

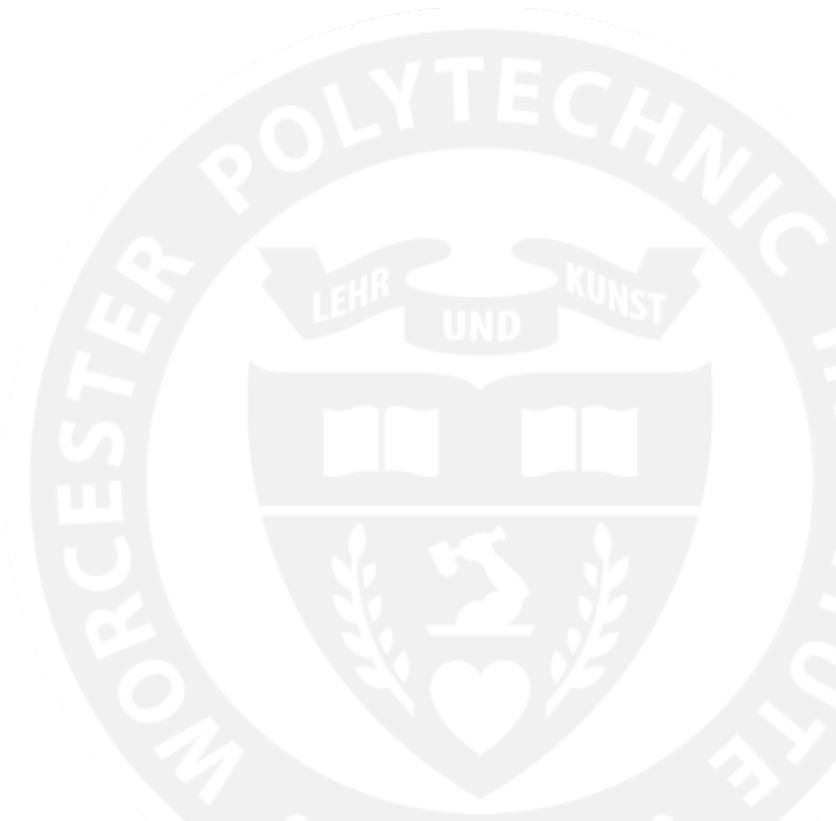


WPI

Artificial Intelligence

CS 534

Week 2



Specifying the task environment

Task environments, are the “problems” to which rational agents are the “solutions.”

Task environment is defined using PEAS descriptors:

- Performance measure
- Environment
- Actuators
- Sensors

Example: Automated taxi

Performance measures: safety, destination, profits, legality, comfort, etc.

Environment: US streets/freeways, traffic, pedestrians, customers, weather, etc.

Actuators: steering, accelerator, brake, horn, speakers, display, etc.

Sensors: cameras, accelerometers, gauges, engine sensors, keyboard, GPS, etc.

Environment types

Fully vs. partially observable: Agent's sensors give it access to the complete (vs. incomplete) state of the environment at each time point

Single agent vs. multiagent: An agent operating by itself in an environment (vs. multiple agents in the same environment)

Deterministic vs. stochastic: The next state of the environment is completely (vs. partially) determined by the current state and the action executed by the agent. If the environment is deterministic **except** for the actions of other agents, then the environment is called **strategic**

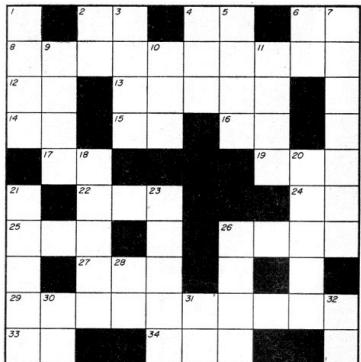
Episodic vs. sequential: Agent's experience is divided (vs. not divided) into atomic 'episodes' (each episode consists of the agent perceiving and then performing a single action). The choice of action in each episode depends **only on the episode itself**

Environment types

Static vs. dynamic: The environment is unchanged while an agent is deliberating. The environment is **semi-dynamic** if the environment itself does not change with the passage of time but the agent's performance score does

Discrete vs. continuous: The way the time is handled: *E.g.*, a limited number of distinct, clearly defined percepts and actions

Task environments: Examples



crossword



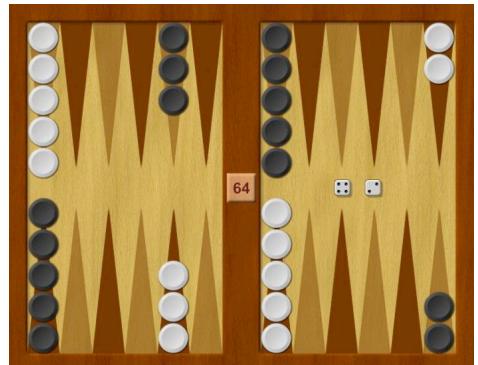
chess with timer



poker



autonomous
taxi



backgammon



medical
diagnostics

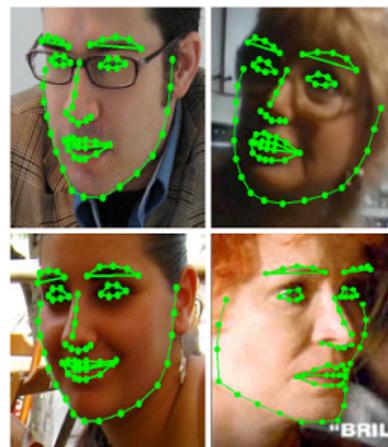


image
processing



interactive
English tutor

Task environments: Examples

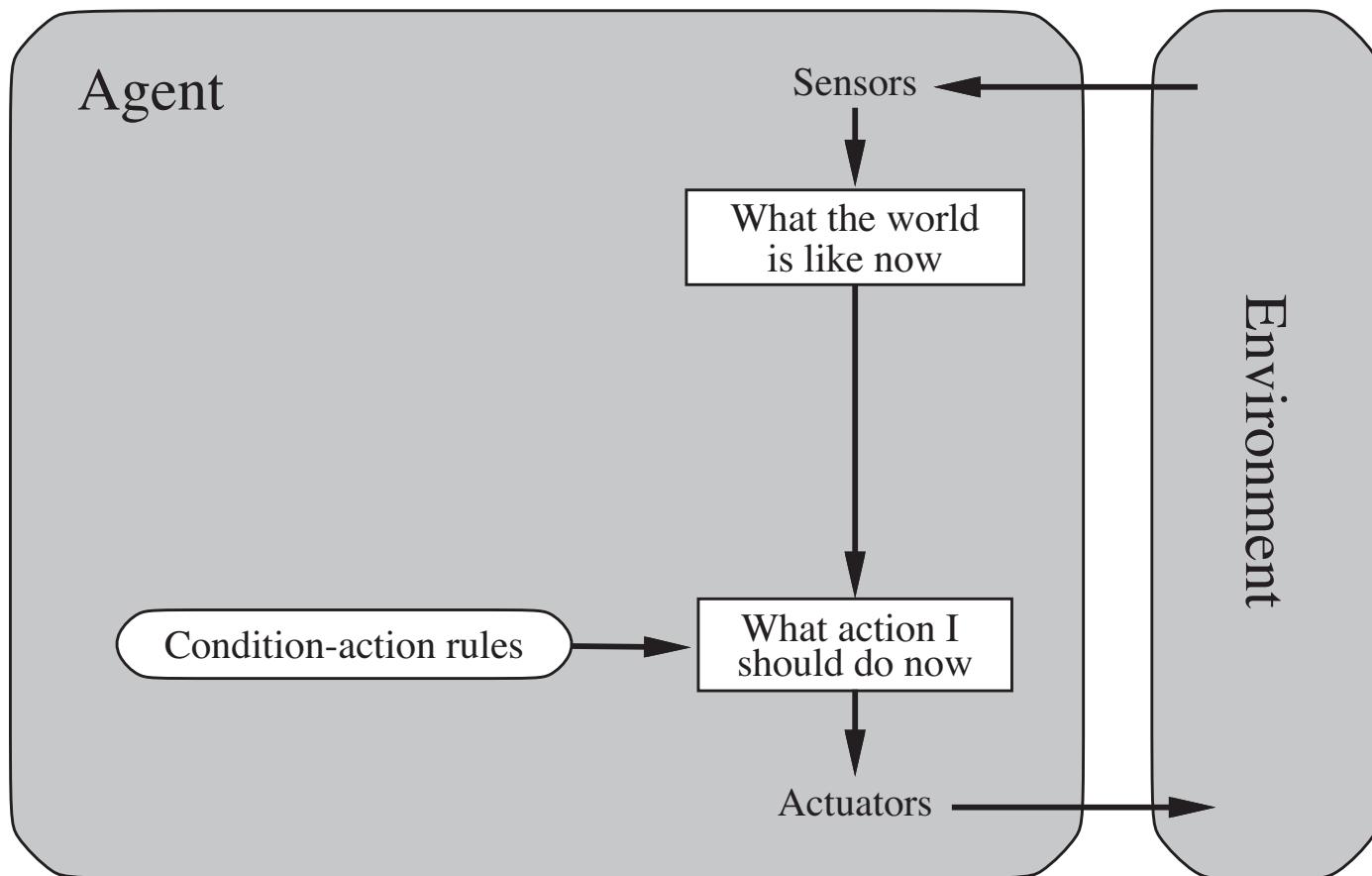
Task Environment	Observ-able	Agent	Determ.	Episodic	Static	Discrete
Crossword	Fully	S	Determ.	Sequen.	Static	Discrete
Chess w. clock	Fully	M	Determ.	Sequen.	Semi	Discrete
Poker	Partial.	M	Stoch.	Sequen.	Static	Discrete
Backgammon	Fully	M	Stoch.	Sequen.	Static	Discrete
Taxi driving	Partial.	M	Stoch.	Sequen.	Dynamic	Continuous
Diagnostics	Partial.	S	Stoch.	Sequen.	Dynamic	Continuous
Image analys.	Fully	S	Determ.	Episod.	Semi	Continuous
English tutor	Partial.	M	Stoch.	Sequen.	Dynamic	Discrete

Agent programs

Four basic types of agent programs:

- Simple reflex agents
 - Model-based reflex agents
 - Goal-based agents
 - Utility-based agents
-
- Note: Each of these can be turned into a **learning agent** (can improve the performance of their components to generate better actions)

Simple reflex agents



Example

```
function REFLEX-VACUUM-AGENT( [location,status]) returns an action
    if status = Dirty then return Suck
    else if location = A then return Right
    else if location = B then return Left
```

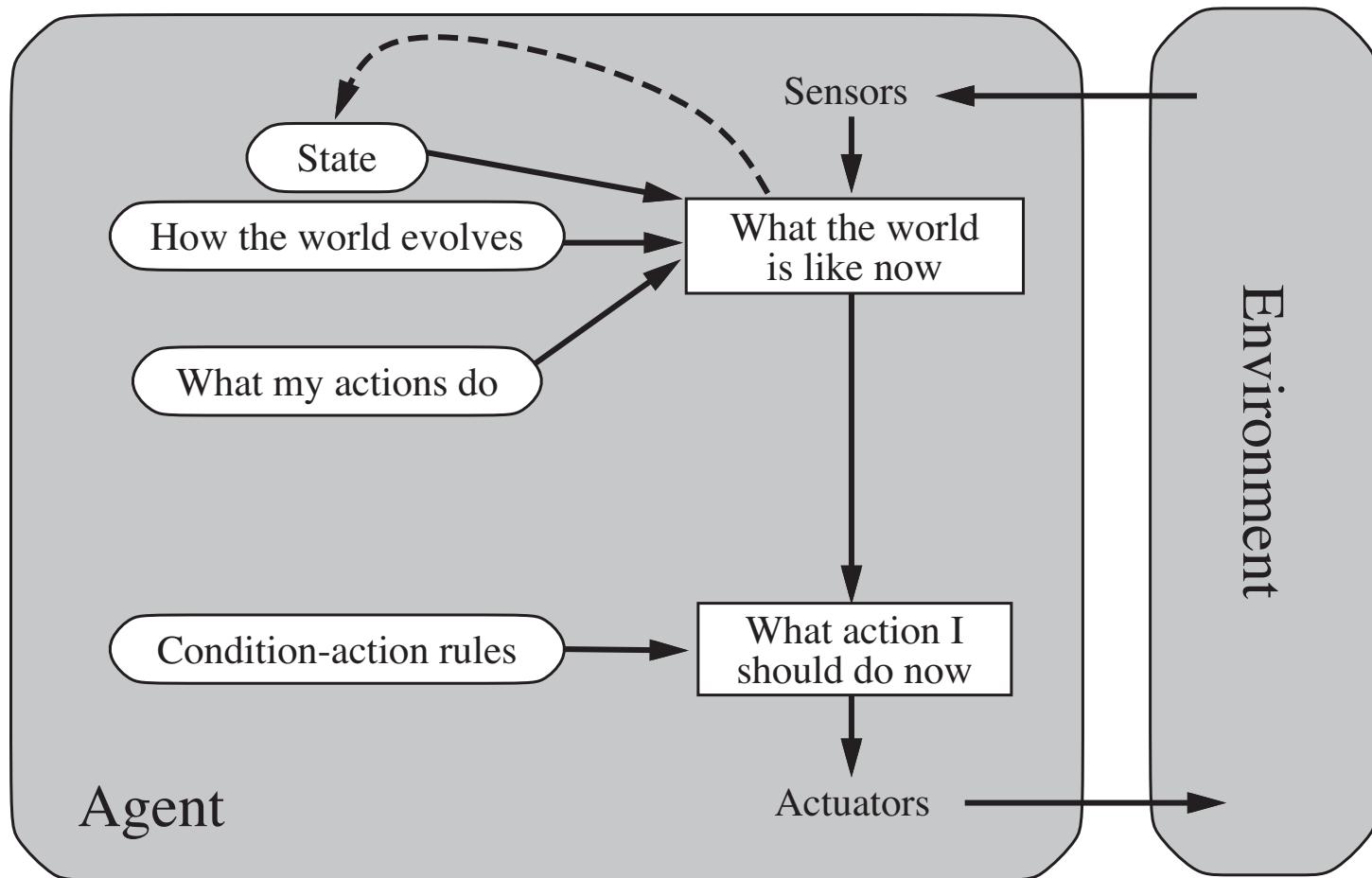
```
function SIMPLE-REFLEX-AGENT(percept) returns action
    static: rules, a set of condition-action rules
    state  $\leftarrow$  INTERPRET-INPUT(percept)
    rule  $\leftarrow$  RULE-MATCH(state, rules)
    action  $\leftarrow$  RULE-ACTION[rule]
    return action
```

- selects rule whose condition matches the current state based on percept
- acts accordingly

Simple reflex agents: Cons and pros

- Very simple
- Will work only if a correct decision can be made on the basis of only the **current percept**
 - *i.e.* environment is fully observable
- Otherwise — lots of problems, with potential infinite loops
- Infinite loops can be escaped from if the agent can randomize its actions

