there for a linkage comprised of N links? The number of combinations possible for N subjects in groups of k, and for $k = 2$ is:

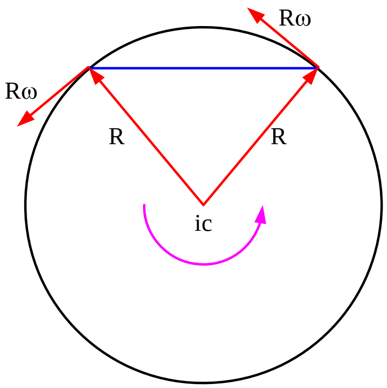
$$C = \frac{\prod_{i=1}^{i=k}(N-i+1)}{k!} \qquad C_{k=2} = \frac{N(N-1)}{2}$$

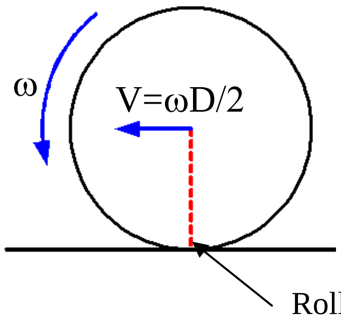
A 4-bar linkage thus has 6 instant centers, and a 6-bar linkage has 15 instant centers. It is not immediately obvious which ones are most useful for the purposes of analyzing a linkage either graphically or analytically; however, being able to identify a point that, for an instant in time, is the common center of rotation of all points on a link is an extremely powerful boundary condition.[1] This raises the delicious potential for creating a linkage where the mechanical advantage is extremely large. Such linkages are often called *toggle linkages*, and they are used on machines ranging from pliers to injection molding machines, to compactors and crushers.

Draw long dashed lines between each of the pivot joints in a linkage on your machine. For sliding joints, draw a dashed line perpendicular to the slider (the instant center is at infinity), and label all the intersections. How do links move with respect to these instant centers?

1. See for example R. Norton, Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines, 2001 Mcgraw-Hill, New York

Instant Centers

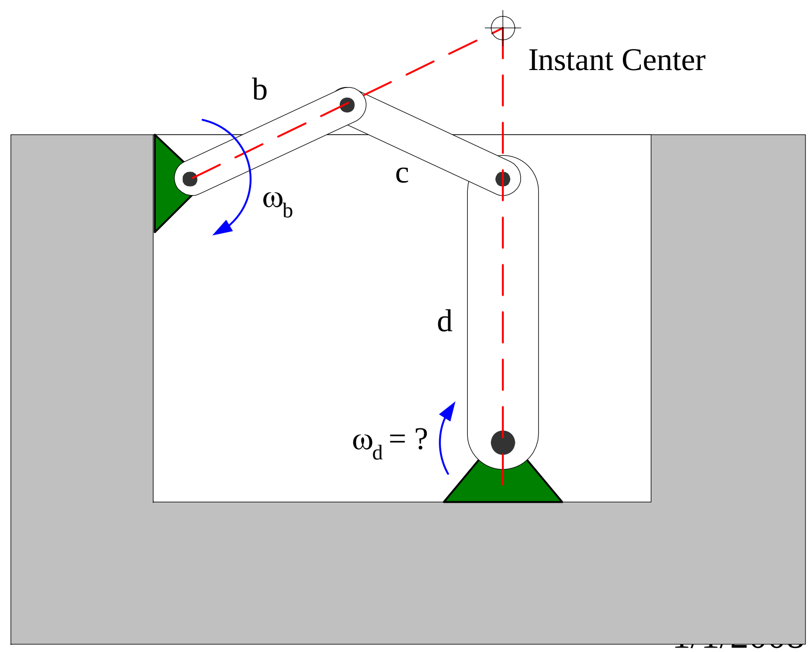




- The instant center of velocity (rotation) for two bodies in plane motion is a point, common to the two bodies, which has the same instantaneous velocity in each body
  - This point may be a virtual point physically located off of the two bodies
- The instant center can be used to determine relative velocities between various links
  - Knowing the relative velocity between links, and the torque input to one link, allows you to use conservation of energy to determine the output torque
  - Can linkages be designed with immense mechanical advantages?

$$\Gamma_{input}\omega_{input} = \Gamma_{output}\omega_{output}$$

$$\Gamma_{output} = \frac{\Gamma_{input}\omega_{input}}{\omega_{output}}$$



Instant Centers: *4-Bar Linkages*

With the instant center located at the intersection of lines drawn along the crank and follower, it is a simple trigonometry problem to find radii lengths R_b and R_d from the instant center to the coupler's pivot points, or the distance R_{ic} to any other point, such as the point on the coupler to where another load may be attached. For geometric compatibility, the product of the distance R_b from the instant center to the coupler link pivot and the instant center's (also the coupler link's) angular velocity ω_{ic} must be equal to the product of the rocker link length b and the rocker angular velocity ω_b. The angular velocity of link d is found in a similar manner:

$$\alpha_1 = \pi - \Omega \qquad e = \sqrt{a^2 + b^2 - 2ab\cos\alpha_1} \qquad \alpha_2 = \cos^{-1}\left(\frac{e^2 + a^2 - b^2}{2ea}\right)$$

$$\alpha_6 = \pi - \alpha_1 - \alpha_2 \qquad \alpha_3 = \cos^{-1}\left(\frac{d^2 + e^2 - c^2}{2de}\right) \qquad \alpha_4 = \pi - \alpha_3 - \alpha_2$$

$$\alpha_5 = \pi - \alpha_4 - \Omega \qquad \alpha_7 = \cos^{-1}\left(\frac{c^2 + e^2 - d^2}{2ce}\right) \qquad t = \sqrt{r^2 + s^2}$$

$$\alpha_8 = \cos^{-1}\left(\frac{r^2 + t^2 - s^2}{2rt}\right) \qquad R_{ic} = \sqrt{R_b^2 + t^2 - 2R_b t\cos(\alpha_6 + \alpha_7 + \alpha_8)}$$

$$d' = \frac{a\sin\Omega}{\sin\alpha_5} \qquad b' = \frac{a\sin\alpha_4}{\sin\alpha_5} \qquad R_d = d + d' \qquad R_b = b + b'$$

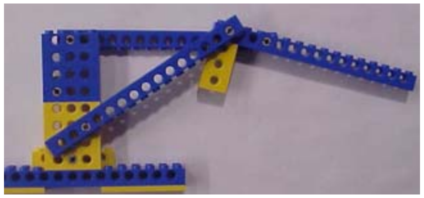
$$\omega_{ic} = \frac{b\omega_b}{R_b} \qquad \omega_d = \frac{R_d\omega_{ic}}{d} \qquad V_L = R_{ic}\omega_{ic}$$

If the crank and the follower are of the same length, and the coupler and the ground link are also of the same length, then the linkage forms a parallelogram. The instant center is at infinity, the velocity vectors of the coupler's pivots are always parallel, and the coupler only translates, it never rotates. In addition, if the input angular acceleration is known, then the accelerations of the other links can be found, and combined with links' inertial properties yields torques and forces. The products of the torques and forces with the velocities gives power, which yields an expression for the motor torque required to accelerate the linkage. The spreadsheet *4baranalysis.xls* implements the above equations and allows a designer to quickly study 4 bar linkage parameters and determine if a gearmotor has sufficient torque and speed.

Instant centers also help to graphically evaluate the stability of a linkage. A mathematical and physical instability occurs if as a linkage moves, the instant center becomes coincident with one of the pivots. Other interesting phenomena also occur. For example, automobile suspension linkage designers must make sure that the line from the instant center to the center of wheel rotation is parallel to the ground (see page 5-18). Else, if one wheel goes over a bump and the suspension deflects, the wheel's axle may see relative motion with respect to the car's forward velocity, which will cause that corner of the car to speed forward. This is called *bump-steer* and it will cause the car to turn, which could result in an accident.

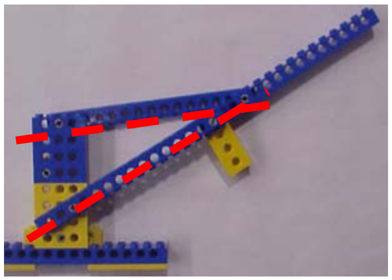
The concept of the instant center, however, must be used with care. For example, assume you have a system that is to pivot about a point in front of itself. You can use a 4-bar linkage or you could use an arc-shaped bearing rail and bearings that ride on the rail. A 4-bar linkage will work just fine for small motions, but for larger motions, the instant center will also translate. Furthermore, if the structure were oriented vertically with the instant center above the coupler, gravity would act to cause the linkage to keep moving and the actuator would have to work to keep the linkage in a stable position. An arc-shaped bearing rail, on the other hand, would have its instant center fixed at the center of curvature of the bearing rail.

Assume the system just described is a new type of rocking chair. Sketch the concept for a 4-bar linkage to locate the instant center in front of the coupler and compare it to a design for a system supported by an arc-shaped bearing rail. What are the strengths and weaknesses of each? How would you choose between these two design options? Find the instant centers of other linkages you have thought of using, as these linkages go through their motions. Are the linkages stable? Should you consider changing the link lengths or attachment points?



Instant Centers: *4-Bar Linkages*

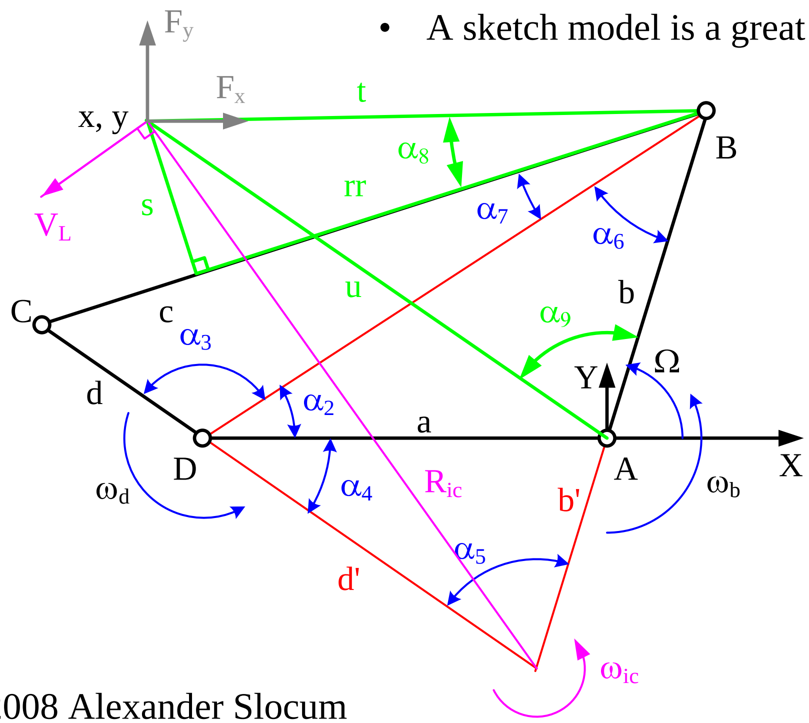
- One can visualize the *instant center* for a point on a moving body as the center of the circular arc that coincides with the point's motion path at an instant in time
  - For a 4-bar linkage, draw lines through the crank and follower pivots, and the point at which the lines intersect is called the *instant center*



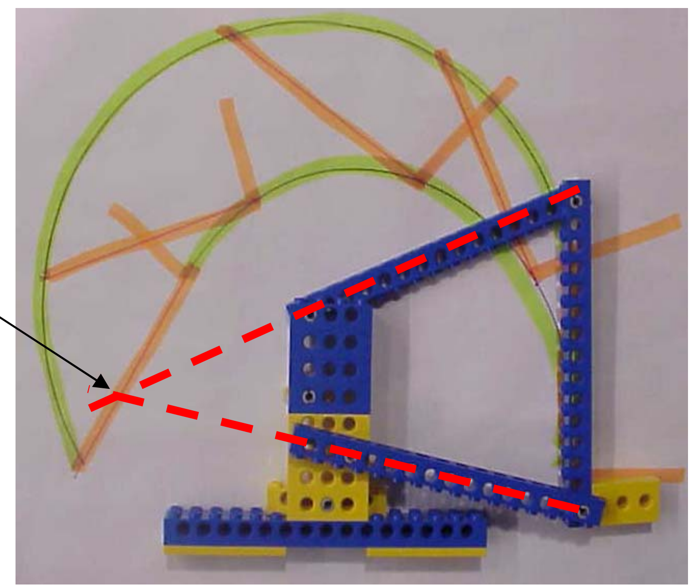
    - The velocity of any point on the coupler at any point is perpendicular to an imaginary line from the instant center to the point of interest
    - The instant center is used to determine the linear and angular velocity and acceleration of the coupler's center of mass or the coupler point

If the instant center is coincident with a joint on the coupler, the linkage can become unstable and can lock up, or the crank must reverse its direction

- A sketch model is a great physical way to check your linkage design!



