Pearson

2.4: The Wave Nature of Matter: The de Broglie Wavelength, the Uncertainty Principle, and Indeterminacy

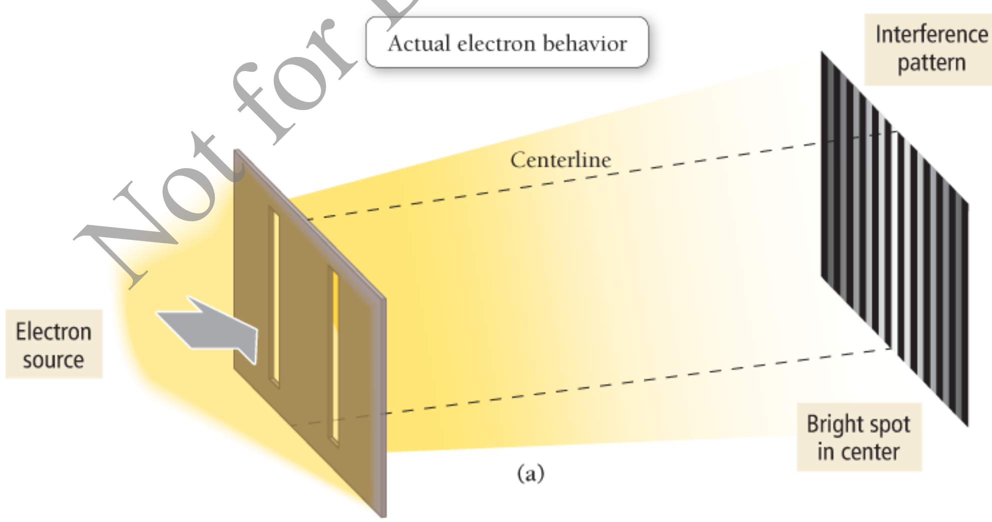
Key Concept Video **The Wave Nature of Matter**

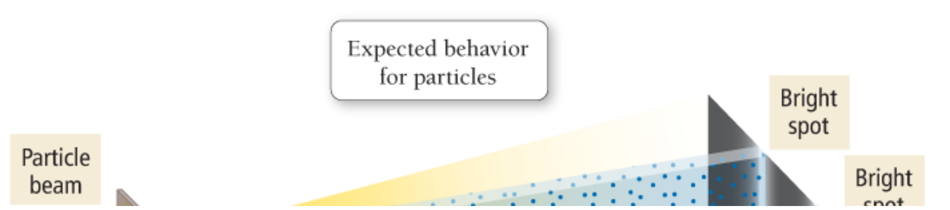
The heart of the quantum-mechanical theory that replaced Bohr's model is the wave nature of the electron, first proposed by Louis de Broglie (1892–1987) in 1924 and later confirmed by experiments in 1927. It seemed incredible at the time, but electrons—which were then thought of only as particles and known to have mass—also have a wave nature. The wave nature of the electron is seen most clearly in its diffraction. If an electron beam is aimed at two closely spaced slits, and a series (or array) of detectors is arranged to detect the electrons after they pass through the slits, an interference pattern similar to that observed for light is recorded behind the slits (Figure 2.16a). The detectors at the center of the array (midway between the two slits) detect a large number of electrons—exactly the opposite of what you would expect for particles (Figure 2.16b). Moving outward from this center spot, the detectors alternately detect small numbers of electrons and then large numbers again and so on, forming an interference pattern characteristic of waves.

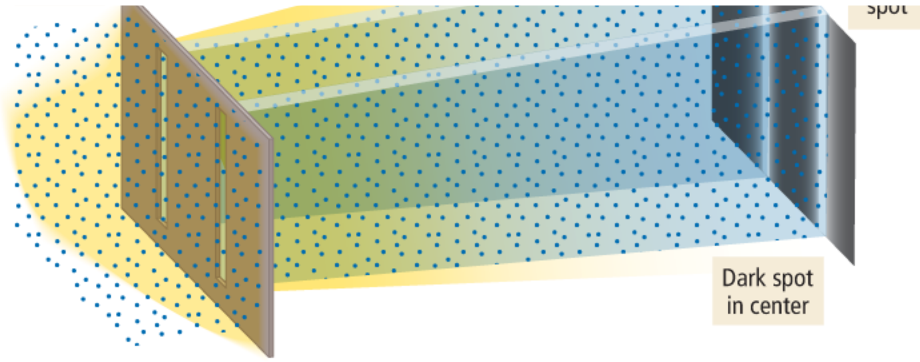
> The first evidence of electron wave properties was provided by the Davisson-Germer experiment of 1927, in which electrons were observed to undergo diffraction by a metal crystal.