Model-based reflex agents



Example

function MODEL-BASED-REFLEX-AGENT(*percept*) **returns** an action

persistent: *state*, the agent's current conception of the world state

model, a description of how the next state depends on current state and action

rules, a set of condition-action rules

action, the most recent action, initially none

state \leftarrow UPDATE-STATE(*state*, *action*, *percept*, *model*)

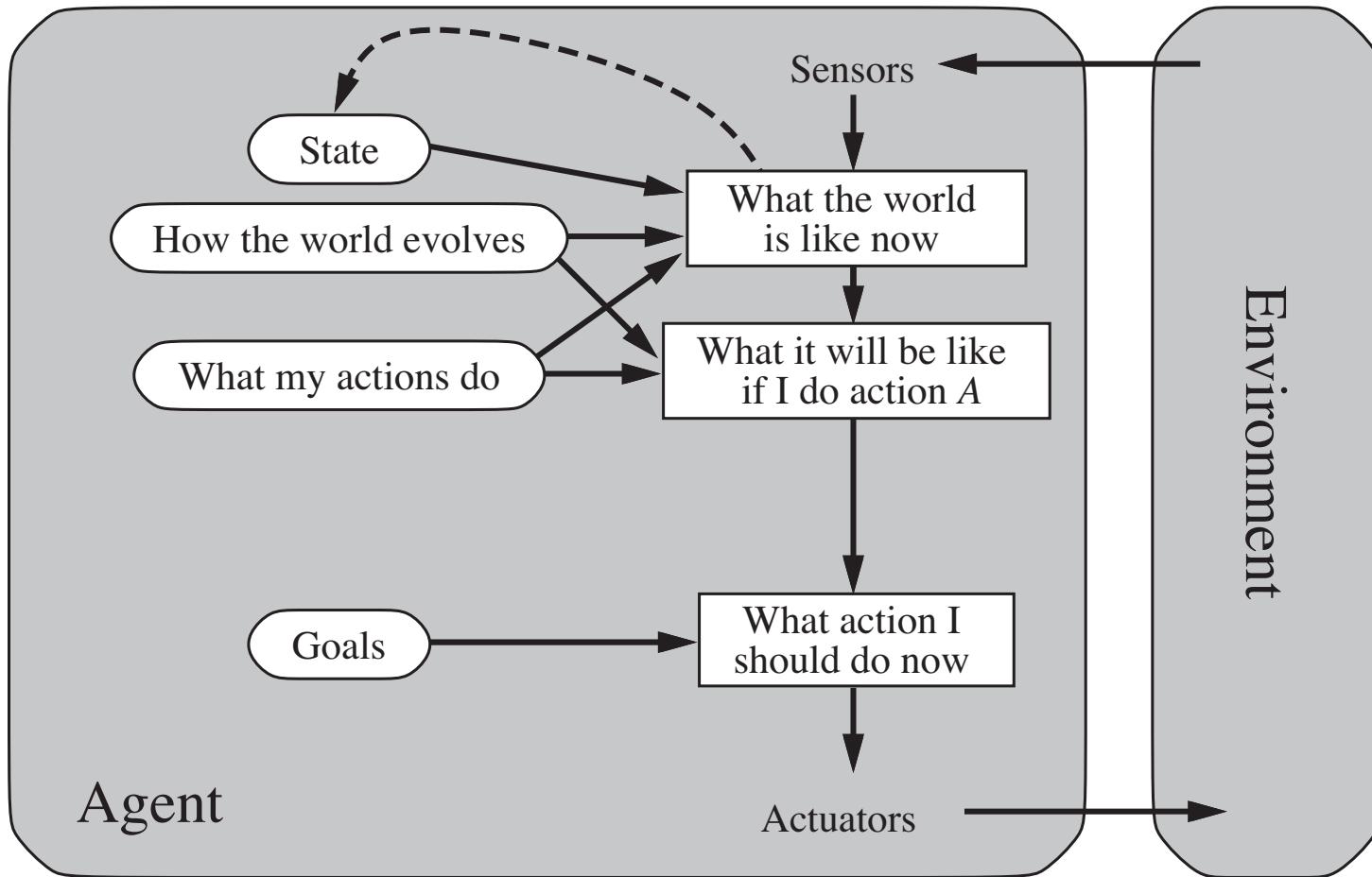
rule \leftarrow RULE-MATCH(*state*, *rules*)

action \leftarrow *rule.ACTION*

return *action*

- Keeps track of the current state of the world using an internal model

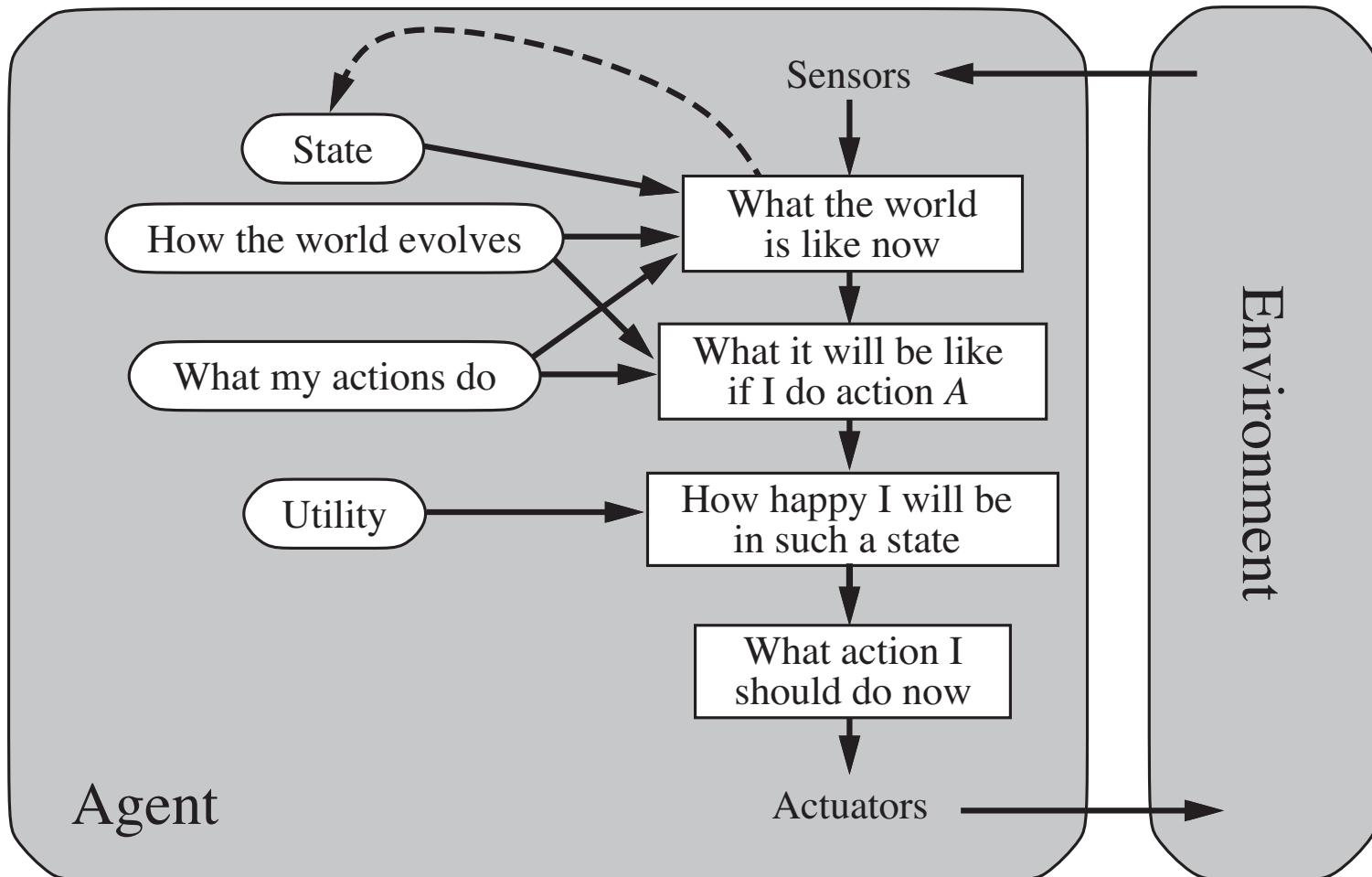
Goal-based agents



Utility-based agents

- Some action sequences are better than others: quicker, safer, more reliable, or cheaper than others, etc..
- Idea: introduce a utility function, an “internalization” of the performance measure. If the internal utility function and the external performance measure are in agreement, then an agent that chooses actions to maximize its utility will be rational according to the external performance measure
- Utility-based agent has many advantages in terms of flexibility and learning
- It allows to resolve the conflicting goals: e.g., speed vs. safety
- When there are several goals, none of which can be achieved with certainty, utility provides a way in which the likelihood of success can be weighed against the importance of the goals

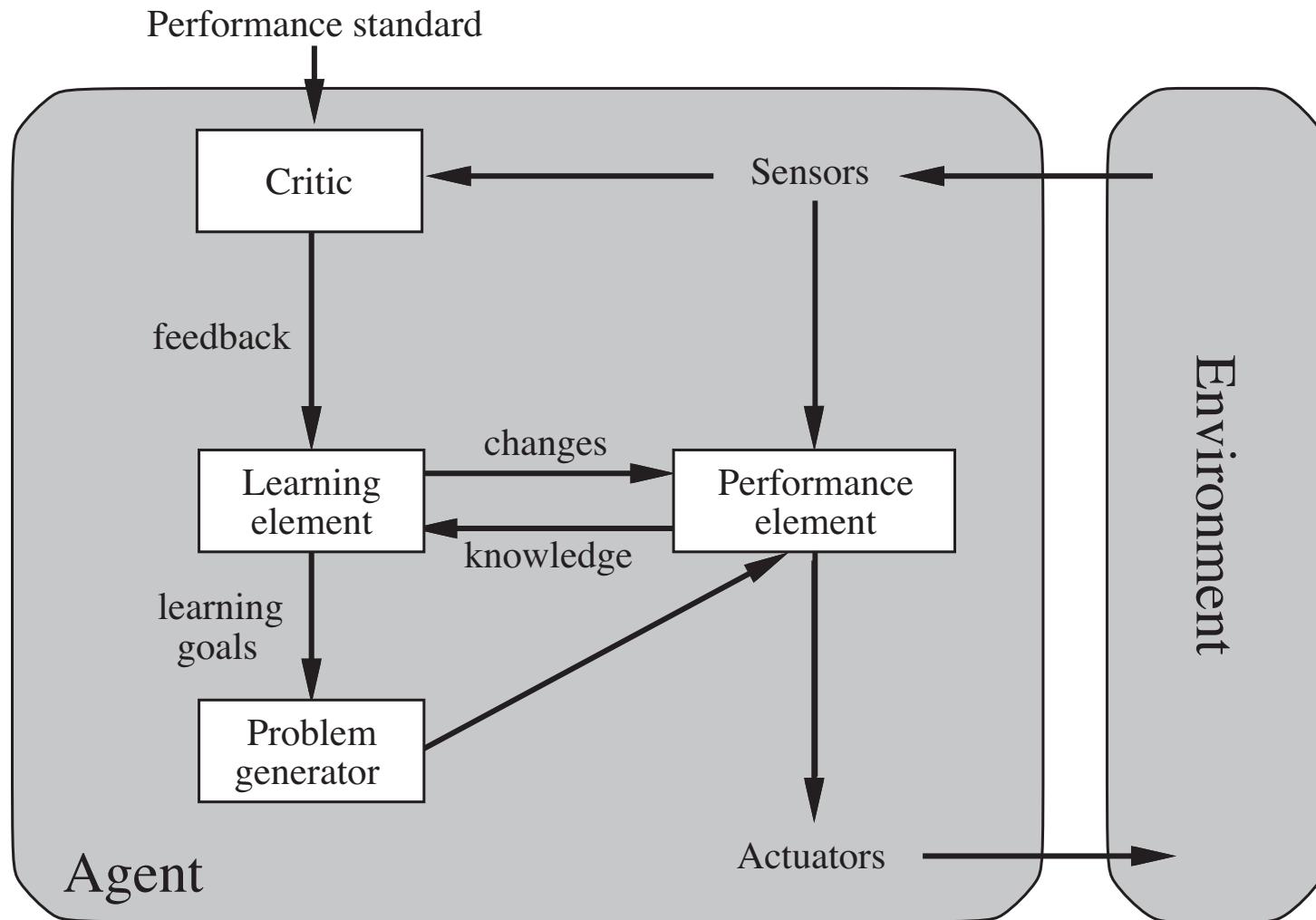
Utility-based agents



Learning agents

- Consists of four basic components: Critic, Learning Element, Performance Element, Problem Generator
- **Learning element:** is responsible for making improvements
- **Performance element:** is responsible for selecting external actions (previously, the entire agent)
- **Problem generator:** is responsible for suggesting actions that will lead to new and informative experiences

Learning agents



Summary

- Agents interact with environments through **actuators** and **sensors**
- The **agent function** describes what the agent does in all circumstances
- The **performance measure** evaluates the environment sequence
- A perfectly **rational** agent maximizes expected performance
- **Agent programs** implement (some) agent functions
- **PEAS** descriptions define task environments
- **Environments** are categorized along several dimensions:
 - observable; deterministic; episodic; static; discrete; single-agent
- Several basic **agent architectures** exist:
 - simple reflex, model-based reflex, goal-based, utility-based

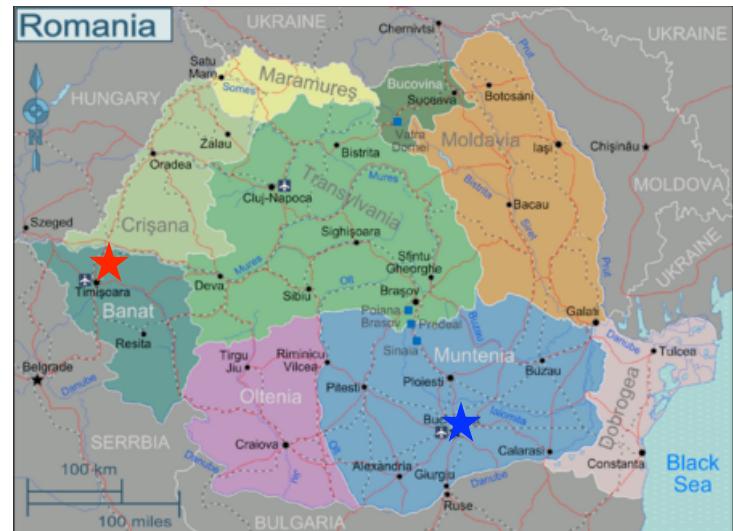
Unit 2. Searching. Problem Solving Agents

Let's define a problem solving agent...

- They use **atomic** representations: states of the world are considered as wholes, with no internal structure visible to problem solving algorithms
- Search algorithms can be:
 - **Uninformed**: no information about its problem rather than its definition is given
 - **Informed**: are provided with some guidance on where to look for solutions

Example: Travelling in Romania

- A touring holiday in the city of Arad, Romania
- Non-refundable ticket to fly out from Bucharest tomorrow
- Must reach Bucharest on time



Note: Goal formulation is therefore extremely simple!