This linkage's instant Center is always behind the pivots



Instant Centers: *Example*

A detailed analysis of a problem can yield detailed results, but it can also take time. Many CAD programs are linked to or are included with mechanism analysis programs that allow a design engineer to play what-if games and obtain plots of link and joint velocities, accelerations etc. So why is it important to be able to use the principle of the instant center to analyze a system? Being able to rapidly graphically evaluate a linkage trains the designer's eye to look at a system, "see" problems. and synthesize solutions. Thus it helps to develop the designer's intuition. A historical analogy is the slide rule verses the calculator. Slide rules allowed an engineer to get two to three significant digits of accuracy, but the user had to know where to place the decimal point and keep track of the exponent. As a result, slide rule using engineers developed amazing intuitive feelings for the order-of-magnitude of a solution. Calculator users, on the other hand, would punch in numbers and get an answer and often blindly move forward.

The detailed analysis for a 4-bar linkage, and the principle of the instant center can be used to study a system at the instant shown. With known dimensions and crank angular velocity ω, what is the magnitude and direction of the linear velocity of the coupler point? Point *C* connected to link *d*, which is the crank turning at ω_c, must move at the same velocity (magnitude and direction) as Point *C* connected to link L_{icC}. Hence the rotation speed ω_{ic} at the instant center is found from:

$$L_{icC}\omega_{ic} = d\,\omega_c$$

$$\omega_{ic} = \frac{d\,\omega_c}{L_{icC}}$$

The magnitude of the velocity of the coupler point is just the product of the instant center rotational velocity and the distance from the instant center to the coupler point (or similar for any other point):

$$V = \omega_{ic}L_{icCP}$$

The direction, is determined by drawing the velocity vector perpendicular to the imaginary line from the instant center to the point of interest. In mechanisms such as suspensions, this graphical check can be a fast and simple method to ensure that the velocity vector of the axle never has a forward component, else when you go over a bump the car can lurch forward. It is nice to know that during design reviews, using the instant center can enable you to do quick design robustness evaluations!

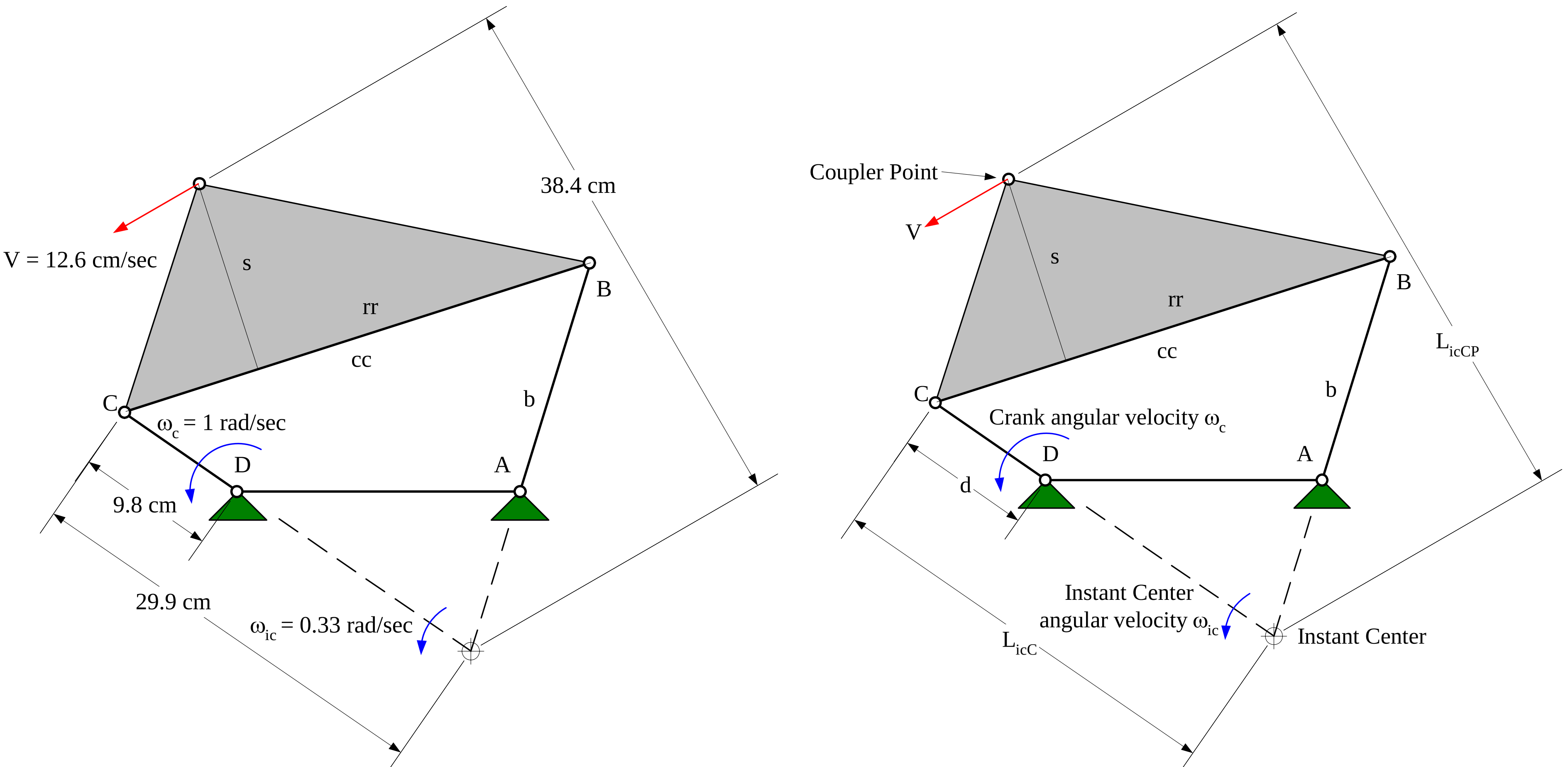
What about acceleration? If you know the angular acceleration of the crank, can you determine the acceleration of the instant center? *Yes!* Can you find the linear and angular accelerations of each of the links' centers of mass, and along with their moments of inertia about their centers of mass, can you determine how much torque is required to accelerate the linkage? *Yes!* Before this level of detail, however, it makes sense from a design layout perspective to assume all the mass of the links is located on the coupler, and just do the calculation based on the coupler. If the design is feasible, you could input the detailed design, via a solid model for example, into a mechanism design package that can be used to determine the exact velocities and accelerations for all the links given a torque input to the crank.

Once again, appropriate analysis is vital to minimizing the cost of developing a product. *Remember, time is expensive!* Many times you may want to do a back of the envelope calculation and then build and try something. However, often you will find in today's world of CAD that the "build and try" phase can best be done with solid models. When is a physical model best? To generalize would be wrong. This is why you need to develop your designer's intuition so you will better be able to determine what is the best way and when!

Look at linkages on familiar mechanisms and identify the "crank" and the "coupler". From pliers to car hoods, how do the instant centers move with the linkage motions? Does the instant center ever get near any of the joints? Is there any position of the linkage that looks like it may go unstable? One of the best ways to build up your "designer's" intuition is to observe and analyze things around you. This can also be a great way to get a date! Try asking someone of interest if they would like to take a walk and look at linkages with you! You will be amazed at the reactions you will get!

Instant Centers: *Example*

- Show the magnitude and direction of the Coupler Point for this 4-bar linkage:
  - Draw lines through the crank and follower pivots, and the point at which the lines intersect is called the *instant center*
      - The velocity of the Coupler Point is perpendicular to an imaginary line from the instant center to the point of interest



5-Bar Linkages

When a 4-bar linkage cannot be synthesized to create the desired motion, one usually makes the jump to a 6-bar linkage. But what about 5-bar linkages? Considering Gruebler's Equation, if we try to solve for the number of joints *f* that give 1 degree of freedom, we get $f = 5.5$. Referring to the types of joints available starting on page 4-6, the only way to get $f = 5.5$ would be to use a combination of links against a plane (half joints) and single joints. In fact, the "***?***" linkage on page 4-5 becomes properly constrained when one of the ground pivots becomes a link sliding on the plane; however, this is not a very useful linkage. Are there any other configurations that might be useful? If you were told "there are no useful single degree of freedom 5-bar linkages" would you try to invent one anyway?! :)>

How about 2 degrees of freedom? In this case, $f = 5$ and we ask ourselves why would we want 2 degrees of freedom? Or perhaps an equally valid questions is what does 5 links get us that 4 or 6 links do not? The primary reason to use 5 instead of six links would be cost, because 6 links requires 7 joints. Carefully compare the bolt cutters in the picture, especially the edge-views. The 4-bar bolt cutters on the right have a single pivot joint formed between the cutting blade links. Even though the center of the joint is essentially in-line with the cutting edges, there will always be some offset and hence some moment on the pin joint. If the joint were to be made as a yoke, a U-shape into which fits in a blade to form a joint such as shown in the upper left hand side of page 4-6, then two identical parts could not be used for the cutting blade links. Thus we could conclude that the design for 4-bar linkage bolt cutters are for smaller size units.

Carefully examine the 5-bar bolt cutters shown. The cutting blade links are joined by connection links, which counts as one link since it functions as a yoke. There are 5 links and 5 pivots and thus there are 3*(5 - 1) - 2*6 = 2 degrees of freedom. In practice, the friction of the pivots in the connection link often makes only one of the cutting blades move with respect to the connection link, so why not eliminate the connection link and one of the joints? This would actually increase complexity, and it would greatly reduce the ergonomics of the design. This is because these large cutters are often used where one handle is placed on the ground, causing one cutting blade to also be parallel to the ground, and one's bulk is applied to the other handle which rises up from the linkage. If the connection link were fixed to one of the cutting blade links, then the design would be asymmetrical, and one would have to think about which side goes down before one applies one's bulk to the handle. It is interesting to note that of the 2 degrees of freedom, only one is used actively, while the other is used momentarily to set up the tool for ergonomic use. Which one becomes dominant depends on how the tool is first picked up.

Returning to the world of synthesis and analysis, let us compare the 4-bar clamping pliers to the 5-bar bolt cutters. Which design can produce more force? The answer lies in which design can have the greatest ratio of handle opening to jaw opening, because ultimately the forces can be calculated from conservation of energy:

$$F_{handles}\delta_{handles}\eta_{efficiency} = F_{jaws}\delta_{jaws}$$

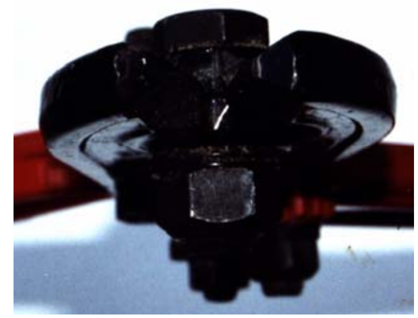
See, that was really easy. You probably thought "this is going to be a painful analytical experience..." The 4-bar design typically allows for larger jaw opening, but it cannot generate anywhere near the force of the 5-bar cutter.

As the picture of the locking pliers with the link outlines added shows, links are not represented merely as straight lines, but often as triangles, where external forces or potential attachments to other links could be applied. The picture also shows a common feature in linkages, that of the *toggle* action. *Toggling* occurs when then angle between two links passes from 180- degrees to 180+ degrees. This means the linkage goes from one stable position to a momentary unstable position, to a new stable position. In general, this means going from an open or free state to a closed or locked state. This is very common in links used to close and clamp on a structure Also note the yellow link and its pivot, which is used as a release lever. When the pliers are first picked up and the handles squeezed, they are functioning as a 4-bar linkage. However, once clamped onto an object, the yellow link and its pivot connection to the purple handle link and its half-joint connection to the black link make the pliers a 1 degree of freedom 5-bar linkage with 5.5 joints! This design was initially patented and marketed as *Vice-Grip*^{TM} pliers.

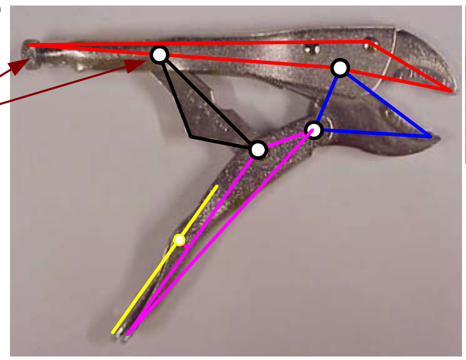
Could you design a toggling linkage to allow you to easily place your machine around the pendulum beam and then with a flick of a lever, clamp onto it so your machine's wheels were now preloaded to the beam and ready to climb it, engage the support shaft and then cause it to spin!?

