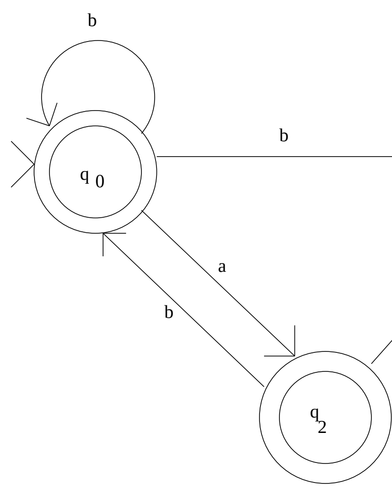
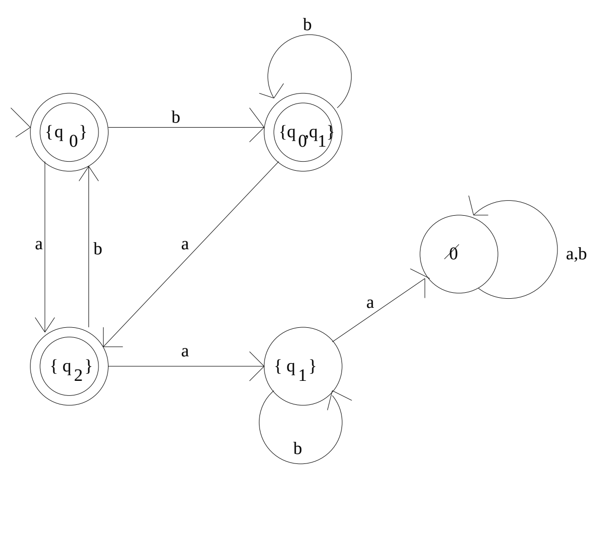
CS 3133 Foundations of Computer Science
C term 2019

Solutions for the Practice Final Exam

1. Construct the state diagram of a DFA equivalent to the following NFA



Solution:



2. Consider the following grammar G:

$$S \rightarrow aSdd|A$$
$$A \rightarrow bAc|bc$$

(a) Give a leftmost derivation of *aabbccdddd*.

(b) Build the derivation tree for the derivation in part (a).

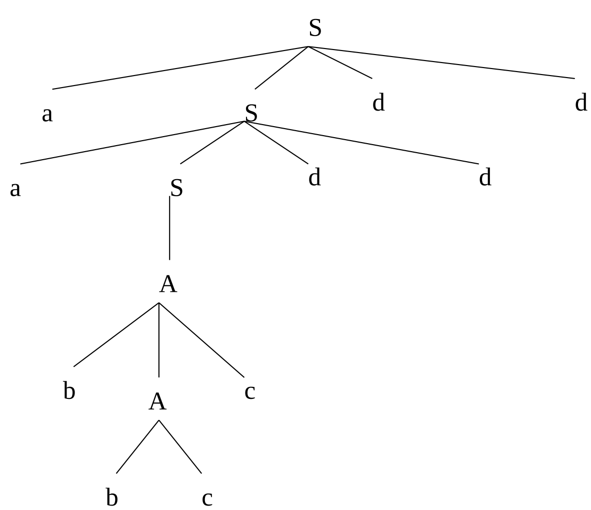
(c) What is $L(G)$?

Solution:

(a) The following is a leftmost derivation of *aabbccdddd*:

$$\begin{aligned} S &\Rightarrow aSdd \\ &\Rightarrow aaSdddd \\ &\Rightarrow aaAdddd \\ &\Rightarrow aabAcdddd \\ &\Rightarrow aabbccdddd \end{aligned}$$

(b) Here is the derivation tree:



(c)

$$L(G) = \{a^n b^m c^m d^{2n} | n \geq 0, m > 0\}$$

3. Consider the following regular grammar G:

$$S \to aB|a$$
$$B \to bS.$$

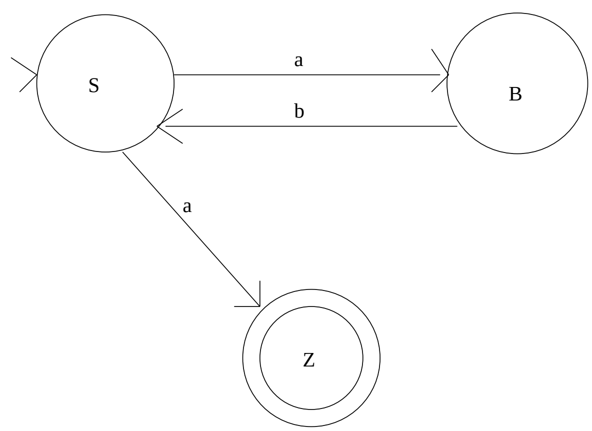
(a) Build an NFA M that accepts $L(G)$.

