E.8: Problem-Solving Strategies

Key Concept Video **Solving Chemical Problems**

Problem solving is one of the most important skills you will acquire in this course. No one succeeds in chemistry—or in life, really—without the ability to solve problems. Although there is no simple formula you can apply to every chemistry problem, you can learn problem-solving strategies and begin to develop some chemical intuition. Many of the problems you will solve in this course are *unit conversion problems*, where you are given one or more quantities in some unit and asked to convert them into different units (see Section E.7). Other problems require that you use *specific equations* to get to the information you are trying to find. In the sections that follow, you will find strategies to help you solve both of these types of problems. Of course, many problems contain both conversions and equations, requiring the combination of these strategies, and some problems require an altogether different approach.

In this book, we use a standard problem-solving procedure that you can adapt to many of the problems encountered in general chemistry and beyond. To solve any problem, you need to assess the information given in the problem and devise a way to get to the information asked for. In other words, you must:

- Identify the starting point (the ***given*** information).
- Identify the endpoint (what we must ***find***).
- Devise a way to get from the starting point to the endpoint using what is given as well as what you already know or can look up. (As we just discussed, we call this the ***conceptual plan***.)

In graphic form, we represent this progression as:

$$\text{Given} \rightarrow \text{Conceptual Plan} \rightarrow \text{Find}$$

One of the principal difficulties beginning students encounter when they try to solve problems in general chemistry is not knowing where to begin. While no problem-solving procedure is applicable to all problems, the following four-step procedure can be helpful in working through many of the numerical problems you encounter in this book:

1. **Sort.** Begin by sorting the information in the problem. *Given* information is the basic data provided by the problem—often one or more numbers with their associated units. *Find* indicates what the problem is asking you to find.
2. **Strategize.** This is usually the most challenging part of solving a problem. In this process, you must develop a *conceptual plan*—a series of steps that will get you from the given information to the information you are trying to find. You have already seen conceptual plans for simple unit conversion problems. Each arrow in a conceptual plan represents a computational step. On the left side of the arrow is the quantity you had before the step; on the right side of the arrow is the quantity you will have after the step; and below the arrow is the information you need to get from one to the other—the relationship between the quantities.

> Most problems can be solved in more than one way. The solutions in this book tend to be the most straightforward but certainly not the only ways to solve the problems.

Often such relationships take the form of conversion factors or equations. These may be given in the problem, in which case you will have written them down under "Given" in Step 1. Usually, however, you will need other information—which may include physical constants, formulas, or conversion factors—to help get you from what you are given to what you must find. This information comes from what you have learned or what you can look up in the chapter or in tables within the book.

In some cases, you may get stuck at the strategize step. If you cannot figure out how to get from the given information to the information you are asked to find, you might try working backwards. For example, you can look at the units of the quantity you are trying to find and try to find conversion factors to get to the units of the given quantity. You may even try a combination of strategies; work forwards, backwards, or some of both. If you persist, you will develop a strategy to solve the problem.

3. **Solve.** This is the easiest part of solving a problem. Once you set up the problem properly and devise a conceptual plan, you follow the plan to solve the problem. Carry out any mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
4. **Check.** This is the step beginning students most often overlook. Experienced problem solvers always ask, does this answer make sense? Are the units correct? Is the number of significant figures correct? When solving multistep problems, errors easily creep into the solution. You can catch most of these errors by simply checking the answer. For example, suppose you are calculating the number of atoms in a gold coin and end up with an answer of 1.1×10^{-6} atoms. Could the gold coin really be composed of one-millionth of one atom?

In Examples E.6 and E.7, we apply this problem-solving procedure to unit conversion problems. The procedure is summarized in the left column, and two examples of the procedure are provided in the middle and right columns. This three-column format is used in selected examples throughout this text. This format allows you to see how you can apply a particular procedure to two different problems. Work through one problem first (from top to bottom) and then apply the same procedure to the other problem. Recognizing the commonalities and differences between problems is a key part of developing problem-solving skills.

Example E.6 Unit Conversion

PROCEDURE FOR Solving Unit Conversion Problems

SORT Begin by sorting the information in the problem into *given* and *find*.

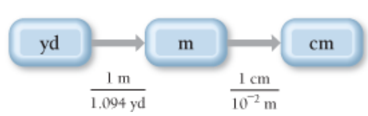
Convert 1.76 yards to centimeters.

