10.5: The Ideal Gas Law

The relationships we discussed in Section 10.4 can be combined into a single law that encompasses all of the simple gas laws. So far, we have shown that:

$$\begin{aligned} V &\propto \frac{1}{P} \quad \text{(Boyle's law)} \\ V &\propto T \quad \text{(Charles's law)} \\ V &\propto n \quad \text{(Avogadro's law)} \end{aligned}$$

Combining these three expressions, we get:

$$V \propto \frac{nT}{P}$$

The volume of a gas is directly proportional to the number of moles of gas and to the temperature of the gas but is inversely proportional to the pressure of the gas. We can replace the proportionality sign with an equals sign by incorporating R, a proportionality constant called the *ideal gas constant:*

$$V = \frac{RnT}{P}$$

> As we shall see in Section 10.11, the particles that compose an ideal gas have two properties: (1) negligible intermolecular forces and (2) low densities (the space between the particles is large compared to the size of the particles themselves).

Rearranging, we get:

[10.5]

$$PV = nRT$$

This equation is the **ideal gas law**, and a hypothetical gas that exactly follows this law is an **ideal gas**. The value of R, the **ideal gas constant**, is the same for all gases and has the value:

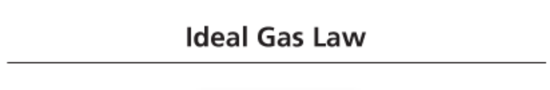
$$R = 0.08206 \, \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$$

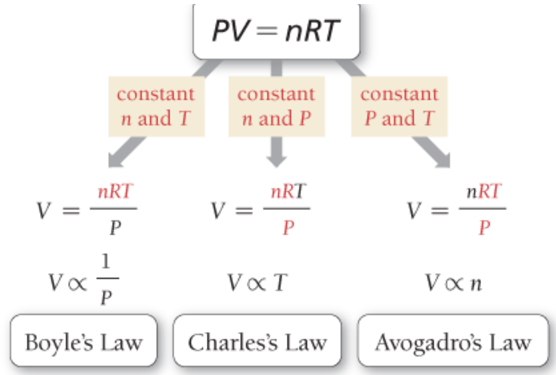
> L= liters
> atm= atmospheres
> mol= moles
> K= kelvins

The Ideal Gas Law Encompasses the Simple Gas Laws

The ideal gas law contains within it the simple gas laws. For example, recall that Boyle's law states that $V \propto 1/P$ when the amount of gas (n) and the temperature of the gas (T) are kept constant. We can rearrange the ideal gas law as follows:

$$PV = nRT$$





The ideal gas law contains the simple gas laws within it.

First, we divide both sides by P:

$$V = \frac{nRT}{P}$$

Then we put the variables that are constant, along with R, in parentheses:

$$V = (nRT)\frac{1}{P}$$

Because n and T are constant in this case, and R is always a constant, we can write:

$$V = (\text{constant}) \times \frac{1}{P}$$

which means that $V \propto 1/P$.

The ideal gas law also shows how other pairs of variables are related. For example, from Charles's law we know that $V \propto T$ at constant pressure and constant number of moles. But what if we heat a sample of gas at constant *volume* and constant number of moles? This question applies to the labels on aerosol cans such as hair spray or deodorants. These labels warn against excessive heating or incineration of the cans, even after the contents are used up. Why? An "empty" aerosol can is not really empty but contains a fixed amount of gas trapped in a fixed volume. What would happen if we were to heat the can? We can rearrange the ideal gas law to clearly see the relationship between pressure and temperature at constant volume and constant number of moles:

$$\begin{aligned} PV &= nRT \\ P &= \frac{nRT}{V} = \left(\frac{nR}{V}\right)T \end{aligned}$$

Because n and V are constant and R is always a constant:

$$P = (\text{constant}) \times T$$

Labels on aerosol cans warn against incineration. Because the volume of the can is constant, an increase in temperature causes an increase in pressure that could result in an explosion.

This relationship between pressure and temperature is known as *Gay-Lussac's law*. As the temperature of a fixed amount of gas in a fixed volume increases, the pressure increases. In an aerosol can, this pressure increase can blow the can apart, which is why aerosol cans should not be heated or incinerated. They might explode.

Calculations Using the Ideal Gas Law

We can use the ideal gas law to determine the value of any one of the four variables (P, V, n, or T) given the other three. To do so, we must express each of the quantities in the ideal gas law in the units within R:

- pressure (P) in atm, volume (V) in L, moles (n) in mol, temperature (T) in K

Example 10.5 Ideal Gas Law I

Calculate the volume occupied by 0.845 mol of nitrogen gas at a pressure of 1.37 atm and a temperature of 315 K.

SORT The problem gives you the number of moles of nitrogen gas, the pressure, and the temperature. You are asked to find the volume.

GIVEN: $n = 0.845 \text{ mol}$, $P = 1.37 \text{ atm}$, $T = 315 \text{ K}$

FIND: V

STRATEGIZE You are given three of the four variables (P, T, and n) in the ideal gas law and asked to find the fourth (V). The conceptual plan shows how the ideal gas law relates the known quantities and the unknown quantity.

