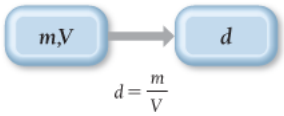
Pearson

E.9: Solving Problems Involving Equations

We can solve problems involving equations in much the same way as problems involving conversions. Usually, in problems involving equations, we are asked to find one of the variables in the equation, given the others. For example, suppose we are given the mass (m) and volume (V) of a sample and asked to calculate its density. A conceptual plan shows how the *equation* takes us from the *given* quantities to the *find* quantity.



Here, instead of a conversion factor under the arrow, the conceptual plan has an equation. The equation shows the *relationship* between the quantities on the left of the arrow and the quantities on the right. Note that at this point, the equation need not be solved for the quantity on the right (although in this particular case it is). The procedure that follows, as well as Examples E.10 and E.11, guides you through solutions to problems involving equations. We again use the three-column format. Work through one problem from top to bottom and then apply the same general procedure to the second problem.

Example E.10

PROCEDURE FOR

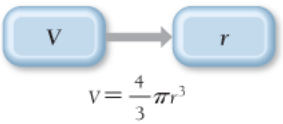
SORT Begin by sorting the information into *given* and *find*.

Problems with Equations

Find the radius (r), in centimeters, of a spherical water droplet with a volume (V) of $0.058\ \text{cm}^2$. For a sphere, $V = (4/3)\,\pi r^3$.

STRATEGIZE Write a conceptual plan for the problem. Focus on the equation(s). The conceptual plan shows how the equation takes you from the *given* quantity (or quantities) to the *find* quantity. The conceptual plan may have several parts, involving other equations or required conversions. In these examples, you use the geometrical relationships given in the problem statements as well as the definition of density, $d = m/V$, which you learned in this chapter.

CONCEPTUAL PLAN



RELATIONSHIPS USED

$V = \frac{4}{3}\pi r^3$

SOLVE Follow the conceptual plan. Solve the equation(s) for the *find* quantity (if it is not solved already). Gather each of the quantities that must go into the equation in the correct units. (Convert to the correct units if necessary.) Substitute the numerical values and their units into the equation(s) and calculate the answer.

Round the answer to the correct number of significant figures.

SOLUTION

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ r^3 &= \frac{3}{4\pi}V \\ r &= \left(\frac{3}{4\pi}V\right)^{1/3} \\ &= \left(\frac{3}{4\pi}0.058\ \text{cm}^3\right)^{1/3} \\ &= 0.24013\ \text{cm} \\ 0.24013\ \text{cm} &= 0.24\ \text{cm} \end{aligned}$$

CHECK Check your answer. Are the units correct? Does the answer make sense?

The units (cm) are correct, and the magnitude makes sense.

The units $\left(\text{g/cm}^3\right)$ are correct. The magnitude of the answer seems correct for one of the lighter metals (see Table E.4).

Example E.11

PROCEDURE FOR

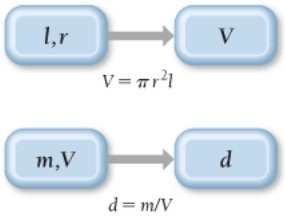
SORT Begin by sorting the information into *given* and *find*.

Problems with Equations

Find the density $\left(\text{in g/cm}^3\right)$ of a metal cylinder with a mass (m) of 8.3 g, a length (l) of 1.94 cm, and a radius (r) of 0.55 cm. For a cylinder, $V = \pi r^2 l$.

STRATEGIZE Write a conceptual plan for the problem. Focus on the equation(s). The conceptual plan shows how the equation takes you from the *given* quantity (or quantities) to the *find* quantity. The conceptual plan may have several parts, involving other equations or required conversions. In these examples, you use the geometrical relationships given in the problem statements as well as the definition of density, $d = m/V$, which you learned in this chapter.

CONCEPTUAL PLAN



RELATIONSHIPS USED

$V = \pi r^2 l$

$d = \dfrac{m}{V}$

SOLVE Follow the conceptual plan. Solve the equation(s) for the *find* quantity (if it is not solved already). Gather each of the quantities that must go into the equation in the correct units. (Convert to the correct units if necessary.) Substitute the numerical values and their units into the equation(s) and calculate the answer.

Round the answer to the correct number of significant figures.

SOLUTION

$$\begin{aligned} V &= \pi r^2 l \\ &= \pi(0.55\ \text{cm})^2\,(1.94\ \text{cm}) \\ &= 1.8436\ \text{cm}^2 \\ d &= \frac{m}{V} \\ &= \frac{8.3\ \text{g}}{1.8436\ \text{cm}^3} = 4.50195\ \text{g/cm}^3 \end{aligned}$$

$$4.50195\ \text{g/cm}^3 = 4.5\ \text{g/cm}^3$$

CHECK Check your answer. Are the units correct? Does the answer make sense?

The units (cm) are correct, and the magnitude makes sense.

The units $\left(\text{g/cm}^3\right)$ are correct. The magnitude of the answer seems correct for one of the lighter metals (see Table E.4).

Find the density, in g/cm^3, of a metal cube with a mass of 50.3 g and an edge length (l) of 2.65 cm. For a cube, $V = l^3$.