10.8: Temperature and Molecular Velocities

According to kinetic molecular theory, particles of different masses have the same average kinetic energy at a given temperature. The kinetic energy of a particle depends on its mass and velocity according to the equation:

$$\mathrm{KE} = \frac{1}{2}mv^2$$

The only way for particles of different masses to have the same kinetic energy is for them to have different velocities, as Conceptual Connection 10.1 demonstrates.

> **In a gas mixture at a given temperature, lighter particles travel faster (on average) than heavier ones.**

In kinetic molecular theory, we define the root mean square velocity (u_{rms}) of a particle as:

[10.22]

$$u_{\mathrm{rms}} = \sqrt{\overline{u^2}}$$

where $\overline{u^2}$ is the average of the squares of the particle velocities. Even though the root mean square velocity of a collection of particles is not identical to the average velocity, the two are close in value and conceptually similar. Root mean square velocity is a special *type* of average. The average kinetic energy of one mole of gas particles is given by:

[10.23]

$$\mathrm{KE}_{\mathrm{avg}} = \frac{1}{2}N_{\mathrm{A}}m\overline{u^2}$$

where N_{A} is Avogadro's number.

Postulate 2 of the kinetic molecular theory states that the average kinetic energy is proportional to the temperature in kelvins. The constant of proportionality in this relationship is (3/2)*R*:

[10.24]

$$\mathrm{KE}_{\mathrm{avg}} = (3/2)\,RT$$

> The (3/2)*R* proportionality constant comes from a derivation that is beyond the current scope of this textbook.

where *R* is the gas constant, but in different units ($R = 8.314\ \mathrm{J/mol \cdot K}$) than those we use in the ideal gas law.

> The joule (J) is a unit of energy discussed in more detail in Section E.6. $\left(1\ \mathrm{J} = 1\ \mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right)$

If we combine Equations 10.23 and 10.24, and solve for $\overline{u^2}$, we get:

$$(1/2)\,N_{\mathrm{A}}m\overline{u^2} = (3/2)\,RT$$

$$\overline{u^2} = \frac{(3/2)\,RT}{(1/2)\,N_{\mathrm{A}}m} = \frac{3RT}{N_{\mathrm{A}}m}$$

Taking the square root of both sides we get:

[10.25]

$$\sqrt{\overline{u^2}} = u_{\mathrm{rms}} = \sqrt{\frac{3RT}{N_{\mathrm{A}}m}}$$

In Equation 10.25, m is the mass of a particle in kg and N_{A} is Avogadro's number. The product $N_{\mathrm{A}}m$, then, is the molar mass in kg/mol. If we call this quantity M, the expression for mean square velocity as a function of temperature is the following important result:

[10.26]

$$u_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}}$$

The root mean square velocity of a collection of gas particles is proportional to the square root of the temperature in kelvins and inversely proportional to the square root of the molar mass of the particles (which because of the units of R, is in kilograms per mole). The root mean square velocity of nitrogen molecules at 25 °C, for example, is 515 m/s (1152 mi/h). The root mean square velocity of hydrogen molecules at room temperature is 1920 m/s (4295 mi/h). Notice that the lighter molecules move much faster at a given temperature.

The root mean square velocity, as we have seen, is a kind of average velocity. Some particles are moving faster, and some are moving more slowly than this average. The velocities of all the particles in a gas sample form distributions such as those shown in Figure 10.19. We can see from these distributions that some particles are indeed traveling at the root mean square velocity. However, many particles are traveling faster and many slower than the root mean square velocity. For lighter particles, such as helium and hydrogen, the velocity distribution is shifted toward higher velocities and the curve becomes broader, indicating a wider range of velocities. Figure 10.20 is the velocity distribution for nitrogen at different temperatures. As the temperature increases, the root mean square velocity increases and the distribution becomes broader.

Figure 10.19 Velocity Distribution for Several Gases at 25 °C

At a given temperature, there is a distribution of velocities among the particles in a sample of gas. The exact shape and peak of the distribution vary with the molar mass of the gas.

