Pearson

20.8: Converting Mass to Energy: Mass Defect and Nuclear Binding Energy

Nuclear fission produces large amounts of energy. But where does the energy come from? The energy comes from the conversion of mass to energy, as described by Einstein's famous equation $E = mc^2$. Here we first look at the conversion of mass to energy in general; then we turn to the topics of mass defect and nuclear binding energy.

The Conversion of Mass to Energy

When a fission reaction occurs, the products have a slightly different mass than the reactants. For example, examine the masses of the reactants and products in the fission equation from Section 20.7:

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \longrightarrow {}^{140}_{56}\text{Ba} + {}^{93}_{36}\text{Kr} + 3{}^{1}_{0}\text{n}$$

Mass Reactants		Mass Products	
${}^{235}_{92}\text{U}$	235.04392 amu	${}^{140}_{56}\text{Ba}$	139.910581 amu
${}^{1}_{0}\text{n}$	1.00866 amu	${}^{93}_{36}\text{Kr}$	92.931130 amu
		$3{}^{1}_{0}\text{n}$	3(1.00866) amu
Total	**236.05258 amu**		**235.86769 amu**

The products of the nuclear reaction have *less mass* than the reactants. The missing mass is converted to energy. In Chapter 1 we learned that matter is conserved in chemical reactions. In nuclear reactions matter can be converted to energy. The relationship between the amount of matter that is lost and the amount of energy formed is given by Einstein's famous equation relating the two quantities:

$$E = mc^2$$

where *E* is the energy produced, *m* is the mass lost, and *c* is the speed of light. For example, in the fission reaction just shown, we calculate the quantity of energy produced as follows:

$$\begin{aligned}
\text{mass lost } (m) &= 236.05258 \text{ amu} - 235.86769 \text{ amu} \\
&= 0.18489 \cancel{\text{amu}} \times \frac{1.66054 \times 10^{-27} \text{ kg}}{1 \cancel{\text{amu}}} \\
\text{energy produced } (E) &= mc^2 \\
&= 3.0702 \times 10^{-28} \text{ kg} \left(2.9979 \times 10^{8} \text{ m/s}^2\right) \\
&= 2.7593 \times 10^{-11} \text{ J}
\end{aligned}$$

> In a chemical reaction, there are also mass changes associated with the emission or absorption of energy. Because the energy involved in chemical reactions is so much smaller than that of nuclear reactions, however, these mass changes are negligible.

The result $\left(2.7593 \times 10^{-11} \text{ J}\right)$ is the energy produced when one nucleus of U-235 undergoes fission. This may not seem like much energy, but it is only the energy produced by the fission of a *single* nucleus. If we calculate the energy produced *per mole* of U-235, we can compare it to a chemical reaction:

$$2.7593 \times 10^{-11} \frac{\text{J}}{\text{U-235 } \cancel{\text{atom}}} \times \frac{6.0221 \times 10^{23} \text{ U-235 } \cancel{\text{atoms}}}{1 \text{ mol U-235}} = 1.6617 \times 10^{13} \text{ J/mol U-235}$$

The energy produced by the fission of 1 mol of U-235 is about 17 billion kJ. In contrast, a highly exothermic chemical reaction produces 1000 kJ per mole of reactant. Fission produces over a million times more energy per mole than chemical processes.

Mass Defect and Nuclear Binding Energy

We can examine the formation of a stable nucleus from its component particles as a nuclear reaction in which mass is converted to energy. For example, consider the formation of helium-4 from its components:

$$2\,{}^{1}_{1}\mathrm{H} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{4}_{2}\mathrm{He}$$

Mass Reactants		Mass Products	
$2\,{}^{1}_{1}\mathrm{H}$	2(1.00783) amu	${}^{4}_{2}\mathrm{He}$	4.00260 amu
$2\,{}^{1}_{0}\mathrm{n}$	2(1.00866) amu		
Total	**4.03298 amu**		**4.00260 amu**

> The electrons are contained on the left side in the $2\,{}^{1}_{1}\mathrm{H}$, and on the right side in ${}^{4}_{2}\mathrm{He}$. If we write the equation using only two protons on the left $\left({}^{1}_{1}\mathrm{p}\right)$, we must also add two electrons to the left.