Figure 2.16 Electron Diffraction

When a beam of electrons goes through two closely spaced slits (**a**), an interference pattern is created, as if the electrons were waves. By contrast, a beam of particles passing through two slits (**b**) produces two smaller beams of particles. Particle beams produce two bright stripes with darkness in between, but waves produce the brightest strip directly in the center of the screen.







(b)

Counter to what might be our initial intuition about electron interference, the interference pattern is *not caused by pairs of electrons interfering with each other, but rather by single electrons interfering with themselves*. If the electron source is turned down to a very low level, so that electrons come out only one at a time, *the interference pattern remains*. In other words, we can design an experiment in which electrons come out of the source singly. We can then record where each electron strikes the detector after it has passed through the slits. If we individually record the positions of thousands of electrons over a long period of time, we find the same interference pattern shown in Figure 2.16a. This leads us to an important conclusion: *The wave nature of the electron is an inherent property of individual electrons*. In this case, the unobserved electron goes through both slits—it exists in two states simultaneously, just like Schrödinger's cat—and interferes with itself. As it turns out, this wave nature explains the existence of stationary states (in the Bohr model) and prevents the electrons in an atom from crashing into the nucleus as predicted by classical physics. We now turn to three important manifestations of the electron's wave nature: the de Broglie wavelength, the uncertainty principle, and indeterminacy.

> For interference to occur, the spacing of the slits has to be on the order of atomic dimensions.

The de Broglie Wavelength

As we have seen, a single electron traveling through space has a wave nature; its wavelength is related to its kinetic energy (the energy associated with its motion). The faster the electron is moving, the higher its kinetic energy and the shorter its wavelength. The wavelength (λ) of an electron of mass m moving at velocity v is given by the **de Broglie relation**:

[2.4]

$$\lambda = \frac{h}{mv} \quad \text{de Broglie relation}$$

where h is Planck's constant. *Notice that the velocity of a moving electron is related to its wavelength—knowing one is equivalent to knowing the other*.

> The mass of an object (m) times its velocity (v) is its momentum. Therefore, the wavelength of an electron is inversely proportional to its momentum.

Example 2.4 de Broglie Wavelength

Calculate the wavelength of an electron traveling with a speed of 2.65×10^6 m/s.

SORT You are given the speed of an electron and asked to calculate its wavelength.

GIVEN: $v = 2.65 \times 10^6$ m/s

FIND: λ

STRATEGIZE The conceptual plan shows how the de Broglie relation relates the wavelength of an electron to its mass and velocity.

CONCEPTUAL PLAN

$$\boxed{\nu} \xrightarrow[\lambda = \frac{h}{m\nu}]{} \boxed{\lambda}$$

RELATIONSHIPS USED

$\lambda = h/m\nu$ (de Broglie relation, Equation 2.4)

SOLVE Substitute the velocity, Planck's constant, and the mass of an electron to calculate the electron's wavelength. To correctly cancel the units, break down the J in Planck's constant into its SI base units $(1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2)$.

SOLUTION

$$\begin{aligned}\lambda &= \frac{h}{mv} = \frac{6.626 \times 10^{-34} \dfrac{\cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}}^{2}}\ \cancel{\text{s}}}{\left(9.11 \times 10^{-31}\ \cancel{\text{kg}}\right) 2.65 \times 10^{6} \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}} \\ &= 2.74 \times 10^{-10}\ \text{m}\end{aligned}$$

CHECK The units of the answer (m) are correct. The magnitude of the answer is very small, as we would expect for the wavelength of an electron.

FOR PRACTICE 2.4 What is the velocity of an electron that has a de Broglie wavelength approximately the length of a chemical bond? Assume this length to be 1.2×10^{-10} m.

