Pearson

9.4: Quantifying Heat and Work

In the previous section, we calculated ΔE based on *given values of q (heat) and w (work)*. We now turn to *calculating q and w* based on changes in temperature and volume.

Heat

Key Concept Video **Heat Capacity**

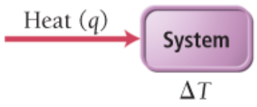
Recall from Section 9.2 that *heat* is the exchange of thermal energy between a system and its surroundings caused by a temperature difference. Notice the distinction we make between heat and temperature. Temperature is a *measure* of the thermal energy within a sample of matter. Heat is the *transfer* of thermal energy. Thermal energy always flows from hot to cold—from matter at higher temperatures to matter at lower temperatures. For example, a hot cup of coffee transfers thermal energy—as heat—to the lower temperature surroundings as it cools down.

Imagine a world where the cooler surroundings actually became colder as they transferred thermal energy to the hot coffee, which became hotter. Such a scenario is impossible because the spontaneous transfer of heat from hot to cold is a fundamental principle of our universe—no exception has ever been observed. The thermal energy in the molecules that compose the hot coffee distributes itself to the molecules in the surroundings. The heat transfer from the coffee to the surroundings stops when the two reach the same temperature, a condition called **thermal equilibrium**. At thermal equilibrium, there is no additional net transfer of heat.

> The reason for this one-way transfer (from hot to cold) is related to the second law of thermodynamics, which we will discuss in Chapter 18.

Temperature Changes and Heat Capacity

When a system absorbs heat (q), its temperature changes by ΔT, where $\Delta T = T_{\text{final}} - T_{\text{initial}}$.



Experiments show that the heat absorbed by a system and its corresponding temperature change are directly proportional: $q \alpha \Delta T$. The constant of proportionality between q and ΔT is the system's *heat capacity* (C), a measure of how much heat a system must absorb to undergo a specified change in temperature:

[9.4]

$$q = \underset{\substack{\uparrow \\ \text{Heat capacity}}}{C} \times \Delta T$$

Notice that the higher the heat capacity of a system, the smaller the change in temperature for a given amount of absorbed heat. We define the **heat capacity (*C*)** of a system as the quantity of heat required to change its temperature by 1 °C. As we can see by solving Equation 9.4 for heat capacity, the units of heat capacity are those of heat (typically J) divided by those of temperature (typically °C):

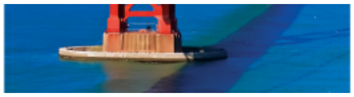
$$C = \frac{q}{\Delta T} = \frac{J}{°C}$$

In order to understand two important concepts related to heat capacity, consider a steel saucepan on a kitchen flame. The saucepan's temperature rises rapidly as it absorbs heat from the flame. However, if we add some water to the saucepan, the temperature rises more slowly. Why? The first reason is that, when we add the water, the same amount of heat must now warm more matter, so the temperature rises more slowly. In other words, heat capacity is an extensive property—*it depends on the amount of matter being heated* (see Section E.5). The second (and more fundamental) reason is that *water is more resistant to temperature change than steel*—water has an intrinsically higher capacity to absorb heat without undergoing a large temperature change. The measure of the *intrinsic capacity* of a substance to absorb heat is its **specific heat capacity** (C_s), the amount of heat required to raise the temperature of *1 gram* of the substance by 1 °C. The units of specific heat capacity (also called *specific heat*) are J/g · °C. Table 9.2 lists the values of the specific heat capacity for several substances. Heat capacity is also sometimes reported as **molar heat capacity**, the amount of heat required to raise the temperature of *1 mol* of a substance by 1 °C. The units of molar heat capacity are J/mol · °C. *Specific* heat capacity and *molar* heat capacity are intensive properties—they depend on the *kind* of substance being heated, not on the amount.

