Pearson

7.5: Stoichiometric Relationships: Limiting Reactant, Theoretical Yield, Percent Yield, and Reactant in Excess

Key Concept Video Limiting Reactant, Theoretical Yield, and Percent Yield

When we carry out a chemical reaction, we combine the reactants in a container and allow the reaction to occur. Because of the stoichiometric relationships we discussed in Section 7.4, the quantities of each reactant present (along with other factors) determine how much product the reaction produces. In this section, we look at these relationships, including the concepts of *limiting reactant, theoretical yield*, *percent yield,* and *reactant in excess*.

Limiting Reactant and Yield

We return to our pizza analogy to understand *limiting reactant*, *theoretical yield*, and *percent yield*. Recall our pizza recipe from Section 7.4:

$$1 \text{ crust} + 5 \text{ ounces tomato sauce} + 2 \text{ cups cheese} \rightarrow 1 \text{ pizza}$$

Suppose that we have 4 crusts, 10 cups of cheese, and 15 ounces of tomato sauce. How many pizzas can we make?

We have enough crusts to make:

$$4 \text{ crusts} \times \frac{1 \text{ pizza}}{1 \text{ crust}} = 4 \text{ pizzas}$$

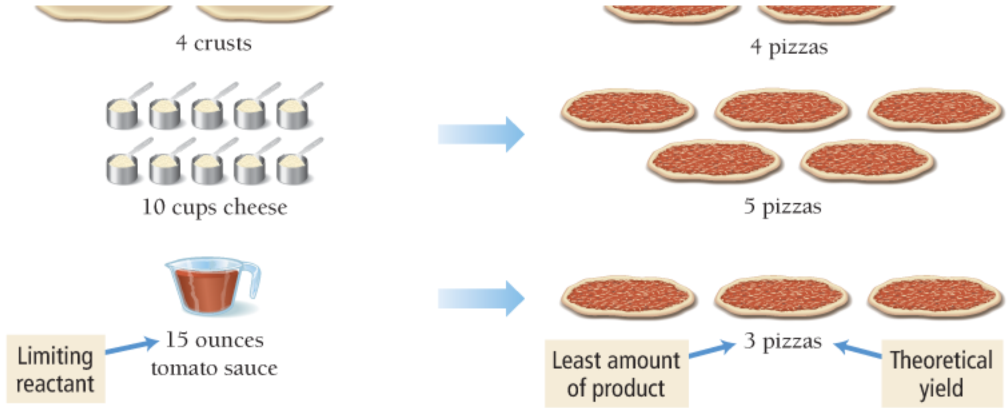
We have enough cheese to make:

$$10 \text{ cups cheese} \times \frac{1 \text{ pizza}}{2 \text{ cups cheese}} = 5 \text{ pizzas}$$

We have enough tomato sauce to make:

$$\underbrace{\cancel{15 \text{ ounces tomato sauce}}}_{\text{Limiting reactant}} \times \frac{1 \text{ pizza}}{\cancel{5 \text{ ounces tomato sauce}}} = \underbrace{3 \text{ pizzas}}_{\text{Smallest number of pizzas}}$$

We have enough crusts for 4 pizzas, enough cheese for 5 pizzas, but enough tomato sauce for only 3 pizzas. Consequently, unless we get more ingredients, we can make only 3 pizzas. The tomato sauce *limits* how many pizzas we can make. If the pizza recipe were a chemical reaction, the tomato sauce would be the **limiting reactant**—the reactant that limits the amount of product in a chemical reaction. Notice that the limiting reactant is the reactant that makes *the least amount of product*. The reactants that *do not* limit the amount of product—such as the crusts and the cheese in this example—are said to be *in excess*. If this were a chemical reaction, 3 pizzas would be the **theoretical yield**—the amount of product that can be made in a chemical reaction based on the amount of limiting reactant.



The ingredient that makes the least amount of pizza determines how many pizzas we can make.

> The term *limiting reagent* is sometimes used in place of *limiting reactant*.