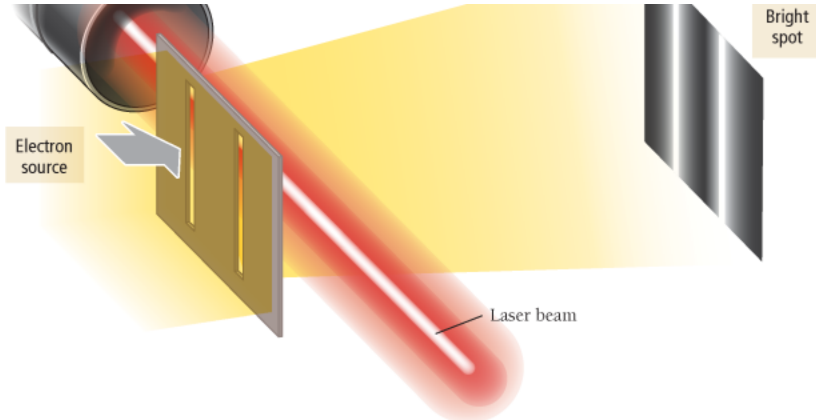
Conceptual Connection 2.3 The de Broglie Wavelength of Macroscopic Objects

The Uncertainty Principle

Interactive

The wave nature of the electron is difficult to reconcile with its particle nature. How can a single entity behave as both a wave and a particle? We can begin to address this question by returning to the single-electron diffraction experiment. Specifically, we can ask the question: How does a single electron aimed at a double slit produce an interference pattern? We stated previously that the electron travels through both slits and interferes with itself. This idea is testable. We simply have to observe the single electron as it travels through both of the slits. If it travels through both slits simultaneously, our hypothesis is correct. But here is where nature gets tricky.

Any experiment designed to observe the electron as it travels through the slits results in the detection of an electron "particle" traveling through a single slit and no interference pattern. Recall from Section 2.1 that an *unobserved* electron can occupy two different states; however, the act of observation forces it into one state or the other. Similarly, the act of observing the electron as it travels through both slits forces it to go through only one slit. The electron diffraction experiment shown here is designed to observe which slit the electron travels through by using a laser beam placed directly behind the slits.



An electron that crosses the laser beam produces a tiny "flash" when a single photon is scattered at the point of crossing. If a flash shows up behind a particular slit, that indicates an electron is passing through that slit. When the experiment is performed, the flash always originates either from one slit *or* the other, but *never* from both at once. Furthermore, the interference pattern, which was present without the laser, is now absent. With the laser on, the electrons hit positions directly behind each slit, as if they were ordinary particles; their wavelike behavior is no longer manifested.

As it turns out, no matter how hard we try, or whatever method we set up, *we can never both see the interference pattern and simultaneously determine which hole the electron goes through*. It has never been done, and most scientists agree that it never will. In the words of P. A. M. Dirac (1902–1984):

> There is a limit to the fineness of our powers of observation and the smallness of the accompanying disturbance—a limit which is inherent in the nature of things and can never be surpassed by improved technique or increased skill on the part of the observer.

The single-electron diffraction experiment demonstrates that we cannot simultaneously observe both the wave nature and the particle nature of the electron. When we try to observe which hole the electron goes through (associated with the particle nature of the electron), we lose the interference pattern (associated with the wave nature of the electron). When we try to observe the interference pattern, we cannot determine which hole the electron goes through. The wave nature and particle nature of the electron are said to be **complementary properties**. Complementary properties exclude one another—the more we know about one, the less we know about the other. Which of two complementary properties we observe depends on the experiment we perform—in quantum mechanics, the observation of an event affects its outcome.