Table 9.2 **Specific Heat Capacities of Some Common Substances**

Substance	Specific Heat Capacity, C_s (J/g · °C)*
Elements	
Lead	0.128
Gold	0.128
Silver	0.235
Copper	0.385
Iron	0.449
Aluminum	0.903
Compounds	
Ethanol	2.42
Water	4.18
Materials	
Glass (Pyrex)	0.75
Granite	0.79
Sand	0.84

*At 298 K.



The high heat capacity of the water surrounding San Francisco results in relatively cool summer temperatures.

Notice that water has the highest specific heat capacity of all the substances in Table 9.2—changing the temperature of water requires a lot of heat. If you have ever experienced the drop in temperature that occurs when traveling from an inland region to the coast during the summer, you have experienced the effects of water's high specific heat capacity. On a summer's day in California, for example, the temperature difference between Sacramento (an inland city) and San Francisco (a coastal city) can be 18 °C (30 °F)—San Francisco enjoys a cool 20 °C (68 °F), while Sacramento bakes at nearly 38 °C (100 °F). Yet the intensity of sunlight falling on these two cities is the same. Why the large temperature difference? San Francisco sits on a peninsula, surrounded by the water of the Pacific Ocean. Water, with its high heat capacity, absorbs much of the sun's heat without undergoing a large increase in temperature, keeping San Francisco cool. Sacramento, by contrast, is about 160 km (100 mi) inland. The land surrounding Sacramento, with its low heat capacity, undergoes a large increase in temperature as it absorbs a similar amount of heat.

Similarly, only two U.S. states have never recorded a temperature above 100 °F. One of them is obvious: Alaska. It is too far north to get that hot. The other one, however, may come as a surprise. It is Hawaii. The high heat capacity of the water that surrounds the only island state moderates the temperature, preventing Hawaii from ever getting too hot.

We can use the specific heat capacity of a substance to quantify the relationship between the amount of heat added to a given amount of the substance and the corresponding temperature increase. The equation that relates these quantities is:

[9.5]

$$\text{Heat (J)} \longrightarrow q = \underset{\text{Mass (g)}}{\underset{\uparrow}{m}} \times \underset{\text{Specific heat capacity J/g • °C}}{\underset{\nwarrow}{C_s}} \times \Delta T \longleftarrow \text{Temperature change (°C)}$$

where q is the amount of heat in J, m is the mass of the substance in g, C_s is the specific heat capacity in J/g · °C, and ΔT is the temperature change in °C. Example 9.2 demonstrates the use of this equation.

> ΔT in °C is equal to ΔT in K, but not equal to ΔT in °F (see Chapter E).

Example 9.2 Temperature Changes and Heat Capacity

You find a penny (minted before 1982, when pennies were made of nearly pure copper) in the snow. How much heat is absorbed by the penny as it warms from the temperature of the snow, which is –8.0 °C, to the temperature of your body, 37.0 °C? Assume the penny is pure copper and has a mass of 3.10 g.

SORT You are given the mass of copper as well as its initial and final temperature. You are asked to find the heat required for the given temperature change.

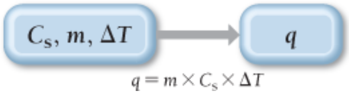
GIVEN:

m = 3.10 g copper
T_i = −8.0 °C
T_f = 37.0 °C

FIND: q

STRATEGIZE Equation 9.5, $q = m \times C_s \times \Delta T$, gives the relationship between the amount of heat (q) and the temperature change (ΔT).

CONCEPTUAL PLAN



RELATIONSHIPS USED

$q = m \times C_s \times \Delta T$ (Equation 9.5)

$C_s = 0.385\ \text{J/g} \cdot {}^\circ\text{C}$ (from Table 9.2)

SOLUTION

SOLVE Gather the necessary quantities for the equation in the correct units and substitute these into the equation to calculate q.

$$\begin{aligned}\Delta T &= T_f - T_i = 37.0\ {}^\circ\text{C} - (-8.0\ {}^\circ\text{C}) = 45.0\ {}^\circ\text{C}\\ q &= m \times C_s \times \Delta T\\ &= 3.10\ g \times 0.385\ \frac{\text{J}}{g \cdot {}^\circ\text{C}} \times 45.0\ {}^\circ\text{C} = 53.7\ \text{J}\end{aligned}$$

CHECK The units (J) are correct for heat. The sign of q is *positive*, as it should be because the penny *absorbed* heat from the surroundings.