Worcester Polytechnic Institute

Example: Travelling in Romania

Goal formulation: be in Bucharest

Problem formulation:

- the process of actions and states to consider given a goal
- **states:** Romanian cities
- **actions:** drive roads between cities

Solution: sequence of actions—a driving route

Example: Arad, Sibiu, Rimnicu, Pitesti, Bucharest

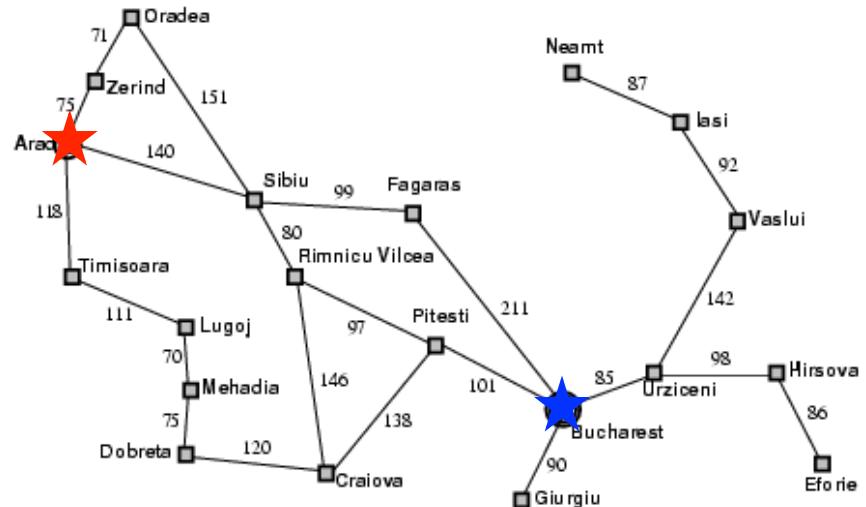
Example: Travelling in Romania

Worst case scenario: Unknown environment, i.e. no additional information

- No choice, but to try one of the actions on random

Better case scenario: Agents has a map of Romania!

- An agent with several immediate options of unknown value can decide what to do by first examining future actions
- Environment is **observable**
- Environment is **discrete**
- Environment is **deterministic**



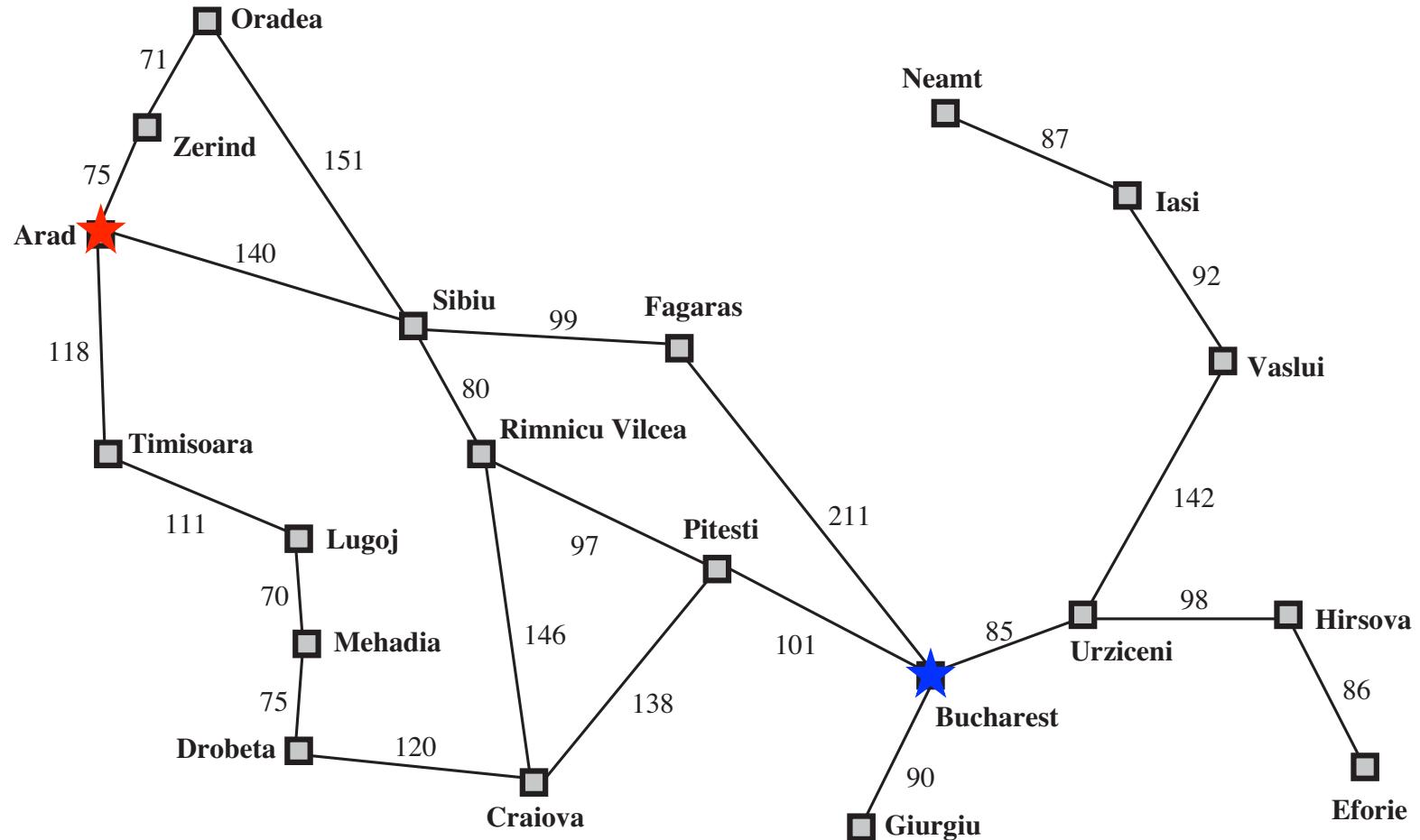
A Simple Problem Solving Agent

Problem can be defined formally by five components:

1. The initial state that the agent starts in
 - $In(Arad)$
2. Description of possible actions. For a state s , $Actions(s)$ returns a set of actions executable from s
 - For $s = In(Arad)$, $Actions(s)$ returns:
 $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$
3. Description of transition model (what an action does). For a state s and action a , $Results(s, a)$ returns the next state
 - For $Results(In(Arad), Go(Zerind))$ returns: $In(Zerind)$

Definition: Initial state + actions + transition model = State space
State space forms a directed graph, where nodes are states and edges are actions.

Travelling in Romania



A Simple Problem Solving Agent

4. The **goal test**: determines if a given state is the goal state
 - $\{In(Bucharest)\}$
5. A **path cost** function. Described as a sum of step costs for individual actions along the path: $c(s, a, s')$
 - For our example the cost the the distance:
 $c(In(Arad), Go(Sibiu), In(Sibiu)) = 140$

Definitions:

Solution: An action sequence from initial state to goal state

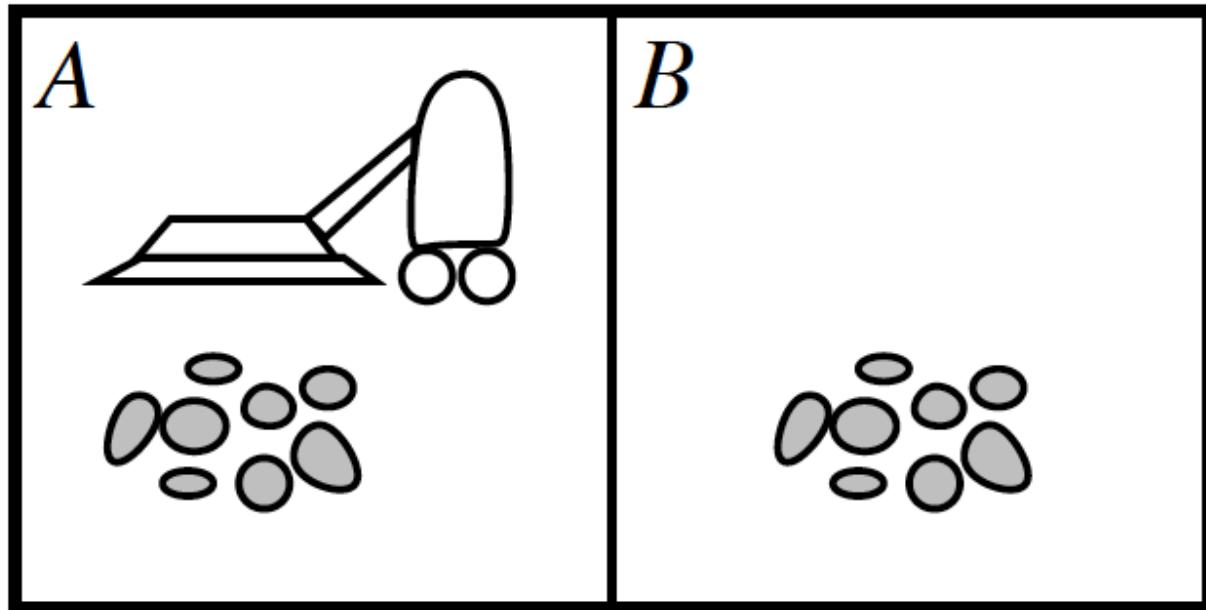
Optimal solution: The one with the lowest path cost among all solutions

A Simple Problem Solving Agent

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  static: seq, an action sequence, initially empty
          state, some description of the current world state
          goal, a goal, initially null
          problem, a problem formulation

