FUN*da*MENTALS of Design

Topic 6

Power Transmission Elements II

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Power Transmission Elements II

Screws & gears are transmission elements that warrant a special place in power transmission systems because of the huge range of power levels at which they are applied and the very high transmission ratios they can achieve.

The first screws were used perhaps by Archimedes in 200 BC as pumps to lift water from a river to a field. It is not known when the first power screw for lifting a solid load was invented, but it was likely over a thousand years ago. Like many technologies, the military was a powerful catalyst for development. One could imagine designers wondering how they could convert rotary motion into linear motion, or how they could amplify force or torque. With gunpowder came the need for the manufacture of better gun barrels which propelled the fledgling machine tool industry. Fortunately, making cannons is like making cylinders for steam engines, and swords begot plowshares...

The need to tell time accurately for navigation, the longitude problem[1], was perhaps the major driving force in the development of accurate gears. The methods developed for creating accurate gears for clocks were scaled to create accurate gears for industry and this also helped power the industrial revolution.

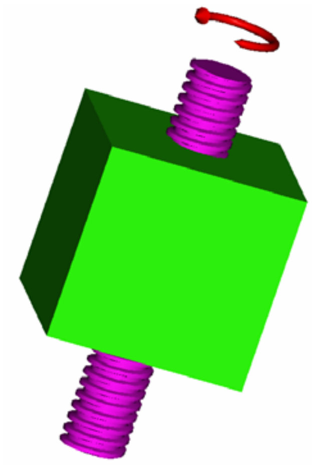
We may never know who the great minds were who actually had the moment of brilliance to invent screws and gears, but we salute these great minds; for without screws and gears, which still serve us today, society would grind to a halt, and industry would be threaded!

By considering *FUNdaMENTAL* principles, you will better be able to separate marketing hype from engineering reality. Remember Maudslay's Maxims, and apply them in the context of:

- Kinematics: motion, accuracy, space
- Dynamics: forces, speeds, life
- Economics: design, build, maintain

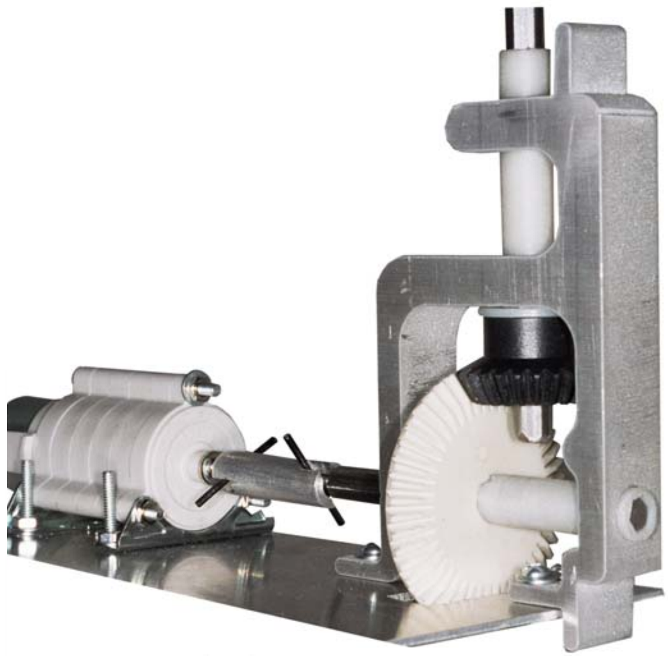
1. See Dava Sobel's book <u>Longitude</u>, ISBN 0802713122

Topic 6
Power Transmission Elements II

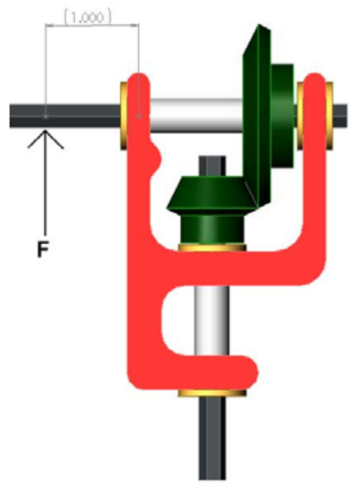


Topics:

- Screws!
- Gears!



www.omax.com



Screws!

If we consider the double helix structure of DNA, perhaps it makes sense that past, present, and future of the human race also depends on leadscrews to move us forward to our destiny![1] Replisomes move along the DNA double helix, dividing it into two strands, and then appropriate matching nucleootides are added to yield two new strands of DNA. What mechanical actions compliment chemical actions, and who will develop the instruments and experiments that will unravel the mysteries of the mechanism of life? Perhaps a mechanical engineer with insight into macro scale mechanisms will play an important role!

To look to the future, consider clues from the past. Was Archimedes the inventor of the first screw, and was there an observation that led to its discovery, or was it playing with things that randomly led to the invention? Is there a deep galactic memory imbedded in DNA that enabled it in its quest for never ending replication on all scales to fire neurons in Archimedes' head to enable him to suddenly imagine the idea of a screw? Is it a coincidence that leadscrew nuts move along leadscrews as they rotate to move a machine tool's axes which are used to make parts for other machines? Throughout history, humans have observed nature and used their observations as catalysts for creativity. So it is with screws, for when used in designs created using the fundemental principles presented in Chapter 3, leadscrews have been at the heart of nearly every type of machine used to make nearly every type of part.

A screw is essentially a rotary wedge: it converts rotary motion into linear motion by the action of the inclined plane of one thread on another. If you walk up a spiral staircase, the spiral staircase remains fixed, but you rise up and turn as you walk up the staircase. The thread on a nut and a screw shaft are also helixes. Iff a screw shaft were fixed, and a nut on it were made to turn, it would move along the screw as the nut's inclined thread planes slide along those of the screw shaft. By reciprocity, if the nut were prevented from turning or translating and the screw shaft was made to turn, the screw would translate. If the nut were prevented from rotating but allowed to translate, and the screw shaft were made to turn, but not translate, rotary motion of the screwshaft would be converted into linear motion of the nut. In each of these cases, what is fixed and what is moving is relative in terms of which element is attached to a component that is to be moved in order to do useful work.

There are two principal types of screws: sliding contact screws and rolling contact screws. The former are the simplest to make, and they have sliding contact between the thread of the screwshaft and the nut,. These are typically called *powerscrews* or sometimes just *leadscrews*; however, simplicity comes at the expense of efficiency. Friction can decrease the efficiency of a sliding contact thread leadscrew to 30% or less. As is generally the case in life, trade-offs abound, and leadscrews can handle large loads and suffer tremendous abuse. This low efficiency also means that once a bolt is tightened, it generally will not unwind. Once again, reciprocity shows us that a problem can be an opportunity.

Rolling contact between the threads of the nut and screw can be obtained using recirculating ball bearings, and can enable *ballscrews* to have 90+% efficiencies. However, their rolling elements can be quickly destroyed by contamination and shock loads. In addition, ballscrews require much more careful machining than do simple leadscrews, and hence they are primarily used where high efficiency and precision are required.

It is easier to create high speed low torque rotary motion from an electric motor, than high torque (or force) ow speed motion. Furthermore, since linear motion is so often required in machines, leadscrews are fundamentally fantastic machine elements. They are the truest of transmissions, and in order to maximize your creativity in using them, it is vital to understand the *phine* points of their *fizziks* of operation. *Reciprocity Rules*! Moreover, seemingly mundane manufacturing issues can cause mechanistic mayhem if overlooked, so once again, attention to detail is a must.

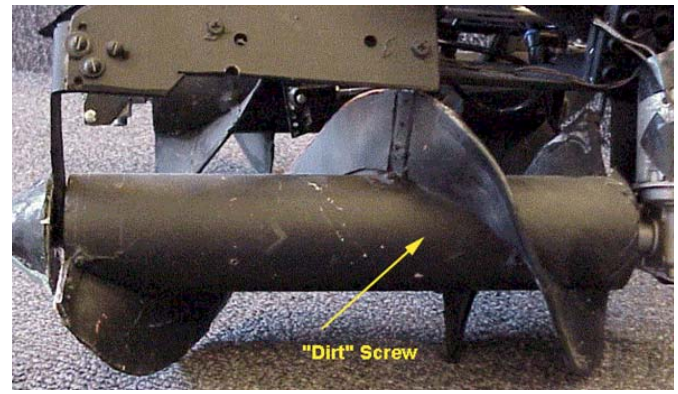
Want some apple pie? Use a screw-based apple peeler! Feeling mellow when you are of age, use a corkscrew to open a bottle of wine! What types of linear motion do you require in your machine, and where might a screw be used? Can linear motion be turned into rotary motion, such as by using a screw to move the boom of a crane? When is it easier to turn the nut instead of the screw shaft?

1. See for example D. DeRosier, "The Turn of the Screw: The Bacterial Flagellar Motor", Cell, Vol. 93, 17-20, April 1998 (http://www.cell.com/cgi/content/full/93/1/17/); and J. Stanley and C. Guthrie, "Mechanical Devices of the Spliceosome: Motors, Clocks, Springs, and Things", Cell, Vol. 92, 315-326, Feb. 1998 (http://www.cell.com/cgi/content/full/92/3/315/

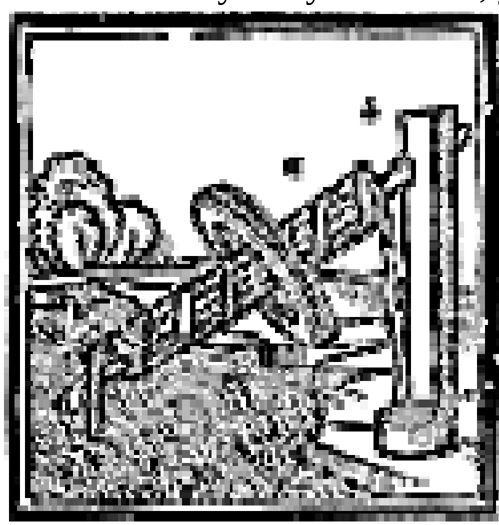
Screws!

- **The screw thread is one of the most important inventions ever made**
- **HUGE forces can be created by screw threads, so they need to be carefully engineered:**
  - Leadscrews
  - Physics of operation
  - Stresses
  - Buckling and shaft whip
  - Mounting

- **When HUGE forces are created by screws**
  - The speed is often slow
  - Always check to make sure you get what you want
  - If you try sometime, you just might get what you need ☺



Mike Schmidt-Lange designed this auger-wheeled vehicle for the "sands" of 1995's 2.007 contest Pebble Beach, and years later, a major government lab "invented" the idea as a Mars rover sand-propulsion device…



Someday, apples will be so plentiful, people will need machines to peel them…!

Screws: *Leadscrews & Ballscrews*

Leadscrews and ballscrews are the two very commonly used transmission elements in machines. They used to transform rotary motion from an electric motor into linear motion of a carriage. In fact, the kinematics of leadscrews' motion are quite simple: they are a helical version of a wedge. Holding a nut so it cannot rotate but allowing it to translate, and conversely rotating a screwshaft while preventing it from translating, causes a nut's thread to slide along the thread of the screwshaft. The kinematics of a ballscrew are similar, except that instead of sliding contact between the threads, rolling elements, balls, are used. However, since the balls only travel half the distance of the threads (try rolling a pen between your hands) a mechanism is needed to collect the balls as the threads roll past them. This mechanism, called a *pickup* or *ball deflector*, diverts the balls into a return tube that carries them back to the beginning of the nut's thread.

A leadscrew' or ballscrew's accuracy can be very great, because multiple thread turns can be simultaneously engaged which can help to average out errors. However, when the threads are first cut, errors in their shape can affect the accuracy of motion. *Lead error* and *thread drunkenness* are the two most common errors. Lead error occurs when the axial distance travelled per rotation of the shaft is not constant. This creates an imperfect overall transmission ratio. Thread drunkenness occurs when the thread does not make a perfect helix. in addition, errors in the profile of the thread also occur, which cause the nut to wobble and also affects the lead accuracy. Thus these errors require that some clearance exist between the threads of the screwshaft and the nut, or else the threads may bind and jamb. Unfortunately, the requirement for clearance between the threads creates a condition known as *backlash*. When rotating the screwshaft in one direction, the nut moves forward; however when the screwshaft rotation is reversed, unless there is a load on the nut, for a small rotation angle there will be no motion of the nut. For precision applications where backlash is a concern, multiple nuts or nuts that are slit can be *preloaded* with spring elements to reduce backlash and provide the local compliance needed.

A leadscrew and nut can fit into a small space. The nut thread can be tapped into a structure itself to further reduce mounting space requirements; however this would eliminate the chance to replace the nut once the thread wears. A ballscrew nut is typically larger in diameter than a leadscrew nut because more space is required for the balls and the return tubes, but it is still small enough overall to generally not be a problem. The key mounting issue is typically where to locate the nut axially along a carriage being moved, for as the figure reminds us, placing the nut at the *center of stiffness* (see page 3-26) reduces pitch moments imposed on the carriage by a non-straight screwshaft.

The dynamics of leadscrews and ballscrews and their load & life calculations are to the first order very simple, but as with many designs, the devil is in the details, and thus will be discussed in depth on following pages. Suffice to say here that in general, there are such a wide variety of components available that one rarely has to resort to a custom manufactured design.

Leadscrews, because of their inherent simplicity, are very simple to design and manufacture. It is their very simplicity that led to their being so widely used at the beginning of the industrial revolution. Even today they are the actuator of choice for many low cost machine tools. In addition, they are also easy to maintain. A simple seal and periodic lubrication are all that is required to keep them running, due to the fact that a nut's sliding contact threads themselves act like scrapers to clear the threads before they are engage. Because leadscrews are so simple to design and manufacture, they can easily be created by students in a robot design contest for their machines!

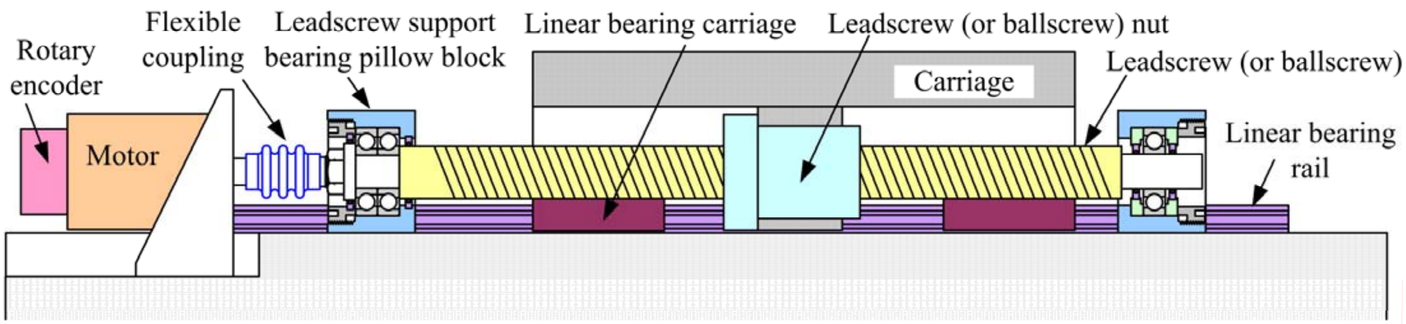
Ballscrews, on the other hand, require extra care in the design of the thread profile, and the design of the ends of the nut's threads where the balls enter and leave the thread circuit. If the entrance and exit paths, as well as the ball return mechanisms, are not properly designed and manufactured, then the balls can bunch up and interfere with each other's rolling. Hence ballscrews should only be designed and manufactured by companies with either the appropriate experience, or the desire to invest significant amounts of money.

Look at the machine tools in the shop you are using to build your robot and see where and how they use leadscrews. Where do you need linear motion in your machine, and could a leadscrew be used? Should your leadscrew be directly driven by a motor or should a belt connect the motor shaft to the leadscrew shaft? If you are using the leadscrew to actuate a linkage, how might you mount the screwshaft and nut so that they are mounted to components that pivot? What sort of life are you expecting? Would you be better off with a gear transmission?

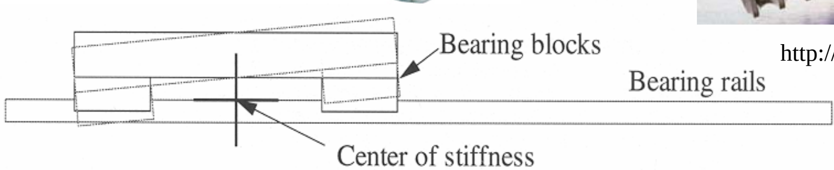
Screws: *Leadscrews & Ballscrews*

- **Leadscrews are essentially accurate screws used to move a nut attached to a load, and they have been used for centuries to convert rotary motion into linear motion**
  - *Leadscrews* are commonly used on rugged economy machine tools
  - Efficiency in a leadscrew system may be 30-50%,
- **Precision machine or those concerned with high efficiency often uses a *ballscrew***
  - Sliding contact between the screw and nut is replaced by recirculating ball bearings and may have 95% efficiency
  - Point contact between the balls and the threads makes them more delicate

A leadscrew integrated with a round shaft slotted linear bearing to create a linear actuator. Note with an anti-rotation segment between the nut and the slot. See http://www.kerkmotion.com/



http://www.thomsonindustries.com/industrl/ti090600.htm



Screws: *Forces*

Leadscrews and ballscrews both rely on what is essentially the action of a helical wedge to cause the nut to translate with respect to the screwshaft as the two rotate with respect to each other. Note that it is the relative motion that is important. It does not matter which element is fixed or allowed to move, just as long as one rotation and one translation are constrained. The kinematics are simple: The product of the rotation θ (radians) of either the screw or the nut (one rotates, the other not) and the thread lead ℓ (distance/revolution) results in the translation of the other element. Similarly, the product of the angular speed (radians/second) and lead equals the translation speed (distance/s):

$$x = \ell\theta / 2\pi$$
$$v = \ell\omega / 2\pi$$

The fundemental principle of c*onservation of energy* means that the input work from the motor, (the product of efficiency, torque (N-m) and angular motion (radians)) must equal the product of nut translation, and nut force; thus the output force can be found:

$$F = \frac{\eta 2\pi\Gamma}{\ell}$$

But how is the efficiency calculated from first principles and the coefficient of friction between the nut and screw materials? Given a differential element of the screw thread, one can calculate the efficiency when raising or lowering a load[1]. The efficiency is higher when lowering a load, because the applied force essentially is sliding down the helix. But this means that there must be a critical state of friction and thread geometry where an axial force on one member can just cause the other member to rotate. This condition, known as *backdriveability*, means that rotation cannot occur when a force is applied if the lead is smaller than a critical value:

$$\ell \leq \frac{\pi\mu D_{pitch}}{\cos\alpha}$$

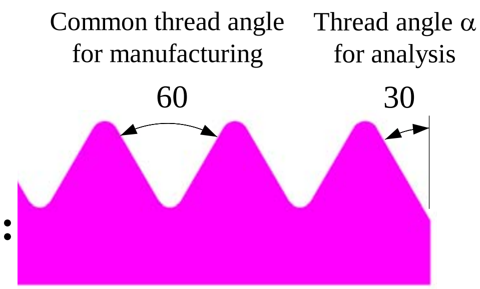
1. A. Slocum, Precision Machine Design, Society of Manufacturing Engineers, 1994, pp 711-714

In fact, as the equation with the figure of the differential screw element shows, the torque required to create a force must not only overcome the friction in the screw threads, it must also overcome the friction in the bearings that are used to support the rotating element (e.g., the screw shaft). In a bolt, this is the friction torque between the bolt head and the structure. As the spreadsheet shows, this contributes to the low efficiency often experienced by sliding contact thread screw systems. Note from the equations how the efficiency decreases with diameter and what other parameters? Play with the spreadsheet *Leadscrew_design.xls* and see what forces you could generate using elements in your robot kit of parts.

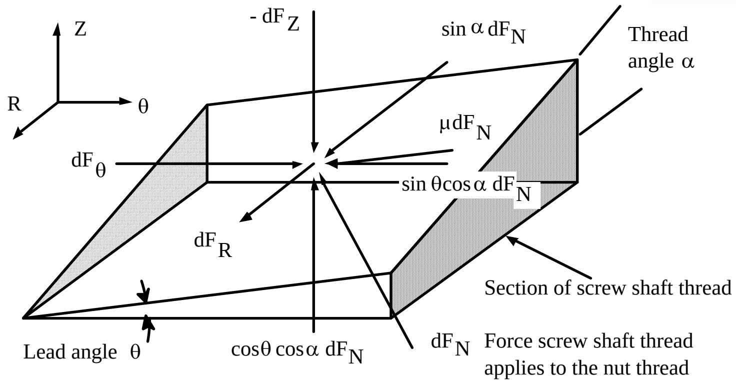
The life of a leadscrew depends on axial forces it generates, and unintended radial forces on the screw created by misalignment. In Chapter 9, sliding contact bearings are discussed in detail, including wear rates. For a leadscrew, the axial force generated can be predicted using the given equations, but what about misalignment forces? The answer is actually very straightforward. It is simple to determine the radial force required to deflect the screwshaft by the amount it is misaligned. Similarly, one can determine the moment caused by imposing an angular deflection along the shaft, and an equivalent radial force couple is just the moment divided by the length of the nut. Divide the net equivalent radial misalignment force by the sine of the thread angle, typically 30 degrees, and you have an equivalent force of misalignment to be added to the generated axial force. A ballscrew suffers from the same type of overloading, but the thread angle is greater, typically 45 degrees. The life of a rolling element bearing is also discussed in Chapter 10, which also takes into account preload forces.

Use the design equations and/or spreadsheet to predict the forces that you can generate for your machine. What assumptions must be made? If you do not know the coefficient of friction, how might you determine it? Can you put one material on the other and then tilt them and note the angle at which they slide ($\mu = \tan\theta$)? Do you have enough motor torque and speed? Can you determine early on if you have enough power to accomplish the desired task without even considering the leadscrew? Does the product of maximum motor power ($0.25\Gamma_{max}\omega_{max}$) and expected leadscrew efficiency of 30% provide a good enough estimate? In Chapter 7, electric motors are discussed including models to enable you to predict the time versus speed for a system.

Screws: *Forces*



- **To move a load with a screw thread:**



$$\beta = \ell / D_{pitch}$$

$$\eta_{raise} = \frac{\left(\cos\alpha - \mu\beta/\pi\right)}{\left(\cos\alpha + \pi\mu/\beta\right)}$$

$$\eta_{lower} = \frac{\left(\cos\alpha + \mu\beta/\pi\right)}{\left(\pi\mu/\beta - \cos\alpha\right)}$$

  - Γ is the applied torque
  - μ is the coefficient of friction (0.1 typical for greased threads)
  - D_{pitch} is the pitch diameter of the screw thread
  - ℓ is the lead of the thread (e.g., mm/revolution)
  - D is the bolt head or thrust bearing diameter
  - α is the thread angle (typically 30 degrees for a standard bolt)

Screwforce.xls		
Spreadsheet for lifting force from a screw		
Written 3/08/01 by Alex Slocum		
Enter numbers in bold		
Be consistant with units! (in, lb or N, m or N, mm)		
Motor torque (input)	**50**	
Motor speed (rpm)	**100**	
Dthrustbearing	**12**	
Dpitch	**5**	
Lead	**1.25**	
alpha, cos(alpha)	**30**	0.8660254
Coefficients of friction		
muthrustbearing	**0.2**	
muthreads	**0.2**	
beta	**0.25**	
To RAISE a load		
screwthread efficiency, etaraise	**25.16%**	
Without thrust bearing		
Force (output)	**63.2**	
With thrust bearing		
Force (output)	**25.1**	
Linear speed (mm/sec)	**2.08**	
To LOWER a load		
screwthread efficiency, etalower	**54%**	
Without thrust bearing		
Force (output)	**134.6**	
With thrust bearing		
Force (output)	**31.8**	

- **Based on a simple work-in=work-out (torque*one rev=Force*lead (distance/rev) with efficiency of η:**

$$\eta_{\text{thrust}} = \frac{\ell}{\ell + \pi\eta_{\text{thread}}\mu D_{\text{mean diameter thrust bearing}}}$$

$$\Gamma_{\text{required}} = \frac{F_{\text{desired}}\ell}{2\pi\eta_{\text{thread}}} + \frac{F_{\text{desired}}\mu D_{\text{mean diameter thrust bearing}}}{2} = \frac{F_{\text{desired}}\ell}{2\pi\eta_{\text{thread}}\eta_{\text{thrust}}}$$

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Screws: *Stresses*

The primary failure mode of a leadscrew or ballscrew is typically that of the thread wearing out or becoming damaged; however, the shaft itself can sometimes be overstressed, particularly in the case of a bolt which is also a form of leadscrew. A rolled thread is the strongest way to make a thread, but the least accurate. Rolling carbide dies plastically deform a smooth shaft to create a subsurface grain structure that greatly increase thread strength and reduce the effect of stress concentrations at the thread root. Ballscrews have precision ground circular thread shapes, so stress concentrations are generally not an issue in the threads. For all screws, stress concentrations also exist between square shoulders and the shaft[1].

When calculating stresses in a threaded shaft, always use the root diameter which is also the minimum diameter of the shaft. Note that the outside diameter of the shaft is just referred to as the "diameter" and the pitch diameter can be estimated to be the average of the diameter and the root diameter. Sliding contact threads typically have a thread angle of 19 degrees, and typically there is a stress concentration on the order of 2 at the thread root.

What about the stress in the nut or section into which a bolt is threaded? How many threads should be engaged in order to ensure that the threads do not shear (strip out)? The thread nearest the load bears most of the stress and subsequent threads bear less and less load; therefore it does not help to engage threads over many bolt diameters. In order to estimate the number of threads that are needed, assume that the threads shear mid-plane, and then assume that the proper strength is when the threads shear just as the shaft breaks. Given nut length L, outer diameter D_{bolt}, root diameter D_{root}, and the shear strength equal to ½ the tensile strength, the threads will shear at the same time the bolt breaks, when the thread engagement length equals the bolt diameter. This is why a nut's thickness is typically equal to the thread diameter.

One could also equate the Von Mises stress in the shaft to the equivalent Von Mises stress associated with the shear of the threads. This level of detail is an example of inappropriate analysis, because the threads are not perfect and the first thread bears a greater proportion of the load than does the last thread. If one really wanted to obtain a more accurate analysis, one would have to do a detailed finite element study, which included the effect of statistically varying thread shapes. It is better to use the first order approximation of making sure the engaged thread length is at least equal to the thread diameter, and then double it as a measure of safety. It is also a good idea in mission critical situations to test the system. Finally, all of the above assumes that the screw shaft and nut threads are made from materials with the same yield strength. It does little good to use a high strength bolt with a mild steel nut!

All the attention has thus far been paid to axial loads, but one must also be concerned with bending loads in some cases. If the screw shaft or nut are misaligned, they can be subject to large bending stresses. With rapid rotation of an actuator's screw shaft, these stresses can lead to low cycle fatigue even if there is only a small axial force on the system.

Ballscrews have the additional issue of Hertz point contact stresses[2] that occur between the balls and the threads. The greatest danger to a ballscrew is an impact load. Impact loads can cause the balls to indent the thread, which causes the ballscrew to run rough and soon fail. Sliding contact thread leadscrews have the fortunate property of being essentially immune to axial impact loads as long as they do not yield the threads.

Additional factors such as shaft buckling and shaft whip, caused by the screwshaft spinning at its natural frequency, are discussed on following pages. These effects can also significantly affect the life of a leadscrew and must be carefully considered. Fortunately, with the use of a spreadsheet or other program, a design engineer can include all these primary effects to help design a robust system.

Preliminarily determine the size of the leadscrew that you might use in your machine. Do you have room for it? Will you be able to mount it so as to not cause bending stresses on it as your machine moves? If all seems right, then proceed to the determination of buckling loads and critical shaft frequencies to ensure it has sufficient dynamic performance.

1. See Pages 5-23 through 5-26

2. Discussed in detail in Chapter 9

Screws: *Stresses*

- **Forces created by screw threads cause tension & torsion**
  - The thread root is a stress concentration area (about 1.5)
  - The stresses, not including the stress concentration, are:

$$\sigma_{tensile} = \frac{4F_{axial}}{\pi D^2_{root\_diameter}} \qquad \tau_{shear} = \frac{16\Gamma}{\pi D^3_{root\_diameter}}$$

- **The Von Mises equivalent stress is:**

$$\sigma_{tensileequivelent} = \sqrt{\sigma^2_{tensile} + 3\tau^2_{shear}}$$

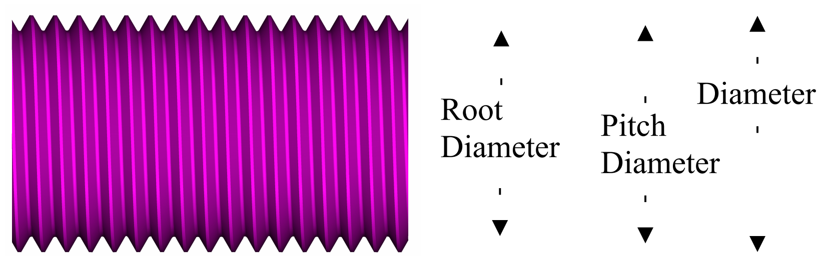
- **Minimum thread engagement length to avoid shearing:**

$$F_{Shear\_Nut\_Threads} = F_{Bolt\_Tensile}$$

$$\left(\frac{L_{Nut}}{2}\right)\pi\left(\frac{D_{Thread\_outside\_diameter} + D_{Thread\_root\_diameter}}{2}\right)\left(\frac{\sigma_{yield}}{2}\right) = \frac{\pi D^2_{Thread\_root\_diameter}\sigma_{yield}}{4}$$

$$L_{Nut} \approx D_{Bolt}$$

leadscrew_design.xls		
Screwthread forces		
Enter numbers in BOLD, output in RED		
Written by Alex Slocum, last updated 1/17/03		
Force (no help from gravity), thrust (N)	**400**	
Lead, (mm)	**2**	
Coefficient of friction, mu	**0.1**	
Screw pitch diameter, dscrew (mm)	**20**	
Thrust bearing diameter, dthrust (mm)	**25**	
Thread angle (deg), alpha (rad)	**30**	0.524
Thread root stress concentration, scf	**1.5**	
Beta	0.1	
Torque required at screw, gamscrew (N-mm)	591	
Torque required at thrust bearing, gamthrust(N-mm)	500	
Thrust bearing efficiency, etathrust	54%	
Total torque, gamtotal (N-mm)	1,091	
Backdriveable?	NO	
Thread efficiency, et	22%	
Total system efficiency, eta	12%	
Estimated torsional stress, tau (N/mm^2)	0.52	
Tensile stress, sig (N/mm^2)	1.57	
Mises equivelant stress, sigma (N/mm^2)	2.71	
Gearbox ratio, n	**1**	
Travel, s (mm)	**50**	
Time to travel, tt (s)	**5**	
Motor speed, w (rpm, rad/s)	300	31
Gearbox efficiency, etagearbox	**90%**	
Motor torque, gammotor (N-mm)	1213	
Power, Preq (watts)	38	



Screws: *Buckling and Shaft Whip*

There are two primary dynamic failure modes of leadscrews: *buckling* and *shaft whip*. Buckling is a common structural failure mode in which a large *compressive* load is applied to a long slender member. The load is never applied exactly on center and the member is never perfectly straight. The result is that a small moment is created which causes more deflection and this compounds the problem. Euler developed a theory and formulas for predicting buckling and as expected, the buckling load is heavily dependant on the manner in which the ends of the member are held (the boundary conditions). As the formulae show, the buckling load is also proportional to the member's length squared.

