10.6: Applications of the Ideal Gas Law: Molar Volume, Density, and Molar Mass of a Gas

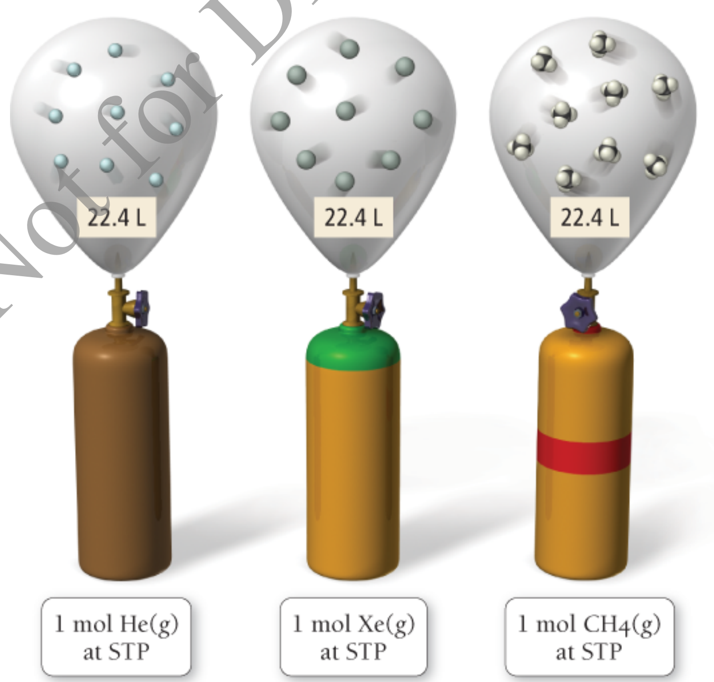
In Section 10.5, we examined how we can use the ideal gas law to calculate one of the variables (P, V, T, or n) given the other three. We now turn to three other applications of the ideal gas law: molar volume, density, and molar mass.

Molar Volume at Standard Temperature and Pressure

The volume occupied by one mole of a substance is its **molar volume**. For gases, we often specify the molar volume under conditions known as **standard temperature** ($T = 0\,^{\circ}\mathrm{C}$ or 273 K) **and pressure** ($P = 1.00$ atm), abbreviated as **STP**. Using the ideal gas law, we can determine that the molar volume of an ideal gas at STP is:

$$
\begin{aligned}
V &= \frac{nRT}{P} \\
&= \frac{1.00\ \cancel{\mathrm{mol}} \times 0.08206 \dfrac{\mathrm{L}\cdot\cancel{\mathrm{atm}}}{\cancel{\mathrm{mol}}\cdot\cancel{\mathrm{K}}} \times 273\ \cancel{\mathrm{K}}}{1.00\ \cancel{\mathrm{atm}}} \\
&= 22.4\ \mathrm{L}
\end{aligned}
$$

The molar volume of an ideal gas at STP is useful because—as we saw in the *Check* steps of Examples 10.5 and 10.6—it gives us a way to approximate the volume of an ideal gas under conditions that are close to STP.



One mole of any gas occupies approximately 22.4 L at standard temperature (273 K) and pressure (1.0 atm).

Conceptual Connection 10.3 **Molar Volume**

Density of a Gas

Because 1 mol of an ideal gas occupies 22.4 L at standard temperature and pressure, we can readily calculate the density of an ideal gas under these conditions. Density is mass/volume, and because the mass of one mole of a gas is simply its molar mass, the *density of a gas* is:

$$\text{density} = \frac{\text{molar mass}}{\text{molar volume}}$$

We can calculate the density of a gas at STP by using 22.4 L as the molar volume. For example, the densities of helium and nitrogen gas at STP are:

$$d_{\text{He}} = \frac{4.00\ \text{g/mol}}{22.4\ \text{L/mol}} = 0.179\ \text{g/L} \quad d_{\text{N}_2} = \frac{28.02\ \text{g/mol}}{22.4\ \text{L/mol}} = 1.25\ \text{g/L}$$

