Pearson

1.10: Atoms and the Mole: How Many Particles?

Key Concept Video **The Mole Concept**

My 7-year-old sometimes asks, "How much eggs did the chickens lay?" or "How much pancakes do I get?" My wife immediately corrects him, "Do you mean how *many* eggs?" The difference between "how many" and "how much" depends on what you are specifying. If you are specifying something countable, such as eggs or pancakes, you say "how many." But if you are specifying something noncountable, such as water or milk, you say "how much."

Although samples of matter may seem noncountable—we normally say how *much* water—we know that *all matter is ultimately particulate and countable*. Even more importantly, when samples of matter interact with one another, they interact *particle by particle*. For example, when hydrogen and oxygen combine to form water, two hydrogen atoms combine with one oxygen atom to form one water molecule. Therefore, as chemists, we often ask of a sample of matter, not only *how much*, but also *how many*—how many particles does the sample contain?

The particles that compose matter are far too small to count by any ordinary means. Even if we could somehow count atoms, and counted them 24 hours a day for as long as we lived, we would barely begin to count the number of atoms in something as small as a grain of sand. Therefore, if we want to know the number of atoms in anything of ordinary size, we must count them by weighing.

As an analogy, consider buying shrimp at your local fish market. Shrimp is normally sold by count, which indicates the number of shrimp per pound. For example, for 41–50 count shrimp there are between 41 and 50 shrimp per pound. The smaller the count, the larger the shrimp. Big tiger prawns have counts as low as 10–15, which means that each shrimp can weigh up to 1/10 of a pound. One advantage of categorizing shrimp this way is that we can count the shrimp by weighing them. For example, two pounds of 41–50 count shrimp contain between 82 and 100 shrimp. A similar concept exists for the particles that compose matter. We can determine the number of particles in a sample of matter from the mass of the sample.

The Mole: A Chemist's "Dozen"

When we count large numbers of objects, we use units such as a dozen (12 objects) or a gross (144 objects) to organize our counting and to keep our numbers more manageable. With atoms, quadrillions of which may be in a speck of dust, we need a much larger number for this purpose. The chemist's "dozen" is the **mole** (abbreviated mol). A mole is the *amount* of material containing 6.02214×10^{23} particles.

$$1 \text{ mol} = 6.02214 \times 10^{23} \text{ particles}$$

This number is **Avogadro's number**, named after Italian physicist Amedeo Avogadro (1776–1856), and is a convenient number to use when working with atoms, molecules, and ions. In this book, we usually round Avogadro's number to four significant figures or 6.022×10^{23}. Notice that the definition of the mole is an *amount* of a substance. We will often refer to the number of moles of substance as the *amount* of the substance.

The first thing to understand about the mole is that it can specify Avogadro's number of anything. For example, 1 mol of marbles corresponds to 6.022×10^{23} marbles, and 1 mol of sand grains corresponds to 6.022×10^{23} sand grains. *One mole of anything is* 6.022×10^{23} *units of that thing.* One mole of atoms, ions, or molecules, however, makes up objects of everyday sizes. Twenty-two copper pennies, for example, contain approximately 1 mol of copper atoms, and one tablespoon of water contains approximately 1 mol of water molecules.

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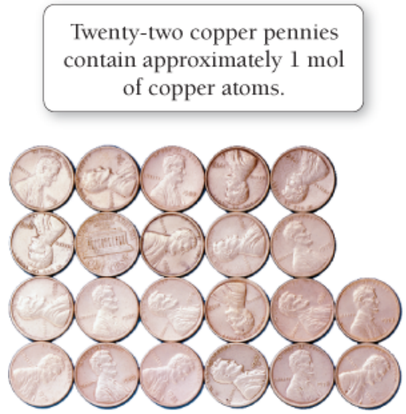


Before 1982, when they became almost all zinc with only a copper coating, pennies were mostly copper.

One tablespoon is approximately 15 mL; one mole of water occupies 18 mL.

The second, and more fundamental, thing to understand about the mole is how it gets its specific value:

> **The value of the mole is equal to the number of atoms in exactly 12 g of pure carbon-12 $\left(12 \text{ g C-12} = 1 \text{ mol C-12 atoms} = 6.022 \times 10^{23} \text{ C-12 atoms}\right)$.**

The definition of the mole gives us a relationship between mass (grams of carbon) and number of atoms (Avogadro's number). This relationship, as we will see shortly, allows us to count atoms by weighing them.