It might seem as if a simple way to avoid buckling in a leadscrew or ballscrew is to anchor both ends in bearings so the screw is always in tension. Nevertheless, leadscrews and ballscrews get warm as they spin because they are not 100% efficient. This temperature increase can cause the leadscrew to expand and overload the bearings if they are rigidly fixed in place. Hence it is best to use one of the design conditions shown and make sure that the buckling load is greater than any applied compressive loads.

Shaft whip occurs when the rotational frequency equals the bending natural frequency of the shaft. Once again, the instability is strongly affected by how the leadscrew is mounted which can greatly affect the shaft natural frequency as shown in the figures. The natural frequency will always be inversely proportional to the leadscrew's length squared. Be aware that the frequencies in the formula for ω have units of radians per second.

Shaft whip is an instantaneous occurrence, so shaft speed can be a function of position. Think of not just an instant along the motion path, but travel along the entire motion path. As the graph shows, if shaft speed is limited to the worst case critical speed when the nut was all the way at one end of the shaft, it would take many times longer for the nut to move from one end of the shaft to the other. It is far more efficient if the control system simply keeps the shaft speed below the critical shaft speed as a function of position!

Another idea might be to rotate the nut while rigidly anchoring the shaft. The nut can still generate considerable heat and still cause the shaft to expand. If one end of the shaft is allowed to expand, as shown in the boundary conditions drawn, this can help a little, but there are still several factors to consider including:

- The nut is still spinning and thus still acts as a vibration source, so some shaft vibration can still occur, although it will be at a reduced level.
- The mass moment of inertia is proportional to the length and the diameter to the fourth power; hence unless the shaft is very long, rotating the nut may take considerably more power.
- Typically the motor and screw shaft are mounted on the bed of the machine, and the non-rotating nut is mounted to the moving carriage; thus rotating the nut can complicate the design. Can the screw shaft be mounted to the carriage and the nut and motor mounted to the bed?

The spreadsheet *leadscrew_design.xls* examines various stability criteria for shafts. Sometimes, as is the case with leadscrews, shafts are placed in tension, and this can increase their natural frequency significantly if the tensile load applied is much greater than the buckling load. In this case, the shaft's natural frequency can be considered to be that of a string in tension[1], which given its length L, mass per unit length q, and pre-tension T is (units of *N, m, kg, s* or *N, mm, g, s*):

$$\omega_{Hz,N,m,s} = \sqrt{\frac{T}{L^2 q}} \quad \omega_{Hz,N,g,mm,s} = 1000\sqrt{\frac{T}{L^2 q}}$$

Where do you have shafts in compression in your machine and what is a conservative estimate of their buckling load? What loads do you plan on applying? What is the lowest cost method of remedying any problems you might have identified? What are the critical shaft speeds, and can you stay below them or do you need to try and run at super critical speeds? For your robot that only has to last a few weeks, is passing through the resonances even a problem? If the rest of your machine is designed and built well, it might not be an issue. Many turbines in many types of machinery operate at super critical speeds!

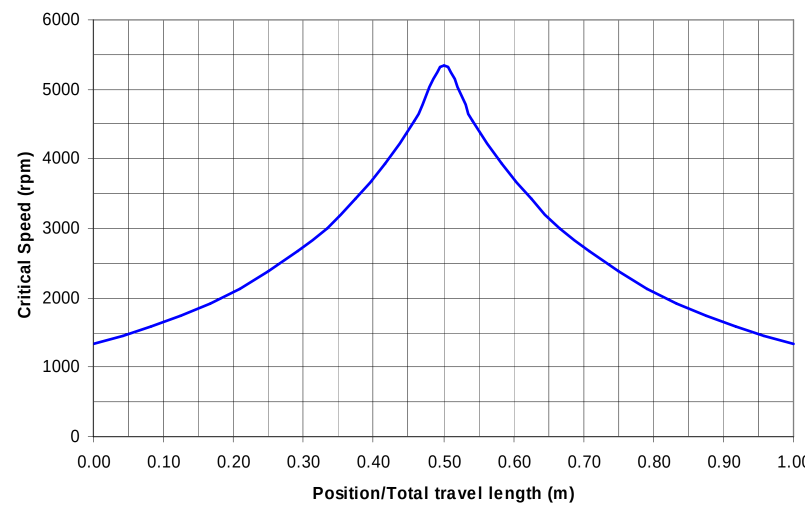
1. See for example Kenneth G. McDonald, Vibration Testing: Theory and Practice, 1995, John Wiley and Sons, New York.

Leadscrews: *Buckling and Shaft Whip*

- **Leadscrews in compression can *buckle***
  - Pull on a straw and it slips out of your hands
  - Push on a straw and it will “snap in half”
    - *Buckling* is a common failure mode in shafts
    - If possible, put shafts in TENSION and avoid the problem!
  - Leadscrews can easily generate forces that will make them buckle
    - Heavily loaded leadscrews should ideally be used to ***PULL*** not ***PUSH*** loads!
    - The calculations are EASY, DO THEM! (use the ROOT diameter and mks units!)
  - Thermal expansion in precision systems can be overcome by pre-stretching a screw
- **Leadscrews that spin too fast can excite shaft bending, *shaft whip*, and cause support bearing failure**

$$\omega_n = k^2\sqrt{\frac{EI}{A\rho L^4}} \qquad F_{buckle} = \frac{cEI}{L^2}$$

								
	Cantilevered		Simply Supported		Fixed-Simple		Fixed-Fixed	
mode n	k	c	k	c	k	c	k	c
1	1.875	2.47	3.142	9.87	3.927	20.2	4.730	39.5
2	4.694		6.283		7.069		7.853	
3	7.855		9.425		10.210		10.996	
4	10.996		12.566		13.352		14.137	
n	(2n-1)π/2		nπ		(4n+1)π/4		(2n+1)π/2	



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Leadscrews: *Mounting*

There are three principle issues in the mounting of a leadscrew: 1) how to select and mount the bearings which will hold the rotating member (typically the screwshaft), 2) how to couple the non-rotating member, which is typically the nut, to the system, and 3) how to achieve alignment between all the elements to minimize error motions and maximize efficiency. The primary concerns are embodied in the admonishment made earlier to always think of machine elements and their mounting in terms of their *kinematics* (motion, accuracy, space requirements), *dynamics* (forces, speeds, life), and *economics* (design, build, maintain). A FRDPARRC table, either in written or mental form, should be used to help ensure that all the issues are addressed. Now, down to details:

Assuming the screwshaft is rotating, in order to mount the support bearings, one must first do the calculations for buckling load and shaft whip to determine what sort of end conditions are required. In all cases where the shaft is moving at higher speeds, make appropriate allowances for thermal expansion. Typically one set of bearings is rigidly held to the screwshaft and the mounting structure, and the other set of bearings is rigidly held to the mounting structure. The screw shaft passes through them for radial support only, and is thus free to grow axially. The figures show the case for one end of a ball bearing supported ballscrew, and for both ends of a simple leadscrew as might be used in a robot design contest where sliding contact bearings are used.

In these figures, how did the designers decide where to place the bearings and how to constrain them? It all goes back to fundamentals: Every rigid body has six degrees of freedom. You merely have to imagine that each body has to have forces applied to resist the translation and rotational motions. Translations are resisted by a single force, and rotations are resisted by two forces spaced apart (a couple). Naturally, Saint-Venant plays a role in determining how far apart to place the forces[1].

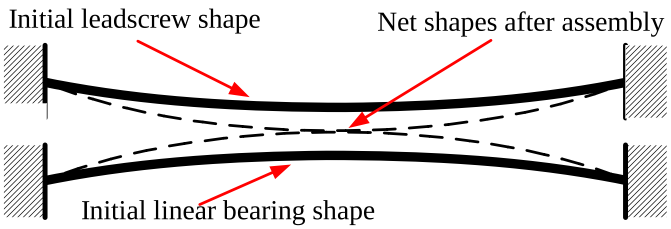
Radial forces on a screw are usually very low as they should be. It is the axial forces that can be very large. This may tempt the designer to use a large thrust bearing; however, the larger the thrust bearing, the greater the radius at which the thrust forces act. With the friction in the bearing, this means greater restraining friction torques. In fact, the thrust bearing can reduce system efficiency by half! Use the spreadsheet *Leadscrew_design.xls* to see the effect! Thus it is very desirable to minimize thrust bearing diameter.

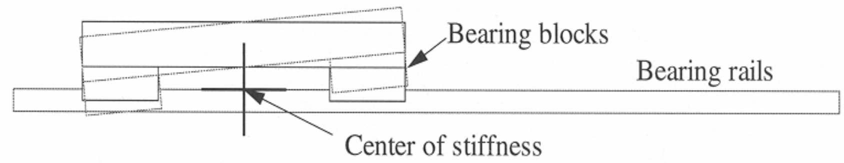
An actuator can never be perfectly aligned with the component it is moving. As a result, there are always force components generated that are perpendicular to the desired direction of motion. In addition an actuator has a finite mechanical stiffness in a direction perpendicular to the direction of motion as does the object being moved. hence the actuator and the object being moved must achieve an equilibrium position along the motion path profile. Physics rules, and the equilibrium is based on the relative stiffness and misalignment between the actuator and object being moved. Even a linear electric motor, where the forcer is mounted to the carriage and the magnets are attached to the structure, experiences small force components perpendicular to the direction of travel. These forces are caused by non uniformities in the magnetic field as they move through the electric field. There will always be plenty of issues to keep engineers busy and happy!

Design engineers learned long ago to use couplings, as discussed on page 5-29, to allow for misalignment. On the other hand, placing a coupling between a leadscrew nut and the object being moved (e.g., the carriage) is not always a simple prospect. The coupling must not only be stiff in tension/compression, it must also be stiff in torsion in order to counteract the torque on the nut from the screw. *For non-precision applications, the backlash in the leadscrew threads, or radial bearing clearance often suffices. How much is enough? You can tell because the turning resistance of the screw will increase dramatically if you do not have enough clearance.* For precision systems, misalignment between the axis of the leadscrew and the linear bearings is accommodated by considering the product of the relative radial stiffness between the leadscrew and the linear bearings and the misalignment. The resulting force must be much less than is tolerable by either the linear bearings or the leadscrew, otherwise, the leadscrew will likely rapidly wear.

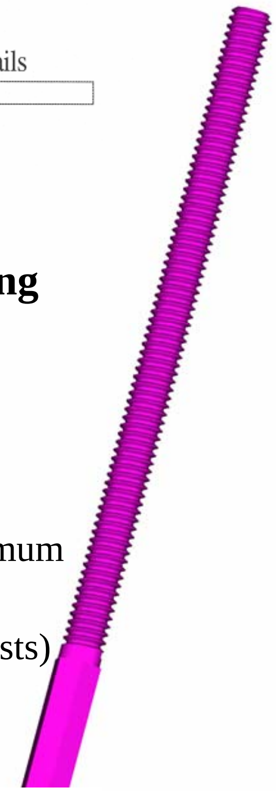
Where might you use a leadscrew in your machine? They can generate huge forces, but as a result also move very slowly if power is limited. Play with the spreadsheet *Leadscrew_design.xls* to determine what is feasible.

1. The use and mounting of bearings is discussed in much greater detail in Chapters 10 and 11

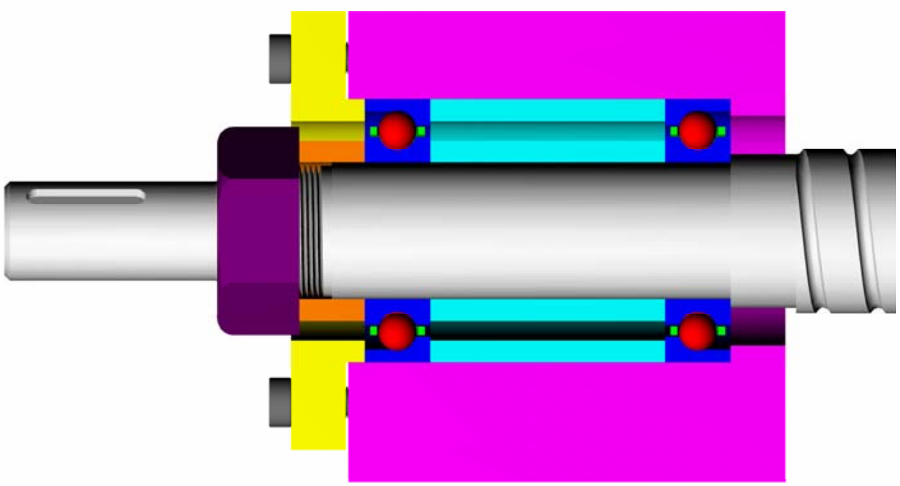


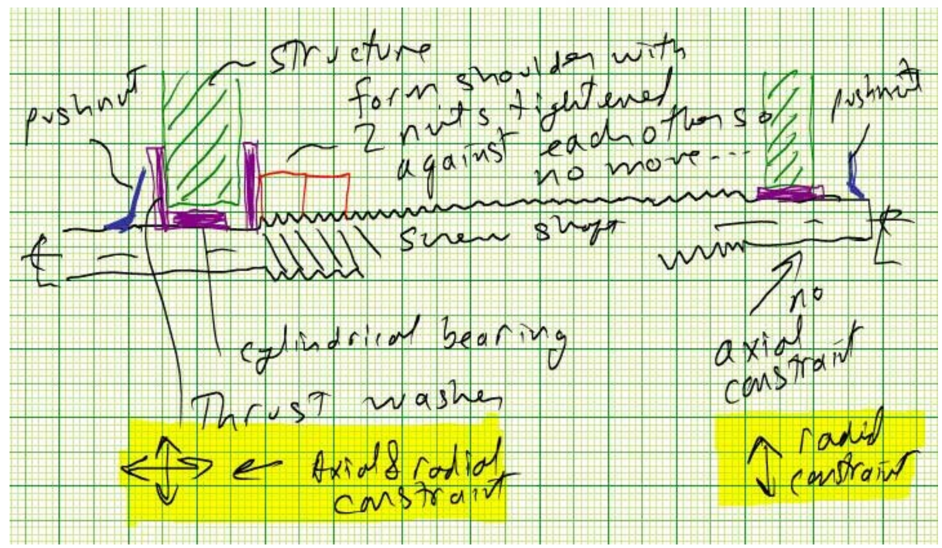


Leadscrews: *Mounting*



- **Leadscrews used in robotics contests are often mounted using a radial sleeve bearing at one end, and journal and thrust bearings at the other end**
  - The bearings in gear motors are generally not designed to take the huge thrust loads that a leadscrew can generate
- **Beware of constraints: either provide precision or compliance**
  - The only way to effectively mount a leadscrew to achieve a zero-slope end condition for maximum buckling resistance is to use a back-to-back arrangement of ball bearings (see page 10-6)
    - This also generally involves the use of a ballscrew (not used in simple robot design contests)
- **It is easy to make a leadscrew**
  - Screw threads can be cut directly into round, square, or hexagonal steel stock
    - A square or hexagonal hole can be broached into a gear or pulley which can then be pressed onto the leadscrew





Leadscrews: *Differential Motion*

A simple leadscrew can be used to move a load, but what happens if an extremely small lead is needed to generate either very high forces, or to give extremely fine motion control? The effective lead of a *differential screw* is determined by realizing that a screw with two different leads (distance traveled per revolution) threaded into two nuts, each with a lead matching a portion of the screw, must move forward the same distance all along its length. The two nuts have different leads, so must move relative to each other as the screw turns. The differential motion between them is the *virtual lead* of the system. If one of the nuts is attached to a fixed reference, for every turn of the screw, the other nut will appear to move forward by:

$$\ell_{differential} = \frac{1}{1/\ell_1 - 1/\ell_2}$$

The result using standard coarse thread pitch (1/lead) screws can be quite dramatic:

Screw diameter (in)	Threads per inch (tpi)	Differential (tpi)
0.190	24	
0.250	20	120
0.313	18	180
0.375	16	144
0.438	14	112
0.500	13	182

With a single pitch leadscrew system, the screw shaft is typically axially constrained and rotates, while the nut axially translates and does not rotate. In a differential screw system, if one of the nuts is axially restrained, the screw shaft will move forward by that nut’s lead, while the other nut moves forward by the differential lead. This means that the motor which is coupled to the screw shaft must also be free to move forward axially with the screw shaft. Alternatively, there must be a coupling between the motor and screw shaft that allows the screw shaft to move axially with respect to the motor while maintaining a rigid torsional connection. Accomplishing this without backlash can be difficult.

Study the solid model of the flexural bearing supported stage system with differential screw drive. How can reciprocity and self-help be used here? Since the motor is being used to drive a flexure, can a second flexure stage be created on which the motor is mounted so it can be rigidly connected to the screw shaft? Over what ranges of motion could this work? What are the differences in the functional requirements for the flexure supported stage and the flexure supports for a motor?

Differential motion can also be created by having a left-hand and a right-hand screw thread on the same screw shaft. A classic example many decades old is the *Jorgenson Handscrew™* clamp commonly used by woodworkers. If one handle is grasped by one hand and held stationary, and the other is grasped by the other hand and spun around the axis of the first, the jaws can be made to open or closed in a parallel manner. If one handle is grasped and the other handle is turned about its own axis, the angle between the jaws is adjusted!

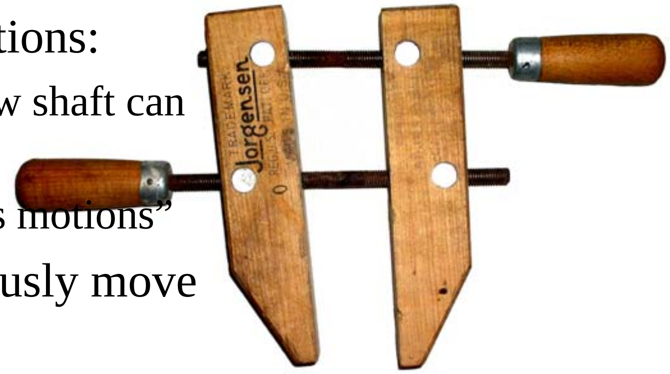
Applying the principle of the use of a left-hand and a right-hand thread to a screw shaft, one can create a robot gripper, as shown, where the fingers can be made to open or close and they are interchangeable. The gripper is very thin and symmetric so a blank part can be held while a finished part is removed from a lathe. The gripper then flips over and the blank part is inserted. While the lathe machines the blank part, the robot drops off the finished part and acquires another blank part. The gripper is also shown mounted to a kinematically designed five axis micromanipulator (mini robot) which is mounted to a large robot. When the large inaccurate robot wants to load a part in a collet on a lathe, where the clearance between the part and the collet is only 0.1 mm or less, the large robot stops in front of the collet and the micromanipulator goes through an insertion search pattern at 20 Hz, just like a human does, until the part is inserted![1]

Differential motions anyone? Can this principle be applied with gears or linkages? Can you use this principle to enable one motor to actuate two different axes and hence accomplish two different functions?

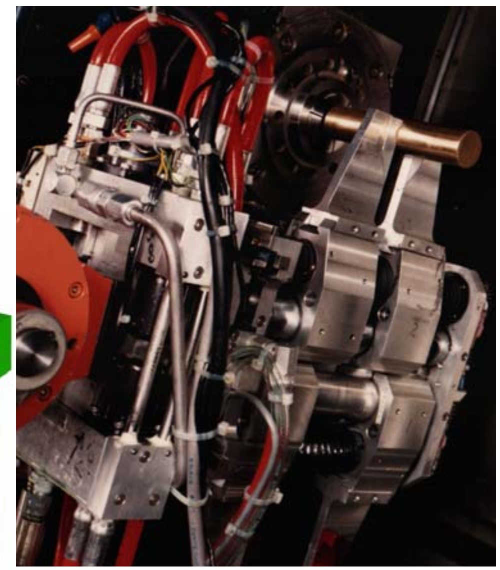
1. This project was led by the author as his first real engineering project after he received his SM thesis (but while he was also working on his Ph.D. thesis) while he was working at the National Institute of Standards and Technology (NIST). See US Patents 4,765,668 and 4,694,230; and Slocum, A. H., Greenspan, L., Peris, J.P., "Design and Implementation of a Five Axis Robotic Micro-manipulator," Int. Jrl. Machine Tool Design, Vol. 28, No. 2, 1988, pp 131-141.

Leadscrews: *Differential Motion*

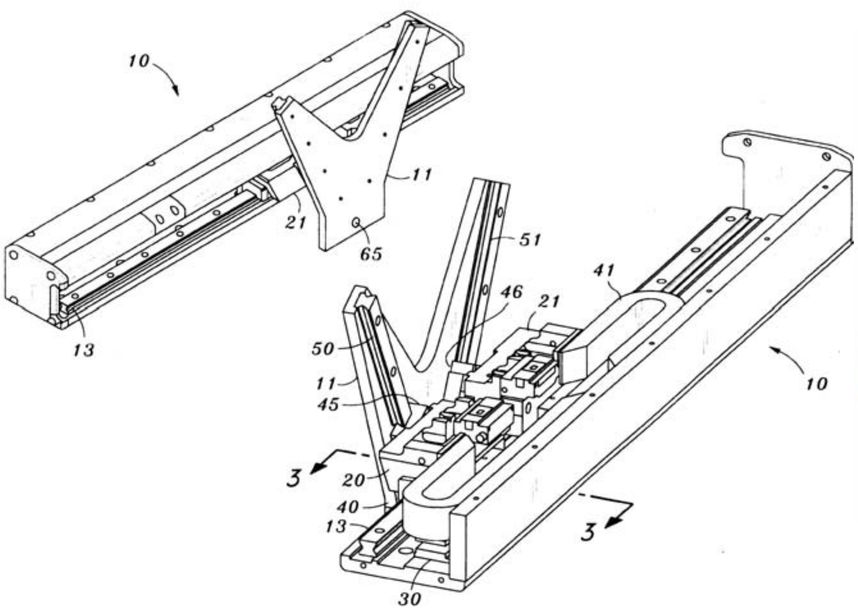
- Differential motion can be used to create most excellent motions:
  - Two independently rotating leadscrew nuts on a common screw shaft can enable components to move in the same or different directions
    - See US patent **6,194,859** “X-Y positioner based on X axis motions”
- A leadscrew with left and right hand threads can simultaneously move components together or apart
  - See US patent 4,765,668 " Double End Effecter“
- A leadscrew with two different leads can create a small virtual lead
- Two ballscrews coupled together with angled linear motion bearings can create a "wedge" driven XY motion platform
  - http://www.bell-everman.com



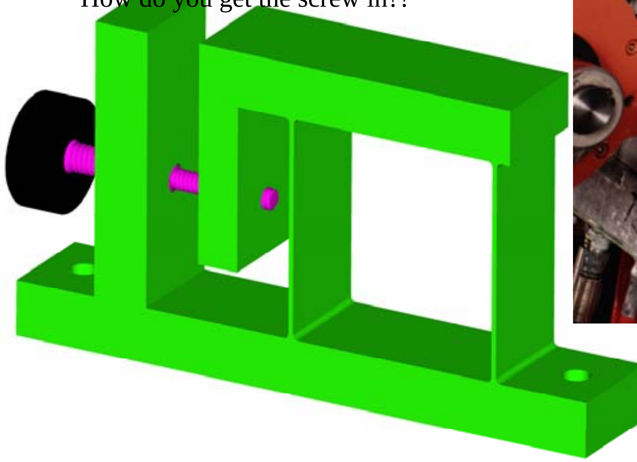
♥ Alex Slocum’s first miniature 6 axis robot and double gripper! ♥



Note the gripper’s left/right hand screws that actuate the fingers. Where did he get the idea?



How do you get the screw in?!



Leadscrews: *Preload*

No threads can be made perfect and exactly the same, and hence when a nut is threaded onto a shaft, there either has to be clearance between the threads, or interference accommodated for by elastic deformation. The latter is referred to as *preload*. An "exact" fit cannot be maintained, because with the slightest bit of wear, clearance is generated between the threads, which is called *backlash*. When a screwshaft is rotated in one direction, and then it stops and starts rotating in the opposite direction, the nut is at first translating in the direction indicated by the screw rotation. When the screw rotation stops, the nut stops. With *backlash*, there is clearance between the nut and screw threads. When the screwshaft starts rotating in the opposite direction, it has to rotate for a bit before the thread translates to contact the thread on the nut. Thus there is a period when the screwshaft is turning and the nut is not translating. This is also sometimes referred to as *lost motion*. Backlash is good in that it helps to prevent overconstraint in the system which leads to increased friction. However, it reduces the accuracy, repeatability and resolution of a machine.

There are several ways to eliminate backlash by preloading the nut and screw thread elements. When preload is provided by a spring, which is soft compared to the spring constant of the threads, as the thread wears, the spring maintains the preload. A sliding contact thread leadscrew's nut can have axial slits and an internal diametrical relief so the threads are only on one end of the nut. Thread segments are then held on cantilever beam ends which are pressed radially into the threads of the screwshaft threads by the circumferential clamping action of a round spring element such as an O-ring. The trapezoidal shape of the threads causes them to engage without backlash as they are radially forced together. The primary concern is that given the size of the nut, relatively few threads are engaged, so life is not as great as if a full length of threads were used. A patented design by Kerk Motion Products uses two nuts that are rotationally preloaded against each other using a torsion spring. This allows a full length compliment of threads to engage along the nut's length.

In precision mechanisms, backlash is particularly important to remove. As described previously, ballscrews use balls as rolling element interfaces between the screwshaft and the nut threads and they can have efficiencies up to 95%. They are widely used in precision, high speed and power-conscious mechanisms, and there are many different ballnut designs that can be used to achieve a preloaded condition. *Tensile preloading* is created by inserting an oversized spacer between two nuts and then clamping the nuts together. One nut takes loads in one direction and the other takes loads in the other direction. This creates a back-to-back mounting effect that is thermally stable for a rotating shaft design. A rotating shaft will generally be hotter than the nut because the shaft is not attached to a large heat sink. As shaft temperature increases, ball contact points on the shaft spread apart axially, which lowers the preload; however the shaft diameter increases which tends to increase the preload. Ideally, the two effects balance, and preload remains constant. However, the nut is very rigid in pitch and yaw directions, so it is sensitive to misalignments. *Compressive preloading* uses an undersized spacer between two nuts. This creates a face-to-face mounting situation and is therefore only thermally stable if the nut is likely to be hotter than the leadscrew. The nut is more compliant in pitch and yaw directions, so it is less sensitive to misalignments.

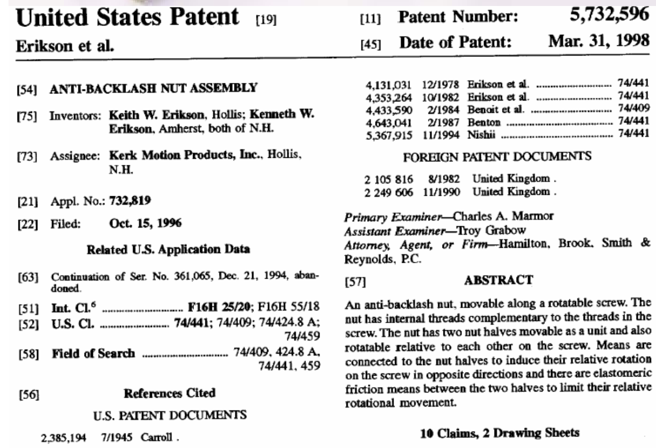
P-type preloading uses oversized balls to obtain preloading by four-point contact between the balls and the Gothic arch thread shape of the shaft and the nut. This greatly increases the amount of skidding the ball is subjected to as discussed on page 10-31, which can decrease life and position controllability. When the forces generated by the screw are large, the balls are forced to one side of the groove, so full-four point contact does not occur and skidding is minimized. *Z-type* preloading is also obtained with a single nut by shifting the lead between ball circuits. This creates two-point contact between the balls and the grooves which is suitable for medium preloads, but is not as accurate or controllable as tensile preloading. *J-type* preloading uses a spring (e.g., disk spring) between the nuts to establish a constant preload. This provides the most constant preload, and hence the most uniform drive torque is obtained. However, the carriage can only be mounted rigidly to one of the nuts.