We can carry this analogy one step further. Suppose we go on to cook our pizzas and accidentally burn one of them. Even though we theoretically have enough ingredients for 3 pizzas, we end up with only 2. If this were a chemical reaction, the 2 pizzas would be our **actual yield**, the amount of product actually produced by a chemical reaction. The actual yield is always equal to or less than the theoretical yield because a small amount of product is usually lost to other reactions or does not form during a reaction.

With this information, we can calculate our **percent yield**, the percentage of the theoretical yield that was actually attained, as the ratio of the actual yield to the theoretical yield:

$$\%\text{ yield} = \frac{\overbrace{2\ \cancel{\text{pizzas}}}^{\text{Actual yield}}}{\underbrace{3\ \cancel{\text{pizzas}}}_{\text{Theoretical yield}}} \times 100\% = 67\%$$

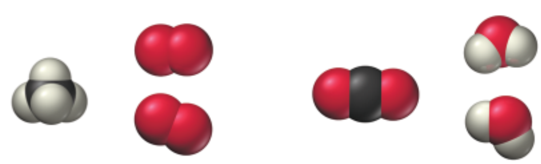
Since one of our pizzas burned, we obtained only 67% of our theoretical yield.

Summarizing Limiting Reactant and Yield:

- **The limiting reactant** (or **limiting reagent**) is the reactant that is completely consumed in a chemical reaction and limits the amount of product.
- **The reactant in excess** is any reactant that occurs in a quantity greater than is required to completely react with the limiting reactant.
- **The theoretical yield** is the amount of product that can be made in a chemical reaction based on the amount of limiting reactant.
- **The actual yield** is the amount of product actually produced by a chemical reaction.
- **The percent yield** is $\frac{\text{actual yield}}{\text{theoretical yield}} \times 100\,\%$.

We can apply these concepts to a chemical reaction. Recall from Section 7.3 our balanced equation for the combustion of methane:

$$CH_4(g) + 2\,O_2(g) \longrightarrow CO_2(g) + 2\,H_2O(l)$$



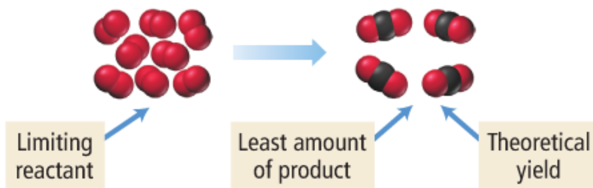
If we start out with 5 CH_4 molecules and 8 O_2 molecules, what is our limiting reactant? What is our theoretical yield of carbon dioxide molecules?

First, we calculate the number of CO_2 molecules that can be made from 5 CH_4 molecules:

$$5\ \cancel{CH_4} \times \frac{1\ CO_2}{1\ \cancel{CH_4}} = 5\ CO_2$$

We then calculate the number of CO_2 molecules that can be made from 8 O_2 molecules:

$$8\ \cancel{O_2} \times \frac{1\ CO_2}{2\ \cancel{O_2}} = 4\ CO_2$$



We have enough CH_4 to make 5 CO_2 molecules and enough O_2 to make 4 CO_2 molecules. Therefore, O_2 is the limiting reactant, and 4 CO_2 molecules is the theoretical yield. The CH_4 is in excess.

An alternative way to calculate the limiting reactant (which we mention here but do not use in this book) is to pick any reactant and determine how much of the *other reactant* is necessary to completely react with it. For the reaction we just examined, we have 5 CH_4 molecules and 8 O_2 molecules. Let's pick the 5 CH_4 molecules and determine how many O_2 molecules are necessary to completely react with them:

$$5\ CH_4 \times \frac{2\ O_2}{1\ CH_4} = 10\ O_2$$

Since we need 10 O_2 molecules to completely react with the 5 CH_4 molecules, and since we have only 8 O_2 molecules, we know that the O_2 is the limiting reactant. We can apply the same method by comparing the amounts of reactants in moles.