Converting between Number of Moles and Number of Atoms

Converting between number of moles and number of atoms is similar to converting between dozens of eggs and number of eggs. For eggs, we use the conversion factor $1 \text{ dozen eggs} = 12 \text{ eggs}$. For atoms, we use the conversion factor $1 \text{ mol atoms} = 6.022 \times 10^{23} \text{ atoms}$. The conversion factors take the form:

$$\frac{1 \text{ mol atoms}}{6.022 \times 10^{23} \text{ atoms}} \text{ or } \frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mol atoms}}$$

Example 1.5 demonstrates how to use these conversion factors in calculations.

Example 1.5 Converting between Number of Moles and Number of Atoms

Calculate the number of copper atoms in 2.45 mol of copper.

SORT You are given the amount of copper in moles and asked to find the number of copper atoms.

GIVEN: 2.45 mol Cu

FIND: Cu atoms

STRATEGIZE Convert between number of moles and number of atoms using Avogadro's number as a conversion factor.

CONCEPTUAL PLAN

$$\text{mol Cu} \xrightarrow{\frac{6.022 \times 10^{23}\ \text{Cu atoms}}{1\ \text{mol Cu}}} \text{number of Cu atoms}$$

RELATIONSHIPS USED

$$6.022 \times 10^{23} = 1\ \text{mol (Avogadro's number)}$$

SOLVE Follow the conceptual plan to solve the problem. Begin with 2.45 mol Cu and multiply by Avogadro's number to get to the number of Cu atoms.

SOLUTION

$$2.45\ \text{mol Cu} \times \frac{6.022 \times 10^{23}\ \text{Cu atoms}}{1\ \text{mol Cu}} = 1.48 \times 10^{24}\ \text{Cu atoms}$$

CHECK Since atoms are small, it makes sense that the answer is large. The given number of moles of copper is almost 2.5, so the number of atoms is almost 2.5 times Avogadro's number.

FOR PRACTICE 1.5 A pure silver ring contains 2.80×10^{22} silver atoms. How many moles of silver atoms does it contain?

Converting between Mass and Amount (Number of Moles)

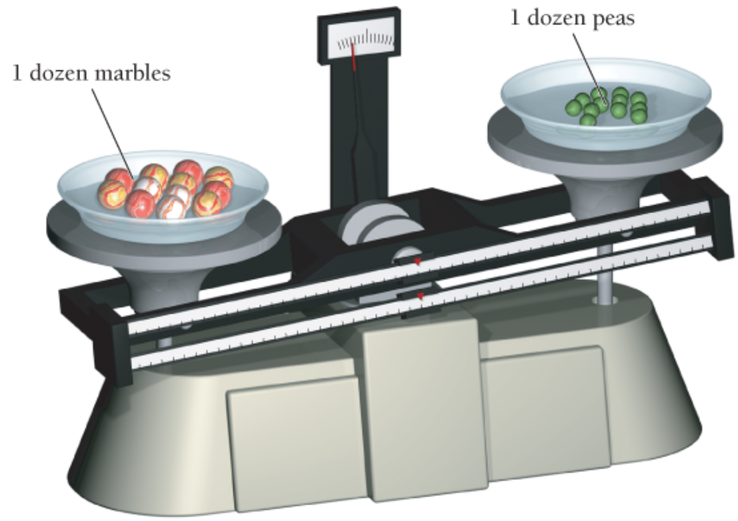
To count atoms by weighing them, we need one other conversion factor—the mass of 1 mol of atoms. For the isotope carbon-12, we know that the mass of 1 mol of atoms is exactly 12 g, which is numerically equivalent to carbon-12's atomic mass in atomic mass units. Since the masses of all other elements are defined relative to carbon-12, the same relationship holds for all elements. The mass of 1 mol of atoms of an element is its **molar mass**.

> **An element's molar mass in grams per mole is numerically equal to the element's atomic mass in atomic mass units.**

For example, copper has an atomic mass of 63.55 amu and a molar mass of 63.55 g/mol. One mole of copper atoms therefore has a mass of 63.55 g. Just as the count for shrimp depends on the size of the shrimp, the mass of 1 mol of atoms depends on the element (Figure 1.13): 1 mol of aluminum atoms (which are lighter than copper atoms) has a mass of 26.98 g; 1 mol of carbon atoms (which are even lighter than aluminum atoms) has a mass of 12.01 g; and 1 mol of helium atoms (lighter yet) has a mass of 4.003 g: The lighter the atom, the less mass in 1 mol of atoms.

Figure 1.13 Molar Mass

The two dishes contain the same number of objects (12), but the masses are different because peas are less massive than marbles. Similarly, a mole of light atoms has less mass than a mole of heavier atoms.