CONCEPTUAL PLAN

$$n, P, T \xrightarrow[PV = nRT]{} V$$

RELATIONSHIP USED

$PV = nRT$ (ideal gas law)

SOLVE To solve the problem, first solve the ideal gas law for V.

Substitute the given quantities to calculate V.

SOLUTION

$$\begin{aligned} PV &= nRT \\ V &= \frac{nRT}{P} \\ V &= \frac{0.845\ \cancel{\text{mol}} \times 0.08206\ \dfrac{\text{L} \cdot \cancel{\text{atm}}}{\cancel{\text{mol}} \cdot \cancel{\text{K}}} \times 315\ \cancel{\text{K}}}{1.37\ \cancel{\text{atm}}} \\ &= 15.9 \text{ L} \end{aligned}$$

CHECK The units of the answer are correct. The magnitude of the answer (15.9 L) makes sense because, as you will see in the next section, 1 mol of an ideal gas under standard temperature and pressure (273 K and 1 atm) occupies 22.4 L. Although this is not standard temperature and pressure, the conditions are close enough for a ballpark check of the answer. This gas sample contains 0.845 mol, so a volume of 15.9

L is reasonable.

FOR PRACTICE 10.5 An 8.50-L tire contains 0.552 mol of gas at a temperature of 305 K. What is the pressure (in atm and psi) of the gas in the tire?

Interactive Worked Example 10.5 **Ideal Gas Law I**

Example 10.6 Ideal Gas Law II

Calculate the number of moles of gas in a 3.24-L basketball inflated to a *total pressure* of 24.3 psi at 25 °C. (*Note:* The *total pressure* is not the same as the pressure read on the type of pressure gauge used for checking a car or bicycle tire. That pressure, called the *gauge pressure*, is the *difference* between the total pressure and atmospheric pressure. In this case, if atmospheric pressure is 14.7 psi, the gauge pressure would be 9.6 psi. However, for calculations involving the ideal gas law, you must use the *total pressure* of 24.3 psi.)

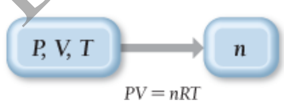
SORT The problem gives you the pressure, the volume, and the temperature. You need to find the number of moles of gas.

GIVEN:
$P = 24.3$ psi,
$V = 3.24$ L,
$T(^\circ\mathrm{C}) = 25\ ^\circ\mathrm{C}$

FIND: n

STRATEGIZE The conceptual plan shows how the ideal gas law provides the relationship between the given quantities and the quantity to be found.

CONCEPTUAL PLAN



RELATIONSHIP USED

$PV = nRT$ (ideal gas law)

SOLVE To solve the problem, first solve the ideal gas law for n.

Before substituting into the equation, convert P and T into the correct units.

Substitute into the equation and calculate n.

SOLUTION

$$\begin{aligned} PV &= nRT \\ n &= \frac{PV}{RT} \\ P &= 24.3\ \cancel{\text{psi}} \times \frac{1\ \text{atm}}{14.7\ \cancel{\text{psi}}} = 1.6\underline{5}31\ \text{atm} \end{aligned}$$

(Because rounding the intermediate answer would result in a slightly different final answer, we mark the least significant digit in the intermediate answer but don't round until the end.)

$T(K) = 25 + 273 = 298$ K

$$T\,(K) = 25 + 273 = 298\ K$$

$$n = \frac{1.6531\ \cancel{atm} \times 3.24\ \cancel{L}}{0.08206 \frac{\cancel{L \cdot atm}}{mol \cdot \cancel{K}} \times 298\ \cancel{K}} = 0.219\ mol$$

CHECK The units of the answer are correct. The magnitude of the answer (0.219 mol) makes sense because, as you will see in the next section, one mole of an ideal gas under standard temperature and pressure (273 K and 1 atm) occupies 22.4 L. At a pressure that is 65% higher than standard pressure, the volume of 1 mol of gas would be proportionally lower. This gas sample occupies 3.24 L, so the answer of 0.219 mol is reasonable.

FOR PRACTICE 10.6 What volume does 0.556 mol of gas occupy at a pressure of 715 mmHg and a temperature of 58 °C?

FOR MORE PRACTICE 10.6 Determine the pressure in mmHg of a 0.133-g sample of helium gas in a 648-mL container at a temperature of 32 °C.

Kinetic Molecular Theory and the Ideal Gas Law

We have just seen how each of the simple gas laws conceptually follows from kinetic molecular theory. We can also *derive* the ideal gas law from the postulates of kinetic molecular theory. The kinetic molecular theory is a quantitative model that *implies* $PV = nRT$. Let's explore this derivation.