Conceptual Connection 7.6 Limiting Reactant and Theoretical Yield

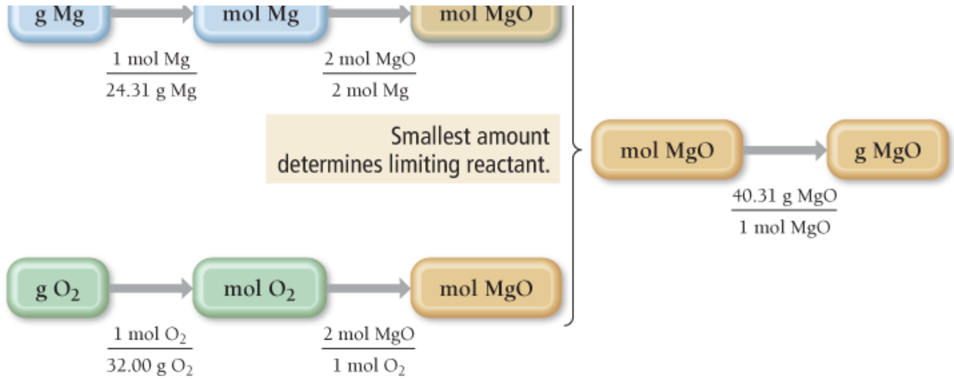
When working in the laboratory, we normally measure the initial quantities of reactants in grams, not in number of molecules. To find the limiting reactant and theoretical yield from initial masses, we must first convert the masses to amounts in moles. Consider the reaction:

$$2\ Mg(s) + O_2(g) \rightarrow 2\ MgO(s)$$

A reaction mixture contains 42.5 g Mg and 33.8 g O_2; what are the limiting reactant and theoretical yield? To solve this problem, we must determine which of the reactants makes the least amount of product.

Conceptual Plan

We can determine the limiting reactant by calculating how much product can be made from each reactant. However, we are given the initial quantities in grams, and stoichiometric relationships are between moles, so we must first convert to moles. We then convert from moles of the reactant to moles of product. The reactant that makes the *least amount of product* is the limiting reactant. The conceptual plan is:



In this conceptual plan, we compare the number of moles of magnesium oxide made by each reactant and convert only the smaller amount to grams. (Alternatively, we can convert both quantities to grams and determine the limiting reactant based on the mass of the product.)

Relationships Used

molar mass Mg = 24.31 g Mg

molar mass O_2 = 32.00 g O_2

2 mol Mg : 2 mol MgO (from chemical equation)

1 mol O_2 : 2 mol MgO (from chemical equation)

molar mass MgO = 40.31 g MgO

Solution

Beginning with the masses of each reactant, we follow the conceptual plan to calculate how much product can be made from each.

$$\boxed{42.5\ \cancel{\text{g Mg}}} \times \frac{1\ \cancel{\text{mol Mg}}}{24.31\ \cancel{\text{g Mg}}} \times \frac{2\ \text{mol MgO}}{2\ \cancel{\text{mol Mg}}} = \boxed{1.7\underline{4}83\ \text{mol MgO}}$$

Limiting reactant

Least amount of product

$$33.8\ \cancel{\text{g O}_2} \times \frac{1\ \cancel{\text{mol O}_2}}{32.00\ \cancel{\text{g O}_2}} \times \frac{2\ \text{mol MgO}}{1\ \cancel{\text{mol O}_2}} = 2.1\underline{1}25\ \text{mol MgO}$$

$$1.7\underline{4}83\ \cancel{\text{mol MgO}} \times \frac{40.31\ \text{g MgO}}{1\ \cancel{\text{mol MgO}}} = 70.5\ \text{g MgO}$$

Because magnesium makes the least amount of product, it is the limiting reactant, and oxygen is in excess. Notice that the limiting reactant is not necessarily the reactant with the least mass. In this case, the mass of O_2 is less than the mass of Mg, yet Mg is the limiting reactant because it makes the least amount of MgO. The theoretical yield is 70.5 g of MgO, the mass of product possible based on the limiting reactant.