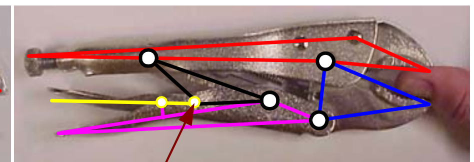
5-Bar Linkages

- Compare a simple 4-bar linkage for pliers or small bolt cutters to a 5-bar linkage (5 bars, 5 joints, *2 DOF*) for bolt cutters
  - Where are the 2 degrees of freedom?
  - – The FRs of the pliers are for wide range of motion and modest clamping force



What effect does the screw have on the pivot?





momentary half-joint

  - – The FRs of the bolt cutters are for modest motion with extreme force
    - A 5-bar linkage can also act like a toggle mechanism

Cutting blade links

Connection link

5-Bar Linkages: *Analysis*

We have seen how the 4-bar linkage pliers revert to a 5-bar linkage with f=5.5 when the handle link pivot angle θ goes from 180- degrees to 180+ degrees. In terms of determining the jaw force F_j as a function of the handle force F_h and the angle θ:

$$e=\sqrt{a^2+b^2-2ab\cos\theta} \qquad \alpha_2=\sin^{-1}\left(\frac{a\sin\theta}{e}\right) \qquad \beta_3=\sin^{-1}\left(\frac{b\sin\theta}{e}\right)$$

$$\beta_1=\cos^{-1}\left(\frac{c^2+d^2-e^2}{2cd}\right) \qquad \alpha_1=\sin^{-1}\left(\frac{d\sin\beta_1}{e}\right) \qquad \beta_2=\sin^{-1}\left(\frac{c\sin\beta_1}{e}\right)$$

$$\alpha_3=\pi-\theta \qquad g=\sqrt{b^2+h^2-2bh\cos\alpha_3}$$

$$\alpha_4=\sin^{-1}\left(\frac{h\sin\alpha_3}{g}\right) \qquad \beta_4=\cos^{-1}\left(\frac{d^2+j^2-k^2}{2dj}\right) \qquad \beta_5=\pi-\beta_1-\beta_4$$

$$x_j=i\cos\beta_5 \qquad x_h=-c+g\cos(\alpha_1+\alpha_2+\alpha_4)$$

$$y_j=-i\sin\beta_5 \qquad y_h=-g\sin(\alpha_1+\alpha_2+\alpha_4) \qquad F_j=\frac{F_h\sqrt{dx_h^2+dy_h^2}}{\sqrt{dx_j^2+dy_j^2}}$$

The spreadsheet *Linkage_4_bar_locking.xls* shows that the locking pliers can generate substantial jaw forces at the toggle point:

Enter numbers in bold	Results in red			
a (mm)	**15**	h (mm)	**60**	
b (mm)	**35**	j (mm)	**30**	
c (mm)	**45**	k (mm)	**30**	
d (mm)	**16**	Fh (N)	**100**	
Linkage motions				
Theta (degrees)	**120**	**135**	**150**	**165**	**180**
Fj (N)	252	463	641	1203	609952
amplification	3	5	6	12	6100

The 5-bar bolt cutters with 5 pivots is assumed to be operated in a symmetric mode, where the person would apply forces to the handles by pressing inwards on the handles. In this case, the bolt cutters can actually be modelled as a 3-bar linkage with 2 pivots and a half joint C, which is constrained to slide along the dotted line; Gruebler's Equation yields 3*(3 - 1) - 2*2.5 = 1 dof:

$$\gamma=\cos^{-1}\left(\frac{b^2+g^2-h^2}{2bg}\right) \qquad \alpha_1=\pi-\theta-\gamma \qquad e=b\cos\alpha_1$$

$$f=b\sin\alpha_1 \qquad \alpha_2=\sin^{-1}\left(\frac{f-d}{a}\right) \qquad m=a\cos\alpha_2$$

$$c=m+e \qquad k=\sqrt{i^2+j^2} \qquad \alpha_3=\sin^{-1}\left(\frac{j}{k}\right)$$

$$\alpha_4=\pi-\beta+\alpha_2-\alpha_3 \qquad x_h=-c-g\cos\theta \qquad y_h=g\sin\theta$$

$$x_j=k\cos\alpha_4 \qquad y_j=d-k\sin\alpha_4 \qquad F_j=\frac{F_h\sqrt{dx_h^2+dy_h^2}}{\sqrt{dx_j^2+dy_j^2}}$$

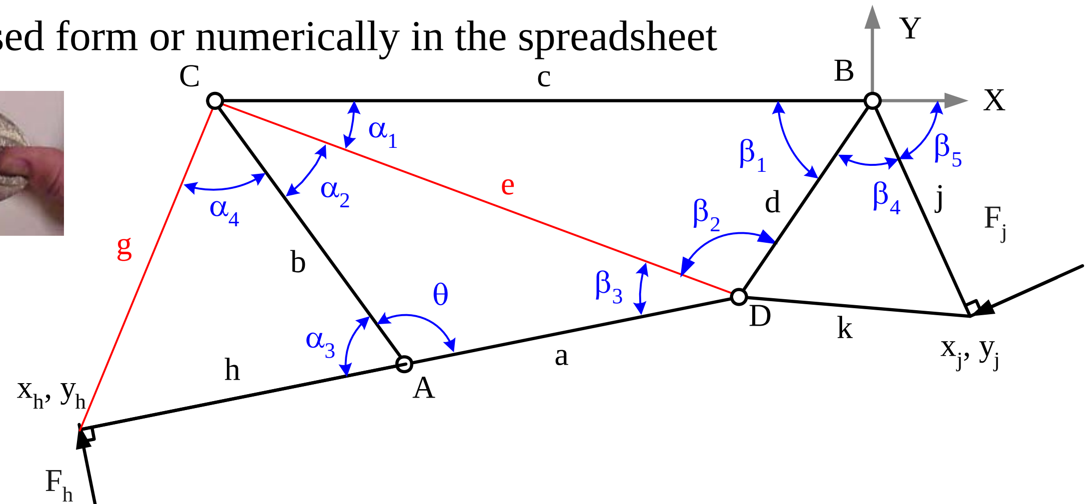
The spreadsheet *Linkage_5_bar.xls* shows how the force amplification ratio is greatest near the end of a cut, but also very high at the beginning, which is where it is needed the most. The force amplification is far greater than that for the 4-bar linkage pliers, but the jaw range of motion is much less. Note that the final angle that yields the maximum force can be found in a spreadsheet using the *solver* tool.

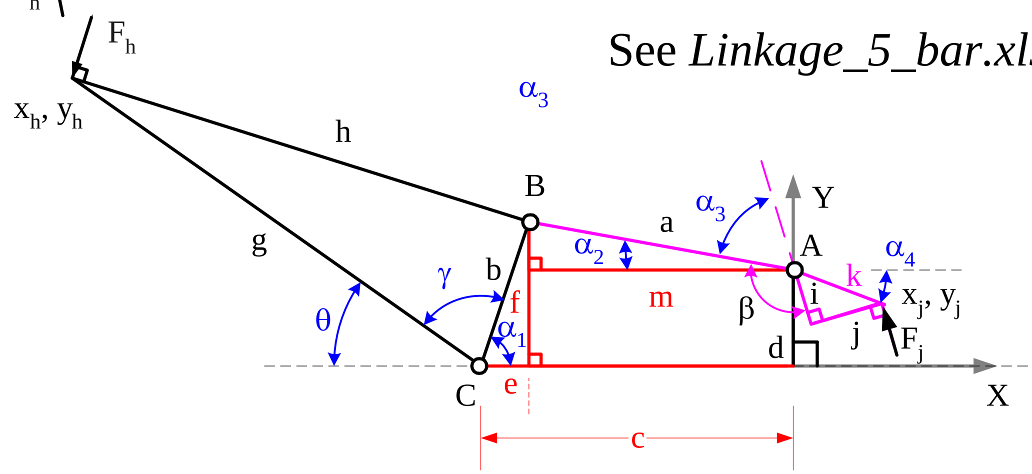
Enter numbers in bold	Results in red			
b (mm)	**40**		a (mm)	**115**	
g (mm)	**802**		d (mm)	**30**	
h (mm)	**800**		i (mm)	**30**	
Fh (N)	**200**		j (mm)	**50**	
beta (deg, rad)	**95**	1.658			
Linkage motions					
Theta (degrees)	**40.00**	**31.07**	**22.15**	**13.22**	**4.29**
Fj (N)	13187	17115	25249	50114	15717809
amplification	66	86	126	251	78589

Can a toggle mechanism or a 5-bar clamping linkage preload your machine's wheels to the pendulum? Sketch it and use the spreadsheet to help determine the forces achievable. Be careful, as these forces act on the links and joints too!

5-Bar Linkages: *Analysis*

- To determine the force on the jaws caused by a force on the handles, equate the work done:
  - The product of the force applied with an incremental input motion equals the product of the jaw force with the incremental jaw (output) motion
    - Because of long lever lengths, pin joint inefficiency is minor
  - The differentiation can be done closed form or numerically in the spreadsheet





See *Linkage_5_bar.xls*

6-Bar Linkages

When a 4-bar linkage cannot create required motions, one typically resorts to a 6-bar linkage. Within most 6-bar linkages, one can find what is essentially an input stage and out output stage; thus the simple linear input becomes a complex input to the second stage thereby producing an even more interesting output. Unfortunately, there are no simple synthesis methods, like 3 precision points, for 6-bar linkages. However, thinking of them as combinations of 4-bar linkages, with some links and joints shared, can help. In addition, there are several standard known 6-bar linkages that can be good synthesis starting points.

The *Watt I* 6-bar linkage is essentially a 4-bar linkage stacked on top of another 4-bar linkage. It starts with a 4-bar linkage, and the coupler and crank links are made as ternary links and their free pivot points become attachment points for links 5 and 6. Links 5 and 6 can also be made as ternary links, and thus either can be output links. With either link 5 or 6 assumed to be a binary link, only 12 link parameters need to be determined. The *Watt II* 6-bar linkage uses one 4-bar linkage to drive the rocker of another 4-bar linkage; this allows continuous rotary input to generate a reciprocating arc motion. The linkage can be optimize to minimize the curve of the arc and create as close to straight-line motion by determining only 11 link parameters. The Lego™ sketch model, shown with its near straight-line motion coupler curve, would be driven by another 4-bar to create a Watt II linkage

The *Stephenson I* 6-bar linkage replaces the Watt I binary follower with a ternary link and thus stacks what would appear to be a 5-bar linkage on top of a 4-bar linkage, but the ternary nature of the crank and follower make the system have one degree of freedom, and only requires the determination of 12 link parameters. The *Stephenson II* 6-bar linkage is a 4-bar linkage on top of a 5-bar linkage, again with the coupling between the two making the system have one degree of freedom; however, it requires the determination of 14 link parameters. The *Stephenson III* 6-bar linkage is a variation of the Watt II, where a 4-bar linkage is used to drive another 4-bar linkage, but instead of driving the rocker, it essentially forms a variable length rocker (or crank)! Hence the system behaves as a 4-bar linkage, where the crank's length varies with its input angle. This can be a very useful design, where synthesis only requires determination of 10 parameters!

These linkages use rotary input and the output can be any point that is attached to the output link. *Another common 6-bar linkage, as has been discussed in the context of the excavator bucket on page 4-8, is where two of the links are actually the rod and cylinder of a hydraulic actuator, or the screw and nut of a leadscrew actuator.* This allows a link to move through 180 degrees of motion, with high moment capacity, which could not be obtained with a simple 4-bar linkage such as shown on page 4-15.

Synthesis of 6-bar linkages can get very complicated very quickly, and this is where analysis programs rapidly take over from intuition and sketch models. An analysis program can allow you to program nested loops that vary each of the link geometries through reasonable ranges by using a *field search*. If each of N parameters are allowed to vary over a range of K values, while the crank link position moves through M increments, then only $J = M*N^K$ calculation steps need to be made. Each step may itself require a couple dozen parameters to be computed to relate the output point to the crank angle and the link length parameters. The plot, where M = 35, shows how painfully large J can become. Compare the number of calculations needed for brute-force synthesis of a 4-bar linkage compared to a 6-bar linkage. This is where brute force calculations can then yield to more sophisticated search algorithms which incorporate logical conditions to direct the search.

A very useful half-step to sophisticated searches is to use a *coarse-fine* field search. In the first *coarse* search, the link lengths are varied over a large range, but with only 4 steps each. Hence only 10^9 cycles are required, which is tolerable for those with a 1 GHz or faster processor[1]. From this coarse search, a set of reasonable link lengths can usually be selected that yields a useful coupler curve. The *fine* search is then run, again with 4 steps, where the total range of change in each link parameter is the step size used in the course search. This allows one to converge on a reasonable design within a few hours. This design can then be input into a CAD program or linkage analysis program for refinement and calculation of other parameters.

Where might you need a 6-bar linkage? Try creating a linkage synthesis program with a nested coarse-fine field search capability!

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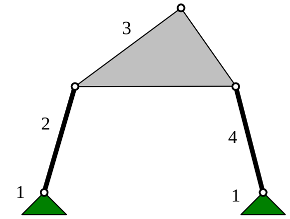
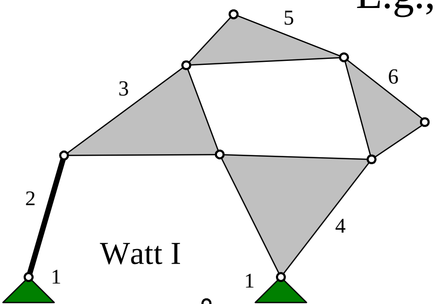
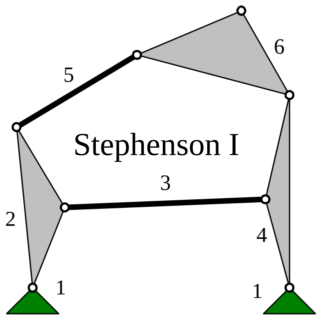
1. If however, as Weird Al points out in his song "Its All About the Pentium", if your computer chip is a Dorito™, then you are going to be out to lunch!

