

**CS 3133 Foundations of Computer Science**  
**C term 2019**

**Solutions for Homework 1**

1. Exercise 23 on page 60.

**Solution:**

The set of strings over  $\{a, b, c\}$  that begin with  $a$ , contain exactly two  $b$ 's, and end with  $cc$  is represented by

$$a(a \cup c)^*b(a \cup c)^*b(a \cup c)^*cc.$$

(15 points)

2. Exercise 26 on page 60.

**Solution:**

The set of strings over  $\{a, b\}$  in which the number of  $a$ 's is divisible by three is represented by

$$(b^*ab^*ab^*ab^*)^*b^*.$$

(15 points)

3. Exercise 39.d on page 61.

**Solution:**

$$(a \cup b)^* = (a \cup ba^*)^* = (a^* \cup ba^*)^*$$

Here in the first equality we have used the third regular expression identity in 12. of Table 2.1, and in the second equality we have used the first identity in 12. (15 points)

4. Let  $\Sigma$  be an alphabet, and  $u, v, w$  regular expressions over  $\Sigma$ . Are the following regular expression identities true?

(a)  $u \cup (vw) = (u \cup v)(u \cup w)$ .

$$(b) \quad u^*(v \cup w) = u^*v \cup u^*w.$$

If yes, explain why, if no, give a counterexample.

**Solution:**

(a) False. For example if  $\Sigma = \{a, b, c\}$  and  $u = \mathbf{a}, v = \mathbf{b}, w = \mathbf{c}$ , then the string  $ac$  is contained in  $(u \cup v)(u \cup w)$  but it is not contained in  $u \cup (vw)$ .

(b) True. This is just regular expression identity 9. of Table 2.1 with  $u^*$  playing the role of  $u$ . (20 points)

5. Exercise 44 on page 39.

**Solution:**

The set of ancestors of a node  $x$  in a tree is defined recursively in the following way:

**basis:**  $x$  is an ancestor of itself.

**recursive step:** If  $y$  is an ancestor of  $x$  and  $z$  is the parent of  $y$  (if there is any) then  $z$  is also an ancestor of  $x$ .

**closure:**  $y$  is ancestor of  $x$  only if it can be obtained by finitely many applications of the recursive step. (15 points)

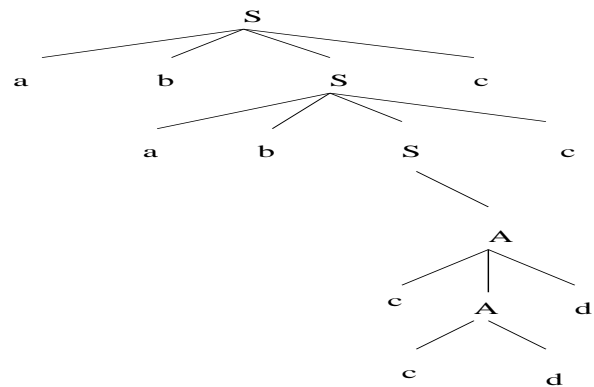
6. Exercise 1 on page 97.

**Solution:**

(a) The derivation of  $ababccddcc$  and the applied rules are the following:

$$\begin{array}{ll} S & \Rightarrow abSc & S & \rightarrow abSc \\ & \Rightarrow ababScc & S & \rightarrow abSc \\ & \Rightarrow ababAcc & S & \rightarrow A \\ & \Rightarrow ababcAdcc & A & \rightarrow cAd \\ & \Rightarrow ababccddcc & A & \rightarrow cd \end{array}$$

(b)



(c)  $L(G) = \{(ab)^n c^m d^m c^n \mid n \geq 0, m > 0\}$ . (20 points)