GIVEN: 1.76 yd

FIND: cm

STRATEGIZE Devise a *conceptual plan* for the problem. Begin with the *given* quantity and symbolize each conversion step with an arrow. Below each arrow, write the appropriate conversion factor for that step. Focus on the units. The conceptual plan should end at the *find* quantity and its units. In these examples, the other information you need consists of relationships between the various units as shown.

CONCEPTUAL PLAN



RELATIONSHIPS USED

$$\begin{aligned} 1.094\ \text{yd} &= 1\ \text{m} \\ 1\ \text{cm} &= 10^{-2}\ \text{cm} \end{aligned}$$

(These conversion factors are in the inside back cover of your book.)

SOLVE Follow the conceptual plan. Begin with the *given* quantity and its units. Multiply by the appropriate conversion factor(s), canceling units, to arrive at the *find* quantity. Round the answer to the correct number of significant figures following guidelines in Section E.4. Remember that exact conversion factors do not limit significant figures.

SOLUTION

$$1.76\ \cancel{\text{yd}} \times \frac{1\ \cancel{\text{m}}}{1.094\ \cancel{\text{yd}}} \times \frac{1\ \text{cm}}{10^{-2}\ \cancel{\text{m}}}$$

$$= 160.8775 \text{ cm}$$
$$160.8775 \text{ cm} = 161 \text{ cm}$$

CHECK Check your answer. Are the units correct? Does the answer make sense?

The units (cm) are correct. The magnitude of the answer (161) makes sense because a centimeter is a much smaller unit than a yard.

FOR PRACTICE E.6

Convert 288 cm to yards.

Example E.7 Unit Conversion

PROCEDURE FOR Solving Unit Conversion Problems

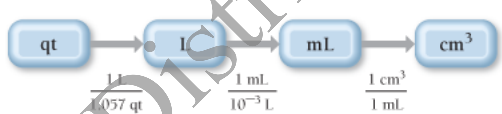
SORT Begin by sorting the information in the problem into *given* and *find*.

GIVEN: 1.8 qt

FIND: cm^3

STRATEGIZE Devise a *conceptual plan* for the problem. Begin with the *given* quantity and symbolize each conversion step with an arrow. Below each arrow, write the appropriate conversion factor for that step. Focus on the units. The conceptual plan should end at the *find* quantity and its units. In these examples, the other information you need consists of relationships between the various units as shown.

CONCEPTUAL PLAN



RELATIONSHIPS USED

$$1.057 \text{ qt} = 1 \text{ L}$$
$$1 \text{ mL} = 10^{-3} \text{ L}$$
$$1 \text{ mL} = 1 \text{ cm}^3$$

(These conversion factors are in the inside back cover of your book.)

SOLVE Follow the conceptual plan. Begin with the *given* quantity and its units. Multiply by the appropriate conversion factor(s), canceling units, to arrive at the *find* quantity. Round the answer to the correct number of significant figures following guidelines in Section E.4. Remember that exact conversion factors do not limit significant figures.

SOLUTION

$$1.8 \cancel{\text{qt}} \times \frac{1 \cancel{\text{L}}}{1.057 \cancel{\text{qt}}} \times \frac{1 \cancel{\text{mL}}}{10^{-3} \cancel{\text{L}}} \times \frac{1 \text{ cm}^3}{1 \cancel{\text{mL}}}$$
$$= 1.70293 \times 10^3 \text{ cm}^3$$
$$1.70293 \times 10^3 \text{ cm}^3 = 1.7 \times 10^3 \text{ cm}^3$$

CHECK Check your answer. Are the units correct? Does the answer make sense?

The units (cm^3) are correct. The magnitude of the answer (1700) makes sense because a cubic centimeter is a much smaller unit than a quart.

is a much smaller unit than a quart.

FOR PRACTICE E.7

Convert 9255 cm^3 to gallons.

