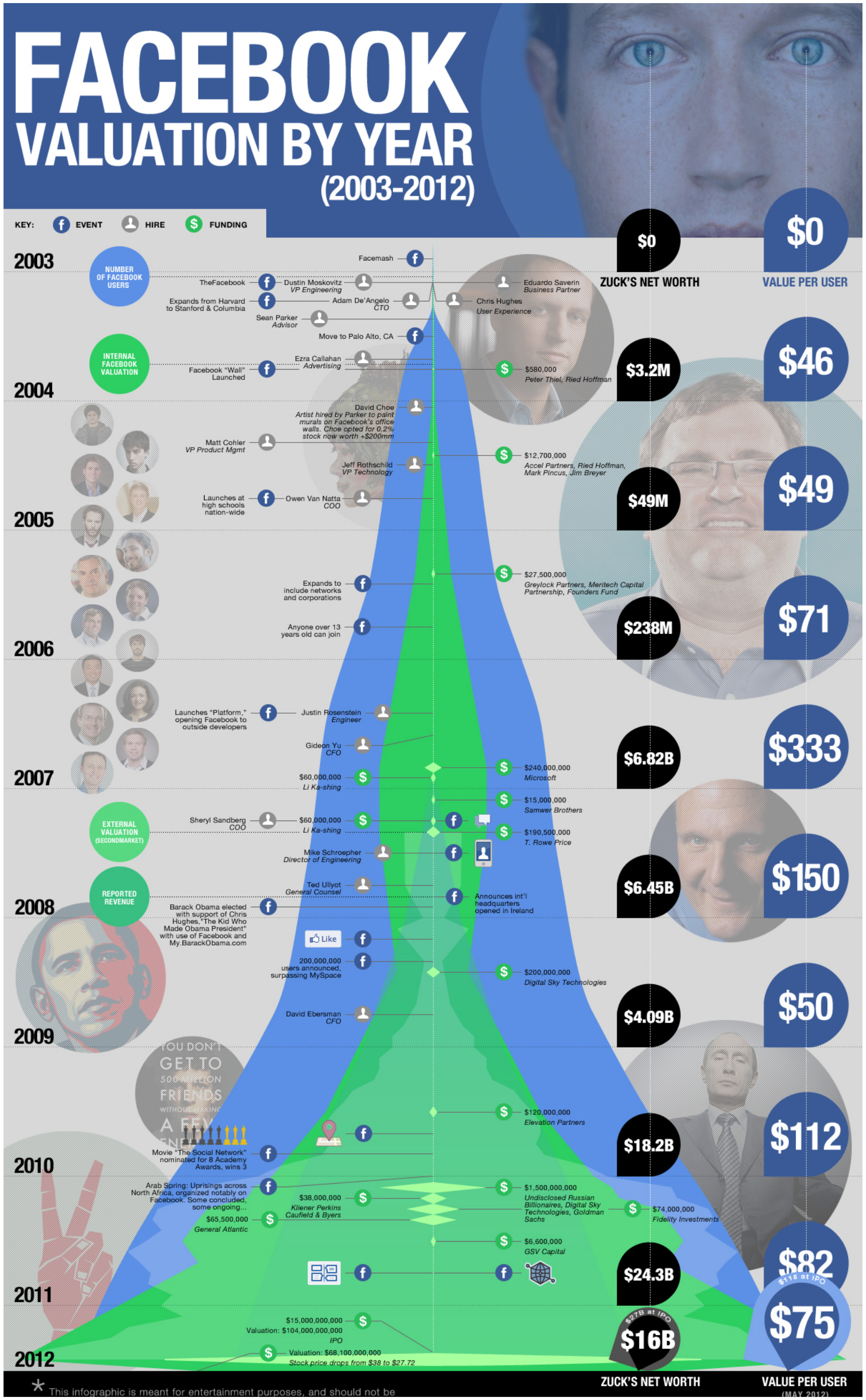
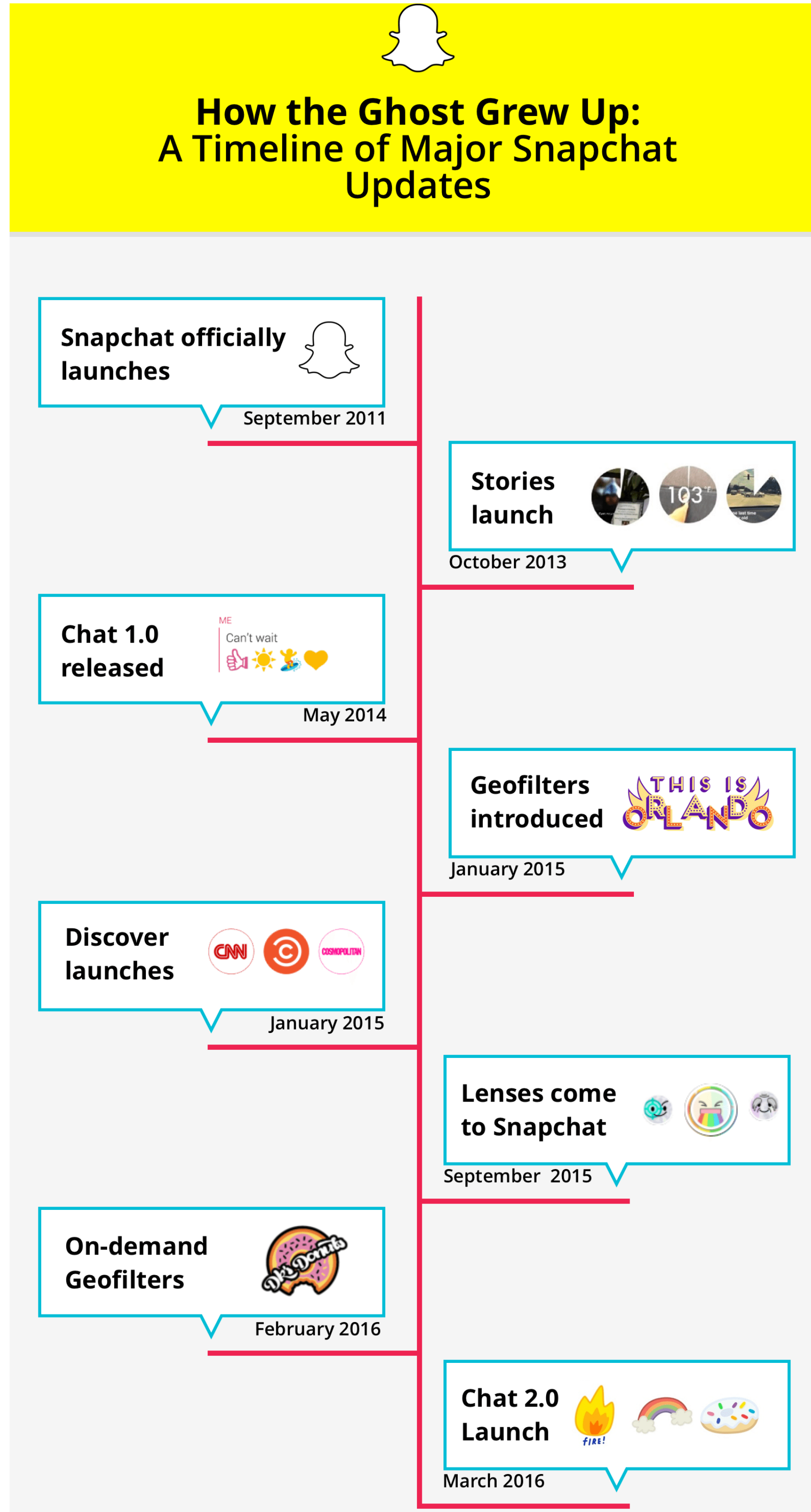
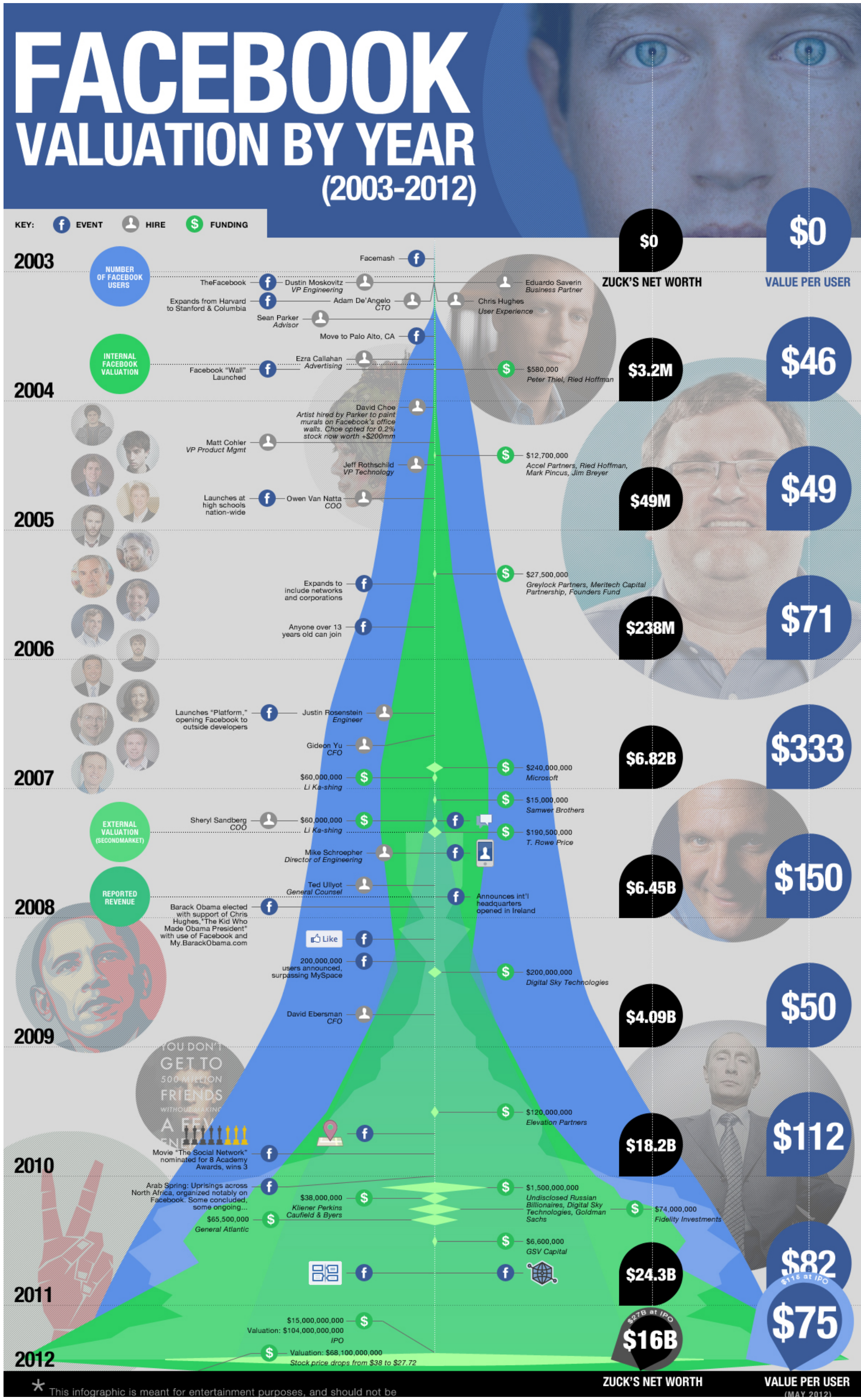
A/B Testing in Social Networks

Spectral Clustering


FACEBOOK
VALUATION BY YEAR
(2003-2012)
KEY: EVENT HIRE FUNDING
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
NUMBER OF FACEBOOK USERS
INTERNAL FACEBOOK VALUATION
EXTERNAL VALUATION (SECONDMARKET)
REPORTED REVENUE
ZUCK'S NET WORTH
VALUE PER USER
$0
$3.2M
$49M
$238M
$6.82B
$6.45B
$4.09B
$18.2B
$24.3B
$16B
$0
$46
$49
$71
$333
$150
$50
$112
$82
$75
ZUCK'S NET WORTH
VALUE PER USER

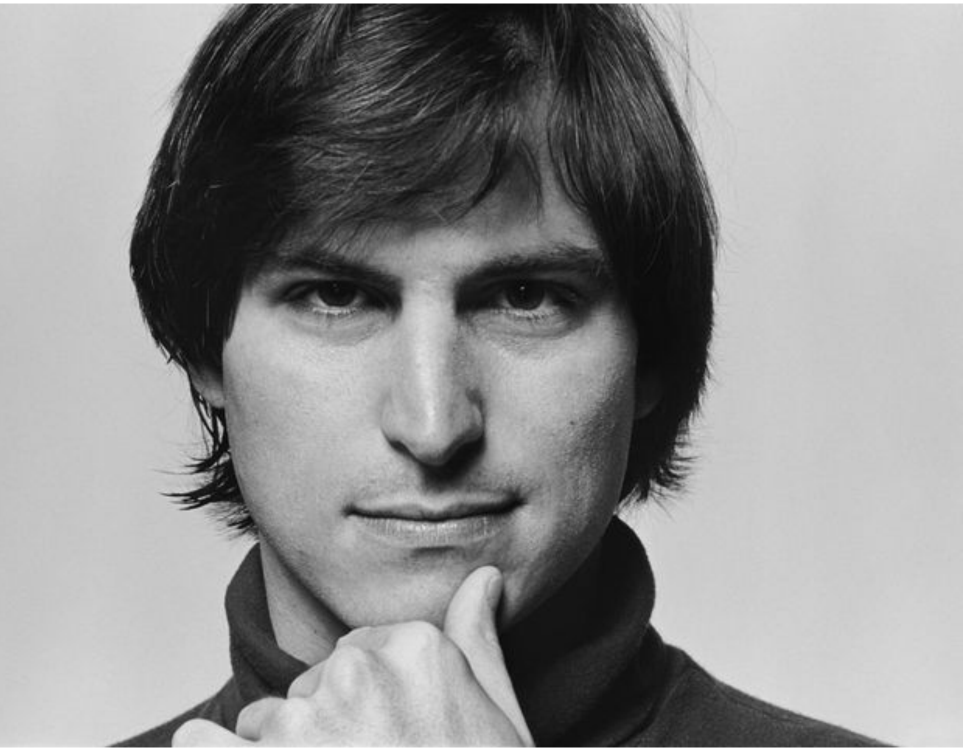

How the Ghost Grew Up:
A Timeline of Major Snapchat Updates
Snapchat officially launches
September 2011
Stories launch
October 2013
Chat 1.0 released
May 2014
Geofilters introduced
January 2015
Discover launches
January 2015
Lenses come to Snapchat
September 2015
On-demand Geofilters
February 2016
Chat 2.0 Launch
March 2016



"Salary" per second?

Assuming 8 hours per day, 5 days per week

$ 23.7



We built [the Mac] for ourselves. We were the group of people who were going to judge whether it was great or not. We weren't going to go out and do market research.

A lot of times, **people don't know what they want** until you show it to them.


NOKIA
N·GAGE
meet
the
N·GAGE


NOKIA
KYIVSTAR
Чт 01.01.2004
N-GAGE QD
Конт-ты
Игра
N-GAGE


NOKIA
N~GægE
Operatio...
MarcelDe...
Ashen
System R...
Tomb Rai...
Pandemo...
VirtuaTe...
Asphalt
Asphalt 2
Options
Back
N·GAGE

Nokia N-Gage


N·GAGE
NOKIA
Menu
Telephone
Contacts
Calendar
Messaging
Media
Extras
Tools
Profiles
THPS
Options
Exit

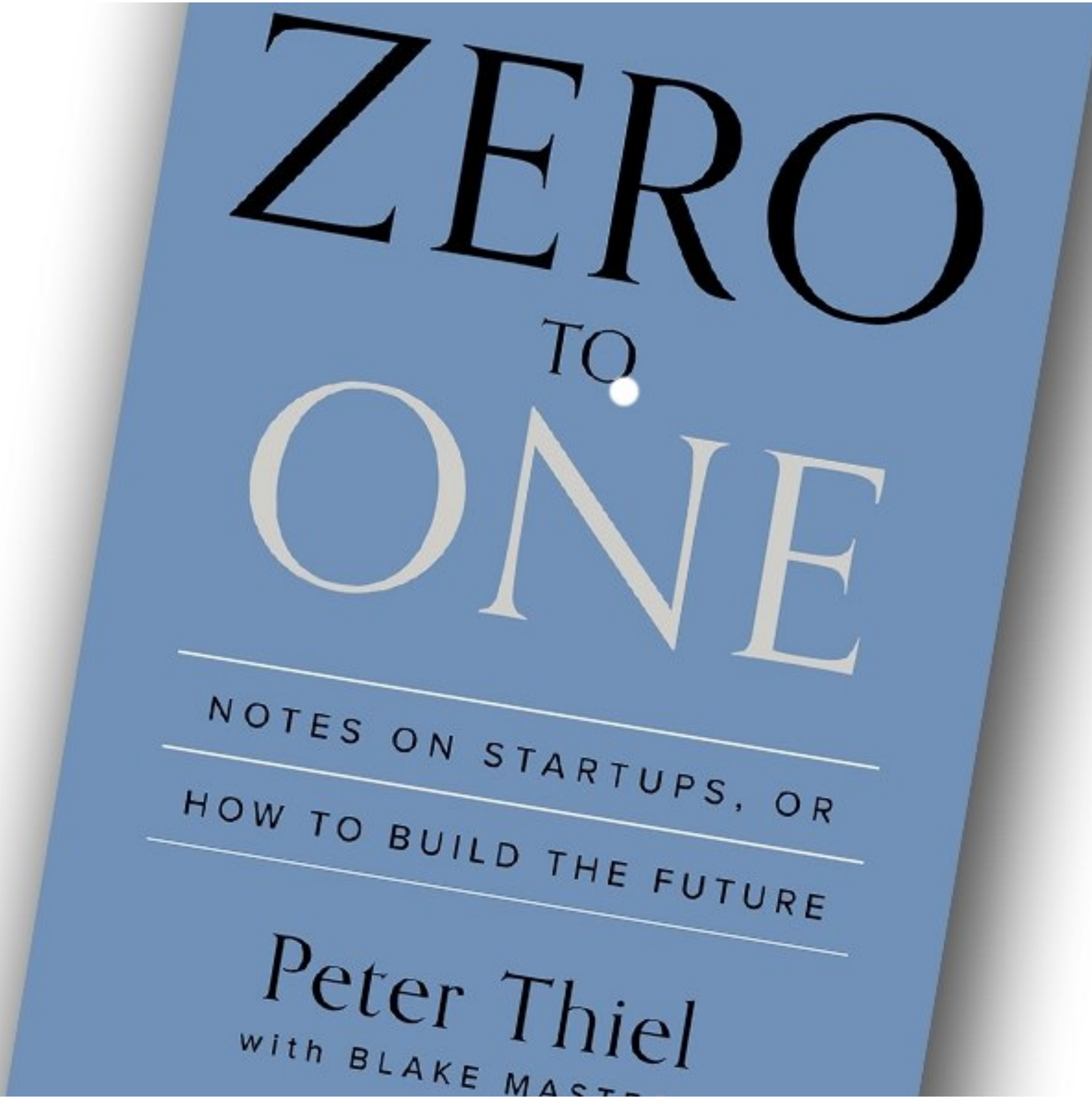

NOKIA
N-GAGE


N·GAGE
NOKIA

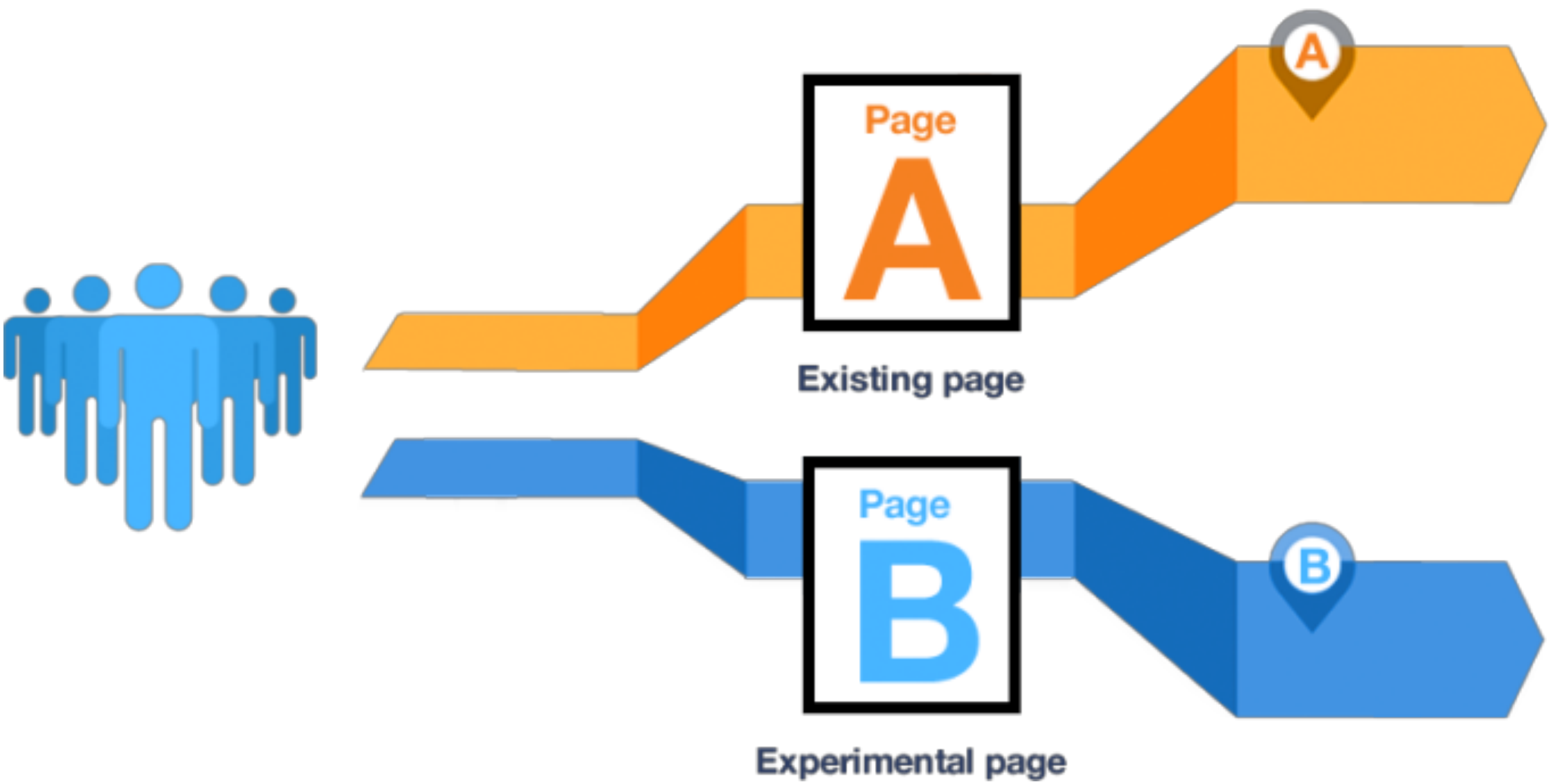

N·GAGE
DECEPÇÕES

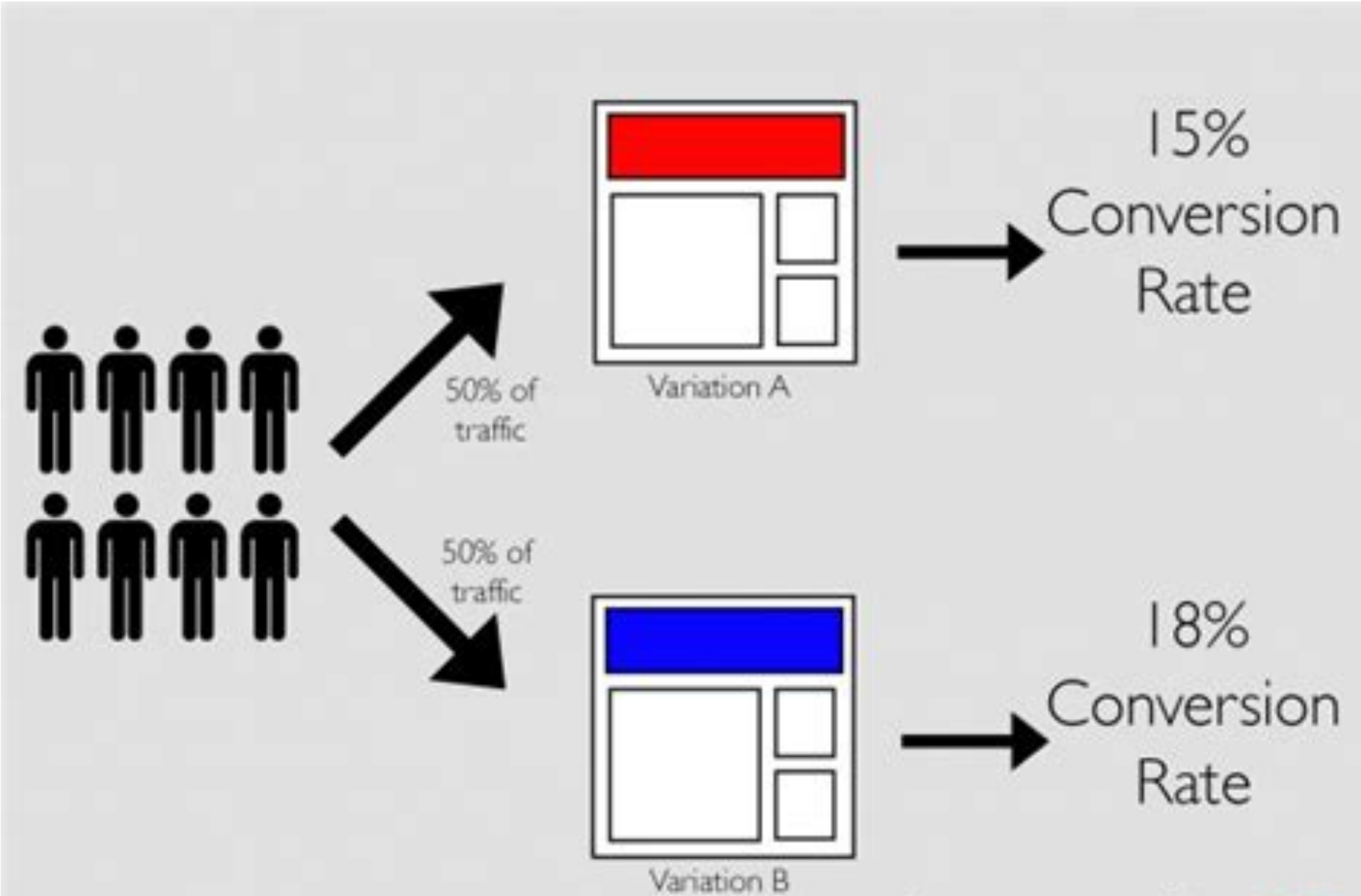
https://www.youtube.com/watch?v=ZdukwH1QAxs