Suppose that after the synthesis, the actual yield of MgO is 55.9 g. What is the percent yield? We calculate the percent yield as follows:

$$\%\ \text{yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100\% = \frac{55.9\ \text{g}}{70.5\ \text{g}} \times 100\% = 79.3\ \%$$

Example 7.6 Limiting Reactant and Theoretical Yield

Ammonia, NH_3, can be synthesized by the reaction:

$$2\ NO(g) + 5\ H_2(g) \rightarrow 2\ NH_3(g) + 2\ H_2O(g)$$

Starting with 86.3 g NO and 25.6 g H_2, find the theoretical yield of ammonia in grams.

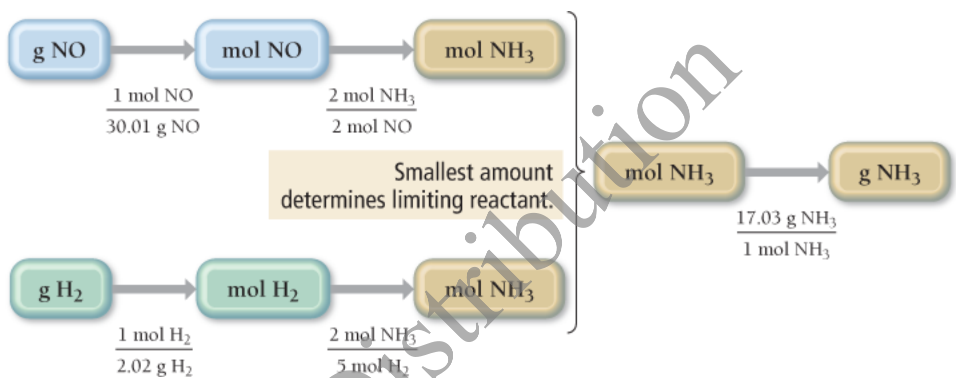
SORT You are given the mass of each reactant in grams and asked to find the theoretical yield of a product.

GIVEN: 86.3 g NO, 25.6 g H_2

FIND: theoretical yield of $NH_3(g)$

STRATEGIZE Determine which reactant makes the least amount of product by converting from grams of each reactant to moles of the reactant to moles of the product. Use molar masses to convert between grams and moles and use the stoichiometric relationships (from the balanced chemical equation) to convert between moles of reactant and moles of product. Remember that the reactant that makes *the least amount of product* is the limiting reactant. Convert the number of moles of product obtained using the limiting reactant to grams of product.

CONCEPTUAL PLAN



RELATIONSHIPS USED

molar mass NO = 30.01 g/mol

molar mass H_2 = 2.02 g/mol

2 mol NO : 2 mol NH_3 (from chemical equation)

5 mol H_2 : 2 mol NH_3 (from chemical equation)

molar mass NH_3 = 17.03 g/mol

SOLVE Beginning with the given mass of each reactant, calculate the amount of product that can be made in moles. Convert the amount of product made by the limiting reactant to grams—this is the theoretical yield.

SOLUTION

$$\boxed{86.3\ \cancel{g\ NO}} \times \frac{1\ \cancel{mol\ NO}}{30.01\ \cancel{g\ NO}} \times \frac{2\ mol\ NH_3}{2\ \cancel{mol\ NO}} = \boxed{2.8\underline{7}57\ mol\ NH_3}$$

Limiting reactant

Least amount of product

$$2.8\underline{7}57\ \cancel{mol\ NH_3} \times \frac{17.03\ g\ NH_3}{\cancel{mol\ NH_3}} = 49.0\ g\ NH_3$$

$$\ldots \frac{1\ \cancel{mol\ H_2}}{} \ldots \frac{2\ mol\ NH_3}{}$$

$$25.6\ \cancel{g\ H_2} \times \frac{}{2.02\ \cancel{g\ H_2}} \times \frac{}{5\ \cancel{mol\ H_2}} = 5.06\underline{9}3\ mol\ NH_3$$

Since NO makes the least amount of product, it is the limiting reactant, and the theoretical yield of ammonia is 49.0 g.