FOR PRACTICE 9.2 To determine whether a shiny gold-colored rock is actually gold, a chemistry student decides to measure its heat capacity. She first weighs the rock and finds it has a mass of 4.7 g. She then finds that upon absorption of 57.2 J of heat, the temperature of the rock rises from 25 °C to 57 °C. Find the specific heat capacity of the substance composing the rock and determine whether the value is consistent with the rock being pure gold.

FOR MORE PRACTICE 9.2 A 55.0-g aluminum block initially at 27.5 °C absorbs 725 J of heat. What is the final temperature of the aluminum?

Interactive Worked Example 9.2 **Temperature Changes and Heat Capacity**

Conceptual Connection 9.3 **The Heat Capacity of Water**

Thermal Energy Transfer

Interactive

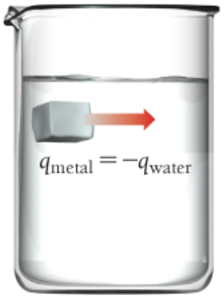
As we noted earlier, when two substances of different temperature are combined, thermal energy flows as heat from the hotter substance to the cooler one. If we assume that the two substances are thermally isolated from everything else, then the heat lost by one substance exactly equals the heat gained by the other (according to the law of energy conservation). If we define one substance as the system and the other as the surroundings, we can quantify the heat exchange as:

$$q_{\text{sys}} = -q_{\text{surr}}$$

Suppose a block of metal initially at 55 °C is submerged into water initially at 25 °C. Thermal energy transfers as heat from the metal to the water. The metal becomes colder and the water becomes warmer until the two substances reach the same temperature (thermal equilibrium). The exact temperature change that occurs depends on the masses of the metal and the water and on their specific heat capacities. Because $q = m \times C_s \times \Delta T$, we can arrive at the following relationship:

$$\begin{aligned} q_{\text{metal}} &= -q_{\text{water}} \\ m_{\text{metal}} \times C_{\text{s, metal}} \times \Delta T_{\text{metal}} &= -m_{\text{water}} \times C_{\text{s, water}} \times \Delta T_{\text{water}} \end{aligned}$$

Example 9.3 demonstrates how to work with thermal energy transfer.


$q_{\text{metal}} = -q_{\text{water}}$

Example 9.3 Thermal Energy Transfer

A 32.5-g cube of aluminum initially at 45.8 °C is submerged into 105.3 g of water at 15.4 °C. What is the final temperature of both substances at thermal equilibrium? (Assume that the aluminum and the water are thermally isolated from everything else.)

SORT You are given the masses of aluminum and water and their initial temperatures. You are asked to find the final temperature.

GIVEN:

$m_{\text{Al}} = 32.5\text{ g} \quad m_{\text{H}_2\text{O}} = 105.3\text{ g}$

$T_{\text{i, Al}} = 45.8\ °\text{C}; T_{\text{i, H}_2\text{O}} = 15.4\ °\text{C}$

FIND: T_{f}

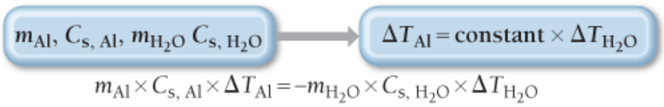
STRATEGIZE The heat lost by the aluminum (q_{Al}) equals the heat gained by the water $(q_{\text{H}_2\text{O}})$.

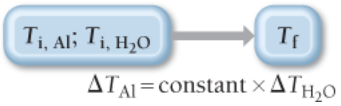
Use the relationship between q and ΔT and the given variables to find a relationship between ΔT_{Al} and $\Delta T_{\text{H}_2\text{O}}$.

