CS 3133 Foundations of Computer Science
C term 2018

Solutions for Homework 5

1. Exercise 17.b. on page 249.

   Solution: (with the pumping lemma for context-free languages) Let us assume indirectly that the language $L = \{a^i b^j c^i d^j \mid i, j \geq 0\}$ is context-free. Then by the pumping lemma for context-free languages there exists a number k, such that every string $z \in L$ of length k or more can be decomposed into substrings u, v, w, x and y such that $length(vwx) \leq k$, $length(v) + length(x) > 0$ and $uv^i wx^i y \in L$ for all $i \geq 0$.

   Consider the string $z = a^k b^k c^k d^k$. Clearly $z \in L$ and $length(z) \geq k$. Using the pumping lemma we decompose z into substrings u, v, w, x and y, where $0 < length(vwx) \leq k$. But in this case uv^2wx^2y cannot be in L, a contradiction. Indeed, consider the possibilities for v and x. If either of these contains more than one type symbol, then uv^2wx^2y is not in L. So v and x must be substrings of one of a^k, b^k, c^k, or d^k and if they contain different symbols, then they have to be two "consecutive" symbols, so a and b, or b and c, or c and d. But then again uv^2wx^2y is not in L, since either the number of a's is different from the number of c's, or the number of b's is different from the number of d's. Thus L is not context-free. (20 points)

2. Let M be the Turing machine defined by

δ	B	a	b	c
q_0	(q_0, B, R)	(q_0, a, R)	(q_0, b, R)	(q_1, c, L)
q_1	(q_2, B, R)	(q_1, b, L)	(q_1, a, L)	-
q_2	-	-	-	-

   (a) Trace the computation for the input string $abcab$.

   (b) Trace the first six transitions of the computation for the input string $abab$.

(c) Give the state diagram of M.

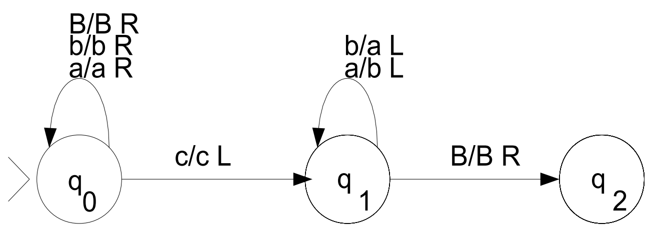
(d) Describe the result of a computation in M.

Solution:

a,b.)

$$\begin{array}{ll} & q_0Babcab \\ \vdash & q_0abcab \\ \vdash & aq_0bcab \\ \vdash & abq_0cab \\ \vdash & aq_1bcab \\ \vdash & q_1aacab \\ \vdash & q_1Bbacab \\ \vdash & q_2bacab \end{array} \qquad \begin{array}{ll} & q_0Babab \\ \vdash & q_0abab \\ \vdash & aq_0bab \\ \vdash & abq_0ab \\ \vdash & abaq_0b \\ \vdash & ababq_0B \\ \vdash & ababBq_0B \end{array}$$

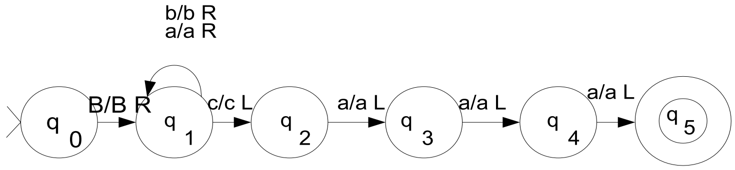
c.)



d.) If there is a c, then swap the a's and b's before the first c. Otherwise, if there is no c, then go to the right infinitely. (20 points)

3. Construct a Turing machine with input alphabet $\{a, b, c\}$ that accepts strings in which the first c is immediately preceded by the substring aaa. A string must contain a c to be accepted by the machine.

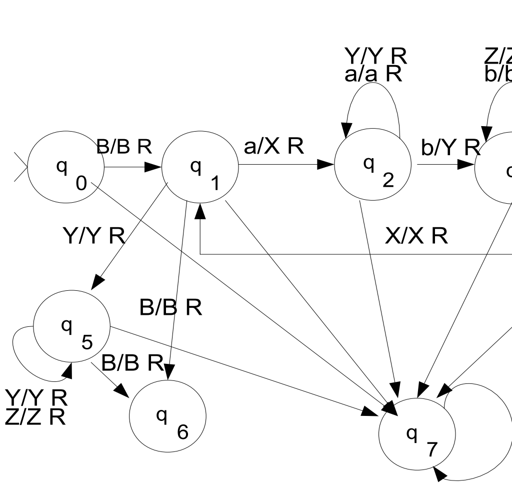
Solution:



(20 points)

4. Construct a Turing machine with input alphabet $\{a, b, c\}$ that accepts the language $L = \{a^i b^i c^i \mid i \geq 0\}$ by halting only.

   Solution:

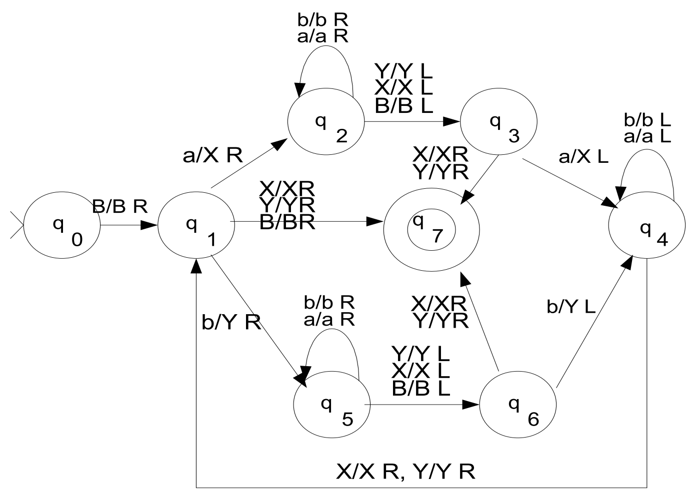


   In this picture, if $\delta(q_i, x)$ was not defined for $q_i \notin F$ in the original machine, then we have $\delta(q_i, x) = [q_7, x, R]$ for every $1 \leq i \leq 5$ and $\delta(q_7, x) = [q_7, x, R]$ for every $x \in \Gamma$. (20 points)

5. Construct a standard Turing machine that accepts the set of palin-

dromes over $\{a, b\}$.

Solution:



(20 points)