

CS 3133 Foundations of Computer Science
C term 2018

Solutions of the Sample Problems for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ that contain the substring ab and the substring ba .

Solution:

$$(a^+b^+a(a \cup b)^*) \cup (b^+a^+b(a \cup b)^*)$$

(20 points)

2. Consider the following grammar G :

$$S \rightarrow SAB|\lambda$$

$$A \rightarrow aA|a$$

$$B \rightarrow bB|\lambda$$

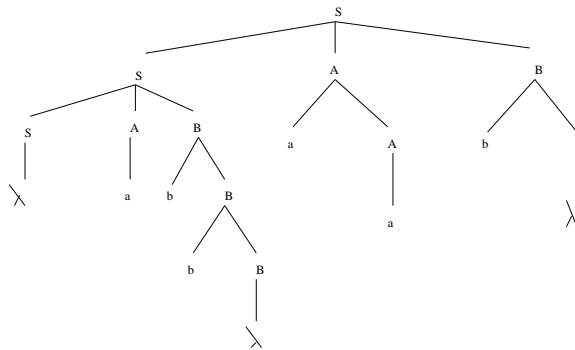
- (a) Give a leftmost derivation of $abbaab$.
- (b) Build the derivation tree for the derivation in part (1).
- (c) What is $L(G)$?

Solution:

(a) The following is a leftmost derivation of $abbaab$:

$$\begin{aligned}
 S &\Rightarrow SAB \\
 &\Rightarrow SABAB \\
 &\Rightarrow ABAB \\
 &\Rightarrow aBAB \\
 &\Rightarrow abBAB \\
 &\Rightarrow abbBAB \\
 &\Rightarrow abbAB \\
 &\Rightarrow abbaAB \\
 &\Rightarrow abbaaB \\
 &\Rightarrow abbaabB \\
 &\Rightarrow abbaab
 \end{aligned}$$

(b)



(c)

$$L(G) = a(a \cup b)^* \cup \lambda$$

(20 points)

3. Construct a regular grammar over the alphabet $\Sigma = \{a, b, c, d\}$ whose language is the set of strings that contain exactly two b -s.

Solution:

The following is a *regular* grammar over $\{a, b, c, d\}$ whose language is the set of strings containing exactly two b -s:

$$\begin{aligned}
 S &\rightarrow aS \mid cS \mid dS \mid bB \\
 B &\rightarrow aB \mid cB \mid dB \mid bC \\
 C &\rightarrow aC \mid cC \mid dC \mid \lambda
 \end{aligned}$$

(20 points)

4. Consider the following grammar G :

$$\begin{aligned} S &\rightarrow aSA|\lambda \\ A &\rightarrow bA|\lambda \end{aligned}$$

- (a) Give a regular expression for $L(G)$.
(b) Is G ambiguous? Explain your answer.

Solution:

- (a) The following is a regular expression for $L(G)$:

$$a^+b^* \cup \lambda$$

- (b) Yes the grammar is ambiguous. Here are two different *leftmost* derivations for the string $aabb$:

$$\begin{aligned} S &\Rightarrow aSA \\ &\Rightarrow aaSAA \\ &\Rightarrow aaAA \\ &\Rightarrow aabAA \\ &\Rightarrow aabbAA \\ &\Rightarrow aabbA \\ &\Rightarrow aabb \end{aligned}$$

and

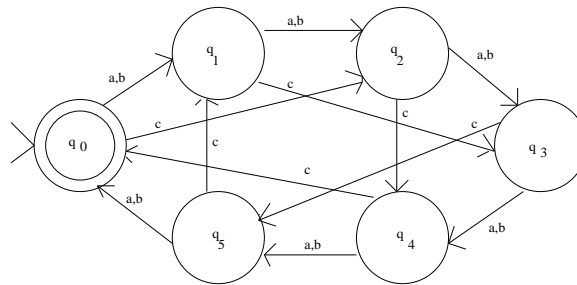
$$\begin{aligned} S &\Rightarrow aSA \\ &\Rightarrow aaSAA \\ &\Rightarrow aaAA \\ &\Rightarrow aaA \\ &\Rightarrow aabA \\ &\Rightarrow aabbA \\ &\Rightarrow aabb \end{aligned}$$

(20 points)

5. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of a 's plus the number of b 's plus twice the number of c 's is divisible by six.

Solution:

The state diagram of a DFA is



(20 points)