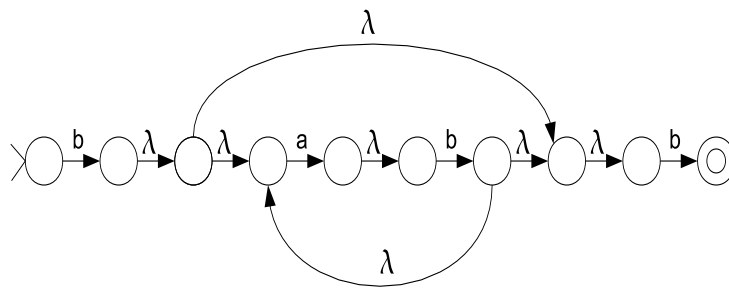**CS 3133 Foundations of Computer Science**
C term 2018

Solutions for Homework 4

1. Use the technique from Section 6.1 in the book (i.e. constructing an NFA-λ for a given regular set by following the recursive definition of the regular set) to build the state diagram of an NFA-λ that accepts the language $b(ab)^*b$.
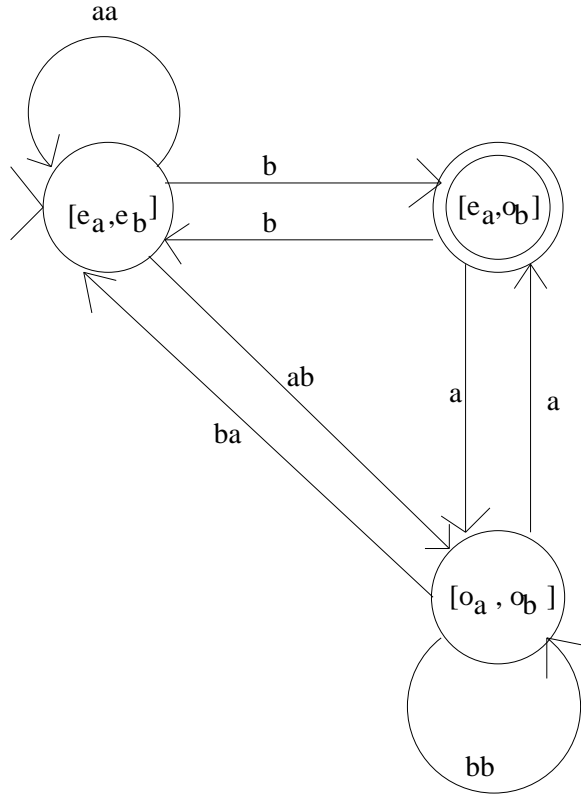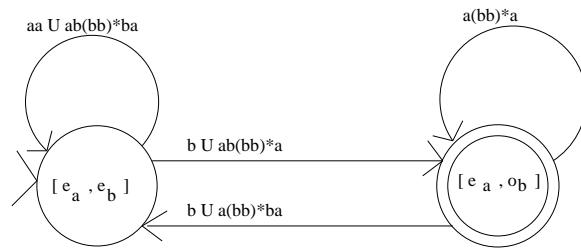
   Solution:

   (20 points)

2. Exercise 3 on page 217. (There is an error in the book, it should say: The language of the DFA M in Example 5.3.7 consists of...)

   Solution:

   After deleting the state $[o_a, e_b]$ we get the following expression graph:

aa

b

b

$[e_a, e_b]$

$[e_a, o_b]$

ab

a

a

ba

$[o_a , o_b]$

bb

Finally after deleting the only remaining state $[o_a, o_b]$ that is not the starting state and the accepting state we get the final expression graph:

aa U ab(bb)*ba

a(bb)*a

b U ab(bb)*a

$[e_a , e_b]$

$[e_a , o_b]$

b U a(bb)*ba

Thus in the second figure on page 195 we have

$$u = aa \cup ab(bb)^*ba, v = b \cup ab(bb)^*a, w = a(bb)^*a \quad \text{and}$$

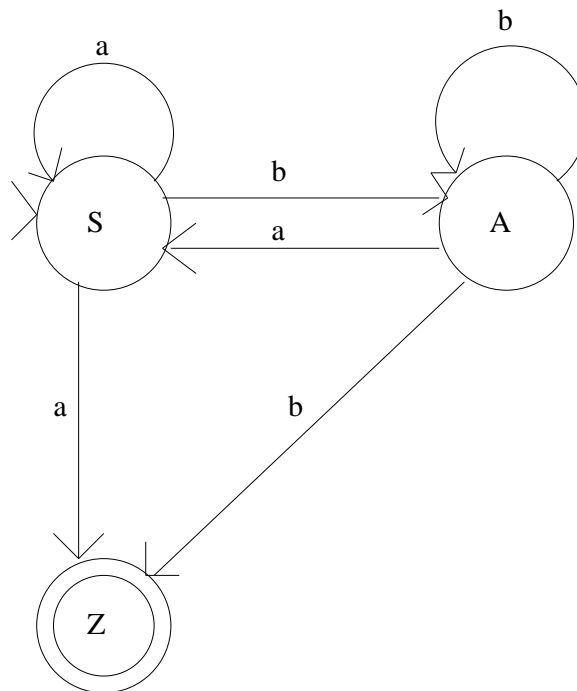$$x = b \cup a(bb)^*ba,$$

and the regular expression is

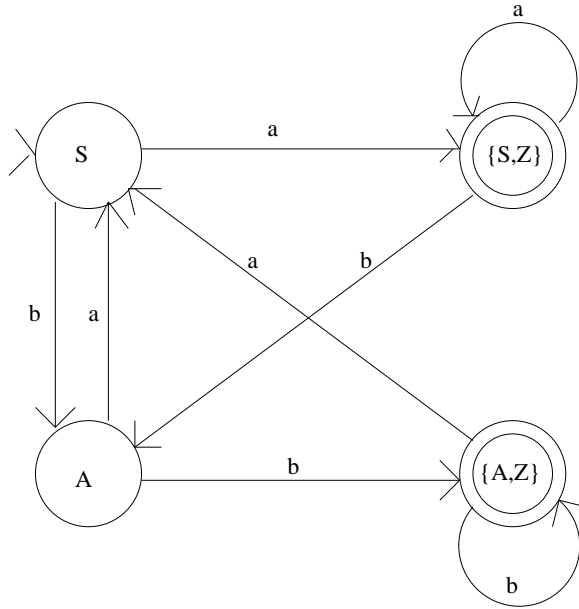$$u^*v(w \cup x(u)^*w)^*.$$

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(20 points)

3. Exercise 4 on page 217.

   Solution:

   a.)

   b.)

c.)
$$S \rightarrow aS|bA|aZ$$
$$A \rightarrow bA|aS|bZ$$
$$Z \rightarrow \lambda$$

d.)
$$S \rightarrow bA|aB$$
$$A \rightarrow aS|bC$$
$$B \rightarrow aB|bA|\lambda$$
$$C \rightarrow aS|bC|\lambda$$

where $\{S, Z\} = B$ and $\{A, Z\} = C$.

e.) $(\boldsymbol{a} \cup \boldsymbol{b}^+\boldsymbol{a})^*(\boldsymbol{a} \cup \boldsymbol{b}^+\boldsymbol{b})$.

(20 points)

4. Exercise 14.b. on page 218.

Solution: (with the pumping lemma) Let us assume indirectly that the language $L = \{a^n b^m | n < m\}$ is regular. This implies that L is accepted by some DFA. Let k be the number of states of the DFA. By the pumping lemma, every string $z \in L$ of length k or more can

