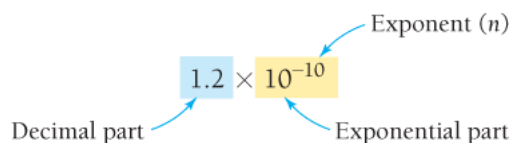**P** Pearson

Appendix I Common Mathematical Operations in Chemistry

A. Scientific Notation

A number written in scientific notation consists of a **decimal part**, a number that is usually between 1 and 10, and an **exponential part**, 10 raised to an **exponent**, n.

Each of the following numbers is written in both scientific and decimal notation:

$$1.0 \times 10^5 = 100,000 \qquad 1.0 \times 10^{-6} = 0.000001$$
$$6.7 \times 10^3 = 6700 \qquad 6.7 \times 10^{-3} = 0.0067$$

A positive exponent means 1 multiplied by 10 n times.

$$10^0 = 1$$
$$10^1 = 1 \times 10$$
$$10^2 = 1 \times 10 \times 10 = 100$$
$$10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

A negative exponent ($-n$) means 1 divided by 10 n times.

$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{1 \times 10} = 0.01$$
$$10^{-3} = \frac{1}{1 \times 10 \times 10} = 0.001$$

To convert a number to scientific notation, we move the decimal point to obtain a number between 1 and 10 and then multiply by 10 raised to the appropriate power. For example, to write 5983 in scientific notation, we move the decimal point to the left three places to get 5.983 (a number between 1 and 10) and then multiply by 1000 to make up for moving the decimal point.

$$5983 = 5.983 \times 1000$$

Since 1000 is 10^3, we write:

$$5983 = 5.983 \times 10^3$$

We can do this in one step by counting how many places we move the decimal point to obtain a number between 1 and 10 and then writing the decimal part multiplied by 10 raised to the number of places we moved the decimal point.

$$5983 = 5.983 \times 10^3$$

If the decimal point is moved to the left, as in the previous example, the exponent is positive. If the decimal is moved to the right, the exponent is negative.

$$0.00034 = 3.4 \times 10^{-4}$$

To express a number in scientific notation:

1. Move the decimal point to obtain a number between 1 and 10.

2. Write the result from step 1 multiplied by 10 raised to the number of places you moved the decimal point.

- *The exponent is positive if you moved the decimal point to the left.*
- *The exponent is negative if you moved the decimal point to the right.*

Consider the following additional examples:

$$290,809,000 = 2.90809 \times 10^8$$
$$0.000000000070 \text{ m} = 7.0 \times 10^{-11} \text{ m}$$

Multiplication and Division

To multiply numbers expressed in scientific notation, multiply the decimal parts and add the exponents.

$$(A \times 10^m)(B \times 10^n) = (A \times B) \times 10^{m+n}$$

To divide numbers expressed in scientific notation, divide the decimal parts and subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{(A \times 10^m)}{(B \times 10^n)} = \left(\frac{A}{B}\right) \times 10^{m-n}$$

Consider the following example involving multiplication:

$$\begin{aligned}(3.5 \times 10^4)(1.8 \times 10^6) &= (3.5 \times 1.8) \times 10^{4+6} \\ &= 6.3 \times 10^{10}\end{aligned}$$

Consider the following example involving division:

$$\begin{aligned}\frac{(5.6 \times 10^7)}{(1.4 \times 10^3)} &= \left(\frac{5.6}{1.4}\right) \times 10^{7-3} \\ &= 4.0 \times 10^4\end{aligned}$$

Addition and Subtraction

To add or subtract numbers expressed in scientific notation, rewrite all the numbers so that they have the same exponent, then add or subtract the decimal parts of the numbers. The exponents remained unchanged.

$$\begin{aligned}A &\times 10^n \\ \pm B &\times 10^n \\ \hline (A \pm B) &\times 10^n\end{aligned}$$

Notice that the numbers *must have* the same exponent. Consider the following example involving addition:

$$\begin{aligned}4.82 &\times 10^7 \\ +3.4 &\times 10^6\end{aligned}$$

First, express both numbers with the same exponent. In this case, we rewrite the lower number and perform the addition as follows:

$$
\begin{array}{r}
4.82 \times 10^7 \\
+0.34 \times 10^7 \\
\hline
5.16 \times 10^7
\end{array}
$$

Consider the following example involving subtraction:

$$
\begin{array}{r}
7.33 \times 10^5 \\
-1.9 \times 10^4 \\
\hline
\end{array}
$$