NOKIA
Menu
Telephone
Contacts
Calendar
Messaging
Media
Tools
THPS
Options
Exit

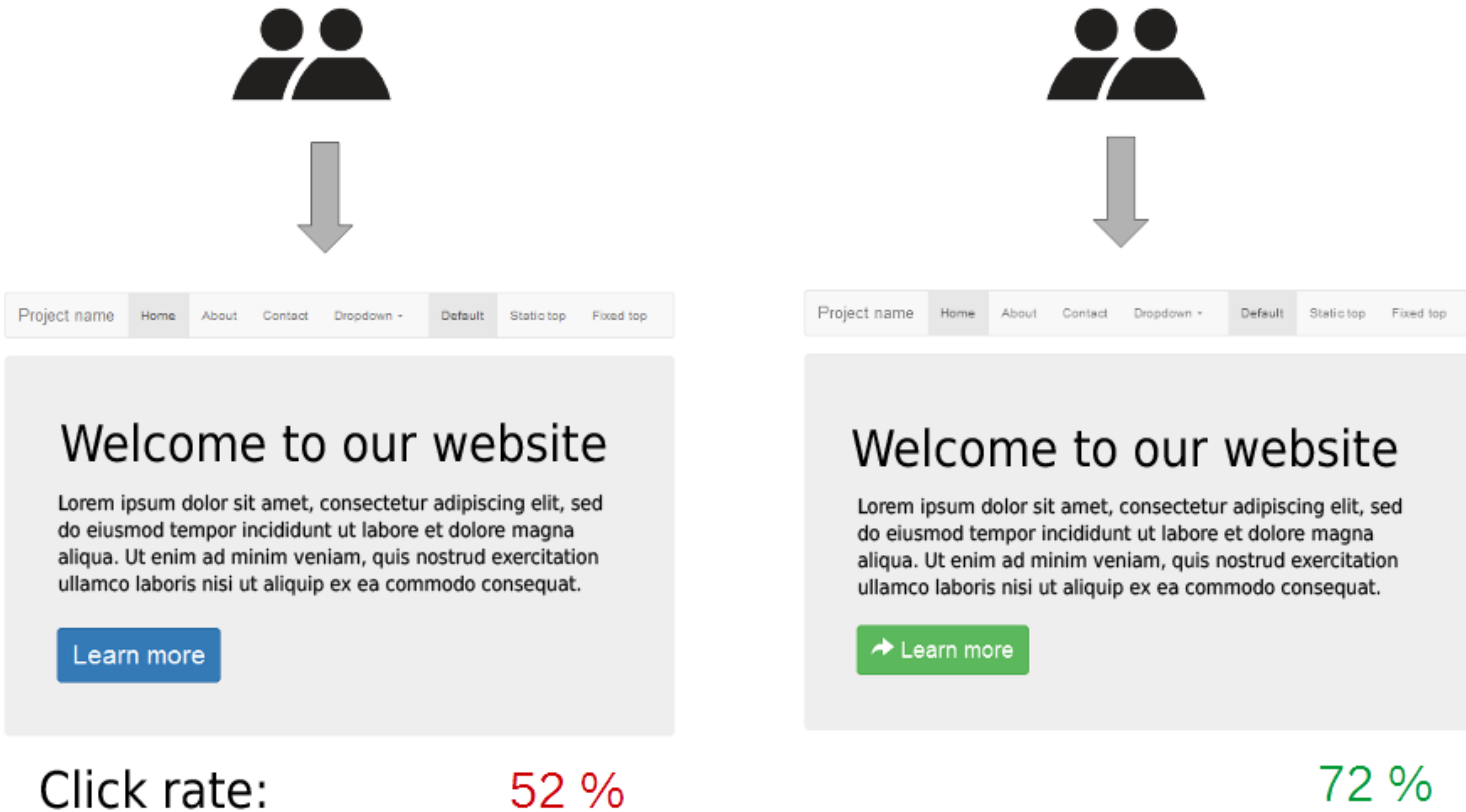

ZERO
TO
ONE
NOTES ON STARTUPS, OR
HOW TO BUILD THE FUTURE
Peter Thiel
with BLAKE

A/B test

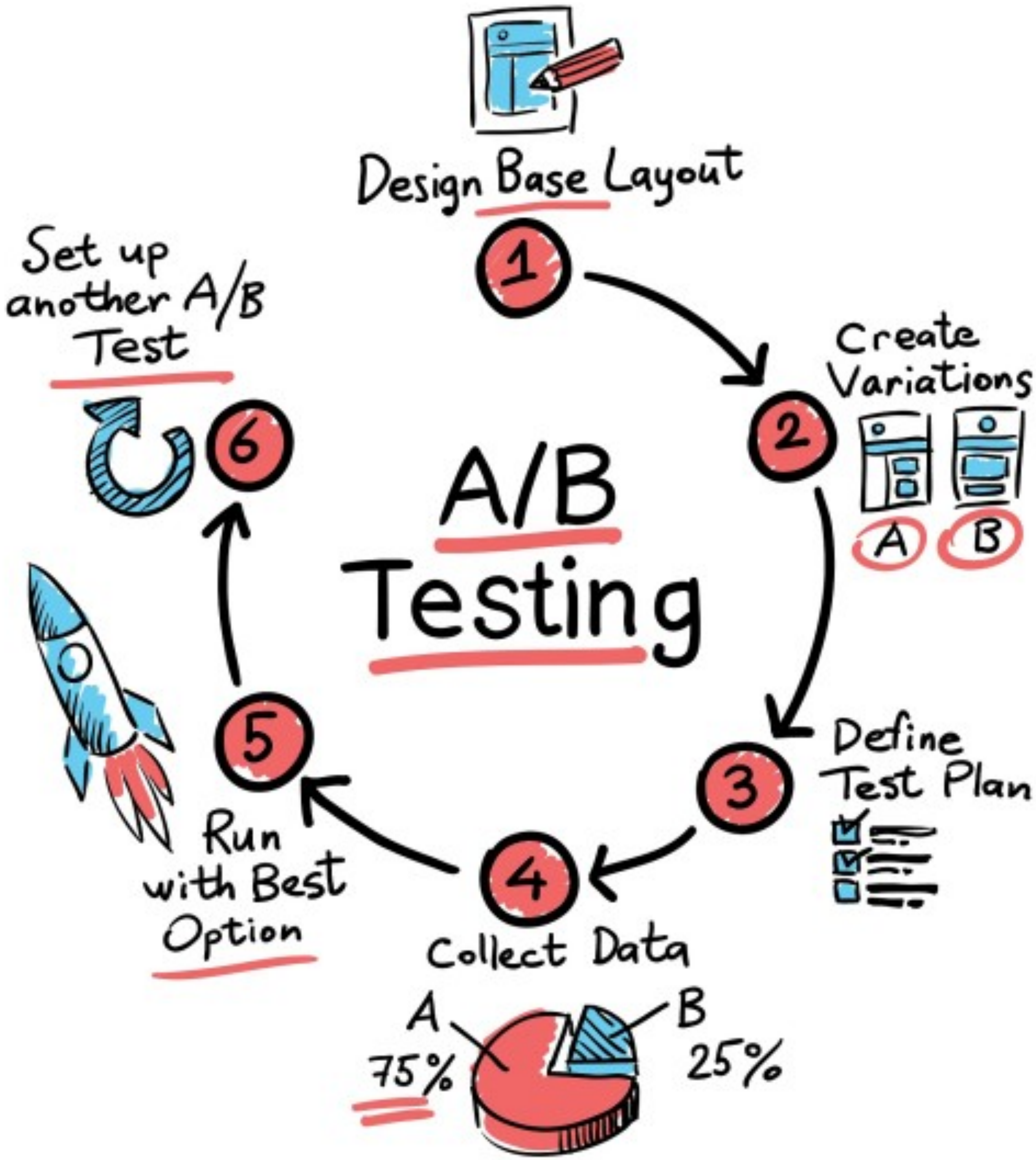



15%
Conversion
Rate
50% of
traffic
Variation A
50% of
traffic
18%
Conversion
Rate
Variation B

A/B test



Project name
Home
About
Contact
Dropdown
Default
Static top
Fixed top
Welcome to our website
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat.
Learn more
Click rate:
52 %
Project name
Home
About
Contact
Dropdown
Default
Static top
Fixed top
Welcome to our website
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat.
Learn more
72 %


A/B Testing
1
Design Base Layout
2
Create Variations
A
B
3
Define Test Plan
4
Collect Data
A 75%
B 25%
5
Run with Best Option
6
Set up another A/B Test

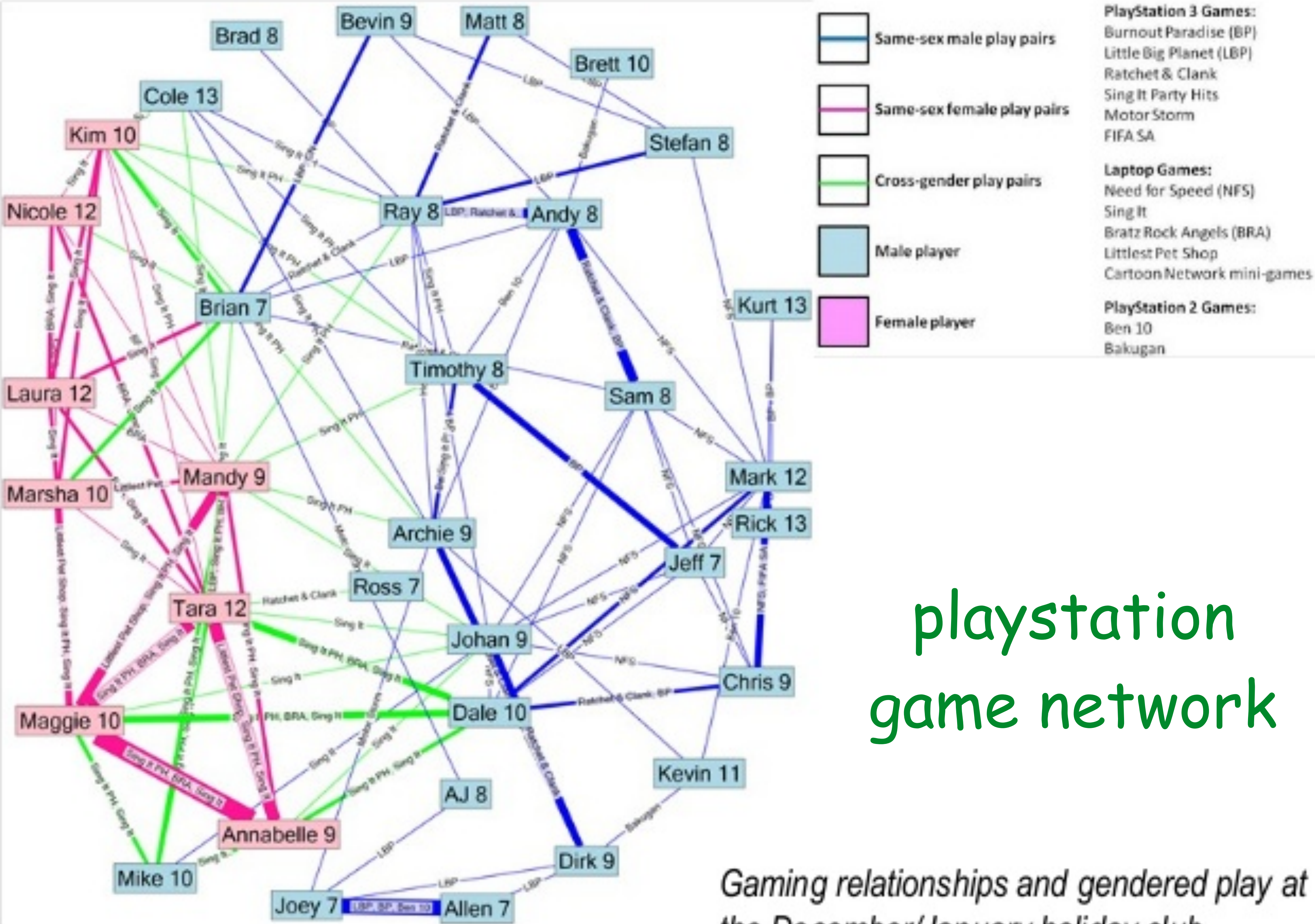
A/B test on Social Network


SMILEYS & PEOPLE
ABC
space


SMILEYS & PEOPLE
ABC
space


End
End

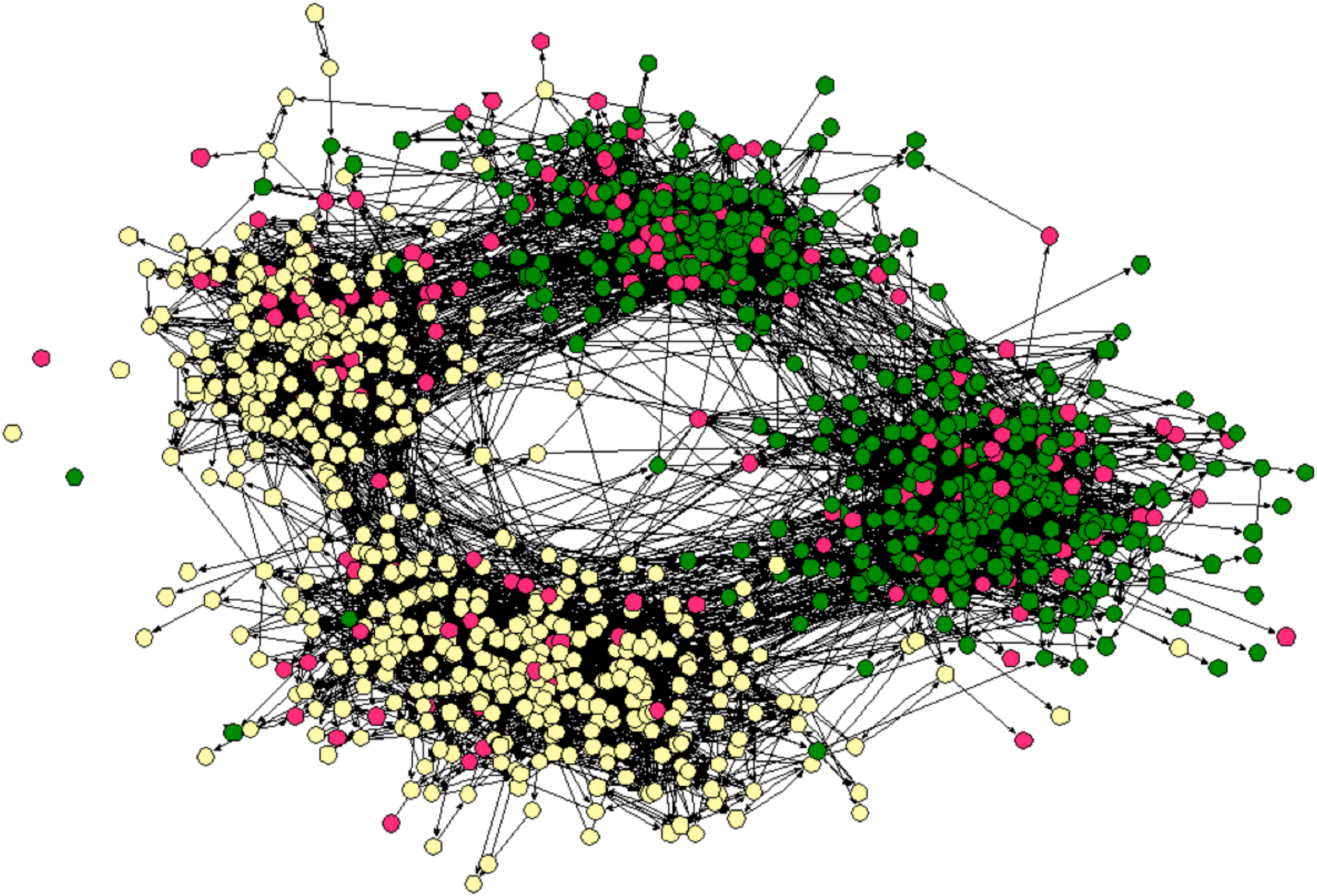

Jamie
Eliza



playstation game network

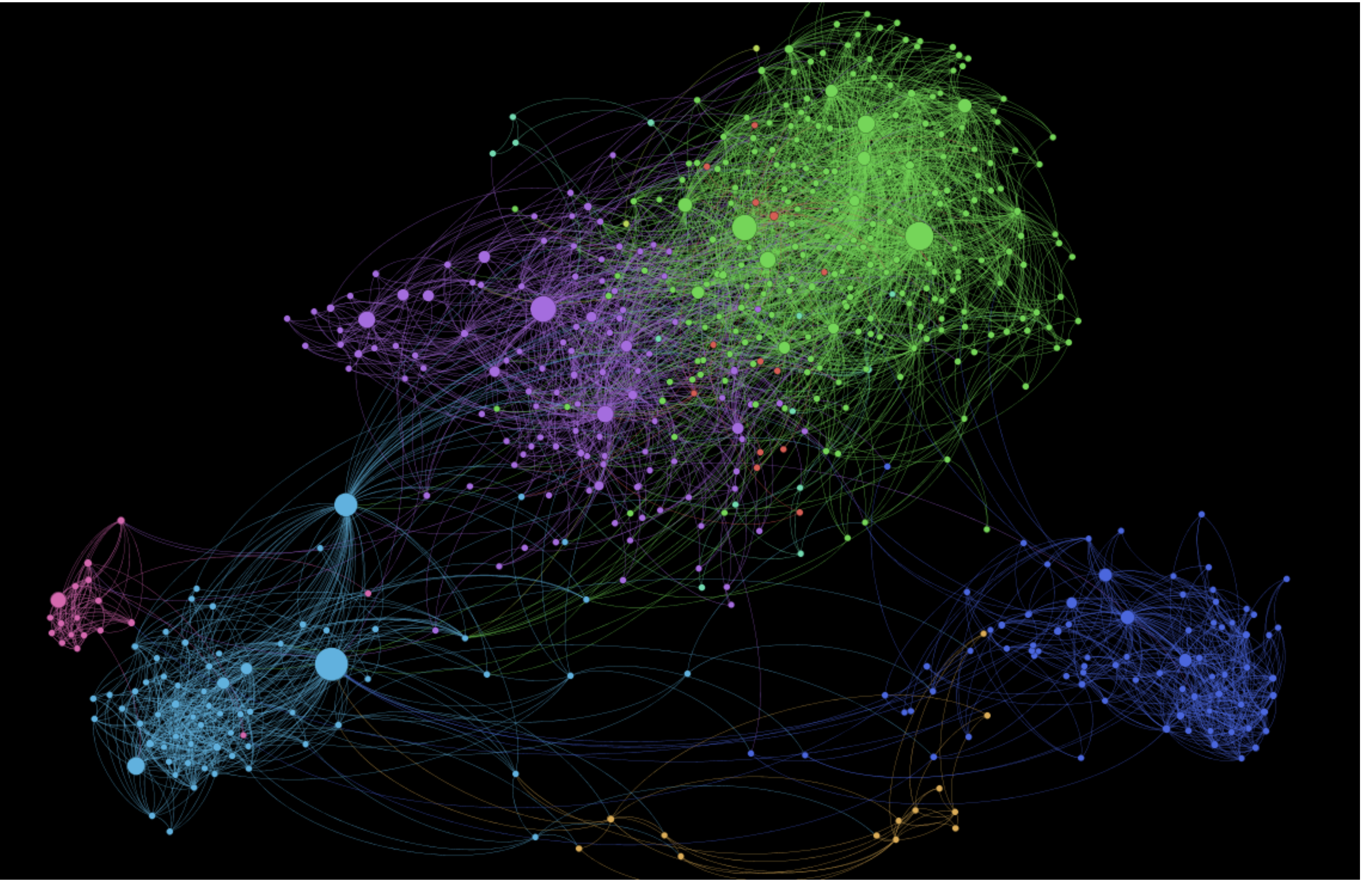
Gaming relationships and gendered play at the December/January holiday club

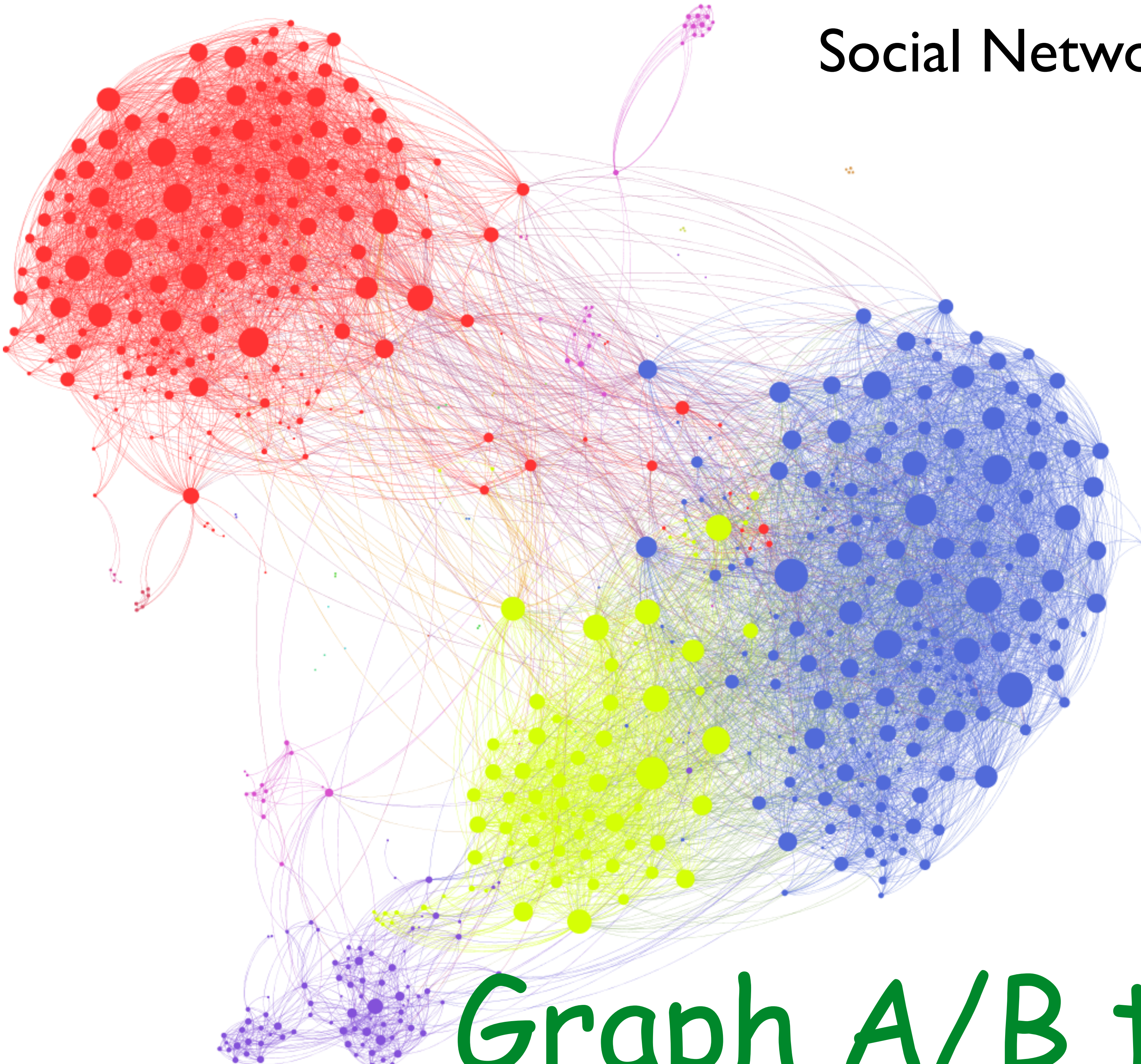
Who-calls-whom network



Email Communication Network

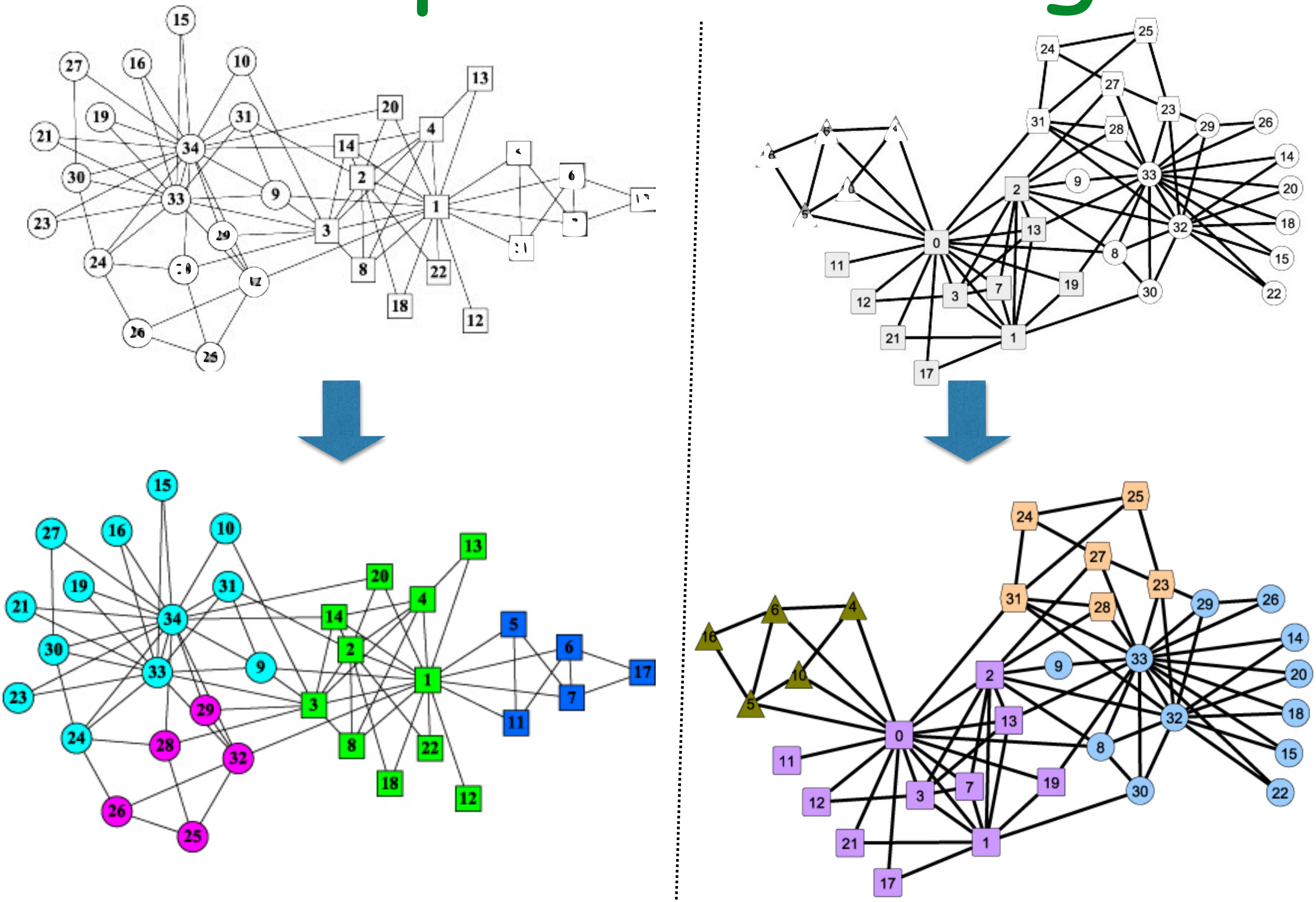
Facebook Network (1000)



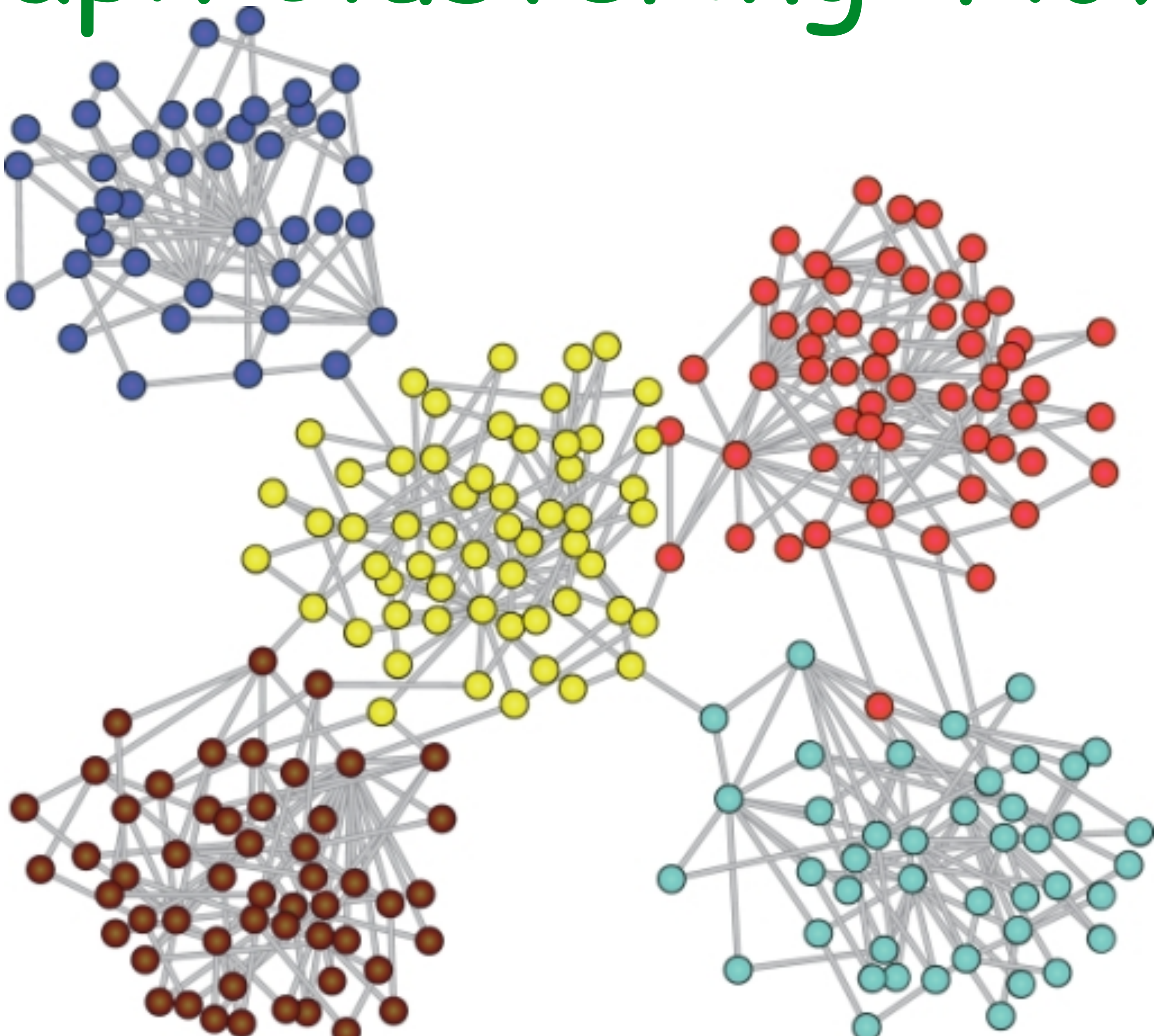


Graph A/B test

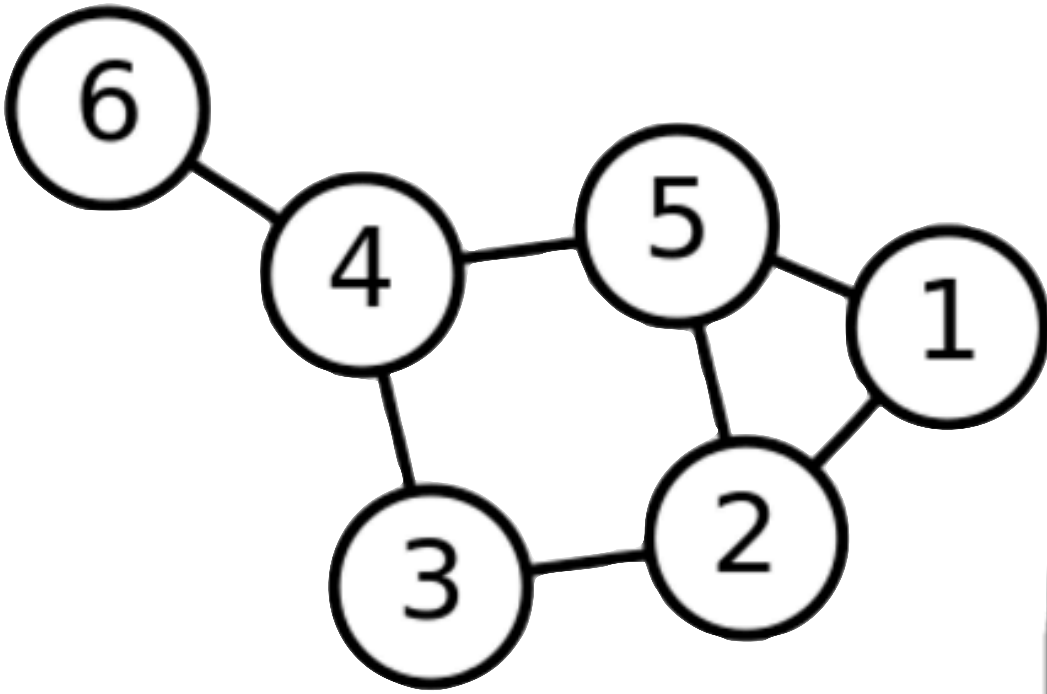
Graph Clustering



Graph Clustering: How?