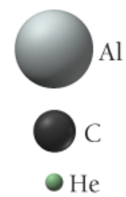


Same number of particles, but different masses

$$26.98 \text{ g aluminum} = 1 \text{ mol aluminum} = 6.022 \times 10^{23} \text{ Al atoms}$$

$$12.01 \text{ g carbon} = 1 \text{ mol carbon} = 6.022 \times 10^{23} \text{ C atoms}$$

$$4.003 \text{ g helium} = 1 \text{ mol helium} = 6.022 \times 10^{23} \text{ He atoms}$$



The molar mass of any element is the conversion factor between the mass (in grams) of that element and the amount (in moles) of that element. For carbon:

$$12.01 \text{ g C or } \frac{12.01 \text{ g C}}{\text{mol C}} \text{ or } \frac{1 \text{ mol C}}{12.01 \text{ g C}}$$

Example 1.6 demonstrates how to use these conversion factors.

Example 1.6 Converting between Mass and Amount (Number of Moles)

Calculate the amount of carbon (in moles) contained in a 0.0265-g pencil "lead." (Assume that the pencil "lead" is made of pure graphite, a form of carbon.)

SORT You are given the mass of carbon and asked to find the amount of carbon in moles.

GIVEN: 0.0265 g C

FIND: mol C

STRATEGIZE Convert between mass and amount (in moles) of an element using the molar mass of the element.

CONCEPTUAL PLAN

$$\text{g C} \xrightarrow{\frac{1 \text{ mol}}{12.01 \text{ g}}} \text{mol C}$$

RELATIONSHIPS USED

12.01 g C =1 mol C (carbon molar mass)

SOLVE Follow the conceptual plan to solve the problem.

SOLUTION

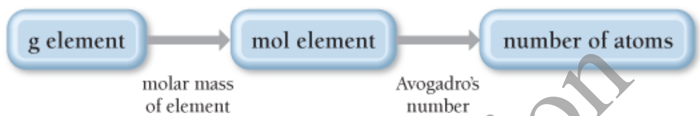
$$0.0265 \text{ g C} \times \frac{1 \text{ mol C}}{12.01 \text{ g C}} = 2.21 \times 10^{-3} \text{ mol C}$$

CHECK The given mass of carbon is much less than the molar mass of carbon, so it makes sense that the answer (the amount in moles) is much less than 1 mol of carbon.

FOR PRACTICE 1.6 Calculate the amount of copper (in moles) in a 35.8-g pure copper sheet.

FOR MORE PRACTICE 1.6 Calculate the mass (in grams) of 0.473 mol of titanium.

We now have all the tools to count the number of atoms in a sample of an element by weighing it. First, we obtain the mass of the sample. Then we convert it to the amount in moles using the element's molar mass. Finally, we convert to number of atoms using Avogadro's number. The conceptual plan for these kinds of calculations is:



Examples 1.7 and 1.8 demonstrate these conversions.

Example 1.7 The Mole Concept—Converting between Mass and Number of Atoms

How many copper atoms are in a copper penny with a mass of 3.10 g? (Assume that the penny is composed of pure copper.)

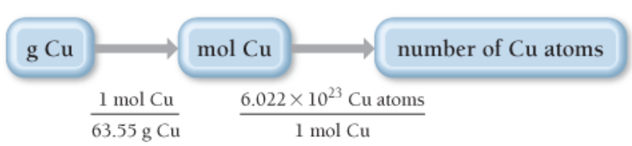
SORT You are given the mass of copper and asked to find the number of copper atoms.

GIVEN: 3.10 g Cu

FIND: Cu atoms

STRATEGIZE Convert between the mass of an element in grams and the number of atoms of the element by first converting to moles (using the molar mass of the element) and then to number of atoms (using Avogadro's number).

CONCEPTUAL PLAN



RELATIONSHIPS USED

$$63.55 \text{ g Cu} = 1 \text{ mol Cu (molar mass of copper)}$$
$$6.022 \times 10^{23} = 1 \text{ mol (Avogadro's number)}$$

SOLVE Follow the conceptual plan to solve the problem. Begin with 3.10 g Cu and multiply by the appropriate conversion factors to arrive at the number of Cu atoms.

SOLUTION

$$3.10 \text{ g Cu} \times \frac{1 \text{ mol Cu}}{63.55 \text{ g Cu}} \times \frac{6.022 \times 10^{23} \text{ Cu atoms}}{1 \text{ mol Cu}} = 2.94 \times 10^{22} \text{ Cu atoms}$$

CHECK The answer (the number of copper atoms) is less than 6.022×10^{23} (one mole). This is consistent with the given mass of copper atoms, which is less than the molar mass of copper.