There are unidirectional means to preload a screw thread, and these include the use of gravity when the leadscrew is mounted in a vertical position. In addition, one can use a hanging weight on a cable running over a pulley and connected to a horizontal leadscrew-driven carriage. One can also use a constant force spring to provide a biased load that takes up backlash.

What are your machine's functional requirements with respect to backlash? How does backlash affect your machine's accuracy, repeatability and resolution? Does it really matter? What is the simplest way in which you can reduce backlash? What are the effects on your machine's performance?

Leadscrews: *Preload*

- Threads can never mate perfectly: *backlash* normally exists
  - When a screw is rotated in one direction, the nut translates
  - When rotation stops and reverses direction, for a small angle there is no translation of the nut (backlash or lost motion)
- Preload is used to remove the backlash
  - Both sides of the threads are loaded with some sort of spring to removed backlash
- Preloading is used for ballscrews and leadscrews and many other relative motion machine elements (see page 10-16)



United States Patent [19]
Erikson et al.

[11] **Patent Number:** **5,732,596**
[45] **Date of Patent:** **Mar. 31, 1998**

[54] **ANTI-BACKLASH NUT ASSEMBLY**

[75] Inventors: **Keith W. Erikson**, Hollis; **Kenneth W. Erikson**, Amherst, both of N.H.

[73] Assignee: **Kerk Motion Products, Inc.**, Hollis, N.H.

[21] Appl. No.: **732,819**

[22] Filed: **Oct. 15, 1996**

Related U.S. Application Data

[63] Continuation of Ser. No. 361,065, Dec. 21, 1994, abandoned.

[51] **Int. Cl.**[6] **F16H 25/20**; F16H 55/18
[52] **U.S. Cl.** .... **74/441**; 74/409; 74/424.8 A; 74/459
[58] **Field of Search** .... 74/409, 424.8 A, 74/441, 459

[56] **References Cited**

U.S. PATENT DOCUMENTS

2,385,194 7/1945 Carroll .
4,131,031 12/1978 Erikson et al. 74/441
4,353,264 10/1982 Erikson et al. 74/441
4,433,590 2/1984 Benoit et al. 74/409
4,643,041 2/1987 Benton 74/441
5,367,915 11/1994 Nishii 74/441

FOREIGN PATENT DOCUMENTS

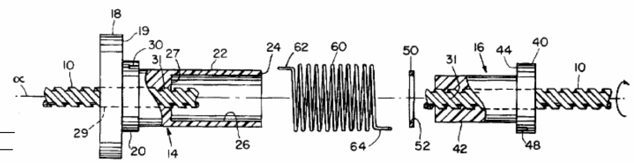
2 105 816 8/1982 United Kingdom .
2 249 606 11/1990 United Kingdom .

Primary Examiner—Charles A. Marmor
Assistant Examiner—Troy Grabow
Attorney, Agent, or Firm—Hamilton, Brook, Smith & Reynolds, P.C.

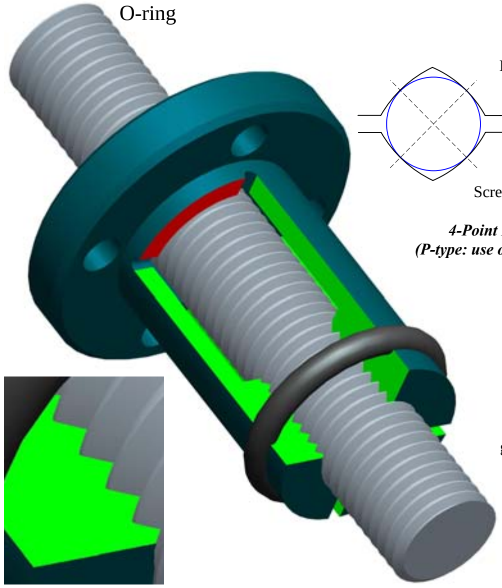
[57] **ABSTRACT**

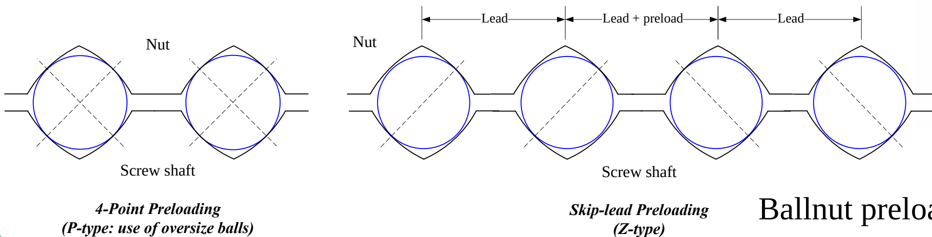
An anti-backlash nut, movable along a rotatable screw. The nut has internal threads complementary to the threads in the screw. The nut has two nut halves movable as a unit and also rotatable relative to each other on the screw. Means are connected to the nut halves to induce their relative rotation on the screw in opposite directions and there are elastomeric friction means between the two halves to limit their relative rotational movement.

10 Claims, 2 Drawing Sheets

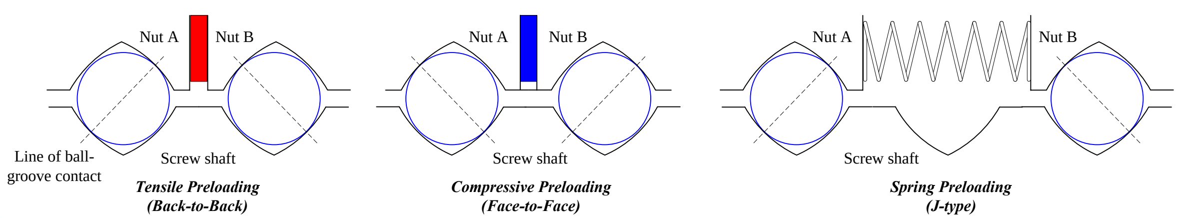


A plastic nut can be axially slit and then the threads radially preloaded with an O-ring





Ballnut preload methods



Leadscrews: *Flexibility*

Leadscrews are amazingly flexible machine elements in terms of the breadth of applications in which they are used. This is due to the fact that they are so easy to manufacture and they serve as transmissions to convert rotary motion, which is easy to achieve in an electric motor, into linear motion, which is so often desired in mechanical systems.

The flexibility of leadscrew applications can also be considered literally! A common problem in many industries, from egg farming to the sorting of packaged semiconductor devices, is how to pick up a tightly (or widely) spaced array of elements, and then deposit them spaced further (closer) apart? Many different devices have been created to achieve this goal, including lazy tong type devices. How can a leadscrew spread apart an array of grippers? A leadscrew with a left hand and a right hand thread could separate a single set of grippers as described previously. Is there a means to separate many such grippers? This would imply that the leadscrew nuts would have different leads to enable the nuts to separate from each other.

Could a leadscrew shaft be created with a variable pitch that would act on many different nuts? Here we invoke the fundamental principle of *reciprocity.* If it is too difficult or impossible to achieve what we want with a thread on the screw, can we achieve what we want with the threads in the nuts? The different nuts can have different pitches, and this would enable their spreading. however, how can the leadscrew shaft have a varying thread? It is not possible. Once again, we invoke reciprocity: If we cannot achieve a variable thread, can we make a threaded surface of a shaft that changes (*complies*) in accordance with the thread lead of the nuts? The answer is make the surface of the leadscrew shaft *compliant*[1] so it deforms to mate with the nut thread that happens to be engaging it. Indeed, there are many different types of elastomers that could be used to cover a smooth shaft. The nut threads could be round in form to engage the elastomer and deform it gently without causing wear.

Another prime example of the utility of leadscrews is seen in their use in CD drives, where a precision leadscrew drives the carriage that moves the optical read head across the CD. From the picture you can see some detail (but perhaps purposefully not too much?!). There is no actual leadscrew nut that typically encircles the leadscrew shaft. Why is this? Why does the "nut segment" appear to be two trapezoids, the distance between them a little farther apart than the pitch of the leadscrew? Does the nut segment appear to be mounted on a thin piece of steel that radially pushes the trapezoidal shapes into the threads so that both sides of the thread pitch are always engaged? Is this an effective means to eliminate backlash?

How about the gears that drive the leadscrew shaft? Do gears not have backlash which means that if the motor reverses there will be a period of time in which the leadscrew nut does not reverse? Why does one set of gears appear to have a helical tooth pattern? What is the purpose of the spring at the end of the leadscrew? Could it be that the spring axially pushes the leadscrew and the helical gear attached to it against the helical teeth of the other gear? Will this cause backlash to be eliminated from the gear train?

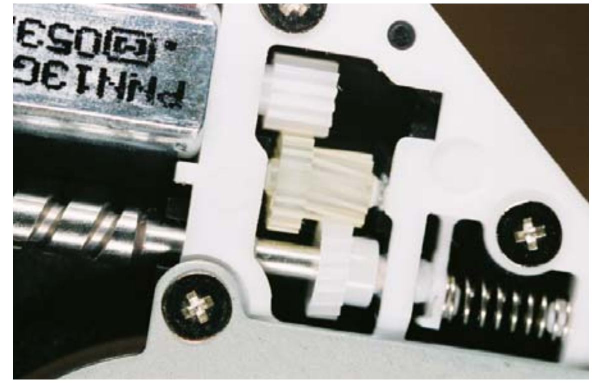
The modern CD drive is indeed a beautiful precision machine whose elegant simplicity can only be matched by its low price! Furthermore, as shall be discussed in Chapter 7, an ingenious flexural bearing supported fine motion stage is the key to its existence. The fine motion stage is carried by the leadscrew powered stage, and it is servo controlled by a Lorentz force motor to enable the read head to respond with incredible speed and precision.

Who were the designers of the ingenious CD drive? Sometimes it seems electrical and computer science engineers have all the fun and get all the glory when it comes to the computer industry. However, mechanical engineers are every bit as critical and challenged as their spark & bits brethren, for without high performance mechanical devices ingeniously achieved for low cost, none would have jobs!

Leadscrews might be very useful in many different parts of your machine. You might only need a conventional leadscrew, or you might need some sort of interesting leadscrew or nut as described here! Carefully think about the different parts of your machine, and how the knowledge presented thus far might be used. Is there some leadscrew or gear-like application where a toothed or threaded component can temporarily elastically deform another component and thus achieve high driving forces?

1. Note how the words describing what is required (FRs) can help trigger an idea for the DP (see pages 1-14 and 6-17)...

Leadscrews: *Flexibility*



- Leadscrews are used in many everyday applications
  - How does a CD drive work?
- Must the pitch of a leadscrew be constant?
  - See "Expanding Gripper with Elastically Variable Pitch Screw", #5,839,769, Nov. 24, 1998

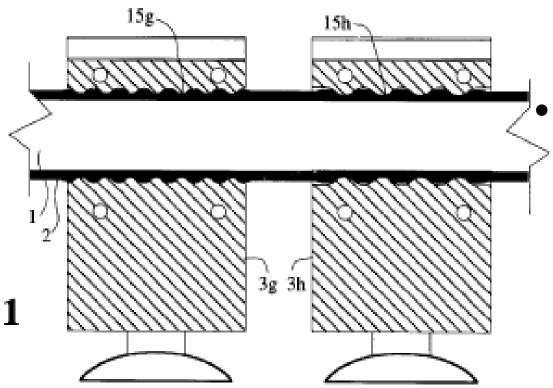


Fig. 1

To reduce friction, could the gripper units' threads be replaced with inclined rollers at different angles to achieve different effective leads? (I bet they could!)

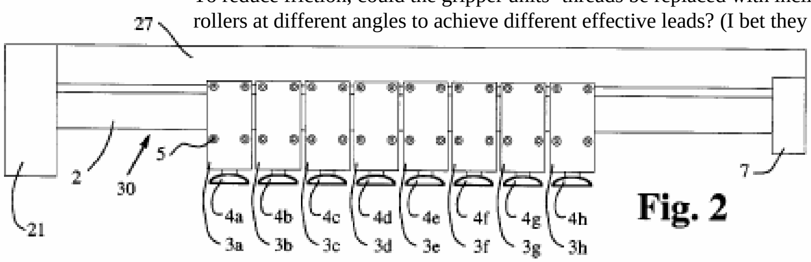


Fig. 2

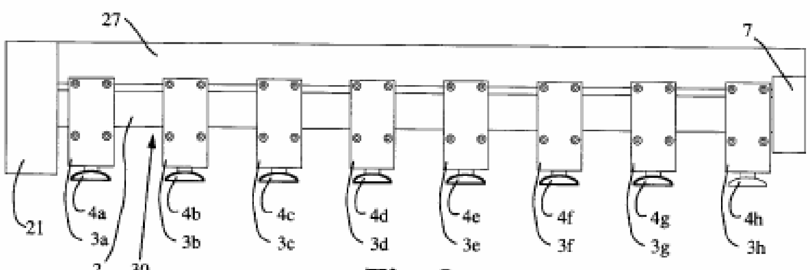
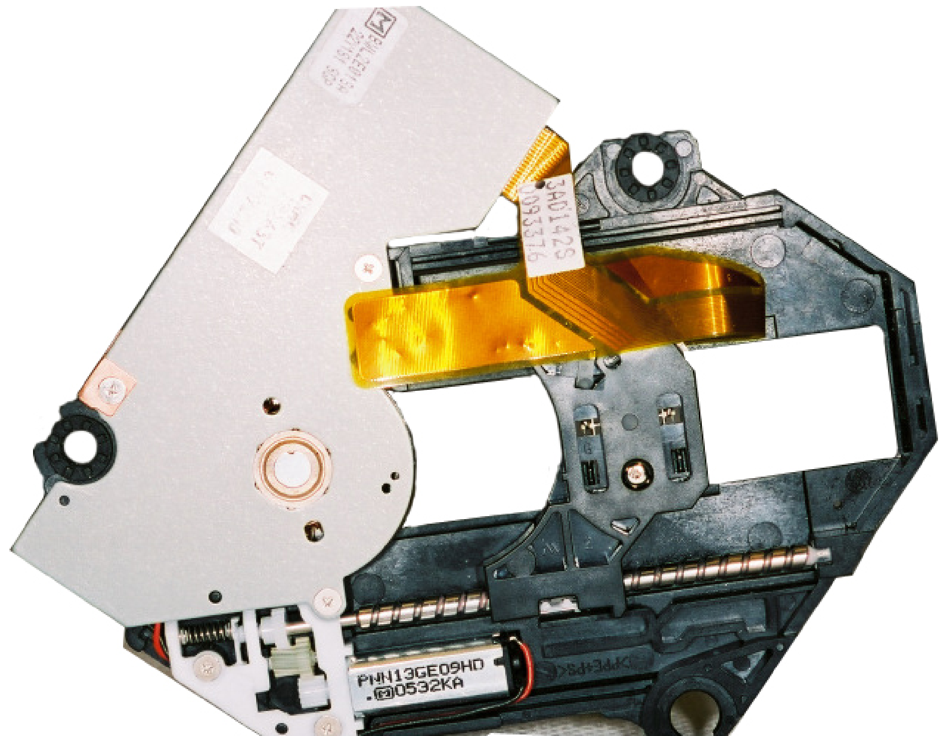


Fig. 3



Leadscrews: *Contest Machine Design Example*

The forces to raise a boom with a load on the end can be very large, and the motion of a point on a boom near the pivot is small compared to the motion at the end of the boom. Why do some cranes use winches and some use hydraulic cylinders. Should you use a winch or a leadscrew to raise or lower the boom? The first step would thus to be to create a spreadsheet to compare the design of a winch and a leadscrew. Assume for now that you cannot get a large enough force from a winch and that you need to use a leadscrew. How should you develop the design? A good place to start is to list the functional requirements for the leadscrew system and the different design parameters (ideas) for the design, as shown in Table 1.

Table 1: FRs, DPs, & As for a leadscrew to lift a boom

Functional Requirements	**Design Parameters** (possibilities)	**Analysis** (dominant physics)
Properly constrain motor end (e.g., Fixed: axial and lateral motion and yaw and rotation Free: pitch)	1) Motor mounts to pin joint supported block 2) Motor fixed and screw drives a carriage onto which a linkage is attached that actuates boom	Clearance holes for pins should provide sufficient constraint and freedom. Pin joint friction induced bending moments. Calculate pin joint bearing stresses
Properly constrain nut end Fixed: axial and lateral motion and rotation Free: pitch, yaw	1) Pivot nut to boom and make sure screw can pass into boom 2) Fix nut at end of tube, and pivot other end of tube (so the system “looks” like a hydraulic cylinder)	1) (same as above) 2) Same joint issues as above, but also consider bending moment on screw shaft caused by clearance between nut and shaft...

Table 2: Rs, Rs, & Cs for a leadscrew to lift a boom

References	**Risks**	**Countermeasures**
1) Cranes 2) Hexaglide (http://www.iwf.bepr.ethz.ch/web/de/forschung/wzm/hexa1.shtml)	1) Low, but pivot moments must be low 2) Medium: added complexity	1) Calculate joint pressures and friction moments. 2) Try option 1 first
1) Cranes and jacks 2) Linear screw actuators	1) Low, but pivot moments must be low. Also, beware of clearances 2) Buckling	1) None required 2) Use two nuts spaced apart to reduce lateral deflection

In particular, note Functional Requirement #1’s Design Parameter #2, which the author conveniently forgot to sketch. What would such a system look like? Check out the website for the Hexaglide (or just type in Hexaglide on www.google.com). What about FR2DP2 which is sketched? Is this a better design than just pivoting the nut on a mount on the boom? How will you decide? Which concept “costs less” but “works great”? How will the motor be coupled to the leadscrew (see page 5-30)? It is almost never allowable to connect a leadscrew directly to a motor, and then rely on the motor’s bearings for supporting that end of the leadscrew! Leadscrews generate huge forces, and motor bearings are sized to resist small radial and axial forces because they assume the user will use a coupling.

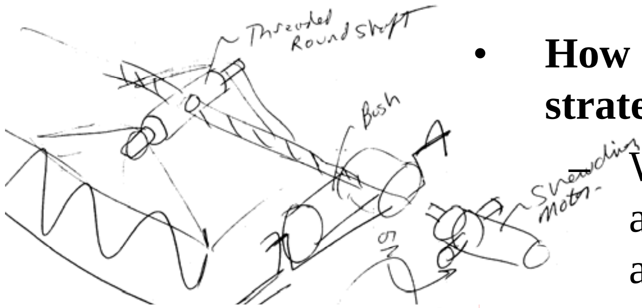
Perhaps the biggest mistakes people make when designing with leadscrews is the incorrect calculation of efficiency (forgetting the thrust bearing is a common mistake), and improper mounting of the components. If the components do not have the required degrees of freedom to allow them to move when misaligned, large radial forces between the screw and nut are generated, which can greatly reduce life and efficiency.

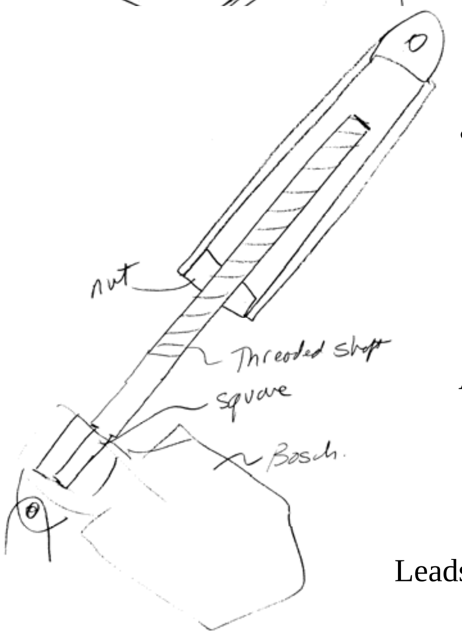
Virtually all discussions have thus far focussed on the leadscrew acting as an actuator to move a carriage supported by bearings. Is it ever acceptable to use leadscrews themselves to also support radial loads?[1] If one considers that a radial load on a standard 30 degree thread is amplified by a factor of 2 in the axial direction (the wedge effect) and considers this in determining the life of the screw, then it can be acceptable, especially for a simple robot competition. However, note that the increased effective axial load will decrease system efficiency. In addition, one has to make sure that the leadscrew(s) will not bend too much. The advantage of such a design is that it can help reduce part count.

Think of mechanisms in your machine which could use leadscrews. How would you mount the motors and support bearings and the nuts? Sketch the mountings and make sure you have properly constrained the required degrees of freedom. Check the connection between the motor and the leadscrew. How will thrust forces be transmitted?

1. Slocum, A.H., Elmouelhi, A., Lawrence, T., How, P., Cattell, J., "Linear Motion Carriage Driven and Guided by Elastically Supported and Preloaded Lead Screw Nuts", presented at the 17th Annual Meeting of the American Society of Precision Engineering, St. Louis, Missouri, October 20-25, 2002.

Leadscrews: *Contest Machine Design Example*





- **How might we evolve a lifting strategy into a boom concept?**

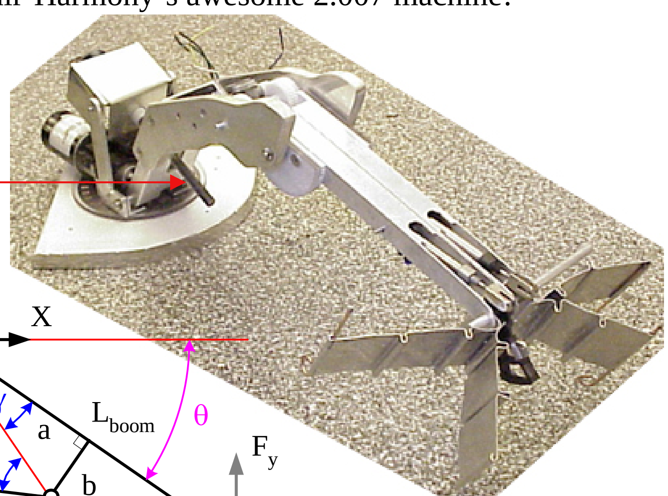
  What are the forces on the boom and where are they applied? What are its ranges of motion? How fast should it move the load? What is the desired resolution of motion?

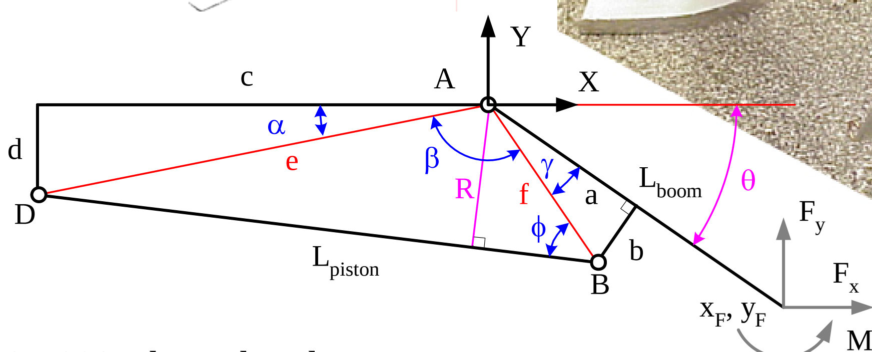
- **Use a spreadsheet to study the effects of different design parameters?**

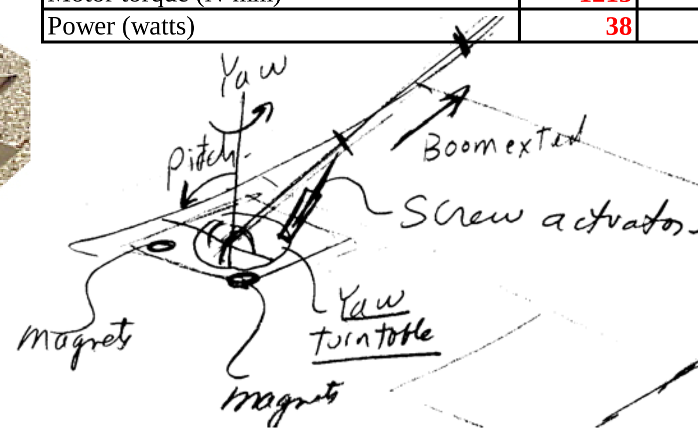
leadscrew_design.xls		
Screwthread forces		
Enter numbers in BOLD, output in RED		
Written by Alex Slocum, last updated 1/17/03		
Force (no help from gravity), thrust (N)	**400**	
Lead, (mm)	**2**	
Coefficient of friction, mu	**0.1**	
Screw pitch diameter, dscrew (mm)	**20**	
Thrust bearing diameter, dthrust (mm)	**25**	
Thread angle (deg), alpha (rad)	**30**	0.524
Thread root stress concentration, scf	**1.5**	
Beta	0.1	
Torque required at screw (N-mm)	591	
Torque required at thrust bearing (N-mm)	500	
Total torque (N-mm)	1,091	
Backdriveable?	NO	
Thread efficiency, et	22%	
Total system efficiency	12%	
Estimated torsional stress (N/mm^2)	0.52	
Tensile stress (N/mm^2)	1.57	
Mises equivalent stress (N/mm^2)	2.71	
Gearbox ratio	**1**	
Travel (mm)	**50**	
Time to travel (s)	**5**	
Motor speed (rpm, rad/s)	300	31
Gearbox efficiency	**90%**	
Motor torque (N-mm)	1213	
Power (watts)	38	

Ayr Muir-Harmony's awesome 2.007 machine!

Leadscrew!







Gears!

Gears function to transfer torque from one axis to another through the successive engagement of their peripheral teeth. The axes about which they rotate can be parallel, intersecting, or non-intersecting and non-parallel. The subject of gearing is thus primarily focused on the transmission of power between rotating shafts, which is one of the most common functional requirements of mechanical systems[1].

In general, there are two principle functional requirements of a geared system: power and precision. The former focuses on transmitting power as efficiently as possible, and is the concern of automotive, heavy equipment, aircraft, and marine systems. Precision systems focus on transmitting motion as accurately or as repeatably as possible, and precision is a primary concern of instrument and machine tool designers. In fact, early computers relied on precision gears to perform differential and integration functions for anti aircraft guns' guidance and control systems in WWII. These systems were also used extensively in inertial guidance systems. Today precision gears make it possible for robotic systems to achieve high accuracy motion. When the lessons of achieving higher precision are applied to power systems, greater efficiency, lower noise and longer life can be achieved.

Some might see gearing as a very mature field not worth further study; however, as stronger and more wear resistant materials and better lubricants are continually developed, there is an ever present need for new gears to be designed to pack more power into less space. There is also the field of Micro Electro Mechanical Systems (MEMS) where researchers are creating gears to solve the same problem that they face in the macro world: how to transform high-speed low-torque motor output into useful high-torque low-speed motion. The systems that are being developed are extremely impressive, except that they do not last very long. In fact, if one looks at relative precision (gear size/surface finish), MEMS gears are commensurate with gears that were made in the 19th century! As Richard Feynman said, "there is plenty of room at the bottom", not only for design, but for manufacturing innovation.

Despite their ubiquitous nature, gears are often misunderstood by students new to mechanical design. Although some students love design courses because they feel they are not as rigid and inflexible as engineering science courses, they must not forget that fundemental principles apply to ALL systems. An amusing anecdote concerns a student design team that was asked during a design review what the power requirements were for their human powered system. They did some quick calculations and said the device required 600 Watts. When they were asked how this could be achieved given that a human can only comfortably continuously generate about 100 Watts of power, they replied "we will use gears". The laws of thermodynamics, as well as all those other pesky laws and rules you learn in engineering science courses, still apply in design courses, and perhaps even more so, because when you do not apply them, they apply themselves, things can go wrong. So it is with gears, which are often seen as mystical devices because they are so hard to draw, manufacture, and difficult to analyze compared to a nice uniform cantilever beam[2].

Following pages focus on the basic fundamentals of gears, and their application to robots that students are likely to design for competitions. Specifically, focus will be on simple spur, bevel, and worm gears. Although as noted earlier, where helical gears formed a critical part of the CD drive system, some special interesting applications will of course be discussed where appropriate. Students who wish to learn more, in particular how to design geared systems for production, should be sure to learn from one or more of the many fine texts on gear design or take an advanced machine elements design course.

Take a look at how Martin Jonikas used gears for his "Bomb Totin' Momma" machine which won the 2002 contest "The MIT & the Pendulum". Are there potentially non-conventional ways in which gears can be used in your machine? Gears are not only transmissions, they are motion generating devices.

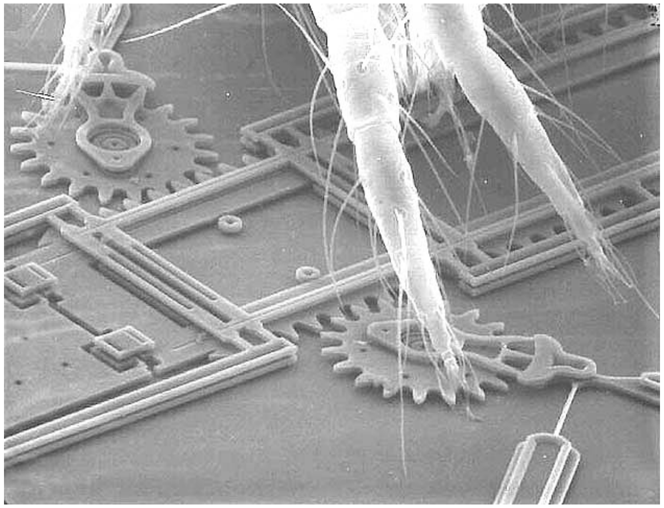
1. For a good introduction to gear design, see J.E. Shigley & J.J. Uicker Theory of Machines & Mechanisms, McGraw-Hill, Inc. New York, 1995, Machinery's Handbook, Industrial Press Inc, New York, and Stock Drive Products' on-line tech library at: http://www.sdp-si.com/Sdptech_lib.htm. For an excellent detailed treatise on the fine points of gearing, see G.W. Michalec Precision Gearing Theory and Practice, John Wiley & Sons, Inc., New York, 1966. Many gear manufacturers also have excellent gear tutorials, see for example http://www.khkgears.co.jp/english/teach/teach.htm. And of course: http://science.howstuffworks.com//gears.htm

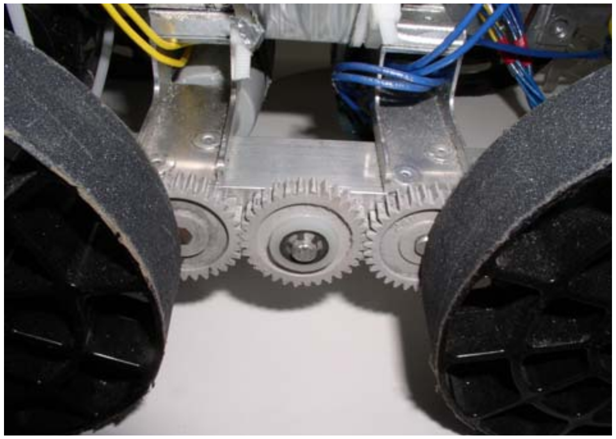
2. New materials are constantly evolving to withstand higher stresses and less wear, and new manufacturing techniques allow ever better and more complex shapes to be obtained. Designers are always trying to shrink package sizes, so standard gears are often not available. It is no wonder that gear designers are often highly coveted and can be as well paid as software engineers!