Adjacency Matrix (A)



Properties:

n X n matrix

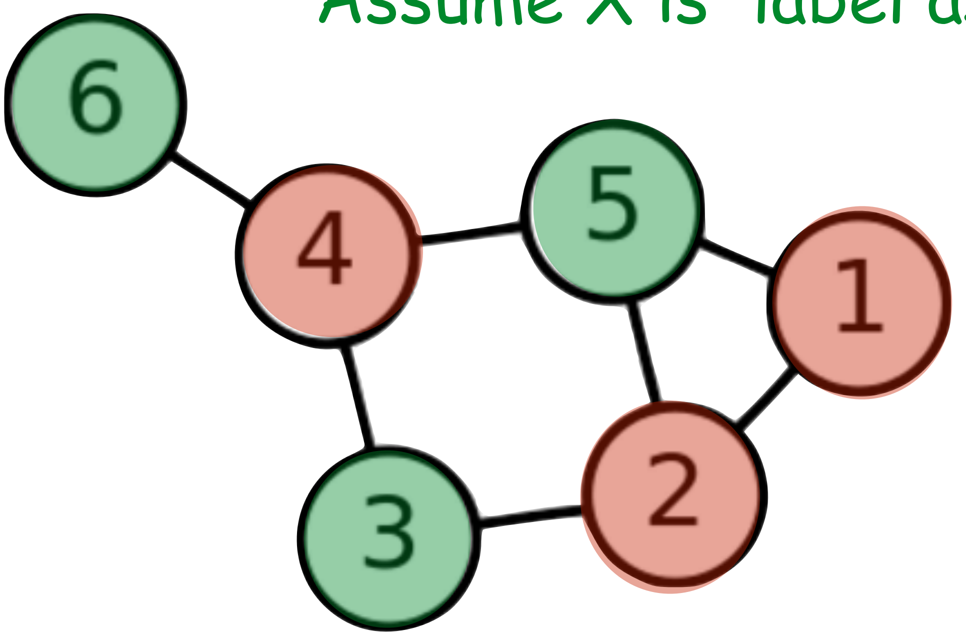
Symmetric matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$Ax = y$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

Assume X is label assignments of the nodes



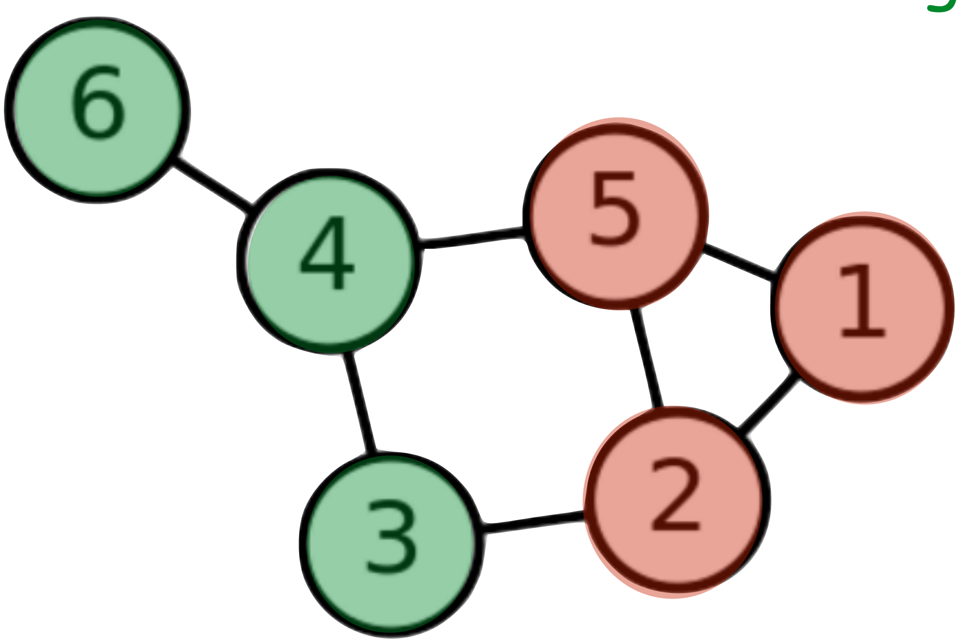
what is y_i ?

sum of labels of Node i's neighbors

$$Ax = \lambda x$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \\ x_n \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ \vdots \\ \\ x_n \end{bmatrix}$$

sum of labels of Node i's neighbors = λ X Node i's label



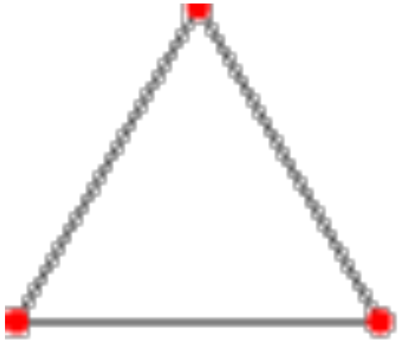
what is λ ?

what is x ?

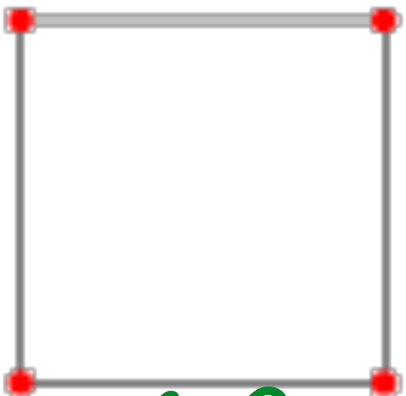
$$Ax = \lambda x$$

if $x = (1,1,\ldots,1)$

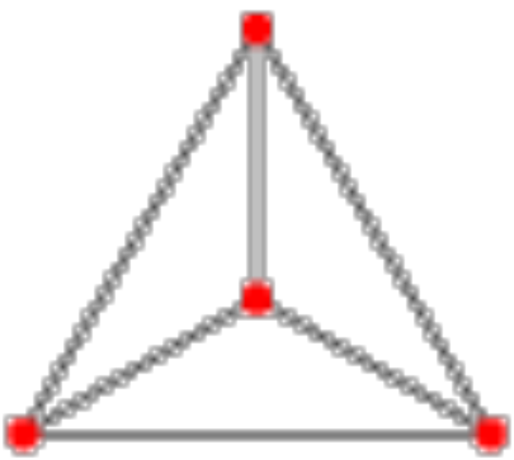
what is λ?



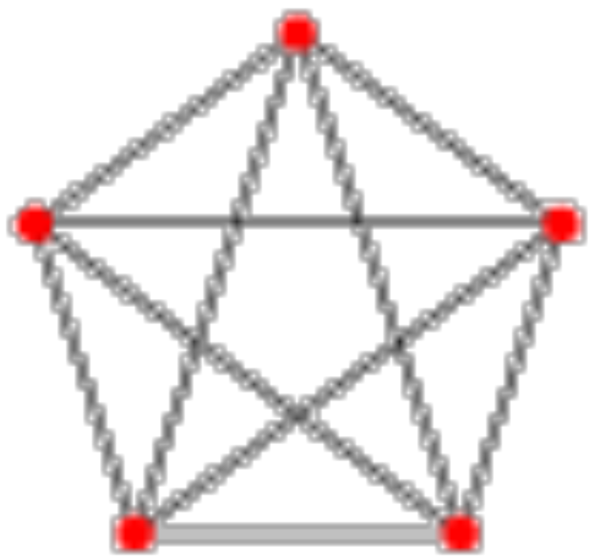
$\lambda=2$



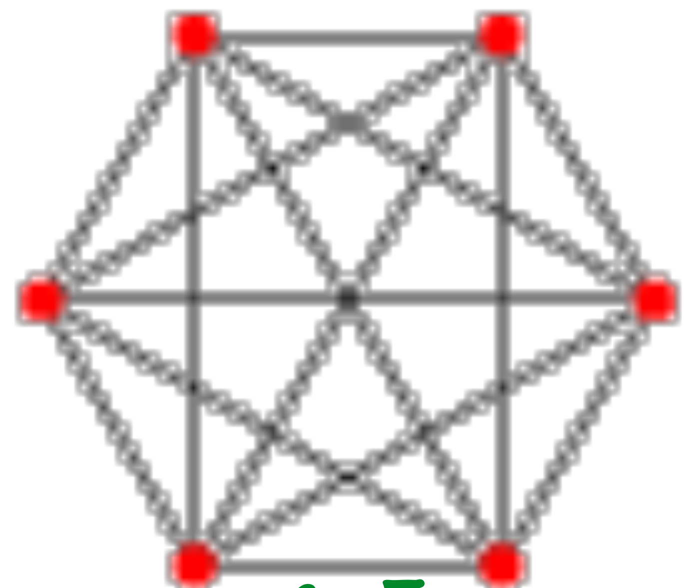
$\lambda=2$



$\lambda=3$

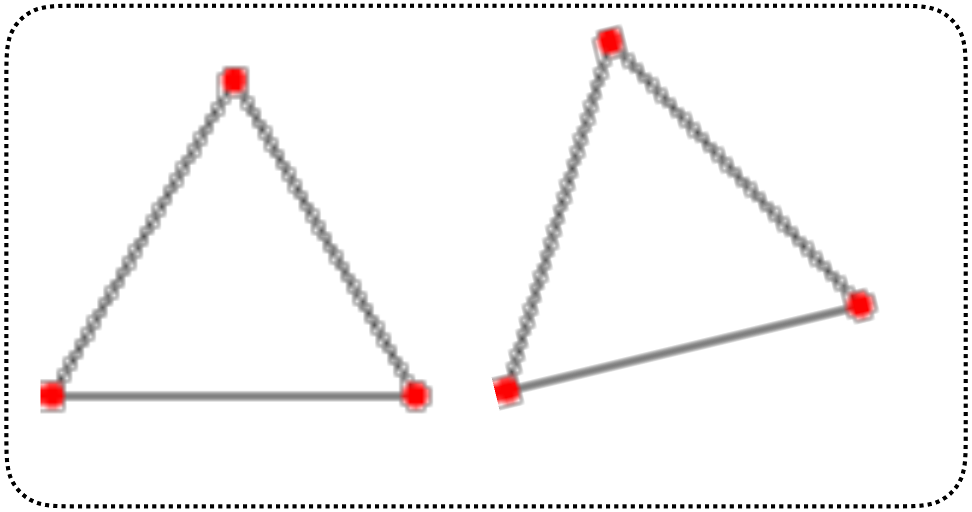


$\lambda=4$



$\lambda=5$

λ - seems to be related to node degrees



A Graph with 2 components

$$Ax = \lambda x$$

What is x?

if $\lambda=0$ ➡ $x = (0,0,0,0,0,0)$

if $\lambda=2$ ➡

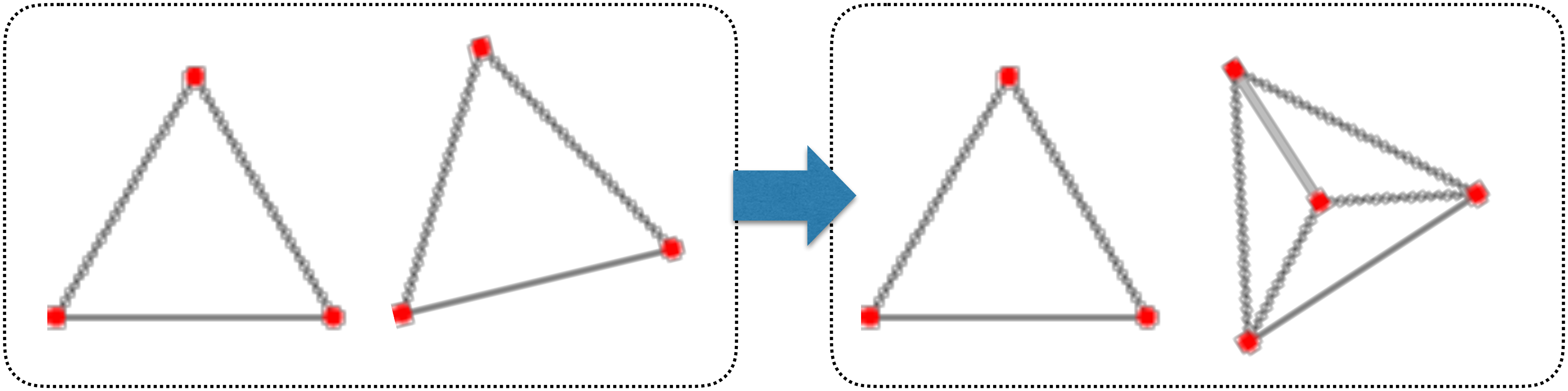
$x = (0,0,0,0,0,0)$

$x = (1,1,1,0,0,0)$

$x = (0,0,0,1,1,1)$ …

$x = (1,1,1,1,1,1)$

What is x?



$\lambda=0$ $x = (0,0,0,0,0,0)$

$\lambda=2$

~~$x = (0,0,0,0,0,0)$~~

$x = (1,1,1,0,0,0)$

$x = (0,0,0,1,1,1)$

~~$x = (1,1,1,1,1,1)$~~

2 binary partitions

$\lambda=0$ $x = (0,0,0,0,0,0,0)$

$\lambda=2$ $x = (1,1,1,\ 0,\ 0,0,0)$

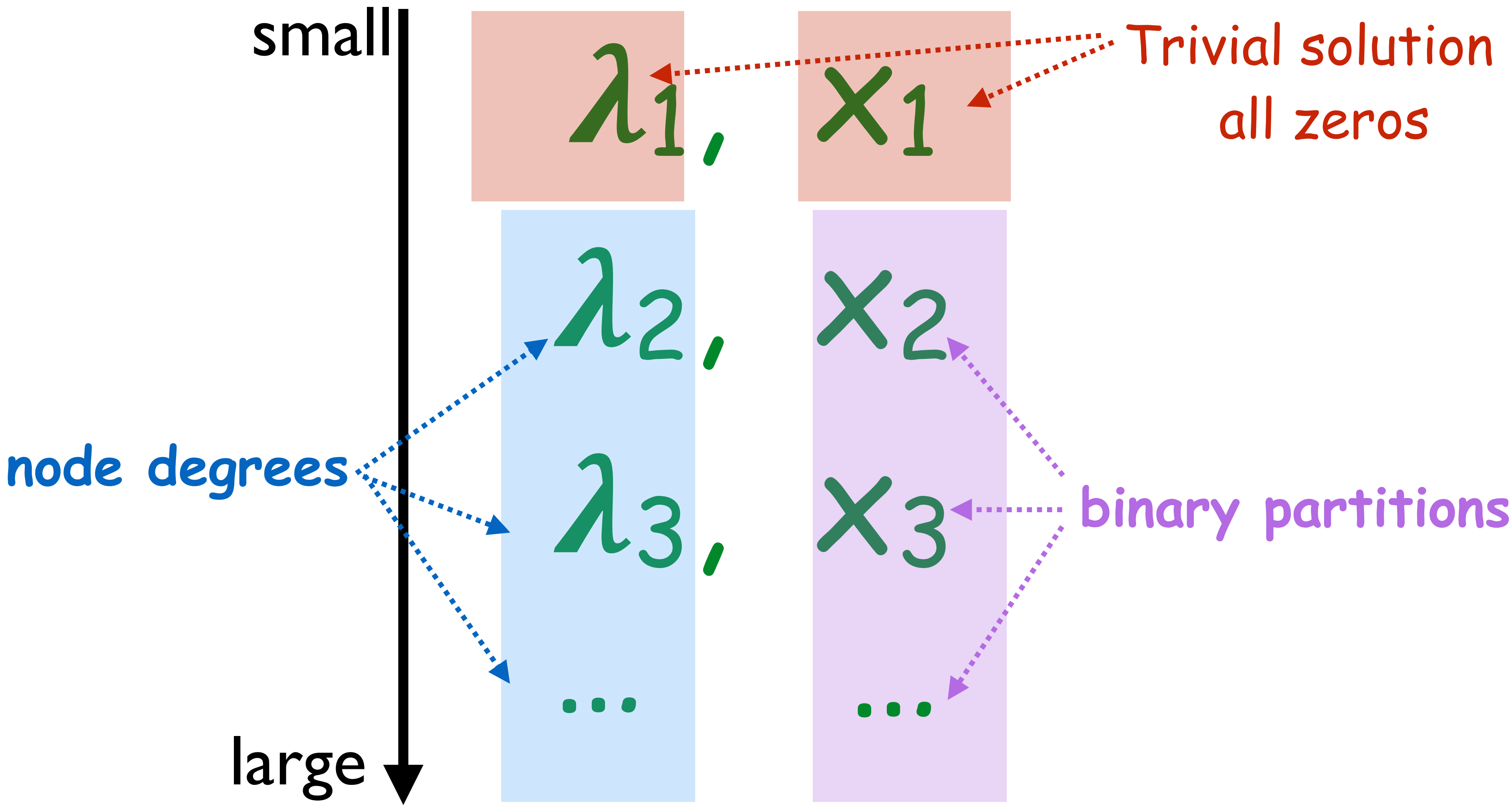
$\lambda=3$ $x = (0,0,0,\ 1,1,1,1)$

2 binary partitions

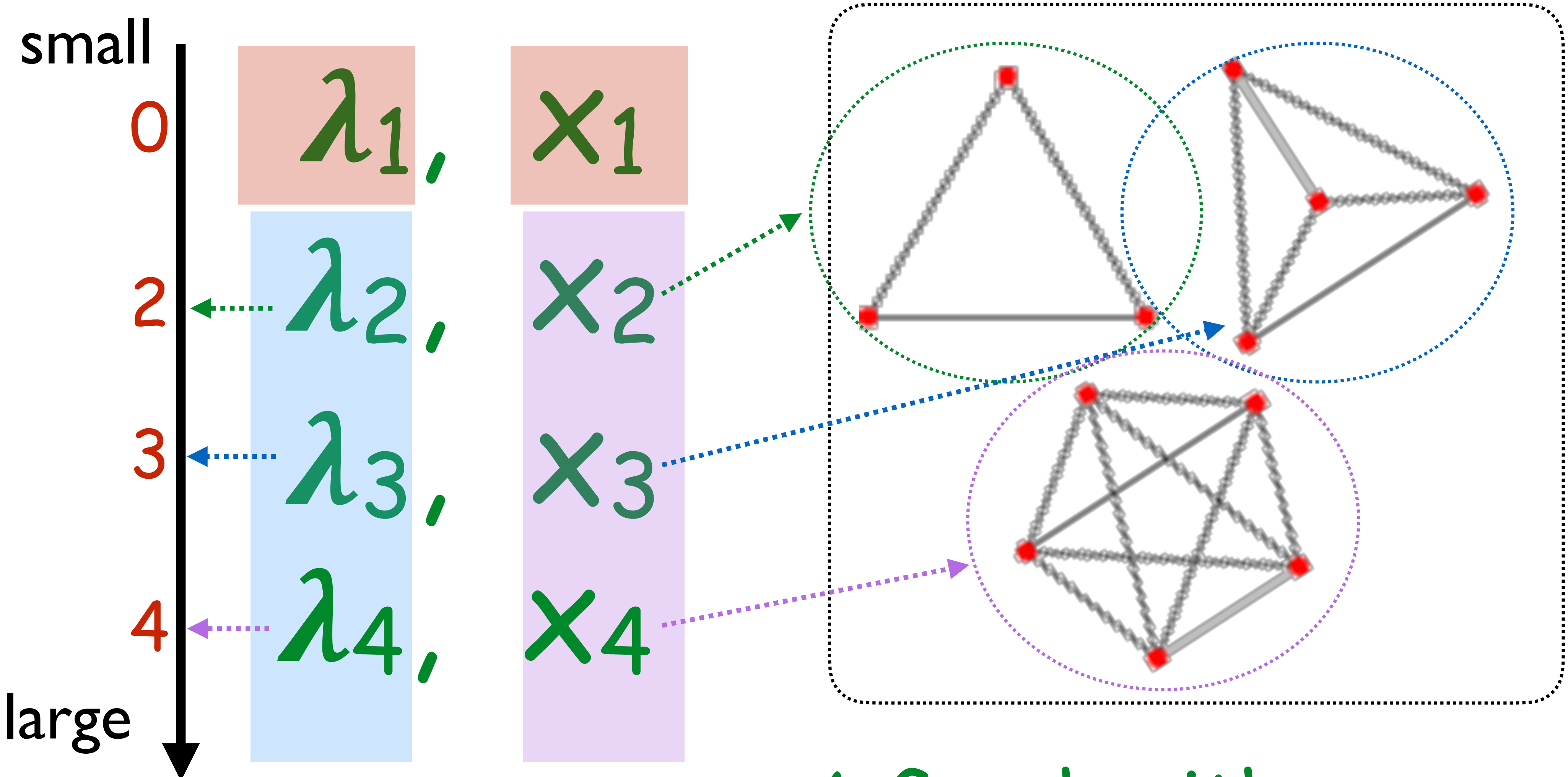
x - seems to be related to graph partition

$$Ax = \lambda x$$

Eigen Pairs of Matrix A



Graph Clustering



$$Ax = \lambda x$$

A Graph with
3 components

Graph Clustering

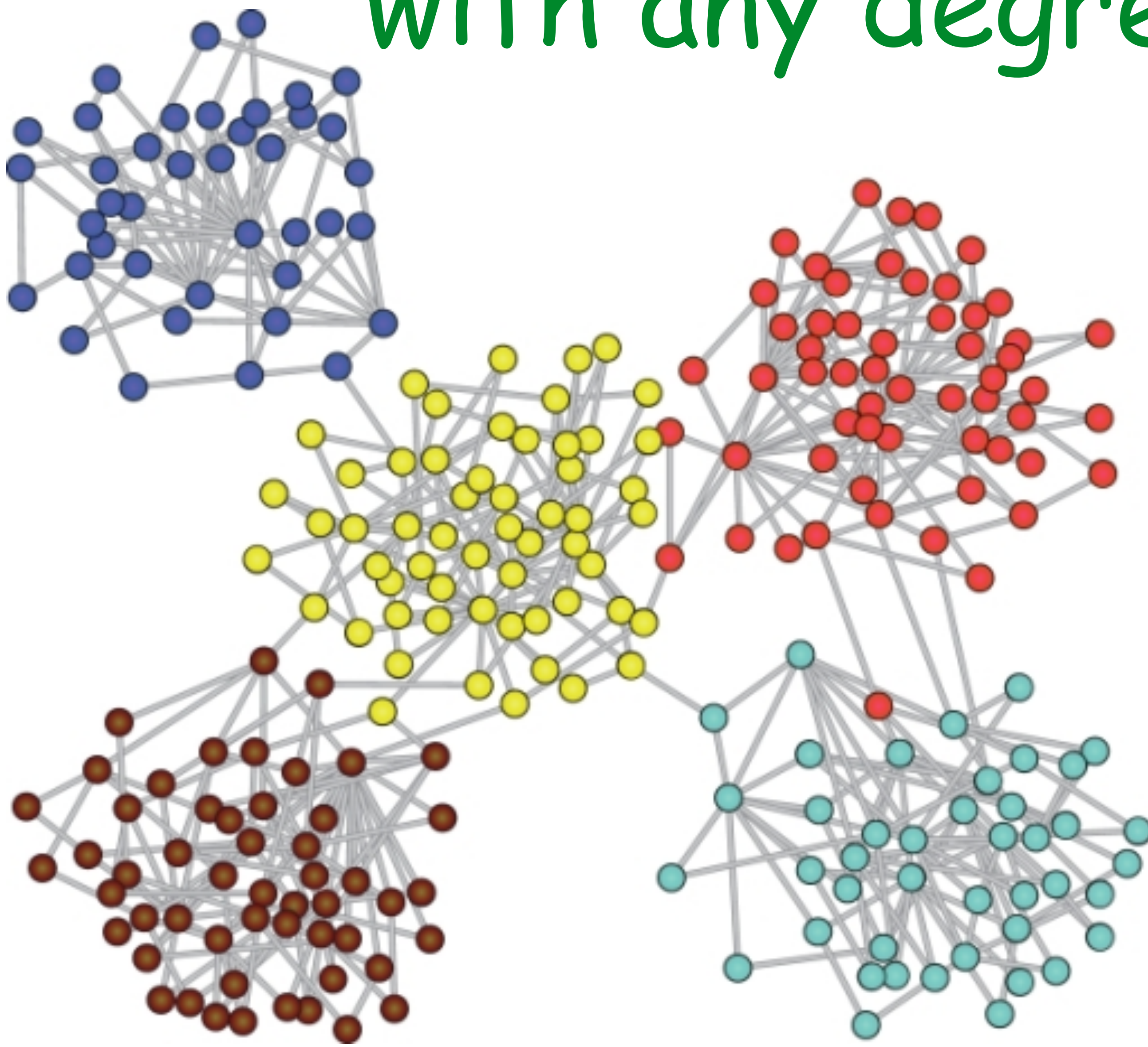
What A Magic Equation!

$$Ax = \lambda x$$

Wait, it only works for graphs:

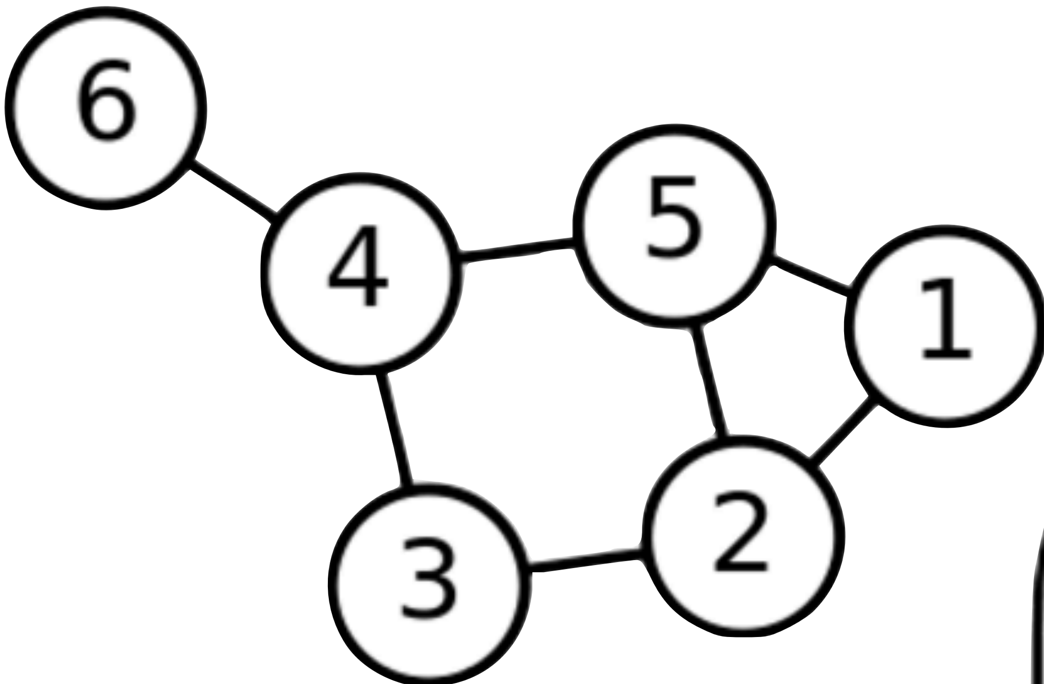
1. with multiple components
2. have the same node degree within each components

How about connected graphs with any degree?