  state  $\leftarrow$  UPDATE-STATE(state, percept)
  if seq is empty then
    goal  $\leftarrow$  FORMULATE-GOAL(state)
    problem  $\leftarrow$  FORMULATE-PROBLEM(state, goal)
    seq  $\leftarrow$  SEARCH(problem)
  action  $\leftarrow$  RECOMMENDATION(seq, state)
  seq  $\leftarrow$  REMAINDER(seq, state)
  return action
```

Recall: Vacuum world



Percepts: location and contents, e.g., [A, Dirty]

Actions: *Left*, *Right*, *Suck*, *NoOp*

Example: Vacuum world

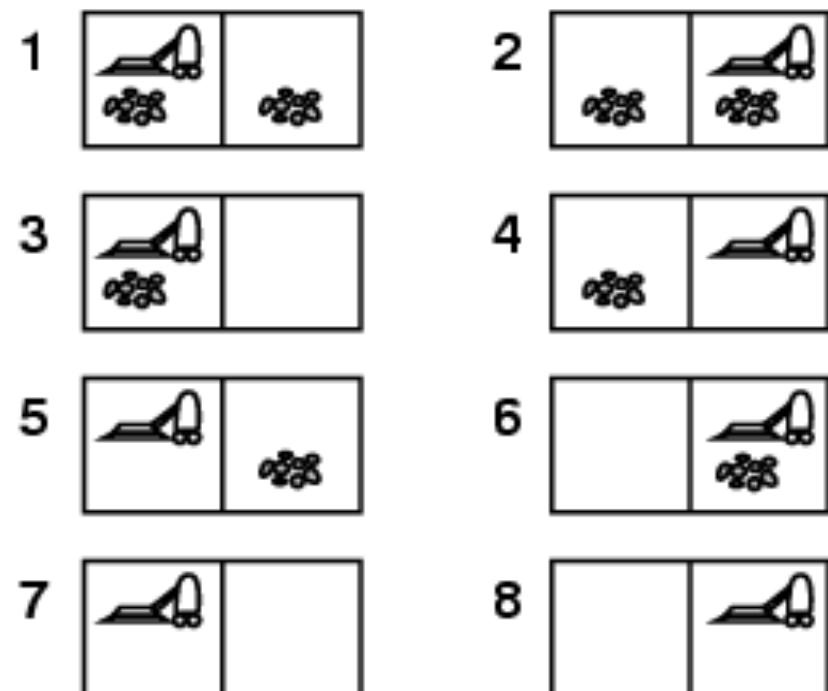
States: robot location, dirt locations \rightarrow 8 possible states

- For n locations, there are $n \cdot 2^n$ states

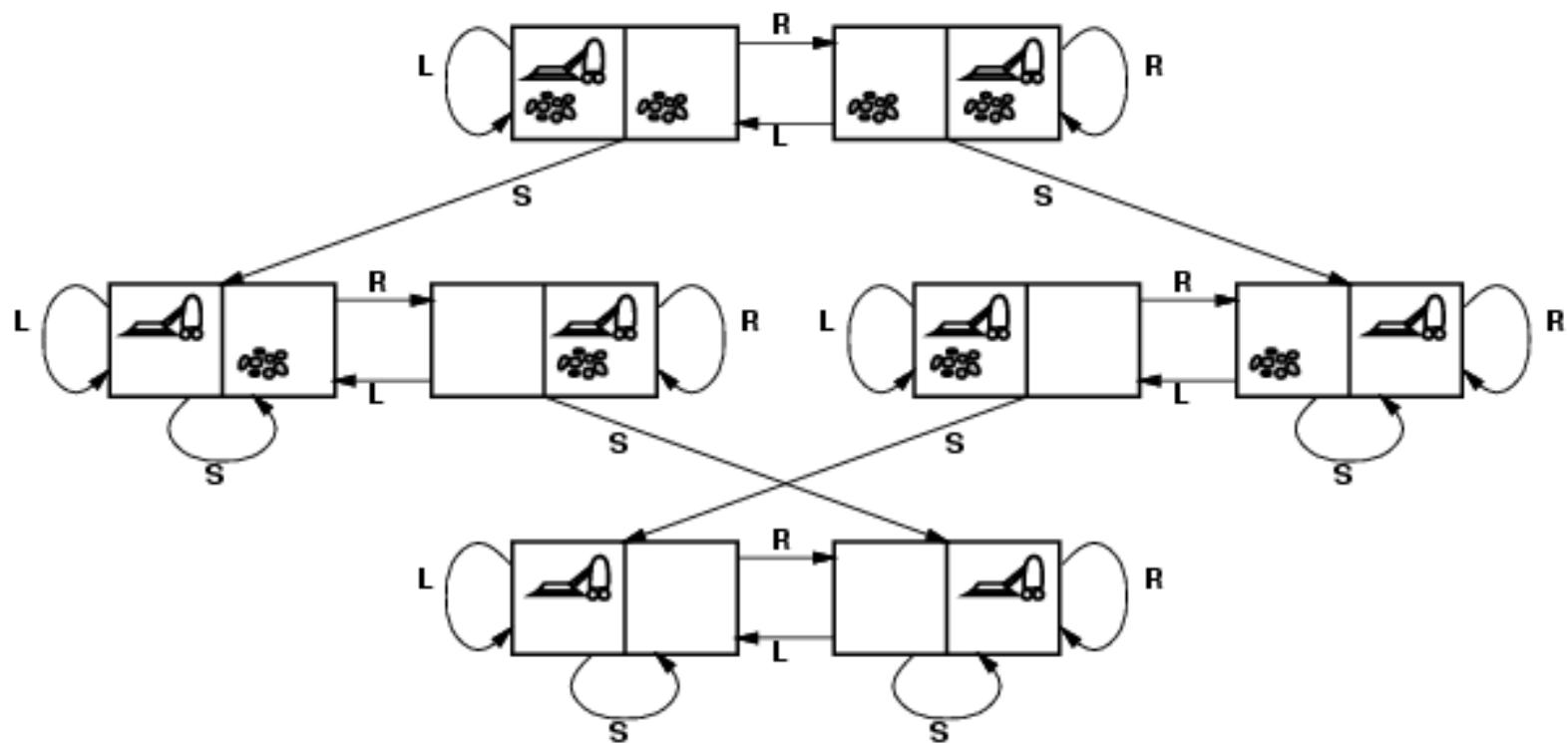
Actions: {Left, Right, Suck}

Goal test: no dirt at all locations

Path cost: $c(s, a, s') = 1$

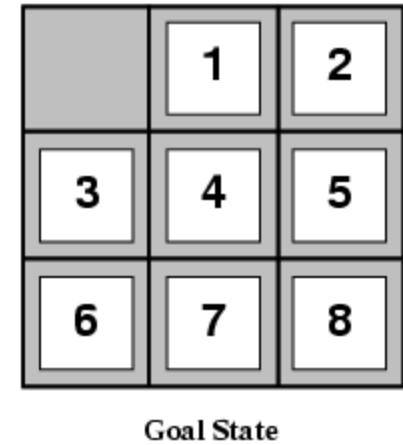
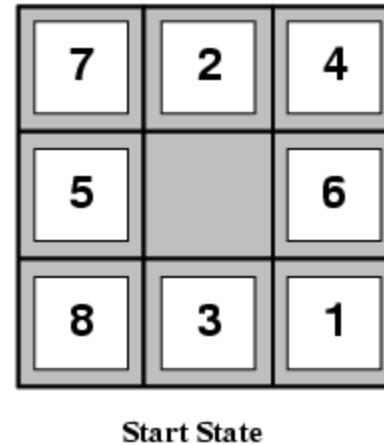


State space of Vacuum world



Example: 8-puzzle

- 3x3 board with 8 number tiles and an empty space
- A tile adjacent to an empty space can slide into the space
- The objective is to reach the goal state
- 8 puzzle belongs to the family of sliding-block puzzles, that are often used to test new search algorithms
- The problem is known to be **NP-complete**
- 8 puzzle has $9!/2=181,440$ reachable states (easy to solve), while the 15-puzzle (4x4 board) has ~ 1.3 trillion states, and 24-puzzle (5x5 board) has 10^{25} states

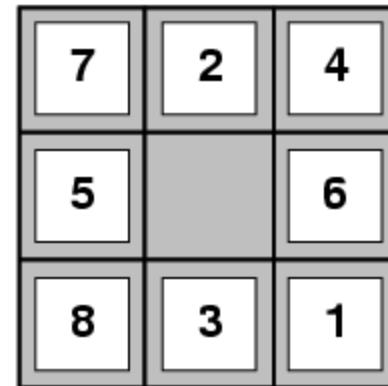


Example: 8-puzzle

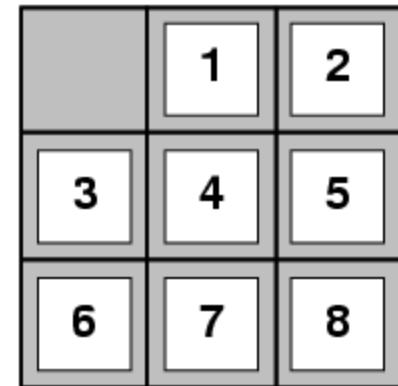
States: Location of each of the tiles

Initial state: Any configuration

Actions: {Left, Right, Up, Down}



Start State



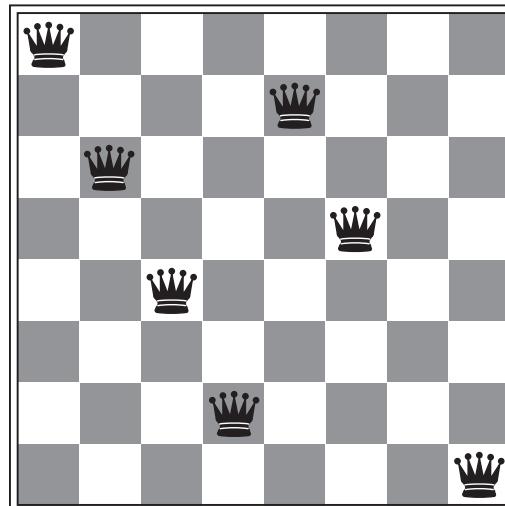
Goal State

Goal test: check if the state matches the goal test configuration

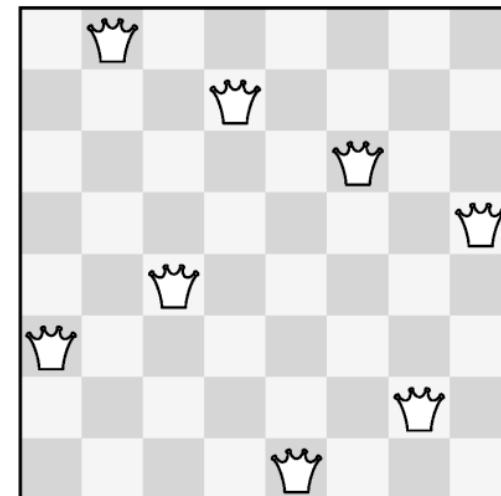
Path cost: $c(s, a, s')$ = 1

Example: 8-queens problem

- Goal: place eight queens on a chessboard such that no queen attacks any other
- A queen attacks another queen on the same row, column, or diagonal



WRONG



CORRECT

8-queens problem: Naïve incremental formulation

States: Any arrangement of 0 to 8 queens

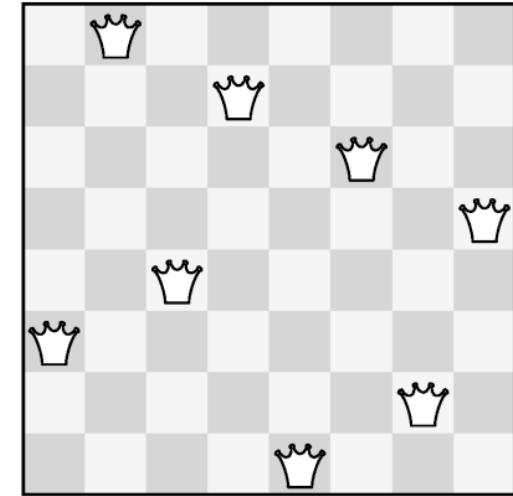
Initial state: No queens on the board

Actions: Add a queen to any empty space

Note: This is an incremental formulation

Transition model: Return the board with a queen added to specified square

Goal test: 8 queens on the board, none are attacked



Note: Sequences to investigate = $64 \times 63 \times \dots \times 57 \sim 1.8 \times 10^{14}$

For 100-queen problem – roughly 10^{400} states !!!

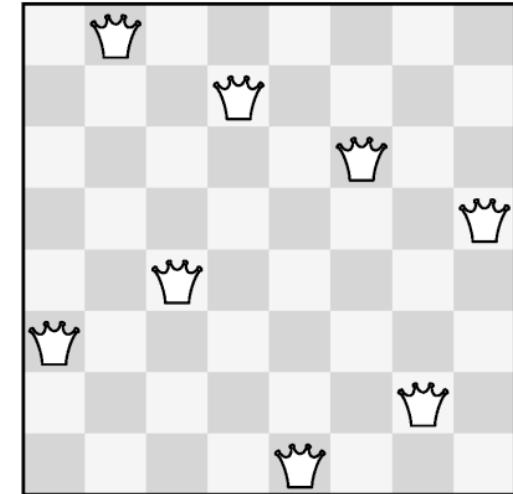
8-queens problem: Advanced incremental formulation