Gears!

- **Gears are most often used in transmissions to convert an electric motor's high speed and low torque to a shaft's requirements for low speed high torque:**
  - *Speed* is easy to generate, because voltage is easy to generate
  - *Torque* is difficult to generate because it requires large amounts of current
- Gears essentially allow positive engagement between teeth so high forces can be transmitted while still undergoing essentially rolling contact
  - Gears do not depend on friction and do best when friction is minimized
- Basic Law of Gearing:
  - *A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point*
  - Any two curves or profiles engaging each other and satisfying the law of gearing are *conjugate curves, and the relative rotation speed of the gears will be constant*

http://mems.sandia.gov/scripts/images.asp





http://www.bostongear.com/

Gears: *Involutes*

The fundamental premise of gearing is to maintain a constant relative rotation rate, which can be achieved, with a tooth shape called an *involute*[1]. Its study is called *involutometry.* Simple *spur* gear involute tooth shapes are generated starting with two base circles with a line drawn tangent to both. Now imagine a thin wire is attached to the base circle at the tangent points, and then cut the wire. Trace out curves with the ends of the wires where the wire can wrap around the circumferences of the base circles. The resulting curves are *involutes* of the *base circles*. The geometry is computed given assuming segment AT is always equal in length to arc AB:

$$p = r_b(\alpha + \varphi) = r_b \tan\varphi$$
$$\alpha = \mathrm{inv}\varphi = \tan\varphi - \varphi$$

- *Addendum:* Radial distance a tooth projects from the *Pitch Circle* (B) outwards towards the outside diameter (also called the addendum circle)
- *Backlash:* The amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles
- *Base Circle:* The circle from which an involute curve is developed
- *Circular Pitch:* Arc length, measured on pitch circle from one point on a tooth to a similar point on an adjacent tooth
- *Dedendum:* Radial distance from the pitch circle towards the root diameter
- *Face Width:* The width of the tooth in the axial direction of the gear
- *Line of Action:* The line tangent to both base circles
- *Module:* Ratio of pitch diameter to number of teeth (mm/teeth only)
- *Pitch or Diametrical pitch:* The total number of teeth divided by the diameter in inches (sorry, teeth-per-inch units only). *A 16 pitch gear will only mate with another 16 pitch gear that also has the same pressure angle!*
- *Pitch Circle (Pitch Diameter):* Theoretical circle upon which all calculations and contact is made. The *gear ratio* is the ratio of pitch diameters
- *Pressure Angle:* Angle between the *line of action* and the perpendicular to the line between the gear centers.
- *Pitch point:* Point of tangency of the two pitch circles
- *Tooth Thickness:* Chordal thickness of the tooth at the pitch diameter
- *Tooth Face:* Tooth contact surface region from the pitch line to the outer diameter
- *Tooth Flank:* Tooth contact surface region from the pitch line towards the root diameter

The *American Gear Manufacturer's Association* (AGMA) develops and maintains standards. For example, fine pitch involute spur gears can have 14.5, 20, or 25 degree pressure angles:

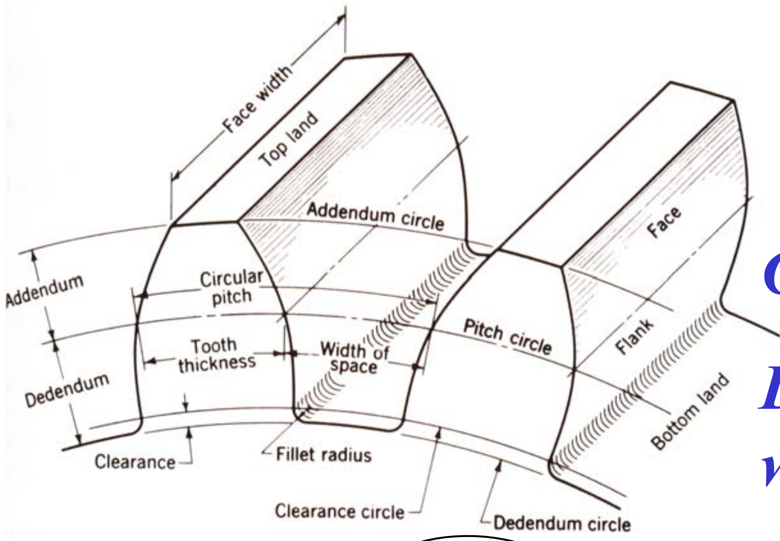
a	Addendum	a (mm) = 25.4/P
a_g	Gear addendum	
a_p	Pinion addendum	
b	Dedendum	b (mm) = 30.48/P+0.05
c	Clearance	c (mm) =5.08/P+.050 (min)
C	Center distance	$C = 0.5(D_P+D_G)$
D	Pitch diameter	$D = N/P = Np/\pi$
D_G	Gear pitch diameter	
D_O	Outside diameter	$D_O = (N+2)/P = D+2a$
D_P	Pinion pitch diameter	
D_B	Base circle diameter	$D_B = D\cos\phi$
D_R	Root diameter	$D_R = D-2b$
ϕ	Pressure angle	
F	Face width (thickness)	
h_k	Working depth of tooth	$h_k = a_g+a_p$
h_t	Whole depth (radial length) of tooth	$h_t = a+b$
$e=1/m_G$	Gear ratio	signΠNinput/ΠNoutput
m	Module (mm only)	m = D/N
N	Number of teeth	N = PD
N_G	Number of teeth on gear	
N_P	Number of teeth on pinion	
p	Circular pitch	$p = \pi D/N = \pi/P$
P	Diametrical pitch (pitch, inches only)	P = N/D
t	Tooth thickness	$t = 0.5\pi/P$

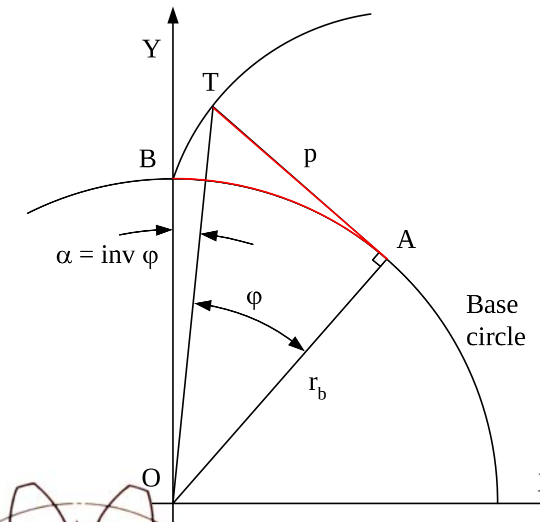
What is the pitch of the gears in your kit of parts? How many different diameter gears are available? How many ways can you use them!?

---

1. In 1673, a mathematician named Christian Huygens published the book Horologium oscillatorium sive de motu pendulorum ad horologia aptato demonstrationes geometrica, which described evolutes and involutes of curves. See http://www.brown.edu/Students/OHJC/hm4/k.htm. The American Gear Manufacturer's Association (AGMA) develops and maintains standards for gears. Involute gear teeth quickly replaced 16th century round-peg gear "teeth" that quickly wore away. However, it appears if the ancient Greeks might have had involute teeth: A precise navigational device was discovered off the island of Antikythera in the early 1900's (see http://www.nordex.com/htmlpages/amazing.html).

Gears: *Involutes*



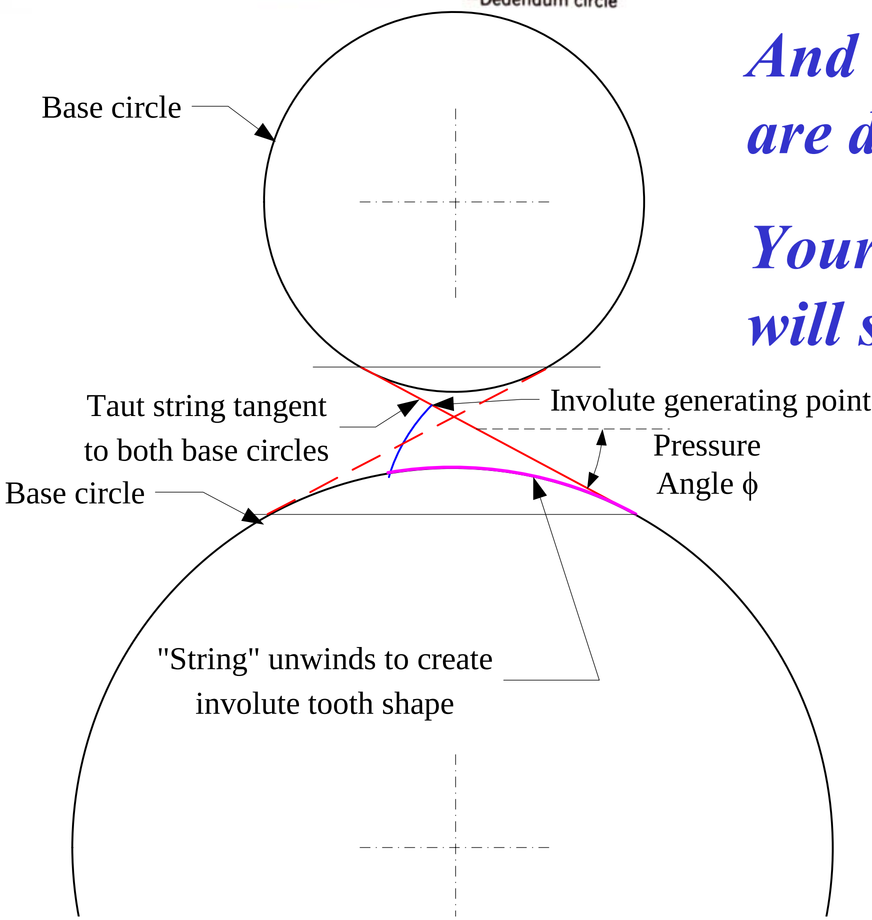


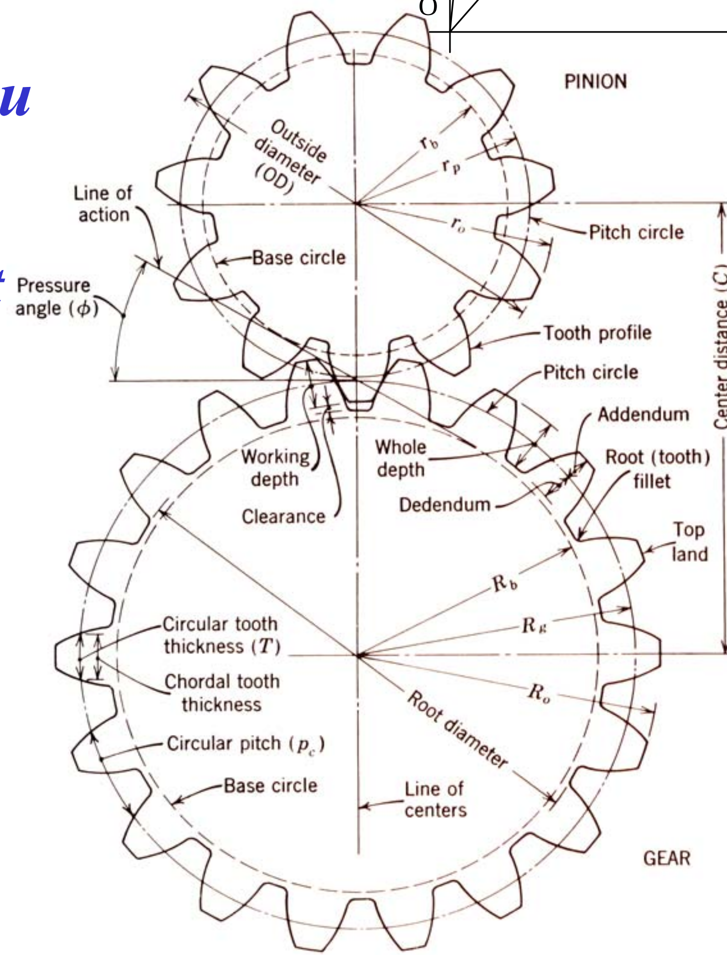
Gears are fun!

Design them well

And when you are done

Your product will sell!





Gears: *Gear Trains*

Recall that there are two fundemental motions in life: rotary and linear; and the latter really is just rotary with an infinite radius! Also recall page 6-8 and the topic of differential leadscrews. You have all the catalysts that you need to create ideas for gear trains to accomplish virtually anything. Start with a simple gear train to decrease the speed and increase the torque from of a motor. Attach the *pinion* (the small gear) to the motor shaft, and attach the *gear* to the output shaft. Given an efficiency of η, the speed and torque of the output shaft are found by applying conservation of energy: power in = motor torque * speed *efficiency = power out:

$$\omega_{output} = \frac{\omega_{input}}{m_g} = e\omega_{input} = \frac{\omega_{input} N_{number\ of\ teeth\ on\ input\ gear}}{N_{number\ of\ teeth\ on\ output\ gear}}$$

$$\Gamma_{output} = \eta m_g \Gamma_{input} = \frac{\eta \Gamma_{input}}{e} = \frac{\eta \Gamma_{input} N_{number\ of\ teeth\ on\ output\ gear}}{N_{number\ of\ teeth\ on\ input\ gear}}$$

Remember that gears transmit torque by generating forces between the teeth. This means if a torque Γ is transmitted at a pitch diameter D_{pitch}, then the shaft on which the gear is mounted also sees a radial load F_{radial} (or $F_{tangent}$) which affects the bearings. In addition, and perhaps most importantly, there is a component of the force, F_{spread}, due to the pressure angle ϕ between the gear teeth that acts to push the gears' center distance apart:

$$F_{radial} = \frac{2\Gamma}{D_{pitch}}$$

$$F_{spread} = F_{radial} \tan\phi$$

This is one of the most overlooked facts of gear train design that dooms many contest robots (and machines designed by inexperienced engineers) to failure. Substantial gear tooth forces can act in the sensitive direction (recall page 3-20) to deform the system which can cause the gear teeth to skip and wear! The force F_{radial} will act to deflect the shafts, but in a *non-sensitive direction* with respect to tooth engagement. The force F_{spread} will act to deflect the shafts in a sensitive direction and if it is too great, the teeth will no longer engage properly and the tips of the teeth could skip over each other and quickly wear off.

What about rack and pinion systems for linear motion? Power out = force * velocity, and the velocity equals the product of the pinion diameter/2 and its speed (remember to convert to radians per second!):

$$F_{rack} = \frac{2\eta\Gamma}{D_{pinion\ pitch}}$$

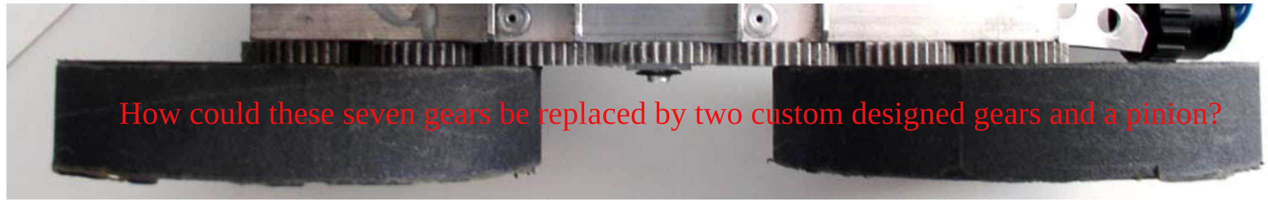
$$v_{rack} = \frac{\omega_{pinion} D_{pinion\ pitch}}{2}$$

What about putting many gears in series? The most important fundemental points to remember about gears for robot design contests is that their relative velocity is constant. Each set of meshing teeth can be on the order of 95% efficient, and the teeth must be strong enough to transmit the torque required. Other things not to forget are that when one gear rotates CW, its mate rotates CCW, and once again, that if gears' center distances are not carefully controlled, and they are not mounted with sufficient radial stiffness, the teeth can skip and wear out very rapidly.

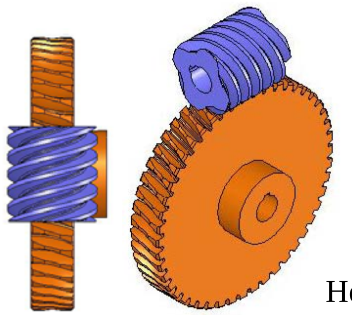
Gears can be made in the form of partial arcs for limited range of motion, they can be non round to generate varying velocities between their shafts, and they can be combined in interesting ways, as will be discussed later, to achieve fantastic ratios in a small space! Your mind should now be abuzz with ideas, and you might be thinking about making your own custom gears for your robot. However, gear designers must also worry about how manufacturing and assembly tolerance will affect performance. Contact stress and bending stresses can cause wear, but they also cause deformations which upsets the ideal tooth motion geometry!

What kinds of gear trains might you use in your machine to achieve the desired motions? Note the picture that shows gears being used to power the front and rear wheels of a robot contest vehicle, thereby enabling obtainment of 4WD without the hassle of belts. Do the gears rotate on the cantilevered shafts or do the shafts rotate? Does it matter?

Gears: *Gear Trains*

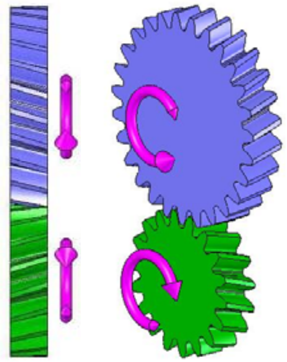


- A *gear train* is used to reduce motor speed and increase output torque:

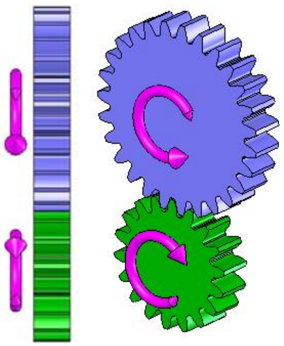


Worm Gear and Worm Wheel

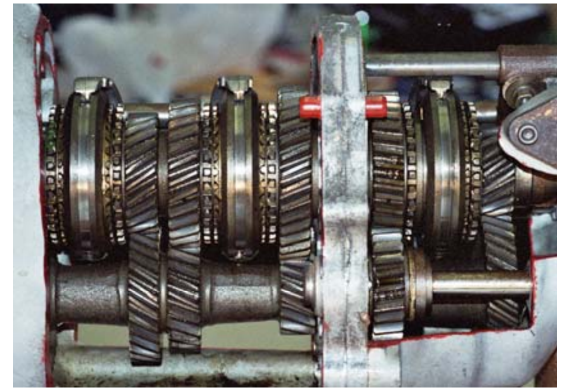
16º RH Helical Gears with crossed axes (32º)



RH and LH Helical Gears

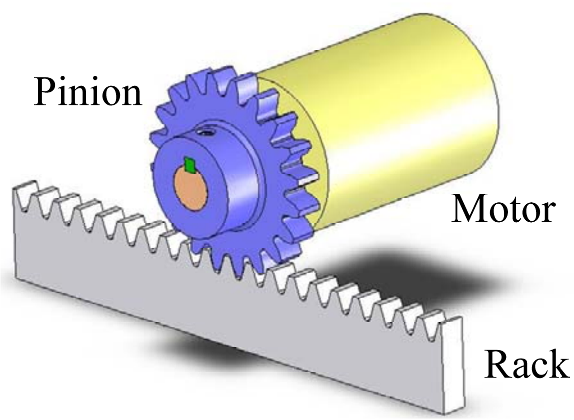


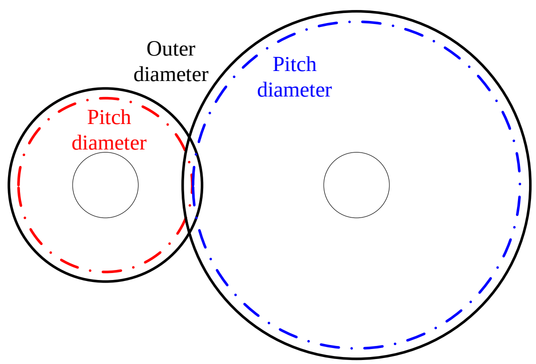
Spur Gears

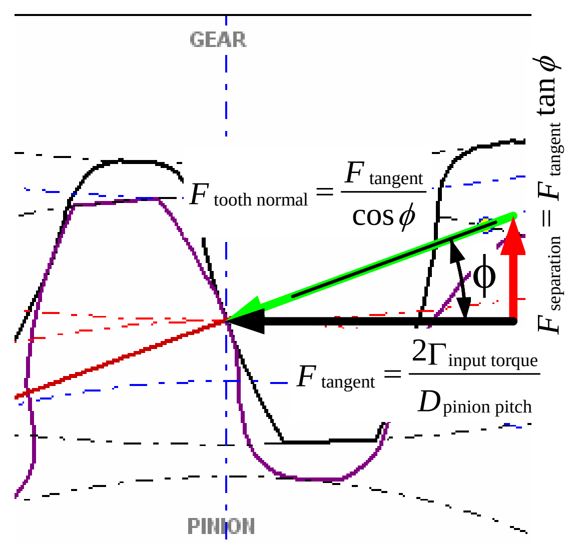


  - *Pinion:* smaller of two gears (typically on the motor) drives a gear on the output shaft
  - *Gear or Wheel:* Larger of the two gears

- Gears are efficient (90-95%) due to mostly rolling contact between the teeth; by conservation of energy: $\Gamma_{output} = \frac{\eta \Gamma_{input} d_{output}}{d_{input}}$ $\omega_{output} = \frac{\omega_{input} d_{input}}{d_{output}}$







Gear Trains: *Serial Gear Train Ratios*

Often there are many more than just two gears between the input and the output shafts in a gear train. This is done to achieve a higher gear ratio (or *train ratio*) than may be practical with just two gears, because otherwise the size of a gear may become too large. Remember, inertia goes with the fourth power of the diameter! In order to compute the transmission ratio, one essentially breaks the transmission up into a series of two-gear systems and then applies the previous formula. This entails identifying all the *driving* (input) gears and all the *driven* (output) gears. As the figure shows, one of the tricks is identifying the sign of the rotation of the final output gear with respect to the input gear. A simple method is to draw arrows: head-to-head and tail-to-tail with arrows on gears attached to the same shaft pointing in the same direction. As the schematic diagrams show, sometimes 4 gears reverse direction and sometimes they do not! Thus the transmission ratio is given by[1]:

$$e = \frac{1}{TR} = \frac{1}{T_R} = (-1)^{(\text{\# independent gears -1 + \# internal gears})} \frac{\prod_{i=1}^{Total\ \#\ of\ driving\ gears} N_{i\ teeth}}{\prod_{i=1}^{Total\ \#\ of\ driven\ gears} N_{i\ teeth}}$$

As an example, consider the simple two-gear system. It is rather obvious that there is one driven and one driving gear, and that rotation is reversed, so the gear ratio is -1. Consider the four gear system, where the driving gears left-to-right are 20-tooth (blue), 10-tooth (red), and 10-tooth (red). The driven gears, and note a gear that is a driver can also be a driven gear, are 10-tooth (red), 10-tooth (red), and 20-tooth (blue). There are 4 independent gears, so the gear ratio is:

$$e = (-1)^3 \frac{20 \times 10 \times 10}{10 \times 10 \times 20} = -1$$

What about the more complex five gear set? Note the trick of attaching the black gear to the same shaft as the red input gear. This is a common method for cascading a series of gears to obtain a very high ratio. In this case, there are five gears, but since two of them are attached to each other and forced to rotate in the same direction, there are only *four* independent gears. The driving gears are: 9-tooth (red), 9-tooth (black), and 67-tooth (green). The driven gears are: 38-tooth (blue), 67-tooth (green), and 33-tooth (purple). The gear ratio is thus:

$$e = (-1)^3 \frac{9 \times 9 \times 67}{38 \times 67 \times 33} = -0.065$$

Why is the large green gear used as the final gear and not another small gear, such as another 9 tooth gear, attached to it and used to drive the final driven gear to obtain an even higher ratio? The equation shows the 67 tooth gear in the numerator which cancels its great gear ratio enhancing effect in the denominator! Who knows what this designer had in mind? However, this shows that the equation gives a quick visual clue as to the effectiveness of achieving a high transmission ratio. A large number in the numerator can signal that there might be a better configuration!

What would happen if the green and purple gear are swung around to the other side to make the gearbox more compact? Will that not change the direction? A vector representing the direction of rotation can be moved parallel to itself without changing direction, so the answer is “no”, which is also shown by the schematic diagram!

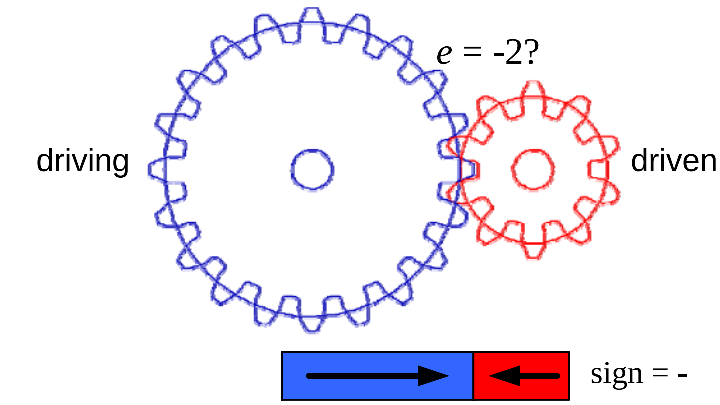
These two methods, graphical and analytical, for determining the transmission ratio and its sign can be applied to the complex of gear trains. They work equally well with gears mounted on non-intersecting axes, such as bevel gears. Thus when faced with what may seem to be a daunting serial collection of gears, just systematically start drawing arrows, or counting independently rotating gears.

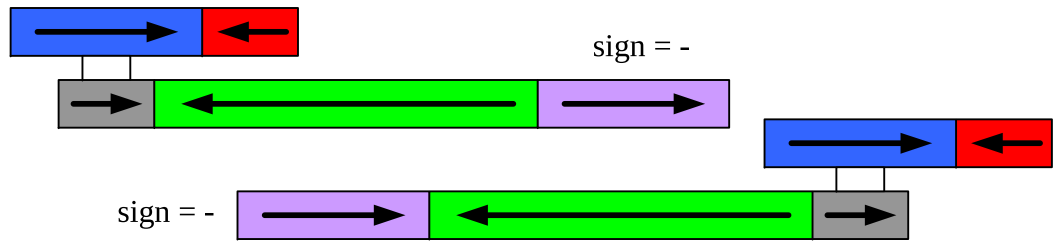
What series gear trains were you considering? Can you make a compact high transmission ratio (low train ratio e) transmission for your system? Can you achieve the ratio you need by attaching a small gear to a large gear?

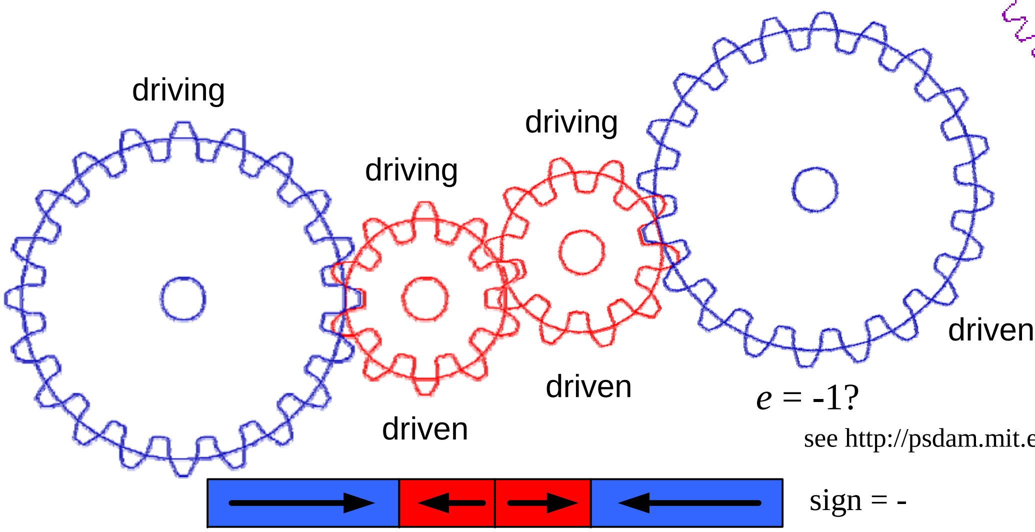
1. Note the term *transmission ratio* TR is also often used, and if a large TR number is given, it denotes that the output shaft spins at a much lower speed than the input speed.

