1 LINEAR ALGEBRA PROJECT

Problem1: Image Classification

1. Download the handwritten digits dataset. Dataset is given as a MATLAB struct with name "data". data.train_8 is the set of images of digit "8", data.train_1 is the set of images of digit "1", data.test is a set of mixed images of digits "1" and "8", and data.labels is the array of labels for samples in data.test. (Label is an integer to categorize the samples (images). Labels of images of digit "8" are 8, labels of images of digit "1" are 1.)

2. "data.train_8" and "data.train_1" are matrices of size [200,256], and "data.test" is a matrix of size [1147, 256]. Each row of these three matrices represents an image of size [16,16] which is reshaped to be an array of size [1,256]. Hence, in data.train_8 and data.train_1 rows represent samples (images), and columns represent pixel intensity values. Each of "data.train_8" and "data.train_1" store 200 images, while "data.test" store 1147 images.

3. Perform the following steps for both data.train_8 and data.train_1:

   (a) Generate a for loop to visualize all samples from the given dataset. Since all samples are stored as array, you should convert each sample array to an image of size [16,16]. MATLAB HINT: In order to convert arrays to images you can use "reshape" command, and to visualize images you can use "imshow" command.

   (b) You now have overall idea about your dataset. Now you will find a basis for the given dataset. What do you expect of elements of basis to be? What is the size of each element in basis of these images?

   (c) Calculate the Singular Value Decomposition (SVD) of the given dataset. What are the sizes of the resulting matrices U, S and V? Interpret the matrices S and V. MATLAB HINT: you can use "svd" command.

   (d) Find a basis for the row space of the given dataset by using SVD decomposition. What information about the given dataset do you get from the basis of rowspace?

   (e) Convert basis elements to matrices of size [16,16] and generate a for loop to visualize them. Sort the basis elements from most informative to least informative. How do you decide that order (Relate with matrix S.)? Now visualize elements. What do you observe about the order of the basis elements.

   (f) Store the most informative 3 basis elements. These 3 elements will give the "best 3-rank approximation".

4. You have 3 most informative basis elements for each of classes (3 basis elements for data.train_8 and 3 basis elements for data.train_1). Now you will

generate a decision strategy to predict label of samples in data.test. Since 3 basis elements can not form a complete basis for any of the given matrices, the representation of samples with these 3 basis elements may not give the exact representation but it will give an approximation. Because of the same reason these representations will not be unique. So you can find multiple representations for each sample. As a reconstruction choose the one which gives the minimum error when it is compared to the sample itself. This representation is called as the minimum error representation (or best k-rank approximation). Check the reference tutorial for "best k-rank approximation" and error calculation of approximations.

5. Create empty arrays as error_8, error_1 and reconstruction_errors with size [1147,1].

6. For each samples in the data.test, find minimum error representation by using 3 basis elements of data.train_8. Store the error in error_8.

7. For each samples in the data.test, find minimum error representation by using 3 basis elements of data.train_1. Store the error in error_1.

8. Compare the calculated errors for each sample and set the label of the sample with the digit (8 or 1) whose basis elements give the minimum error. (If error_8 $\geq$ error_1 set label as 1, if error_1 $\geq$ error_8 set label as 8). Store the labels of samples in a column vector labels_val (clearly labels_val will be in size [1147,1]).

9. Now you will fill the empty array reconstruction_errors by using correct labels: Set

$$\text{reconstruction\_errors\_3(i)} = \text{error\_8(i) if data.labels(i)=8,}$$

$$\text{reconstruction\_errors\_3(i)} = \text{error\_1(i) if data.labels(i)=1.}$$

10. Calculate the overall classification accuracy as

$$\text{accuracy\_3} = \frac{\text{number of correctly labeled samples}}{\text{total number of test samples}}$$

11. Repeat the listed steps in 3 to find the most informative 2 basis elements and 1 basis element. Store the accuracy matrices as accuracy_2 and accuracy_1, and reconstruction error matrices as reconstruction_errors_2 and reconstruction_errors_1 respectively. Compare the three classification accuracies by plotting a bar plot. Compare three average reconstruction errors by plotting a bar plot. How does the classification accuracy change when number of used basis elements are changed? How does the averaged reconstruction error changes? How do you interpret relation between reconstruction error and classification accuracy? Is there a direct relation between them?

12. Give possible strategies to improve the classification accuracy.