States: Any arrangement of n queens, one per column in the leftmost n columns, with no queen attacking another

Actions: Add a queen to any square in the leftmost empty column such that it is not attacked by a queen

Goal test: 8 queens on the board, none are attacked

Note: Sequences to investigate are reduced from 1.8×10^{14} to 2,057



Toy vs. Real-world problems

1. Route-finding (computer networks, transport, military operation planning, etc.)
2. Touring: travel through all cities
3. VLSI Layout
4. Robot navigation
5. Protein design

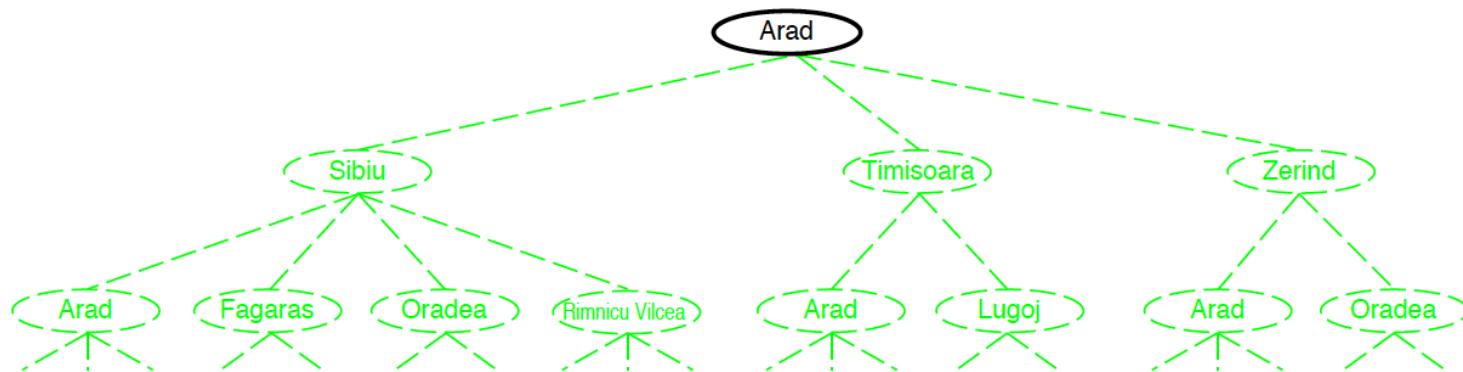
Searching using search tree

Ideas:

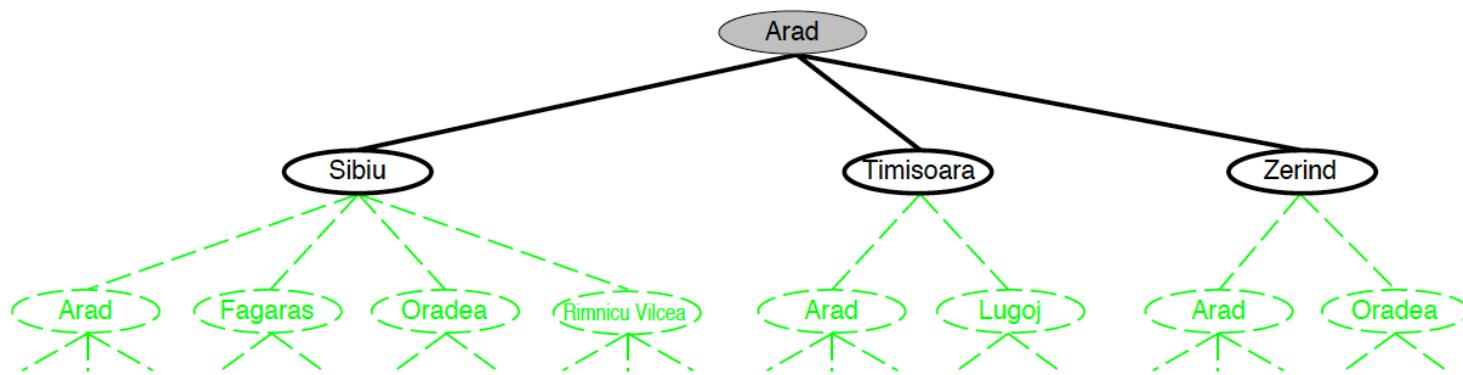
- explore the state space
- expand from the current state
- how to expand the state will depend on the search strategy

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

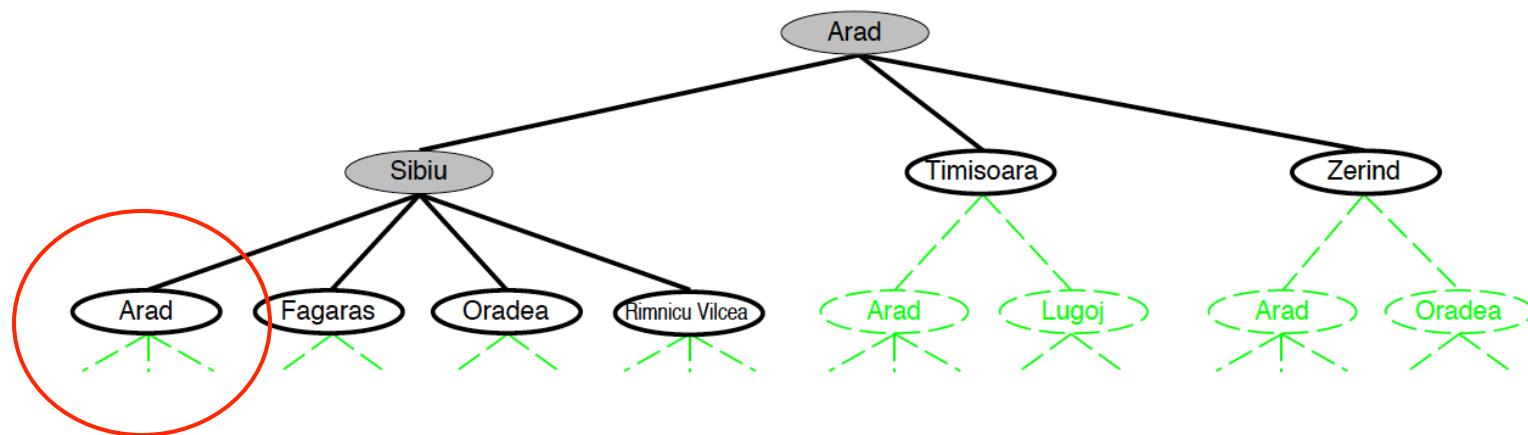
Searching a tree: Example



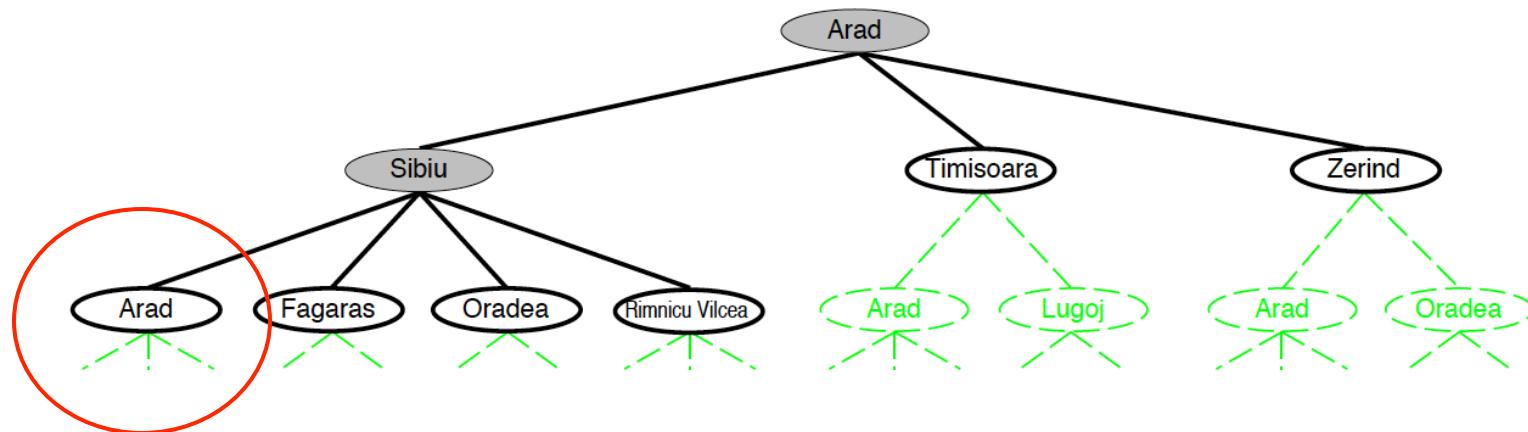
Searching a tree: Example



Searching a tree: Example



Searching a tree: Example



Definition: *In(Arad)* is a **repeated state** generated by a **loopy path**

*This means that the complete search tree is **infinite***

Measuring problem-solving performance

- *Completeness*: Is the algorithm guaranteed to find a solution if there is one?
- *Optimality*: Does the strategy find the optimal solution?
- *Time complexity*: How long does it take to find a solution?
- *Space complexity*: How much memory is needed to perform the search?

Uninformed search

- Aka **the blind search**: the strategy has no additional information about the states beyond that one provided in the problem definition
- *Breadth-first search*
- *Uniform-cost search*
- *Depth first search*
 - *Backtracking search*
- *Depth-limited search*
- *Iterative deepening depth-first search*
- *Bidirectional search*

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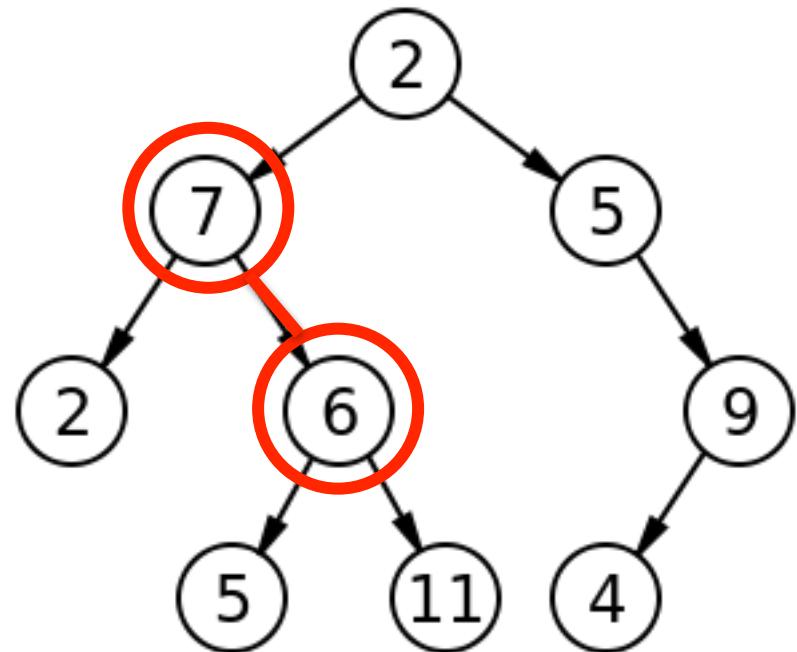
All search strategies are distinguished by the order in which nodes are expanded

Measuring problem-solving performance

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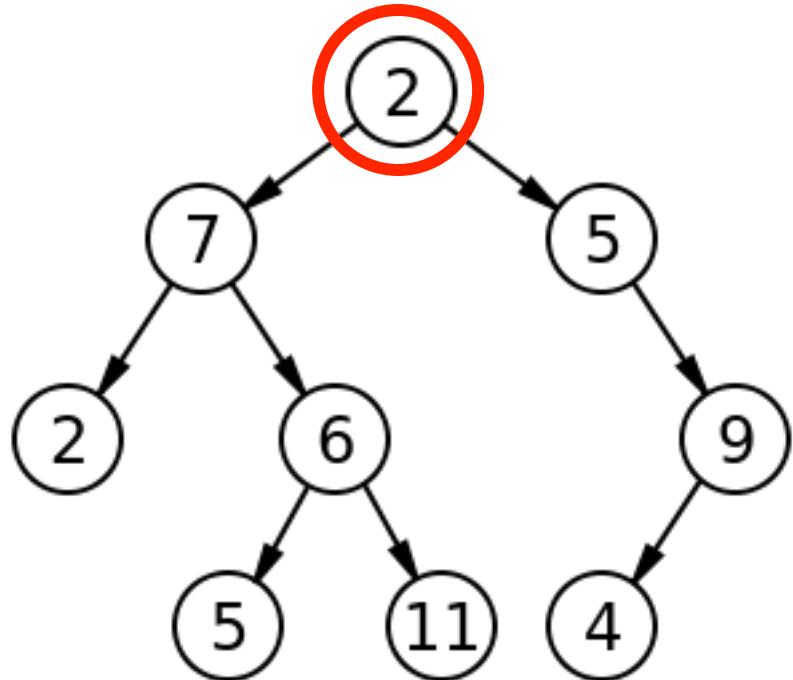
Tree terminology

- *Parent/Child*: Relationship



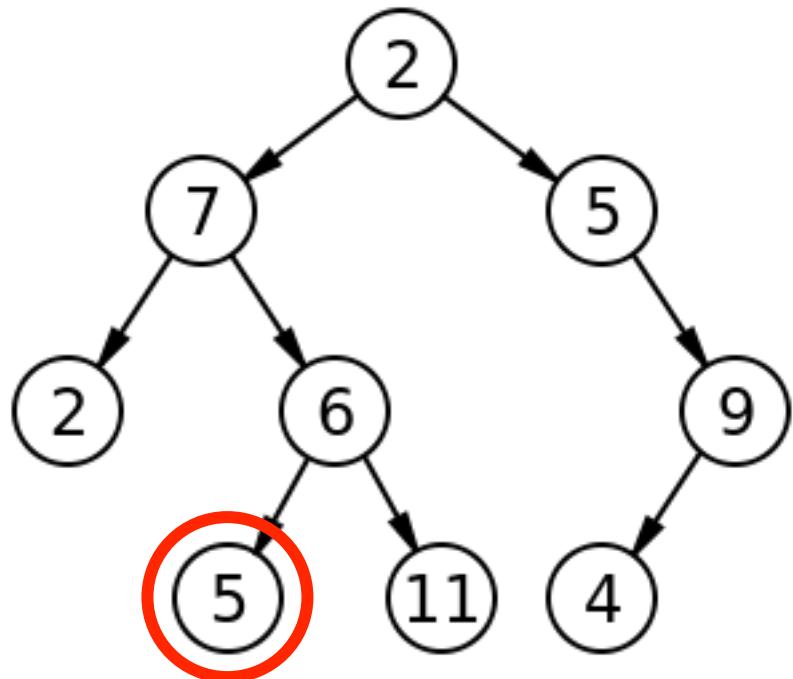
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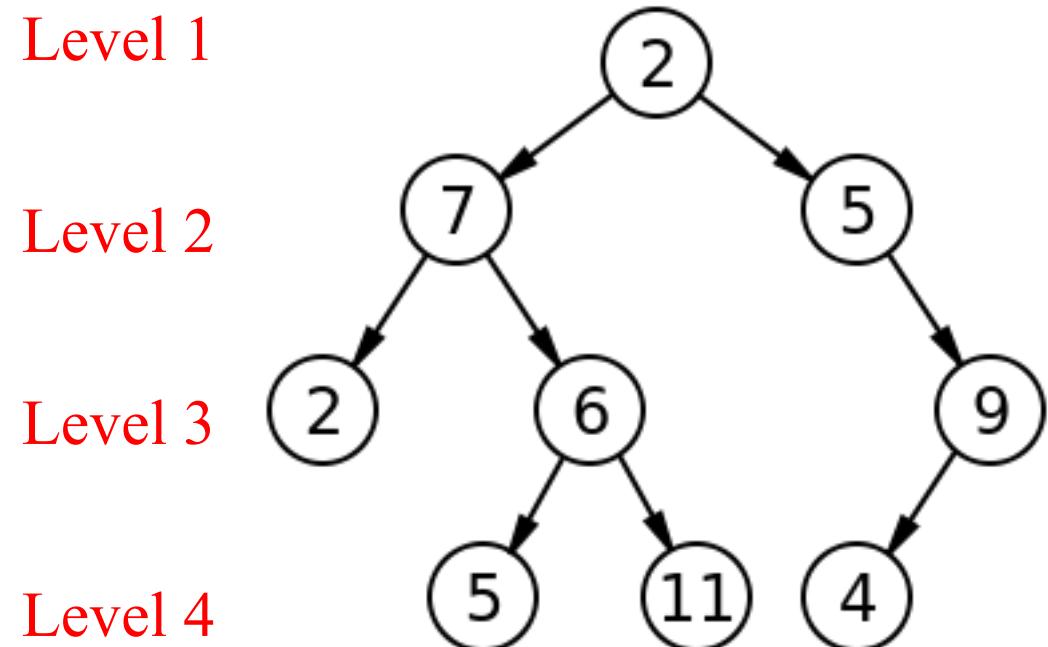
Tree terminology

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- *Leaf*: No child



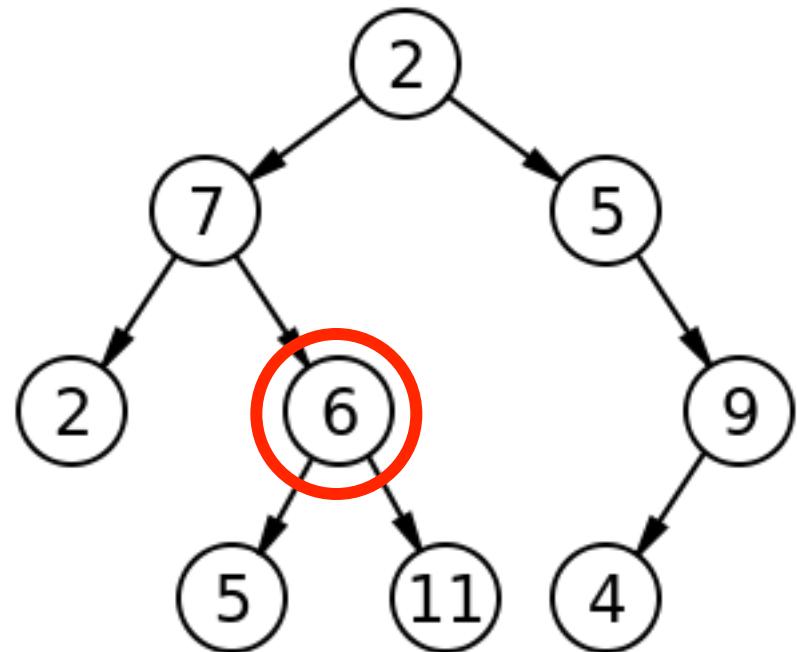
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- *Level*: Depth



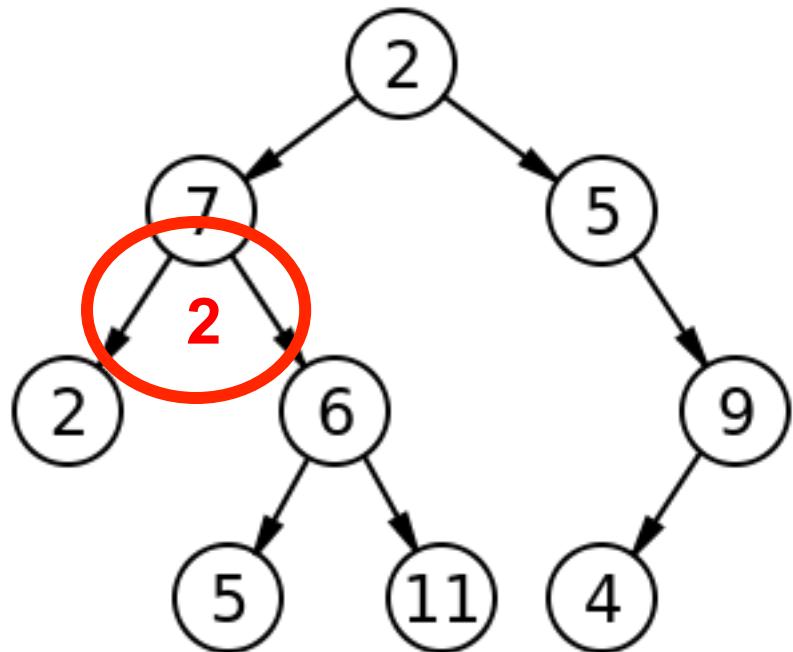
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- *Leaf*: No child
