

Quiz 4 (Solution)

→ Each question is 25 Points ←

Given the following relation $R = (A, B, C, D, E, F)$, and the following dependencies
 $F = \{AB \rightarrow DE, \quad CD \rightarrow E, \quad B \rightarrow EF, \quad DF \rightarrow AC, \quad BD \rightarrow AF\}$

Question 1: Do the following functional dependencies hold or not (Yes or No). Show your work.

Using transitivity, augmentation, union and decomposition rules we get the following dependencies:

$AB \rightarrow DE$

$AB \rightarrow D$

$AB \rightarrow E$

$AB \rightarrow AE$

$AB \rightarrow AD$

$CD \rightarrow E$

$CD \rightarrow CE$

$CD \rightarrow DE$

$B \rightarrow EF$

$B \rightarrow E$

$B \rightarrow F$

$BC \rightarrow CE$

$BC \rightarrow CF$

$BD \rightarrow DF$

$BD \rightarrow AC$

$AB \rightarrow AE$

$AB \rightarrow AF$

$DF \rightarrow AC$

$DF \rightarrow A$

$DF \rightarrow C$

$DF \rightarrow AD$

$DF \rightarrow AF$

$BD \rightarrow A$

$BD \rightarrow C$

$B \rightarrow BE$

$B \rightarrow BF$

$BD \rightarrow AF$

$$BD \rightarrow F$$
$$BD \rightarrow F, BD \rightarrow C \Rightarrow \underline{BD \rightarrow CF}$$

BC \rightarrow CDF **NO**

BD \rightarrow CF **YES**

Start:	{DF}
Adding attributes for D	{DFEAC}
Adding attributes for F	{DFEAC}
Adding attributes for E	{DFEAC}
Adding attributes for A	{DFEAC}
Adding attributes for C	{DFEAC}

$$\{\text{DF}\}^+ = \{\text{ACDEF}\}$$

- Since B is not determined by anything else, then it must be part of the key.
- Also since B determines EF, then no need to check BE and BF.
- What's left is to check BA, BC, and BD. You will find that only BA and BD are keys.

Using union:
 $AB \rightarrow AB EF \rightarrow AB DE EF \rightarrow AB DE FC$ ($AB \rightarrow DE, DF \rightarrow AC$)
 $BC \rightarrow CE F$
 $\textcolor{red}{BD} \rightarrow DB EF \rightarrow DB EF AC$ ($DF \rightarrow AC$)

$\{AB\}$, where $\{AB\}^+ = \{ABCDEF\}$ and none of A^+ or B^+ contains all keys

$\{BD\}$, where $\{BD\}^+ = \{ABCDEF\}$ and none of D^+ or B^+ contains all keys

Question 4: If R is decomposed into two relations $R1 = (A, B, D, F)$ and $R2 = (C, D, E, F)$, is it lossy or lossless decomposition, and Why?

The common columns are $\{DF\}$, Is it candidate key in either of $R1$ or $R2$??

In $R1 \rightarrow \{DF\}^+ = D F A$ (cannot get B, not it is not key)

In $R2 \rightarrow \{DF\}^+ = D F C E$ (so DF is a candidate key in $R2$)

So this decomposition is lossless