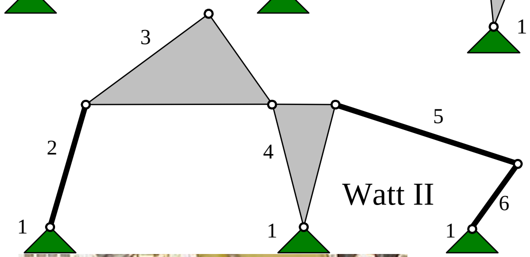
6-Bar Linkages

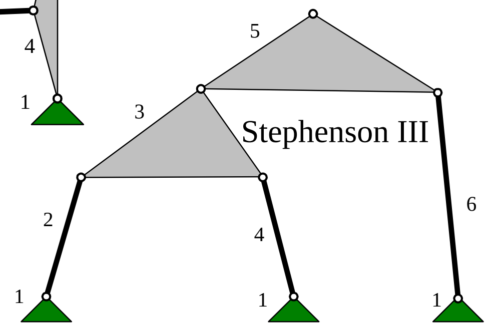


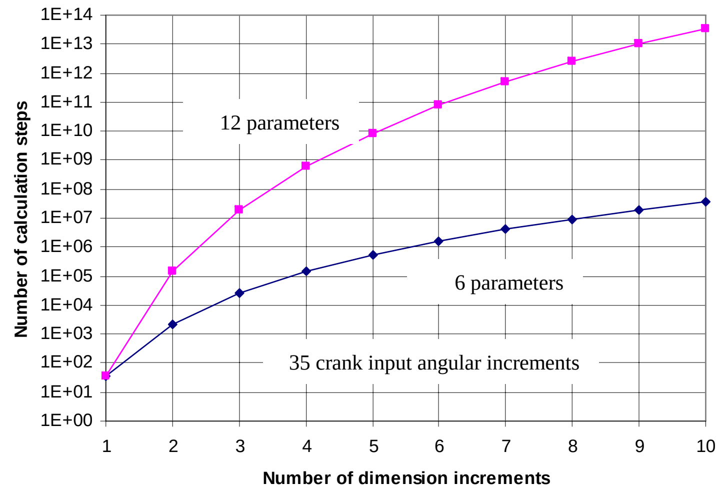
- Simple linkages often cannot meet FRs for large motion or extreme force
  - A linear actuator cannot be effectively (huge stroke or huge forces are required) attached to a point on a 4-bar linkage to allow the output link to move through 180 degree motion
    - E.g., excavator bucket, concrete pump booms….use a ***6-bar*** linkage



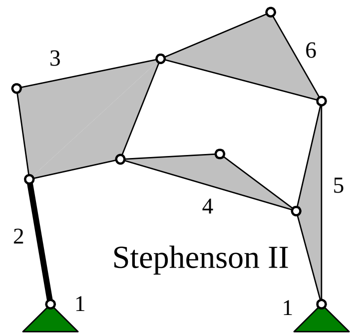








See *Linkage_6_bar.xls*



Extending Linkages

Linkages have been shown that convert simple input motions to more complex motions, or to magnify forces. Another common use of linkages is to magnify displacements, the reciprocal of magnifying forces. These linkages convert large forces applied over small distances to small forces applied over large distances. There are three types of extending linkages commonly encountered: *folding, telescoping,* and *scissor*. All meet the functional requirements of compactness for transport, ready-to-use with minimal set up (outriggers deployed to increase vehicle stability), and long reach capability.

Folding linkages use the same type of 6-bar linkage as used for excavator buckets. The equations relating piston length and force to output position and force were given on page 4-8, and the spreadsheet *6barpistonlinkage.xls* shows how large moments can be supported throughout the range of motion of the piston. Although the results here are by no means optimized, they show that 180 degrees of boom motion can be obtained.

Enter numbers in bold	Results in red				
a (mm)	**30**			g (mm)	**40**
b (mm)	**0**			h (mm)	**35**
cc (mm)	**140**			Fx (N)	**0**
d (mm)	**5**			Fy (N)	**0**
e (mm)	**20**			M (N-mm)	**-1000**
f (mm)	**0**		**Piston (actuator) length**		
Lboom (mm)	**200**			Contracted (mm)	**120**
				Extended (mm)	**180**
Results					
Lpiston (mm)	120	135	150	165	180
R	41.6	45.0	41.7	32.1	2.0
theta (deg)	106.3	81.5	54.3	18.3	-81.8
Fpiston (N)	24	22	24	31	505

Folding linkages are commonly used on concrete pump trucks because each stage can be actuated independently; hence the boom need not be straight, and this gives it the capability of being deployed to reach into buildings. Note that steel technology made large structures practical, but the ability to pump concrete made them truly economical.

Telescoping booms typically use nested tubes with the first segment mounted with a pivot to the base, and a hydraulic cylinder to actuate it as discussed on page 4-11. Some systems, such as battery-operated portable lifts, use nested cables or chains and pulleys or sprockets respectively. This is also a good solution for contest robots.

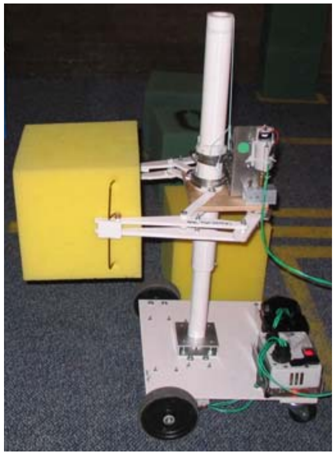
Sliding bearing surfaces between the telescoping sections should typically be spaced at 3-5x the tube size (see *Saint-Venant's principle* on page 3-5). The number of required sections is a function of how much they overlap each other. If one gets too greedy and lets the sections extend too far, the ratio γ of the bearing spacing $L_{bearing}$ to the section length L decreases too much. As shown in the graph, as this ratio decreases, the ratio of the front bearing force F_{Bf} to the applied force F increases dramatically. Considering that other sections also impose a moment M, the bearing forces can become quite large:

$$F_{Br} = \frac{F(1-\gamma)}{\gamma} + \frac{M}{L_{bearing}} \qquad F_{Bf} = \frac{F}{\gamma} + \frac{M}{L_{bearing}}$$

It may seem like a straightforward calculation to determine the force required to extend the booms. However, since sliding contact bearings are used, the frictional forces can become significant, especially as the boom nears horizontal. In order to determine the cable force required to extend the boom, the product of the coefficient of friction with the bearing reaction forces, which the engineer has of course calculated, must be added to the weight of the telescoping sections and the supported load.

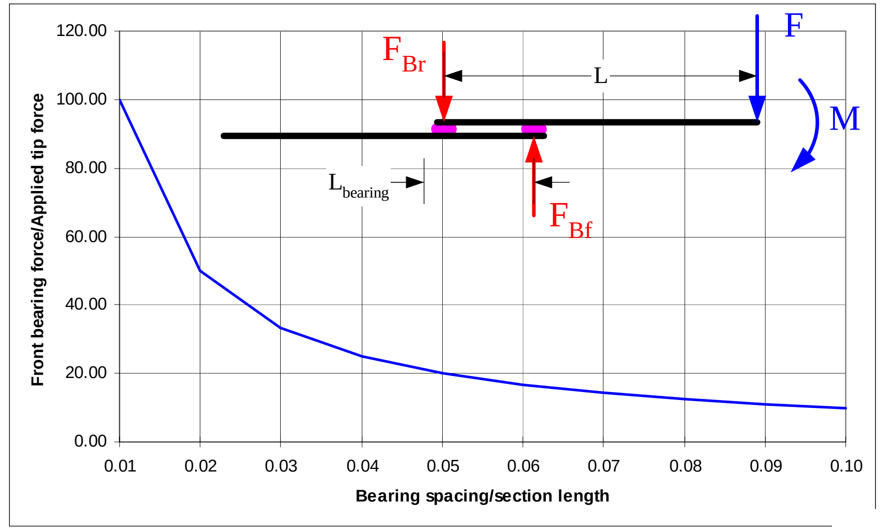
What might be the functional requirements of a defensive or an offensive extending linkage module? Should it extend across the table and clamp on to the other side and block all other motions? When you design a boom to withstand forces, you are also designing a means to defeat the boom! How might you counteract a boom extended at you? The 2001 2.007 winners used extending booms that came up under their opponents' side of the tilting table beam. Few of the opponents were listening in Prof. Slocum's lectures where he specifically said it was likely an extending boom would be used. So do as Sami Busch did in the 1996 contest Niagara Balls and use the fact that you have a huge moment advantage when the boom gets near you. It is simple to design a module that uses a sweeping arm to knock away booms!

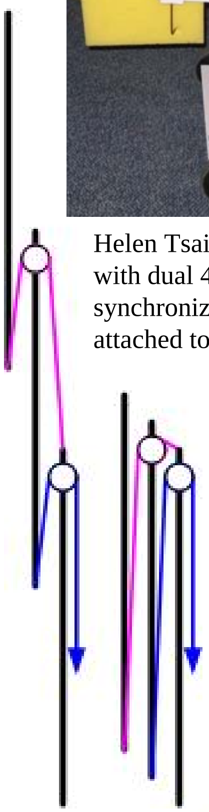
Extending Linkages



Helen Tsaih's extending column with dual 4-bar gripper (cranks synchronized with gears attached to their ends)

- Many extending systems have the following FRs
  - Compact for transport
  - Rapid set-up
  - Long reach during use





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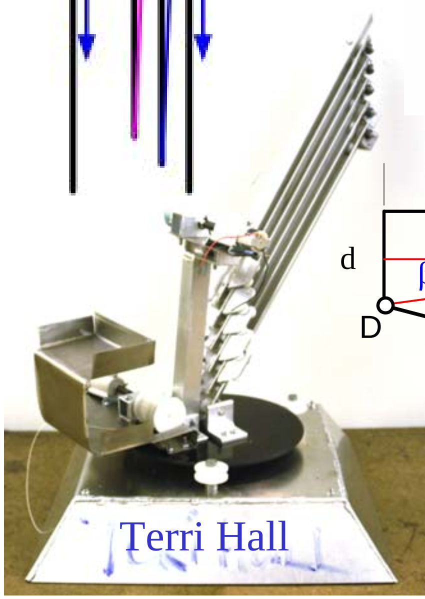
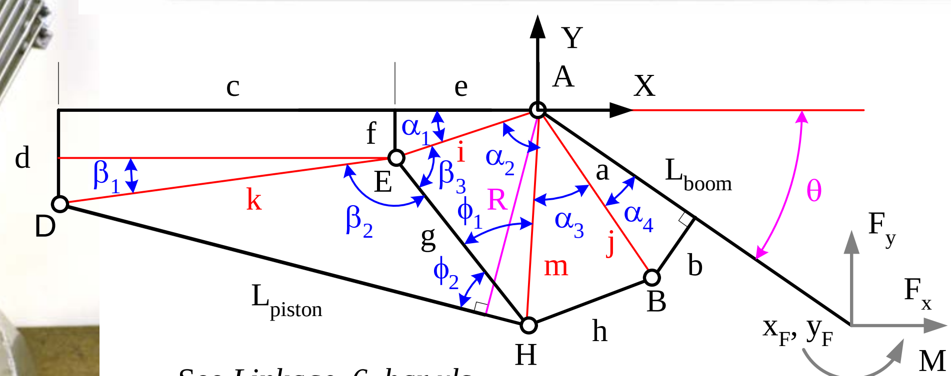




See *Linkage_6_bar.xls*

http://www.terexlift.com/catolog/catopdfs/rt300ser.pdf

Extending Linkages: *Scissor Linkages*

Because of the difficulty in making telescoping segments, and the complexity of cabling or telescoping cylinders required to actuate them, *scissor linkages*, or *lazy tongs*, are often used in both industry and in robot contests! They are called scissor linkages because the basic modular element of the system is a pair of ternary links joined at the middle to each other, and their ends are joined to the ends of another set of ternary links...

Like any other linkage, a scissor linkage's input force can be related to the output force by the work done, although with all the pin joints, one can typically account for the friction in the joints by assuming an efficiency of η per section. Thus for N sections:

$$\alpha = \frac{\pi - \sin^{-1}\left(\dfrac{2h_{link}}{L_{link}}\right)}{2} \qquad \beta = \frac{\pi}{2} - \alpha$$

$$D_{retracked} = L_{link}\sin\alpha \qquad D_{extended} = L_{link}\sin\beta$$

$$\lambda_{retracted} = L_{link}\cos\alpha \qquad \lambda_{extended} = L_{link}\cos\alpha$$

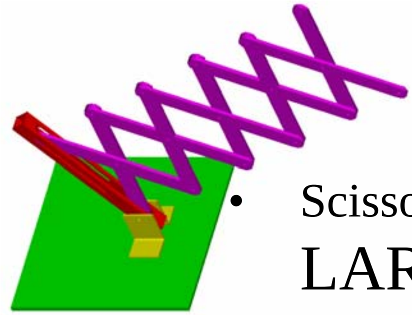
$$F_{extend} = \frac{\eta^N F_{squeeze}\left(D_{retracked} - D_{extended}\right)}{N\left(\lambda_{extended} - \lambda_{retracted}\right)}$$

The spreadsheet *lazy_tongs.xls* calculates the extended and retracted conditions for a lazy tongs linkage, and can be used to size members and determine design feasibility.

