

CS 3133 Foundations of Computer Science
C term 2018

Solutions for the Midterm Exam

1. Give a regular expression that represents the set of all non-empty strings over $\Sigma = \{a, b, c\}$ in which all the a 's precede the b 's, which in turn precede the c 's.

Solution:

$$a^+b^*c^* \cup a^*b^+c^* \cup a^*b^*c^+$$

(20 points)

2. Consider the following grammar G :

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aX \mid bX \mid a \\ Y &\rightarrow Ya \mid Yb \mid b \end{aligned}$$

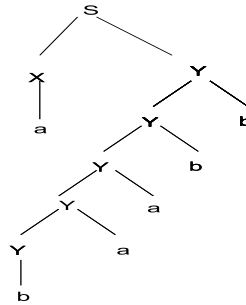
- (a) Give a leftmost derivation of $abaabb$.
(b) Build the derivation tree for the derivation in part (a).
(c) What is $L(G)$? Give a regular expression!

Solution:

- (a) The following is a leftmost derivation of $abaabb$:

$$\begin{aligned} S &\Rightarrow XY \\ &\Rightarrow aY \\ &\Rightarrow aYb \\ &\Rightarrow aYbb \\ &\Rightarrow aYabb \\ &\Rightarrow aYaabb \\ &\Rightarrow abaabb \end{aligned}$$

(b)



(c)

$$L(G) = (a \cup b)^* ab(a \cup b)^*$$

(20 points)

3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language determined in the previous problem 2(c).

Solution:

Unambiguous regular grammar:

$$\begin{aligned} S &\rightarrow bS \mid aA \\ A &\rightarrow aA \mid bB \\ B &\rightarrow aB \mid bB \mid \lambda \end{aligned}$$

Ambiguous regular grammar:

$$\begin{aligned} S &\rightarrow bS \mid aA \\ A &\rightarrow aA \mid bB \mid bC \\ B &\rightarrow aB \mid bB \mid \lambda \\ C &\rightarrow aC \mid bC \mid \lambda \end{aligned}$$

It is ambiguous because there are two different leftmost derivations for the string ab :

$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow abB \\ &\Rightarrow ab \end{aligned}$$

and

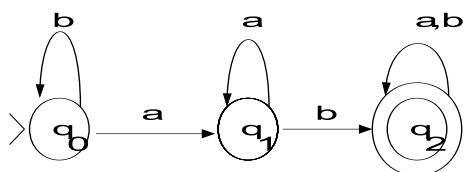
$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow abC \\ &\Rightarrow ab \end{aligned}$$

(20 points)

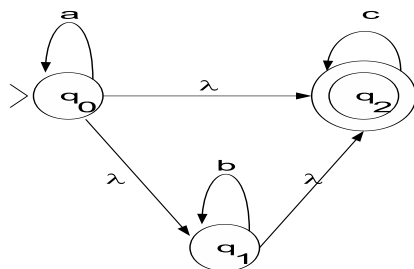
4. Design a DFA that accepts the language determined in problem 2(c).
(20 points)

Solution:

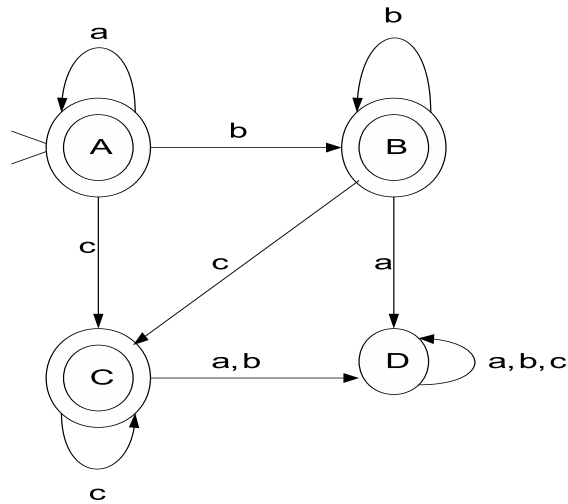
The state diagram of a DFA is



5. Design the state diagram of a DFA equivalent to the following NFA- λ by using the subset construction method. What is the language accepted by these machines? (20 points)



Solution:



Here the states correspond to the following subsets: $A = \{q_0, q_1, q_2\}$, $B = \{q_1, q_2\}$, $C = \{q_2\}$ and $D = \emptyset$. The language is $\mathbf{a^*b^*c^*}$.