- *Level*: Depth
- *Internal nodes*: Parents and children



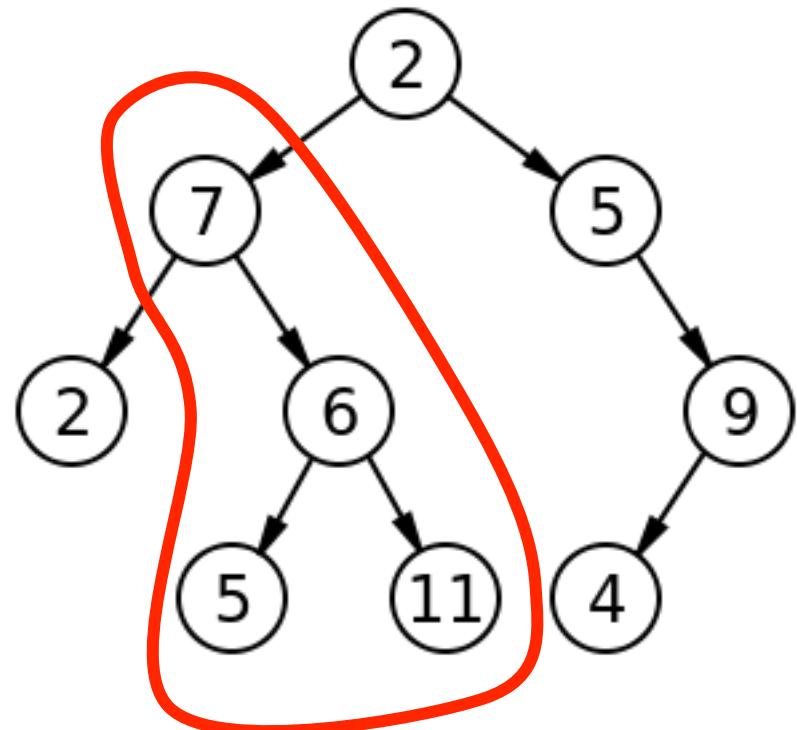
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- *Branching factor*: N of children



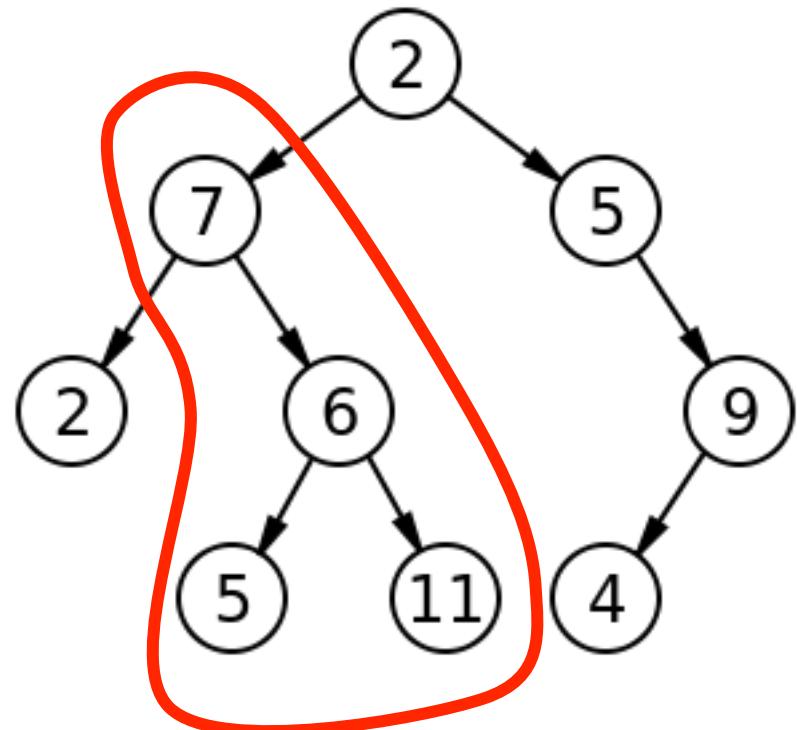
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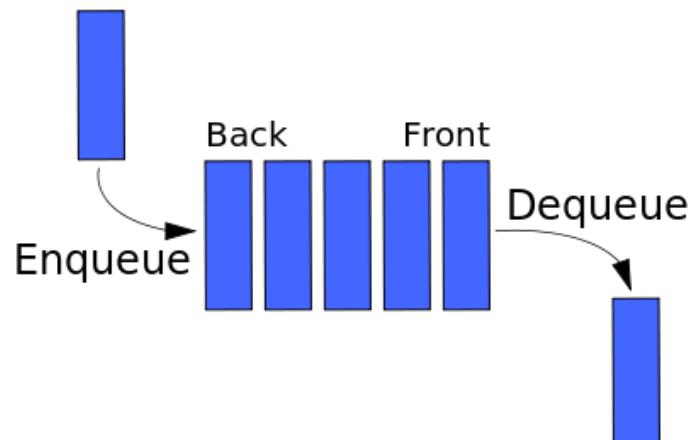
So the complexity of a search algorithm may depend on

- b : maximum branching factor of the search tree
- d : depth of the least-cost (shallowest) solution
- m : maximum depth of the state space (can be infinite)

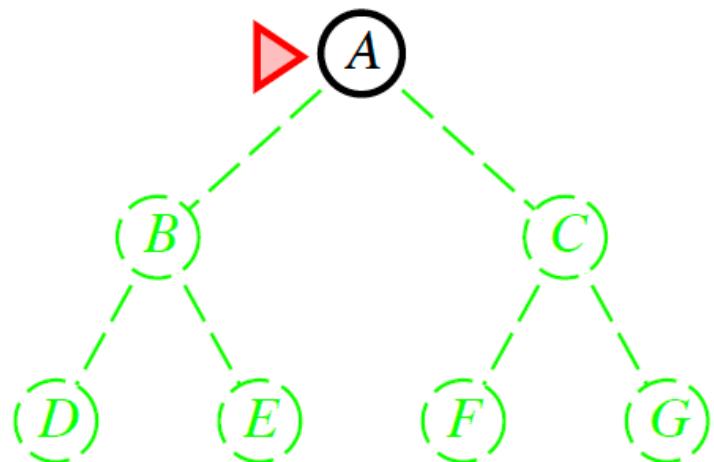
Algorithm 1

Breadth-first search (BFS): Ideas

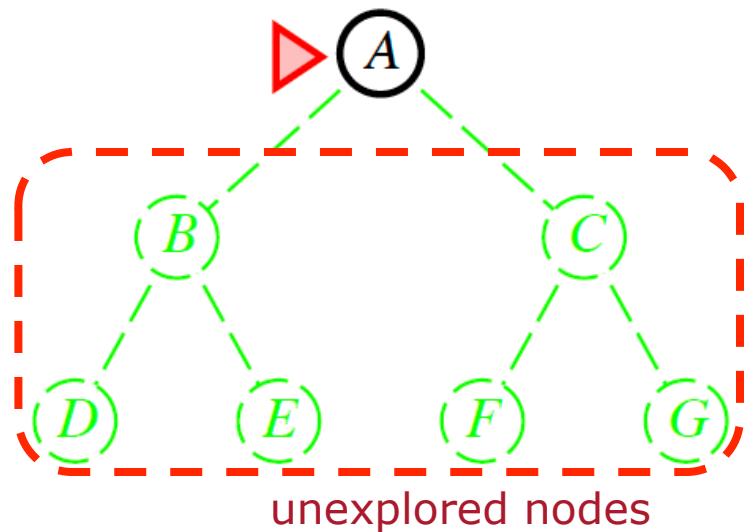
- A simple strategy in which the root node is expanded first, then all successors are expanded, then successors of the successors, etc
- All the nodes are expanded at a given depth in the search tree before the any nodes at the next level are expanded
- Expand the shallowest unexpanded node
- Implementation as a First In First Out (*FIFO*) queue: new successors go at end



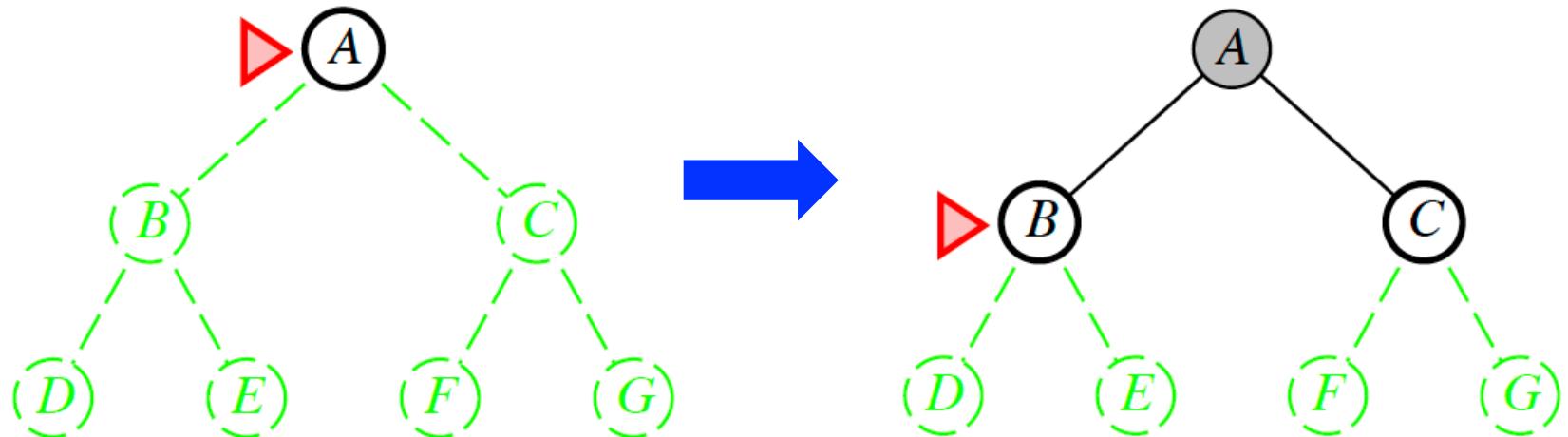
Breadth-first search: Method



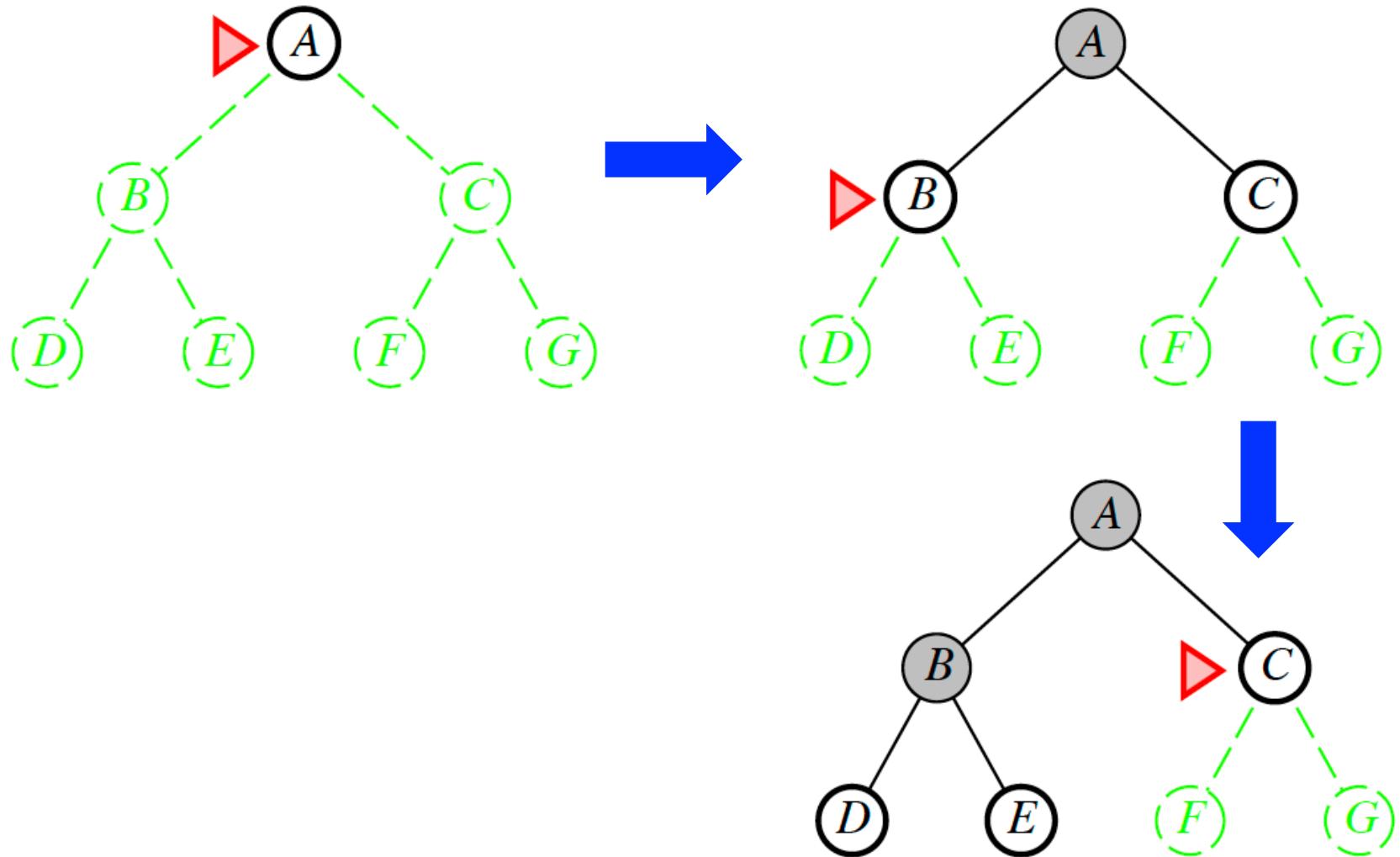
Breadth-first search: Method



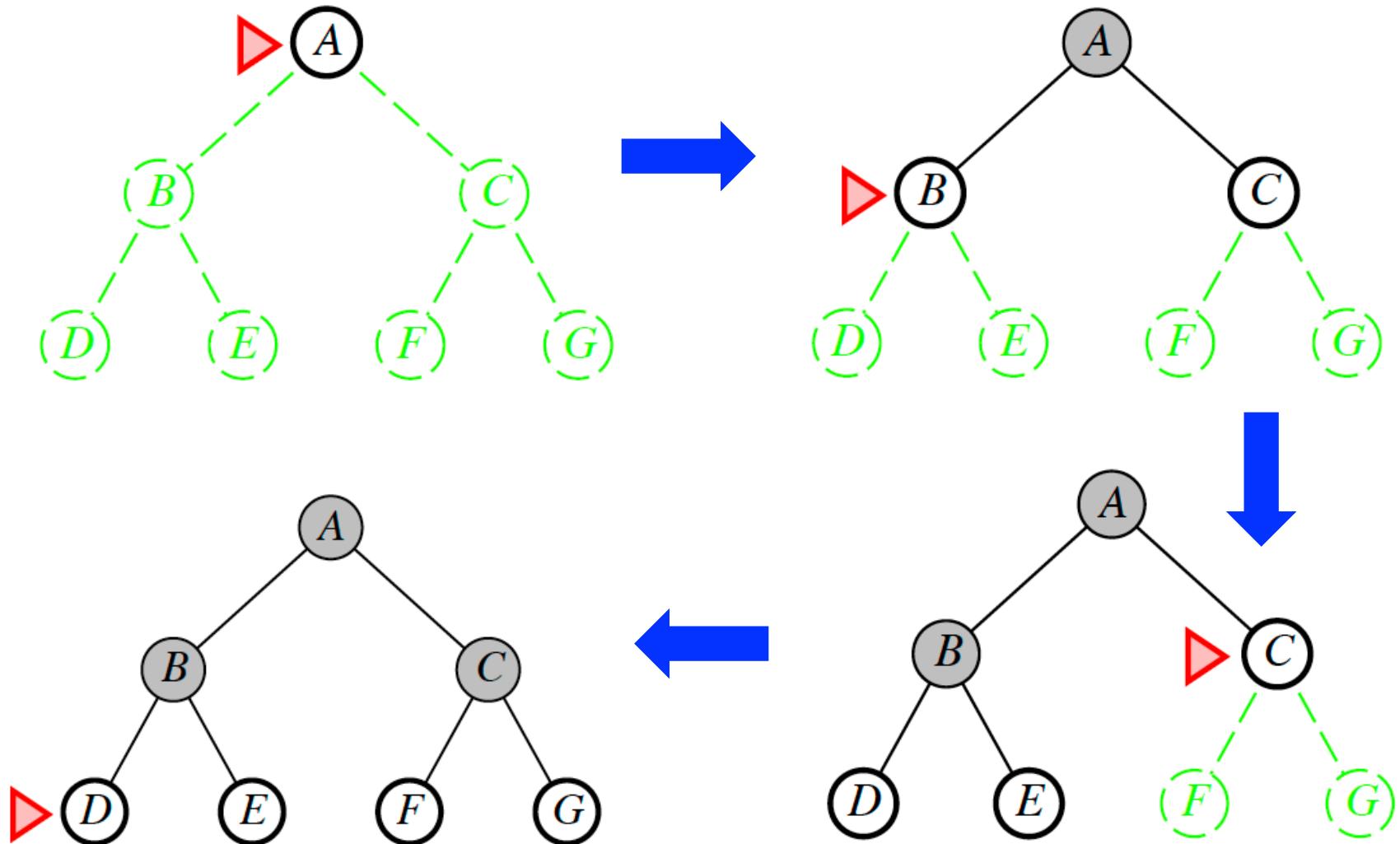
Breadth-first search: Method



Breadth-first search: Method



Breadth-first search: Method



Measuring performance of breadth-first

- *Completeness*: Complete if b is finite

Measuring performance of breadth-first

- *Completeness*: Complete if b is finite
- *Optimality*: Optimal if cost is universally c (e.g. 1) per each step

Measuring performance of breadth-first search

- *Completeness*: Complete if b is finite
- *Optimality*: Optimal if cost is universally c (e.g. 1) per each step
- *Time complexity*: Goal test is applied to each node when it is selected for expansion

$$b + b^2 + b^3 + \dots + b^d + b(b^d - 1) = O(b^{d+1})$$

Measuring performance of breadth-first search

- *Completeness*: Complete if b is finite
- *Optimality*: Optimal if cost is universally c (e.g. 1) per each step
- *Time complexity*:
 - (1) Goal test is applied to each node when it is selected for expansion
$$b+b^2+b^3+\dots+b^d + b(b^d-1) = O(b^{d+1})$$
 - (2) Goal test is applied to each node when it is generated rather than when it is selected for expansion
$$b+b^2+b^3+\dots+b^d = O(b^d)$$
- *Space complexity*: $O(b^{d+1})$ —keeps every node in memory
 - A much bigger problem than time

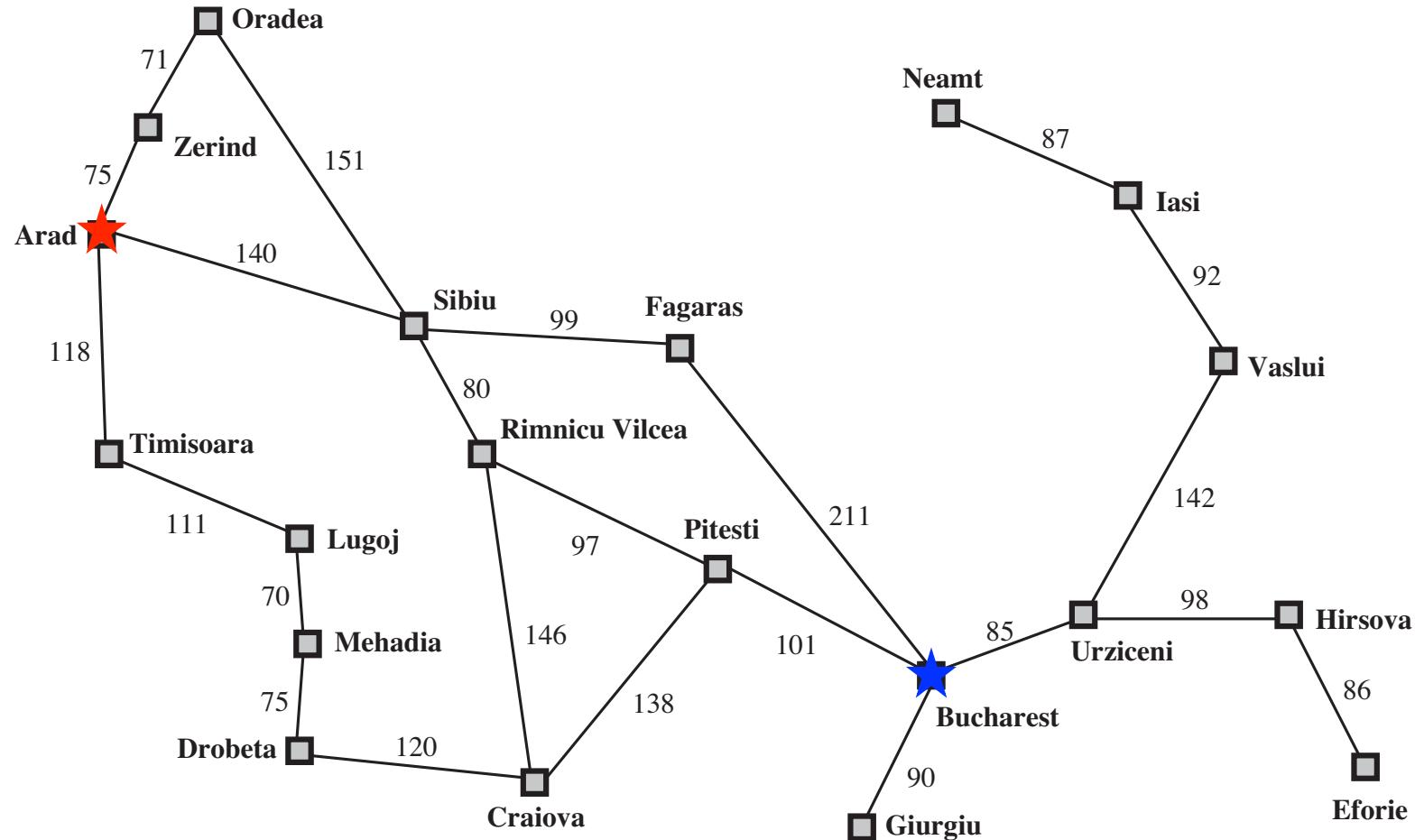
Time vs. memory requirements for BFS

Depth	Nodes	Time	Memory
2	1100	.11 seconds	1 megabyte
4	111,100	11 seconds	106 megabytes