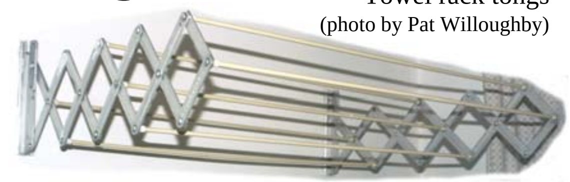
In addition to designing a scissor linkage to have the desired reach and force, one must consider the accuracy of the linkage, and its repeatability. Scissor linkages can suffer from large deflections perpendicular to the plane of the operation, which are caused by clearances in the bearings that make up the pivots, as well as deflection of the links. Recall the lesson of *Abbe's Principle* on page 3-11, and the example of Legos™, which, when stacked together in a long chain, can curve due to all the little micro spaces between each brick amplified by the length of each brick. Each little displacement adds together to create a curved section. In addition, as shown, all the clearances in the joints will prevent the linkage from fully retracting, hence reducing its repeatability.

Geometry	**Enter Numbers in BOLD, Results in Red**	
Nsections	**6**	Number of sections
Llink (mm)	**150**	link length (end hole center to end hole center)
hlink (mm)	**35**	link beam height
tlink (mm)	**3**	link beam thickness
Dpin (mm)	**4**	pin joint diameter
Density (kg/mm^3)	**2.00E-06**	Density of link material
alpha (deg)	**76.09**	Retracted link inclination angle (from horizontal)
beta (deg)	**13.91**	Extended link inclination angle (from horizontal)
Retracted_pitch (mm)	**36.06**	Horizontal space per link in retracted position
Extended_pitch (mm)	**145.60**	Horizontal space per link in extended position
Lretracked (mm)	**216**	Retracked length
Lextended (mm)	**874**	Extended Length
Dretracked (mm)	**145.6**	Retracted distance between ends of a link pair
Dextended (mm)	**36.1**	Extended distance between ends of a link pair
Lsqueeze (mm)	**109.5**	Distance to "squeeze" ends of a link pair to extend
Ioc (mm^4)	**613**	Moment of inertia/distance from NA to outer fiber
Forces, Moments, and stresses (oh my!) (assuming horizontal deployment)		
Fload (N)	**0**	load on end of links
Fpush (N)	**100**	Average desired pushing force
Efficiency	**90%**	Efficiency of linkage with pinned joints
Fsqueeze (N)	**1501**	Average desired squeezing force
Wlinks (N)	**3.7044**	Weight of links
Msqueeze (N-mm)	**109,247**	Moment on first link from squeeze force
Mload (N-mm)	**0**	Moment on first link from end load
Mweight (N-mm)	**1618**	Moment on first link from linkage weight
Mtotal (N-mm)	**110865**	Total worst case moment on first link
Link bending stress	**181**	
ESTIMATED leadscrew actuator to provide squeeze force		
Lead (mm)	**1**	Leadscrew lead
Sefficiency	**0.3**	screw efficiency
Motor torque (N-mm)	**796**	

Is it feasible to create a lazy tongs linkage to reach out and touch someone, and assist them in not having to worry about how they will do in later rounds of the competition? Can the spreadsheet help you to determine feasibility? Analysis is an awesome creativity catalyst and reality barometer!

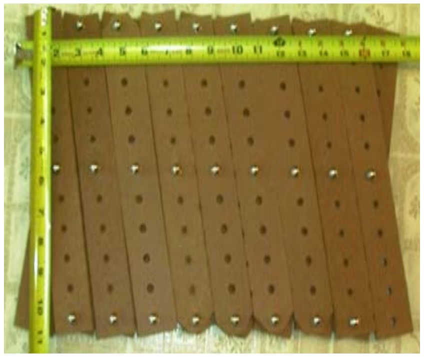


Extending Linkages: *Scissor Linkages*



Towel rack tongs
(photo by Pat Willoughby)

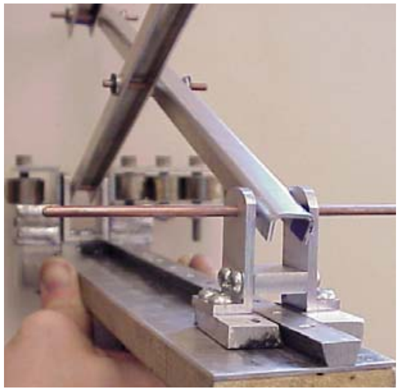
- Scissor Linkages (*Lazy Tongs*) are a great way to get a LARGE range of motion in a small package



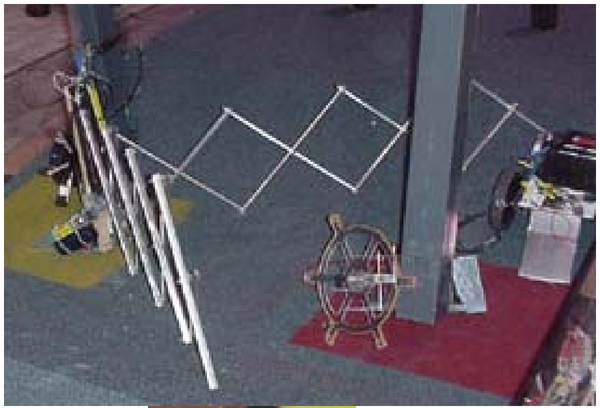
- How does one develop a system as simple in principle, but as complex in detail as the Lazy Tongs?
  - The devil is in the details

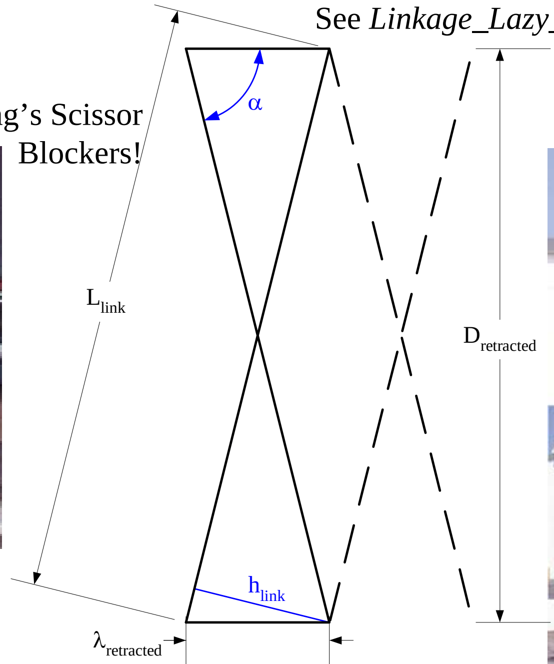
See *Linkage_Lazy_Tongs.xls*

Bryan Ruddy's dovetail bearings to guide his scissors



Eric Tung's Scissor Blockers!





http://www.terexlift.com/catolog/catopdfs/tsm202630.pdf

Tolerances lead to scissor wobble…

Extending Linkages: *Scissor Linkage Example*

Tables 1 and 2 below show the development of concepts for an offensive module to be attached to a stationary scoring module that focuses on spinning its own pendulum for *The MIT and the Pendulum* contest.

Table 1: FRs, DPs, & As for *Offensive Module*

Functional Requirements	**Design Parameters** (possibilities)	**Analysis** (dominant physics)
Rapidly get across the table to pin opponent	1) Vehicle 2) Projectile 3) Linkage	1) F = ma and traction 2) F=ma and angle of projection 2) Fsqueeze*Xsqueeze = ηFpush*Dpush
Locks after being deployed	1) Clamp onto pendulum and opposite wall 2) Tether line and grapple hook 3) One-way gripper at end, non-backdriveable leadscrew	1) Need to do experiments 2) Need to do experiments 3) Need to do experiments
Hard to block	1) Massive force 2) Aim high, and then descend to lower level	1) Impact or static 2) Same as above, but must also push against gravity

Table 2: Rs, Rs, & Cs for *Offensive Module*

References	**Risks**	**Countermeasures**
1) 2.007 notes & past contests 2) 2.007 notes & past contests 3) 2.007 notes & past contests	1) Forces too small, too easily blocked 2) Too easy to block, or tether string causes entanglement and disqualification 3) Complexity	1) Super high speed from spring launcher 2) No tether 3) Scissor linkage is made from many of same parts
1) Past contests 2) Past contests 3) Past contests	1) Too easily deflected 2) Entanglement 3) Minimal if done right	1) Make it very maneuverable 2) Do not use this idea 3) Lots of testing
1) Freshman physics text 2) Freshman physics text	1) Hard to generate 2) Lateral stability, length change, aiming	1) Maximize velocity 2) Make sure system is reversible for another try, OR make gripper also able to lock on to opponent's wall!

The result is the *Attacking Scissor Links*[1] shown on the opposite FRDPARRC sheet. These table entries are frugal, and thus they should get you thinking "wait a minute, what about..." Once complete, the details of the design must be worked out by dividing the *Attacking Scissor Links* module into its own sub modules for development and testing:

Links module: The sketch shows the plane of the linkage horizontal, but reciprocity tells us to also ask what would be the performance if it was vertical? It would sag less, but the gripper design would be a little more complex. This seems worth the trade-off! The spreadsheet will tell you how many sets of links are needed, and you will have to see if you have enough material. Do you need thickness to provide some lateral stability? Do you want to laminate wood between sheet metal, or do you want to bend the edges of sheet metal links to form them into channels? The former would increase the length/diameter ratio of the pins in the joints and reduce cumulative sine errors. If you are to use a laminated design, do you laminate an entire sheet and then cut into links, or do you laminate each link?

Force module: One of the links can be pinned, and the other one needs to be in a pin-in-slot joint and be pulled toward the pivot as shown in the solid model on the previous page. A leadscrew nut can have an integral pivot to which the pin-in-slot joint attaches. The nut should have a square outside shape so it can slide inside the square tube which thus resists the leadscrew actuation torque. The leadscrew would be driven by a motor. Evaluate motor/leadscrew calculations to select the proper motor.

Gripper module: The one-way gripper will probably take some iteration to get it right and robust, but since nothing else is attached to it, it is the least critical. If your opponent's pendulum is missed, make the gripper also have ability to grab onto the opponent's wall?!

Aiming module: If we aim high and engage the other beam, perhaps we can passively disengage a ramp that will let the initial linkage inclination angle decrease so the links end up more horizontal? The complexity of this and the requirement for a longer travel means we should probably just launch from a slightly inclined angle to accommodate linkage sag, and move fast and minimize complexity.

1. As Dave Barry would note, a GREAT name for a rock band!

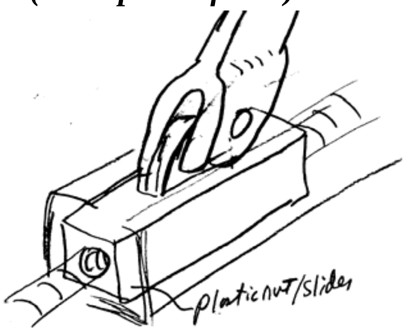
Extending Linkages: *Scissor Linkage Example*

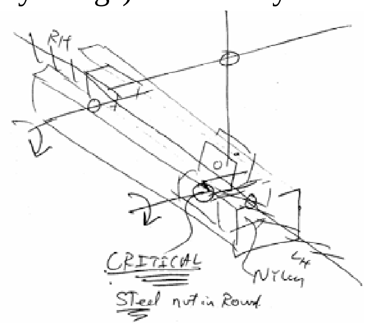
FRDPARRC Sheet Topic: Long Reach System

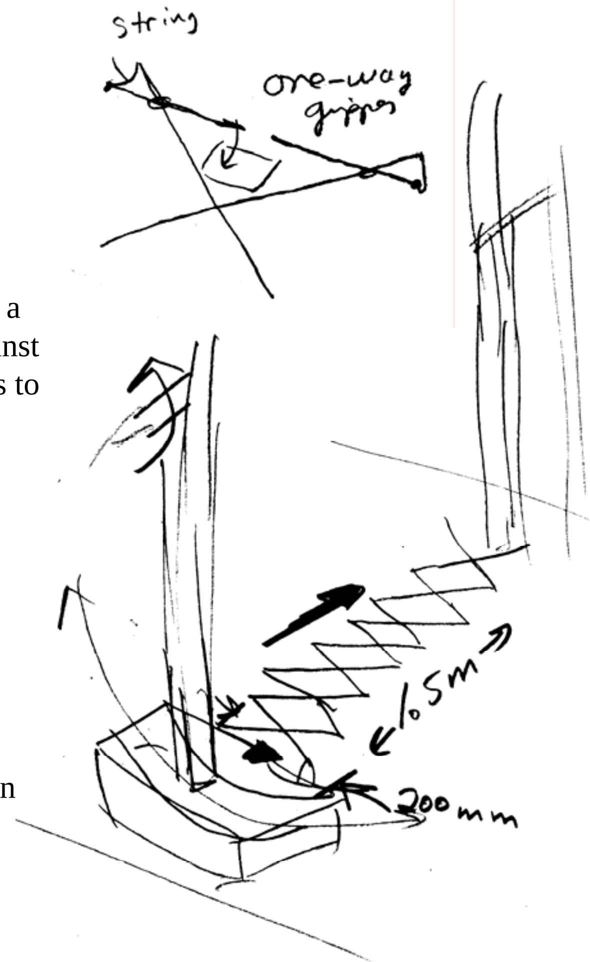
Functional Requirement (Event) Long reach system to rapidly zoom over to opponent and jam their machine and pendulum, and also prevent them from moving around on the table (create a baby gate!)