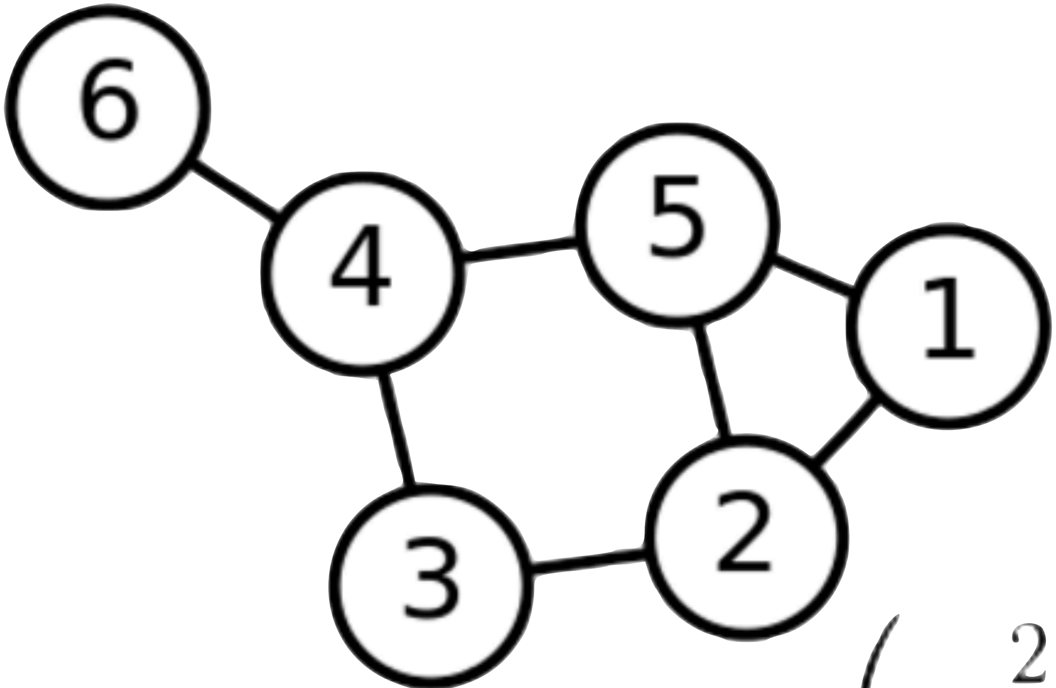
We need another Matrix other than A

Degree Matrix (D)



$$D = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Laplacian Matrix



$$L = D - A$$

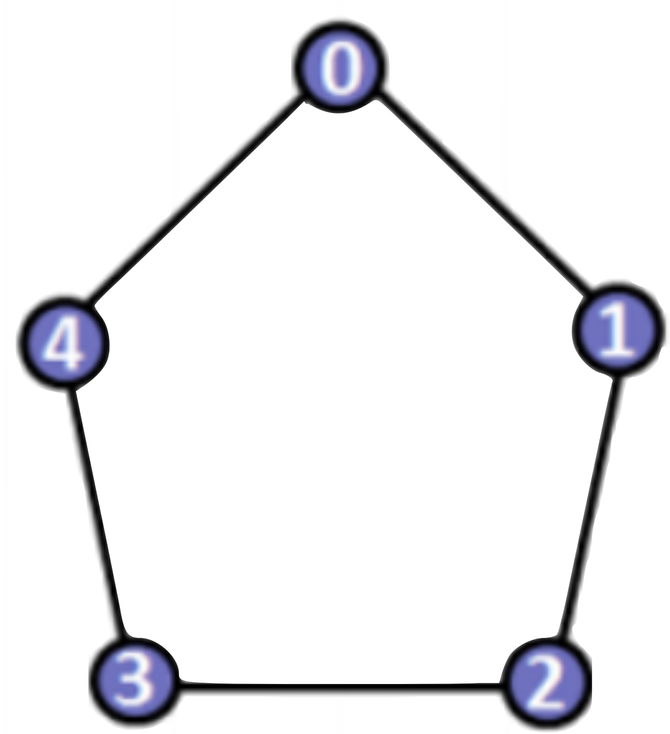
$$L = \begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\boxed{L = D - A}$$

$$L\begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}}_{D} - \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}}_{A}$$

Laplacian Matrix



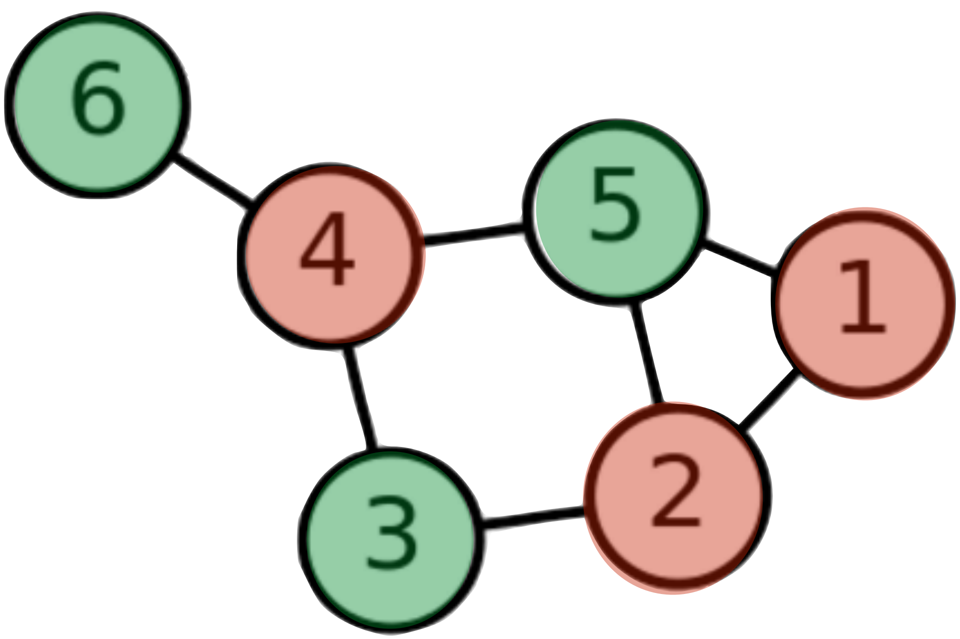
$$\begin{bmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ -1 & & & -1 & 2 \end{bmatrix}$$

Eigenvalues are non-negative real numbers
Eigenvectors are real and orthogonal

$$Lx = y$$

$$\begin{bmatrix} \ell_{11} & \cdots & \ell_{1n} \\ \vdots & & \vdots \\ \ell_{n1} & \cdots & \ell_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

what is y_i ?



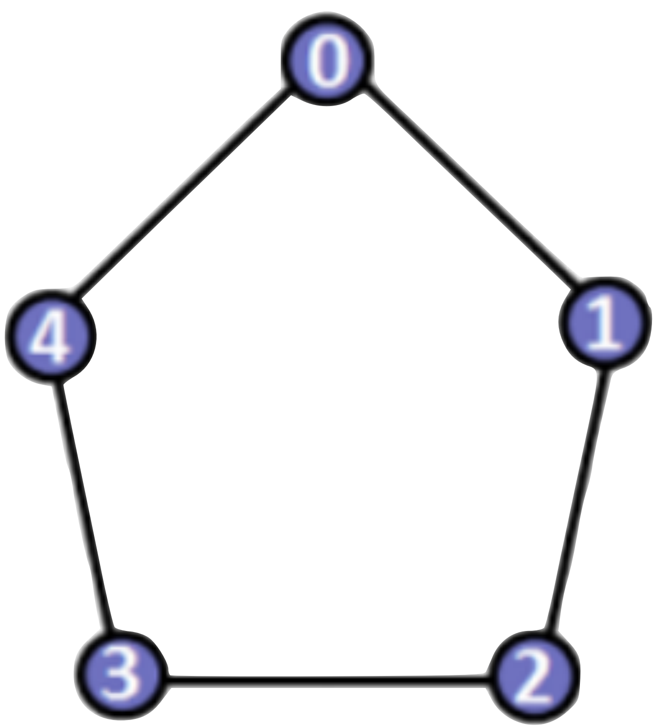
$$Lx = Dx - Ax$$

y_i = Degree X Node i's label - sum of labels of Node i's neighbors

$$Lx = \lambda x$$

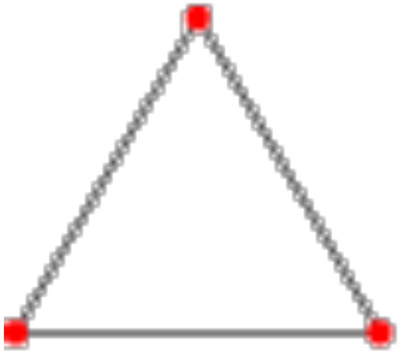
if $x = (1,1,1,1,1)$

what is λ ?



$$\begin{bmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ -1 & & & -1 & 2 \end{bmatrix}$$

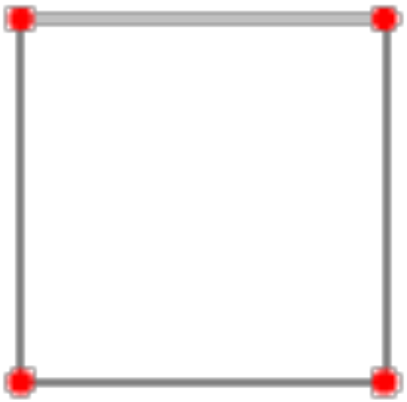
$\lambda = 0$



$\lambda=0$

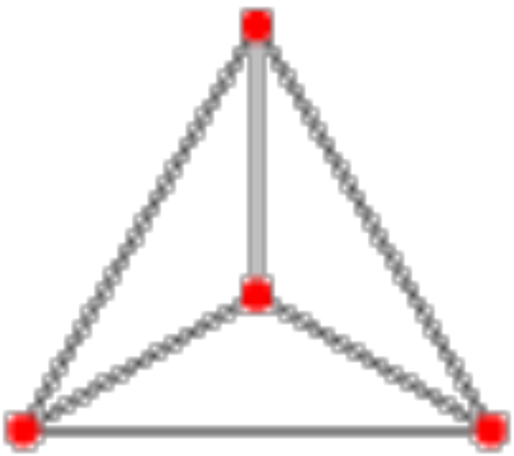
$$L\,x = \lambda\,x$$

if $x = (1,1,\ldots,1)$

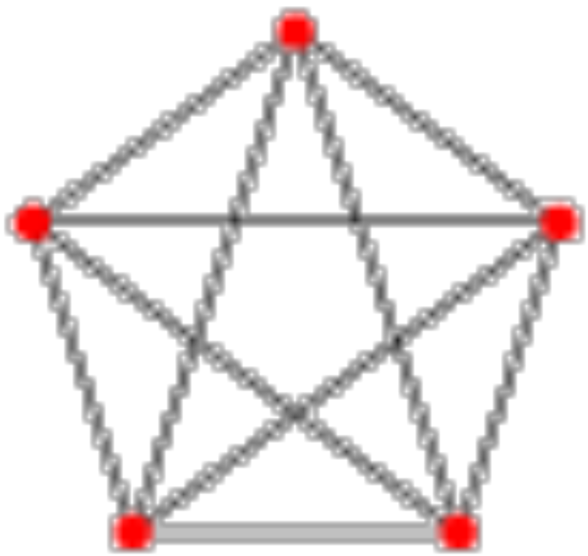


$\lambda=0$

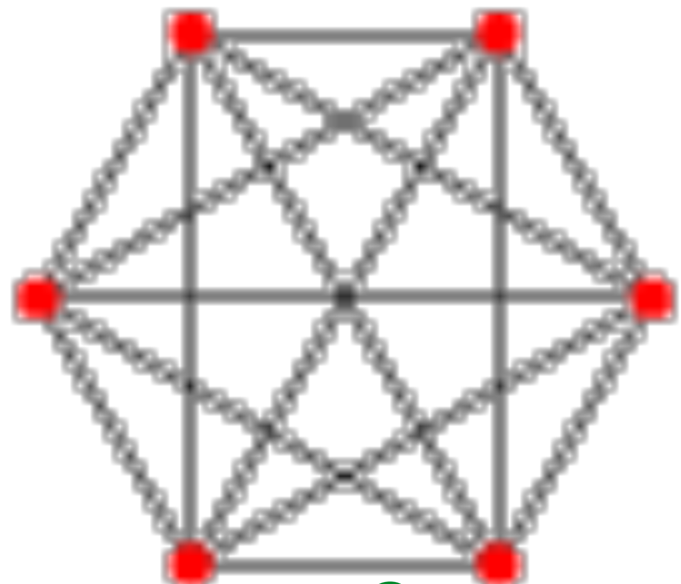
what is λ?



$\lambda=0$



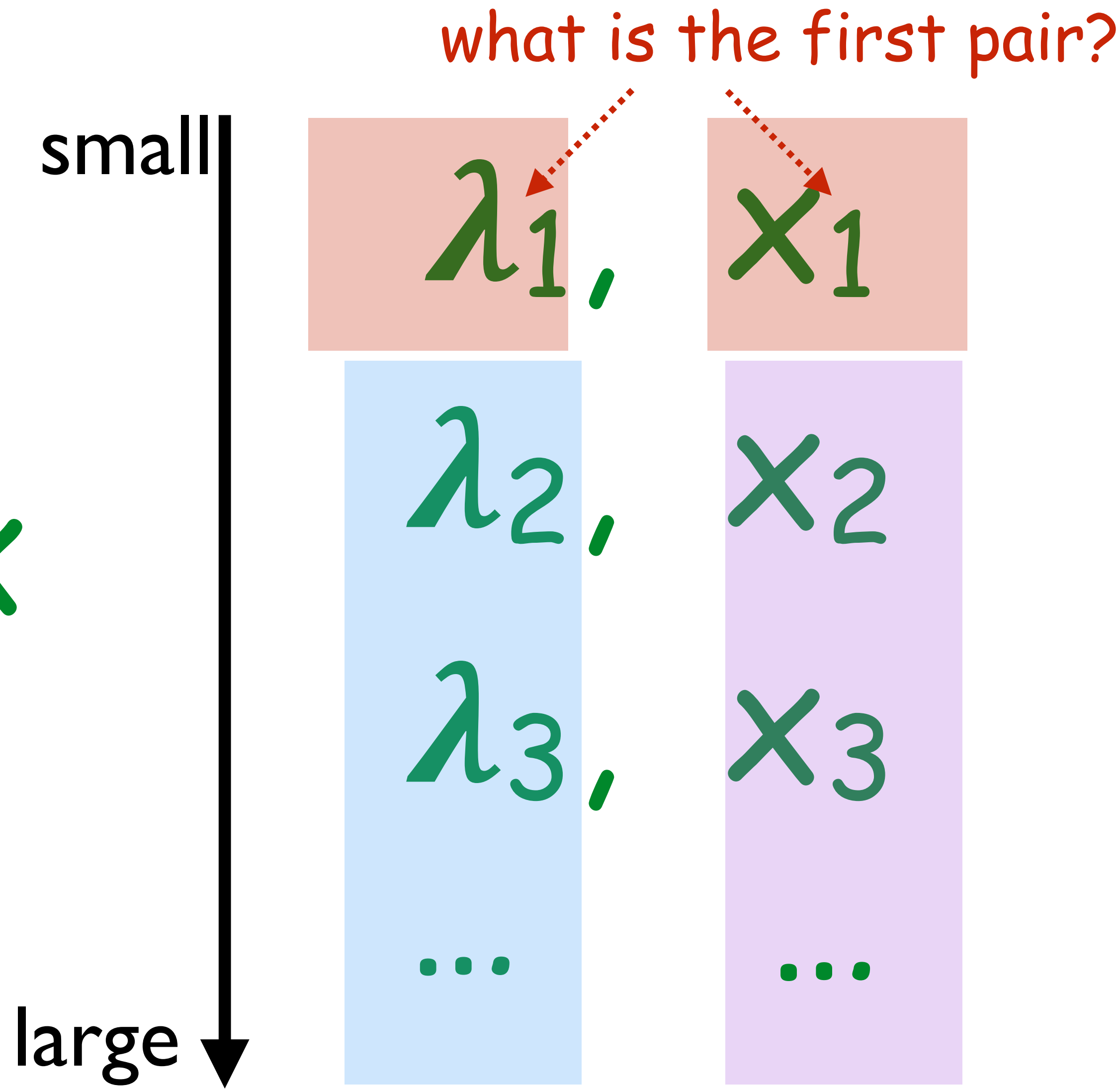
$\lambda=0$



$\lambda=0$

Eigen Pairs of Matrix L

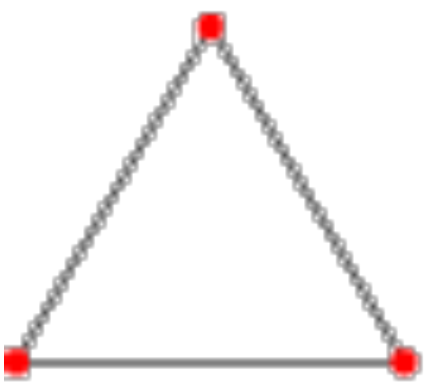
$$Lx = \lambda x$$



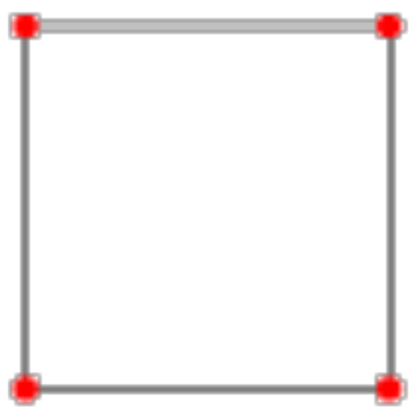
$$L\,x = \lambda\,x$$

the 1st Eigen pair (smallest)

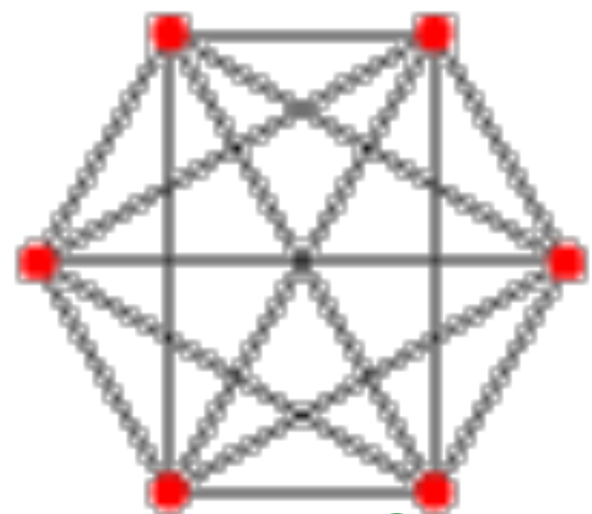
$$\lambda_1 = 0 \qquad x_1 = (1,1,\ldots,1)$$



$\lambda_1=0$

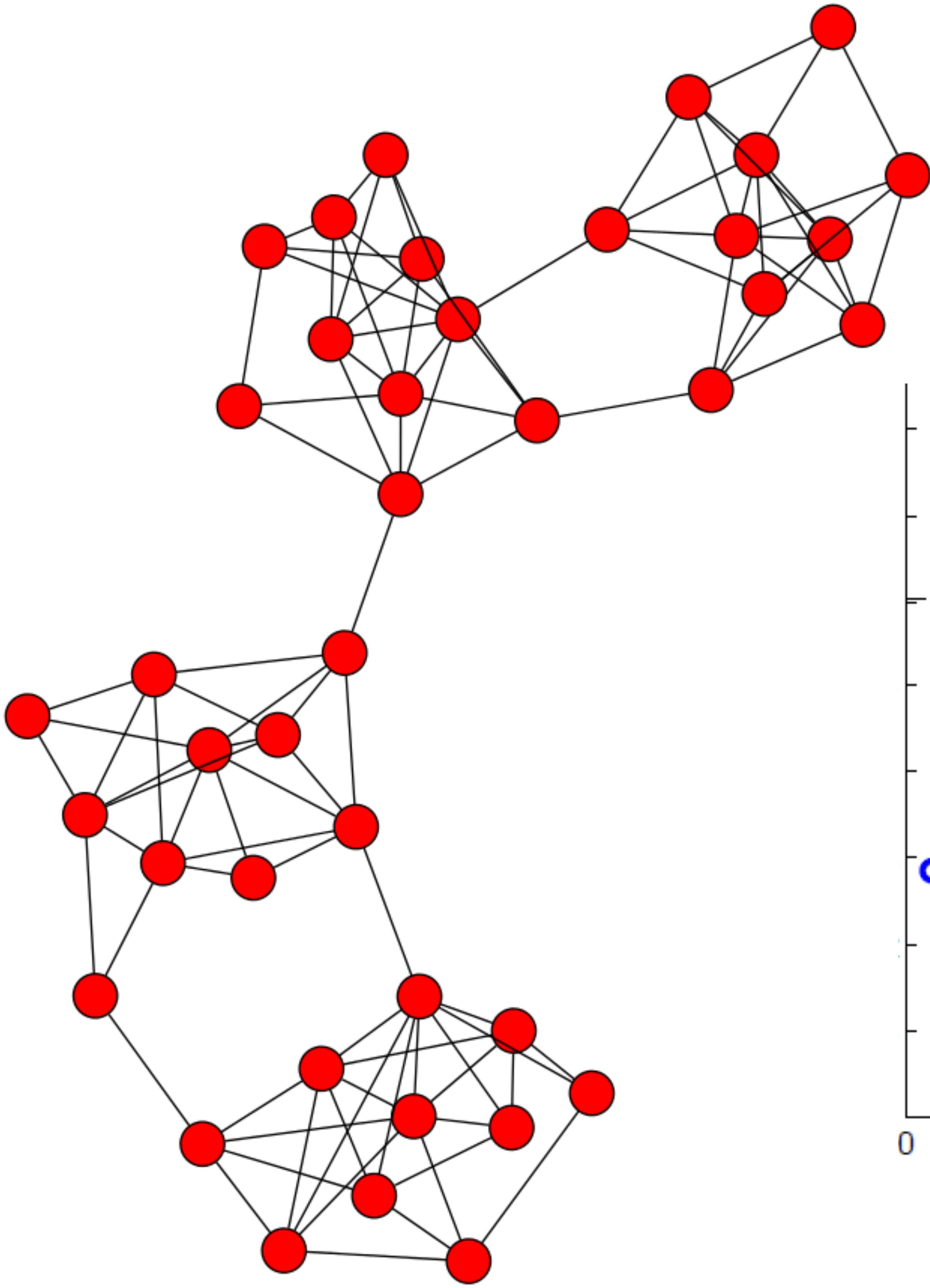


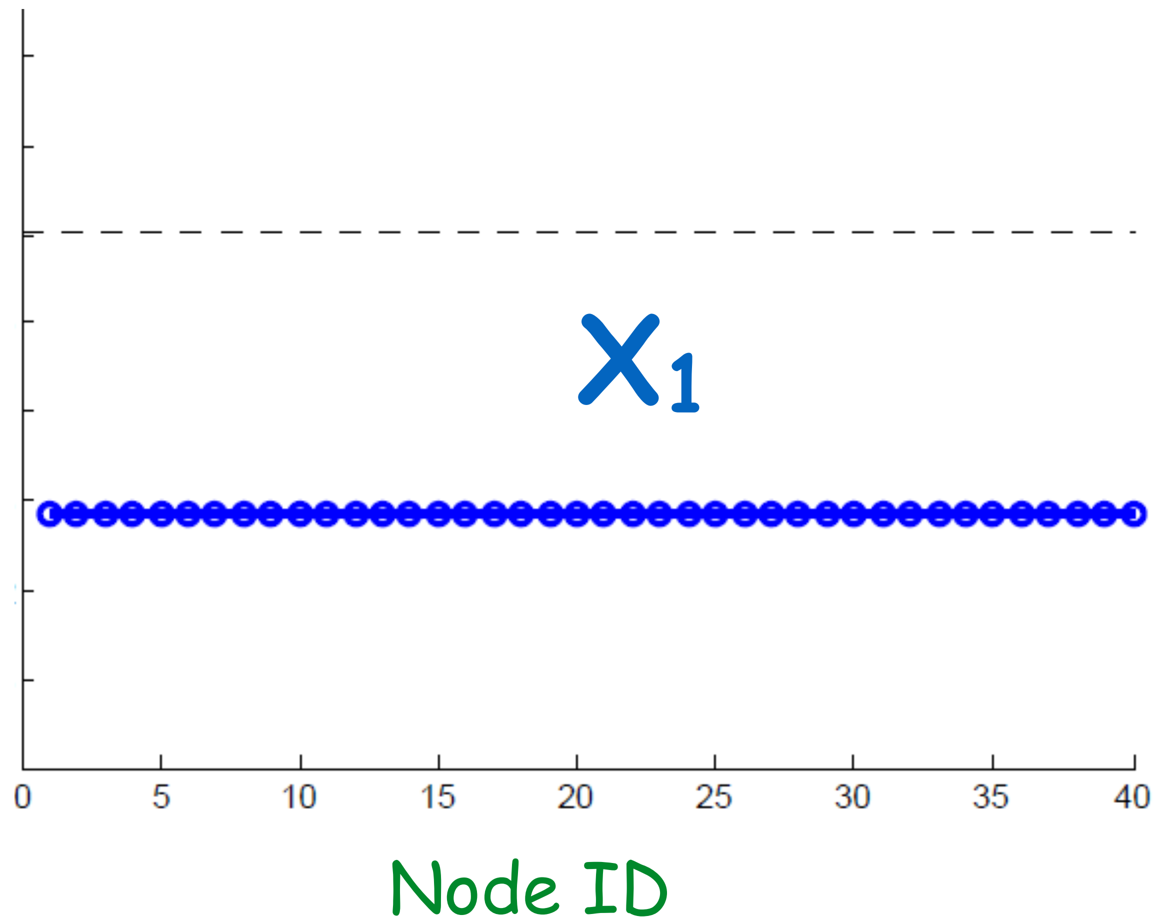
$\lambda_1=0$



$\lambda_1=0$

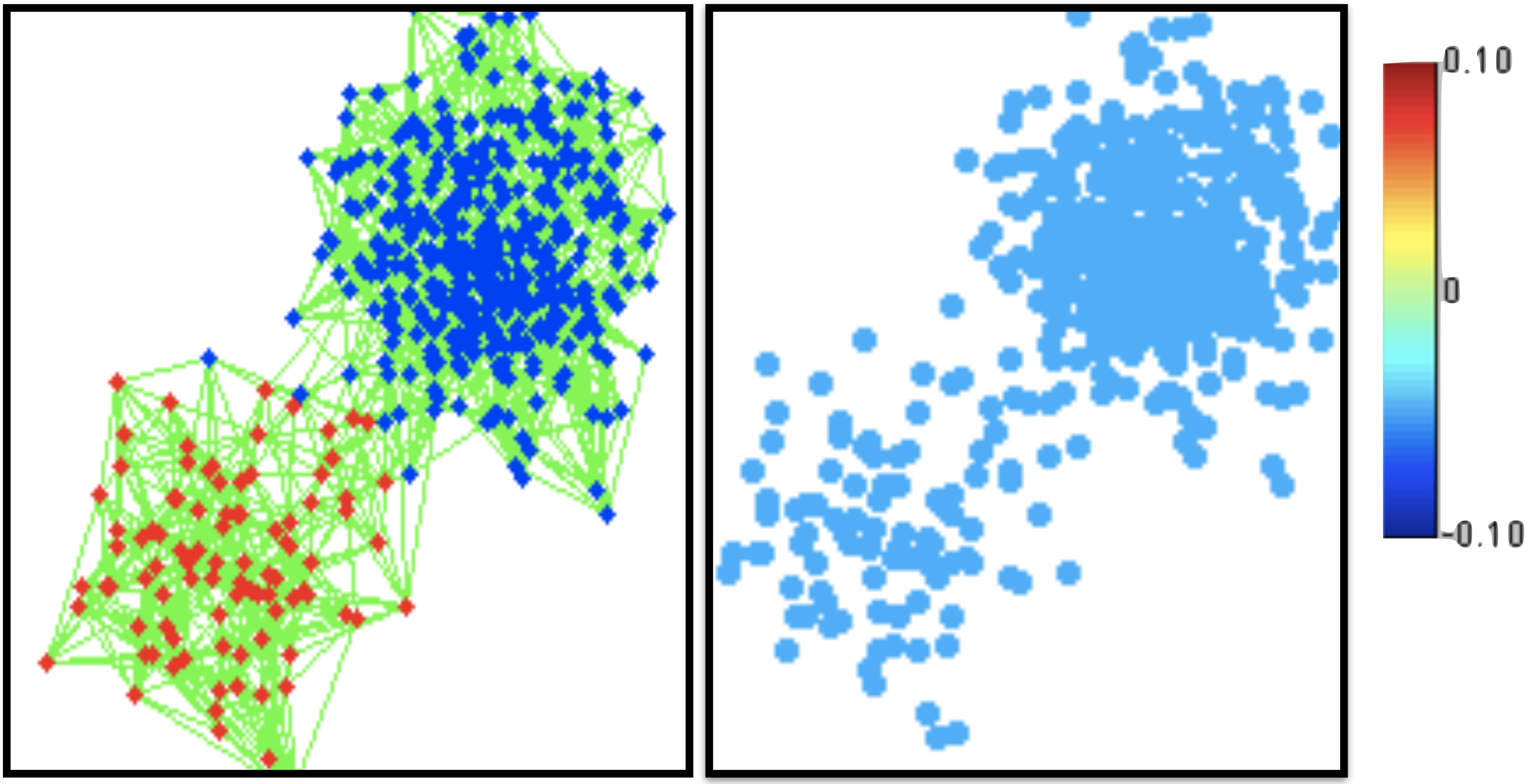
1st Eigen Vector





J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

1st Eigen Vector



Just like Matrix A

the 1st Eigen pair of L
is a trivial solution

1st Eigen pair of A:

$\lambda_1 = 0$

$x_1 = (0,0,0,0,0,0)$

1st Eigen pair of L:

$\lambda_1 = 0$

$x_1 = (1,1,1,1,1,1)$

Cool, but not useful.

how about the other Eigen pairs?