Werner Heisenberg (1901–1976)

As we just saw in the de Broglie relation, the *velocity* of an electron is related to its *wave nature*. The *position* of an electron, however, is related to its *particle nature*. (Particles have well-defined positions, but waves do not.) Consequently, our inability to observe the electron simultaneously as both a particle and a wave means that *we* *cannot simultaneously measure its position and its velocity with infinite precision*. Werner Heisenberg formalized this idea with the equation:

[2.5]

$$\Delta x \times m\Delta v \geq \frac{h}{4\pi} \quad \text{Heisenberg's uncertainty principle}$$

where Δx is the uncertainty in the position, Δv is the uncertainty in the velocity, m is the mass of the particle, and h is Planck's constant. **Heisenberg's uncertainty principle** states that the product of Δx and $m\Delta v$ must be greater than or equal to a finite number ($h/4\pi$). In other words, the more accurately you know the position of an electron (the smaller Δx), the less accurately you can know its velocity (the bigger Δv) and vice versa. The complementarity of the wave nature and particle nature of the electron results in the complementarity of velocity and position.

> Remember that velocity includes speed as well as direction of travel.

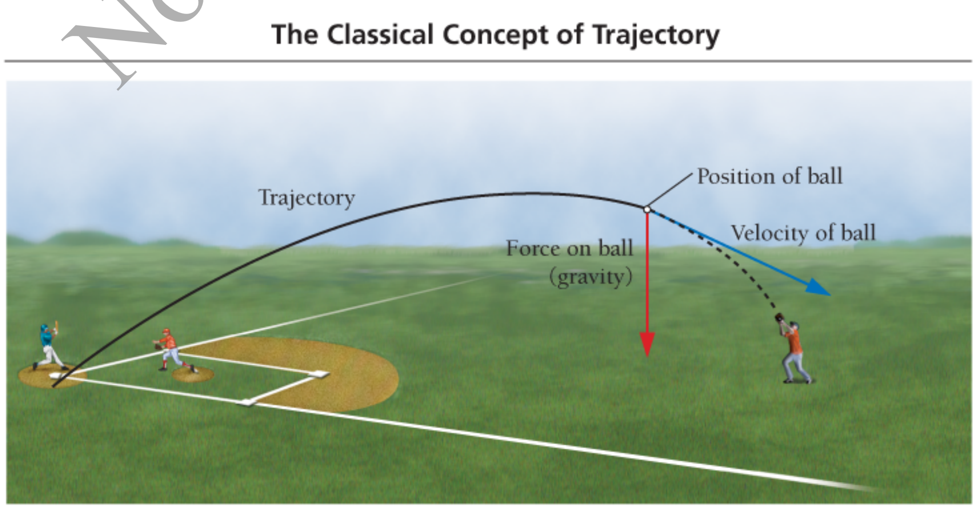
Although Heisenberg's uncertainty principle may seem puzzling, it actually solves a great puzzle. Without the uncertainty principle, we are left with a paradox: How can something be *both* a particle and a wave? Saying that an object is both a particle and a wave is like saying that an object is both a circle and a square—a contradiction. Heisenberg solved the contradiction by introducing complementarity—an electron is observed as *either* a particle or a wave, but never both at once. This idea is captured by Schrödinger's thought experiment about the cat explained in Section 2.1: When observed, the cat is either dead or alive, not both.

Indeterminacy and Probability Distribution Maps

According to classical physics, and in particular Newton's laws of motion, particles move in a *trajectory* (or path) that is determined by the particle's velocity (the speed and direction of travel), its position, and the forces acting on it. Even if you are not familiar with Newton's laws, you probably have an intuitive sense of them. For example, when you chase a baseball in the outfield, you visually predict where the ball will land by observing its path. You do this by noting its initial position and velocity, watching how these are affected by the forces acting on it (gravity, air resistance, wind), and then inferring its trajectory, as shown in Figure 2.17. If you knew only the ball's velocity, or only its position (imagine a still photo of the baseball in the air), you could not predict its landing spot. In classical mechanics, both position and velocity are required to predict a trajectory.

