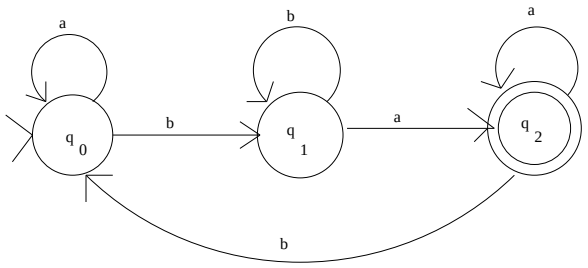
CS 3133 Foundations of Computer Science
C term 2019

Solutions for Homework 3

1. Exercise 1 on page 184.

   Solution:

   (a) The state diagram of M is



   (b)

$$
\begin{array}{ll}
i) & [q_0, abaa] \\
\vdash & [q_0, baa] \\
\vdash & [q_1, aa] \\
\vdash & [q_2, a] \\
\vdash & [q_2, \lambda]
\end{array}
\qquad
\begin{array}{ll}
ii) & [q_0, bbbabb] \\
\vdash & [q_1, bbabb] \\
\vdash & [q_1, babb] \\
\vdash & [q_1, abb] \\
\vdash & [q_2, bb] \\
\vdash & [q_0, b] \\
\vdash & [q_1, \lambda]
\end{array}
$$

$$
\begin{array}{ll}
iii) & [q_0, bababa] \\
\vdash & [q_1, ababa] \\
\vdash & [q_2, baba] \\
\vdash & [q_0, aba] \\
\vdash & [q_0, ba] \\
\vdash & [q_1, a] \\
\vdash & [q_2, \lambda]
\end{array}
\qquad
\begin{array}{ll}
iv) & [q_0, bbbaa] \\
\vdash & [q_1, bbaa] \\
\vdash & [q_1, baa] \\
\vdash & [q_1, aa] \\
\vdash & [q_2, a] \\
\vdash & [q_2, \lambda]
\end{array}
$$

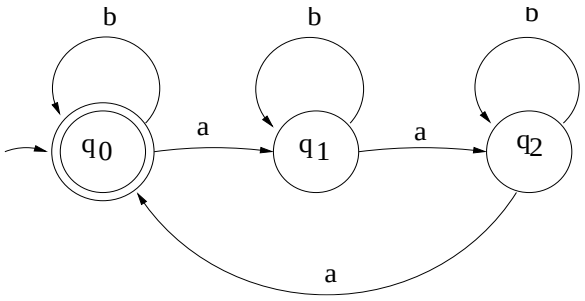
   (c) The computations in i, iii and iv terminate in the accepting state q_2. Therefore the strings $abaa$, $bababa$ and $bbbaa$ are in $L(M)$.

(d) Two regular expressions describing $L(M)$ are $\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+(\boldsymbol{b}\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+)^*$ and $(\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+\boldsymbol{b})^*\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+$. (20 points)

2. Exercise 11 on page 185.

   Solution:

   The state diagram of a DFA is

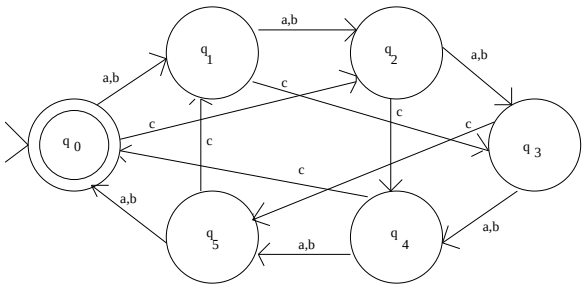


   (20 points)

3. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of a's plus the number of b's plus twice the number of c's is divisible by six.

   Solution:

   The state diagram of a DFA is



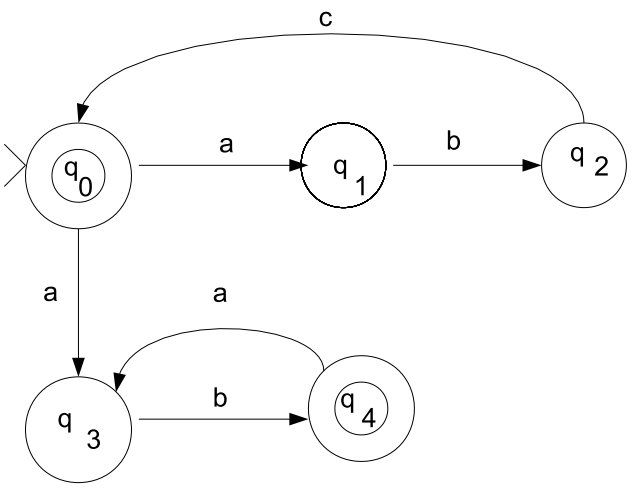
   (20 points)

4. Design an NFA that accepts the following language over the alphabet $\{a, b\}$:

$$(\boldsymbol{abc})^*(\boldsymbol{ab})^*$$

Solution:

The state diagram of an NFA is



(20 points)

5. Exercise 36 on page 187.

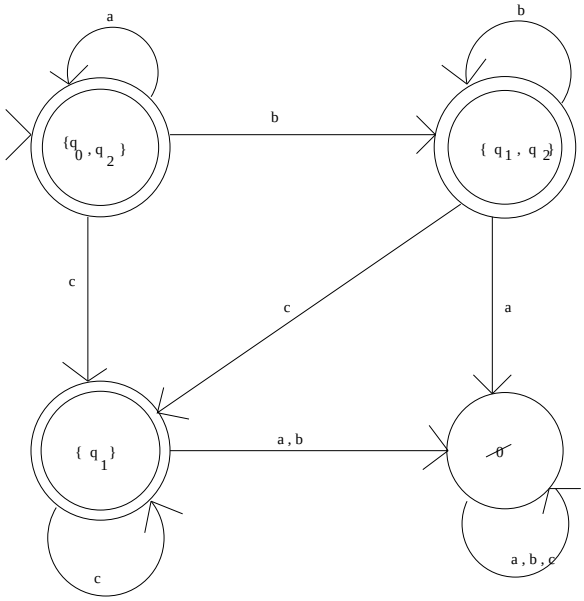
Solution:

(a) $\lambda - closure(q_0) = \{q_0, q_2\}$.

(b) The input transition function t is the following:

t	a	b	c
q_0	$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_1\}$
q_1	$\emptyset$	$\emptyset$	$\{q_1\}$
q_2	$\emptyset$	$\{q_1, q_2\}$	$\emptyset$

(c) The equivalent DFA:



(d) A regular expression is $\boldsymbol{a}^*\boldsymbol{b}^*\boldsymbol{c}^*$. (20 points)