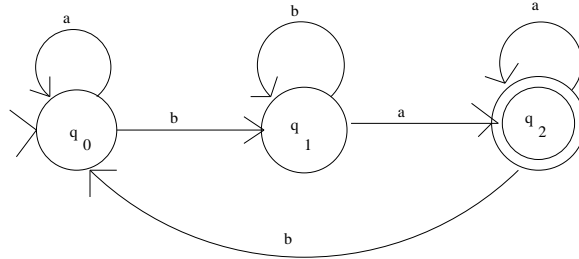**CS 3133 Foundations of Computer Science**
C term 2018

Solutions for Homework 3

1. Exercise 1 on page 184.

   Solution:

   (a) The state diagram of M is

   (b)

	$i)$	$[q_0, abaa]$		$ii)$	$[q_0, bbbabb]$
	$\vdash$	$[q_0, baa]$		$\vdash$	$[q_1, bbabb]$
	$\vdash$	$[q_1, aa]$		$\vdash$	$[q_1, babb]$
	$\vdash$	$[q_2, a]$		$\vdash$	$[q_1, abb]$
	$\vdash$	$[q_2, \lambda]$		$\vdash$	$[q_2, bb]$
				$\vdash$	$[q_0, b]$
				$\vdash$	$[q_1, \lambda]$

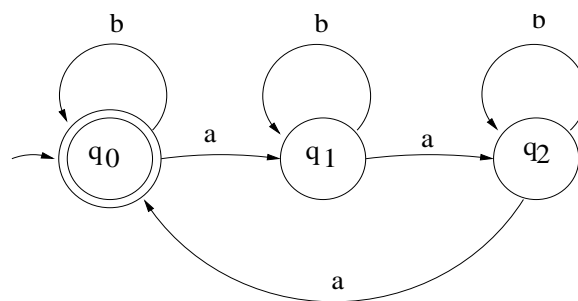 | | $iii)$ | $[q_0, bababa]$ | | $iv)$ | $[q_0, bbbaa]$ |
   |---|---|---|---|---|---|
   | | $\vdash$ | $[q_1, ababa]$ | | $\vdash$ | $[q_1, bbaa]$ |
   | | $\vdash$ | $[q_2, baba]$ | | $\vdash$ | $[q_1, baa]$ |
   | | $\vdash$ | $[q_0, aba]$ | | $\vdash$ | $[q_1, aa]$ |
   | | $\vdash$ | $[q_0, ba]$ | | $\vdash$ | $[q_2, a]$ |
   | | $\vdash$ | $[q_1, a]$ | | $\vdash$ | $[q_2, \lambda]$ |
   | | $\vdash$ | $[q_2, \lambda]$ | | | |

   (c) The computations in i, iii and iv terminate in the accepting state q_2. Therefore the strings $abaa$, $bababa$ and $bbbaa$ are in $L(M)$.

(d) Two regular expressions describing $L(M)$ are $\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+(\boldsymbol{b}\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+)^*$ and $(\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+\boldsymbol{b})^*\boldsymbol{a}^*\boldsymbol{b}^+\boldsymbol{a}^+$. (20 points)

2. Exercise 11 on page 185.

   Solution:

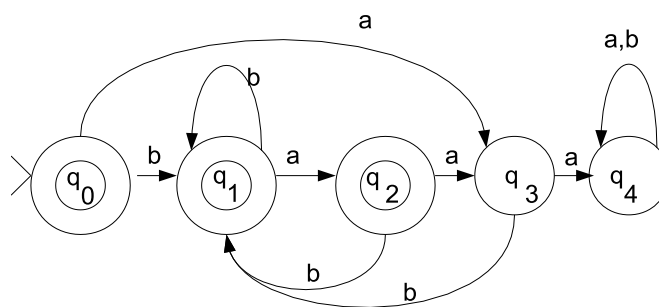   The state diagram of a DFA is

   (20 points)

3. Exercise 12 on page 185.

   Solution:
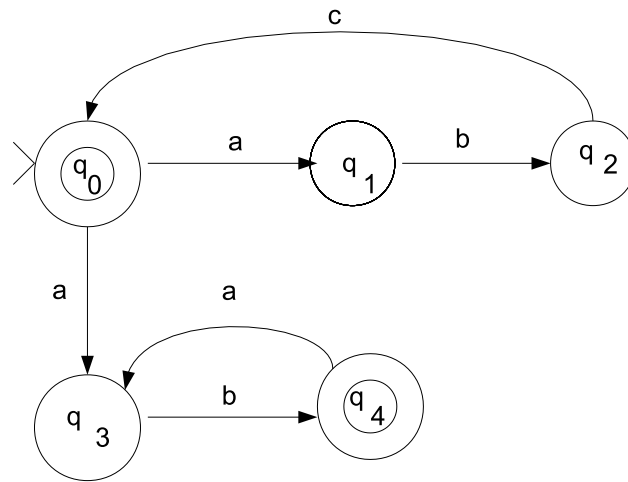
   The state diagram of a DFA is

   (20 points)

4. Design an NFA that accepts the following language over the alphabet $\{a, b\}$:

$$(\boldsymbol{abc})^*(\boldsymbol{ab})^*$$

Solution:

The state diagram of an NFA is

(20 points)

5. Exercise 36 on page 187.

Solution:

(a) $\lambda - closure(q_0) = \{q_0, q_2\}$.

(b) The input transition function t is the following:

t	a	b	c
q_0	$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_1\}$
q_1	$\emptyset$	$\emptyset$	$\{q_1\}$
q_2	$\emptyset$	$\{q_1, q_2\}$	$\emptyset$

(c) The equivalent DFA:

3
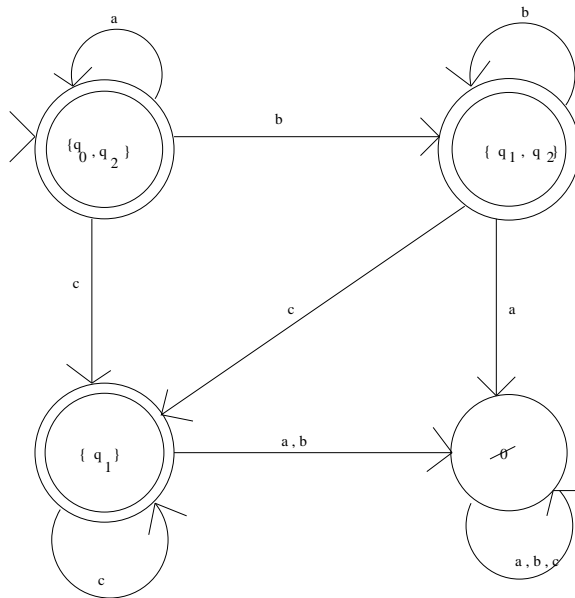
(d) A regular expression is $\boldsymbol{a}^*\boldsymbol{b}^*\boldsymbol{c}^*$. (20 points)