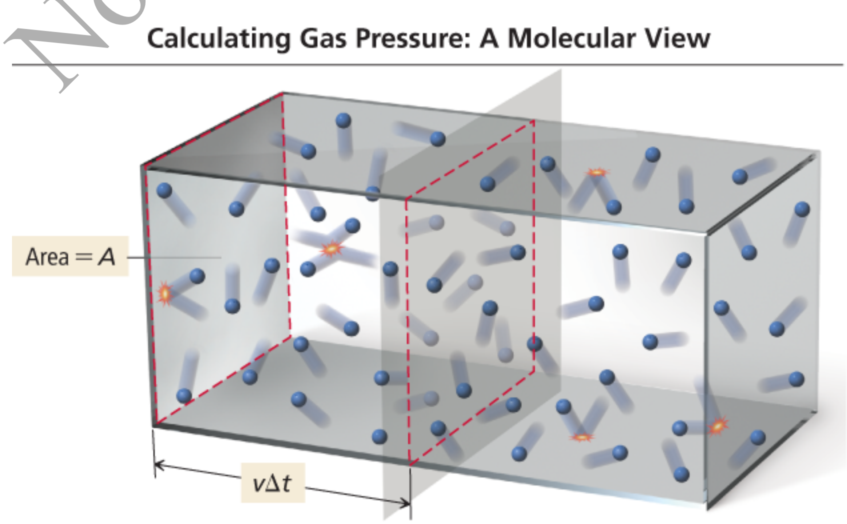
The pressure on a wall of a container (Figure 10.16) occupied by particles in constant motion is the total force on the wall (due to the collisions) divided by the area of the wall:

[10.6]

$$P = \frac{F_{total}}{A}$$

Figure 10.16 The Pressure on the Wall of a Container

We can calculate the pressure on the wall of a container by determining the total force due to collisions of the particles with the wall.



According to Newton's second law, the force (F) associated with an individual collision is given by $F = ma$, According to Newton's second law, the force (F) associated with an individual collision is given by $F = ma$, where m is the mass of the particle and a is its acceleration as it changes its direction of travel due to the collision. The acceleration for each collision is the change in velocity (Δv) divided by the time interval (Δt), so the force imparted for each collision is:

[10.7]

$$F_{\text{collision}} = m\frac{\Delta v}{\Delta t}$$

If a particle collides elastically with the wall, it bounces off the wall with no loss of energy. For a straight-line collision, the change in velocity is $2v$ (the particle's velocity was v before the collision and $-v$ after the collision; therefore, the change is $2v$). The force per collision is given by:

[10.8]

$$F_{\text{collision}} = m\frac{2v}{\Delta t}$$

The total number of collisions in the time interval Δt on a wall of surface area A is proportional to the number of particles that can reach the wall in this time interval—in other words, all particles within a distance of $v\ \Delta t$ of the wall. These particles occupy a volume given by $v\ \Delta t \times A$, and their total number is equal to this volume multiplied by the density of particles in the container (n/V):

[10.9]

$$\begin{aligned} \text{Number of collisions} &\propto \text{number of particles within } v\ \Delta t \\ &\propto \underbrace{v\ \Delta t \times A}_{\text{Volume}} \times \underbrace{\frac{n}{V}}_{\text{Density of particles}} \end{aligned}$$

The *total force* on the wall is equal to the force per collision multiplied by the number of collisions:

[10.10]

$$\begin{aligned} F_{\text{total}} &= F_{\text{collision}} \times \text{number of collisions} \\ &\propto m\frac{2v}{\cancel{\Delta t}} \times v\ \cancel{\Delta t} \times A \times \frac{n}{V} \\ &\alpha\propto mv^2 \times A \times \frac{n}{V} \end{aligned}$$

The pressure on the wall is equal to the total force divided by the surface area of the wall:

[10.11]

$$\begin{aligned} P &= \frac{F_{\text{total}}}{A} \\ &\propto \frac{mv^2 \times \cancel{A} \times \frac{n}{V}}{\cancel{A}} \\ P &\propto mv^2 \times \frac{n}{V} \end{aligned}$$

Notice that Equation 10.11 contains within it Boyle's law ($P \propto 1/V$) and Avogadro's law ($P \propto n$). We can get the complete ideal gas law from postulate 2 of the kinetic molecular theory (see Section 10.2), which states that the average kinetic energy $\left(\frac{1}{2}mv^2\right)$ is proportional to the temperature in kelvins (T):

[10.12]

$$mv^2 \propto T$$

Combining Equations 10.11 and 10.12, we get:

[10.13]

$$P \propto \frac{T \times n}{V}$$

$$PV \propto nT$$

The proportionality can be replaced by an equals sign if we provide the correct constant, *R*:

[10.14]

$$PV = nRT$$

In other words, the kinetic molecular theory (a model for how gases behave) predicts behavior that is consistent with our observations and measurements of gases—the theory agrees with the experiments. Recall from Chapter 1 that a scientific theory is the most powerful kind of scientific knowledge. In the kinetic molecular theory, we have a model for what a gas is like. Although the model is not perfect—indeed, it breaks down under certain conditions, as we shall see later in this chapter—it predicts a great deal about the behavior of gases. Therefore, the model is a good approximation of what a gas is actually like. A careful examination of the conditions under which the model breaks down (see Section 10.11) gives us even more insight into the behavior of gases.