(b) Construct a regular grammar G' from M that generates $L(M)$. What is the difference between G and G'?

(c) Give a regular expression for $L(G)$.

Solution:

(a)



(b) The grammar G' is the following:

$$S \to aB|aZ$$
$$B \to bS$$
$$Z \to \lambda$$

Thus the only difference is that $S \to a$ is replaced by $S \to aZ$ and $Z \to \lambda$.

(c)

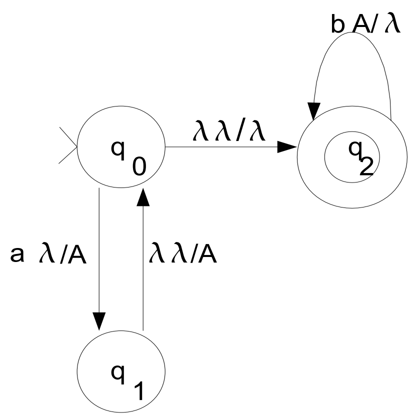
$$L(G) = (\boldsymbol{ab})^*\boldsymbol{a}$$

4. Construct a PDA with $\Sigma = \{a, b\}$ that accepts the language

$$\{a^i b^{2i} | i \geq 0\}.$$

Is this language context-free? Is this language regular? Justify your answers!

Solution:



Of course it is context-free since it is accepted by a PDA. However, it is not regular as can be seen by applying the Pumping Lemma for regular languages. Let us assume indirectly that the language $L = \{a^i b^{2i} | i \geq 0\}$ is regular. This implies that L is accepted by some DFA. Let k be the number of states of the DFA. By the pumping lemma, every string $z \in L$ of length k or more can be decomposed into substrings u, v and w such that $length(uv) \leq k$, $length(v) > 0$ and $uv^i w \in L$ for all $i \geq 0$.

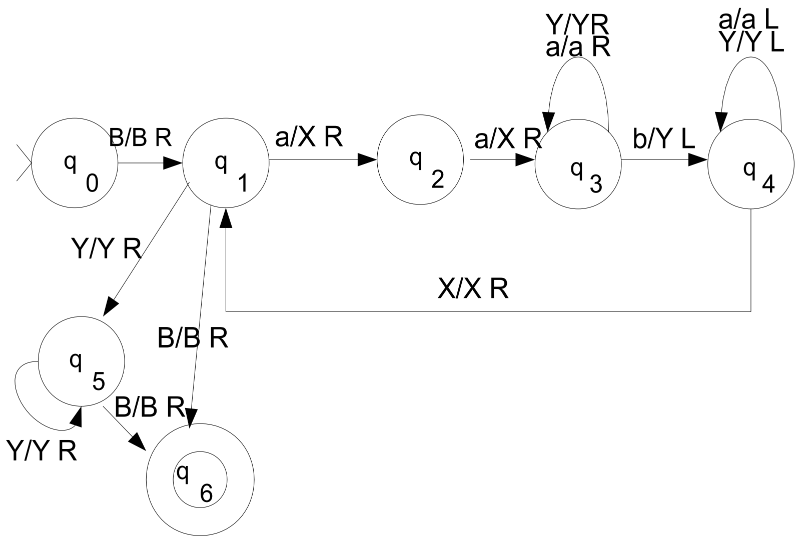
Consider the string $z = a^k b^{2k}$. Clearly $z \in L$ and $length(z) \geq k$. Using the pumping lemma we decompose z into substrings u, v and w, where $0 < length(uv) \leq k$. Then v is a substring of the a^k. But in this case $uv^2 w$ cannot be in L (since the number of a's is more than half of the number of b's), a contradiction. L is non-regular.

5. Design a standard Turing machine with $\Sigma = \{a, b\}$ that accepts the language

$$\{a^{2i} b^i | i \geq 0\}.$$

Is this language recursive?

Solution:



Of course it is recursive since it is accepted by a Turing machine that halts on every input.