After all, they are useful in matrix A

$$L x = \lambda x$$

Eigen Pairs of Matrix L

zero

Trivial solution

all ones

small

λ_1, x_1

λ_2, x_2

what's this?

λ_3, x_3

... ...

large

What is the 2nd Eigen pair of L ?

Fact: for symmetric matrix M

$$\lambda_2 = \min_x \frac{x^T M\ x}{x^T x}$$

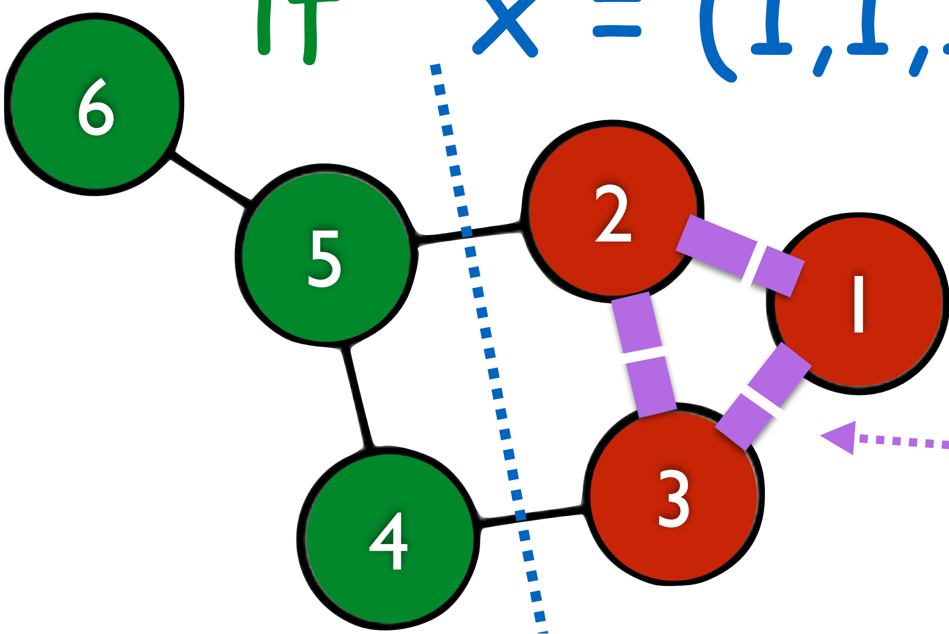
What is $x^T M x$?

What is $x^T A x$?

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

y_i = sum of labels of Node i's neighbors

if $x = (1,1,1,0,0,0)$

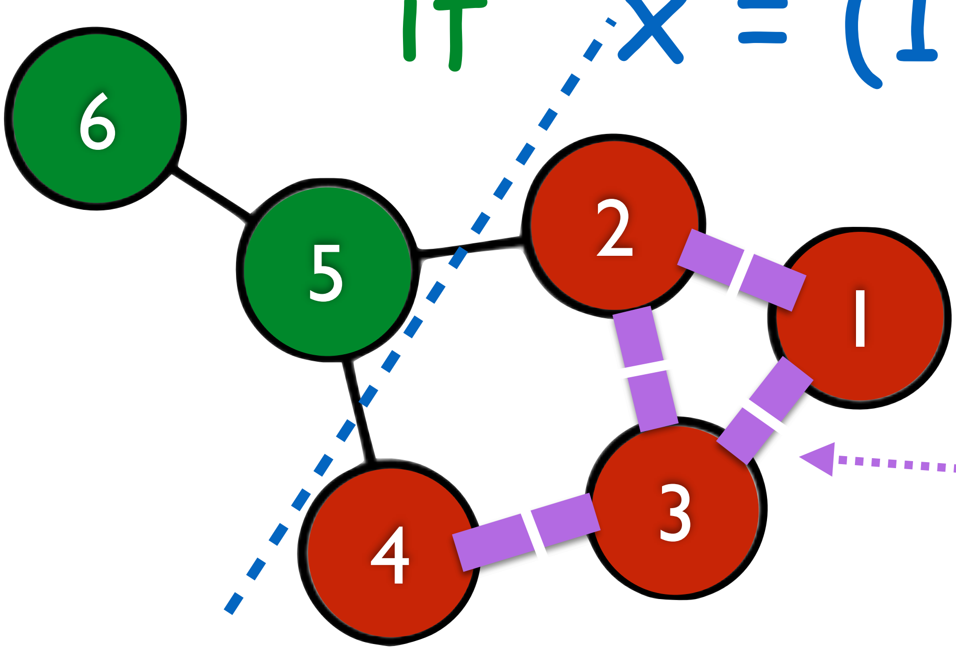


$x^T A x = 6$

What is $x^T A x$?

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

if $x = (1,1,1,1,0,0)$

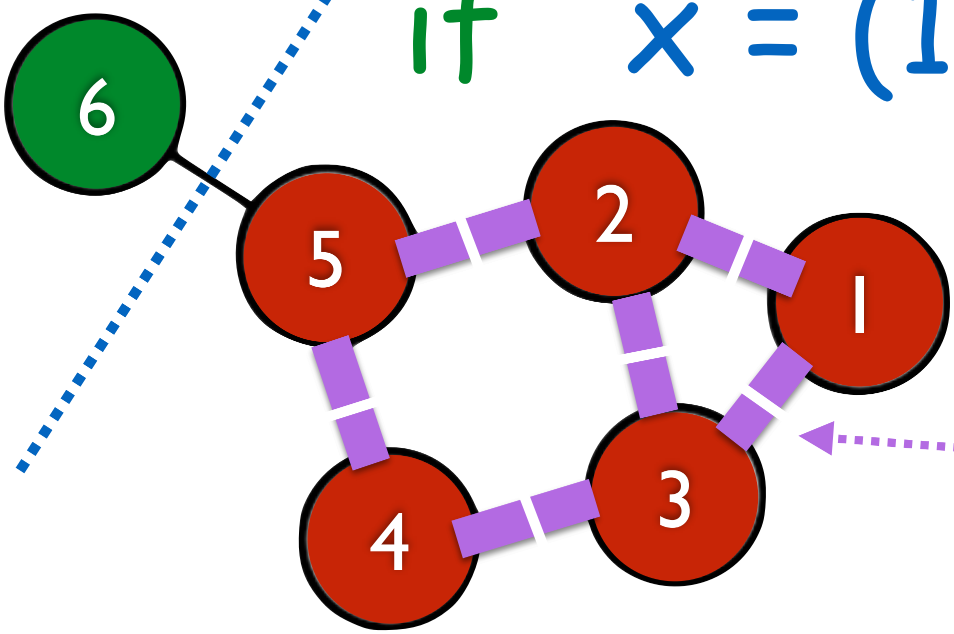


$x^T A x = 8$

What is $x^T A x$?

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

if $x = (1,1,1,1,1,0)$

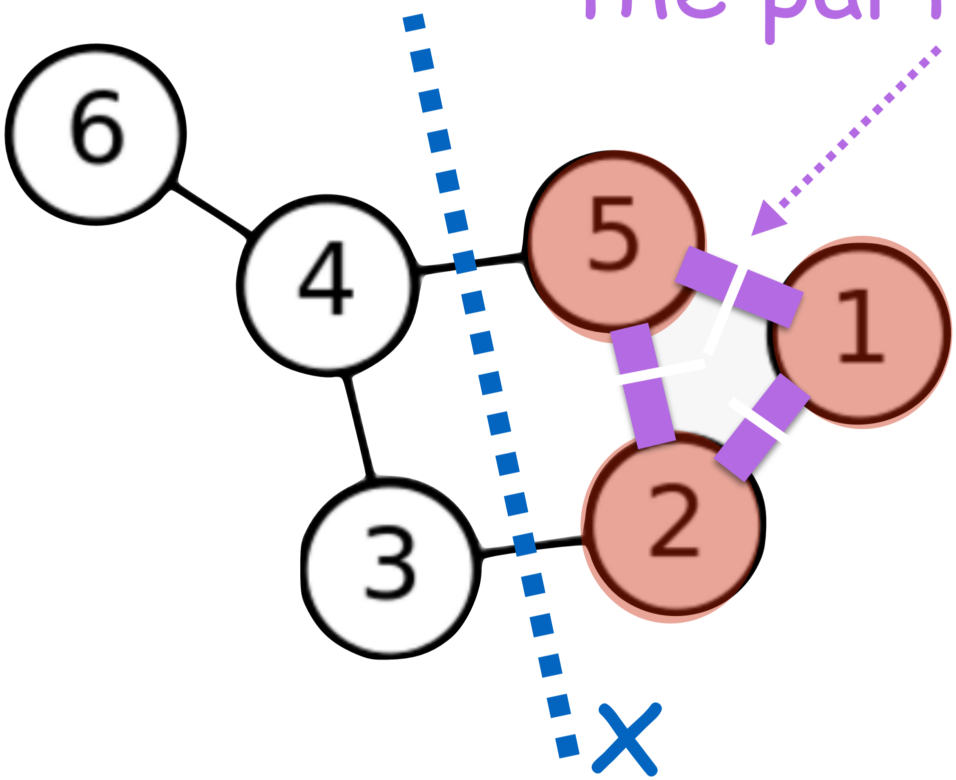


$x^T A x = 12$

What is $x^T A x$?

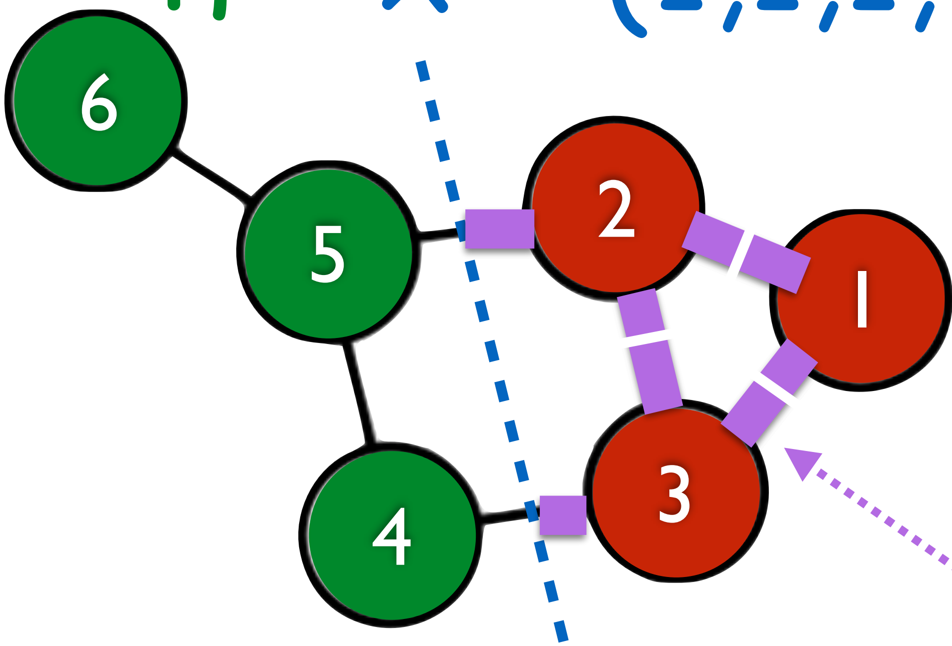
if x is a binary partition

$x^T A x$ = 2 X number of edges **within** the partition



What is $x^T D x$?

if $x = (1,1,1,0,0,0)$

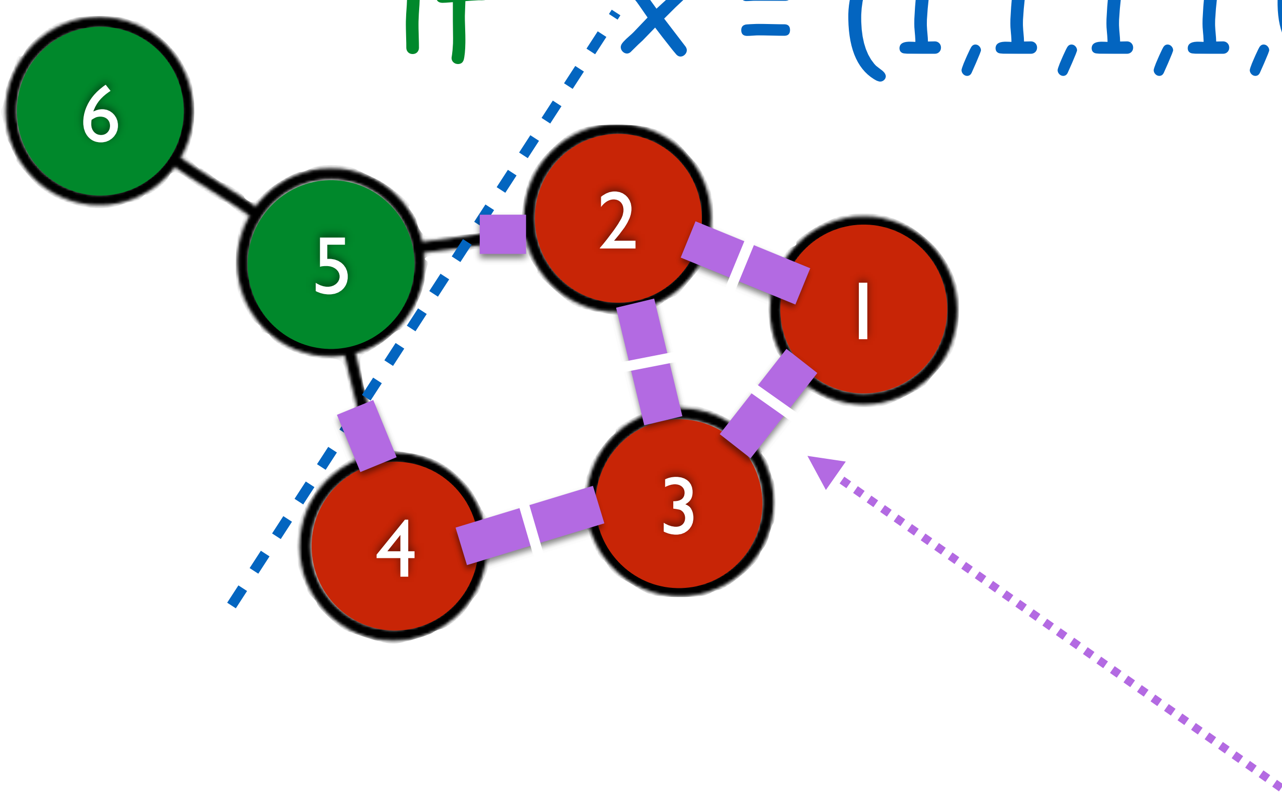


d_i = degree of Node i

$x^T D x = 8$

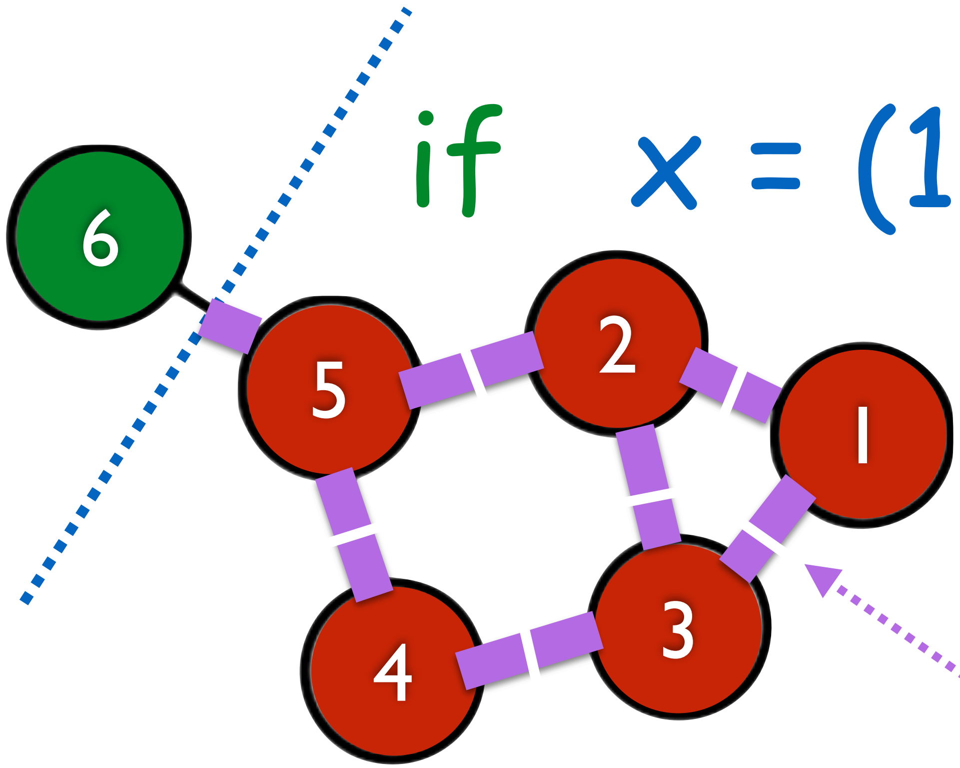
What is $x^T D x$?

if $x = (1,1,1,1,0,0)$



$x^T D x = 10$

What is $x^T D x$?



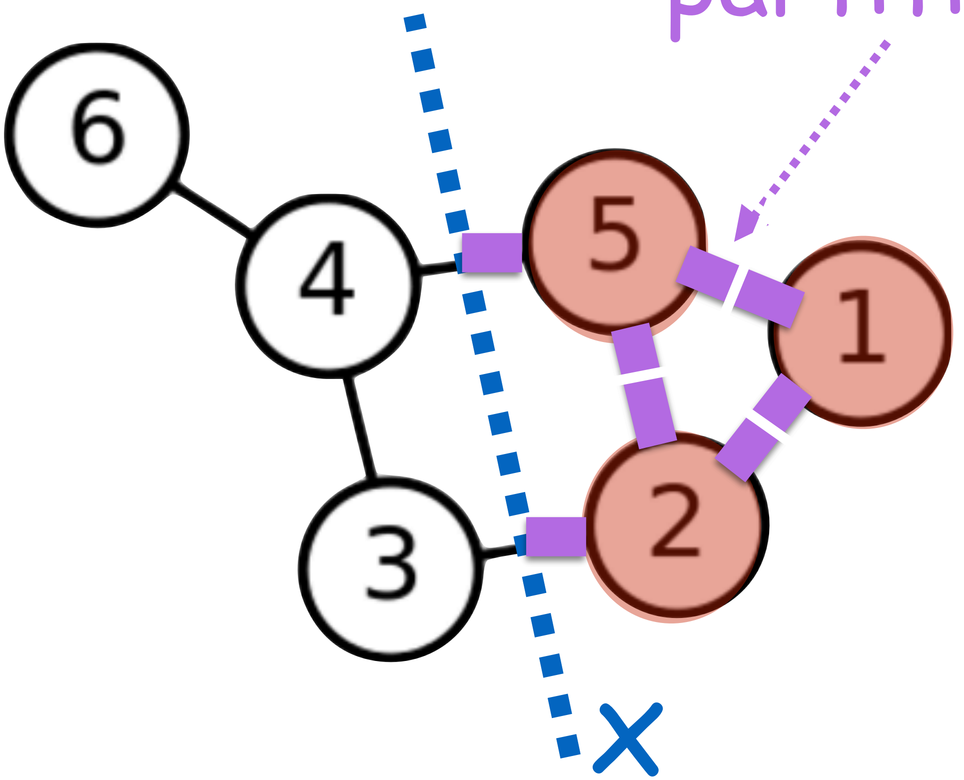
if $x = (1,1,1,1,1,0)$

$$x^T D x = 13$$

What is x^TDx ?

if x is a binary partition

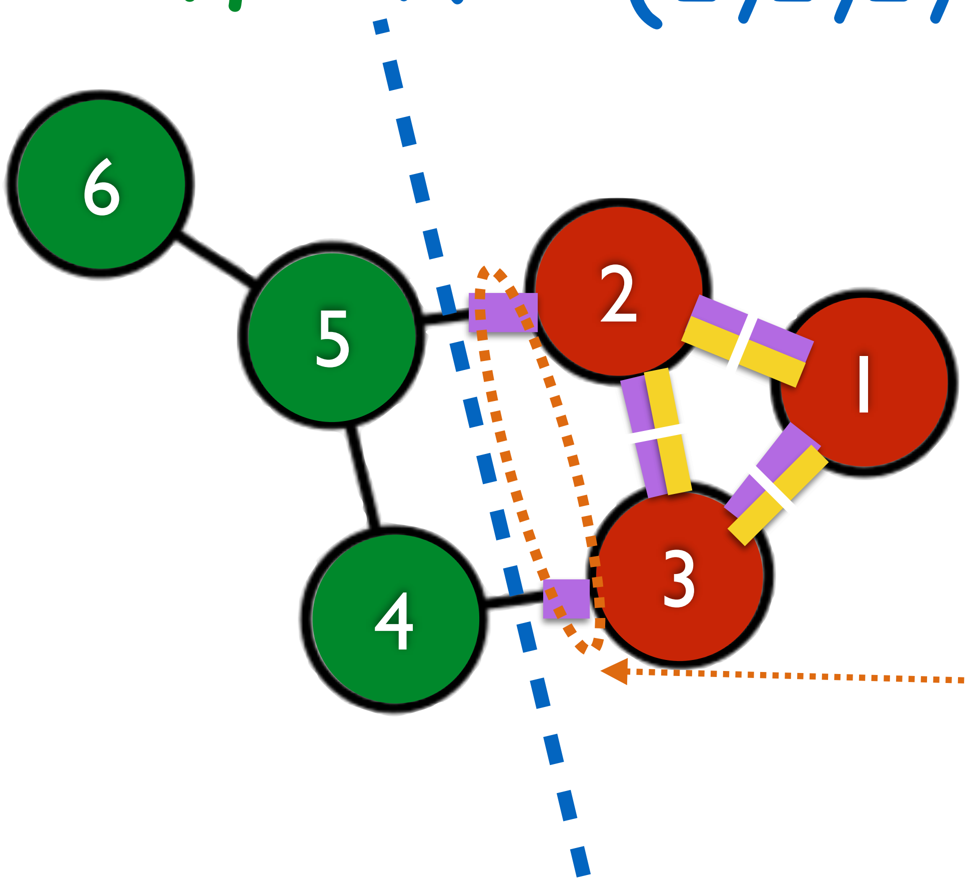
x^TDx = sum of node degrees in the partition



What is $x^T L x$?

$$x^T L x = x^T D x - x^T A x$$

if $x = (1,1,1,0,0,0)$



$$x^T D x = 8$$

$$-$$

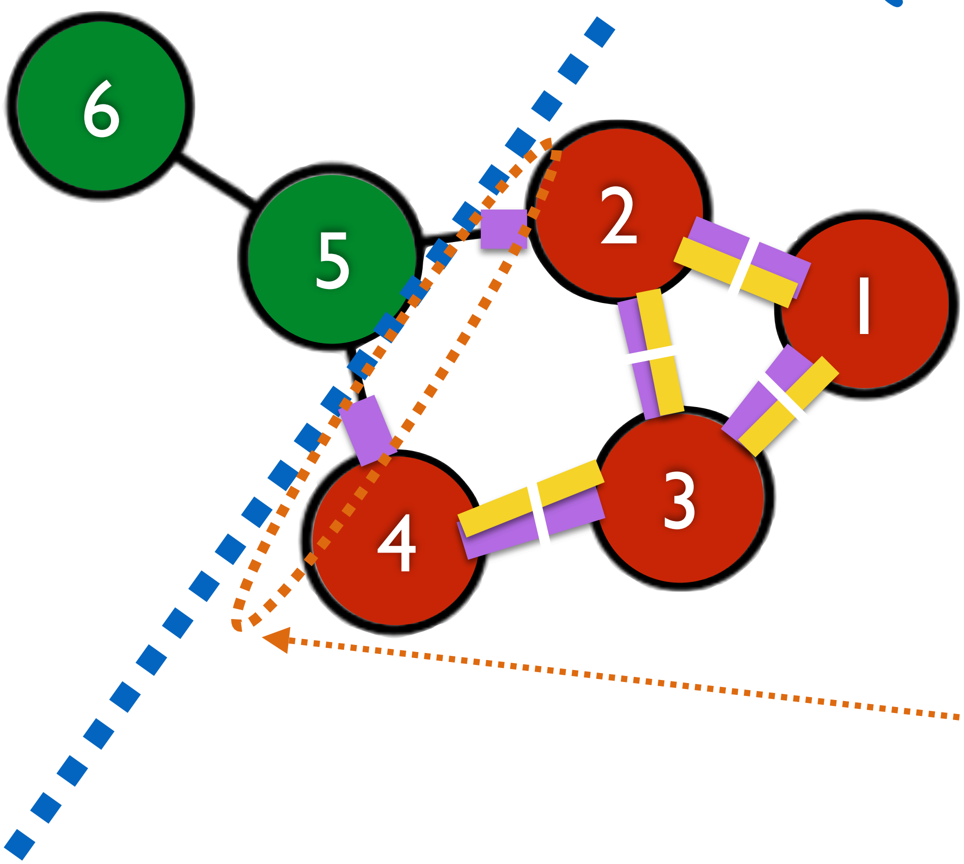
$$x^T A x = 6$$

$$=$$

$$x^T L x = 2$$

What is $x^T L x$?