Design Parameter (description of idea) Scissor linkage (Lazy Tongs) actuated by one base link pin fixed and the other sliding using a pin-in-slot joint

Sketch:







Analysis (physics in words) The input force will act over a short distance, and the output over a long distance, so it will be much less. However, to resist collapsing by opponent pushing against us will be easier since friction in the pins now will work against opponent. Use lazy_tongs.xls to determine feasibility and optimize.

Analysis

See spreadsheet output. To first order, we have a 7.5:1 ratio, so if we generate 300 N of force with a screw to compress the ends, with 50% efficiency we can get 20N of force to jam our opponent and their pendulum.

References: Topic 4 notes and lazy_tongs spreadsheet and *Niagara Balls* contest

Risks: Not enough force, opponent will deflect or push us away.

Countermeasures: shoot high and passive on-way gripper captures beam, and then slide down their beam; thus we form a tether to their pendulum to gumfoozilate them.

Compliant Mechanisms

Linkages are really useful and wonderful; however, their many joints sometimes are the source of errors which are amplified to create unacceptable overall performance. One way to minimize unwanted motions in linkage joints is to use preloaded rolling element bearings. This is often the case for precision robots; however, for mechanisms with limited travel, a more effective alternative is to use structures with local compliances at the joints whose deflections emulate small joint motions. These locally compliant elements are called *flexures*, and mechanisms that use flexures for their joints are called *compliant mechanisms* or *flexural linkages*[1] (*flinkages!*). The compliant elements can either be long thin *blades* that bend along their length, or they can be *hourglass shaped hinges*. The former allow for more deflection, but are also more compliant in out-of-plane directions.

Monolithic compliant mechanisms can be made from a solid block of material, which can be made on a macro scale most cheaply by using an abrasive waterjet machining process. For blades thinner than $^3/_4$ mm, the taper from the waterjet becomes too great, and the blades can be cut by electro-discharge machining (EDM). Hourglass flexures can easily by cut on a milling machine. Clamped compliant mechanisms can be made by clamping thin material sheets to rigid structures.

Consider the robot gripper shown for use in a hostile environment. Robot grippers are designed for dedicated systems, such as picking up standard sized trays of parts. Only a small gripping motion is required, so a compliant mechanism is an ideal way to minimize the number of parts. The flexures are not affected by dirt, and alloys can be selected that are not affected by oven temperatures. The entire linkage system shown was designed to operate in an oven in a 200 ^{o}C environment where a single rotary actuator located outside of the environment was used to actuate two grippers at once. This double gripper allows the robot to pick up a part from an input and operation station, rotate 90 degrees, and place the parts on operation and output stations respectively

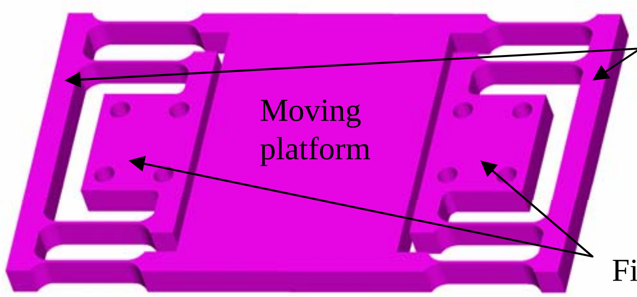
1. Prof. Sridhar Kota at UMI has an entire laboratory devoted to the design of compliant mechanisms. From staplers to windshield washer blades to sophisticated MEMs devices, he has created field-search algorithms to find “optimum” compliant mechanism designs to meet user defined FRs constraints. See http://www.engin.umich.edu/labs/csdl/index.htm. Sandia National laboratories have also created an amazing array of compliant mechanisms. See www.sandia.gov.

A problem that affects compliant mechanisms in the same way that affects linkages with conventional pivots, is that of parasitic error motions as discussed on pages 4-11 and 3-8. As has been shown, in a parallelogram linkage, when the members are nominally orthogonal to each other, relatively pure translation of a platform can occur which is essentially a *sine motion* that is equal to the product of the rocker arm length and the rocker arm angle. However, some small vertical parasitic *cosine error motion* also occurs.

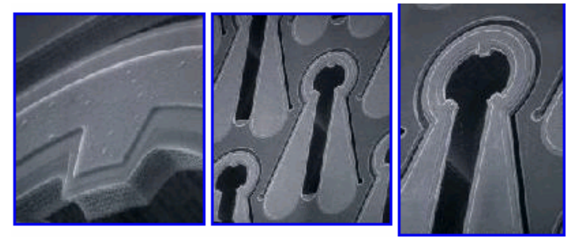
Parasitic error motions can often be dealt with using the principle of reciprocity, as discussed on pages 3-11 - 3-13, to create a *folded beam flexure*. Four 4-bar compliant mechanisms in which the error motions of one are accommodated by another, are used together support a moving platform in a manner similar to a simply supported beam. This design is called a *folded flexure* and it is a relatively common type of 12-bar flexure with two pairs of first stage flexural elements, two intermediate floating platforms that move in mainly in the Y direction and have parasitic X error motions, two pairs of final stage flexural elements, and a platform that moves only in the Y direction. The final two flexural links’ parasitic error motions have been cancelled by the intermediate floating platforms’ error motions! By Gruebler, this system has 3*(12 - 1) - 2*16 = 1 degree of freedom.

Micro Electro Mechanical Systems (MEMS) are miniature devices typically made from etched silicon using standard photolithographic techniques developed by the semiconductor industry, and they are creating a linkage renaissance. From micro mirrors developed by Texas Instruments for projectors to accelerometers developed by Analog Devices for automotive air bag sensors, to countless pressure gauges that use thin film diaphragms and capacitive or piezoresistive sensing elements, MEMs systems are having a huge impact on our lives. In the future, we will likely experience them with energy harvesters that use ambient vibration to power tiny sensors, circuits, and radio transmitters. Flexures can also be used to create bistable devices for miniature relays[2] that will likely change the nature of electronics and power systems!

2. For his Ph.D. thesis, Jin Qiu created a bistable double-beam flexure, which flexuralistically quasi emulates Watt’s 4-Bar linkage, see Qiu, J. Lang, A. Slocum, "A Centrally-Clamped Parallel-Beam Bistable MEMS Mechanism" MEMS 2001 Digest 353-356, Interlaken, Switzerland, January 2001

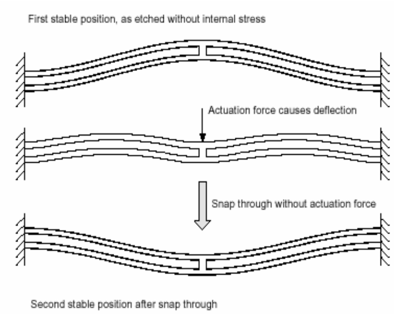


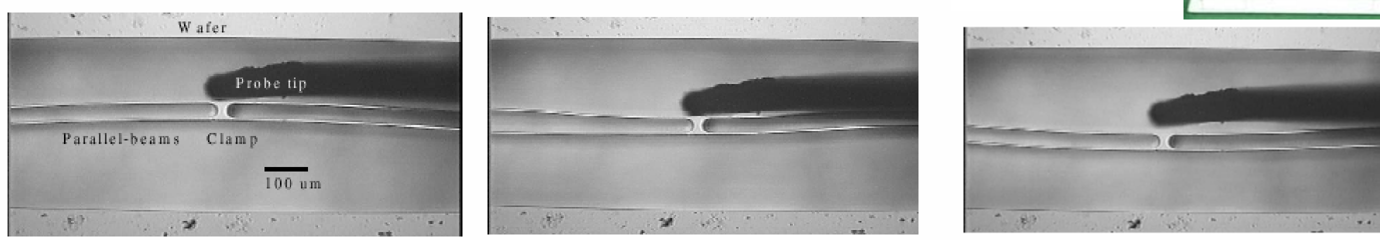
Compliant Mechanisms



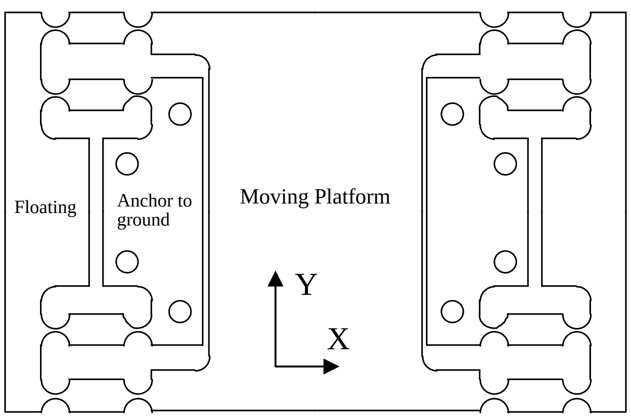
What is this thing used for?

- The pin joints in linkages are often the major source of error motions
  - See page 10-24 and the flexure design spreadsheets!
- When only small motions are required, the pin joints can be replaced with flexural elements, thus forming a *compliant mechanism*
  - Extremely high accuracy small range of motion devices can be made this way
  - Many *Micro Electro Mechanical Systems* (MEMS) use tiny silicon flexures





(a) The mechanism as etched; the probe is ready to pusl *(b) Deflection as the probe pushes the mechanism.* (c

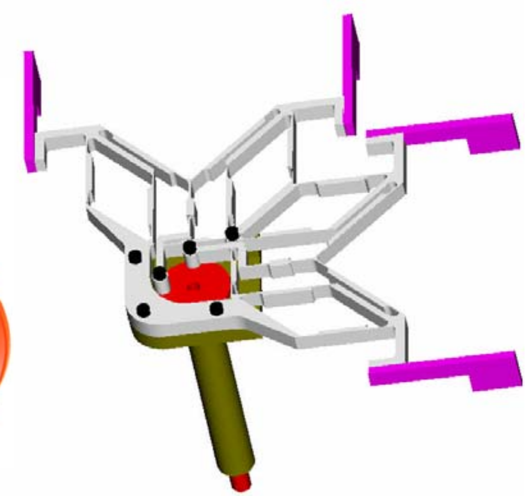


CableCuff® US Patent 6,101,684 (www.cableclamp.com)

a) Note the nifty flexural pawl/ratchet

b) Could the pivot have been made as a snap-fit or a "living" (flexural) hinge

c) If patented and so simple (machines could make and assemble) can it be made domestically?



Compliant Mechanisms: *Analysis*

Compliant mechanism design requires careful analysis to maximize desired deflections while minimizing link stress. The ratio of work done to create motion to the amount of useful work the linkage does in a system is a measure of its *efficiency*. Hourglass-shaped flexures are the least efficient, but they can be made by drilling holes close to each other... *HourglassFlex.xls* computes the moment and angular deflection θ (typically only a few degrees) for radii *R*, web thickness *t*, width *w*, and maximum allowable stress:

$$\theta = \frac{3MR}{2Ew\left[(0.5t+R)^2-R^2\right]}\left\{\frac{1}{0.5t+R}+\frac{1}{(0.5t+R)^2-R^2}\times\left[\frac{2R^2+(0.5t+R)^2}{0.5t+R}+\frac{3R(0.5t+R)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{-R}{\sqrt{(0.5t+R)^2-R^2}}\right)\right)}{\sqrt{(0.5t+R)^2-R^2}}\right]\right\}$$

Beams, or blades, are very efficient. Consider the following cantilevered beam comparison illustrated by the spreadsheet *taperedbeam.xls*:

	Thickness	Width	Staright
Force F (N)	10		
Modulus E (N/mm^2)	7.00E+04		
Maximum safe stress (N/mm^2)	1.00E+02		
Width at base wb (mm)	6	6	6
Width at end we (mm)	6	1	6
Thickness at end te (mm)	1	2	2
Thickness at base tb (mm)	2	2	2
Length L (mm)	25		
Stress at base (N/mm^2)	62.5		62.5
Stress ratio actual/max	0.625		0.625
Slope (radians)	-0.022		-0.011
Deflection (mm)	0.304	0.249	0.186
Efficiency tapered/#		1.22	1.64

These different types of links can be used in a 4-bar flexure to support a platform. As the FEA plots of deflected blades show, a uniform thickness blade of length *L* deflected in a 4-bar flexural linkage, behaves like two cantilever beams each of length *L/2* that are placed end-to-end so their slopes match and they are essentially springs in series. The force *F* applied is the same in the two beams causing each beam to deflect $FL^3/24EI$, so the compliance of a single blade is $L^3/12EI$. Thus the platform, supported by 2 blades that act in parallel, will have a stiffness of:

$$K_{4-bar\_flexure\_platform} = \frac{24E_{blade}I_{blade}}{L^3_{blade}}$$

A curious effect is that the moments created by anchoring the ends of the blades causes deflections in the blades. To prevent actuator misalignment forces from causing further error motions, the drawing shows a *wobble pin coupling* pushing on a flexure supported stage at a height *a* that is one-half the blade length. The vertical and pitch error motions are:

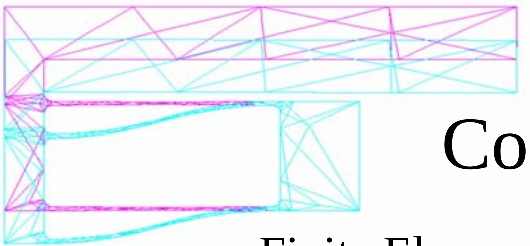
$$\delta_{vertical\_error} = \frac{x^2_{translation}}{2\ell} \qquad \theta_{pitch\_error} = \left(\frac{x_{translation}}{\ell}\right)\left(\frac{6(\ell-2a)t^2}{3b^2\ell-3t^2\ell+6at^2}\right)$$

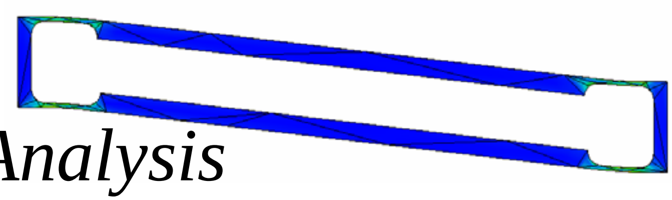
Folded flexure platforms use symmetry to cancel out these errors. A simple folded flexure with blades of equal modulus *E*, length *L*, and moment of inertia *I* behaves like 2 sets of 4-bar flexures in parallel acting in series with 2 sets of 4-bar flexures in parallel, and hence the stiffness is still just:

$$K_{folded\_flexure\_platform} = \frac{24E_{blade}I_{blade}}{L^3_{blade}}$$

If one really wanted to optimize the efficiency of a folded beam flexure supported platform, one would taper the folded beam flexure's blades so they are thinnest in the middle.

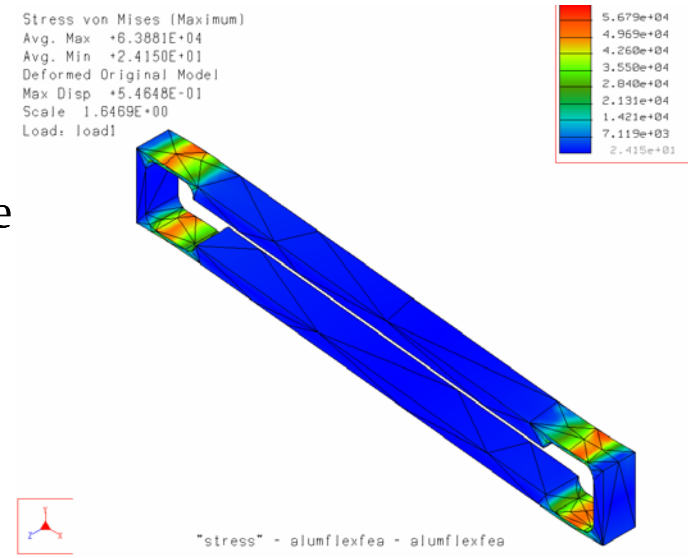
A compliant mechanism would probably be an ideal mechanism to use to allow you to preload a machine around the pendulum. Try to design a compliant mechanism to preload a module's wheels to the pendulum so it can climb up and engage the axles and spin the pendulum!

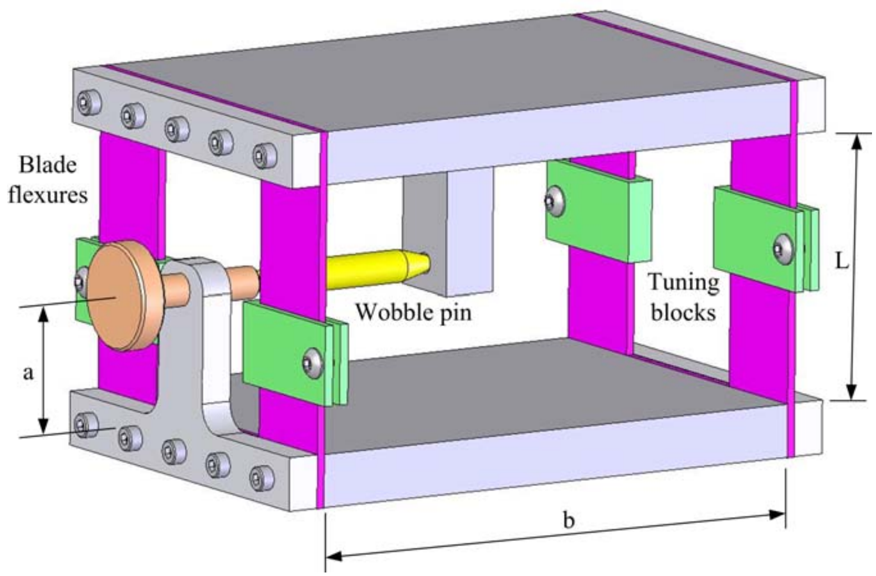


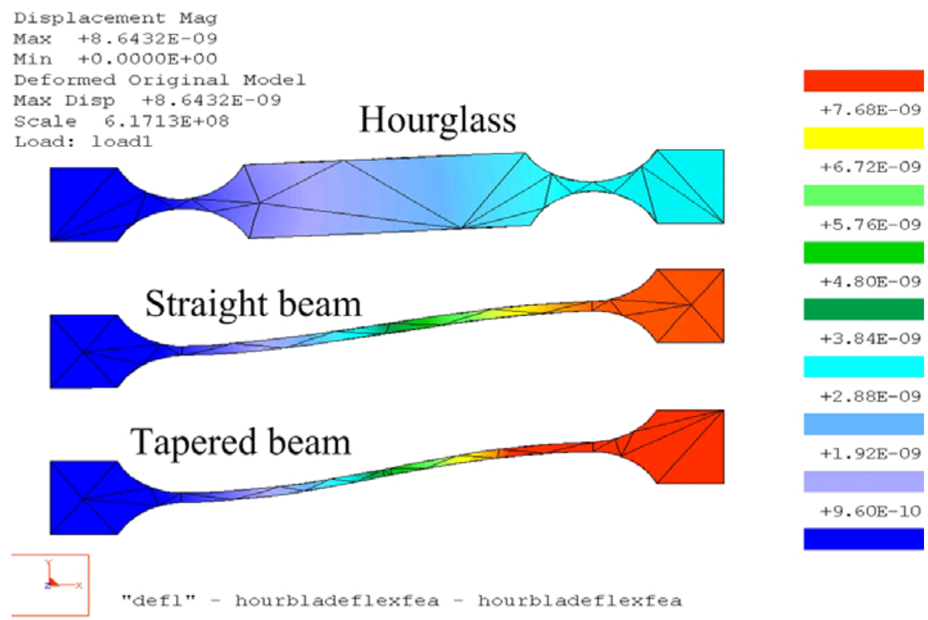


Compliant Mechanisms: *Analysis*

- Finite Element Analysis is a powerful design tool
  - Design Parameters (dimensions) are changed until desired performance (stress, deflection) is obtained
- A 4-bar compliant mechanisms was designed to be made from titanium for strength, weight, and temperature resistance
  - It was machined by an OMAX abrasive jet machining center for $200 ($1000 to wire EDM)
  - When installed, it was strong enough to resist damage by the controls engineers!







Manufacturing & Robust Design

Even though most linkages are planar (their motion is confined to a plane), forces are never exactly in a plane, so slight out-of-plane moments are created. Scissor linkages represent the most extreme case of this problem, where many small *sine errors*, as discussed on page 3-11 - 3-113, add up to potentially cause the scissor linkage to severely deflect out of its plane of motion. One must first ask if this really matters, is this a *sensitive direction* (see page 3-23)? Then one must identify the most sensitive parameter, and operate on that parameter. This is the essence of manufacturing and robust design, which go hand-in-hand.

Consider the case of a scissor linkage, and how it might be made more robust. One can seek to minimize each incremental sine error by decreasing the clearance between the pivots' pins and the links' bores, and increasing the thickness of the links. However, Saint-Venant, see pages 3-6 - 3-9, indicates that this is not an efficient use of resources. To prevent out-of-plane motion of the tip, mechanism must act on the tip. If the tip deflects out of the plane, apply *reciprocity*, as discussed on page 3-14 - 3-16. <u>One linkage causes errors, so if one is not sufficient, try two?</u> Next <u>use errors to cancel errors</u>. The result is the idea to flip one scissor linkage over to cancel the errors of another; thus make two thin and simple scissor linkages and join them to form an isosceles triangle whose height changes as the links extend.

Robust linkage design is also heavily concerned with avoiding over-constraint and singularities. Both can place large loads on the bearings in the joints. Rotary motion joints often use pins in bores, and these can typically handle very large loads with proper lubrication. Linear sliding elements are often the biggest cause of jamming, so one should make use of Saint-Venant's Principle when designing sliders. Furthermore, it is often the unanticipated out-of-plane loads that can overload linkage bearings. Fortunately, Chapters 10 and 11 focus on the details of designing systems with bearings.

Actuators that create linkage motion can also damage themselves or the linkages if the forces or torques are not applied using proper couplings, such as a clevis for linear actuators or a rotary coupling. Clearance in a joint can be used to accommodate misalignment only when loads are low, or the intended life of the device is limited anyway. Out-of-plane forces or moments must not be transferred between the actuator and the linkage! This is where *exact constraint design*, as discussed on page 3-24, becomes really importance.

When a linkage design is synthesized and then drawn on a solid modeler, the part drawings are easily generated. Since links are typically planer, it becomes a simple task to cut them, often all in one set up, using an Abrasive Waterjet Machining Center™ such as the OMAX[1] 2626 shown. In fact, the *wiffle tree* components shown were cut with a waterjet and were used to distribute a centrally applied force to each of 32 points for a special processing machine. Recall wiffle trees accomplish even load distribution by using the principle of *elastic averaging* as discussed on page 3-28.

Whatever methods are used to make the components, manufacture and assembly must be done with sufficient precision to maintain alignment between elements, else the motion of the linkage itself may create the forces that wear it out. Hence manufacturing the parts to ensure pivot bores are perpendicular to links, is of very high importance. Punching holes with a hand drill is not recommended.

To achieve long life, linkage joint stress must be controlled and the joint must be lubricated. For lightly loaded systems, solid lubrication can typically be incorporated into the joint design by using self-lubricated bearings. For heavily loaded systems, liquid lubrication is typically required either in the form of grease or oil. The former must typically be reapplied comparatively frequently, depending on the loads. In the case of a car, the undercarriage joints typically are re-lubed every time the engine oil is changed. For extremely heavily loaded systems, such as construction equipment, the joints may have to be greased every week or more. The challenge, however, is that lubricants can leak out of joints causing damage to the environment, as well as damage to the joint. Fortunately, most robots for design contests can typically use lubricious plastic bearings running on smooth surfaces, and thus really should not need any lubricant applied other than an initial dab of grease.

Conduct a manufacturing review of all linkages you have planned on using, and make sure that they are minimally complex and easy to manufacture. If they present challenges, make sure to consider them for early development as most-critical-modules!

1. See www.omax.com and www.waterjets.org

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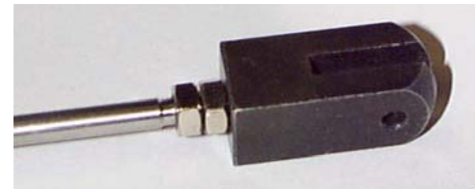
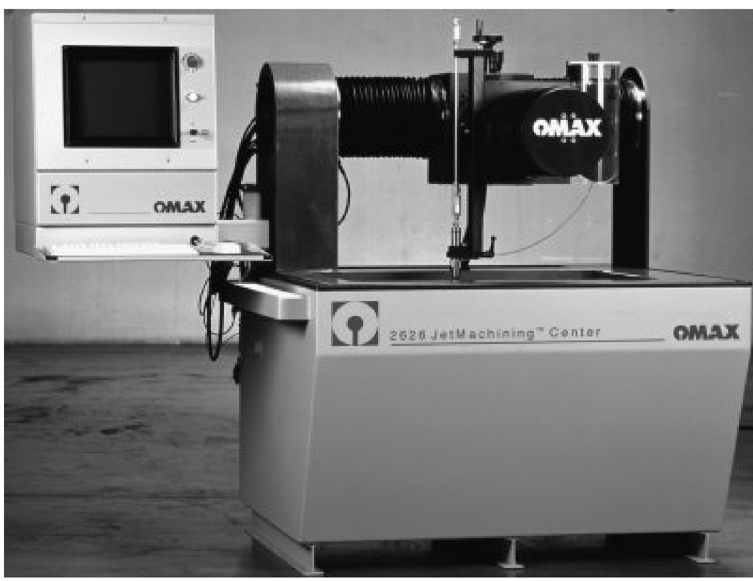
Jorge Renjifo's 2005 2.007 "Tic-Tech-Toe" machine

- Most machining processes can be used (milling, abrasive waterjet, wire EDM)
- Abrasive waterjet machining allows for very complex compliant mechanisms to be machined very rapidly
  - Example: A *wiffle tree* evenly spreads out a point load
    - Windshield wipers are the most common example
    - A wiffle tree was cut on an abrasive waterjet machine in only an hour
      - NOTE the use of a large blade flexure to support the right side of the OMAX machine's X axis, to allow for thermal growth
        - » Example of a machine-tool-precision very-small-range-of-motion 4-bar linkage





Mechanism Mania!