Use the relationship between ΔT_{Al} and $\Delta T_{\text{H}_2\text{O}}$ (that you just found) along with the initial temperatures of the aluminum and the water to determine the final temperature. Note that at thermal equilibrium, the final temperature of the aluminum and the water is the same, that is, $T_{\text{f, Al}} = T_{\text{f, H}_2\text{O}} = T_{\text{f}}$.

CONCEPTUAL PLAN

$q_{\text{Al}} = -q_{\text{H}_2\text{O}}$


$m_{\text{Al}}, C_{\text{s, Al}}, m_{\text{H}_2\text{O}}\ C_{\text{s, H}_2\text{O}}$
$\Delta T_{\text{Al}} = \text{constant} \times \Delta T_{\text{H}_2\text{O}}$
$m_{\text{Al}} \times C_{\text{s, Al}} \times \Delta T_{\text{Al}} = -m_{\text{H}_2\text{O}} \times C_{\text{s, H}_2\text{O}} \times \Delta T_{\text{H}_2\text{O}}$


$T_{\text{i, Al}}; T_{\text{i, H}_2\text{O}}$
T_{f}
$\Delta T_{\text{Al}} = \text{constant} \times \Delta T_{\text{H}_2\text{O}}$

RELATIONSHIPS USED

$C_{s,\,H_2O} = 4.18\ J/g \cdot °C$; $C_{s,\,Al} = 0.903\ J/g \cdot °C$ (Table 9.2)

$q = m \times C_s \times \Delta T$ (Equation 9.5)

SOLVE Write the equation for the relationship between the heat lost by the aluminum $\left(q_{Al}\right)$ and the heat gained by the water $\left(q_{H_2O}\right)$ and substitute $q = m \times C_s \times \Delta T$ for each substance.

Substitute the values of m (given) and C_s (from Table 9.2) for each substance and solve the equation for ΔT_{Al}. (Alternatively, you can solve the equation for ΔT_{H_2O}.)

Substitute the initial temperatures of aluminum and water into the relationship from the previous step and solve the expression for the final temperature $\left(T_f\right)$. Remember that the final temperature for both substances is the same.

SOLUTION

$$\begin{aligned} q_{Al} &= -q_{H_2O} \\ m_{Al} \times C_{s,\,Al} \times \Delta T_{Al} &= -m_{H_2O} \times C_{s,\,H_2O} \times \Delta T_{H_2O} \\ 32.5\ g \times \frac{0.903\ J}{g \cdot °C} \cdot \Delta T_{Al} &= -105.3\ g \times \frac{4.18\ J}{g \cdot °C} \cdot \Delta T_{H_2O} \\ 29.\bar{3}48 \cdot \Delta T_{Al} &= -44\bar{0}.15 \cdot \Delta T_{H_2O} \\ \Delta T_{Al} &= -14.\bar{9}98 \cdot \Delta T_{H_2O} \end{aligned}$$

$$\begin{aligned} T_f - T_{i,\,Al} &= -14.\bar{9}98\left(T_f - T_{i,\,H_2O}\right) \\ T_f &= -14.\bar{9}98 \cdot T_f + 14.\bar{9}98 \cdot T_{i,\,H_2O} + T_{i,\,Al} \\ T_f + 14.\bar{9}98 \cdot T_f &= 14.\bar{9}98 \cdot T_{i,\,H_2O} + T_{i,\,Al} \\ 15.\bar{9}98 \cdot T_f &= 14.\bar{9}98 \cdot T_{i,\,H_2O} + T_{i,\,Al} \end{aligned}$$

$$\begin{aligned} T_f &= \frac{14.\bar{9}98 \cdot T_{i,\,H_2O} + T_{i,\,Al}}{15.\bar{9}98} = \frac{14.\bar{9}98 \cdot 15.4\ °C + 45.8\ °C}{15.\bar{9}98} \\ &= 17.3\ °C \end{aligned}$$

CHECK The units °C are correct. The final temperature of the mixture is closer to the initial temperature of the *water* than the *aluminum*. This makes sense for two reasons: (1) Water has a higher specific heat capacity than aluminum; and (2) there is more water than aluminum. Because the aluminum loses the same amount of heat that the water gains, the greater mass and specific heat capacity of the water make the temperature change in the water *less than* the temperature change in the aluminum.