if $x = (1,1,1,1,0,0)$



$$x^T D x = 10$$

$$-$$

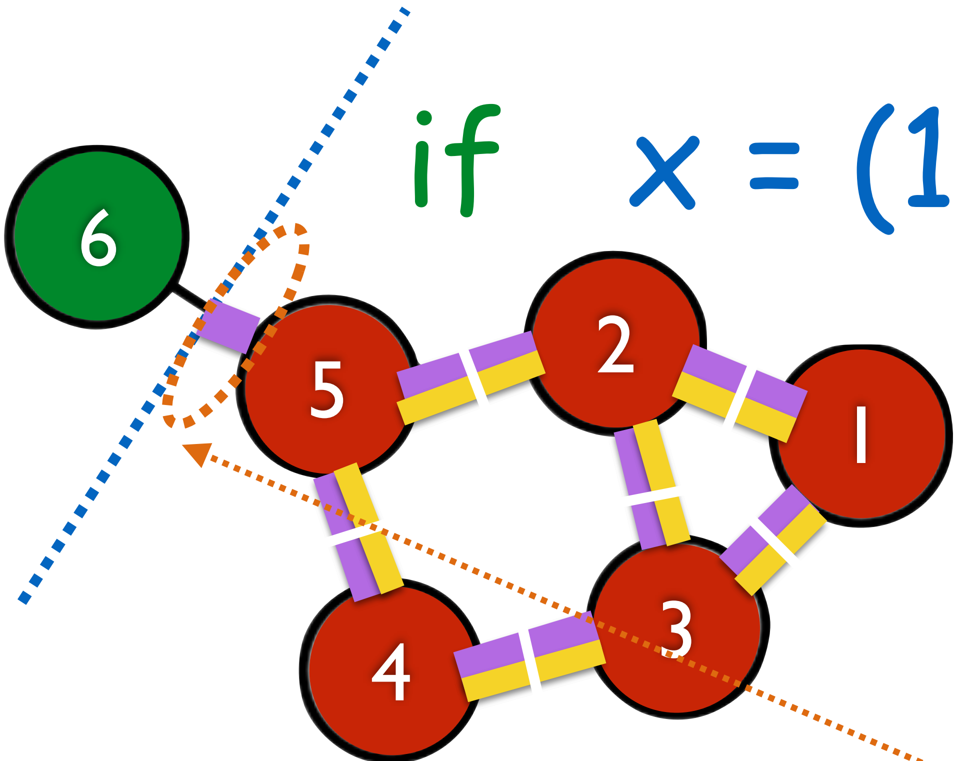
$$x^T A x = 8$$

$$=$$

$$x^T L x = 2$$

What is $x^T L x$?

if $x = (1,1,1,1,1,0)$



$x^T D x = 13$

$-$

$x^T A x = 12$

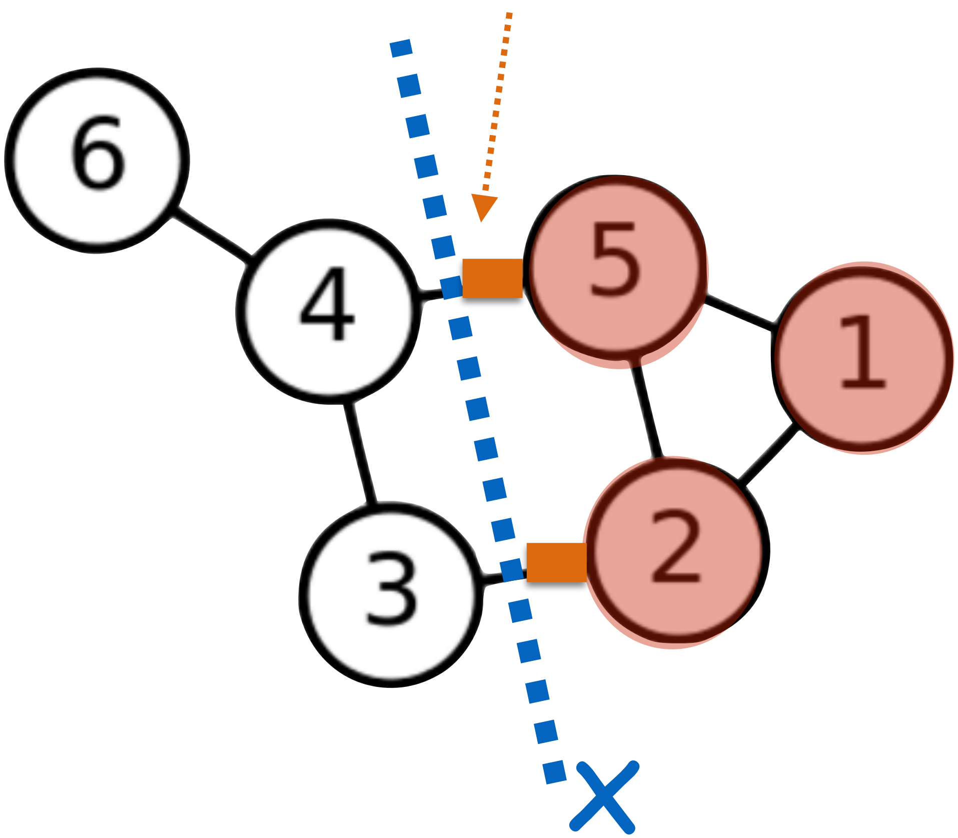
$=$

$x^T L x = 1$

What is $x^T L x$?

if x is a binary partition

$x^T L x$ = number of edges being cut



What is the 2nd Eigen pair of L ?

Fact: for symmetric matrix M

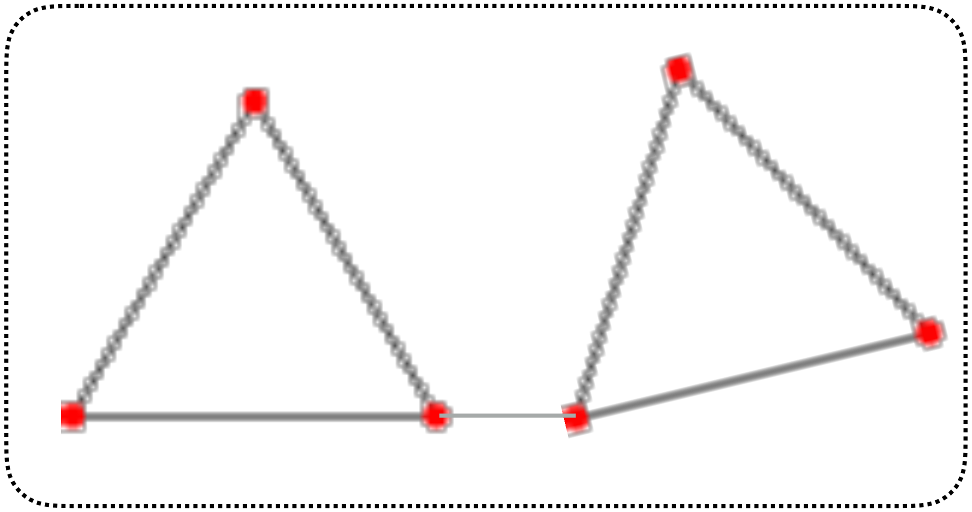
$$\lambda_2 = \min_x \frac{x^T M x}{x^T x}$$

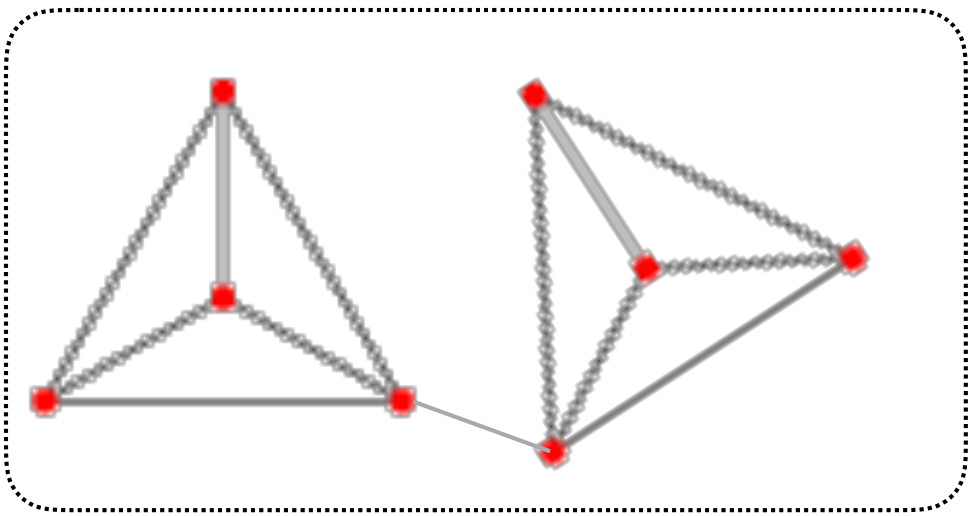
$x^T L x$ = number of edges being cut

$x^T x$ = number of nodes in the partition

$$\min \quad x^T L\, x$$

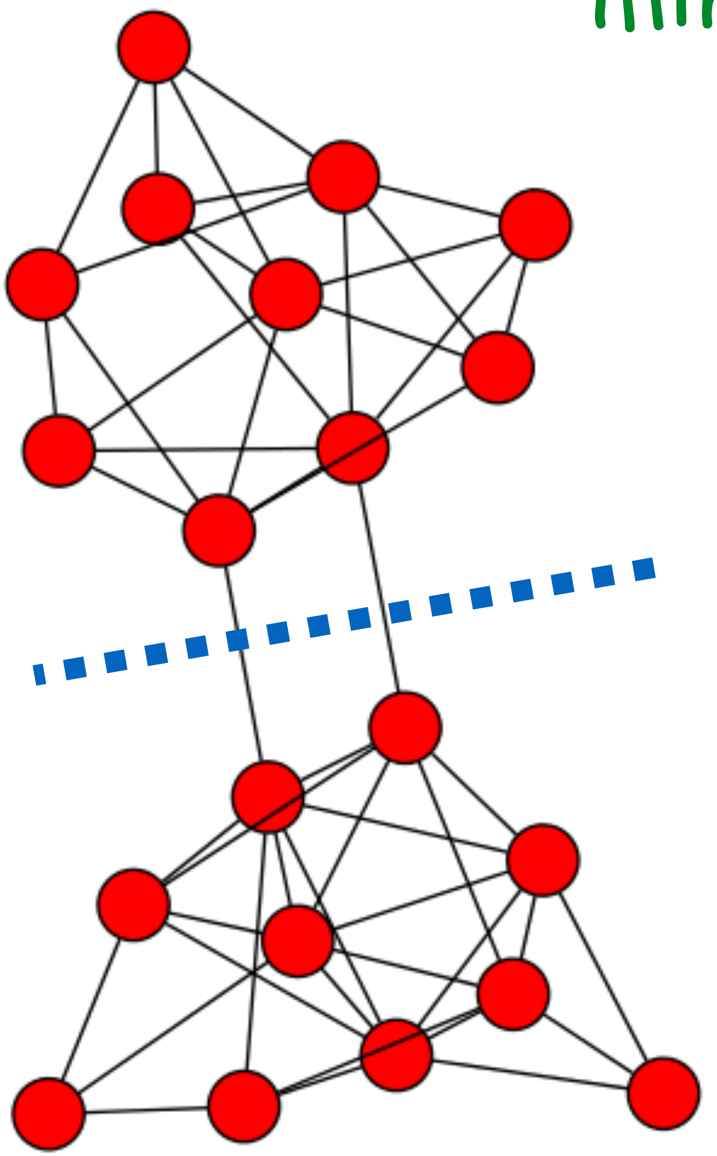
= minimize number of edges being cut

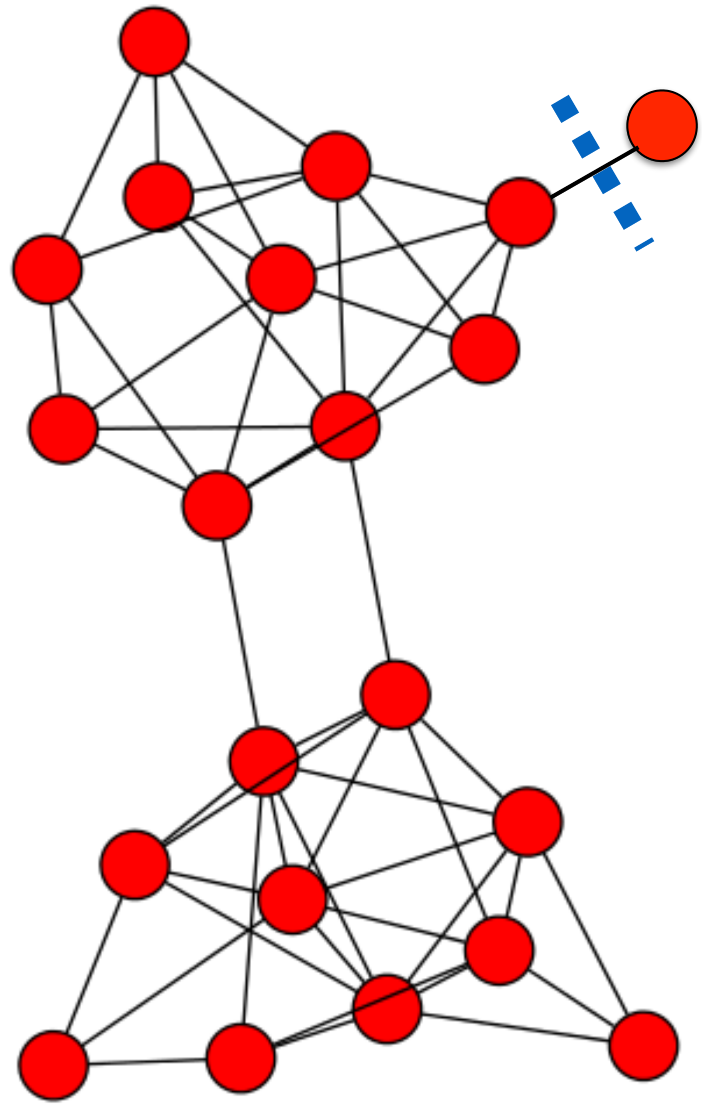




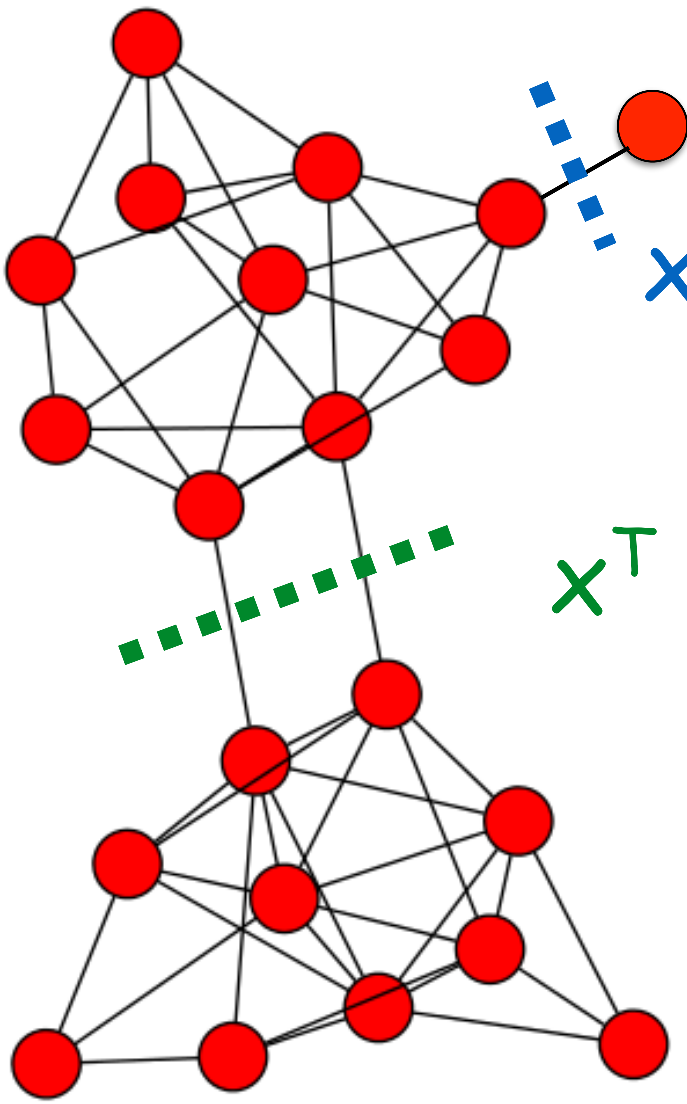
MinCut - graph clustering

$$\min \quad x^T L\, x$$





$x^T x$ = number of nodes in the partition



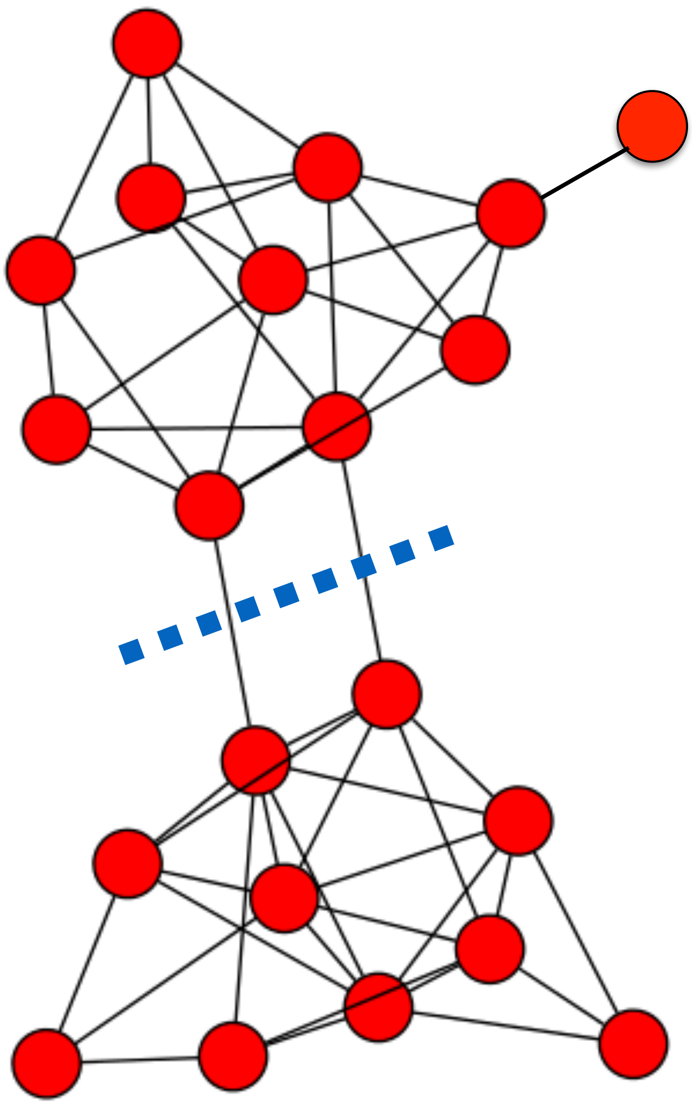
$$x^T x = 1$$

$$x^T x = 11$$

$$\frac{x^T L x = 1}{x^T x = 1}$$

$$\frac{x^T L x = 2}{x^T x = 11}$$

$$\text{Normalized Cut} = \frac{x^T L x}{x^T x}$$



What is the 2nd Eigen pair of L ?

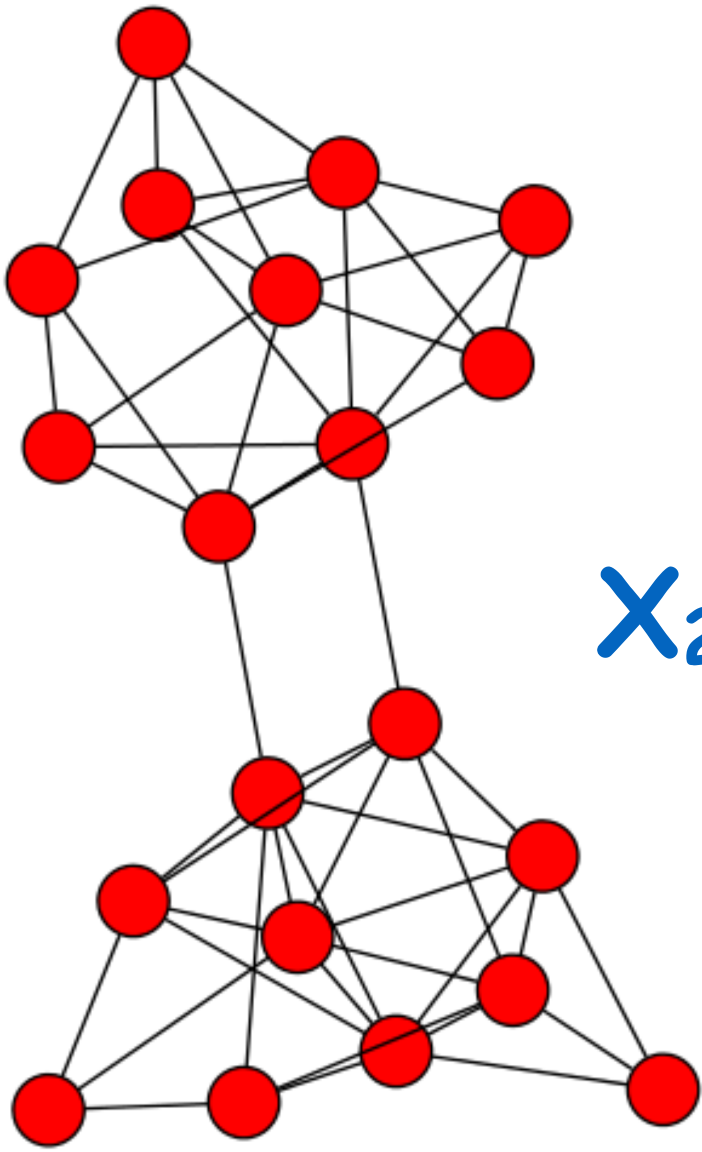
Fact: for symmetric matrix M

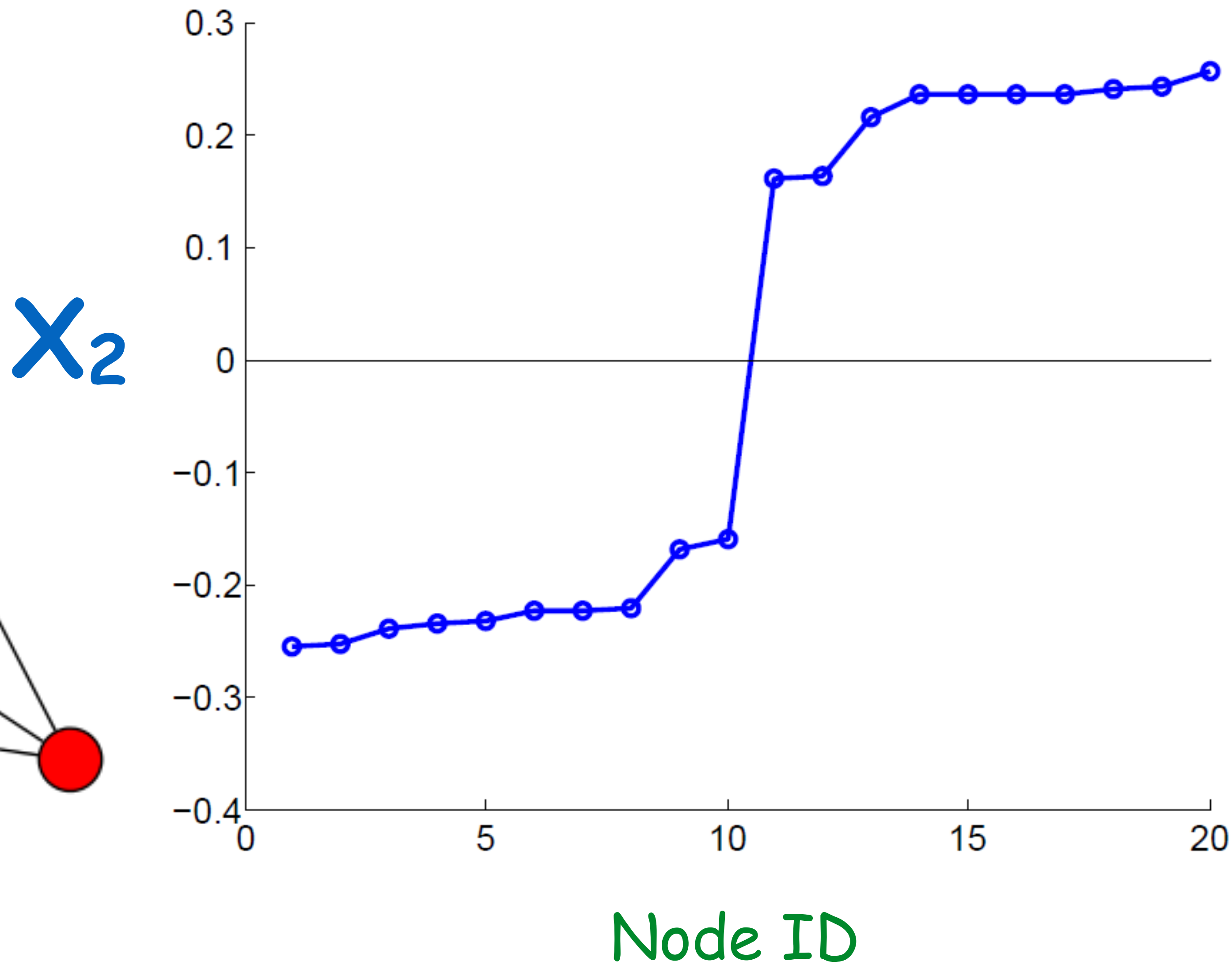
$$\lambda_2 = \min_x \frac{x^T M x}{x^T x}$$

2nd Eigen value is the minimum normalized cut on a graph

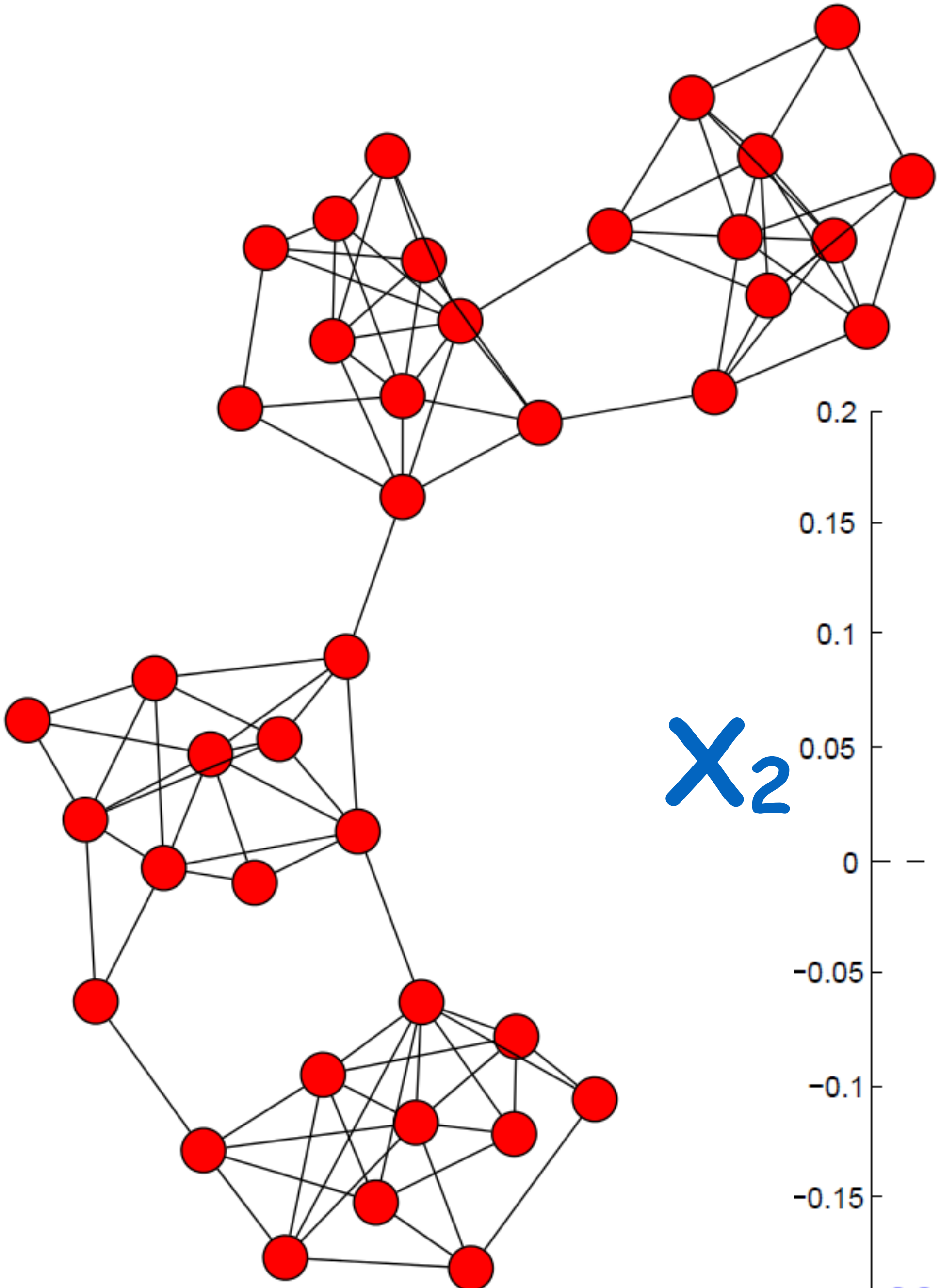
2nd Eigen vector is the cut (partition)

