Pearson

# E.4: Significant Figures in Calculations

The precision of a measurement must be preserved, not only when recording the measurement, but also when performing calculations that use the measurement. We preserve this precision by using *significant figures*.

## Counting Significant Figures

In any reported measurement, the non-place-holding digits—those that are not simply marking the decimal place—are called **significant figures** (or **significant digits**). For example, the number 23.5 has three significant figures while the number 23.56 has four. *The greater the number of significant figures, the greater the certainty of the measurement*.

To determine the number of significant figures in a number containing zeroes, we distinguish between zeroes that are significant and those that simply mark the decimal place. For example, in the number 0.0008, the leading zeroes (zeroes to the left of the first nonzero digit) mark the decimal place but *do not* add to the certainty of the measurement and are therefore not significant; this number has only one significant figure. In contrast, the trailing zeroes (zeroes at the end of a number) in the number 0.000800 *do add* to the certainty of the measurement and are therefore counted as significant; this number has three significant figures.



**To determine the number of significant figures in a number, follow these rules.**

### Examples Significant Figure Rules

**1.** All nonzero digits are significant.

28.03 0.0540

**2.** Interior zeroes (zeroes between two nonzero digits) are significant.

408 7.0301

**3.** Leading zeroes (zeroes to the left of the first nonzero digit) are not significant. They only serve to locate the decimal point.

0.0032 0.00006

not significant

**4.** Trailing zeroes (zeroes at the end of a number) are categorized as follows:

- Trailing zeroes after a decimal point are always significant.
  45.000 3.5600
- Trailing zeroes before a decimal point (and after a nonzero number) are always significant.
  140.00 2500.55
- Trailing zeroes before an implied decimal point are ambiguous and should be avoided by using scientific notation.

| | |
|---|---|
| 1200 | ambiguous |
| $1.2 \times 10^3$ | 2 significant figures |
| $1.20 \times 10^3$ | 3 significant figures |
| $1.200 \times 10^3$ | 4 significant figures |

- Some textbooks put a decimal point after one or more trailing zeroes if the zeroes are to be considered

- Some textbooks put a decimal point after one or more trailing zeroes if the zeroes are to be considered significant. We avoid that practice in this book, but you should be aware of it.

  1200. 4 significant figures (common in some textbooks)

## Exact Numbers

**Exact numbers** have no uncertainty and thus do not limit the number of significant figures in any calculation. We regard an exact number as having an unlimited number of significant figures. Exact numbers originate from three sources:

- From the accurate counting of discrete objects. For example, 14 pencils means 14.0000 … pencils and 3 atoms means 3.00000 … atoms.
- From defined quantities, such as the number of centimeters in 1 m. Because 100 cm is defined as 1 m,

$$100\ \text{cm} = 1\ \text{m means } 100.00000\ \ldots\ \text{cm} = 1.0000000\ \ldots\ \text{m}$$

- From integral numbers that are part of an equation. For example, in the equation, $radius = \frac{diameter}{2}$, the number 2 is exact and therefore has an unlimited number of significant figures.

### Example E.3 Determining the Number of Significant Figures in a Number

How many significant figures are in each number?

**a.** 0.04450 m
**b.** 5.0003 km
**c.** 10 dm = 1 m
**d.** $1.000 \times 10^5$ s
**e.** 0.00002 mm
**f.** 10,000 m

**SOLUTION**

**(a)** 0.04450 m

*Four significant figures*. The two 4s and the 5 are significant (Rule 1). The trailing zero is after a decimal point and is therefore significant (Rule 4). The leading zeroes only mark the decimal place and are therefore not significant (Rule 3).

**(b)** 5.0003 km

*Five significant figures.* The 5 and 3 are significant (Rule 1), as are the three interior zeroes (Rule 2).

**(c)** 10 dm = 1 m

*Unlimited significant figures*. Defined quantities have an unlimited number of significant figures.

**(d)** $1.000 \times 10^5$ s

*Four significant figures*. The 1 is significant (Rule 1). The trailing zeroes are after a decimal point and therefore significant (Rule 4).

**(e)** 0.00002 mm

*One significant figure*. The 2 is significant (Rule 1). The leading zeroes only mark the decimal place and are therefore not significant (Rule 3).

**(f)** 10,000 m

*Ambiguous*. The 1 is significant (Rule 1), but the trailing zeroes occur before an implied decimal point and are therefore ambiguous (Rule 4). Without more information, you would assume one significant figure. It is better to write this as $1 \times 10^5$ to indicate one significant figure or as $1.0000 \times 10^5$ to indicate five (Rule 4).

**FOR PRACTICE E.3**

How many significant figures are in each number?

**a.** 554 km
**b.** 7 pennies
**c.** $1.01 \times 10^5$ m
**d.** 0.00099 s
**e.** 1.4500 km
**f.** 21,000 m

Interactive Worked Example E.3 **Determining the Number of Significant Figures in a Number**

## Significant Figures in Calculations



When we use measured quantities in calculations, the results of the calculation must reflect the precision of the measured quantities. We should not lose or gain precision during mathematical operations. Follow these rules when carrying significant figures through calculations.