CHECK The units of the answer $(g\ NH_3)$ are correct. The magnitude (49.0 g) seems reasonable given that 86.3 g NO is the limiting reactant. NO contains one oxygen atom per nitrogen atom, and NH_3 contains three hydrogen atoms per nitrogen atom. Three hydrogen atoms have less mass than one oxygen atom, so it is reasonable that the mass of NH_3 obtained is less than the mass of NO.

FOR PRACTICE 7.6 Ammonia can also be synthesized by the reaction:

$$3\ H_2(g) + N_2(g) \rightarrow 2\ NH_3(g)$$

What is the theoretical yield of ammonia, in kg, that we can synthesize from 5.22 kg of H_2 and 31.5 kg of N_2?

Interactive Worked Example 7.6 **Limiting Reactant and Theoretical Yield**

Example 7.7 Limiting Reactant and Theoretical Yield

We can obtain titanium metal from its oxide according to the following balanced equation:

$$TiO_2(s) + 2\ C(s) \rightarrow Ti(s) + 2\ CO(g)$$

When 28.6 kg of C reacts with 88.2 kg of titanium(IV) oxide, 42.8 kg of titanium is produced. Find the limiting reactant, theoretical yield (in kg), and percent yield.

SORT You are given the mass of each reactant and the mass of product formed. You are asked to find the limiting reactant, theoretical yield, and percent yield.

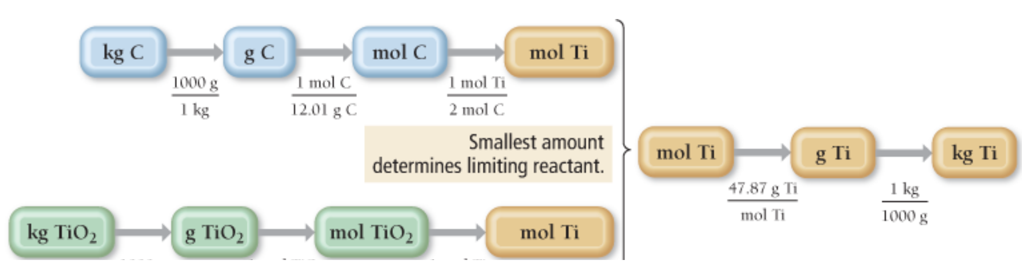
GIVEN: 28.6 kg C, 88.2 kg TiO_2, 42.8 kg Ti produced

FIND: limiting reactant, theoretical yield, % yield

STRATEGIZE Determine which of the reactants makes the least amount of product by converting from kilograms of each reactant to moles of product. Convert between grams and moles using molar mass. Convert between moles of reactant and moles of product using the stoichiometric relationships derived from the balanced chemical equation. The reactant that makes the *least amount of product* is the limiting reactant.

Determine the theoretical yield (in kg) by converting the number of moles of product obtained with the limiting reactant to kilograms of product.

CONCEPTUAL PLAN



$$\frac{1000\ \text{g}}{1\ \text{kg}} \qquad \frac{1\ \text{mol TiO}_2}{79.87\ \text{g TiO}_2} \qquad \frac{1\ \text{mol Ti}}{1\ \text{mol TiO}_2}$$

RELATIONSHIPS USED

$1000\ \text{g} = 1\ \text{kg}$	1 mol TiO_2 : 1 mol Ti (from chemical equation)
molar mass of $C = 12.01\ \text{g/mol}$	2 mol C : 1 mol Ti (from chemical equation)
molar mass of $TiO_2 = 79.87\ \text{g/mol}$	molar mass of $Ti = 47.87\ \text{g/mol}$
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SOLVE Beginning with the actual amount of each reactant, calculate the amount of product that can be made in moles. Convert the amount of product made by the limiting reactant to kilograms—this is the theoretical yield.

Calculate the percent yield by dividing the actual yield (42.8 kg Ti) by the theoretical yield.