A helium-4 atom has less mass than the sum of the masses of its separate components. This difference in mass, known as the **mass defect**, exists in all stable nuclei. The energy corresponding to the mass defect—obtained by substituting the mass defect into the equation $E = mc^2$—is the **nuclear binding energy**, the amount of energy required to break apart the nucleus into its component nucleons.

Although chemists often report energies in joules, nuclear physicists often use the electron volt (eV) or megaelectron volt (MeV): $1\ \mathrm{MeV} = 1.602 \times 10^{-13}\ \mathrm{J}$. Unlike energy in joules, which is usually reported per mole, energy in electron volts is reported per nucleus. A particularly useful conversion for calculating and reporting nuclear binding energies is the relationship between amu (mass units) and MeV (energy units).

$$1\ \mathrm{amu} = 931.5\ \mathrm{MeV}$$

> An electron volt is defined as the kinetic energy of an electron that has been accelerated through a potential difference of 1 V.

A mass defect of 1 amu, when substituted into the equation $E = mc^2$, gives an energy of 931.5 MeV. Using this conversion factor, we can calculate the binding energy of the helium nucleus:

$$\begin{aligned} \text{mass defect} &= 4.03298\ \mathrm{amu} - 4.00260\ \mathrm{amu} \\ &= 0.03038\ \mathrm{amu} \\ \text{nuclear binding energy} &= 0.03038\ \cancel{\mathrm{amu}} \times \frac{931.5\ \mathrm{MeV}}{1\ \cancel{\mathrm{amu}}} \\ &= 28.30\ \mathrm{MeV} \end{aligned}$$

The binding energy of the helium nucleus is 28.30 MeV. In order to compare the binding energy of one nucleus to that of another, we calculate the *binding energy per nucleon*, which is the nuclear binding energy of a nuclide divided by the number of nucleons in the nuclide. For helium-4, we calculate the binding energy per nucleon as follows:

$$\begin{aligned} \text{binding energy per nucleon} &= \frac{28.30\ \mathrm{MeV}}{4\ \text{nucleons}} \\ &= 7.075\ \mathrm{MeV\ per\ nucleon} \end{aligned}$$

We can calculate the binding energy per nucleon for other nuclides in the same way. For example, the nuclear binding energy of carbon-12 is 7.680 MeV per nucleon. Since the binding energy per nucleon of carbon-12 is greater than that of helium-4, we conclude that the carbon-12 nuclide is more *stable* (it has lower potential energy).

Example 20.7 Calculating Mass Defect and Nuclear Binding Energy

Calculate the mass defect and nuclear binding energy per nucleon (in MeV) for C-16, a radioactive isotope of carbon with a mass of 16.014701 amu.

SOLUTION

Calculate the mass defect as the difference between the mass of one C-16 atom and the sum of the masses of 6 hydrogen atoms and 10 neutrons.

$$\begin{aligned}\text{mass defect} &= 6\left(\text{mass } {}^{1}_{1}\text{H}\right) + 10\left(\text{mass } {}^{1}_{0}\text{n}\right) - \text{mass } {}^{16}_{6}\text{C} \\ &= 6\,(1.00783\text{ amu}) + 10\,(1.00866\text{ amu}) - 16.014701\text{ amu} \\ &= 0.1188\underline{7}9\text{ amu}\end{aligned}$$

Calculate the nuclear binding energy by converting the mass defect (in amu) into MeV. (Use $1\text{ amu} = 931.5\text{ MeV}$.)

$$0.1188\underline{7}9\ \cancel{\text{amu}} \times \frac{931.5\text{ MeV}}{\cancel{\text{amu}}} = 110.\underline{7}4\text{ MeV}$$