First, express both numbers with the same exponent. In this case, we rewrite the lower number and perform the subtraction as follows:

$$
\begin{array}{r}
7.33 \times 10^5 \\
-0.19 \times 10^5 \\
\hline
7.14 \times 10^5
\end{array}
$$

Powers and Roots

To raise a number written in scientific notation to a power, raise the decimal part to the power and multiply the exponent by the power:

$$
\begin{aligned}
\left(4.0 \times 10^6\right)^2 &= 4.0^2 \times 10^{6 \times 2} \\
&= 16 \times 10^{12} \\
&= 1.6 \times 10^{13}
\end{aligned}
$$

To take the nth root of a number written in scientific notation, take the nth root of the decimal part and divide the exponent by the root:

$$
\begin{aligned}
\left(4.0 \times 10^6\right)^{1/3} &= 4.0^{1/3} \times 10^{6/3} \\
&= 1.6 \times 10^2
\end{aligned}
$$

B. Logarithms

Common (or Base 10) Logarithms

The common or base 10 logarithm (abbreviated log) of a number is the exponent to which 10 must be raised to obtain that number. For example, the log of 100 is 2 because 10 must be raised to the second power to get 100. Similarly, the log of 1000 is 3 because 10 must be raised to the third power to get 1000. The logs of several multiples of 10 are shown below:

$$
\begin{aligned}
\log 10 &= 1 \\
\log 100 &= 2 \\
\log 1000 &= 3 \\
\log 10,000 &= 4
\end{aligned}
$$

Because $10^0 = 1$ by definition, $\log 1 = 0$.

The log of a number smaller than one is negative because 10 must be raised to a negative exponent to get a number smaller than one. For example, the log of 0.01 is –2 because 10 must be raised to –2 to get 0.01. Similarly, the log of 0.001 is –3 because 10 must be raised to –3 to get 0.001. The logs of several fractional numbers are shown below:

$$
\begin{aligned}
\log 0.1 &= -1 \\
\log 0.01 &= -2 \\
\log 0.00 &= -3
\end{aligned}
$$

$$\log 0.0001 = -4$$

The logs of numbers that are not multiples of 10 can be computed on your calculator. See your calculator manual for specific instructions.

Inverse Logarithms

The inverse logarithm or invlog function is exactly the opposite of the log function. For example, the log of 100 is 2 and the inverse log of 2 is 100. The log function and the invlog function undo one another.

$$\log 100 = 2$$
$$\text{invlog } 2 = 100$$
$$\text{invlog} \left(\log 100 \right) = 100$$

The inverse log of a number is 10 rasied to that number.

$$\text{invlog } x = 10^x$$
$$\text{invlog } 3 = 10^3 = 1000$$

The inverse logs of numbers can be computed on your calculator. See your calculator manual for specific instructions.

Natural (or Base e) Logarithms

The natural (or base e) logarithm (abbreviated ln) of a number is the exponent to which e (which has the value of 2.71828…) must be raised to obtain that number. For example, the ln of 100 is 4.605 because e must be raised to 4.605 to get 100. Similarly, the ln of 10.0 is 2.303 because e must be raised to 2.303 to get 10.0.

The inverse natural logarithm or invln function is exactly the opposite of the ln function. For example, the ln of 100 is 4.605 and the inverse ln of 4.605 is 100. The inverse ln of a number is simply e raised to that number.

$$\text{invln } x = e^x$$
$$\text{invln } 3 = e^3 = 20.1$$

The invln of a number can be computed on your calculator. See your calculator manual for specific instructions.

Mathematical Operations Using Logarithms

Because logarithms are exponents, mathematical operations involving logarithms are similar to those involving exponents as follows:

$$\log \left(a \times b \right) = \log a + \log b \qquad \ln \left(a \times b \right) = \ln a + \ln b$$
$$\log \frac{a}{b} = \log a - \log b \qquad \ln \frac{a}{b} = \ln a - \ln b$$
$$\log a^n = n \log a \qquad \ln a^n = n \ln a$$

C. Quadratic Equations

A quadratic equation contains at least one term in which the variable x is raised to the second power (and no terms in which x is raised to a higher power). A quadratic equation has the following general form:

$$ax^2 + bx + c = 0$$

A quadratic equation can be solved for x using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic equations are often encountered when solving equilibrium problems. Below we show how to use the quadratic formula to solve a quadratic equation for x.

$$3x^2 - 5x + 1 = 0 \quad \text{(quatratic equaion)}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$$
$$= \frac{5 \pm 3.6}{6}$$
$$x = 1.43 \quad \text{or} \quad x = 0.233$$

As you can see, the solution to a quadratic equation usually has two values. In any real chemical system, one of the values can be eliminated because it has no physical significance. (For example, it may correspond to a negative concentration, which does not exist.)