6	10^7	19 minutes	10 gigabytes
8	10^9	31 hours	1 terabyte
10	10^{11}	129 days	101 terabytes
12	10^{13}	35 years	10 petabytes
14	10^{15}	3,523 years	1 exabyte

Figure 3.11 Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 10,000 nodes/second; 1000 bytes/node.

Algorithm 2

BFS did not use the immediately available road length information



Uniform-cost search: Ideas

- Branches of the tree (steps) have different weights (step costs)
- Expand least-cost unexpanded node
- Equivalent to BFS if step costs all equal
- *Implementation*: queue ordered by path cost

Measuring performance of uniform-cost search

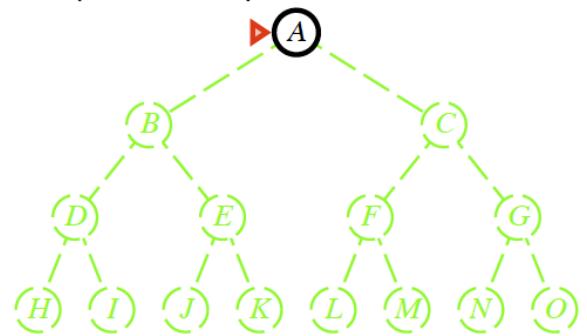
- *Completeness*: Complete if step cost is $\geq \varepsilon$
- *Optimality*: Yes: nodes are expanded in increasing order of $g(n)$
- *Time complexity*: Number of nodes with $g \leq \text{cost}$ of optimal solution, $O(b^{1+\lceil C^*/\varepsilon \rceil})$ where C^* is the cost of the optimal solution
- *Space complexity*: Number of nodes with $g \leq \text{cost}$ of optimal solution, $O(b^{1+\lceil C^*/\varepsilon \rceil})$

Algorithm 3

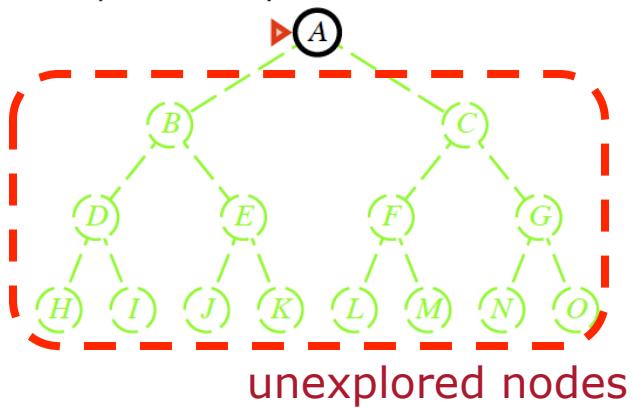
Depth first search (DFS): Ideas

- Always expand the deepest node in the current frontier
- As those nodes are expanded, they are dropped from the frontier, so the search backups to the next deepest unexplored node.
- *Implementation*: Last-in-first-out (LIFO) queue—put successors in front (most recently generated node is chosen for expansion)
- It is common to implement DFS with a recursive function that calls itself on each child

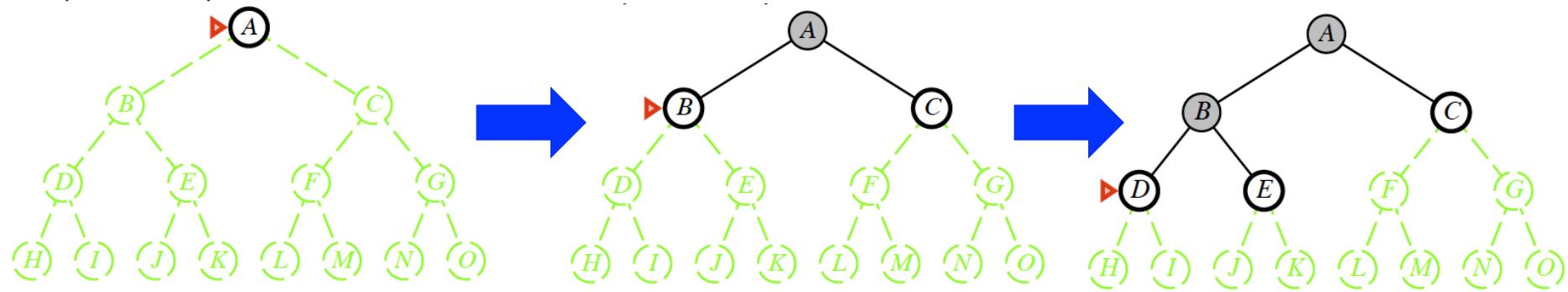
Depth first search (DFS)



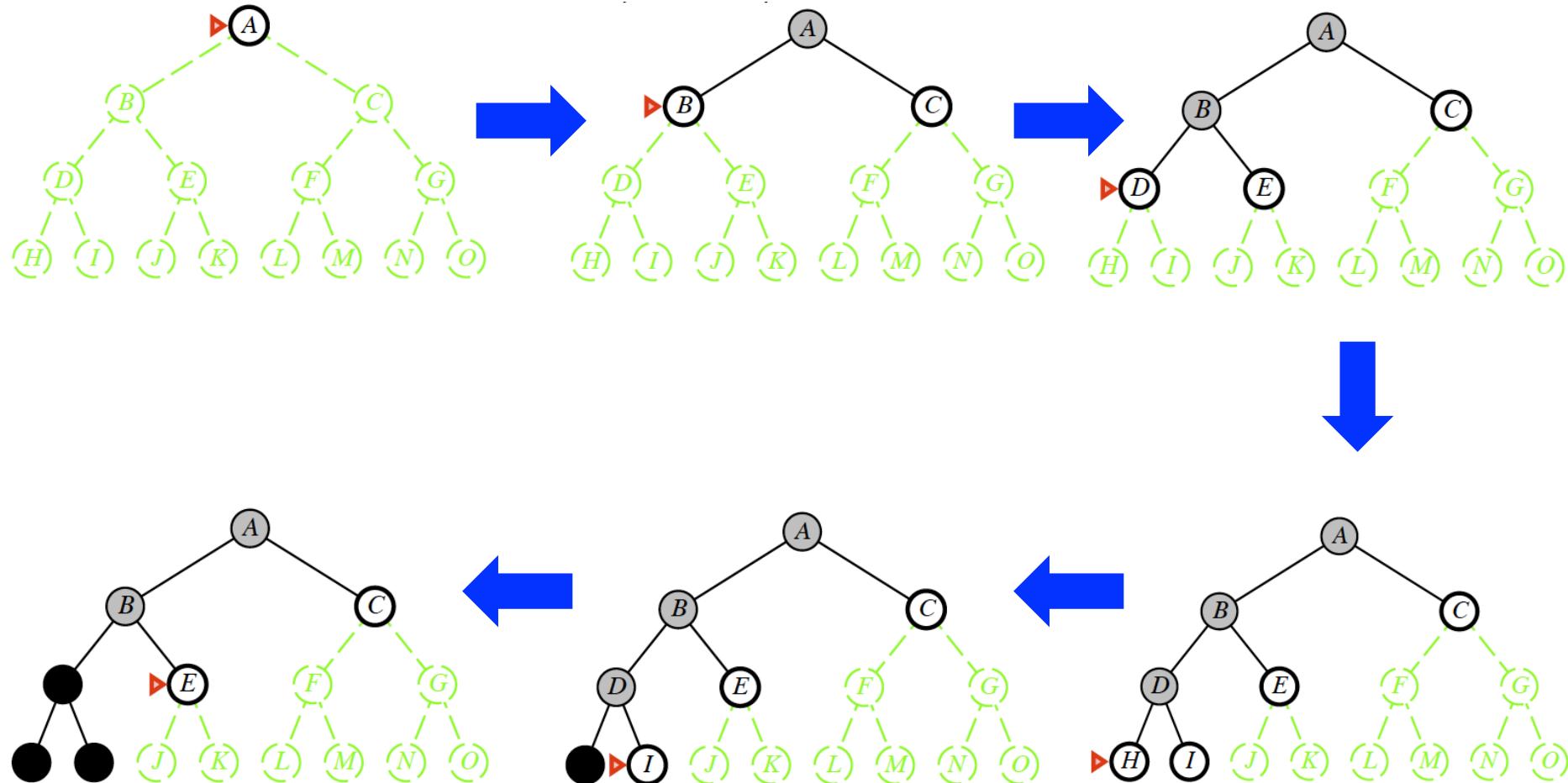
Depth first search (DFS)



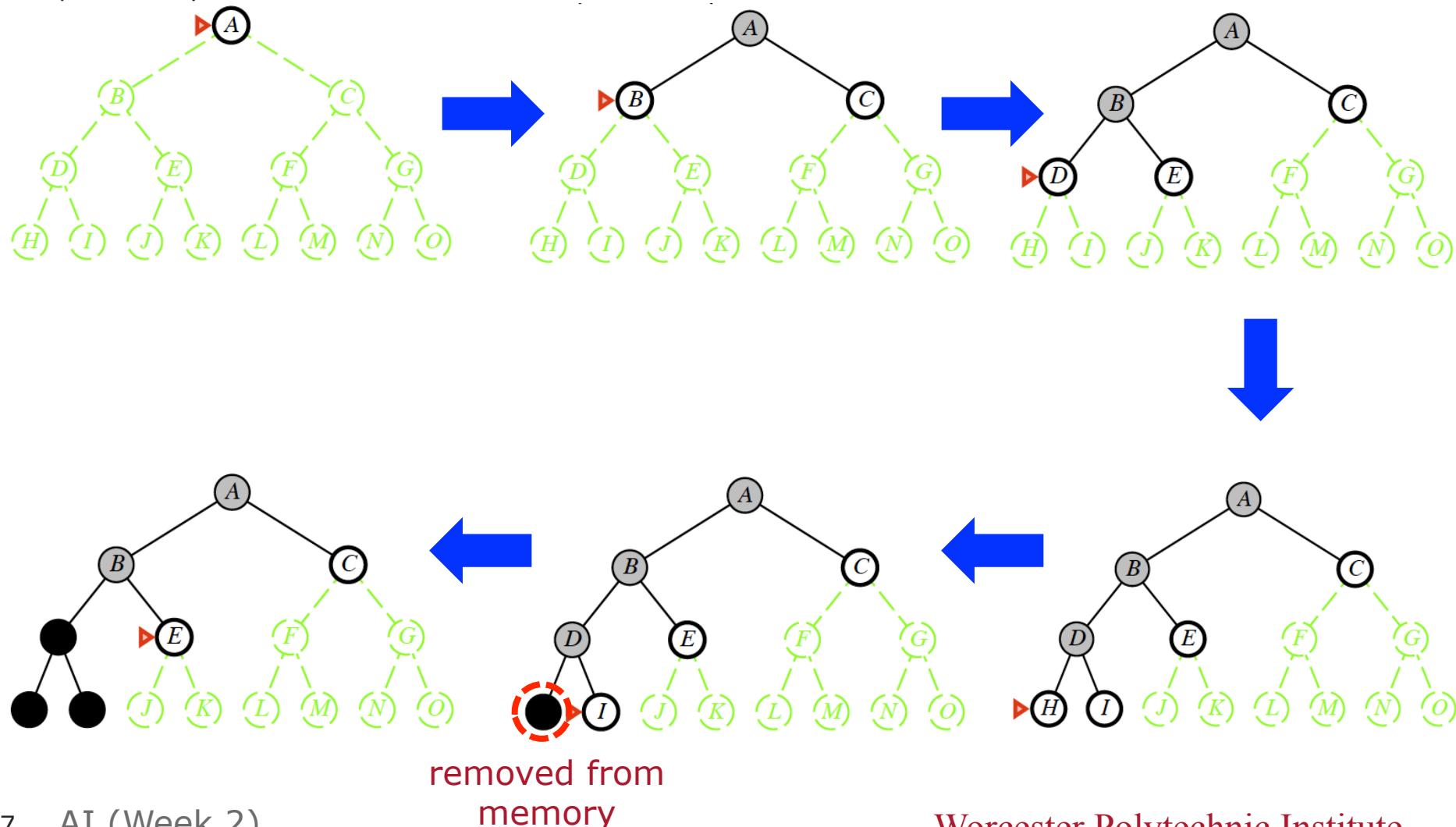
Depth first search (DFS)



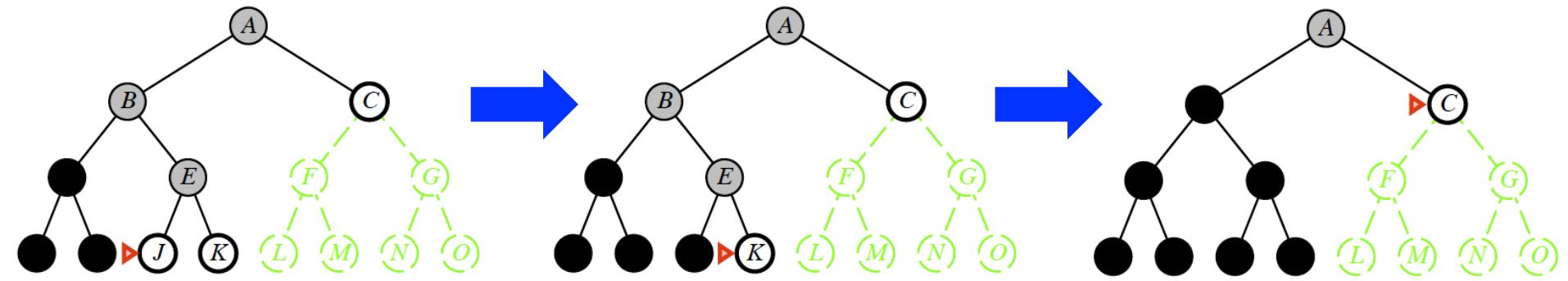
Depth first search (DFS)



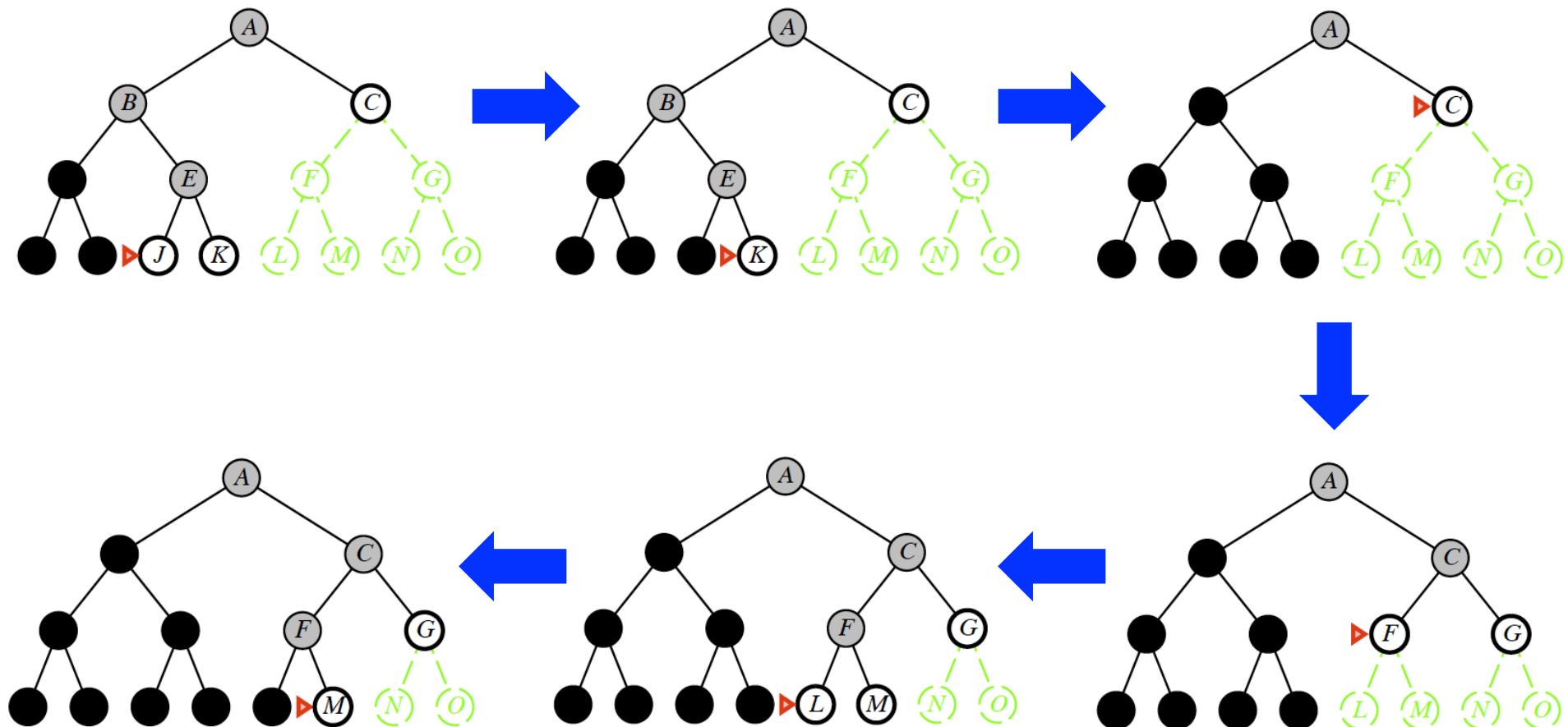
Depth first search (DFS)



Depth first search (DFS)



Depth first search (DFS)

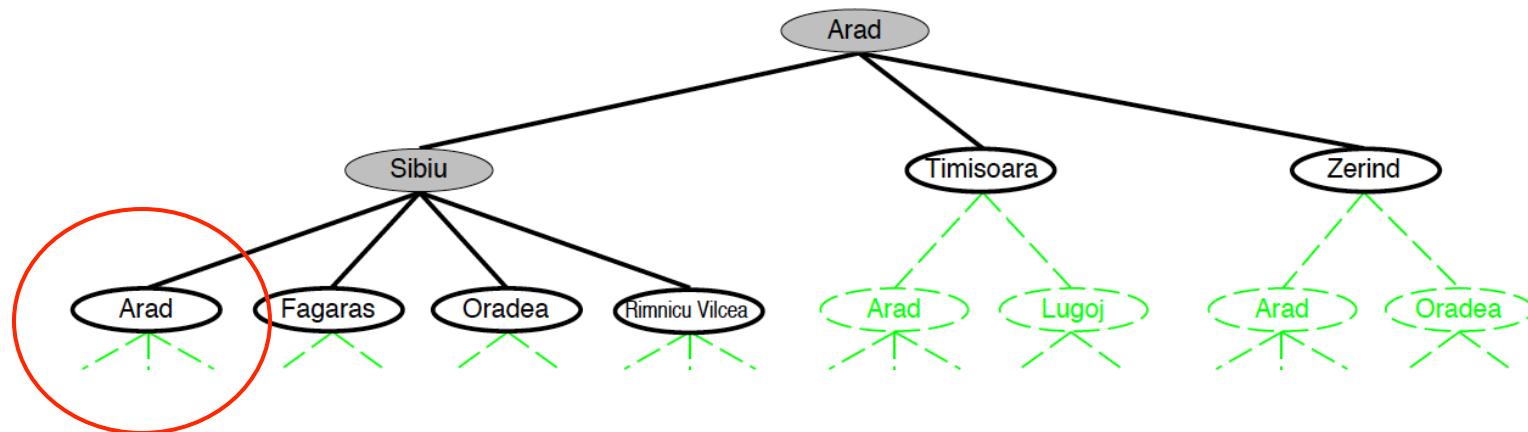


-
- b : maximum branching factor of the search tree
 - d : depth of the least-cost (shallowest) solution
 - m : maximum depth of the state space (can be infinite)

Measuring performance of DFS

- *Completeness*: No, fails in infinite-depth spaces (spaces with loops)
 - Needs to be modified to avoid repeated states along path
 - As a result: complete in finite spaces
- *Optimality*: No
- *Time complexity*: $O(b^m)$ only useful if m is much smaller than d
 - If solutions are dense, may be much faster than BFS
- *Space complexity*: $O(bm)$ — linear!!!

Searching a tree: Example