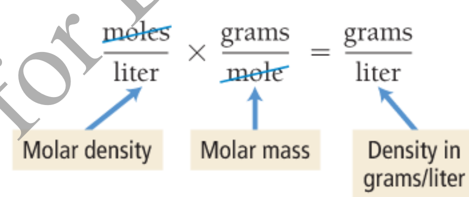
Notice that *the density of a gas is directly proportional to its molar mass*. The greater the molar mass of a gas, the more dense the gas. For this reason, a gas with a molar mass lower than that of air tends to rise in air. For example, both helium and hydrogen gas (molar masses of 4.00 and 2.01 g/mol, respectively) have molar masses that are lower than the average molar mass of air (approximately 28.8 g/mol). Therefore a balloon filled with either helium or hydrogen gas floats in air.

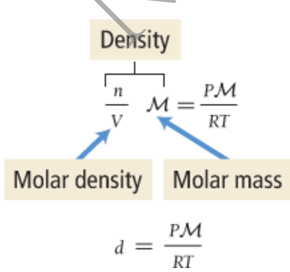
> The primary components of air are nitrogen (about four-fifths) and oxygen (about one-fifth). We discuss the detailed composition of air in Section 10.6.

We can calculate the density of a gas more generally (under any conditions) by using the ideal gas law. To do so, we arrange the ideal gas law as:

$$\begin{aligned} PV &= nRT \\ \frac{n}{V} &= \frac{P}{RT} \end{aligned}$$

Because the left-hand side of this equation has units of moles/liter, it represents the *molar* density. We can obtain the density in grams/liter from the molar density by multiplying by the molar mass (M):





Therefore,

[10.15]

$$d = \frac{PM}{RT}$$

Notice that, as expected, density increases with increasing molar mass. Notice also that as we discussed in

Section 10.4, density decreases with increasing temperature.

Example 10.7 Density of a Gas

Calculate the density of nitrogen gas at 125 °C and a pressure of 755 mmHg.

SORT The problem gives you the temperature and pressure of a gas and asks you to find its density. The problem states that the gas is nitrogen.

GIVEN: $T(^\circ\mathrm{C}) = 125\ ^\circ\mathrm{C}, P = 755\ \mathrm{mmHg}$

FIND: d

STRATEGIZE Equation 10.15 provides the relationship between the density of a gas and its temperature, pressure, and molar mass. The temperature and pressure are given. You can calculate the molar mass from the formula of the gas, N_2.

CONCEPTUAL PLAN

$$\boxed{P, T, \mathcal{M}} \longrightarrow \boxed{d}$$

$$d = \frac{P\mathcal{M}}{RT}$$

RELATIONSHIPS USED

$$d = \frac{P\mathcal{M}}{RT} \text{(density of a gas)}$$
$$\text{molar mass } N_2 = 28.02\ \mathrm{g/mol}$$

SOLVE To solve the problem, gather each of the required quantities in the correct units. Convert the temperature to kelvins and the pressure to atmospheres.

Substitute the quantities into the equation to calculate density.

SOLUTION

$$T(\mathrm{K}) = 125 + 273 = 398\ \mathrm{K}$$

$$P = 755\ \cancel{\mathrm{mmHg}} \times \frac{1\ \mathrm{atm}}{760\ \cancel{\mathrm{mmHg}}} = 0.99342\ \mathrm{atm}$$

$$\begin{aligned} d &= \frac{P\mathcal{M}}{RT} \\ &= \frac{0.99342\ \cancel{\mathrm{atm}}\left(28.02\dfrac{\mathrm{g}}{\cancel{\mathrm{mol}}}\right)}{0.08206\dfrac{\mathrm{L}\cdot\cancel{\mathrm{atm}}}{\cancel{\mathrm{mol}}\cdot\cancel{\mathrm{K}}}\left(398\ \cancel{\mathrm{K}}\right)} \\ &= 0.852\ \mathrm{g/L} \end{aligned}$$

CHECK The units of the answer are correct. The magnitude of the answer (0.852 g/L) makes sense because earlier you calculated the density of nitrogen gas at STP as 1.25 g/L. The temperature is higher than standard temperature, so it follows that the density is lower.