FOR PRACTICE 1.7 How many carbon atoms are there in a 1.3-carat diamond? Diamonds are a form of pure carbon. $(1 \text{ carat} = 0.20 \text{ g})$

FOR MORE PRACTICE 1.7 Calculate the mass of 2.25×10^{22} tungsten atoms.

Interactive Worked Example 1.7 **The Mole Concept—Converting between Mass and Number of Atoms**

Notice that numbers with large exponents, such as 6.022×10^{23}, are almost unbelievably large. Twenty-two copper pennies contain 6.022×10^{23} or 1 mol of copper atoms, but 6.022×10^{23} pennies would cover the Earth's entire surface to a depth of 300 m. Even objects that are small by everyday standards occupy a huge space when we have a mole of them. For example, a grain of sand has a mass of less than 1 mg and a diameter less than 0.1 mm, yet 1 mol of sand grains would cover the state of Texas to a depth of several feet. For every increase of 1 in the exponent of a number, the number increases by a factor of 10, so 10^{23} is incredibly large. Of course, one mole has to be a large number if it is to have practical value because atoms are so small.

Example 1.8 The Mole Concept

An aluminum sphere contains 8.55×10^{22} aluminum atoms. What is the sphere's radius in centimeters? The density of aluminum is 2.70 g/cm^3.

SORT You are given the number of aluminum atoms in a sphere and the density of aluminum. You are asked to find the radius of the sphere.

GIVEN: 8.55×10^{22} Al atoms

$d = 2.70 \text{ g/cm}^3$

FIND: radius (r) of sphere

STRATEGIZE The heart of this problem is density, which relates mass to volume, and though you aren't given the mass directly, you are given the number of atoms, which you can use to find mass.

1. Convert from number of atoms to number of moles using Avogadro's number as a conversion factor.
2. Convert from number of moles to mass using molar mass as a conversion factor.
3. Convert from mass to volume (in cm^3) using density as a conversion factor.
4. Once you calculate the volume, find the radius from the volume using the formula for the volume of a sphere.

CONCEPTUAL PLAN

number of Al atoms → mol Al → g Al → V (in cm^3)

$$\frac{1 \text{ mol Al}}{6.022 \times 10^{23} \text{ Al atoms}} \qquad \frac{26.98 \text{ g Al}}{1 \text{ mol Al}} \qquad \frac{1 \text{ cm}^3}{2.70 \text{ g Al}}$$

$$V \text{ (in cm}^3) \xrightarrow{\quad} r$$

$$V = \frac{4}{3}\pi r^3$$

RELATIONSHIPS AND EQUATIONS USED

$6.022 \times 10^{23} = 1$ mol (Avogadro's number)
26.98 g Al = 1 mol Al (molar mass of aluminum)

2.70 g/cm^3 (density of aluminum)
$V = \frac{4}{3}\pi r^3$ (volume of sphere)

SOLVE Finally, follow the conceptual plan to solve the problem. Begin with 8.55×10^{22} Al atoms and multiply by the appropriate conversion factors to arrive at volume in cm^3.

Then solve the equation for the volume of a sphere for *r* and substitute the volume to calculate *r*.

SOLUTION

$$8.55 \times 10^{22} \text{ Al atoms} \times \frac{1 \text{ mol Al}}{6.022 \times 10^{23} \text{ Al atoms}} \times \frac{26.98 \text{ g Al}}{1 \text{ mol Al}} \times \frac{1 \text{ cm}^3}{2.70 \text{ g Al}} = 1.41\bar{8}7 \text{ cm}^3$$

$$V = \frac{4}{3}\pi r^3$$

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3\left(1.41\bar{8}7 \text{ cm}^3\right)}{4\pi}} = 0.697 \text{ cm}$$

CHECK The units of the answer (cm) are correct. The magnitude cannot be estimated accurately, but a radius of about one-half of a centimeter is reasonable for just over one-tenth of a mole of aluminum atoms.

FOR PRACTICE 1.8 A titanium cube contains 2.86×10^{23} atoms. What is the edge length of the cube? The density of titanium is 4.50 g/cm^3.

FOR MORE PRACTICE 1.8 Find the number of atoms in a copper rod with a length of 9.85 cm and a radius of 1.05 cm. The density of copper is 8.96 g/cm^3.

Interactive Worked Example 1.8 **The Mole Concept**

Conceptual Connection 1.9 **The Mole**

Interactive