Interactive Worked Example E.7 **Unit Conversion**

Units Raised to a Power

When building conversion factors for units raised to a power, remember to raise both the number and the unit to the power. For example, to convert from in^2 to cm^2, you construct the conversion factor as follows:

$$
\begin{aligned}
2.54\ \text{cm} &= 1\ \text{in} \\
(2.54\ \text{cm})^2 &= (1\ \text{in})^2 \\
(2.54)^2\ \text{cm}^2 &= 1^2\ \text{in}^2 \\
6.45\ \text{cm}^2 &= 1\ \text{in}^2 \\
\frac{6.45\ \text{cm}^2}{1\ \text{in}^2} &= 1
\end{aligned}
$$

Example E.8 demonstrates how to use conversion factors involving units raised to a power.

Example E.8 Unit Conversions Involving Units Raised to a Power

Calculate the displacement (the total volume of the cylinders through which the pistons move) of a 5.70-L automobile engine in cubic inches.

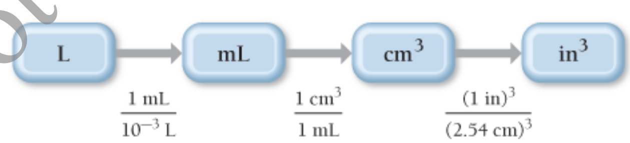
SORT Sort the information in the problem into *given* and *find*.

GIVEN: 5.70 L

FIND: in^3

STRATEGIZE Write a conceptual plan. Begin with the given information and devise a path to the information that you are asked to find. Notice that for cubic units, you must cube the conversion factors.

CONCEPTUAL PLAN



RELATIONSHIPS USED

$$
\begin{aligned}
1\ \text{mL} &= 10^{-3}\ \text{L} \\
1\ \text{mL} &= 1\ \text{cm}^3 \\
2.54\ \text{cm} &= 1\ \text{in}
\end{aligned}
$$

(These conversion factors are in the inside back cover of your book.)

SOLVE Follow the conceptual plan to solve the problem. Round the answer to three significant figures to reflect the three significant figures in the least precisely known quantity (5.70 L). These conversion factors are all exact and therefore do not limit the number of significant figures.

SOLUTION

$$5.70\ \cancel{\text{L}} \times \frac{1\ \cancel{\text{mL}}}{10^{-3}\ \cancel{\text{L}}} \times \frac{1\ \cancel{\text{cm}^3}}{1\ \cancel{\text{mL}}} \times \frac{(1\ \text{in})^3}{(2.54\ \cancel{\text{cm}})^3} = 347.835\ \text{in}^3$$
$$= 348\ \text{in}^3$$

CHECK The units of the answer are correct, and the magnitude makes sense. The unit cubic inches is smaller than liters, so the volume in cubic inches should be larger than the volume in liters.

FOR PRACTICE E.8

How many cubic centimeters are there in 2.11 yd^3?

FOR MORE PRACTICE E.8

A vineyard has 145 acres of Chardonnay grapes. A particular soil supplement requires 5.50 g for every square meter of vineyard. How many kilograms of the soil supplement are required for the entire vineyard? $(1\ \text{km}^2 = 247\ \text{acres})$

Interactive Worked Example E.8 **Unit Conversions Involving Units Raised to a Power**

Example E.9 Density as a Conversion Factor

The mass of fuel in a jet must be calculated before each flight to ensure that the jet is not too heavy to fly. A 747 is fueled with 173,231 L of jet fuel. If the density of the fuel is $0.768\ \text{g/cm}^3$, what is the mass of the fuel in kilograms?

SORT Begin by sorting the information in the problem into *given* and *find*.

GIVEN: fuel volume $= 173,231\ \text{L}$
density of fuel $= 0.768\ \text{g/cm}^3$

FIND: mass in kg

STRATEGIZE Draw the conceptual plan beginning with the given quantity, in this case the volume in liters (L). Your overall goal in this problem is to find the mass. You can convert between volume and mass using density (g/cm^3). However, you must first convert the volume to cm^3. Once you have converted the volume to cm^3, use the density to convert to g. Finally, convert g to kg.