### **Examples** Rules for Calculations

**1.** In multiplication or division, the result carries the same number of significant figures as the factor with the fewest significant figures.

$$\underset{(4 \text{ sig. figures})}{1.052} \times \underset{(5 \text{ sig. figures})}{12.054} \times \underset{(2 \text{ sig. figures})}{0.53} = 6.7208 = \underset{(2 \text{ sig. figures})}{6.7}$$

$$\underset{(5 \text{ sig. figures})}{2.0035} \div \underset{(3 \text{ sig. figures})}{3.20} = 0.626094 = \underset{(3 \text{ sig. figures})}{0.626}$$

**2.** In addition or subtraction, the result carries the same number of decimal places as the quantity with the fewest decimal places.

$$\begin{array}{r} 2.345 \\ 0.07 \\ +2.9975 \\ \hline 5.4125 \end{array} = 5.41 \qquad \begin{array}{r} 5.9 \\ -0.221 \\ \hline 5.679 \end{array} = 5.7$$

In addition and subtraction, it is helpful to draw a line next to the number with the fewest decimal places. This line determines the number of decimal places in the answer.

**3.** When rounding to the correct number of significant figures, round down if the last (or leftmost) digit dropped is 4 or less; round up if the last (or leftmost) digit dropped is 5 or more.

To two significant figures:

5.37 rounds to 5.4
5.34 rounds to 5.3
5.35 rounds to 5.4
5.349 rounds to 5.3

Notice in the last example that only the *last (or leftmost) digit being dropped* determines in which direction to round—ignore all digits to the right of it.

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A few books recommend a slightly different rounding procedure for cases where the last digit is 5. However, the procedure presented here is consistent with electronic calculators, and we use it throughout this book.

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**4.** To avoid rounding errors in multistep calculations, round only the final answer—do not round intermediate steps. If you write down intermediate answers, keep track of significant figures by underlining the least significant digit.

$$\begin{aligned} &6.78 \times 5.903 \times (5.489 - 5.01) \\ &= 6.78 \times 5.903 \times 0.47\underline{9} \\ &= 19.1707 \\ &= 19 \end{aligned}$$

underline least significant digit

Notice that for multiplication or division, the quantity with the fewest *significant figures* determines the number of *significant figures* in the answer, but for addition and subtraction, the quantity with the fewest *decimal places* determines the number of *decimal places* in the answer. In multiplication and division, we focus on significant figures, but in addition and subtraction we focus on decimal places. When a problem involves addition or subtraction, the answer may have a different number of significant figures than the initial quantities. Keep this in mind in problems that involve both addition or subtraction and multiplication or division. For example,

$$\frac{1.002 - 0.999}{3.754} = \frac{0.003}{3.754} = 7.99 \times 10^{-4} = 8 \times 10^{-4}$$

The answer has only one significant figure, even though the initial numbers had three or four significant figures.



## Example E.4 Significant Figures in Calculations

Perform each calculation to the correct number of significant figures.

**a.** $1.10 \times 0.5120 \times 4.0015 \div 3.4555$

**b.** $\begin{array}{r} 0.355 \\ +105.1 \\ \underline{-100.5820} \end{array}$

**c.** $4.562 \times 3.99870 \div (452.6755 - 452.33)$

**d.** $(14.84 \times 0.55) - 8.02$

**SOLUTION**

**a.** Round the intermediate result (in blue) to three significant figures to reflect the three significant figures in the least precisely known quantity (1.10).

$$
\begin{aligned}
&1.10 \times 0.5120 \times 4.0015 \div 3.4555 \\
&\quad = 0.65219 \\
&\quad = 0.652
\end{aligned}
$$

**b.** Round the intermediate answer (in blue) to one decimal place to reflect the quantity with the fewest decimal places (105.1). Notice that 105.1 is not the quantity with the fewest significant figures, but it has the fewest decimal places and therefore determines the number of decimal places in the answer.

$$
\begin{array}{r}
0.355 \\
+105.1 \\
-\ 100.5820 \\
\hline
4.8730
\end{array} = 4.9
$$

**c.** Mark the intermediate result to two decimal places to reflect the number of decimal places in the quantity within the parentheses having the smallest number of decimal places (452.33). Round the final answer to two significant figures to reflect the two significant figures in the least precisely known quantity (0.3$\underline{4}$55).

$$
\begin{aligned}
&4.562 \times 3.99870 \div (452.6755 - 452.33) \\
&\quad = 4.562 \times 3.99870 \div 0.3\underline{4}55 \\
&\quad = 52.79904 \\
&\quad = 53
\end{aligned}
$$

2 places of the decimal

**d.** Mark the intermediate result to two significant figures to reflect the number of significant figures in the quantity within the parentheses having the smallest number of significant figures (0.55). Round the final answer to one decimal place to reflect the one decimal place in the least precisely known quantity (8.$\underline{1}$62)
.

$$
\begin{aligned}
&(14.84 \times 0.55) - 8.02 = 8.\underline{1}62 - 8.02 \\
&= 0.142 \\
&= 0.1
\end{aligned}
$$

**FOR PRACTICE E.4**

Perform each calculation to the correct number of significant figures.

**a.** $3.10007 \times 9.441 \times 0.0301 \div 2.31$

**b.**
$$
\begin{array}{r}
0.881 \\
+132.1 \\
-12.02 \\
\hline
\end{array}
$$

**c.** $2.5110 \times 21.20 \div (44.11 + 1.223)$

**d.** $(12.01 \times 0.3) + 4.811$

Interactive Worked Example E.4 **Significant Figures in Calculations**