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be decomposed into substrings u, v and x such that $length(uv) \leq k$, $length(v) > 0$ and $uv^i x \in L$ for all $i \geq 0$.
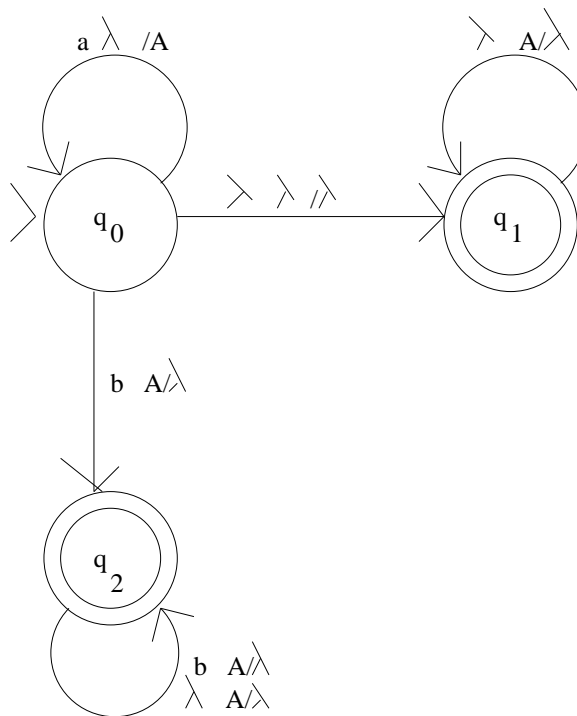
Consider the string $z = a^k b^{k+1}$. Clearly $z \in L$ and $length(z) \geq k$. Using the pumping lemma we decompose z into substrings u, v and x, where $0 < length(uv) \leq k$. Then v is a substring of the first a^k. But in this case $uv^2 x$ cannot be in L, since we have at least as many a's as b's, a contradiction. L is non-regular. (20 points)

5. Exercise 1 on page 247.

   Solution:

   a.) $L(M) = \{a^i b^j | i \geq j \geq 0\}$.

   b.)

   c.) Here are *all* computations for the string *aab*:

$$[q_0, aab, \lambda] \quad [q_0, aab, \lambda] \quad [q_0, aab, \lambda] \quad [q_0, aab, \lambda]$$
$$\vdash [q_0, ab, A] \quad \vdash [q_1, aab, \lambda] \quad \vdash [q_0, ab, A] \quad \vdash [q_0, ab, A]$$
$$\vdash [q_0, b, AA] \quad reject \qquad \vdash [q_1, ab, A] \quad \vdash [q_0, b, AA]$$
$$\vdash [q_2, \lambda, A] \qquad\qquad\qquad \vdash [q_1, ab, \lambda] \quad \vdash [q_1, b, AA]$$
$$\vdash [q_2, \lambda, \lambda] \qquad\qquad\qquad\quad reject \qquad\quad \vdash [q_1, b, A]$$
$$accept \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \vdash [q_1, b, \lambda]$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad reject$$

For the string *abb*:

$$[q_0, abb, \lambda] \quad [q_0, abb, \lambda] \quad [q_0, abb, \lambda]$$
$$\vdash [q_0, bb, A] \quad \vdash [q_1, abb, \lambda] \quad \vdash [q_0, bb, A]$$
$$\vdash [q_2, b, \lambda] \quad\quad reject \qquad\quad \vdash [q_1, bb, A]$$
$$reject \qquad\qquad\qquad\qquad\qquad\quad \vdash [q_1, bb, \lambda]$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad reject$$

For the string *aba*:

$$[q_0, aba, \lambda] \quad [q_0, aba, \lambda] \quad [q_0, aba, \lambda]$$
$$\vdash [q_0, ba, A] \quad \vdash [q_1, aba, \lambda] \quad \vdash [q_0, ba, A]$$
$$\vdash [q_2, a, \lambda] \quad\quad reject \qquad\quad \vdash [q_1, ba, A]$$
$$reject \qquad\qquad\qquad\qquad\qquad\quad \vdash [q_1, ba, \lambda]$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad reject$$

d.)

$$[q_0, aabb, \lambda]$$
$$\vdash [q_0, abb, A]$$
$$\vdash [q_0, bb, AA]$$
$$\vdash [q_2, b, A]$$
$$\vdash [q_2, \lambda, \lambda]$$
$$accept$$

$$[q_0, aaab, \lambda]$$
$$\vdash [q_0, aab, A]$$
$$\vdash [q_0, ab, AA]$$
$$\vdash [q_0, b, AAA]$$
$$\vdash [q_2, \lambda, AA]$$
$$\vdash [q_2, \lambda, A]$$
$$\vdash [q_2, \lambda, \lambda]$$
$$accept$$

(20 points)