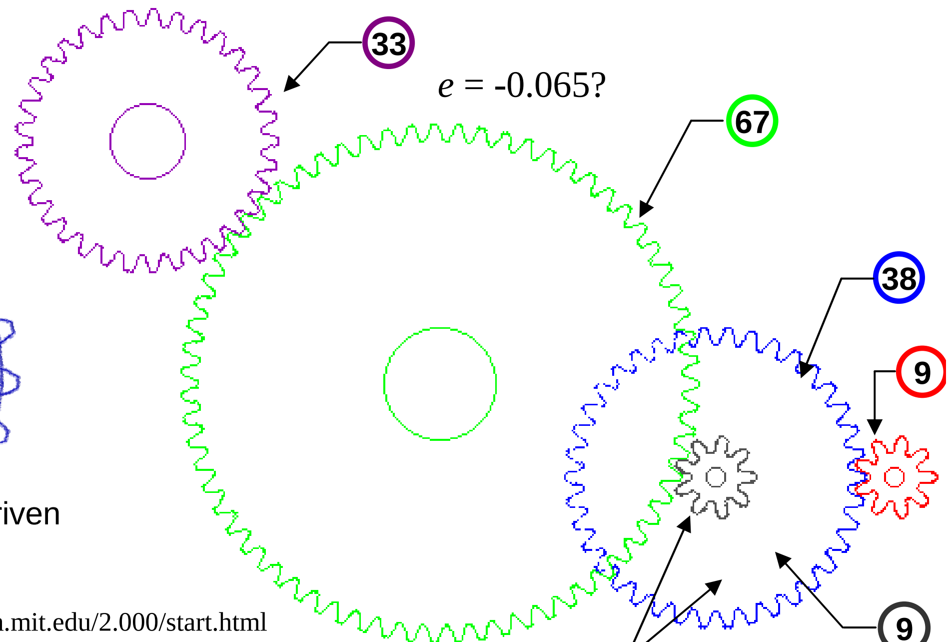
Gear Trains: *Serial Gear Train Ratios*

- For gears arranged in series (serial trains), identify the *driving* and *driven* gears and the relative direction of rotation (sign) between the input and output gears
  - Draw arrows on the gears: head-to-head or tail-to-tail, head to right is +
  - A negative transmission ratio (*TR* or *e*) means that the output rotation direction is opposite the input rotation direction









These two gears are rigidly attached to each other

see http://psdam.mit.edu/2.000/start.html

Thanks to Prof. Marty Culpepper for creating these images

Gears: *Planetary Gear Trains*

Recall page 6-8 where a leadscrew with two different leads was attached to a stage with one nut grounded and the other nut attached to the moving element. As the screw was turned, it moved forward through the fixed nut, but it also moved forward through the nut attached to the carriage. The result was that the carriage only moved forward by the difference in motion of the screw through the two nuts. The same type of clever transmission effect is obtained by planetary gear trains.

In a *planetary* gear train, the goal is to create differential motion. This is typically accomplished with an outer *ring gear*, whose teeth are on the inside diameter and an inner *sun gear* coupled together by *planet gears* held by a *planet carrier.* The planet carrier has shafts on which rotate planet gears that engage both the ring gear and the sun gear. The sun gear can also be integral (attached or made part of) a planet carrier that was driven from a previous stage, allowing the system to be *cascaded* to achieve a very high ratio.

There are several arrangements for planetary or *epicyclic* speed reducers that design engineers are likely to encounter[1]. Perhaps the most common is shown where the ring gear is stationary and the motor drives the sun gear. As the sun gear rotates, the small planet gears on the planet arm rotate as they roll and mesh with the teeth on both the ring and sun gears. The distance the planet gears roll on the inside of the ring gear is the product of the rotation angle of the planet arm and the pitch diameter of the ring gear. The planets also mesh with the sun gear; however, the sun gear has a smaller pitch diameter than the ring gear. The planet gears are geometrically constrained to roll on two surfaces at once. Thus the amount the planet carrier gear rotates must equal the difference in distances the planet gears would rotate if they only made contact with the ring and sun gears, respectively. The transmission ratio of the gear train is the ratio of input to output rotation angles (sun gear to planet arm rotation angle):

$$TR = \frac{\mathrm{D}_{ring\ gear\ pitch\ diameter} + \mathrm{D}_{sun\ gear\ pitch\ diameter}}{\mathrm{D}_{sun\ gear\ pitch\ diameter}}$$

1. There are actually 12 different types of planetary gear trains. These are shown schematically and their transmission ratios tabulated in J. Shigley and C. Mischke, Standard Handbook of Machine Design, McGraw-Hill Book Co, New York.

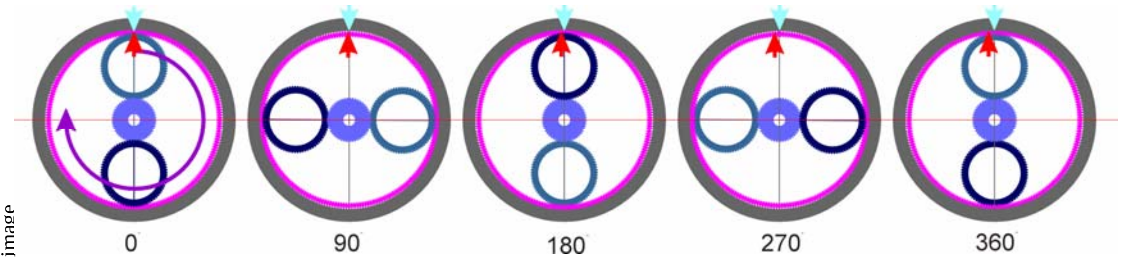
This type of transmission is among the most compact available. The ring gear can be molded or broached on the inside of a tube. A sun gear is attached to a motor output shaft and it engages planets held by a planet carrier. The planet carrier also has a sun gear on it, which in turn drives another set of planet gears, and so on. In this manner it is easy to build up a series gearbox with a tremendously high gear ratio. In addition, the sun gear is making contact with several teeth so the contact ratio is much higher than for a conventional multi-stage gearbox. This enables planetary speed reducers to carry very large loads. Although economical to manufacture, this type of series gear train can be noisy at high speeds because the radial position of inner planet assemblies is usually not fixed with bearings. It often must rely to some extent on the meshing of the three planets with the ring gear to centralize the planet arm.

Note the pictures of molded plastic parts and the box for a small modular planetary transmission often used in robot design contests and by hobbyists. When assembling such systems, it is vital to follow the instructions, but perhaps even more important, remember Maudslay's maxims! Every piece in the kit is there for a reason (else it would not be there!), so:

- Make sure that all the mold flash is removed from the parts
- Take care to observe alignment marks when assembling the pieces
- Sparingly apply a little lubrication (often in a blue tube) to each moving surface (gear teeth and shafts on which the gears spin)
- Do not tighten the bolts too tight, or else they can cause deformations and reduce efficiency
- If the efficiency is good, then the gearbox will be back-drivable
- Make sure you have a torque coupling method figured out, as the output shafts are rarely designed for large radial loads (see pages 5-29 and 5-30)
- Select the proper transmission ratio (see page 7-4) to achieve the desired output torque and speed, but not such a high ratio that the final stage teeth shear off!

Assemble your modular gearboxes one stage at a time, and make sure they are backdrivable at each step. Connect the motor up to a power source and run the system in. How will you mount the motor/gearbox, and how will you couple its output to wheels, pulleys etc.?

Special thanks to Micromotion GmbH for this image



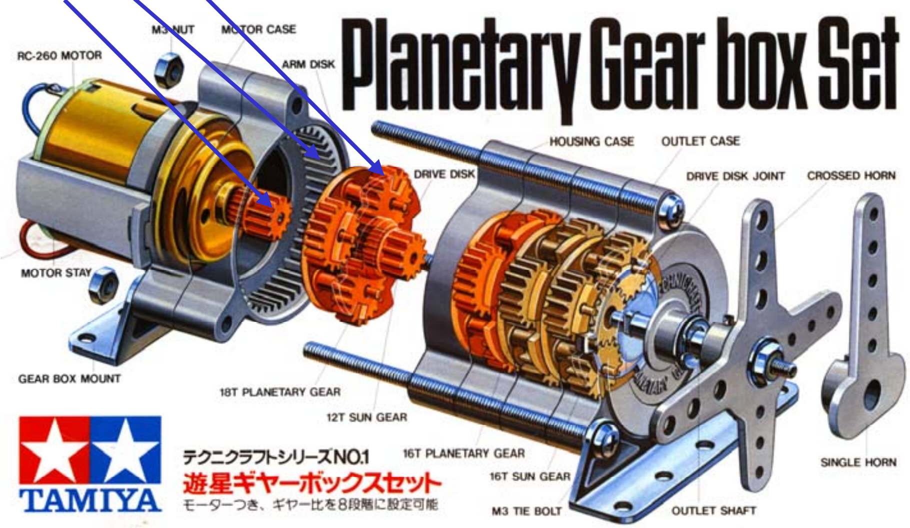
Gears: *Planetary Gear Trains*

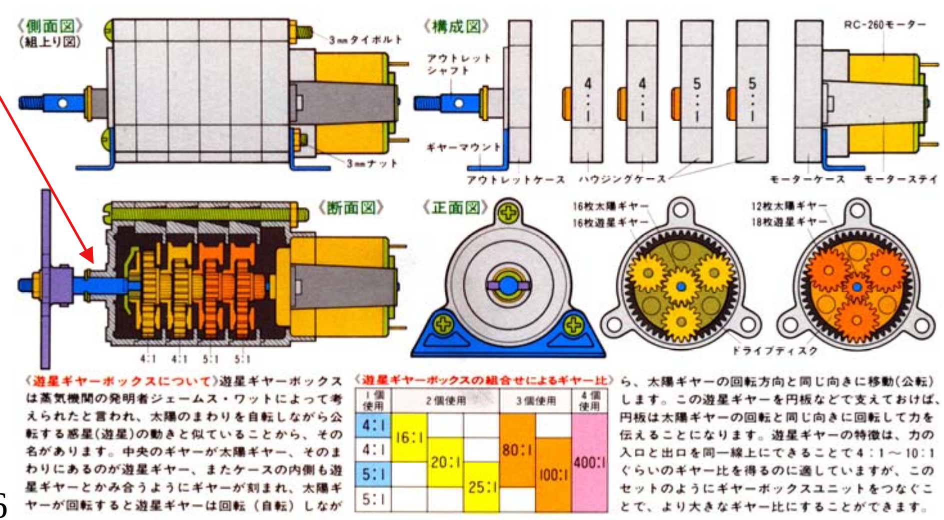
- Planetary (epicyclic) gear trains enable a high reduction ratio to be obtained in a small place
  - With a fixed ring gear, as the planet carrier rotates, the planet gears must simultaneously roll on both the surfaces of the sun gear and the ring gear (review page 6-8)
  - The *difference* in the path length must be accommodated by rotation of the sun gear:

For each stage of the common planetary system shown below: $$TR_{Transmission\ ratio} = \frac{D_{Sun} + D_{Ring}}{D_{Sun}}$$

- The size of the teeth and the torque transmitted limit the minimum size of the sun gear
  - A sun gear can be mounted to a planet carrier's stem…..and a multistage system can be created
  - Very high ratios can thus be achieved but beware of high applied torques that can strip teeth!
  - Think of Saint Venant (page 3-5): Can the shaft support bending loads, or only transmit torque?

Planet
Ring
Sun





Epicyclic Drives: *Gear Train Ratios*

The use of multiple planets between the sun and the ring gear increases the number of teeth that effectively transmit torque, and thus a very strong gear train can be obtained in a very small space. The use of more than one planet, however, does not change the gear train ratio. The gear train ratio, depends on which element is the input, which is the output, and what is held fixed. Consider the general case shown. The red sun gear (1) drives the green planet gear (3) which in turn drives the blue ring gear[1] (4). The purple planet carrier arm (2) is free rotate. If we try to physically envision what is moving and what is not, and what is maybe moving a little bit, we might get confused, so let analysis guide us. The first steps are to find the speed of gear 1 (the sun) with respect to gear 2 (the arm), and the speed of gear 4 (the ring) with respect to gear 2 (the arm). This is done in terms of absolute velocities with respect to ground:

$$\omega_{12} = \omega_1 - \omega_2$$

$$\omega_{42} = \omega_4 - \omega_2$$

The relative velocity of the ring (gear 4) to the sun (gear 1) is just the ratio of these two velocities, because the reference frame cancels out:

$$\frac{\omega_{42}}{\omega_{12}} = \frac{\omega_4 - \omega_2}{\omega_1 - \omega_2} = e = \frac{\omega_{Last} - \omega_{Arm}}{\omega_{First} - \omega_{Arm}}$$

$$\frac{\omega_{Last} - \omega_{Arm}}{\omega_{First} - \omega_{Arm}} = (-1)^{(\text{\# independent gears -1 + \# internal gears})} \frac{\prod_{i=1}^{Total\ \#\ of\ driving\ gears} N_{i\ teeth}}{\prod_{i=1}^{Total\ \#\ of\ driven\ gears} N_{i\ teeth}}$$

This is a very powerful result. Different ratios are obtained by selecting the sizes of the sun (the driven gear) and the ring gear, and whether the ring gear or the arm are stationary. Note that the expression for the velocities also equals the expression for the train ratio obtained from the number of teeth. The sign (+ or -) indicates where the output direction is the same (+) or opposite (-) the input direction of rotation. Given the use of an internal ring gear, it is sometimes easier to determine the sign of the gear ratio by graphical means, but the formula works as well. For a typical planetary transmission comprised of a sun, planet, and internal ring gear, *sign = -1*. There are some type of planetary gears, such as the double planet system shown schematically, where the output gear axis is colinear with the input gear, and then *sign=1*.

This formula is applied in two steps. The first step is to determine the train ratio and its sign using the number of teeth as given by the formula. The next step is to determine which are the first (input), last (output) and arm gears and which are held stationary. Of course there is a spreadsheet for this called *planetary.xls*. Analyzing the three cases for a internal ring gear planetary transmission shown in the figure, assume the driving sun gear has 30 teeth and the stationary (fixed) ring gear has 72 teeth. For the sun as the first gear and the driver, e = -0.42 (look at the cells in the spreadsheet), and with the last gear (the ring gear) stationary, the angular velocity relative to ground of the planet carrier arm (the output) is:

$$e = \frac{-N_{Sun}}{N_{Ring}} = \frac{0 - \omega_{Arm}}{\omega_{Sun} - \omega_{Arm}} \qquad \omega_{Arm} = \frac{e\omega_{Sun}}{e-1} = \frac{N_{Sun}\omega_{Sun}}{N_{Sun} + N_{Ring}}$$

For the case of a *fixed* sun gear and planet carrier arm input, the ring gear is considered the first driving gear, and e = -2.40, and:

$$e = \frac{-N_{Ring}}{N_{Sun}} = \frac{0 - \omega_{Arm}}{\omega_{Ring} - \omega_{Arm}} \qquad \omega_{Ring} = \frac{\omega_{Sun}(e-1)}{e} = \frac{\omega_{Sun}(N_{Sun} + N_{Ring})}{N_{Ring}}$$

How are the formulas derived for the case where the sun gear is the input, the planet carrier arm is stationary and the ring gear is the output?

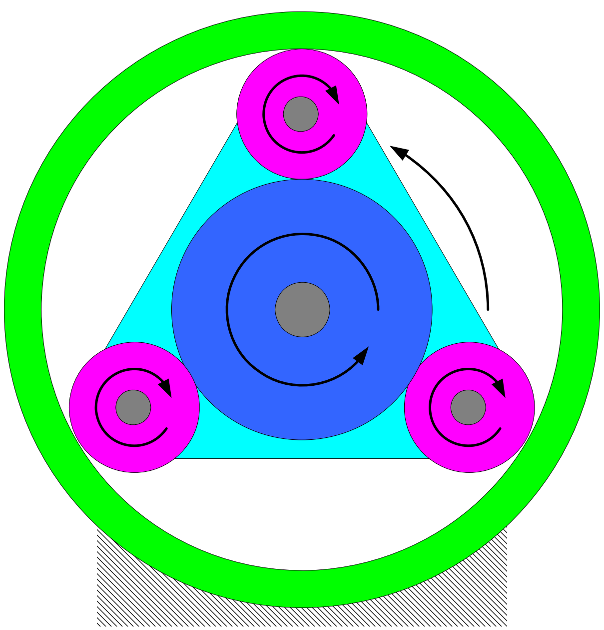
It is most likely that you will use a modular planetary transmission in the most common mode where the sun is the driver and the ring is fixed, but you may decide to create your own planetary transmission. What might be the advantages of this? Could you make it stronger? Will you need to use different pitch gears, or can all the gears have the same pitch? If you can make your own gears, you could create some very useful gear trains!

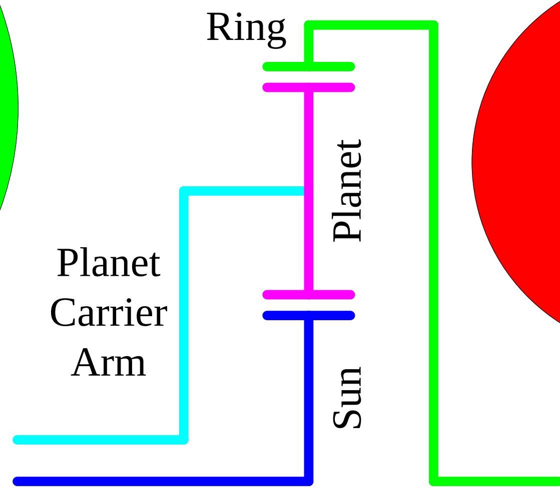
1. The ring gear is shown as a generic gear, it could just as well have been drawn as an internal tooth ring, but that would take up a lot more space!

Epicyclic Drives: *Gear Train Ratios*

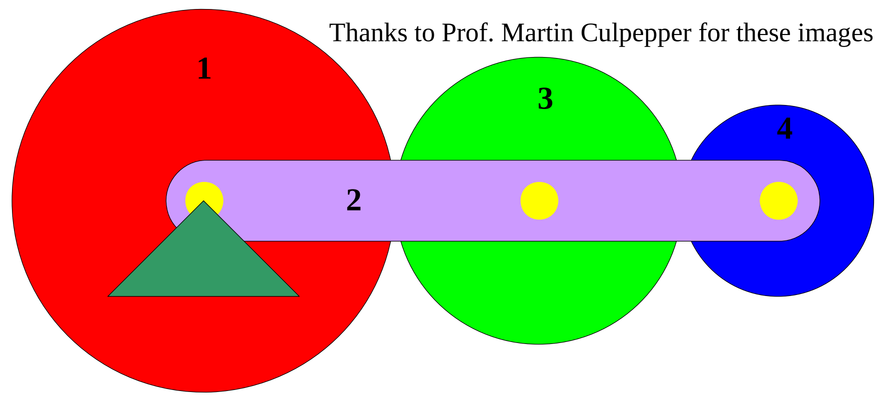
- The transmission ratio for an *epicyclic* gear train can be determined by considering the relative velocities of the components
  - There are 12 unique planetary gear transmissions

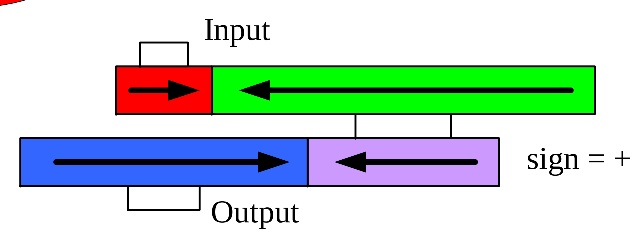
planetary.xls				
Enter numbers in BOLD, output in RED				
Written by Alex Slocum, last updated 3/05/03				
Gears	Generic	Type A (sun, planets on carrier, ring)		
Number of teeth on 1st driving gear	**20**	**30**	**72**	**30**
Number of teeth on 2nd driving gear (or enter 1)	**16**	**1**	**1**	**1**
Number of teeth on 1st driven gear	**30**	**72**	**30**	**72**
Number of teeth on 2nd driven gear (or enter 1)	**34**	**1**	**1**	**1**
relative direction of rotation (first to last gear)	**1**	**-1**	**-1**	**-1**
Train ratio	**0.31**	**-0.42**	**-2.40**	**-0.42**
Speed of first gear	**250**	**100**	**141.7**	**100**
Speed of last gear	**0**	**0**	**0**	**-41.7**
Speed of planet carrier arm	**-114.3**	**29.4**	**100**	**0**
Transmission ratio	**-2.19**	**3.40**	**0.71**	**-2.40**
Input		Sun	Planet carrier	Sun
Output		Planet carrier	Ring	Ring
Stationary		Ring	Sun	Planet carrier





Thanks to Prof. Martin Culpepper for these images





Gears: *Modular Epicyclic Drives*

Another type of planetary gear train is sometimes referred to as a *perpetual wedge*. In this design, gears 1 and 3 are attached to a common shaft that is supported in the planet arm by bearings. Usually three planet arms exist and are attached to the input shaft. Gear 1 meshes with the fixed gear 2, and gear 3 meshes with gear 4, which is attached to the output shaft. Tracing through the amounts each gear rotates and travels circumferentially, it is not difficult to show that the ratio of the input to output rotation is:

$$TR = \frac{D_1 D_4}{D_1 D_4 - D_2 D_3}$$

It is not difficult to make the product of D_1 and D_4 approach that of D_2 and D_3, thereby obtaining a very high transmission ratio with a single stage. However, since the shear stress of lubrication is proportional to velocity, and power is proportional to the product of shear stress and velocity, the extreme velocity experienced in the single stage planets cause excessive losses. In order to maximize efficiency and life, gears are typically supported by precision journal (hydrodynamically lubricated) bearings, although ball bearing support can also be used. The input and output shafts' bearing bores are also line bored (bored at the same time).

A *harmonic drive* has a large fixed ring gear with internal teeth. It achieves the differential effect with a *wave generator*, which is an elliptical cam enclosed in a ball bearing assembly with a flexible outer ring. The wave generator is pressed into the inside of flexible external tooth spline (*flexspline*), causing the external teeth of the flexspline to engage with the internal teeth of the rigid internal gear at two equally spaced regions 180 degrees apart on their respective circumferences. The flexible spline is attached to the output shaft through a rigid back plate. As the input shaft rotates the wave generator, the flexible spline orbits within the ring gear while slowly rotating in the opposite direction as the teeth mesh to account for the difference in teeth. However, because the teeth are forced into contact by radial pressure, the efficiency is low (60%) unless it is heavily loaded, which then causes one side of the teeth to be primarily loaded and the efficiency can then be 90%. The transmission ratio is derived from above by setting $D_1 = D_3$:

$$TR = \frac{N_{Number\ of\ teeth\ on\ spline}}{N_{Number\ of\ teeth\ on\ \mathrm{ring}} - N_{Number\ of\ teeth\ on\ spline}}$$

In order to achieve high ratios the gear teeth must be very small which limits their stiffness and strength. There may be 202 teeth on the ring and 200 teeth on the spline, which gives a transmission ratio of 100:1. Harmonic drives can be made very compact and lightweight and thus have been popular with robot manufacturers and in other applications where weight is critical.

A *cycloidal drive*[1] (or an epitrochoidal drive) uses cam rollers on a fixed housing, a dual trochoidal-shaped cam, and cam rollers on the output housing. The cam is made to orbit inside the input and output housings by an eccentric cam attached to the input shaft. There is one less lobe on each track of the epitrochoidal-shaped[2] cam than on the input and output housings, respectively; thus as the cam orbits, it also rotates. The shape of the dual track cam allows it to be in contact with all rollers at all times, with each roller contacting the cam at a different point on the cam profile. As a result, the device has tremendous stiffness and overload capacity. The transmission ratio for this type of drive is:

$$TR = \frac{\left(\mathrm{N}_{\text{Number of lobes on input cam}} - 1\right)\mathrm{N}_{\text{Number of lobes on output cam}}}{\mathrm{N}_{\text{Number of lobes on input cam}} - \mathrm{N}_{\text{Number of lobes on output cam}}}$$

Typically, there may be 11 input rollers and 10 output rollers, which gives a transmission ratio of 100:1. Ratios from 10:1 to 225:1 are commonly available. Although this type of drive is physically much more complicated than a worm gear drive, it can achieve greater transmission ratios with higher efficiency and is thus becoming more commonly used.

Do you not wish you had some of these drives?

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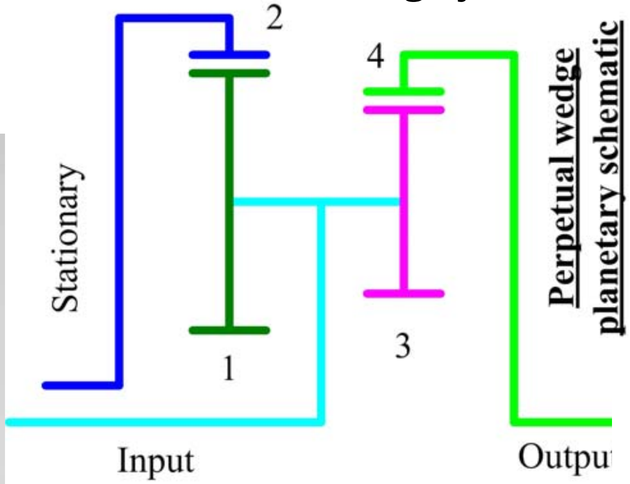
1. See, for example, M. Seneczko, "Gearless Speed Reducers," Mach. Des., Oct. 14, 1984. Also see the website http://cyclo.shi.co.jp/eng/product/gmoter/saikuro6000/sa60020.html
2. A sinusoid superimposed on the circumference of a circle

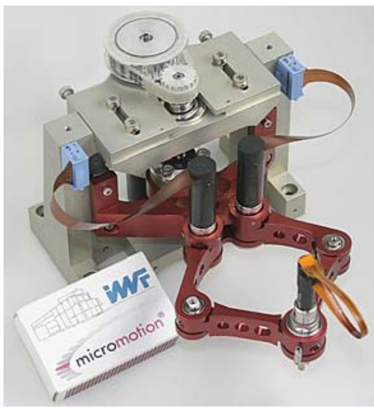
Gears: *Modular Epicyclic Drives*

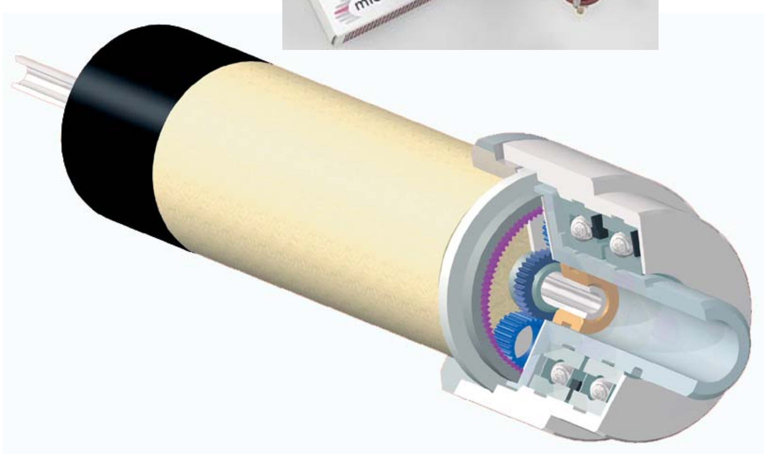
- The concept of differential motion can also be exploited using a wave generator to convert rotary motion from a motor into rotary motion of wave generator
  - The wave generator is forced to roll on two different surfaces at once which thus causes it to revolve and drive an output shaft
- Several different types of commercial systems are available, and are often used in industrial robots and indexing systems
  - Harmonic drives
  - Cycloidal drives

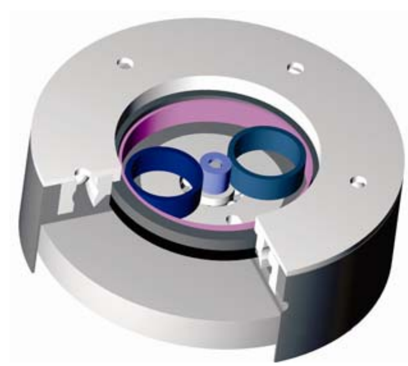


www.hdsi.net

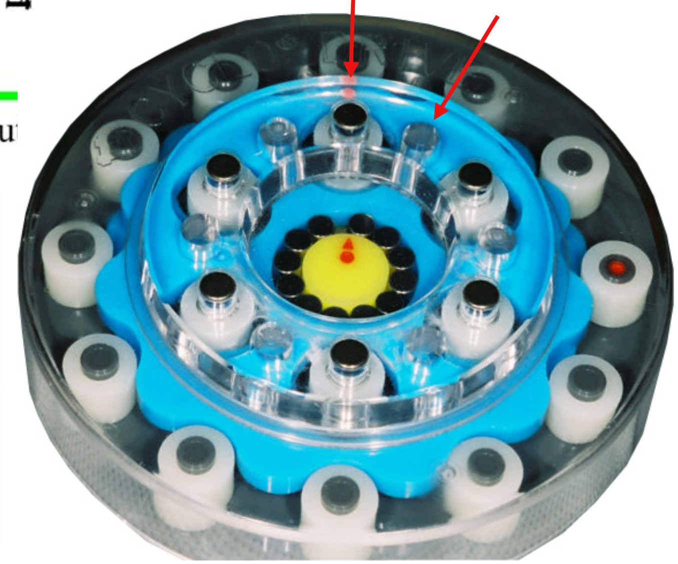
Special thanks to Micromotion GmbH for these micro-gearmotor images. See www.micromotion-gmbh.de for detailed catalogs







1 input revolution causes this much output



http://cyclo.shi.co.jp/eng/product/gmoter/saikuro6000/

Gears: *Automotive Transmissions*

Without a low cost, long life, and efficient transmission, automobiles would never have come into being. How then does a manual transmission work? How does an automatic transmission work?[1] By looking at how other things work, you might be able to better create your own machine. A 3 speed manual transmission, as shown in the figure, has two parallel shafts. The first (upper) shaft is connected directly to the engine via the clutch. Located on this shaft, but free to rotate, are three gears which are always engaged to a secondary shaft that is coupled to the output shaft of the transmission either directly by another gear set for forward motion, or via two other gears for reverse. The shifter's job is to move internally splined collars that selectively engage different gears. The trick is to do so without grrr-inding the gears, and with minimal effort by the driver.