Determine the nuclear binding energy per nucleon by dividing by the number of nucleons in the nucleus.

$$\text{nuclear binding energy per nucleon} = \frac{110.\underline{7}4\text{ MeV}}{16\text{ nucleons}} = 6.921\text{ MeV/nucleon}$$

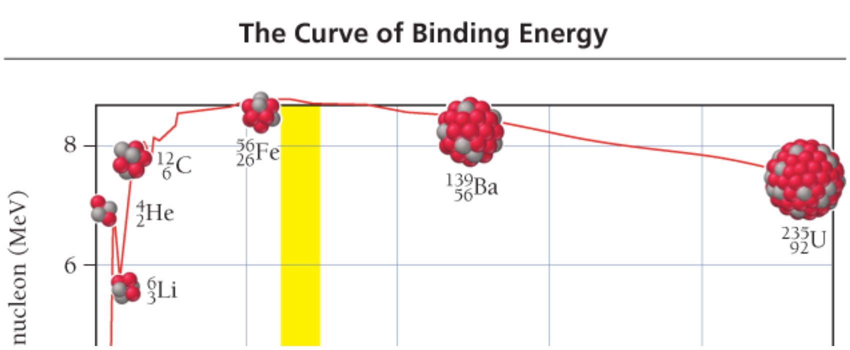
FOR PRACTICE 20.7 Calculate the mass defect and nuclear binding energy per nucleon (in MeV) for U-238, which has a mass of 238.050784 amu.

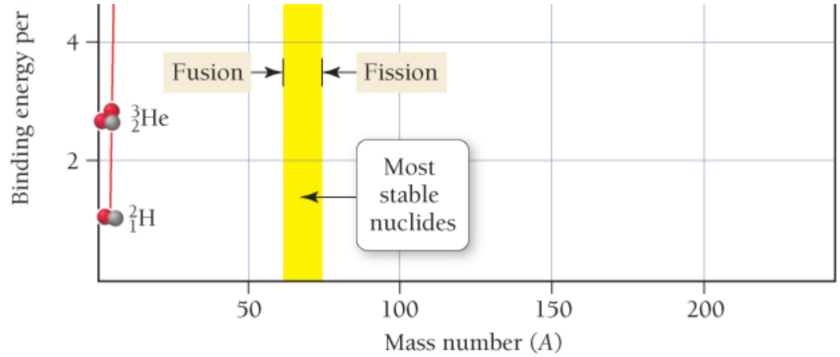
The Nuclear Binding Energy Curve

Figure 20.12 shows the binding energy per nucleon plotted as a function of mass number (A). The binding energy per nucleon is relatively low for small mass numbers and increases until about $A = 60$, where it reaches a maximum. Nuclides with mass numbers of about 60, therefore, are among the most stable. Beyond $A = 60$, the binding energy per nucleon decreases again. The figure illustrates why nuclear fission is a highly exothermic process. When a heavy nucleus, such as U-235, breaks up into smaller nuclei, such as Ba-140 and Kr-93, the binding energy per nucleon increases. This is analogous to a chemical reaction in which weak bonds break and strong bonds form. In both cases, the process is exothermic. Figure 20.12 also reveals that *combining* two lighter nuclei (below $A = 60$) to form a heavier nucleus is exothermic as well. This process is called *nuclear fusion*, which we discuss in the next section of this chapter.

Figure 20.12 Nuclear Binding Energy per Nucleon

The nuclear binding energy per nucleon (a measure of the stability of a nucleus) reaches a maximum at about $A = 60$. Energy can be obtained either by breaking a heavy nucleus up into lighter ones (fission) or by combining lighter nuclei into heavier ones (fusion).




Binding energy per
4
2
Fusion
Fission
$^{3}_{2}$He
$^{2}_{1}$H
Most
stable
nuclides
50
100
150
200
Mass number (A)