SOLUTION

$$28.6\ \cancel{\text{kg C}} \times \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \times \frac{1\ \cancel{\text{mol C}}}{12.01\ \cancel{\text{g C}}} \times \frac{1\ \text{mol Ti}}{2\ \cancel{\text{mol C}}} = 1.1\underline{9}07 \times 10^3\ \text{mol Ti}$$

Limiting reactant

Least amount of product

$$\boxed{88.2\ \cancel{\text{kg TiO}_2}} \times \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \times \frac{1\ \cancel{\text{mol TiO}_2}}{79.87\ \cancel{\text{g TiO}_2}} \times \frac{1\ \text{mol Ti}}{1\ \cancel{\text{mol TiO}_2}} = \boxed{1.1\underline{0}43 \times 10^3\ \text{mol Ti}}$$

$$1.1\underline{0}43 \times 10^3\ \cancel{\text{mol Ti}} \times \frac{47.87\ \cancel{\text{g Ti}}}{1\ \cancel{\text{mol Ti}}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}} = 52.9\ \text{kg Ti}$$

Since TiO_2 makes the least amount of product, it is the limiting reactant, and 52.9 kg Ti is the theoretical yield.

$$\%\ \text{yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100\% = \frac{42.8\ \text{g}}{52.9\ \text{g}} \times 100\% = 80.9\%$$

CHECK The theoretical yield has the correct units (kg Ti) and has a reasonable magnitude compared to the mass of TiO_2. Because titanium has a lower molar mass than titanium(IV) oxide, the amount of Ti made from TiO_2 should have a lower mass. The percent yield is reasonable (under 100% as it should be).

FOR PRACTICE 7.7 Mining companies use this reaction to obtain iron from iron ore:

$$Fe_2O_3(s) + 3\ CO(g) \rightarrow 2\ Fe(s) + 3\ CO_2(g)$$

The reaction of 167 g Fe_2O_3 with 85.8 g CO produces 72.3 g Fe. Determine the limiting reactant, theoretical yield, and percent yield.

Reactant in Excess

We have now seen how the limiting reactant limits the amount of product that a chemical reaction produces. If the reaction goes to completion, the limiting reactant is completely consumed. The reactant in excess, by contrast, is not completely consumed; some of it remains in the reaction mixture after the reaction is complete.

Returning to the pizza analogy from earlier in this section, recall that if we have 4 crusts, 10 cups of cheese, and 15 ounces of tomato sauce, the tomato sauce is the limiting reactant. The tomato sauce is completely consumed while the other ingredients are not. We can calculate how much of the excess reactant (or reactants) remains after the reaction is complete from the relationships in the recipe.

$$1 \text{ crust} + 5 \text{ ounces tomato sauce} + 2 \text{ cups cheese} \rightarrow 1 \text{ pizza}$$

For example, to calculate the amount of crust remaining, we first calculate how many crusts are consumed by the amount of limiting reactant. We use the relationship between the crust and the tomato sauce from the recipe.

$$15 \text{ ounces tomato sauce} \times \frac{1 \text{ crust}}{5 \text{ ounces tomato sauce}} = 3 \text{ crusts}$$

Then we subtract the number of crusts used from the number of crusts we had initially to determine how many crusts remain.

$$4 \text{ crusts} - 3 \text{ crusts} = 1 \text{ crust}$$

One crust remains after we have used up all the tomato sauce. Similar concepts apply to a chemical reaction. To determine how much of a reactant in excess remains after the reaction has gone to completion, we figure out the amount of that reactant needed to react with the limiting reactant and then subtract that amount from the initial amount as shown in Example 7.8.

Example 7.8 Reactant in Excess

Recall from Example 7.7 that we can extract titanium metal from its oxide according to the following balanced equation:

$$TiO_2(s) + 2\,C(s) \rightarrow Ti(s) + 2\,CO(g)$$

Recall further that in a reaction mixture containing 28.6 kg of C and 88.2 kg of TiO_2, TiO_2 is the limiting reactant. Calculate the mass of the reactant in excess (which is carbon) that remains after the reaction has gone to completion.