Figure 2.17 The Concept of Trajectory

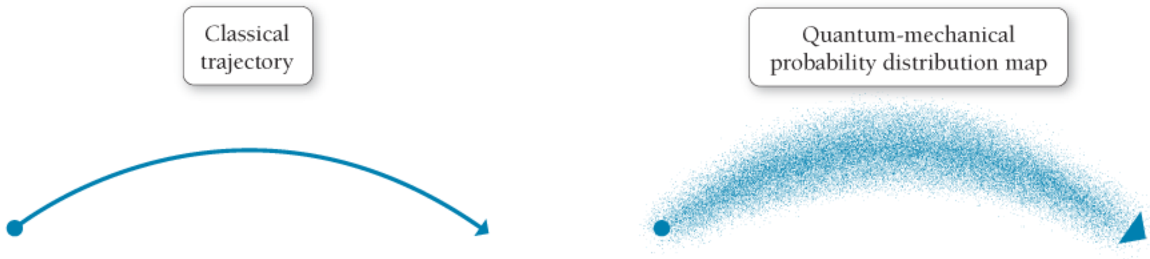
In classical mechanics, the position and velocity of a particle determine its future trajectory, or path. Thus, an outfielder can catch a baseball by observing its position and velocity, allowing for the effects of forces acting on it, such as gravity, and estimating its trajectory. (For simplicity, air resistance and wind are not shown.)



Newton's laws of motion are **deterministic**—the present *determines* the future. This means that if two baseballs are hit consecutively with the same velocity from the same position under identical conditions, they will land in exactly the same place. The same is not true of electrons. We have just seen that we cannot simultaneously know the position and velocity of an electron; therefore, we cannot know its trajectory. In quantum mechanics, trajectories are replaced with *probability distribution maps*, as shown in Figure 2.18. A probability distribution map is a statistical map that shows where an electron is likely to be found under a given set of conditions.

Figure 2.18 Trajectory versus Probability

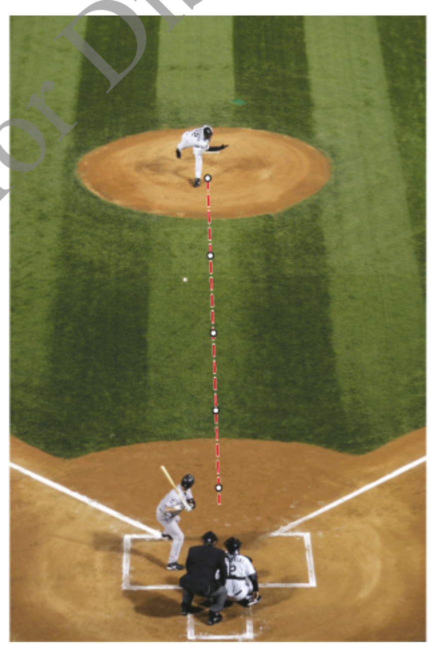
In quantum mechanics, we cannot calculate deterministic trajectories. Instead, it is necessary to think in terms of probability maps: statistical pictures of where a quantum-mechanical particle, such as an electron, is most likely to be found. In this hypothetical map, darker shading indicates greater probability.



To understand the concept of a probability distribution map, let us return to baseball. Imagine a baseball thrown from the pitcher's mound to a catcher behind home plate (Figure 2.19). The catcher can watch the baseball's path, predict exactly where it will cross home plate, and place his mitt in the correct place to catch it. As we have seen, the same predictions cannot be made for an electron. If an electron were thrown from the pitcher's mound to home plate, it would land in a different place every time, even if it were thrown in exactly the same way. This behavior is called **indeterminacy**. Unlike a baseball, whose future path is *determined* by its position and velocity when it leaves the pitcher's hand, the future path of an electron is indeterminate and can only be described statistically.

Figure 2.19 Trajectory of a Macroscopic Object

A baseball follows a well-defined trajectory from the hand of the pitcher to the mitt of the catcher.



In the quantum-mechanical world of the electron, the catcher cannot know exactly where the electron will cross the plate for any given throw. However, if he were to record hundreds of identical electron throws, the catcher would observe a reproducible, *statistical pattern* of where the electron crosses the plate. He could even draw a map of the strike zone showing the probability of an electron crossing a certain area, as shown in Figure 2.20.

This would be a probability distribution map. In the sections that follow, we discuss quantum-mechanical

This would be a probability distribution map. In the sections that follow, we discuss quantum-mechanical electron *orbitals*, which are essentially probability distribution maps for electrons as they exist within atoms.

Figure 2.20 The Quantum-Mechanical Strike Zone

An electron does not have a well-defined trajectory. However, we can construct a probability distribution map to show the relative probability of it crossing home plate at different points.