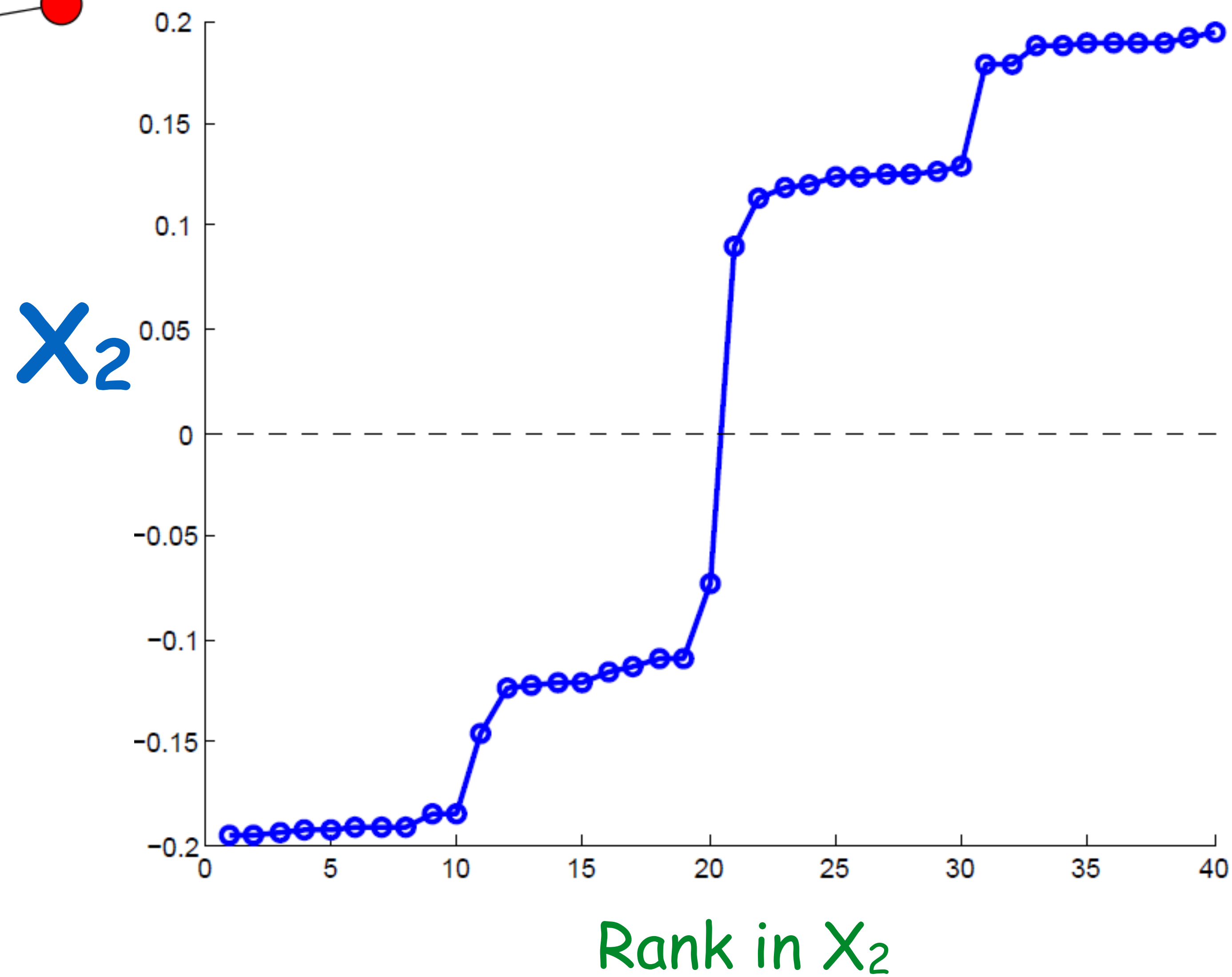
2nd Eigen Vector





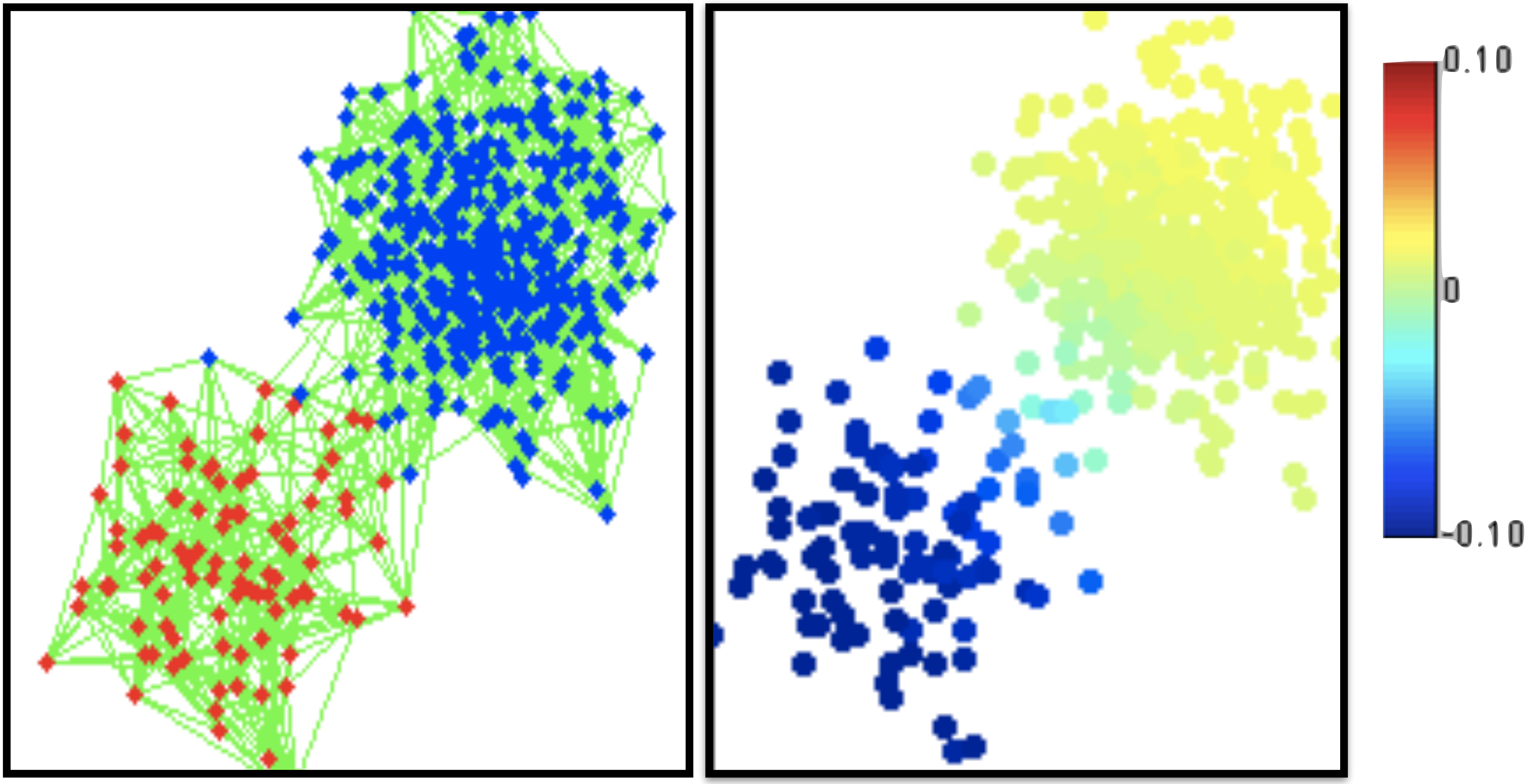
2nd Eigen Vector

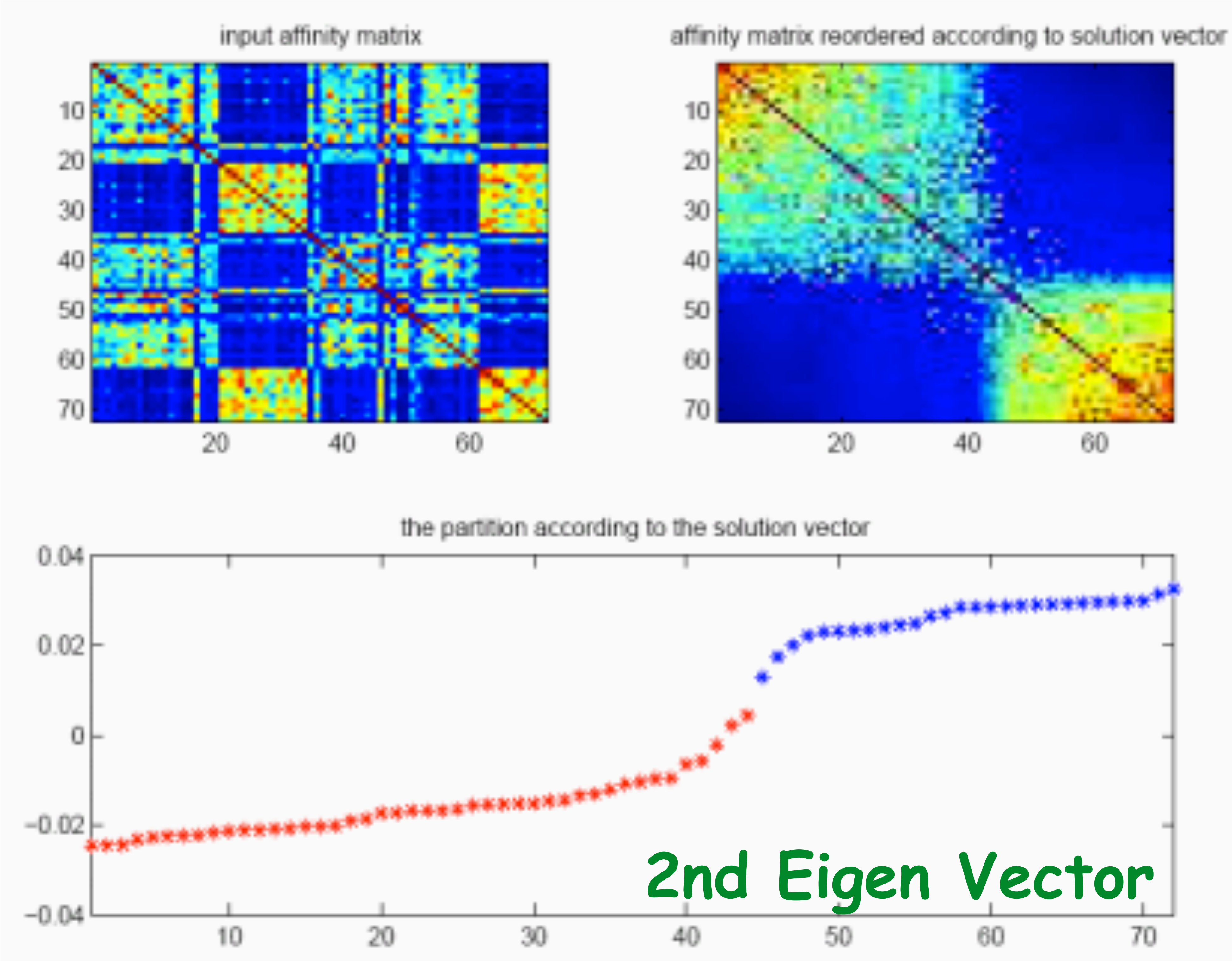




J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

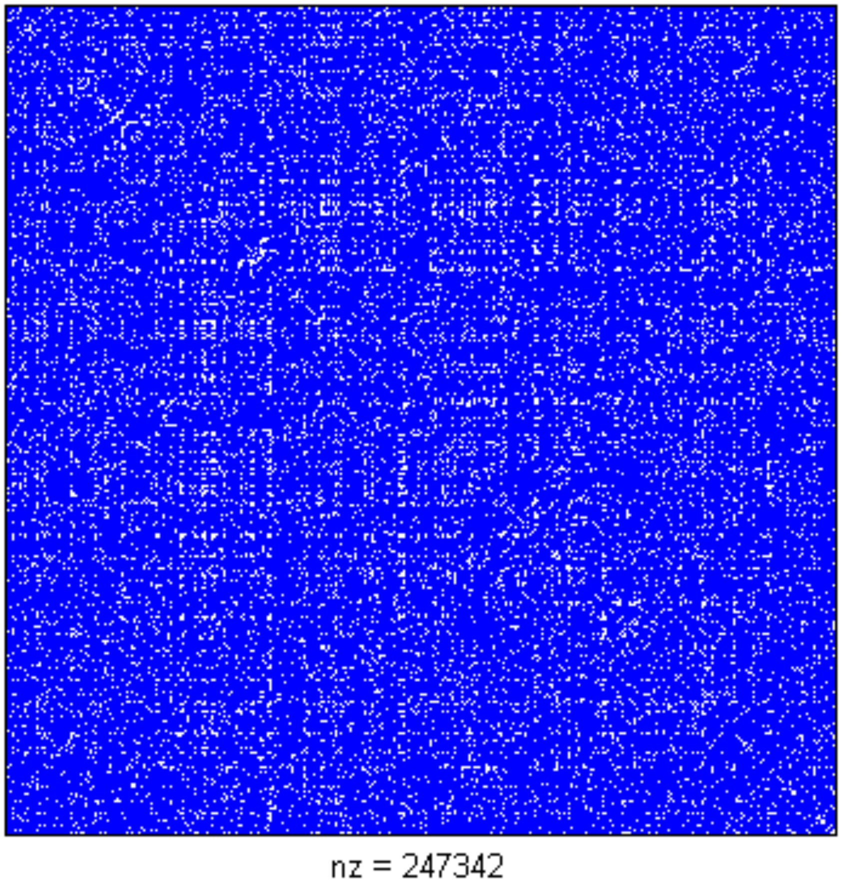
2nd Eigen Vector

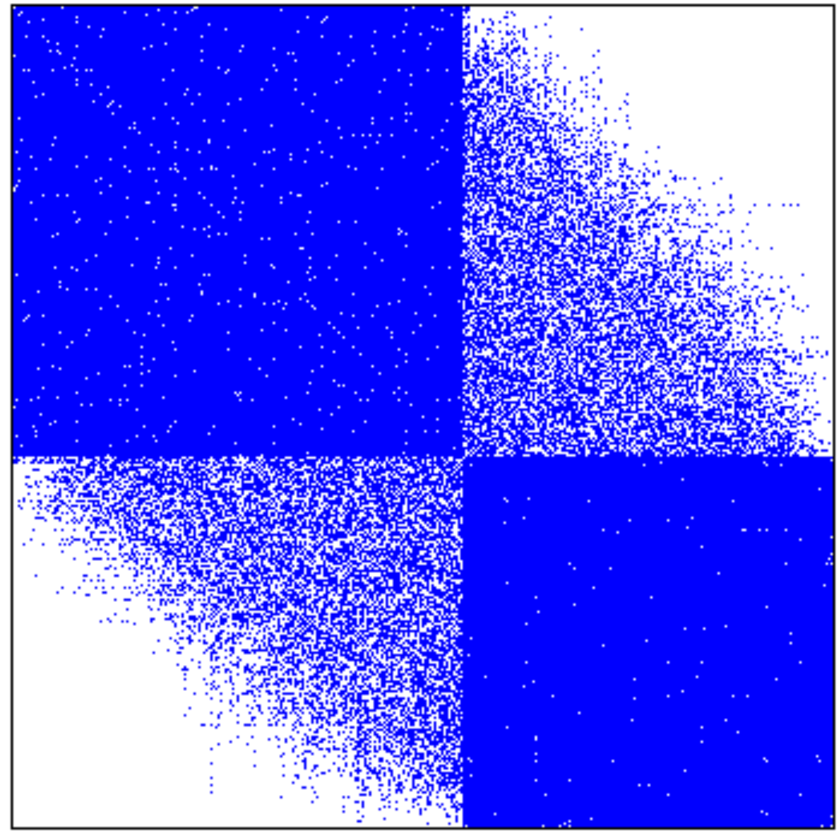


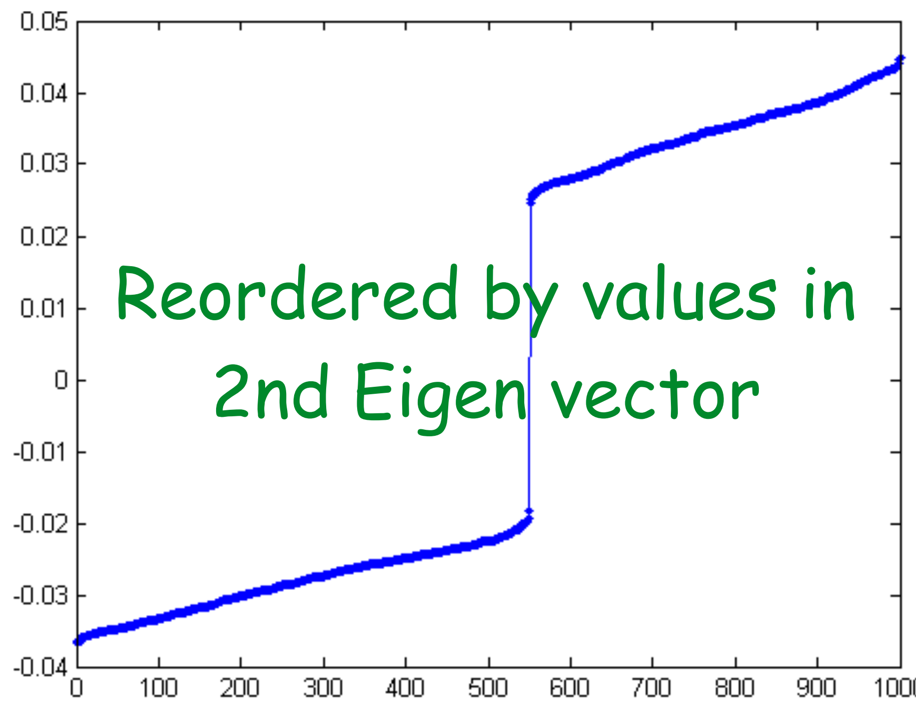


input affinity matrix
affinity matrix reordered according to solution vector
the partition according to the solution vector
2nd Eigen Vector

Original Adjacency Matrix



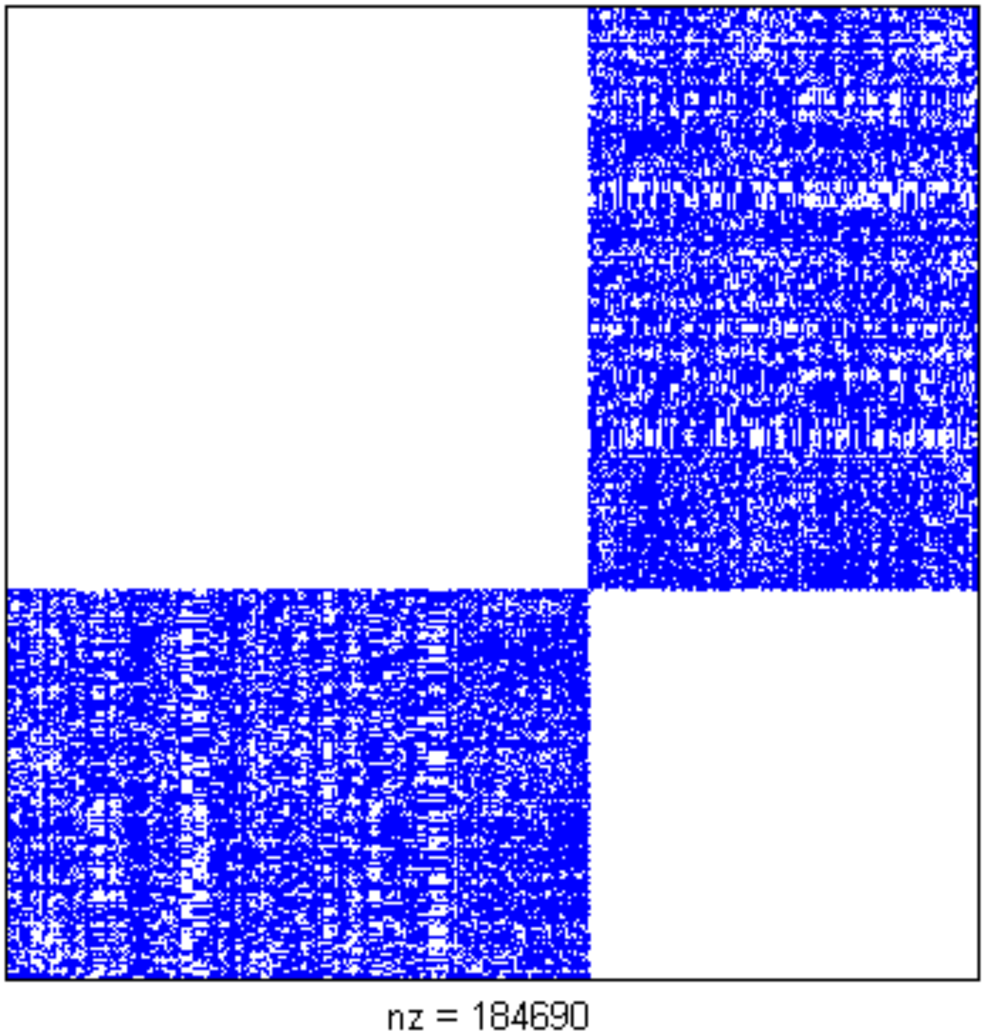


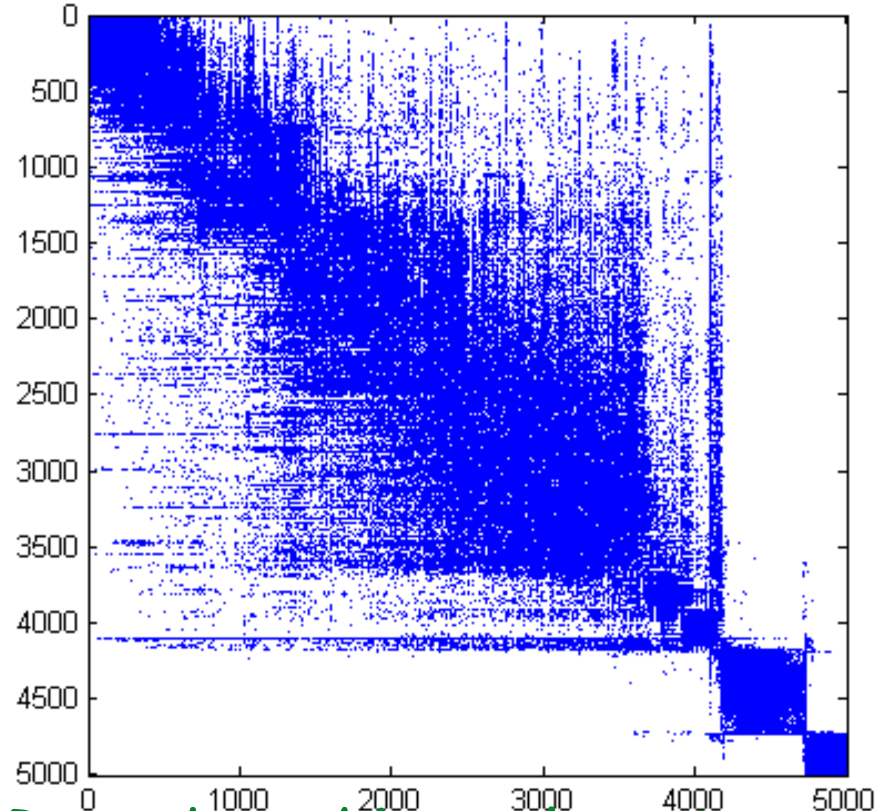


Reordered by values in 2nd Eigen vector

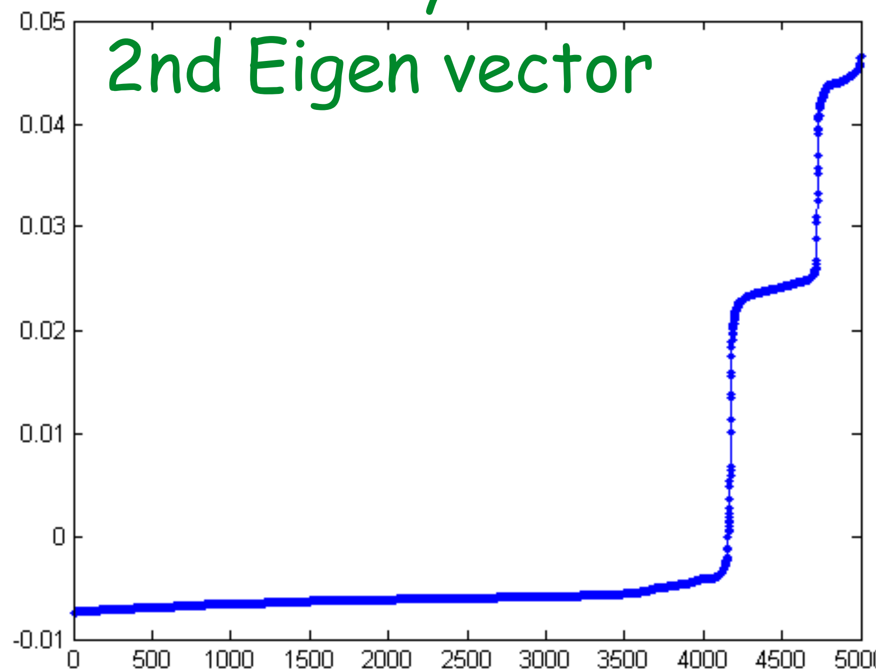
https://www.cs.purdue.edu/homes/dgleich/demos/matlab/spectral/spectral.html

