CS 3133 Foundations of Computer Science
C term 2019

Solutions for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ with an even number of a's or an odd number of b's.

   Solution:

   $$(\boldsymbol{b}^*\boldsymbol{a}\boldsymbol{b}^*\boldsymbol{a})^*\boldsymbol{b}^* \cup (\boldsymbol{a}^*\boldsymbol{b}\boldsymbol{a}^*\boldsymbol{b})^*\boldsymbol{a}^*\boldsymbol{b}\boldsymbol{a}^*$$

   (20 points)

2. Consider the following grammar G:

   $$\begin{aligned} S &\to XY \\ X &\to aX \mid bX \mid a \\ Y &\to Ya \mid Yb \mid a \end{aligned}$$

   (a) Give a leftmost derivation of $abaabb$.

   (b) Build the derivation tree for the derivation in part (a).

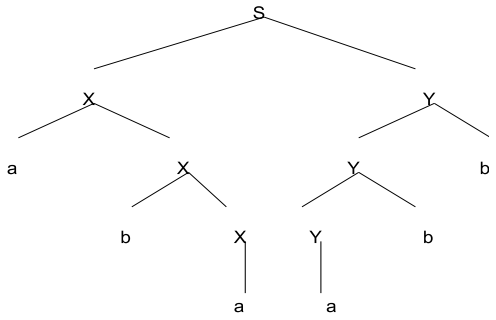
   (c) What is $L(G)$?

   Solution:

   (a) The following is a leftmost derivation of $abaabb$:

   $$\begin{aligned} S &\Rightarrow XY \\ &\Rightarrow aXY \\ &\Rightarrow abXY \\ &\Rightarrow abaY \\ &\Rightarrow abaYb \\ &\Rightarrow abaYbb \\ &\Rightarrow abaabb \end{aligned}$$

   (b)



(c)

$$L(G) = (\boldsymbol{a} \cup \boldsymbol{b})^* \, \boldsymbol{aa} \, (\boldsymbol{a} \cup \boldsymbol{b})^*$$

(20 points)

3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language consisting of the set of strings over $\Sigma = \{a, b\}$ in which the number of a's is divisible by three.

Solution:

Unambiguous regular grammar:

$$\begin{aligned} S &\rightarrow bS \mid aA \mid \lambda \\ A &\rightarrow bA \mid aB \\ B &\rightarrow bB \mid aS \end{aligned}$$

Ambiguous regular grammar:

$$\begin{aligned} S &\rightarrow bS \mid aA \mid \lambda \\ A &\rightarrow bA \mid aB \mid aC \\ B &\rightarrow bB \mid aS \\ C &\rightarrow bC \mid aS \end{aligned}$$

It is ambiguous because there are two different leftmost derivations for the string aaa:

$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow aaB \\ &\Rightarrow aaaS \\ &\Rightarrow aaa \end{aligned}$$

and

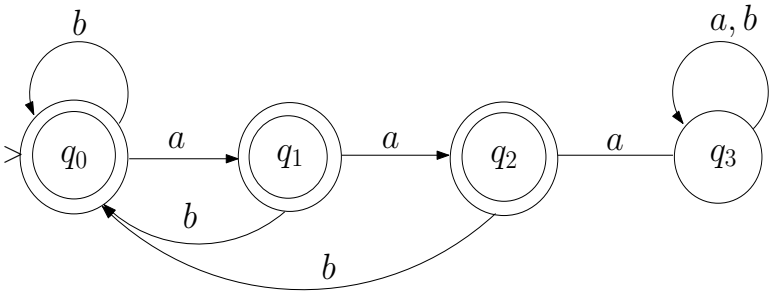
$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow aaC \\ &\Rightarrow aaaS \\ &\Rightarrow aaa \end{aligned}$$

(20 points)

4. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b\}$ that do not contain the substring aaa.

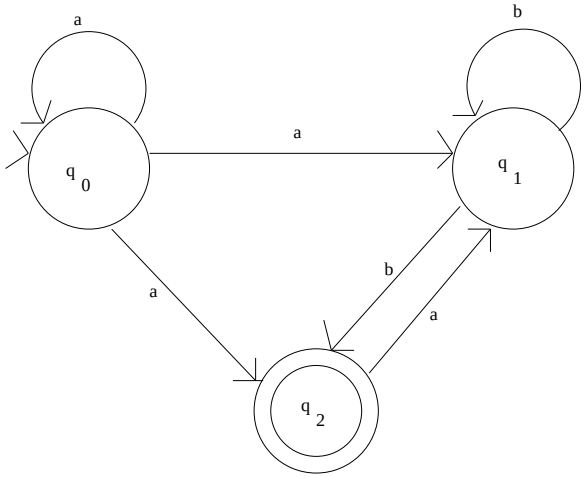
   Solution:

   The state diagram of a DFA is

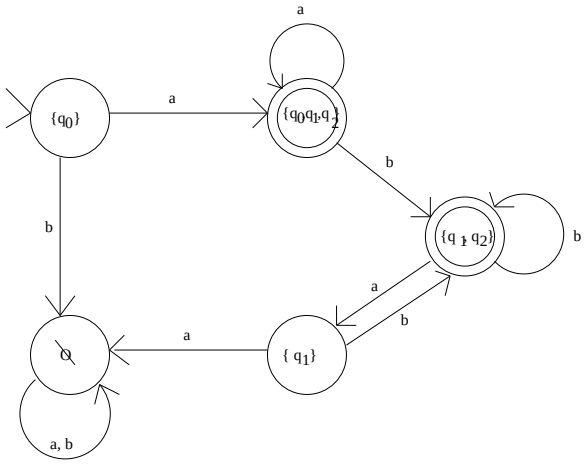


   (20 points)

5. Construct the state diagram of a DFA equivalent to the following NFA. What is the language accepted by these machines?



Solution:



The language is

$$\boldsymbol{a}^{+}\boldsymbol{b}^{*}(\boldsymbol{a}\boldsymbol{b}^{+})^{*}$$

(20 points)