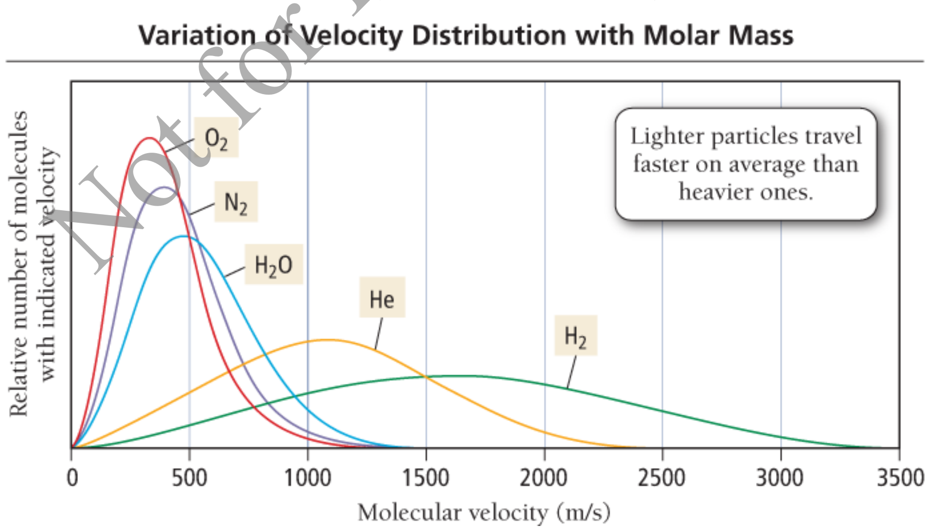
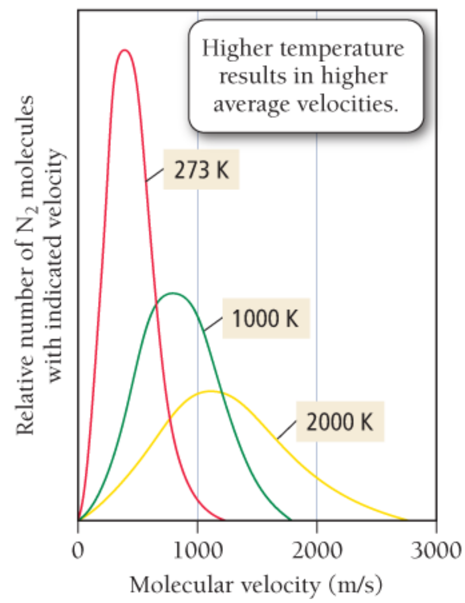


Figure 10.20 Velocity Distribution for Nitrogen at Several Temperatures

As the temperature of a gas sample increases, the velocity distribution of the molecules shifts toward higher velocity and becomes less sharply peaked.

Variation of Velocity Distribution with Temperature



Example 10.12 Root Mean Square Velocity

Calculate the root mean square velocity of oxygen molecules at 25 °C.

SORT The problem describes the kind of molecule and its temperature and asks you to find the root mean square velocity.

GIVEN: O_2, $t = 25\ ^\circ C$

FIND: u_{rms}

STRATEGIZE The conceptual plan for this problem illustrates how to use the molar mass of oxygen and the temperature (in kelvins) with the equation that defines the root mean square velocity to calculate root mean square velocity.

CONCEPTUAL PLAN

$$\boxed{\mathcal{M}, T} \longrightarrow \boxed{u_{rms}}$$

$$u_{rms} = \sqrt{\frac{3RT}{\mathcal{M}}}$$

RELATIONSHIP USED

$u_{rms} = \sqrt{\frac{3RT}{M}}$ (Equation 10.26)

SOLVE Gather the required quantities in the correct units. Note that molar mass must be in kg/mol.

Substitute the quantities into the equation to calculate root mean square velocity. Note that $1\ J = 1\ kg \cdot m^2/s^2$.

SOLUTION

$$T = 25 + 273 = 298\ K$$

$$M = \frac{32.00\ \cancel{g}\ O_2}{1\ mol\ O_2} \times \frac{1\ kg}{1000\ \cancel{g}} = \frac{32.00 \times 10^{-3}\ kg\ O_2}{1\ mol\ O_2}$$

$$u_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.314\frac{\mathrm{J}}{\cancel{\mathrm{mol}}\cdot\cancel{\mathrm{K}}}\right)(298\ \cancel{\mathrm{K}})}{\frac{32.00\times10^{-3}\ \mathrm{kg}\ \cancel{\mathrm{O_2}}}{1\ \cancel{\mathrm{mol}}\ \cancel{\mathrm{O_2}}}}}$$

$$= \sqrt{2.32\times10^{5}\frac{\mathrm{J}}{\mathrm{kg}}}$$

$$= \sqrt{2.32\times10^{5}\frac{\frac{\cancel{\mathrm{kg}}\cdot\mathrm{m}^2}{\mathrm{s}^2}}{\cancel{\mathrm{kg}}}} = 482\ \mathrm{m/s}$$

CHECK The units of the answer (m/s) are correct. The magnitude of the answer is reasonable because oxygen is slightly heavier than nitrogen and should therefore have a slightly lower root mean square velocity at the same temperature. Recall that the root mean square velocity of nitrogen is 515 m/s at 25 °C.

FOR PRACTICE 10.12 Calculate the root mean square velocity of gaseous xenon atoms at 25 °C.

Conceptual Connection 10.6 **Kinetic Molecular Theory and Particle Mass**

Interactive