FOR PRACTICE 9.3 A block of copper of unknown mass has an initial temperature of 65.4 °C. The copper is immersed in a beaker containing 95.7 g of water at 22.7 °C. When the two substances reach thermal equilibrium, the final temperature is 24.2 °C. What is the mass of the copper block?

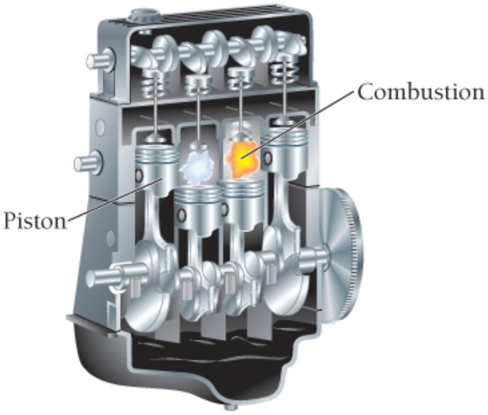
Interactive Worked Example 9.3 **Thermal Energy Transfer**

Conceptual Connection 9.4 **Thermal Energy Transfer**

Interactive

Work: Pressure–Volume Work

Recall that energy transfer between a system and its surroundings can occur via heat (q) or work (w). We just discussed how to calculate the *heat* associated with an observed *temperature* change. We now turn to calculating the *work* associated with an observed *volume* change. Although a chemical reaction can do several different types of work, for now we limit our discussion to **pressure–volume work**. Pressure–volume work occurs, for example, in the cylinder of an automobile engine. The combustion of gasoline causes gases within the cylinders example, in the cylinder of an automobile engine. The combustion of gasoline causes gases within the cylinders to expand, pushing the piston outward (against an external pressure) and ultimately moving the wheels of the car. See Section 10.2 for a detailed description of pressure.



The combustion of gasoline within an engine's cylinders does pressure–volume work that results in the motion of the car.

We can derive an equation for the value of pressure–volume work from the definition of work as a force (F) acting through a distance (D):

[9.6]

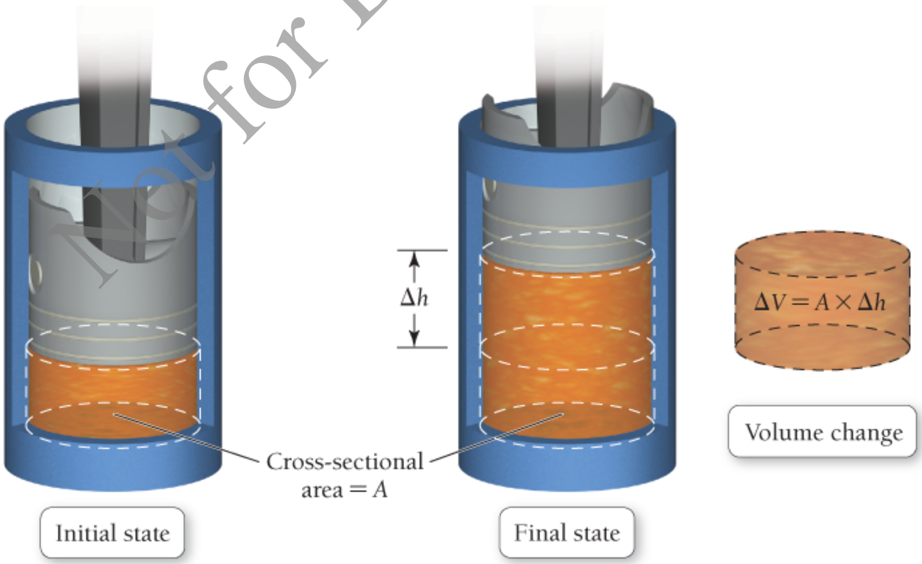
$$w = F \times D$$

> The force in this equation must be a constant force.

