CS 3133 Foundations of Computer Science
C term 2019

Solutions of the Sample Problems for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ that contain the substring ab and the substring ba.

   Solution:

   $$\left(\boldsymbol{a}^{+}\boldsymbol{b}^{+}\boldsymbol{a}(\boldsymbol{a} \cup \boldsymbol{b})^{*}\right) \cup \left(\boldsymbol{b}^{+}\boldsymbol{a}^{+}\boldsymbol{b}(\boldsymbol{a} \cup \boldsymbol{b})^{*}\right)$$

   (20 points)

2. Consider the following grammar G:

   $$S \to SAB|\lambda$$
   $$A \to aA|a$$
   $$B \to bB|\lambda$$

   (a) Give a leftmost derivation of $abbaab$.

   (b) Build the derivation tree for the derivation in part (1).

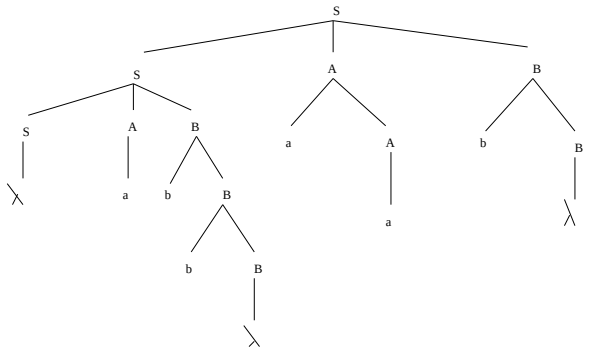
   (c) What is $L(G)$?

   Solution:

(a) The following is a leftmost derivation of $abbaab$:

$$\begin{aligned} S &\Rightarrow SAB \\ &\Rightarrow SABAB \\ &\Rightarrow ABAB \\ &\Rightarrow aBAB \\ &\Rightarrow abBAB \\ &\Rightarrow abbBAB \\ &\Rightarrow abbAB \\ &\Rightarrow abbaAB \\ &\Rightarrow abbaaB \\ &\Rightarrow abbaabB \\ &\Rightarrow abbaab \end{aligned}$$

(b)



(c)

$$L(G) = \boldsymbol{a}(\boldsymbol{a} \cup \boldsymbol{b})^* \cup \lambda$$

(20 points)

3. Construct a regular grammar over the alphabet $\Sigma = \{a, b, c, d\}$ whose language is the set of strings that contain exactly two b-s.

Solution:

The following is a *regular* grammar over $\{a, b, c, d\}$ whose language is the set of strings containing exactly two b-s:

$$\begin{aligned} S &\rightarrow aS \mid cS \mid dS \mid bB \\ B &\rightarrow aB \mid cB \mid dB \mid bC \\ C &\rightarrow aC \mid cC \mid dC \mid \lambda \end{aligned}$$

(20 points)

4. Consider the following grammar G:

$$S \to aSA|\lambda$$
$$A \to bA|\lambda$$

   (a) Give a regular expression for $L(G)$.

   (b) Is G ambiguous? Explain your answer.

   Solution:

   (a) The following is a regular expression for $L(G)$:

$$\boldsymbol{a}^{+}\boldsymbol{b}^{*} \cup \lambda$$

   (b) Yes the grammar is ambiguous. Here are two different *leftmost* derviations for the string $aabb$:

$$\begin{aligned} S &\Rightarrow aSA \\ &\Rightarrow aaSAA \\ &\Rightarrow aaAA \\ &\Rightarrow aabAA \\ &\Rightarrow aabbAA \\ &\Rightarrow aabbA \\ &\Rightarrow aabb \end{aligned}$$

   and

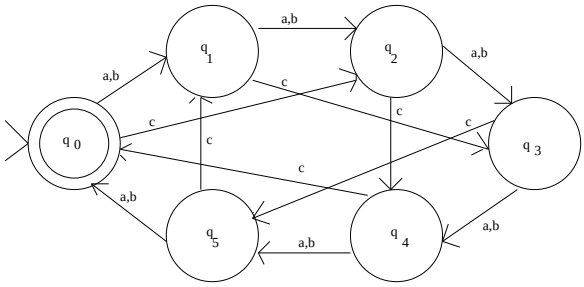
$$\begin{aligned} S &\Rightarrow aSA \\ &\Rightarrow aaSAA \\ &\Rightarrow aaAA \\ &\Rightarrow aaA \\ &\Rightarrow aabA \\ &\Rightarrow aabbA \\ &\Rightarrow aabb \end{aligned}$$

   (20 points)

5. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of a's plus the number of b's plus twice the number of c's is divisible by six.

Solution:

The state diagram of a DFA is



(20 points)