Original Adjacency Matrix



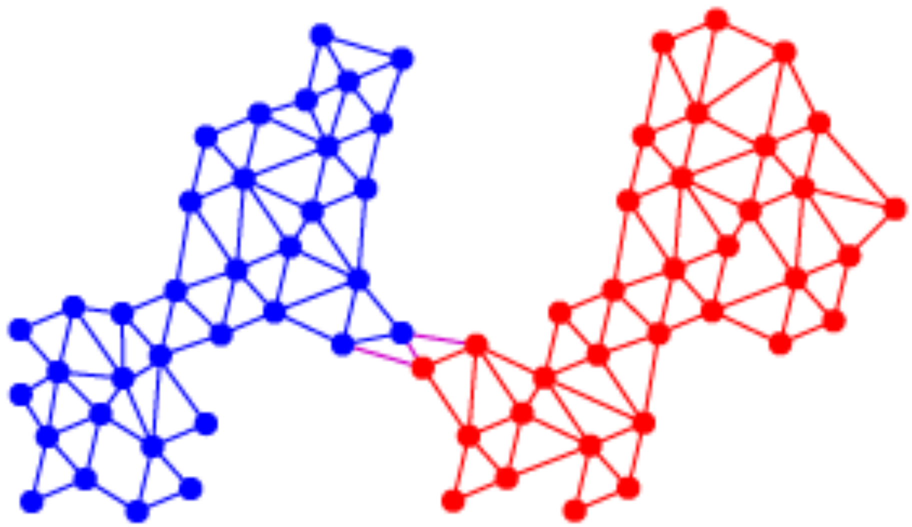


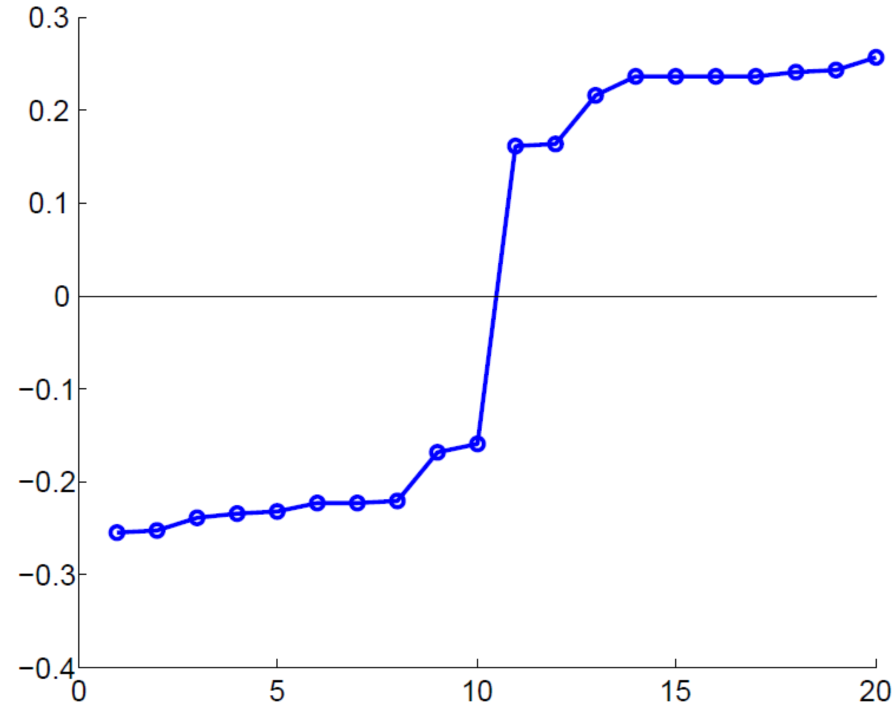
Reordered by values in 2nd Eigen vector



https://www.cs.purdue.edu/homes/dgleich/demos/matlab/spectral/spectral.html

Spectral Clustering





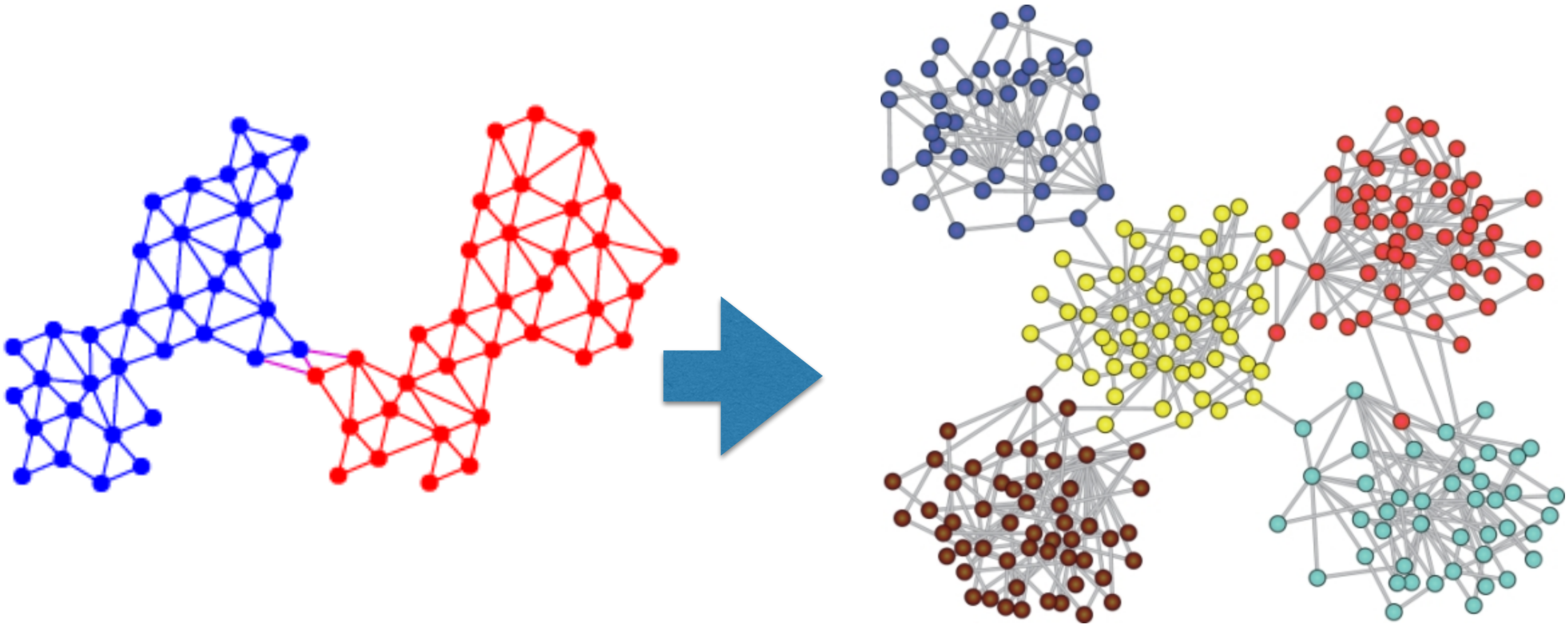
1. Compute $L = D - A$

2. Compute Eigen pairs of Matrix L

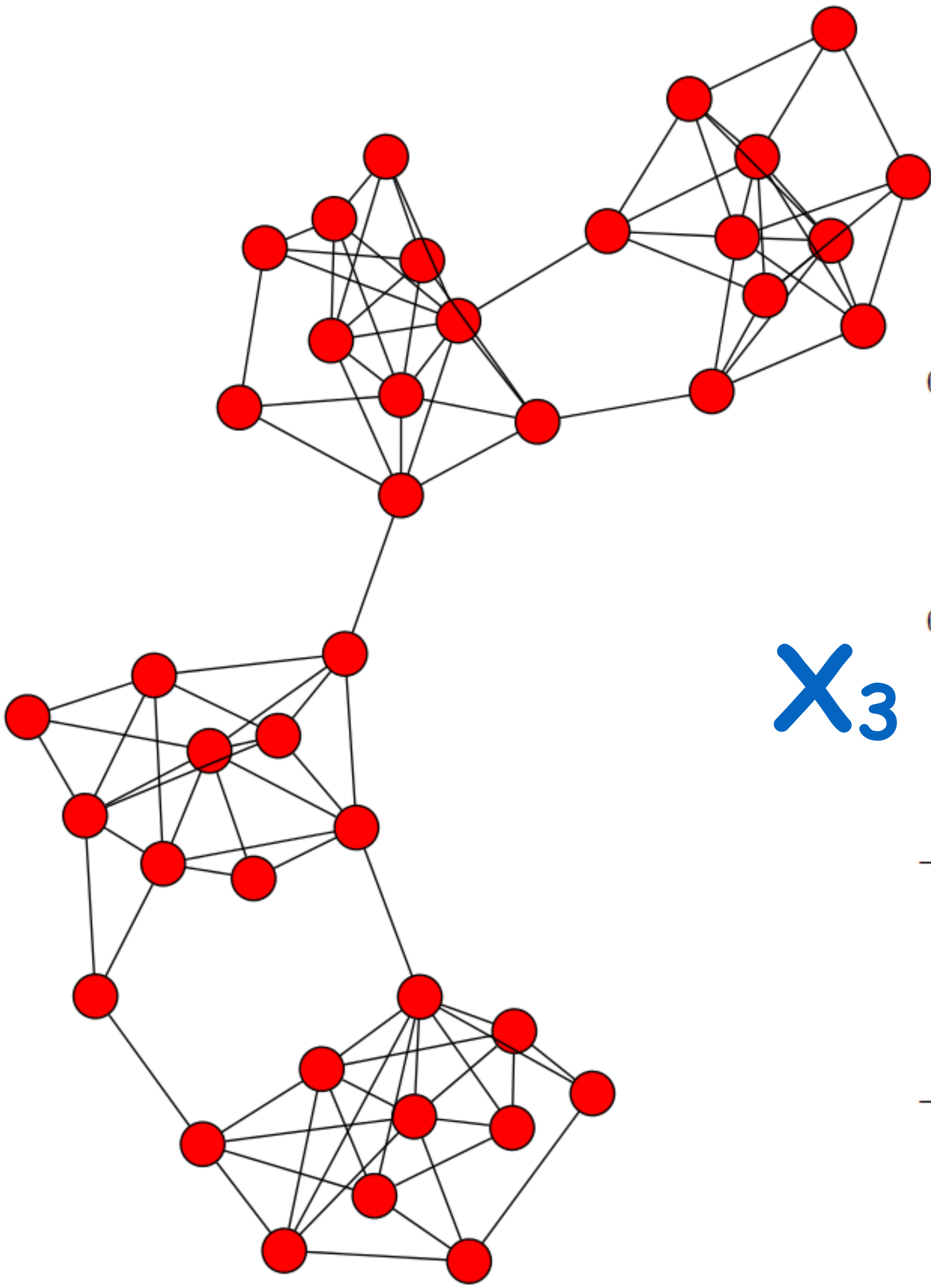
3. use the Eigen vector with smallest non-zero positive Eigen values

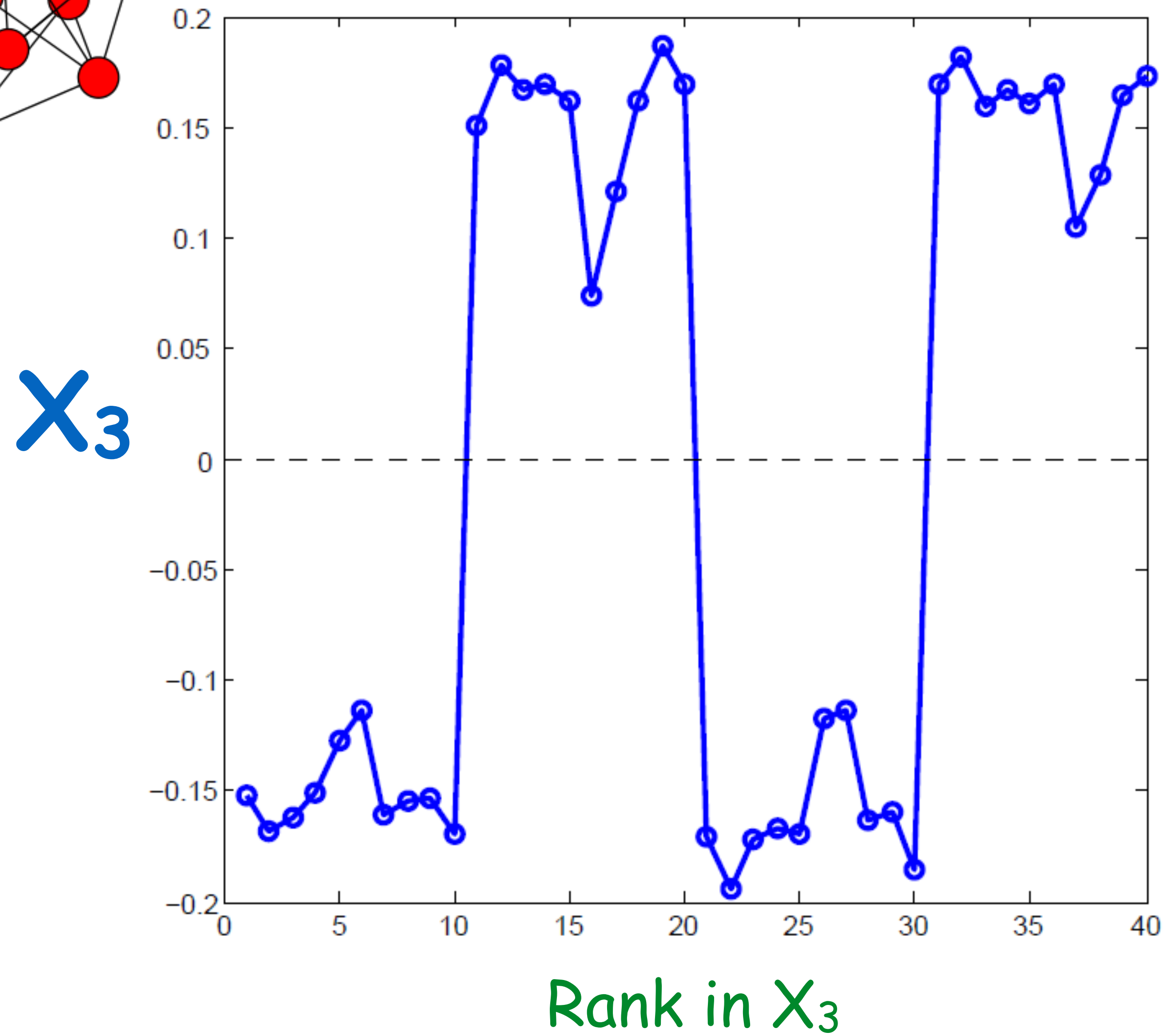
4. use zero to cut

Beyond Binary Partition

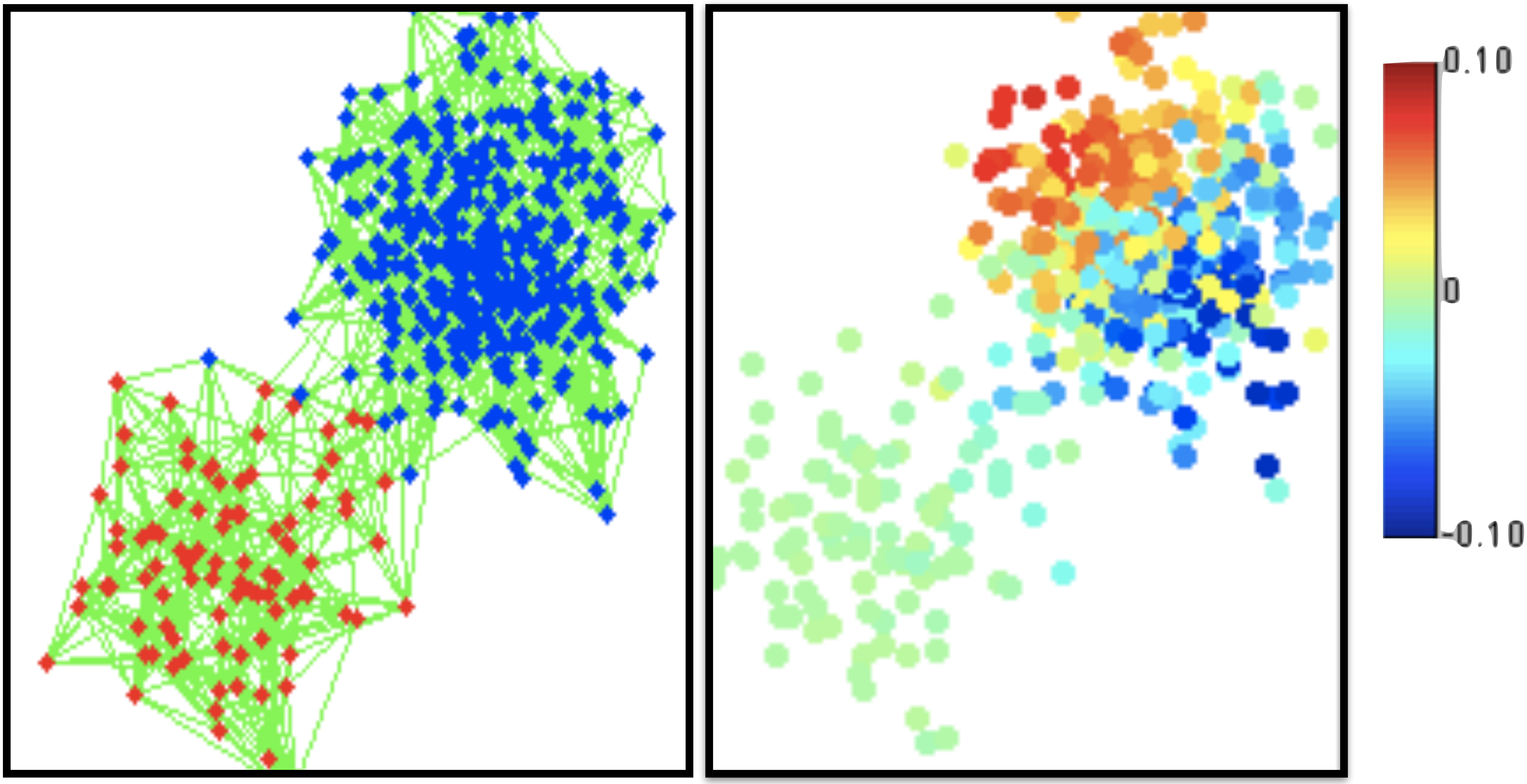


3rd Eigen Vector

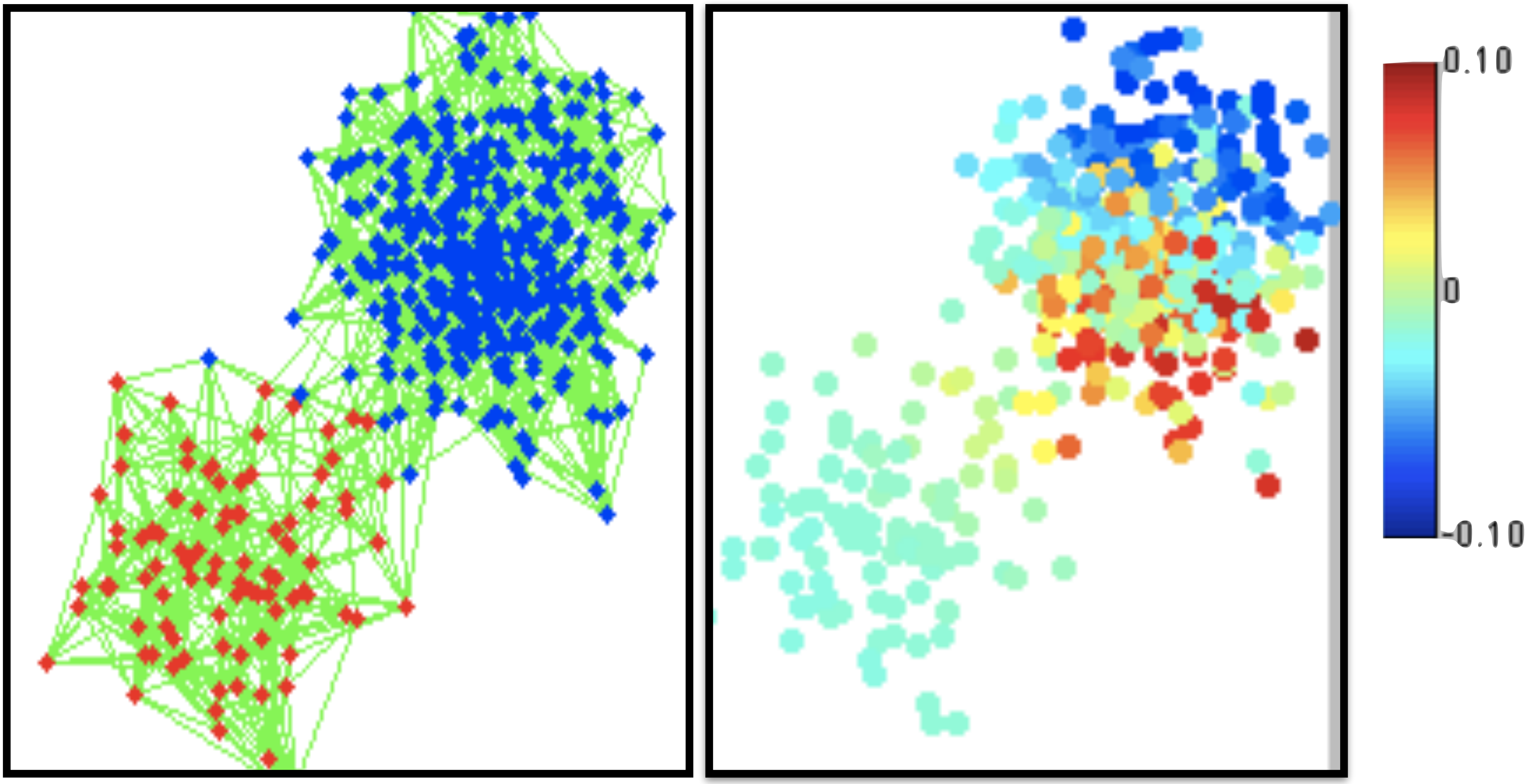




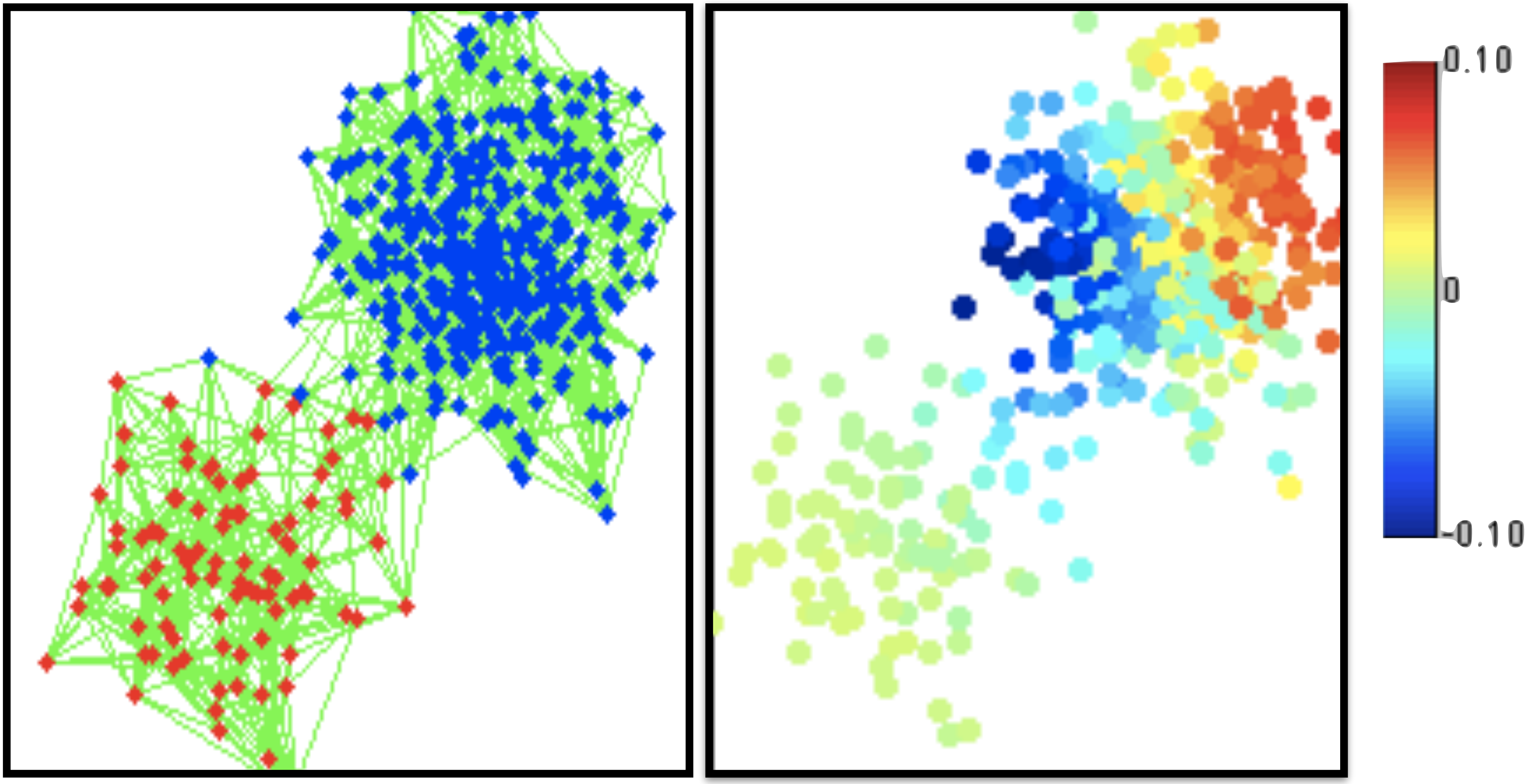
3rd Eigen Vector

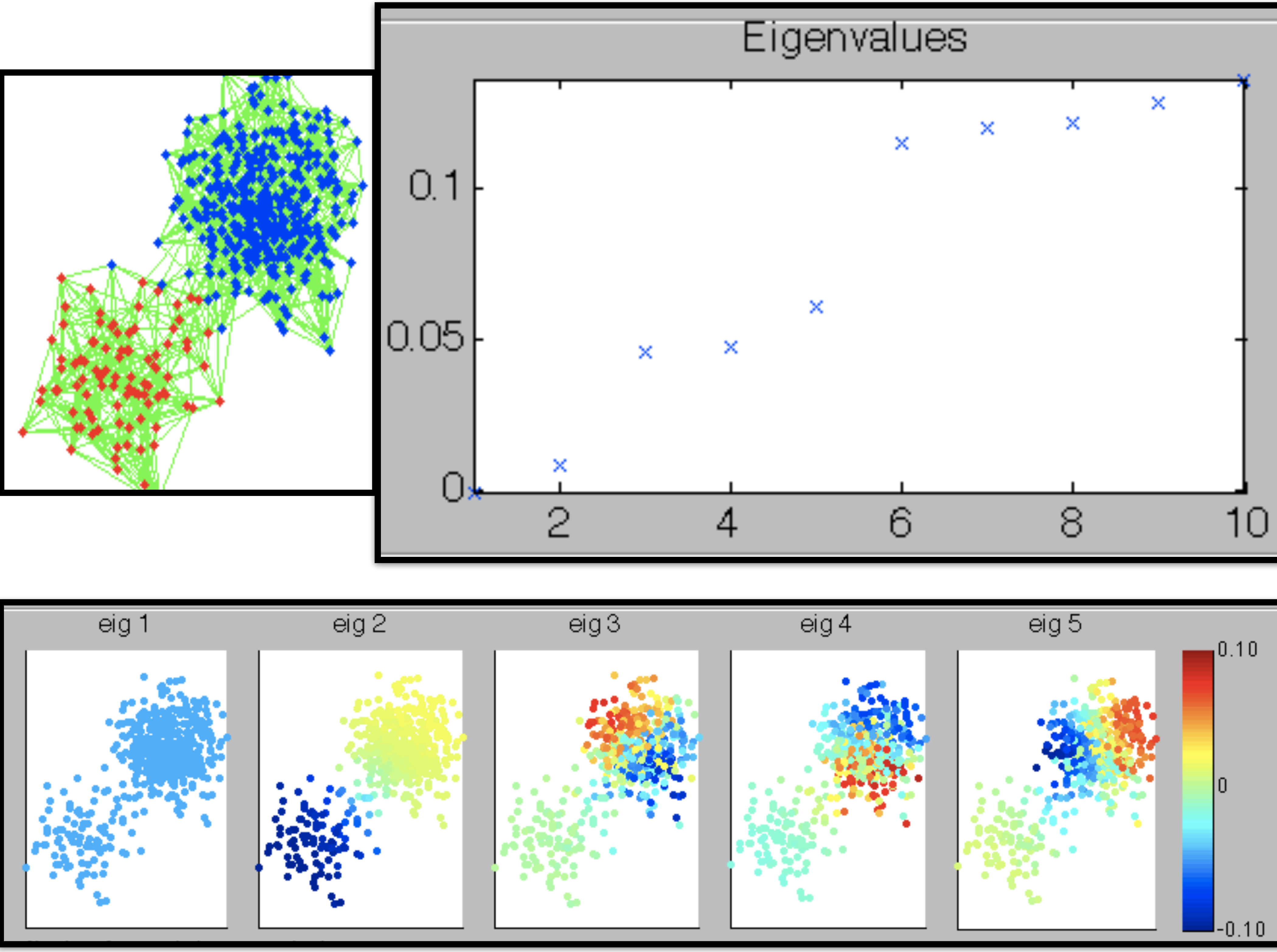


4th Eigen Vector



5th Eigen Vector




Eigenvalues
0.1
0.05
0
2
4
6
8
10
eig 1
eig 2
eig 3
eig 4
eig 5
0.10
0
-0.10

Done!

Welcome to the World of Network Analysis