FOR PRACTICE 10.7 Calulate the density of xenon gas at a pressure of 742 mmHg and a temperature of 45 °C.

FOR MORE PRACTICE 10.7 A gas has a density of 1.43 g/L at a temperature of 23 °C and a pressure of 0.789 atm. Calculate its molar mass.

Interactive Worked Example 10.7 **Density of a Gas**

Conceptual Connection 10.4 **Density of a Gas**

Molar Mass of a Gas

We can use the ideal gas law in combination with mass measurements to calculate the molar mass of an unknown gas. First, we measure the mass and volume of an unknown gas under conditions of known pressure and temperature. Then we determine the amount of the gas in moles from the ideal gas law. Finally, we calculate the molar mass by dividing the mass (in grams) by the amount (in moles), as shown in Example 10.8.

Example 10.8 Molar Mass of a Gas

A sample of gas has a mass of 0.311 g. Its volume is 0.225 L at a temperature of 55 °C and a pressure of 886 mmHg. Find its molar mass.

SORT The problem gives you the mass of a gas sample, along with its volume, temperature, and pressure. You are asked to find the molar mass.

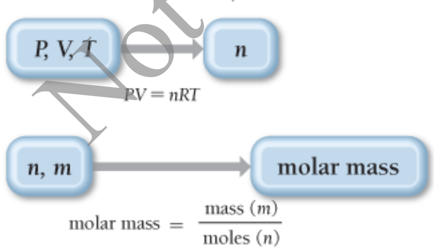
GIVEN:
$m = 0.311\ g, V = 0.225\ \mathrm{L},$
$T(^\circ\mathrm{C}) = 55\ ^\circ\mathrm{C}, P = 886\ \mathrm{mmHg}$

FIND: molar mass (g/mol)

STRATEGIZE The conceptual plan has two parts. In the first part, use the ideal gas law to find the number of moles of gas.

In the second part, use the definition of molar mass to find the molar mass.

CONCEPTUAL PLAN



RELATIONSHIPS USED

$$PV = nRT$$

$$\text{molar mass} = \frac{\text{mass } (m)}{\text{moles } (n)}$$

SOLVE To find the number of moles, first solve the ideal gas law for n.

Before substituting into the equation for n, convert the pressure to atm and the temperature to K.

Substitute into the equation and calculate n, the number of moles.

Finally, use the number of moles (n) and the given mass (m) to find the molar mass.

SOLUTION

$$PV = nRT$$
$$n = \frac{PV}{RT}$$

$$P = 886\ \cancel{\text{mmHg}} \times \frac{1\ \text{atm}}{760\ \cancel{\text{mmHg}}} = 1.1\underline{6}58\ \text{atm}$$
$$T(K) = 55 + 273 = 328\ \text{K}$$
$$n = \frac{1.1\underline{6}58\ \text{atm} \times 0.225\ \text{L}}{0.08206 \frac{\cancel{\text{L}} \cdot \cancel{\text{atm}}}{\text{mol} \cdot \cancel{\text{K}}} \times 328\ \cancel{\text{K}}} = 9.7\underline{4}54 \times 10^{-3}\ \text{mol}$$

$$\text{molar mass} = \frac{\text{mass}\ (m)}{\text{moles}\ (n)}$$
$$= \frac{0.311\ g}{9.7\underline{4}54 \times 10^{-3}\ \text{mol}} = 31.9\ \text{g/mol}$$

CHECK The units of the answer are correct. The magnitude of the answer (31.9 g/mol) is a reasonable number for a molar mass. If your answer is some very small number (such as any number smaller than 1) or a very large number, you solved the problem incorrectly. Most gases have molar masses between one and several hundred grams per mole.

FOR PRACTICE 10.8 A sample of gas has a mass of 827 mg. Its volume is 0.270 L at a temperature of 88 °C and a pressure of 975 mmHg. Find its molar mass.

Interactive Worked Example 10.8 **Molar Mass of a Gas**