This chapter has just introduced the concept of simple linkages that act in planes. More advanced linkages can work together to create complex motions such as walking or full three dimensional motion! Indeed, walking machines have been the dream of engineers for centuries, and perhaps it is ironic that simple walking action is readily achieved by many toys. Real walking motion that correctly emulates bipeds or quadrupeds is an extremely difficult task to accomplish, and represents an exciting area of robotics. However, note the neato walker that Linus park built for his 2.007 robot; thus proving that the most important thing in any student "contest" is that you should create a design about which you personally are most passionate. The design may not "win" in terms of points scored, but you will "win" in terms of showing your engineering prowess!

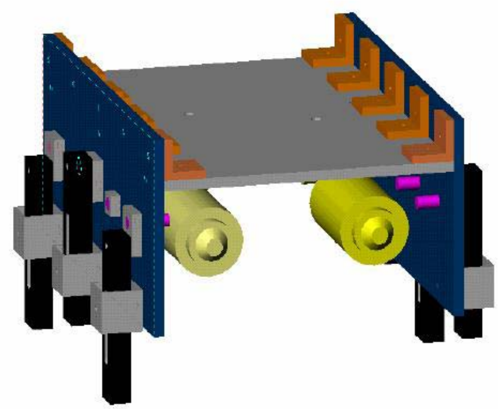
What about 3D motion? Spatial linkages move a coupler point in three dimensions, and are extremely challenging to synthesize. An interesting spatial linkage was discovered in the age of steam engines when James Watt was creating complex planar linkages to try and achieve straight-line motion to guide steam engine pistons. Pure translational motion with only rotary joints was considered an impossibility, and so engineers made do with clever complex planer linkages. Then along came Sarrus and his mechanism in 1853! Did his bio-neural net feel it could not achieve what it wanted to in a plane, and did he jump out of the plane to create the simple and ingenious device shown?[1]

We also have not even begun to touch on the subject of cams, which can also be used to convert simple rotary motion into interesting reciprocating motion. Like linkages, cams require careful analytical modelling to create the exact desired motion, and to calculate parameters such as position, velocity, and acceleration as a function of the input angular velocity. Like linkages, the calculations required are straight-forward, a geometrist's delight; thus you should be confident that you can design cams!

Looking at machines in the world around you (and on the web) can help make up for a lack of experience, and will help you become experienced! Having said this, remember, to become good at linkage design, or any other type of design, requires experience. Those who succeed, do so with practice, and an in-depth understanding of fundamental principles and analytical modelling skills catalyzed and tempered with manufacturing knowledge, *and a high degree of professional ethics.*

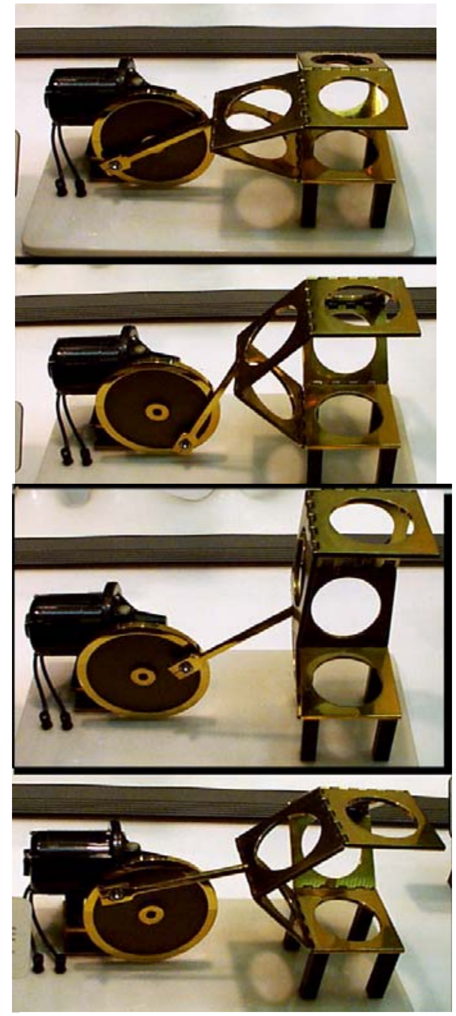
Dive into your machine's design with delight, dream what you want it to do, and then synthesize the linkages you need to realize your dreams. There is always a mechanism that can be created to give you motion happiness!

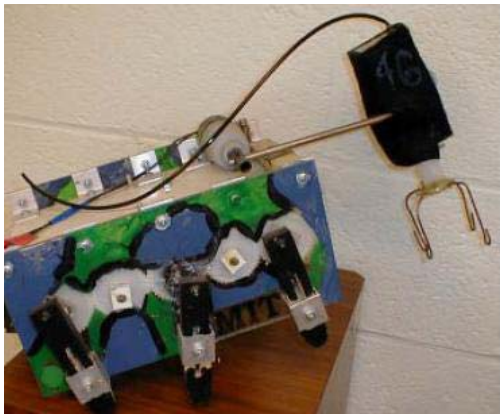
1. The beautiful model shown is on display in the *Mathematical Models* section of *Boston Museum of Science.*



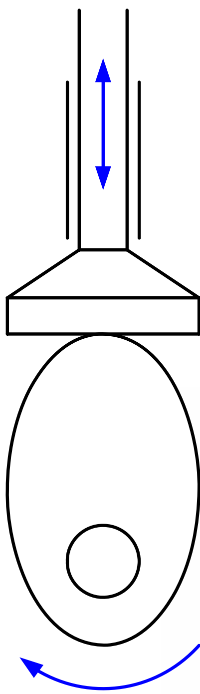
Mechanism Mania!

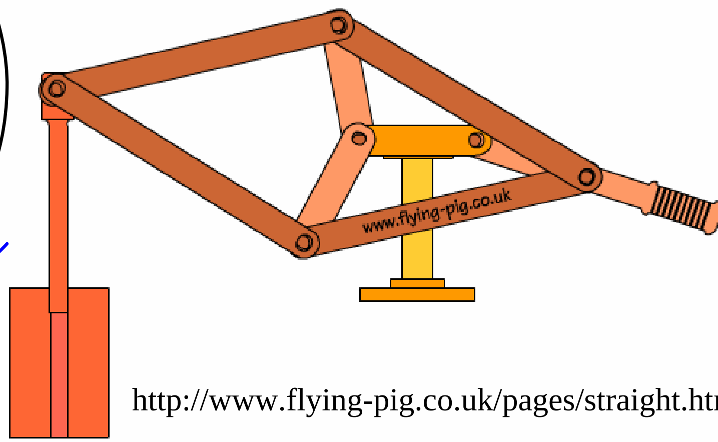
- Mechanisms can be created to accomplish virtually any task
  - They are essentially mechanical computers
  - Seek and you will find!





Linus Park created an awesome walker for 2.007 2000 MechaImpactAgeddon!





http://www.flying-pig.co.uk/pages/straight.htm

Pictures taken at
Boston Museum of Science

Topic 4 Study Questions

Which suggested answers are correct (there may be more than one, or none)? Can you suggest additional and/or better answers?

1. The weaving of cloth gave rise to the need for more complex machines to convert waterwheels' rotary motion into complex motions
   - True
   - False
2. The invention of the steam engine helped to create a great need for new mechanisms and machines
   - True
   - False
3. James Watt (1736-1819) applied thermodynamics (though he did not know it) and rotary joints and long links to create efficient straight line motion, although many other more efficient linkages were later discovered by others:
   - True
   - False
4. James Watt invented the flyball governor, the first servomechanism, which made steam engines safe and far more useful
   - True
   - False
5. Leonard Euler (1707-1783) was one of the first mathematicians to study the mathematics of linkage design (synthesis):
   - True
   - False
6. Conservation of energy (work) can be used to determine the output force of a linkage if the input force and displacement are known as well as the output displacement and efficiency:
   - True
   - False
7. Degrees of Freedom (DOF) are:
   - The number of input motions that must be provided in order to provide the desired output
   - The number of independent coordinates required to define the position & orientation of an object
8. For a planar mechanism, the degree of freedom (mobility) is given by Gruebler's Equation

   $$F = 3(n-1) - 2f_1$$

   - n = Total number of links (including a fixed or single ground link)
   - f_1 = Total number of joints (some joints count as f = ½, 1, 2, or 3)
9. Links can have different numbers of nodes (i.e., joints) to which other links are attached:
   - True
   - False
10. Lower pairs (first order joints) or full-joints (counts as f = 1 in Gruebler's Equation) have one degree of freedom (only one motion can occur), and they include:
    - Revolute (R): Also called a pin joint or a pivot, and a rolling contact joint also counts as a one-degree-of-freedom revolute joint
    - Prismatic (P): Also called a slider or sliding joint, beware Saint-Venant!
    - Helical (H): Also called a screw
11. Lower Pair joints with multiple degrees of freedom include:
    - Cylindrical (C) 2 DOF (translates and rotates) multiple-joint (f = 2)
    - Spherical (S) 3 DOF multiple-joint (f = 3)
    - Planar (F) 3 DOF multiple-joint (f = 3)
12. Higher Pair joints with multiple degrees of freedom include:

Link against a plane where a force is required to keep the joint closed (force closed) is a half-joint (f = 2 in Gruebler's equation)
Pin-in-slot where the slot geometry keeps the joint closed (form closed) is a multiple-joint (f = 2 in Gruebler's equation)
Second order pin joint, has 3 links joined together and thus has 2-DOF and is a multiple-joint (f = 2 in Gruebler's equation)

13. The sum of the shortest (S) and longest (L) links of a planar four-bar linkage cannot be greater than the sum of the remaining two links (P, Q) if there is to be continuous relative motion between two links
    True
    False
14. Two-bar linkages are simply levers
    True
    False
15. Two bar linkages should never be used in triggers
    True
    False
16. A three bar linkage usually has three degrees of freedom
    True
    False
17. Four-bar parallelogram linkage type supported stage provides mostly error free translation of the coupler:
    True
    False
18. 4-Bar linkages are commonly used for moving platforms, clamping, and for actuating buckets on construction equipment
    True
    False
19. 4-Bar linkages typically include:
    Ground link
    Crank link to which power is applied, which has joints between it and the ground and coupler link
    Follower link which has joints between it and the ground and coupler link
    Coupler link which has joints between it and the follower and coupler links
    Driver link which connects the crank link to the power source
20. 4-Bar linkage motion can be developed using kinematic synthesis:
    3 Precision Point Circle Construction
    Spreadsheet or other synthesis software
    Experimentation
    Copy another design and hope it works for your application
21. The parasitic error motions of a four-bar linkage parallelogram type flexure supported stage include:
    Pitch error caused by the applied load or actuation force not being applied through the system center of stiffness
    Even though it may be a large radius arc, the stage is still moving along an arc-shaped trajectory which includes a component perpendicular to the desired motion
22. The center of stiffness of a four-bar linkage parallelogram type flexure supported stage is typically located halfway between the bottom of the moving stage and the top of the anchoring structure:
    True
    False
23. The instant center of velocity (rotation) for two bodies in plane motion is a point, common to the two bodies, which has the same instantaneous velocity in each body:
    True
    False
24. The instant center is always located inside the perimeter of a linkage:
    True
    False
25. The instant center can be used to determine relative velocities between various links

True
False

26. Knowing the relative velocity between links, and the torque input to one link, allows you to use conservation of energy to determine the output torque
    True
    False
27. The instant center of a link will be located at the intersection of lines colinear with links on either side of the link
    True
    False
28. One can visualize the instant center for a point on a moving body as the center of the circular arc that coincides with the point's motion path at an instant in time
    True
    False
29. For a 4-bar linkage, the instant center is found by drawing lines through the crank and follower pivots:
    True
    False
30. The velocity of any point on the coupler at any point is perpendicular to an imaginary line from the instant center to the point of interest
    True
    False
31. The instant center can be used to determine the linear and angular velocity and acceleration of the coupler's center of mass or the coupler point
    True
    False
32. If the instant center is coincident with a joint on the coupler, the linkage can become unstable and can lock up, or the crank must reverse its direction
    True
    False
33. A linear actuator cannot be effectively (huge stroke or huge forces are required) attached to a point on a 4-bar linkage to allow the output link to move through 180 degree motion, and so a 6-bar linkage should be used:
    True
    False
34. Linear motion bearings are typically used to achieve long range rotary motion
    True
    False
35. The output force of a scissor linkage equals the product of the input force and the output stroke divided by the input stroke
    True
    False
36. When only small motions are required, linkages' pin joints can be replaced with flexural elements, to form a compliant mechanism
    True
    False
37. Abrasive waterjet machining allows for very complex compliant mechanisms to be machined very rapidly
    True
    False