SORT You are given the mass of each reactant and asked to find the mass of the excess reactant remaining after the reaction has gone to completion. You know from Example 7.7 that TiO_2 is the limiting reactant.

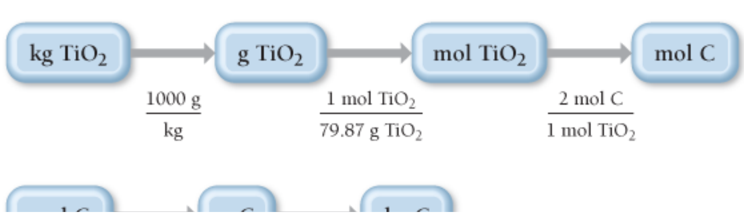
GIVEN: 28.6 kg C, 88.2 kg TiO_2(limiting)

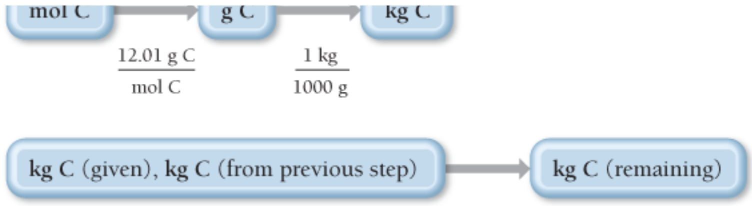
FIND: kg C remaining

STRATEGIZE Determine how much carbon is needed to completely react with the given amount of titanium dioxide. Begin with kg TiO_2 and convert first to mol TiO_2 and then to mol C. Convert between grams and moles using molar mass. Convert between mol TiO_2 and mol C using the stoichiometric relationship from the balanced chemical equation. Then convert mol C to kg C. This is the amount of C needed to react with the given amount of TiO_2 (which is the limiting reactant).

Now you use the given kg C. Subtract the kg C needed to react with the limiting reactant (from the previous step) to determine the kg remaining.

CONCEPTUAL PLAN





RELATIONSHIPS USED

1000 g = 1 kg

1 mol TiO_2 : 2 mol C (from chemical equation)

molar mass of C = 12.01 g/mol

molar mass of TiO_2 = 79.87 g/mol

SOLVE Begin with kg TiO_2 and carry out the conversions to arrive at mol C.

Then convert mol C to kg C. This is the mass of C that reacts with the limiting reactant.

Finally, subtract the kg C you just calculated from the initial kg C given in the problem to determine the kg C remaining.

SOLUTION

$$88.2 \text{ kg } TiO_2 \times \frac{1000 \text{ g}}{\text{kg}} \times \frac{1 \text{ mol } TiO_2}{79.87 \text{ g } TiO_2} \times \frac{2 \text{ mol C}}{1 \text{ mol } TiO_2} = 2.20\text{¯}86 \times 10^3 \text{ mol C}$$

$$2.20\text{¯}86 \times 10^3 \text{ mol C} \times \frac{12.01 \text{ g C}}{\text{mol C}} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 26.5\text{¯}25 \text{ kg C}$$

$$\begin{aligned} \text{mass C remaining} &= 28.6 \text{ kg C} - 26.5\text{¯}25 \text{ kg C} \\ &= 2.1 \text{ kg C} \end{aligned}$$

CHECK The mass of carbon has the right units (kg C). It also seems to be a reasonable mass given that the reaction mixture initially had almost 29 kg C—most of the carbon reacted to leave about 2 kg C.

FOR PRACTICE 7.8 Recall from Example 7.8 that mining companies extract iron from iron ore according to the following balanced equation:

$$Fe_2O_3(s) + 3\,CO(g) \rightarrow 2\,Fe(s) + 3\,CO_2(g)$$

In a reaction mixture containing 167 g Fe_2O_3 and 85.8 g CO, CO is the limiting reactant. Calculate the mass of the reactant in excess (which is Fe_2O_3) that remains after the reaction has gone to completion.

Conceptual Connection 7.7 **Reactant in Excess**

Interactive