When the volume of a cylinder increases (Figure 9.6), it pushes against an external force. That external force is pressure (P), which is defined as force (F) divided by area (A):

$$P = \frac{F}{A} \quad \text{or} \quad F = P \times A$$

Figure 9.6 Piston Moving within a Cylinder against an External Pressure



If we substitute this expression for force into the definition of work given in Equation 9.6, we arrive at:

$$\begin{aligned} w &= F \times D \\ &= P \times A \times D \end{aligned}$$

The distance through which the force acts is the change in the height of the piston as it moves during the expansion (Δh). Substituting Δh for D, we get:

$$w = P \times A \times \Delta h$$

Because the volume of a cylinder is the area of its base times its height, $A \times \Delta h$ is actually the change in volume (ΔV) that occurs during the expansion (see Figure 9.6). Thus, the expression for work becomes:

$$w = P\Delta V$$

Still missing from the equation is the *sign* of the work done by the expanding gases. As the volume of the cylinder increases, work is done *on* the surroundings *by* the system, so w should be negative. However, upon expansion, V_2 (the final volume) is greater than V_1 (the initial volume) so ΔV is positive. In order for w to be negative for a positive expansion, we need to add a negative sign to our equation. In other words, w and ΔV must be opposite in sign:

[9.7]

$$w = -P\Delta V$$

So the work caused by an expansion of volume is the negative of the pressure that the volume expands against multiplied by the change in volume that occurs during the expansion. The units of the work we obtain by using this equation depend on the units of pressure and volume. Most commonly, the units of pressure are atmospheres (atm) (see Section 10.2) and those of volume are liters (L), so the resulting units are L · atm. To convert between L · atm and J, we use the conversion factor $101.3\ \text{J} = 1\ \text{L} \cdot \text{atm}$.

Example 9.4 Pressure–Volume Work

To inflate a balloon you must do pressure–volume work on the surroundings. If you inflate a balloon from a volume of 0.100 L to 1.85 L against an external pressure of 1.00 atm, how much work is done (in joules)?

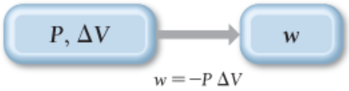
SORT You know the initial and final volumes of the balloon and the pressure against which it expands. The balloon and its contents are the system.

GIVEN: $V_1 = 0.100\ \text{L}$, $V_2 = 1.85\ \text{L}$, $P = 1.00\ \text{atm}$

FIND: w

STRATEGIZE The equation $w = -P\Delta V$ specifies the amount of work done during a volume change against an external pressure.

CONCEPTUAL PLAN



SOLVE To solve the problem, calculate the value of ΔV and substitute that value, together with P, into Equation 9.7.

SOLUTION

$$\begin{aligned}\Delta V &= V_2 - V_1\\ &= 1.85\ \text{L} - 0.100\ \text{L}\\ &= 1.75\ \text{L}\\ w &= -P\Delta V\\ &= -1.00\ \text{atm} \times 1.75\ \text{L}\\ &= -1.75\ \text{L} \cdot \text{atm}\end{aligned}$$

Convert the units of the answer (L · atm) to joules using $101.3\ \text{J} = 1\ \text{L} \cdot \text{atm}$.

$$-1.75\ \text{L} \cdot \text{atm} \times \frac{101.3\ \text{J}}{1\ \text{L} \cdot \text{atm}} = -177\ \text{J}$$

CHECK The units (J) are correct for work. The sign of the work is negative, as it should be for an expansion: Work is done on the surroundings by the expanding balloon.

FOR PRACTICE 9.4 A cylinder equipped with a piston expands against an external pressure of 1.58 atm. If the initial volume is 0.485 L and the final volume is 1.245 L, how much work (in J) is done?

FOR MORE PRACTICE 9.4 When fuel is burned in a cylinder equipped with a piston, the volume expands from 0.255 L to 1.45 L against an external pressure of 1.02 atm. In addition, 875 J is emitted as heat. What is ΔE for the burning of the fuel?

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