The primary functional requirement is to enable the driver to shift easily when they push in the clutch. Since all the gears are always meshing, the shifter's job is to use the desired gear to bring the upper (input) shaft up to the same speed as the desired gear. The desired gear is connected to output shaft and ultimately the car's wheels. A splined collar couples the desired gear to a spline on the upper shaft. The clutch can then be let out and because it allows for slip as the speed equilibrates. It will gradually couple the engine to the wheels and bring the two to equilibrium through the desired gear.

In order to accomplish this, the input shaft has a series of male splines attached to it on which slide a series of splined collars which can be selectively axially moved by the shifter. Each splined section has an axially spring loaded "dog" that is 1 spline-tooth-circular pitch wide that can engage a slot in the synchronizer which is 1.5 spline-tooth-circular pitches wide. This causes the teeth of the collar's spline to be out of sync with the synchronizer's teeth by 1/2 tooth pitch. The shifter pushes the collar forward to engage the desired gear, and the dog also pushes on the synchronizer, forcing its tapered inside diameter onto a tapered outside diameter on the flange of the desired gear. However, the collar cannot slide forward over the synchronizer, because its teeth are engaging the slight angle taper of the synchronizer teeth and because the dog is pushing hard on the synchronizer, the wedge effect locks them. As the synchronizer ID taper engages the gear flange taper harder and harder, due to the dog pushing on it, it spins up the input shaft until the two shafts reach the same speed. At that point, the collar teeth can slide over the synchronizer teeth and since they are going at the same speed. Since they are within 1/2 pitch of the spline attached to the gear, they can cause a small rotation and allow the collar spline to slide over and couple the spline on the input shaft to the spline on the desired gear. *Self help rules!*

An automatic transmission, on the other hand, uses an ingenious device called a *torque convertor* to couple the engine to the transmission at above-idle speeds. The torque convertor is essentially a turbine that is fluidically coupled to another turbine connected to the transmission. Below a certain speed, the fluid coupling is not strong, but soon above idle, the coupling becomes very strong and very efficient. Perhaps the most amazing thing is the way that the torque convertor is made from formed sheet metal!

The torque converter output is used to drive a hydraulic pump and a planetary gear transmission. The trick, however, is what is driven, what is fixed, and what is the output! Clutches activated by hydraulic fluid at predetermined speeds engage either the sun gear, planet carrier, or the ring gear to be either the input or output of the transmission. In addition, steel bands (see page 5-5), activated by hydraulic cylinders couple the desired part of the planetary gear train to the housing. *Overdrive* has a 1:1 or lower transmission ratio ($e > 1$). If more than one planetary gear set is used, 4 speeds, plus reverse and overdrive are obtained.

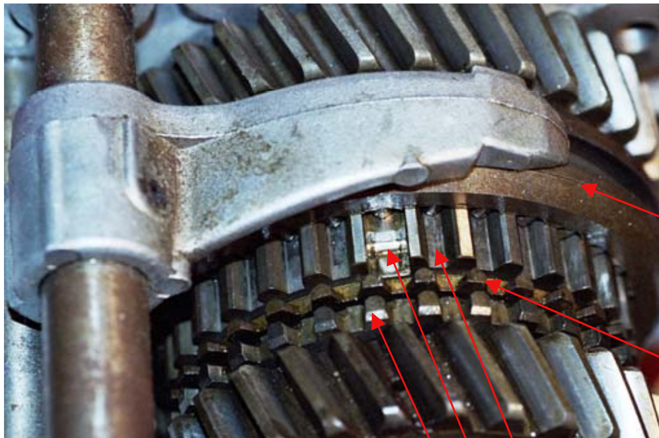
Automotive transmissions started out simple and customer desires helped to catalyze invention. The result is today's automatic transmissions which are just as efficient as manual transmissions, despite all the use of hydraulic fluid circuits. But what about the fun of shifting? That desire too is now met with automatic transmissions that allow the user to shift as if they are driving a manual transmission, but then when they get tired of it, or have other things to do, they can let the transmission do the shifting. Good electronics meets good mechanics to satisfy customer needs!

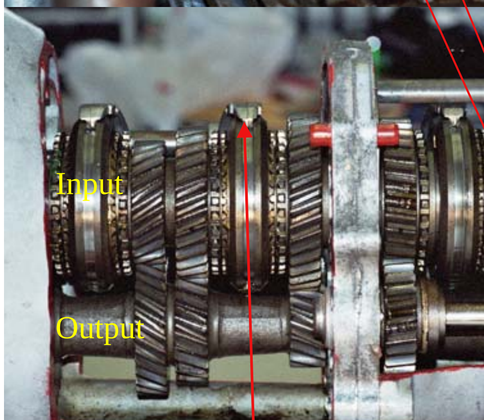
What do you like about the transmission in your car? What do you dislike about it? How can you imagine making the transmission better? Could you achieve your goals purely mechanically? A great social and learning activity is to take apart a transmission with your friends!

1. For nice animations and a more detailed discussion, see http://auto.howstuffworks.com/transmission3.htm

Gears: *Automotive Transmissions*

- An automotive transmission is a truly amazing system
  - The shifter controls linkages that slide internal-toothed collars (*synchronizer sleeve*) over splined shafts connected to different gears and the input shaft to engage corresponding gears on the output coupling shaft
    - The synchronizer brings the drive gear up to speed before allowing the spline to engage it (no grrr-inding!)





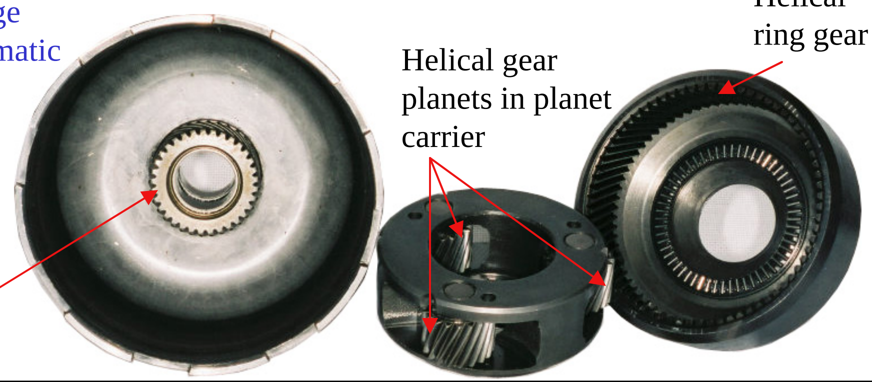
Spline (*synchronizer hub*) attached to input shaft

Spring loaded "dog"

Engaging spline (*blocking ring*) attached to gear

From the other side, note the shifter forks

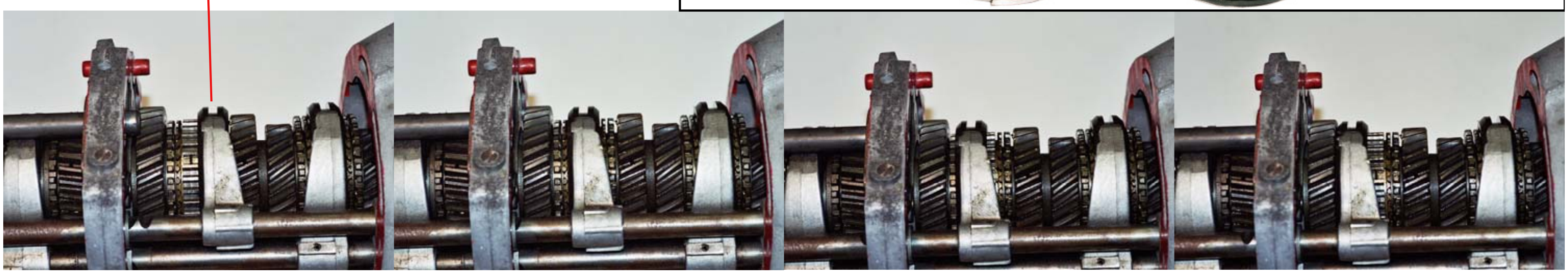
Planetary stage from an automatic transmission



Helical ring gear

Helical gear planets in planet carrier

Helical sun gear



Gears: *Differentials*

The differential effect can be used not only to achieve very high gear ratios, it can literally allow for differential motion between shafts that are subject to varying speed and load conditions. For example, any high performance driving machine must address the fact that as it goes around a corner, the outside wheels must travel a further distance than the inside wheels. Some race cars actually used a solid rear axle to maximize torque transmission to the pavement at all times, but it is tough on tires and can be difficult to steer! So differentials are used on most cars, from full size to radio controlled hobby machines.

Recall the discussion on page 5-19, where it was shown that a conventional *open differential* allows differential motion between the output shafts, but it transfers torque to the wheel which needs it least, which lets a car go around a corner, but will also allow a wheel to spin on the ice while the other wheel stands still on dry pavement. This problem can be addressed with the addition of an electromagnetic or spring-loaded clutch to control torque flow between the axles, or to lock them together. But does this add complexity and susceptibility to wear-induced changes in performance?

In 1958, Vernon Gleasman invented a new type of differential (US Patent 2,896,541) that uses the principle of *self-help* (page 3-14) to enable differential motion to occur between the output shafts, while multiplying the torque on the wheel with least traction and applying it to the wheel with the most traction. He called his invention a *Torsen differential*, and it essentially uses the thrust forces generated by helical gears to apply pressure to friction washers to direct torque to the wheel with the greatest tractive capability. The more torque that an axle can utilize, the greater the thrust force from the helical gears... Since the first patent, many more have been issued as the design has evolved.

“Invex™” gearing in a Torsen differential includes two or more pairs of helical satellite gears (*element gears*) mounted on chordal shafts in the carrier, which mesh with central helical gears (*side gears*) that are attached to the output shafts. The pairs of element gears are interconnected with each other by spur gears at their ends. The figure shows six element gears and two side gears. The number of element gear pairs used in a specific design is a function of overall torque capacity and space requirements. The shafts that support the element gears are supported by the carrier which is attached to the ring gear that is driven by the driveshaft. As the carrier turns, the element gears’ helical teeth apply forces to the mating helical side gears to create torque. The helical gears’ mating angles, however, also generate axial thrust forces (F_{spread}) which push the side gears against friction washers that act to couple the side gears to the carrier. Relative motion between the output shafts is enabled by the spur gears on the ends of the element gears, which causes them to counter rotate in accordance with the velocity difference required between the output shafts. The *torque bias ratio* is the relative torque between the two output shafts, and it is a function of the friction washer properties and the helical gear angles.[1] The simplicity and effectiveness of the Torsen differential is why they are used on Hummer vehicles, and by more and more automobile and truck manufacturers.

Other than just its ingenious design, what made the Torsen differential a viable component for the mass automotive market? The answer includes great manufacturing, including computer controlled gear manufacturing equipment made by Gleason Corp. which enables all the seemingly complex gears to be mass produced with high accuracy for low cost. Still, it took almost 25 years for the Torsen differential to reach the “mainstream”. What took so long and what could have been done to speed things up? Will the need for a differential ever be eliminated by each wheel being driven by its own electric motor? If this were possible, why do electric motors today still often use gearboxes?

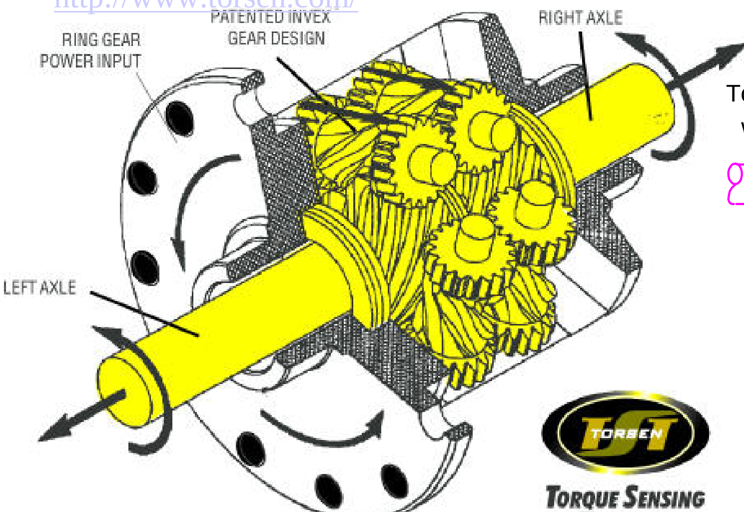
Even in a simple robot design contest, never forget the importance of creating a design that is manufacturable. Good physics*good fundementals*good creativity*manufacturability = good design! So what most excellent brilliantly clever concept have you created? Is it manufacturable? What is the greatest risk you will face in bringing it to life? Do you have appropriate countermeasures?

1. An analytical model for the operation of the Torsen differential is described at http://www.sonic.net/garyg/zonc/TechnicalInformation/TorsenDifferential.html. Also see S. E. Chocholek “The development of a differential for the improvement of traction control”, C368/88, IMechE, 1988 pp. 75-82.

Gears: *Differentials*

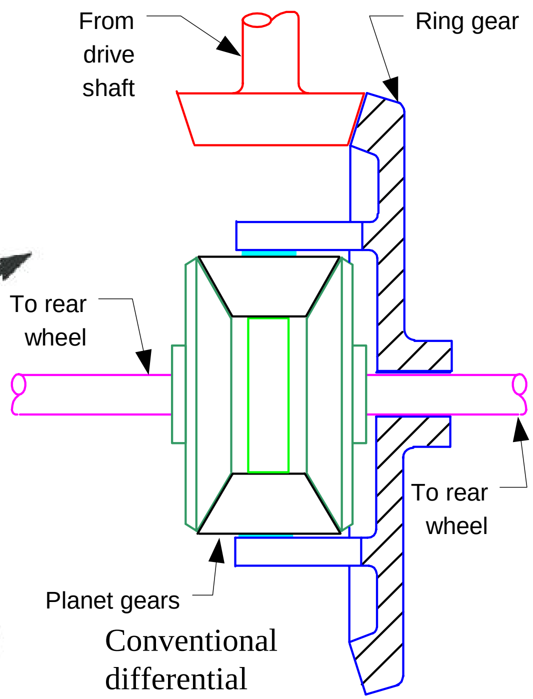
- A differential allows for differential motion between output shafts
  - See page 5-17
- The Torsen™ differential was invented in by Vern Gleasman (US Patent 2,896,541), and uses helical gears and the principle of self-help
  - Helical gears' thrust loads apply forces to friction clutches torque is delivered to the wheel that can use the torque, so wheels never spin as they can with a conventional open differential

http://www.torsen.com/



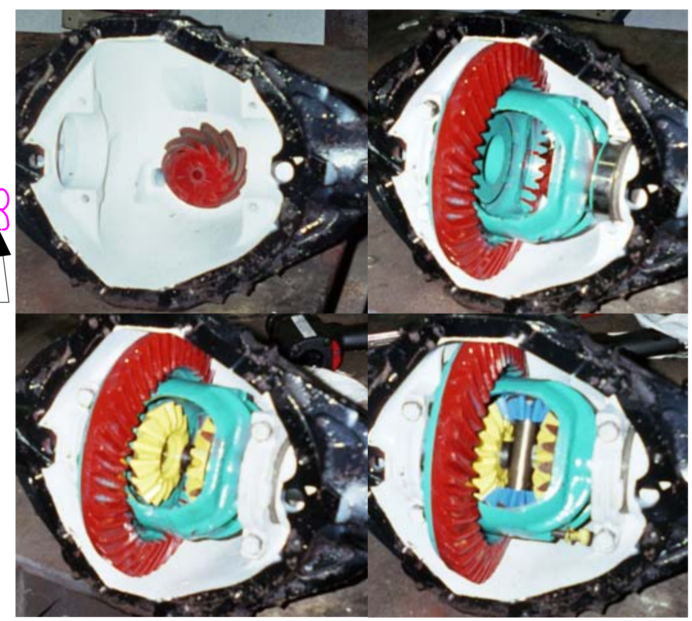
TORQUE SENSING TRACTION

Torsen™ differential (US Patent 4,491,035)



Conventional differential

Assembly of a classic open differential (Thanks Bill Miskoe!)



Gears: *Robot Design Contest Kits*

One of the most fun aspects of robot design contests is that students get to "play" with gears which are one of the most interesting mechanical components on the face of the Earth. The very notion of gears conjures up images of machines and inspires engineers to get mechanical! Even the field of MEMS (Micro Electro mechanical Systems) needs gears to convert high speed low torque power into low speed high torque (for them) power.

Just like in sports, to best sink your *teeth* into the challenge, you have to have the appropriate *gear*! In the case of a robot design contest, this means maintaining a keen awareness of the gears with which you are provided, the functional requirements of the tasks, and whatever capabilities you may have for making gears. In addition, what design tools do you have available? In the latter case, read all of this chapter to learn how to do basic gear design calculations, because if you strip your gear teeth, it can be very embarrassing!

So the first task is to examine all the different gears that have been made available to you. Play with them by examining how they mesh. How accurately do the teeth need to be aligned? How critical is the center distance between them? What sort of tolerances will be required to hold when manufacturing your machine? Can these be achieved on a drill press or do you need to use a milling machine? What does the torque speed curve of your motor look like, and how might you attach gears to its output shaft?

Next carefully think of the tasks required. Specifically, what motion (circular, linear?), speed, and torque (force) are to be accommodated? This enables you to determine the power required and to make sure that your electric motor is powerful enough to begin with. Each stage in a gear train is typically 90+% efficient, although if sliding contact bearings are used to support the shafts, efficiency might drop 5-10% or more per supported shaft.

You now have all the information you need to determine if you can create a gear train to meet the functional requirements of the required task in a straightforward manner[1].

1. More on doing the detailed engineering calculations on the following pages, but at this point you already have all the knowledge you need in order to determine feasibility!

But what if the gears you require are not in the kit of parts? What if you need a transmission ratio of 6:1 and you have only 24 and 12 tooth gears? If you have lots of space, would you use a single 12:1 reduction stage? Furthermore, do you need full 360 degree rotation, or is rotation over an arc sufficient? What minimizes cost (including time to design and manufacture) while achieving the desired performance? In many cases, it would be better to have a single 6:1 reduction, for example, a large slewing ring[2] on a crane or excavator that allows it to swing around.

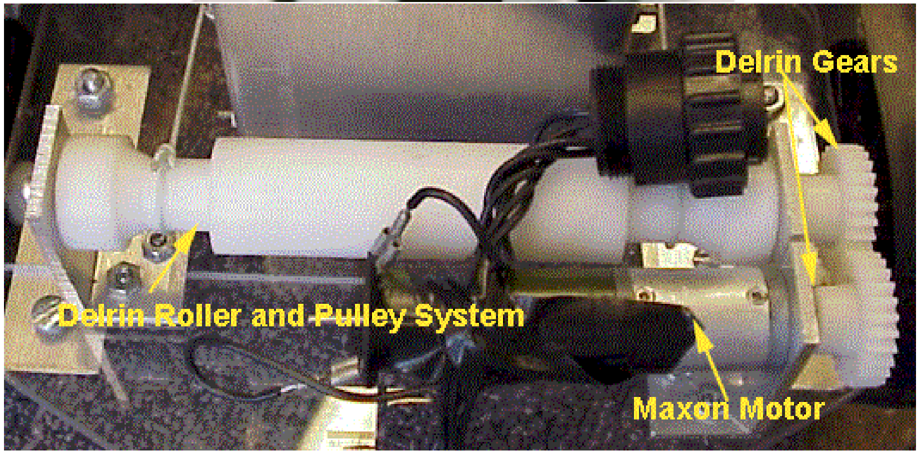
You may have been given a 25 mm diameter gear, but where can you get a 150 mm diameter gear (or gear segment?). If your shop has a laser or abrasive waterjet machining center, you could design and manufacture your own custom gear or gear segment! If it does not, making the gear would be very difficult. Are their other alternatives? Think of the spreading gripper shown on page 6-10. Can you use the principle of a toothed form temporarily elastically deforming a surface to form virtual gear teeth? You may also recall drawings of early mechanisms where instead of gear teeth, round pegs were used. These devices did not last long because they did not have the involute shape which makes for primarily rolling contact, and hence round-peg-teeth are too crude even for a short term robot design contest. Yet there are many other alternatives that are often available. Are you given a plastic gear rack or toothed belt that can be bent over an arc and form a good-enough large gear? Will the implementation last long enough?

Many machine elements books can teach you about the details of gear engineering, and this text will also introduce the fundamentals. However, perhaps most importantly, it is important that you do not assume that just because gears have been around for so long that you are restricted to the use of gears you find at your immediate disposal! You can be every bit as creative with gears as you can with anything else in your kit of parts!

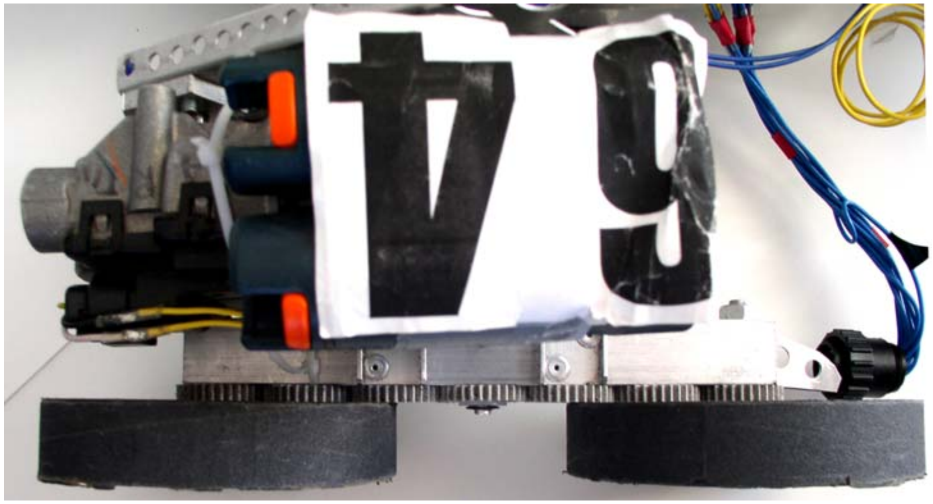
2. A *slewing ring* is a bearing with bolt holes in the inner and gear teeth integral with the inner or outer race that is driven by a pinion attached to the drive motor. Ayr Muir-Harmony cut a large diameter gear into the base of his robot and made his own bearing and slewing ring!

Gears: *Robot Design Contest Kits*

- There are usually a large number of gears available for a design contest
- Spur gears are the most commonly used gears, & they have straight *involute* teeth
  - Justify your designs with basic engineering calculations
    - Show the system will achieve the desired speed and torque requirements
    - Determine the stresses in the gear teeth
    - Students who strip gear teeth should not be given replacement gears until they fix their calculations and adjust the design accordingly!
  - In addition to spur gears, bevel gears may also be available



Martin Jonikas' machine, winner of 2002 *The MIT and the Pendulum*



Gears: *Spur and Straight Bevel Gears*

Spur gears have straight teeth with an involute profile, and they are easy to design and manufacture (see page 6-29) even for use in robot design contests. Spur gears have the following interesting properties:

- The center distance between the gears is chosen so the gears' pitch diameters are tangent. If the gears are too close together, the tip of one tooth can jam into the root of the other and cause failure. If the centers are too far apart, then the tips of the teeth may be loaded too heavily[1].
- The teeth first come into contact below the pitch circle, and their line of contact follows a trajectory that lies along the *line of action*. The *approach phase* is where the pinion tooth first makes contact with the gear tooth. The *contact line* (across the width of the teeth) between the teeth rolls and slides up the tooth until it reaches the point of tangency between the pitch diameters which is called the *pitch point*. The contact is purely rolling only at the pitch point. The *recess phase* is where the line of contact leaves the pitch point and continues to roll and slide up the gear teeth. In some designs, more than one pair of teeth are in contact, but for conservative stress calculations assume only one pair of teeth are in contact.
- The involute tooth profile has the property that the effective gear ratio between the two gears is a constant, even if the center distance varies. However, this profile also generates significant separation forces, so supporting shafts must have sufficient rigidity to prevent deflections that could cause the teeth to separate shear the tips.

Helical gears also use involute tooth profiles, but the teeth are formed along a helix on an imaginary cylinder through the pitch diameter to form an *involute helicoid*. The contact between spur gears is always a line, but in a helical gear, the contact starts as a point and becomes a line; thus engagement of the teeth is more gradual and more teeth are engaged simultaneously. This enables helical gears to run more quietly and at higher speeds while carrying greater loads. Their shafts can also intersect at an angle equal to the sum of their tooth helix angles. However, they generate axial thrust loads equal to the product of the radial load and the sine of the helix angle.[2]

Straight-tooth bevel gears are essentially cone-shaped spur gears that transmit torque between intersecting axes. S*pirol bevel gears* allow for higher torques and speeds with less noise and wear. Bevel gear velocity (gear) ratio is a function of the number of teeth on each gear, and the pitch (cone) angles are a function of the shafts' axes intersection angle θ and the number of teeth:

$$\tan\gamma_2 = \frac{\sin\theta}{r_3/r_2+\cos\theta} = \frac{\sin\theta}{N_3/N_2+\cos\theta}$$

$$\tan\gamma_3 = \frac{\sin\theta}{r_2/r_3+\cos\theta} = \frac{\sin\theta}{N_3/N_2+\cos\theta} \qquad \text{and} \quad \theta=\gamma_2+\gamma_3$$

Bevel gears are designed using *Tredgold's* approximation that determines equivalent spur gears' pitch radii and number of teeth. The number of equivalent teeth is based on the circular pitch of the bevel gears at the large end of the teeth, and it is typically not an integer number:

$$r_{e2}=\frac{r_2}{\cos\gamma_2} \qquad r_{e3}=\frac{r_3}{\cos\gamma_3} \qquad N_e=\frac{2\pi r_e}{p}$$

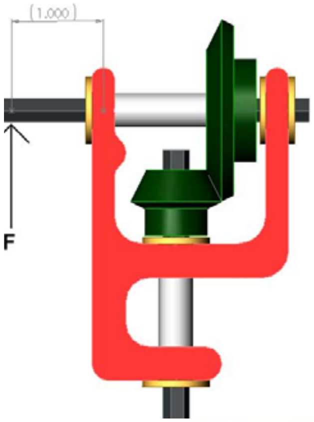
Internal gears have concave shaped spur or helical teeth on the inside of a cylinder. Internal gears are generally more efficient since the sliding velocity along the tooth profile is lower than for an equivalent external set. An internal gear also operates at a closer center distance with its mating pinion than an external gear of the same size. The internal gear rotates in the same direction as the pinion, and it forms its own guard to help prevent objects (fingers and clothing!) from being pulled into the mesh.

Where do you want to transfer power between parallel shafts? Would you be better off using bevel gears to locate the power source perpendicular to the output? How will you determine if the gears are strong enough? Play with the spreadsheets *SpurGears.xls* and *BevelGears.xls*

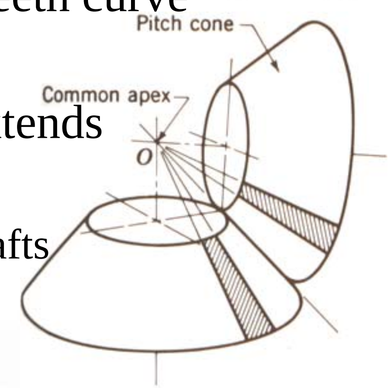
1. For some pitch and pressure angles, the teeth may be cut below the base circle to avoid interference between the tip of one gear's tooth and the base of the mating tooth. The teeth only have an involute shape at a radius larger than the base circle. Too big teeth on too small a gear requires too much undercut which weakens the teeth.