D. Graphs

Graphs are often used to visually show the relationship between two variables. For example, in Chapter 11 we show the following relationship between the volume of a gas and its pressure:

Volume versus Pressure

A plot of the volume of a gas sample—as measured in a J-tube—versus pressure. The plot shows that volume and pressure are inversely related.
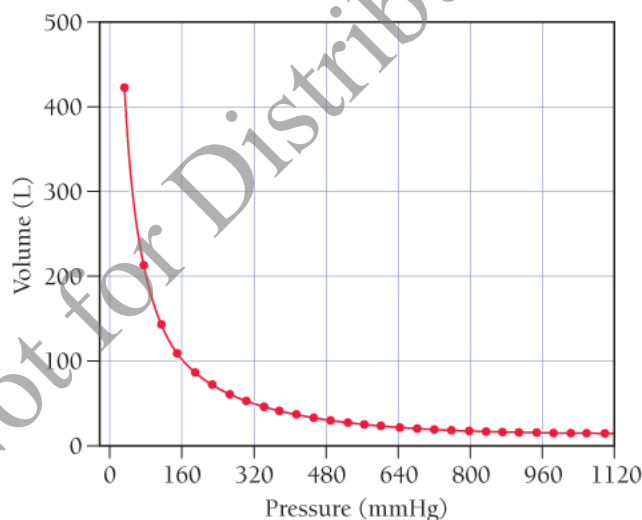

The horizontal axis is the x-axis and is normally used to show the independent variable. The vertical axis is the y-axis and is normally used to show how the other variable (called the dependent variable) varies with a change in the independent variable. In this case, the graph shows that as the pressure of a gas sample increases, its volume decreases.

Many relationships in chemistry are *linear*, which means that if you change one variable by a factor of n the other variable will also change by a factor of n. For example, the volume of a gas is linearly related to the number of moles of gas. When two quantities are linearly related, a graph of one versus the other produces a straight line. For example, the graph below shows how the volume of an ideal gas sample depends on the number of moles of gas in the sample:

Volume versus Number of Moles

The volume of a gas sample increases linearly with the number of moles of gas in the sample.
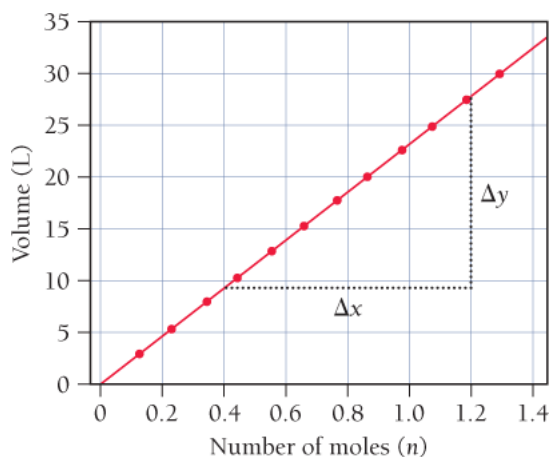
A linear relationship between any two variables x and y can be expressed by the following equation:

$$y = mx + b$$

where m is the slope of the line and b is the y-intercept. The slope is the change in y divided by the change in x.

$$m = \frac{\Delta x}{\Delta y}$$

For the graph above, we can estimate the slope by simply estimating the changes in y and x for a given interval. For example, between $x = 0.4$ and 1.2 mol, $\Delta x = 0.80$, and we can estimate that $\Delta y = 18$ L. Therefore, the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{18 \text{ L}}{0.80 \text{ mol}} = 23 \text{ L/mol}$$

In several places in this book, logarithmic relationships between variables can be plotted in order to obtain a linear relationship. For example, the variables $[A]_t$ and t in the following equation are not linearly related, but the natural logarithm of $[A]_t$ and t are linearly related.

$$\begin{aligned} \ln[A]_t &= -kt + \ln[A]_0 \\ y &= mx + b \end{aligned}$$

A plot of $\ln[A]_t$ versus t will therefore produce a straight line with slope $= -k$ and y-intercept $= \ln[A]_0$.