CS 3133 Foundations of Computer Science
C term 2019

Solutions for Homework 5

1. Exercise 17.b. on page 249.

   Solution: (with the pumping lemma for context-free languages) Let us assume indirectly that the language $L = \{a^i b^j c^i d^j \mid i, j \geq 0\}$ is context-free. Then by the pumping lemma for context-free languages there exists a number k, such that every string $z \in L$ of length k or more can be decomposed into substrings u, v, w, x and y such that $length(vwx) \leq k$, $length(v) + length(x) > 0$ and $uv^i wx^i y \in L$ for all $i \geq 0$.

   Consider the string $z = a^k b^k c^k d^k$. Clearly $z \in L$ and $length(z) \geq k$. Using the pumping lemma we decompose z into substrings u, v, w, x and y, where $0 < length(vwx) \leq k$. But in this case uv^2wx^2y cannot be in L, a contradiction. Indeed, consider the possibilities for v and x. If either of these contains more than one type symbol, then uv^2wx^2y is not in L. So v and x must be substrings of one of a^k, b^k, c^k, or d^k and if they contain different symbols, then they have to be two "consecutive" symbols, so a and b, or b and c, or c and d. But then again uv^2wx^2y is not in L, since either the number of a's is different from the number of c's, or the number of b's is different from the number of d's. Thus L is not context-free. (20 points)

2. Let M be the Turing machine defined by

δ	B	a	b	c
q_0	(q_0, B, R)	(q_0, a, R)	(q_0, b, R)	(q_1, c, L)
q_1	(q_2, B, R)	(q_1, b, L)	(q_1, a, L)	-
q_2	-	-	-	-

   (a) Trace the computation for the input string *abcab*.

   (b) Trace the first six transitions of the computation for the input string *abab*.

(c) Give the state diagram of M.

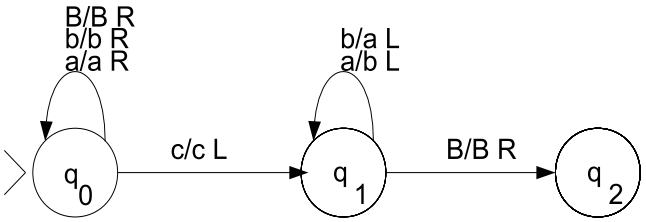
(d) Describe the result of a computation in M.

Solution:

a,b.)

$$
\begin{aligned}
& q_0Babcab \\
\vdash\ & q_0abcab \\
\vdash\ & aq_0bcab \\
\vdash\ & abq_0cab \\
\vdash\ & aq_1bcab \\
\vdash\ & q_1aacab \\
\vdash\ & q_1Bbacab \\
\vdash\ & q_2bacab
\end{aligned}
\qquad\qquad
\begin{aligned}
& q_0Babab \\
\vdash\ & q_0abab \\
\vdash\ & aq_0bab \\
\vdash\ & abq_0ab \\
\vdash\ & abaq_0b \\
\vdash\ & ababq_0B \\
\vdash\ & ababBq_0B
\end{aligned}
$$

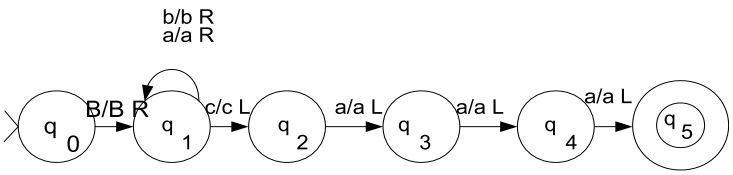
c.)



d.) If there is a c, then swap the a's and b's before the first c. Otherwise, if there is no c, then go to the right infinitely. (20 points)

3. Construct a Turing machine with input alphabet $\{a, b, c\}$ that accepts strings in which the first c is immediately preceded by the substring aaa. A string must contain a c to be accepted by the machine.

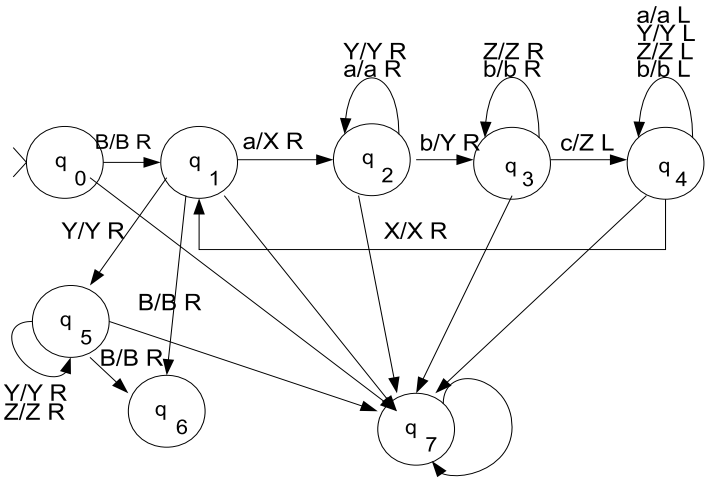
Solution:



(20 points)

4. Construct a Turing machine with input alphabet $\{a, b, c\}$ that accepts the language $L = \{a^i b^i c^i \mid i \geq 0\}$ by halting only.

   Solution:

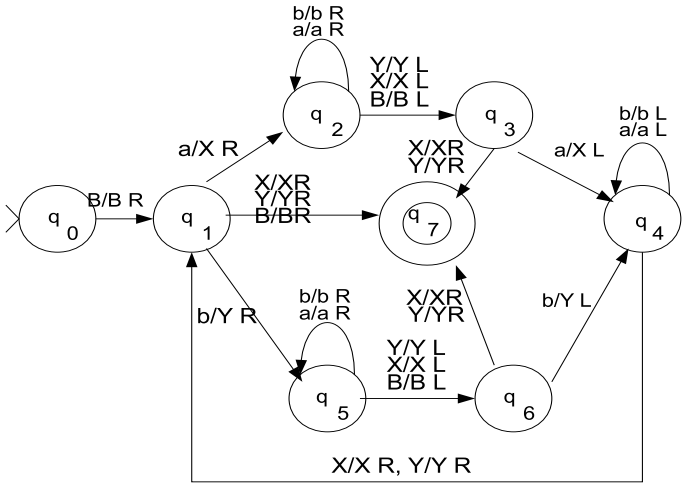


   In this picture, if $\delta(q_i, x)$ was not defined for $q_i \notin F$ in the original machine, then we have $\delta(q_i, x) = [q_7, x, R]$ for every $1 \leq i \leq 5$ and $\delta(q_7, x) = [q_7, x, R]$ for every $x \in \Gamma$. (20 points)

5. Construct a standard Turing machine that accepts the set of palin-

dromes over $\{a, b\}$.

Solution:



(20 points)