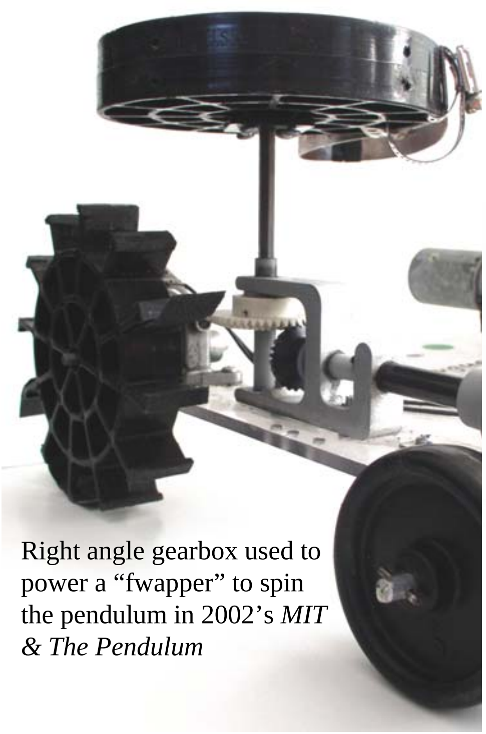
2. See for example http://www.roymech.co.uk/Useful_Tables/Drive/Hellical_Gears.html

Gears: *Spur and Straight Bevel Gears*

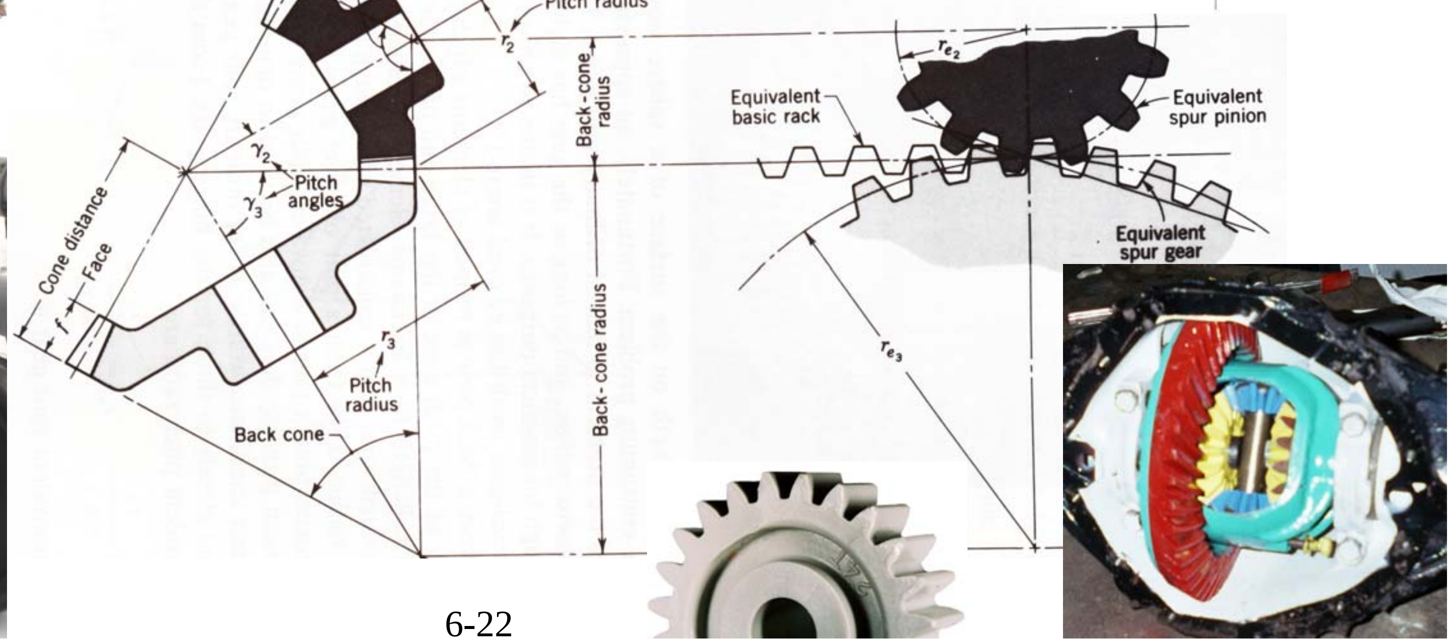


- Spur gears have an involute cross section that extends linearly along the gear's axial direction
  - They are the most common type of gear
- Helical gears also have an involute cross section, but the teeth curve around on a helical trajectory
- Straight bevel gears have an involute cross section that extends linearly on the surface of a cone towards the apex
  - They can be used to transmit torque between intersecting shafts





Right angle gearbox used to power a "fwapper" to spin the pendulum in 2002's *MIT & The Pendulum*



Gears: ***Rack & Pinion***

Imagine a pinion that is driving a gear of infinite radius, and you have a pinion driving a linear gear, which is called a *rack*. The involute degenerates into a shape with planar contact surfaces that are inclined at an angle equal to the pressure angle. Even though the rack's teeth are straight, the system is dimensioned with respect to the pitch line and, in general, the strength formulas for spur gears apply to racks. For racks, the transmission effect is:

$$F_{rack} = \frac{\Gamma_{motor}}{R_{pinion\ pitch}}$$

$$V_{linear} = \omega_{motor} R_{pinion\ pitch}$$

The rack will mate with any pinion that has the same pressure angle and the same diametrical pitch (or module if using metric gears). However, there will be no mechanical advantage achieved, and the force exerted on the shaft will simply be equal to the torque on the pinion divided by the pinion's pitch radius. And as with spur gears, there will also be a sinusoidally varying force component normal to the direction of motion that will push the pinion away from the rack unless a sufficiently rigid system is designed:

$$F_{separation} = \frac{\Gamma_{motor} \tan \alpha_{pressure\ angle}}{R_{pinion\ pitch}}$$

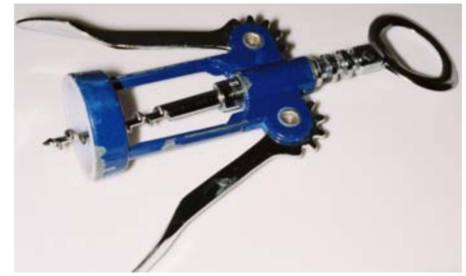
Earlier in this chapter leadscrews were shown to be effective means of converting rotary torque into linear force; however, they required support bearings and were subject to shaft whip, as discussed on page 6-6. Because racks are so simple to implement, they are often used instead of leadscrews, particularly for long strokes. In addition, they are easily designed into a system that must extend a member. If large axial forces are required, the pinion cannot be made arbitrarily small, because then the teeth will break off too easily; therefore a gearbox may also used with a rack and pinion.

The *pitch P* of a rack is defined so it will mate with a pinion. It does not mean the number of teeth per inch of rack. The actual number of teeth per inch is π/P and is equal to the circular pitch of the pinion. All the other rack dimensions are computed using the formulas on Page 6-12. The stresses in the rack teeth can then be computed based on the teeth shearing at the pitch line, and on bending at the root. For example, consider the use of a molded plastic nylon rack with 24 pitch, and from *RackPinion.xls*:

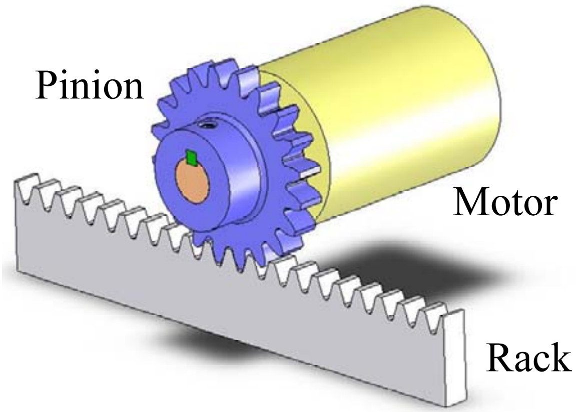
Inputs		
Pitch, P	**24**	
Pressure angle, alpha (degrees, rad)	**14.5**	0.253
Safety factor, sf	**2**	
Number of teeth on pinion, N	**24**	
Tooth material	Nylon	
Allowable bending stress, sb (psi)	**6000**	
Tooth width, wr (in)	**0.250**	
Tooth geometry		
Circular pitch (in)	**0.1309**	
Tooth height (root to tip), hr (in)	**0.104**	
Addendum, ar (in)	**0.042**	
Dedendum, br (in)	**0.052**	
Clearance, cr (in)	**0.010**	
Tooth thickness, tr (in)	**0.065**	
Tooth thickness at root, trr (in)	**0.097**	
Shear area, Arear (in^2)	**0.016**	
I/C at root, Iocr (in^3)	**3.9E-04**	
Distance pitch line to root, hl (in)	**0.062**	
Rack		
Maximum tangential force to shear failure, Fsr (lbs)	**49.1**	
Maximum tangential force to bending failure, Fb (lbs)	**37.5**	
Maximum allowable tangential (rack) force, Fmax (lbs)	**18.7**	
Resulting force along line-of-action, FLA (lbs)	**19.4**	
Resulting spreading force, Fspread (lbs)	**4.8**	
Pinion		
Pitch diameter, PD (inches)	**1.000**	
Torque to create axial force, Tmin (in-lbs, N-mm)	**9.7**	**1094**

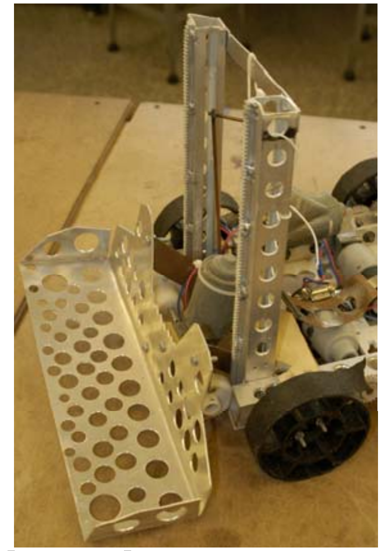
What about the pinion? What diameter should it be? What motor torque is required to drive the pinion? How much force is actually needed from the rack anyway? The interesting thing to note is how large a force this small nylon rack can actually be used to generate! Would you ever need this much force in your machine?

Gears: *Rack & Pinion*



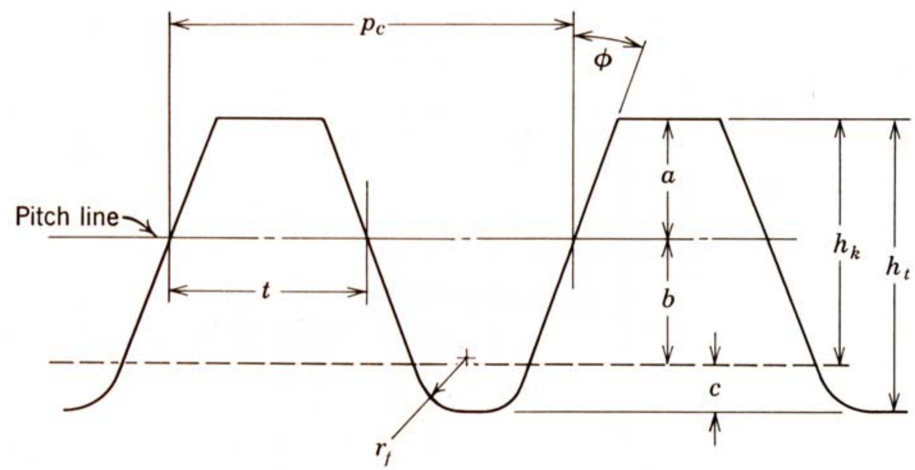
- A rack and pinion is one of the least expensive methods of converting rotary motion to linear motion (what about reciprocity!)





  – It does not provide a mechanical advantage like a leadscrew

- Tooth forces acting at the pressure angle push the pinion away from the rack



Gears: ***Worm***

Recall the earlier discussions on leadscrews and you will have a good idea of how a worm gear transmission works. Instead of a nut that advances along the helix of the screw, a worm's helix is used to advance the teeth of a gear (or worm wheel). The result is that the transmission ratio is given by:

$$TR = \frac{D_{gear\ pitch\ diameter}}{2\ell_{worm\ lead}}$$

A worm gear transmission's efficiency, like that of a sliding contact thread leadscrew, is limited by sliding contact friction. Whereas involute tooth gears can have 90%+ efficiencies, a typical worm gear may only have 30-50% efficiency. However, there can be a great advantage to this, because a worm gear is NOT backdrivable, so it does not need a brake or any external power to hold a load.

Furthermore, just like with ballscrews, recirculating balls can be used to reduce friction between the worm helix and the worm gear. However, unlike a ballscrew, the engagement of the worm with the worm gear is limited to only a few teeth, and thus its load capacity will be more limited. For this reason, more focus has been placed on development of epicyclic drives. But what if a company wanted to develop a recirculating ball worm gear transmission? What would you do? One would search an engineering library (e.g., journals), the web, and US and foreign patent websites to find out what is available and what has been thought of before. In each reference found, there will also be other references cited which must also be investigated.

Although you may not always be given worm gears in a kit of robot contest parts, you can make your own worm in the form of a conventional lead-screw whose linear pitch is equal to the circular pitch of the gear which it will drive. However, given the tremendous gear ratios that can be achieved, and the known tremendous forces that leadscrews can generate, you should be con-cerned about shearing the teeth on the gear! Why do you not have to worry about shearing the thread on the worm? Of course, the spreadsheet *WormGear.xls* can help you design a worm gear transmission. Note that it even calculates the loss in efficiency caused by the thrust bearings. Should you add terms to determine the radial deflection of the worm caused by spreading forces? A small portion of the spreadsheet is shown below:

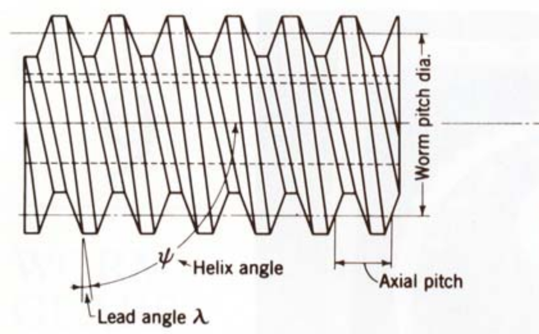
Primary System Inputs		
Desired output torque, To (in-lb, N-mm)	**5**	**565**
Desired output speed, wout (rpm, rad/sec)	**100**	**10.47**
Gear pitch, P	**24**	
Number of teeth on gear, Np	**48**	
Gear yeild stress, sigp (psi)	**6000**	
Calculated Primary System Parameters		
Gear pitch diameter, Dp (inches)	**2.000**	
Required motor torque (in-lb, N-mm))	**0.4**	**48**
Required input motor speed, ww (rpm)	**4800**	
Transmission ratio, TR	**48.00**	
Output power (Watts)	**6**	
Required input power (Watts)	**24**	
Maximum tooth bending stress (psi)	**5811**	

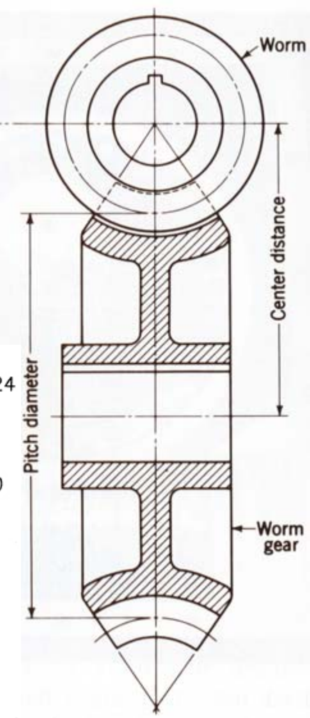
There are many other required detailed inputs and resulting outputs in the spreadsheet. This spreadsheet was set up so the desired output torque and speed are specified, as well as the gear size. Why the gear size and not the motor parameters? If we input the gear size, we are specifying a parameter that will be a primary driver for the overall physical size of the gearbox. The motor size can then be determined. Or we could have specified the motor, and then checked to see if we have the proper gear, etc. Design is often a merry-go-round of parameters, and you can get on or off at many points. However, in order to maximize your fun, you should not risk getting on or off while it is moving at high speed!

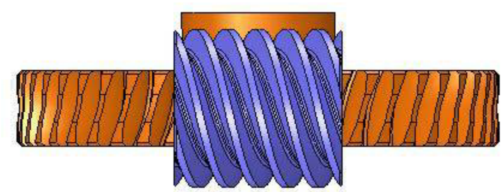
Do you have a need for a large transmission ratio in your machine? Can you design a worm gear transmission using the materials in your kit? Try the spreadsheet, it is easy to use and will enable you to rapidly assess the feasi-bility of your concept.

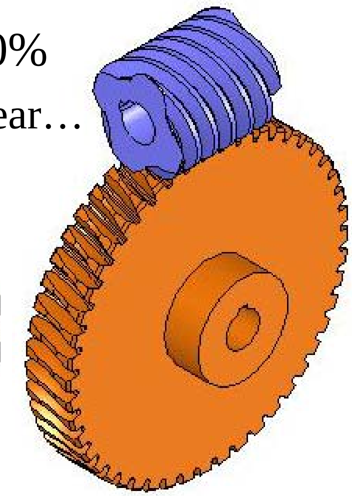
Gears: *Worm*

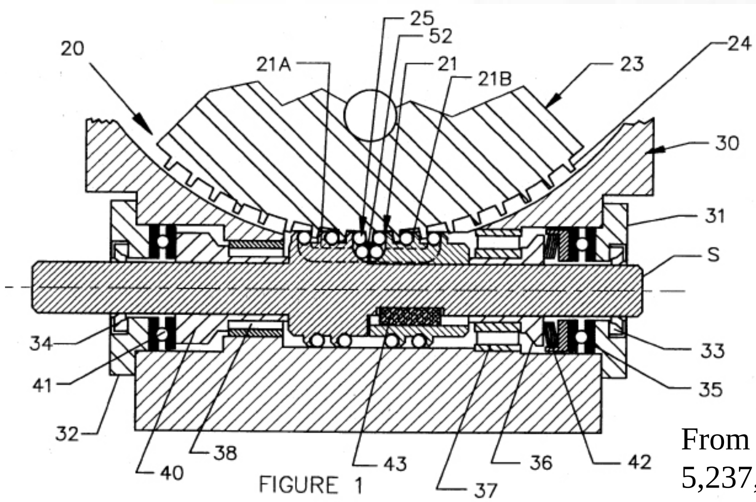
- The transmission ratio is a function of the worm pitch and the worm gear pitch diameter
  - As the worm rotates, its thread pushes the teeth on the worm gear (wheel, or driven gear)
  - Given the lead ℓ_{worm} of the worm and the diameter D of the driven gear, the transmission ratio of a single worm gear set is just $TR = \pi D_{pitch} / \ell_{worm}$
- The contact between the teeth is sliding, so the efficiency may only be 30-50%
  - You can create a worm using a leadscrew and it can contact the teeth of a spur gear… *(this is called blacksmithing!)*



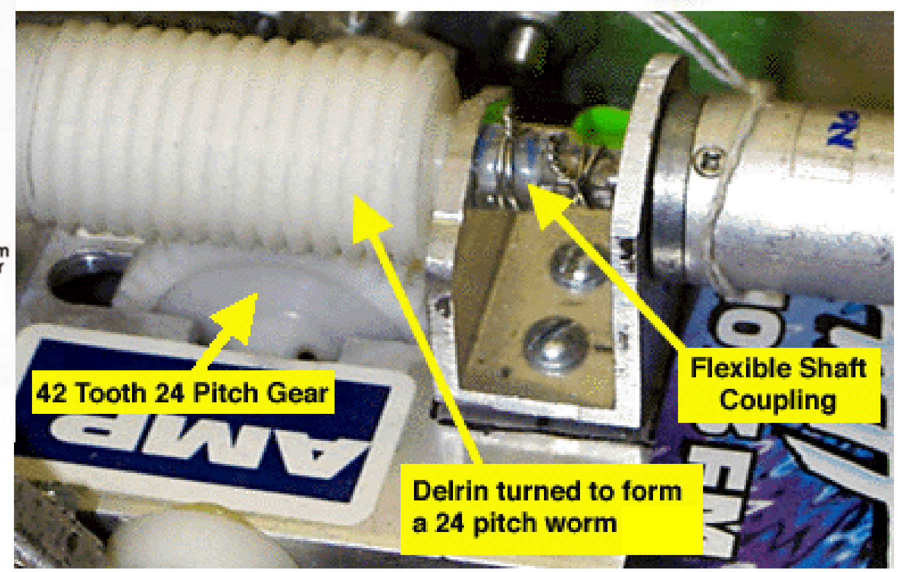








From US Patent 5,237,886



Gears: *Selection of Parameters*

The design spreadsheet presented for rack and pinions gave a hint at how a design engineer can initially determine the loads that a gear train can support. This can be done without ever doing any sort of drawing. The spreadsheet on the opposite page for the design of spur gears, *spurgears.xls*, shows similar inputs and outputs. But where should the design engineer start? Fortunately, once the pitch and pressure angle are selected according to general guidelines, the vast majority of other parameters are automatically defined as shown in the table on page 6-13. You only have to consider a few parameters:

- *Pitch:* The larger the pitch (finer), the smaller the teeth, and hence the less relative sliding motion and the more efficient the gear train. However, smaller teeth are more likely to break, and thus to maximize load capacity, as small (coarse) a pitch as possible should be selected.
- *Pressure Angle: Standard pressure angles include* 14.5, 20, and 25 degrees. The smaller the pressure angle, the smaller the separation force, and hence the smaller the deflections and errors in the system. However, the larger the pressure angle, the wider the base of the tooth, and the more force it can withstand. Hence to maximize load capacity, as large a pressure angle as possible should be selected.
- *Tooth Width:* It seems obvious that the wider the tooth the great the load it can support; however, due to misalignment issues, the edges of the teeth may contact first. As a result, gear teeth cannot be made arbitrarily wide.
- *Gear Material:* Hard steel provides the strongest material for gear teeth, but for wear purposes, it is good to have dissimilar materials. The pinion is usually made from the harder (and stronger) material, while the gear is made from a softer material because its teeth experience fewer cycles.
- Safety Factor: There are a number of methods for determining gear-tooth stress analytically, although with today's fast and easy finite element programs, one would never go into production without checking the stresses using FEA. Even when "exactly" determining the stresses analytically by one of the known methods, there is still the uncertainty associated with knowing the actual loads. Impacts are perhaps the biggest factor, and so if we assume we know the loads and the allowable bending stress, then we can calculate the shear stress at the pitch diameter, and the bending stress at the root, and make sure that the product of the safety factor and the shear or bending stress never exceeds the maximum allowable shear ($\sigma_{yield}/2$) or bending (σ_{yield}) stress for the material. The safety factor can also include the effects of stress concentration at the tooth root. Note that the teeth usually fail in bending.

For example, the spreadsheet *bevelgears.xls* can be used to determine the maximum torque that can safely be transmitted without shear or bending failure in the teeth. The teeth are not uniform in cross section, but as noted before, the *Tredgold spur gear tooth shape approximation method* can be used to create an equivalent spur gear, and then the stress calculations can be done as for spur gears. The spreadsheet also calculates the stress using an AGMA geometry factor. The former is most often more conservative:

Inputs	
Number of Teeth, subject gear, N	**24**
Number of Teeth, mating gear, Nm	**48**
Pitch diameter of subject gear, D (inches)	**1**
Face Width, subject gear, F (inches)	**0.25**
Torque, subject gear, T (in-lbs)	**42**
AGMA Geometry Factor (from chart), J	**0.25**
Stress Outputs	
AGMA Bending Stress (psi)	**32256**
Shear of the tooth (F/A) (psi)	**5780**
Tooth Bending Stress (psi)	**41269**
Tooth Bending Stress/AGMA Stress	**1.2794**

One factor that has not been discussed is the Hertz contact stresses[1] that exist between the teeth which ideally are in line contact (for helical gears, the initial engagement point contact stresses also have to be considered). However, for standard production gears, particularly the plastic gears used in robot design contests, Hertz contact failure is usually not a problem. On the other hand, if you are ever designing a gear train for a production system, Hertz contact stresses can be the primary cause of fatigue and failure, and thus must be carefully evaluated during the design phase!

Are there any custom manufactured gears needed in your machine? Got parameters?

1. See Chapter 9

Gears: *Selection of Parameters*

- Spreadsheet *spur_gears.xls* for conservative estimations of spur gear tooth stress
- Note that the pinion stress is at its limit
  - You will have to think of ways to prevent a single gear's teeth from being stripped!
- For long life in real products, service factors and many other critical geometry checks need to be performed
  - Consult the *Machinery's Handbook*, or a gear design handbook or AGMA standards
  - Proper tooth design involves more careful assessment of the tooth geometry and loads using the *Lewis Form Factor*
  - Improper lubrication is often the greatest cause of gear failure

	Maximum Von Mises stress (MPa)	
Plastic	**Unfilled**	**Glass-filled**
ABS	21	41
Acetal	35	48
Nylon	41	83
Polycarbonate	41	62
Polyester	24	55
Polyurethane	17	

Spur_Gears_Metric.xls		
Spreadsheet to *estimate* gear tooth strength		
Production gears must be designed using the Lewis Form Factor or FEA		
By Alex Slocum 1/18/01, last modified 9/28/2007 by Alex Slocum		
Enters numbers in **BOLD,** Results in **RED**		
Inputs		
Desired torque transmission, T (N-mm, in-lb)	**12**	0.1
Maximum conservative achievable torque transmission (C35 stress ratio = 1) (N-mm, in-lb)	654	5.8
Pressure angle, f (deg, rad)	**20**	0.3491
Module, M	**1**	
Pitch, P	25.4	
Number of teeth on pinion, Np	**24**	
Number of teeth on gear, Ng	**96**	
Center distance tolerance, Ctol (mm)	**0.01**	
Face width, w (mm)	**5**	
Pinion yeild stress, sigp (N/mm^2, psi)	**41**	5944
Gear yield stress, sigg (N/mm^2, psi)	**41**	5944
Stress concentration factor at tooth root, scf	**1.25**	
Outputs		
Gear ratio, mg	4	
Pinion pitch diameter, Dp (mm)	24.00	
Gear pich diameter, Dg (mm)	96.00	
Center distance, C (mm)	60.01	
Tooth thickness at base tt (mm)	1.73	
Addendum, a (mm)	1.00	
Dedendum, b (mm)	1.25	
Clearance, cl (mm)	0.25	
Pinion tooth force (normal to radius), Fp (N)	1.00	
Gear tooth separation force (parallel to radius) Fs (N)	0.36	
Tooth section parameters		
Chordal area, Ac (mm^2)	8.6394	
First Moment, Q (mm^3)	3.73E+00	
2nd moment of area ("moment of inertia") I (mm^4)	2.15E+00	
Pinion tooth stresses (stress ratio must be less than 1)		**σ ratio**
Transverse shear stress of the tooth (F/A) (N/mm^2)	0.14	0.01
Bending induced shear stress (FQ/wI) (N/mm^2)	0.43	0.02
Bending stress (F*b*c/I) (psi)	0.50	0.01

Gears: *Accuracy, Repeatability, & Resolution*[1]

In addition to the function of transmitting and amplifying torque, gears also must often maintain accuracy of position and velocity. The former is required for achieving a desired position. For example, recalling page 3-11, what would happen to the position of a load at the tip of a large crane if the operator could not rely on the large gear (called a *slewing ring*) to orient the body of the crane? What is important here, the accuracy, repeatability, or resolution?[2] How important is it for a gear train to maintain accurate velocity? Accurate velocity may be required for the control of a process, and the velocity should not vary rapidly because this can cause vibrations in the machine. Many machine vibration problems have been traced to $F_{spread}@\omega_{gear}*N_{teeth}$!

Gear accuracy has two primary factors: The first is the accuracy of the teeth themselves in terms of individual tooth shape, their relative spacing (circular pitch), and the pitch diameter (size and roundness). The second is the accuracy of the gears' mounting, and the associated accuracy of the components that support them. Both factors affect how well gears can position a load to a desired position, or maintain constant speed.

Gear repeatability is greatly influenced by the way in which the gears are mounted, and hence depends strongly on the repeatability of the supporting mechanisms and applied loads. However, space between the gear teeth, called *backlash*, also plays a very important role in gear repeatability. These factors affect how repeatably can the gears come to the same position each time.

Gear resolution is also greatly influenced by the way in which the gears are mounted, and by backlash. These factors affect the smallest motion that the gears can transmit. Lubrication and friction are also very important.

Backlash is the amount δ by which the width of the tooth space exceeds the width of a tooth. *Linear backlash* is the normal distance between the teeth along the pitch circle. *Angular backlash* is the linear backlash divided by the gear radius

1. A most excellent book on this topic is G.W. Michalec Precision Gearing Theory and Practice, 1966 John Wiley & Sons, Inc., new York.
2. Now would be a good time to review page 3-18 and related pages (perhaps all of Chapter 3!)

Backlash is the condition where there is clearance between the teeth of mating gears, such that if one gear reverses direction, there can be a period where there is no contact between the gear teeth until the other side of the teeth come in contact. In many cases, backlash is a dominant position or velocity performance limiting factor. It is affected by inaccurate manufacture of the teeth and center distance mounting error. By the principle of reciprocity, some backlash must be present, because whatever parameters lead to backlash, if reversed (opposite sign or direction) they could cause the gear teeth to not be separated but to interfere with each other[3]. Like fire, backlash can harm or help you, and so it must merely be controlled by either allowing some to physically exist, which is acceptable in most situations, or by one of several techniques that eliminate its bad effects while maintaining its ability to prevent tooth interference.

The dominant factor that affects backlash is an error in the center distance between the gears which acts in accordance with the pressure angle and the pitch radius:

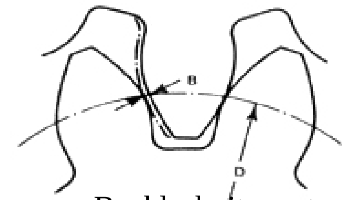
$$\varepsilon_{angular\_backlash} = \frac{\Delta_{centers}\tan(\phi)}{r_{pitch\_radius}}$$

The best way to ensure that the proper amount of backlash is present is through careful manufacturing, as noted in the accompanying pictures, and this typically means precision milling to achieve the proper center distance between bearing bores. Also, the gears must be supported by shafts that are properly supported (review page 3-5). Finally, for precision machines and instruments, anti-backlash gear systems can be obtained. These may use spring loaded split gears for light loads, or two pinions, each of which drives opposite faces of the gear's teeth[4].

How can too much backlash affect your machine? What risks does it raise and what are your countermeasures? Can you adjust the center distance, or will this add too much complexity? Can you use gravity or a spring to always have your mechanism preloaded against one side of the gear teeth?

3. Should this thus then be called *frontsmash*?
4. Hale, L.C., Slocum, A.H., "Design of Anti-Backlash Transmissions for Precision Position Control Systems", Precision Eng., Vol. 16, No. 4, Oct. 1994, pp. 244-258.