CONCEPTUAL PLAN

$$\boxed{\text{L}} \xrightarrow[\frac{1\ \text{mL}}{10^{-3}\ \text{L}}]{} \boxed{\text{mL}} \xrightarrow[\frac{1\ \text{cm}^3}{1\ \text{mL}}]{} \boxed{\text{cm}^3} \xrightarrow[\frac{0.768\ \text{g}}{\text{cm}^3}]{} \boxed{\text{g}} \xrightarrow[\frac{1\ \text{kg}}{1000\ \text{g}}]{} \boxed{\text{kg}}$$

RELATIONSHIPS USED

$$\begin{aligned} 1\ \text{mL} &= 10^{-3}\ \text{L} \\ 1\ \text{mL} &= 1\ \text{cm}^3 \\ d &= 0.768\ \text{g/cm}^3 \\ 1000\ \text{g} &= 1\ \text{kg} \end{aligned}$$

(These conversion factors are in the inside back cover of your book.)

SOLVE Follow the conceptual plan to solve the problem. Round the answer to three significant figures to reflect the three significant figures in the density.

SOLUTION

$$173{,}231\ \cancel{\text{L}} \times \frac{1\ \cancel{\text{mL}}}{10^{-3}\ \cancel{\text{L}}} \times \frac{1\ \cancel{\text{cm}^3}}{1\ \cancel{\text{mL}}} \times \frac{0.768\ \cancel{\text{g}}}{1\ \cancel{\text{cm}^3}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}$$
$$= 1.33 \times 10^5\ \text{kg}$$

CHECK The units of the answer (kg) are correct. The magnitude makes sense because the mass $(1.33 \times 10^5\ \text{kg})$ is similar in magnitude to the given volume $(173{,}231\ \text{L or } 1.73231 \times 10^5\ \text{L})$, as you would expect for a density close to one $(0.768\ \text{g/cm}^3)$.

FOR PRACTICE E.9

Backpackers often use canisters of white gas to fuel a cooking stove's burner. If one canister contains 1.45 L of white gas and the density of the gas is $0.710\ \text{g/cm}^2$, what is the mass of the fuel in kilograms?

FOR MORE PRACTICE E.9

A drop of gasoline has a mass of 22 mg and a density of $0.754\ \text{g/cm}^3$. What is its volume in cubic centimeters?

Interactive Worked Example E.9 **Density as a Conversion Factor**

Order-of-Magnitude Estimations

Calculation is an integral part of chemical problem solving, but precise numerical calculation is not always necessary, or even possible. Sometimes data are only approximate; other times we do not need a high degree of precision—a rough estimate or a simplified "back of the envelope" calculation is enough. We can also use approximate calculations to get an initial feel for a problem, or as a quick check to see whether our solution is in the right ballpark.

One way to make such estimates is to simplify the numbers so that they can be manipulated easily. In the technique known as *order-of-magnitude estimation*, we focus only on the exponential part of numbers written in scientific notation, according to these guidelines:

- If the decimal part of the number is less than 5, we drop it. Thus, 4.36×10^5 becomes 10^5 and 2.7×10^{-3} becomes 10^{-3}.
- If the decimal part of the number is 5 or more, we round it up to 10 and rewrite the number as a power of 10. Thus, 5.982×10^7 becomes $10 \times 10^7 = 10^8$, and 6.1101×10^{-3} becomes $10 \times 10^{-3} = 10^{-2}$.

After we make these approximations, we are left with powers of 10, which are easier to multiply and divide. Of course, our answer is only as reliable as the numbers used to get it, so we should not assume that the results of an order-of-magnitude calculation are accurate to more than an order of magnitude.

Suppose, for example, that we want to estimate the number of atoms that an immortal being could have counted in the 13.7 billion (1.37×10^{10}) years that the universe has been in existence, assuming a counting rate of 10 atoms per second. Since a year has 3.2×10^7 seconds, we can approximate the number of atoms counted as:

$$10^{10}\ \cancel{\text{years}} \times 10^7 \frac{\cancel{\text{seconds}}}{\cancel{\text{year}}} \times 10^1 \frac{\text{atoms}}{\cancel{\text{second}}} \approx 10^{18}\ \text{atoms}$$

(number of years) (number of seconds (number of atoms (number of years) (number of seconds per year) (number of atoms counted per second)

A million trillion atoms (10^{18}) may seem like a lot, but a speck of matter made up of a million trillion atoms is nearly impossible to see without a microscope.

In our general problem-solving procedure, the last step is to check whether the results seem reasonable. Order-of-magnitude estimations can often help us to catch mistakes that we may make in a detailed calculation, such as entering an incorrect exponent or sign into a calculator, or multiplying when we should have divided.