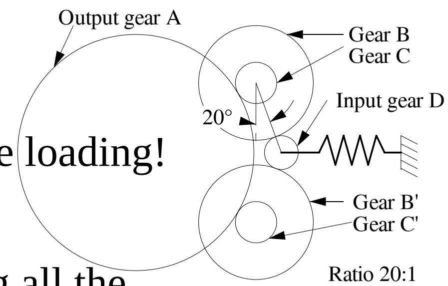
Gears: *Accuracy, Repeatability, & Resolution*



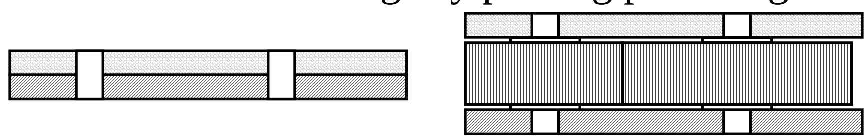
Backlash, it must be managed!

- The shafts and bearings that support them must be carefully spaced and aligned
  - Center distance is half the sum of the pitch diameters + a small amount (0.1 mm):

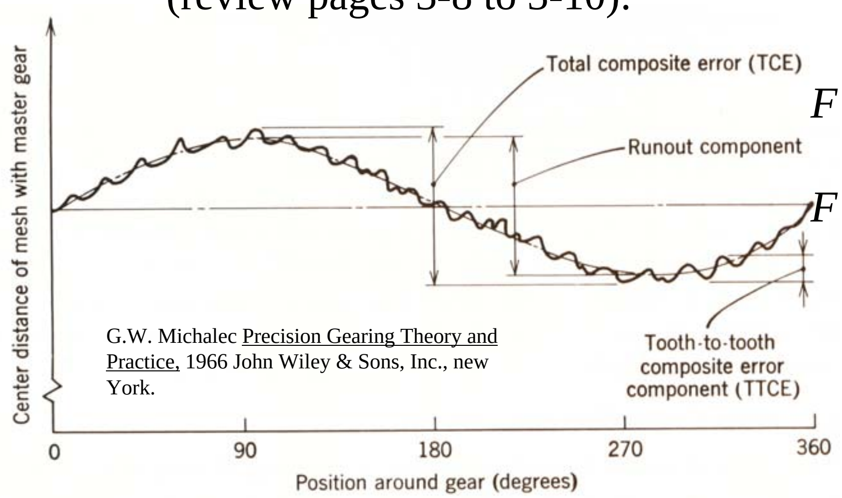
$$L_{dis\tan ce\_between\_shafts} = \frac{\left(D_{output\ pitch\ diameter} + D_{input\ pitch\ diameter}\right)}{2} + \delta$$



  - No wobble!: The axes of rotation must be kept parallel to prevent tooth edge loading!
- Manufacturing is key!
  - Line-bore holes for shafts and bearings by pinning plates together & drilling all the holes at once!



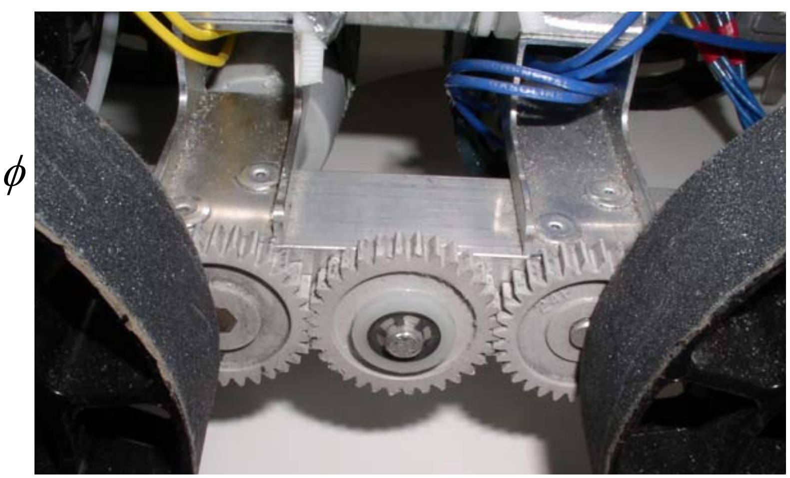
- The bearings and shaft must withstand the speed and loads generated
  - Angular deflection are amplified by distance and can lead to tooth skip and backlash (review pages 3-8 to 3-10):



G.W. Michalec Precision Gearing Theory and Practice, 1966 John Wiley & Sons, Inc., new York.

$$F_{radial} = \frac{2\Gamma}{D_{pitch}}$$

$$F_{spread} = F_{radial} \sin\phi$$



Gears: *CAD Modeling*

There are two aspects to the modeling of gears: graphical and analytical. The former is concerned with representing gears on a drawing to ensure there is space for them in an assembly. The latter is concerned with the design of the gear to ensure it will perform as desired. In terms of useful information, the minimum information that needs to be conveyed includes:

- *Outer diameter:* the outer diameter is typically shown as a dashed line in order to check for clearances between the gear and other machine elements and structures. For bevel gears, show the outer outline of the teeth.
- *Inner diameter:* The inner diameter is typically shown as a solid line. The connection method to the shaft on which the gear is mounted also needs to be shown. Sometimes the inner diameter of a gear acts as the bearing surface on a shaft. This is the case, for example, with pinion gears in a differential. In design contest robots, rotating gears often are also mounted on stationary shafts.
- *Pitch diameter:* The pitch diameter is typically shown as a dashed line, and it is the diameter at which ideal contact is made; hence the gear ratio is the ratio of pitch diameters. For bevel gears, show the average pitch diameter.
- *Root Diameter:* The root diameter is typically shown as a dashed line, and it shows the maximum depth of the tooth. It is helpful to check that the tooth tip of one gear will not impact the root of the other gear, and thereby cause the two gears to jam. *Look at the sample drawing. Are the diameters specified correctly?* Will the tip of one tooth jam against the root of the mating gear? If the outer and pitch diameters are specified correctly, then along with the center distance, you should not have to show the root diameter. Doing so is a good double check! For bevel gears, show the inner outline of the tooth roots.
- *Width:* It should be apparent that the side profile of the gear, including any hub detail, should be shown using solid lines. To indicate teeth, lines are drawn from one side of the gear to the other.
- *Hub detail:* How the gear is attached to the shaft which supports it, or in many cases how it is free to spin on the shaft, must be shown in detail. As described in Chapter 5, keyways are effective and common. Splines are also used, as are tapered hub fittings. When gears are required to spin on a shaft, polished surfaces are often required.

Consider the images shown on the accompanying page. Simple line drawings can use dashed lines to indicate different diameters, or simple shapes selected from a 2D drawing package can use fixed-spacing hatching. Both get across the idea that gears are being used when during the concept phase the goal is to indicate design intent. For early stage design you merely have to show what you want to do and accurately model the space required.

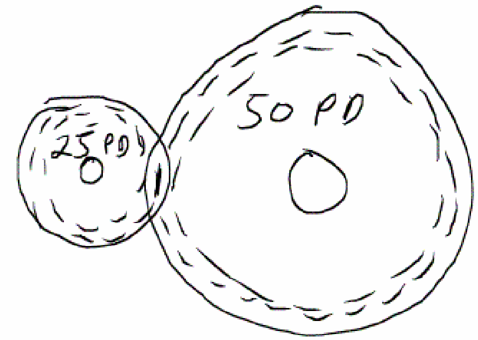
What about solid models? Solid models are especially valuable for the conceptual design phase, because they allow you to block out overall sizes of components and thus enable you to rapidly virtually construct your machine. But should you take the time to model the gear teeth themselves? If you are given a model of a gear that includes the teeth, o the solid model program easily creates gear teeth, go ahead and show the detail. If all you need to do is convey that a gear is being used and to reserve space for its profile, then use a simple model as shown, where the pitch and root diameters are differentiated with the use of slight steps. In fact, given the function of the solid model for creating an engineering representation of a machine, there generally is no reason to show the individual teeth which can greatly slow down the display time. CAD programs create series of points connected by lines or planes and do not accurately model the involute profile as is accomplished mathematically by dedicated gear design software[1]. Hence using a CAD program to show a gear will not necessarily result in a better design.

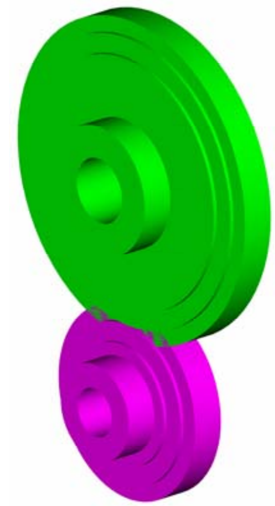
What arrangements of gears are you thinking of? How can you sketch them quickly to show the intent of your design, while minimizing the time required so you can focus your attention on issues that really matter (like perhaps creating a quick spreadsheet to see if the gear teeth are strong enough to withstand the loads you are contemplating!) Make sure you are comfortable with the gear design spreadsheets that are provided. If you strip the teeth on your gears, you will have wasted 10x more time than it would have taken to do the calculations in the first place! (This is where most students make mistakes they later wish they hadn't.).

---

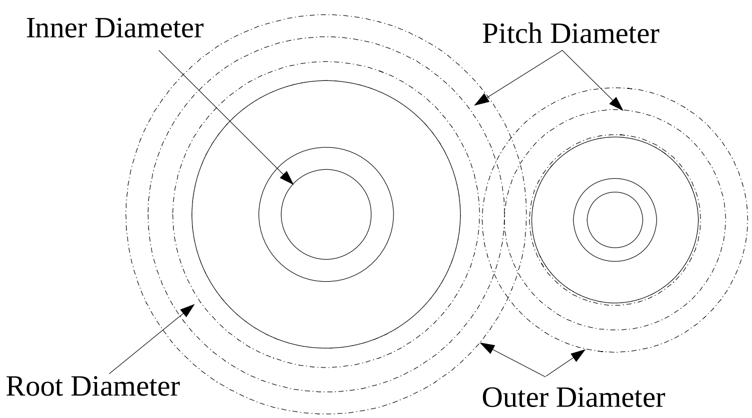
1. For an in-depth discussion, see http://www.geartechnology.com/mag/gt-solid.htm, and for detailed how-to modelling instructions for modeling gears in Pro/ENGINEER, see http://www.geartechnology.com/mag/gt-model.htm. On the other hand, if the gear is designed using a program like GearTrax, which also creates a SolidWorks part file of the gear, the gear can be used in an assembly ad the motion of the mechanism can be animated to check the kinematics of operation.

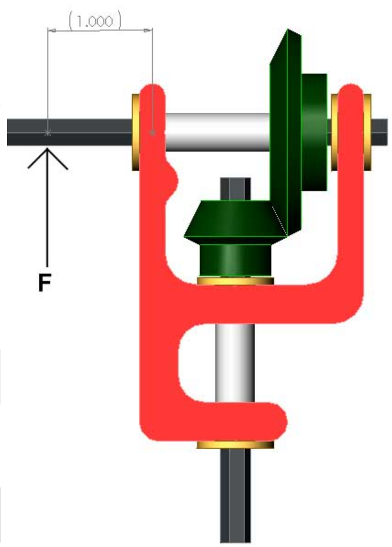
Gears: *CAD Modeling*

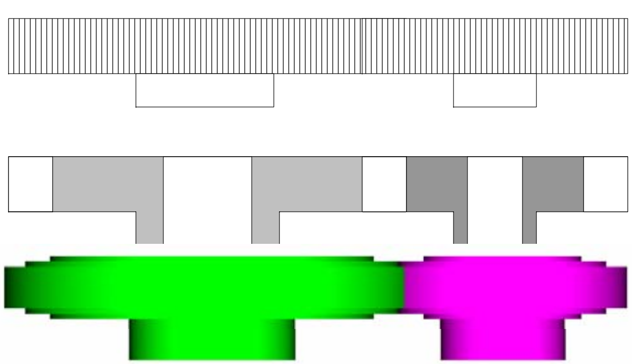


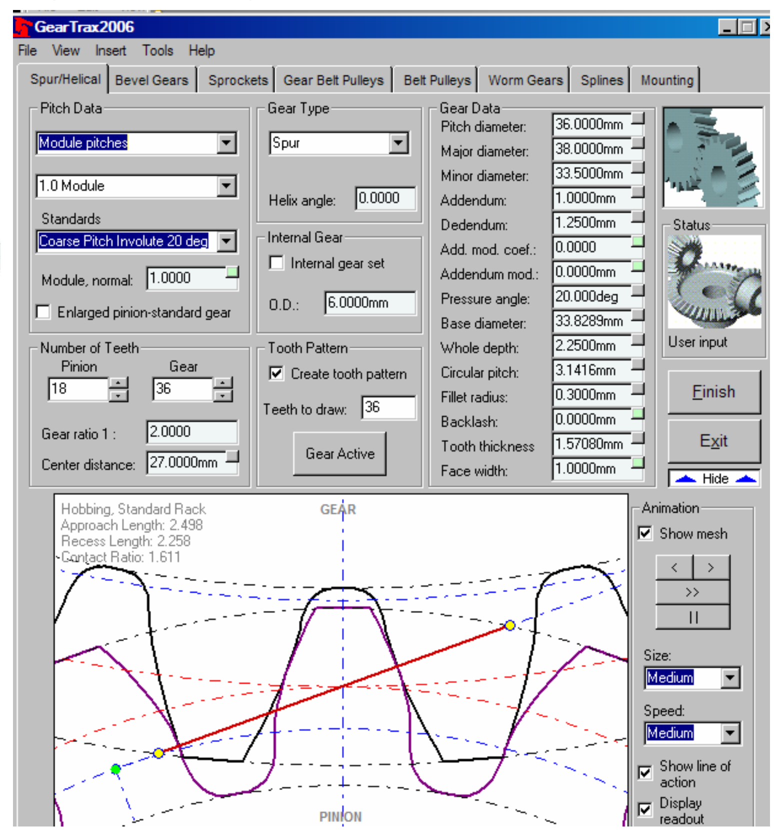


- There are two types of gear models:
  - A geometry placeholder in a drawing shows the gea pitch diameters
    - It can be hand-sketched or shown with a CAD system, and does not include tooth detail
  - An accurate mathematical representation of the gear created by gear design software which allows for the examination of the contact region as the gears rotate









Gears: *Prototyping*

Several spreadsheets have been shown, and these are very useful for initial design feasibility calculations, but what would you do as an engineer if you had to design a custom production gear? For simple spur gears, it is not hard to write the equations that form the involute from the base circle, and then using the other parameter definitions (and equations) on page 6-13, use the relations to drive the generation of features in a solid modeling program.[1] In general when creating your own gears, start with a spreadsheet to determine the pitch (or modulus) of the gear in order to have a tooth capable of transmitting the desired torque. Then use gear deign software, such as GearTraxTM, to create the geometry and a solid model of the gear. Finite element analyses of the gear tooth can then be conducted to check the strength. The solid model can be sent to a rapid prototyping company, or a 2D drawing can be used with a laser or waterjet cutter to cut simple gears.

Ideally you would need to input the exact gear shape into a finite element program to check the stresses and deflections. You would have to do the Hertz contact stress calculations. You would want to evaluate the backlash in the system as a function of center distance variation and manufacturing error. You would want to... Fortunately, there are a number of dedicated gear design software programs[2] available as can be found by searching the internet. The best programs will give predictions for backlash and gear life.

The ability to design and make your own gears means they can be made integral with other parts of the structure. For example, scissor linkages (see page 4- 23) can be made with internal and external gears on the ends of the first links. A pinion can then drive both gears simultaneously to extend the tongs. But what if the teeth break? Is it really wise to make the gear section and the links from one part? Or perhaps use the waterjet to cut mating features on the gears so they can easily be attached to the links? As with so many brilliant designs, manufacturing detail is a critical complement to design detail![3]

1. For specific instructions for modeling spur and helical gears using Pro/ENGINEER, see http://www.geartechnology.com/mag/gt-model.htm
2. See for example http://www.hexagon.de/zar1_e.htm and http://www.camnetics.com
3. Remember MISS and KISS from page 4-2?!

For rapid manufacture of prototype gears, the OMAX Abrasive Waterjet Machining Center™ comes with gear modelling and manufacturing software. Spur gears are two-dimensional objects, and thus they are easily manufactured from metal or plastic using a high pressure abrasive waterjet.[4] An abrasive waterjet[5] uses a high pressure stream of water, typically about 3000 atm., which is focussed into a fine stream through a jewel orifice. The stream of water enters a tungsten carbide tube at supersonic velocity where an abrasive, such as fine grit garnet, flows into it and effectively encases the stream of water. The stream then exits the tube, with a diameter that is typically 1/2 mm, and will cut through just about anything. In order to stop the jet after it cuts through the material, a tank of water is located beneath the cutting zone. The part is supported by curved thin tall stainless steel slats.

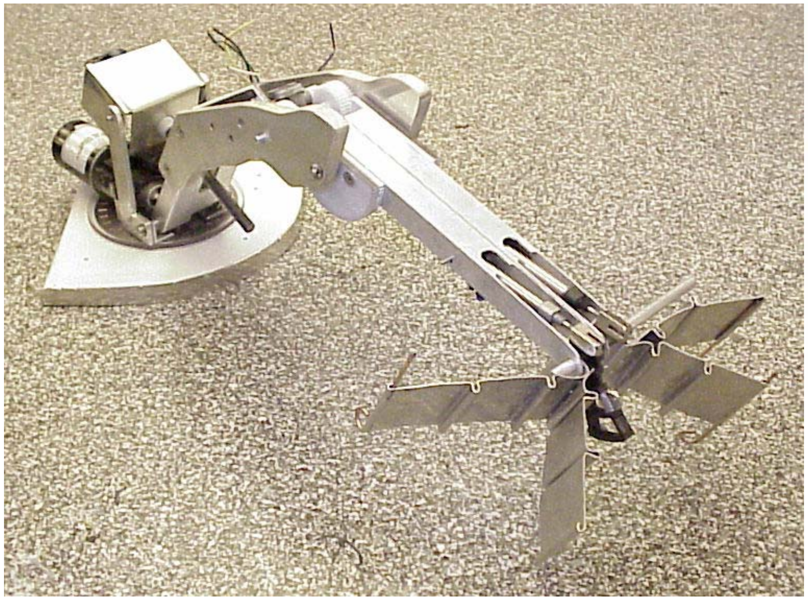
When you create a solid model of your machine, particularly when it uses sheet metal bent up to form the chassis etc., you can easily cut the entire two dimensional form using the abrasive waterjet. Simply take a .dxf file of the two dimensional part to be made to the waterjet, and after a few minutes, you will be ready to cut (after receiving the appropriate training of course). You do not have to have drawn the gears in your solid model; rather just know the pitch, number of teeth, and pressure angle, and the OMAX control software will do the rest. Do not forget to add appropriate details such as center holes or keyways. However, if the gear is to rotate on a shaft, the surface finish of a waterjet-cut hole will be too rough, and thus a smaller pilot hole should be cut using the waterjet. The finished hole can then be drilled or bored.

Have you any custom gears you need to manufacture? Use the spreadsheets to design the gears, and get the design done early, because there may be far more people who want to use the Abrasive Waterjet Machining Center than there is machine time available.

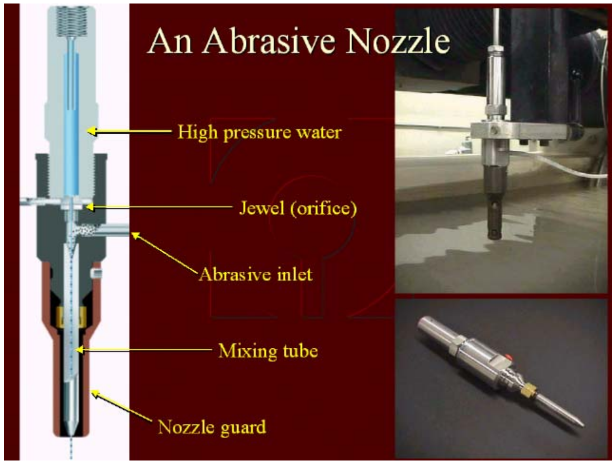
4. Lasers can also be used, particularly for cutting plastic gears. See what your shop has to offer!
5. The website www.waterjets.org has a great variety of information about using waterjets. www.omax.com shows Abrasive Waterjet Machining Centers™ that were designed by Dr. John Olson and Prof. Alex Slocum after they met at an alumni lunch. These machines were designed using the principles described in this book!

Gears: *Prototyping*

- Lasers and waterjets can be used to manufacture prototype gears for low speed and low cycle applications
  - Lasers are great for cutting plastics prototype gears
  - Waterjets are great for cutting metal or plastic prototype gears
- All that is needed is to specify pitch, pressure angle, and pitch diameter
    - You must have previously calculated the proper design parameters to make sure the gears do not fail in bending or shear
  - Your solid model in your assembly can show the gears without teeth, just model them using the pitch diameter
    - Ayr Muir-Harmony designed and built his own large diameter needle bearings, leadscrew, and planetary gear system for his robot's turntable
  - Some gear design programs (e.g., GearTrax™) will generate solid models with the teeth



www.omax.com



Gears: *Prototyping by Waterjet*[1]

Many rapid prototyping machines come with software to help the user rapidly create complex but common parts such as gears. OMAX Corp. Abrasive Waterjet Machining Centers™ have user friendly software for laying out and cutting spur gears and racks. Once the material is located on the waterjet and held down with weights or clamps, the steps are quite simple:

Step 1: Define the Parameters: Number of teeth, pitch, and pressure angle. The software will display a solid gear. If the gear is part of a structure, parts of the gear can be deleted and other lines drawn to form the rest of the structure. if the gears are small, breakoff tabs may be added to hold them to the surrounding material so they do not fall to the bottom of the tank where they will be lost for a very long time.

Step 2: Add a center pilot hole, to be precisely drilled out later so the gear can receive a bearing or be mounted on a shaft; and add lead-in/out lines for the jet to start and end its cut. A keyway can also be added.

Step 3: Define the path quality for cutting. The waterjet can cut fast and leave rough surfaces, or it can cut slowly and form precise surfaces. In general, it is best to use a quality setting of 4 or 5 for gears.

Step 4: “Order” the tool path (make a .ord file) to tell the waterjet where to start and stop its cuts, and on which side of the line to cut.

Step 5: Cut the gears and then enjoy using them!

Racks are made in a very similar fashion:

Step 1: Define the Parameters: Number of teeth, pitch, and pressure angle. The software will display the tooth form, but just as a continuous line.

Step 2: Add lines for the rest of the rack and lead-in/out lines

Step 3: Define the path quality for cutting. Use a quality setting of 4 or 5 for gears.

Step 4: “Order” the tool path (make a .ord file) to tell the waterjet where to start and stop its cuts, and on which side of the line to cut.

Step 5: Cut the gears and then enjoy using them!

The cutting process takes place under water, which helps reduce noise and splatter. However, plastics and light metals are buoyant in water, and the turbulence induced by the jet as it enters the water can thus act to displace lightweight pieces with large surface areas into the cutting stream. In addition, small parts once cut can fall through the support slats, and once they disappear into the bottom of the tank, they are essentially gone. For larger pieces, weights can be placed on top of the materials. For smaller pieces, support tabs that are later manually cut away, can be incorporated into the design, or a mesh-type grating is also available.

Using the waterjet is easy, but like any machine tool, safety is of paramount important. Always wear safety glasses when the machine is running, and keep your hands away from the nozzle when cutting!

1. This section on making gears using an Abrasive Waterjet Machining Center™ was created with the generous help of Carl Olson from Omax, Inc. (www.omax.com).

Topic 6 Study Questions

Which suggested answers are correct (there may be more than one, or none)? Can you suggest additional and/or better answers?

1. Conservation of energy is the same thing as the conservation of work times efficiency:
   True
   False
2. For a leadscrew (or linkage), the product of efficiency, input torque, and the angle (radians) through which the torque is applied equals the product of force out and the distance over which the force acts:
   True
   False
3. Backlash (lost motion) is caused by clearance between the threads of a screw and a nut
   True
   False
4. Sliding contact thread leadscrews will always have some backlash:
   True
   False
5. Sliding contact thread leadscrews can be preloaded to reduce backlash using a split-nut with a circumferential clamp as a preload:
   True
   False
6. Sliding contact thread's low efficiency, on the order of 30%, means that they are usually backdrivable:
   True
   False
7. The force output from the nut of a leadscrew equals the product of efficiency, input torque, 2π, and the lead of the screw:
   True
   False
8. Sliding contact thread's low efficiency, on the order of 30%, means that they are usually backdrivable:
   True
   False
9. A leadscrew must at least be supported by bearings at one end that can withstand radial and thrust loads:
   True
   False
10. If a leadscrew is supported at both ends by bearings, it can generally spin faster than if supported at only one end:
    True
    False
11. If a leadscrew turns, the nut must also be allowed to freely turn in order to efficiently generate axial force:
    True
    False
12. The nut on a leadscrew must be accurately mounted or radial compliance is needed to ensure that the system does not bind as the nut nears the end of travel:
    True
    False
13. A large part of the inefficiency of a leadscrew comes from the thrust bearing:
    True
    False
14. A leadscrew will not buckle if it is properly threaded:
    True
    False
15. For precision machines, ballscrews should always be used instead of sliding contact thread leadscrews:
    True
    False
16. All ballscrews are inherently preloaded:
    True
    False
17. Face-to-face preloaded ballscrew nuts have greater misalignment capability, but are more sensitive to thermal expansion causing preload changes:

True
False

18. Back-to-Back preloaded ballscrew nuts have greater misalignment capability, but are more sensitive to thermal expansion causing preload changes:
    True
    False
19. Primary causes of leadscrew and ballscrew life-reduction include:
    Misalignment between the screw shaft and the carriage support bearings
    Misalignment between the nut face and the carriage surface onto which it is attached
    Radial loads and moments applied to the nut
    Dirt and chips
    Improper lubrication
    Overloading
20. Shock loading can cause of damage to ballscrews:
    True
    False
21. Supporting a shaft by clamping its ends, or mounting them in a quad-set of back-to-back mounted ballscrew support bearings, can increase the first natural frequency by a factor of about 6:
    True
    False
22. When calculating the natural frequency of a leadscrew (or ballscrew) shaft, to be conservative, use the root diameter for determining the cross sectional area, and the outer diameter for determining the moment of inertia:
    True
    False
23. The high rotation rates of ballscrews can create heat, which can become a dominant source of error:
    True
    False
24. Pre-stretching a ballscrew can increase its natural frequency significantly:
    True
    False
25. When a ballscrew heats up, it expands and the lead, distance traveled by the nut per revolution, can also increase:
    True
    False
26. Stretching a leadscrew in its mounts by applying axial forces through its mounting bearings causes mechanical strain to be replaced by thermal strain as the leadscrew heats up, and as a result, the lead is not subject to significant thermal error:
    True
    False
27. Stretching a leadscrew can place huge forces on a machine structure and this could cause deformations in the machine which might be larger than the thermal lead error, unless the machine is carefully designed:
    True
    False
28. A major contributor to leadscrew lead error is thermal growth (in leadscrews that are not pre-stretched and run at high speed):
    True
    False
29. Hollow leadscrews cooled with oil running down their centers are an effective means to remove heat from leadscrews and thereby control thermal error:
    True
    False
30. Primary limiting factors in the speed of a ballscrew include:
    Shaft whip
    The limiting DN value of the balls in the ball nut
    The bearings used to support the ballscrew
31. The *lead error* in a ballscrew can be mapped and thereby reduced by an order of magnitude
    True
    False

32. A good way to ensure that a ballscrew causes NO parasitic errors in a carriage is to attach it to the carriage at its center of stiffness via a *paddle-type* non-influencing coupling:
    True
    False
33. A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point
    True
    False
34. Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves, and the relative rotation speed of the gears will be constant
    True
    False
35. Gears are highly efficient (90-95%) due to primarily rolling contact between the teeth; thus by conservation of energy the product of the output torque and output gear pitch diameter equals the product of efficiency, input torque, and input gear pitch diameter:
    True
    False
36. The pressure angle causes gears to be forced apart as torque is applied, so it is critical that proper radial constraint be provided to shafts that support gears:
    True
    False
37. Saint Venant's principle can be very useful for initially determining if gears are properly supported:
    True
    False
38. For two gears, the output speed equals the product of the input speed and input gear pitch diameter divided by output gear pitch diameter:
    True
    False
39. Gears can never be made perfectly accurate, and thus must be preloaded if ALL backlash is to be removed:
    True
    False
40. The philosophical guidelines for preloading gears are the same as for preloading bearings:
    True
    False
41. The involute tooth form enables two gears to mesh and have the same gear ratio even if the distance between their centers is not exactly the average of their pitch diameters:
    True
    False
42. The involute tooth form can have an efficiency of 90-95%:
    True
    False
43. A single stage (two gears) involute tooth gear train can have an efficiency of 90-95%:
    True
    False
44. A two stage involute tooth gear train can have an efficiency of 81-90%:
    True
    False
45. A five stage involute tooth gear train can have an efficiency of 60-77%:
    True
    False
46. The pressure angle is the angle at which the torque is applied to a gear
    True
    False
47. The pitch of a gear is the angle at which the force is applied to the tooth
    True
    False
48. Metric gears use the term "MOD" to indicate the thickness of the gear tooth, so a MOD 1 gear has a tooth thickness of 1 mm at the base:
    True

False

49. A MOD 1 gear with 24 teeth has a pitch diameter of 24 mm
    - True
    - False
50. Planetary (epicyclic) gear trains enable a high reduction ratio to be obtained in a small place:
    - True
    - False
51. With a fixed ring gear, as the planet carrier rotates, the planet gears must simultaneously roll on both the surfaces of the sun gear and the ring gear, and the difference in the path length must be accommodated by rotation of the sun gear which gives rise to the transmission ratio for the system of TR = $(D_{sun}+D_{ring})/D_{sun}$:
    - True
    - False
52. A differential nominally divides the torque from an input shaft between the two output shafts but allows for differential motion between two output shafts:
    - True
    - False
53. A worm gear is essentially a gear whose teeth mesh with the threads of a leadscrew, and thus a high transmission ratio can be achieved of TR = $(\pi * D_{gear})$(leadscrew lead):
    - True
    - False