Definition: *In(Arad)* is a **repeated state** generated by a **loopy path**

*This means that the complete search tree is **infinite***

Algorithm 4

Backtracking search: Ideas

- Only one successor is generated at a time rather than all successors
- Each partially expanded node remembers which successor to generate next
- Generate a successor by modifying the current state description directly rather than copying it first
- However, we must able to “undo” each modification when we go back to generate the next successor

Measuring performance of backtracking

- *Completeness*: No, fails in infinite-depth spaces (spaces with loops)
 - Needs to be modified to avoid repeated states along path
 - As a result: complete in finite spaces
- *Optimality*: No
- *Time complexity*: $O(b^m)$
- *Space complexity*: $O(m)$ — linear, independent of b !!!

Algorithm 5

Depth limited search: Ideas

- Depth-first search with depth limit l , i.e., nodes at depth l have no successors
- Allows to avoid the failure of DFS in infinite state spaces
- Sometimes the depth limit can be based on the knowledge of the problem

Algorithm

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)
function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred?  $\leftarrow$  false
    if GOAL-TEST(problem, STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result  $\leftarrow$  RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred?  $\leftarrow$  true
        else if result  $\neq$  failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Measuring performance of depth-limited search

- *Completeness*: No—Introduces an additional source of incompleteness, when $l < d$: the shallowest goal is beyond the depth limit
- *Optimality*: No
- *Time complexity*: $O(b^l)$ only useful if l is much smaller than d
 - If solutions are dense, may be much faster than BFS
- *Space complexity*: $O(bl)$ — linear!!!

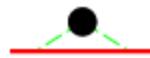
Algorithm 6

Iterative deepening DFS: Ideas

- Strategy that finds the best depth limit
- It gradually increases the limit from 0 to 1, to 2, to 3, until a goal is found
- While it may seem to be wasteful because states are generated multiple time, it is actually not too costly
- Indeed, in a search tree with the same (nearly the same) branching factor at each level, most of the nodes are in the bottom level, but the bottom level nodes are generated once

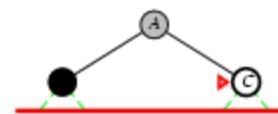
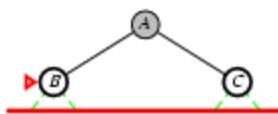
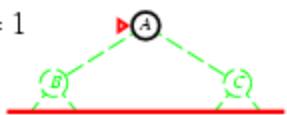
Iterative deepening DFS

Limit = 0



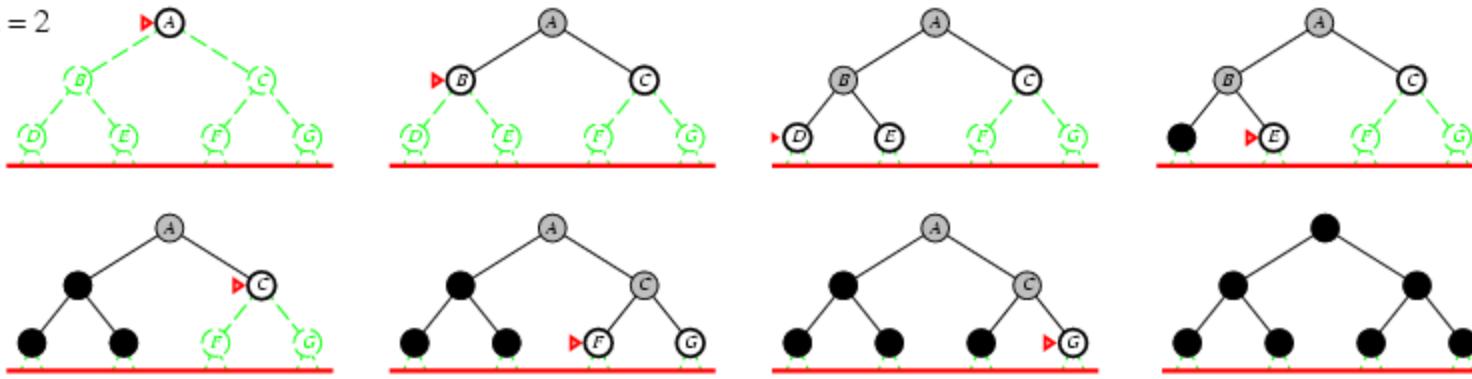
Iterative deepening DFS

Limit = 1

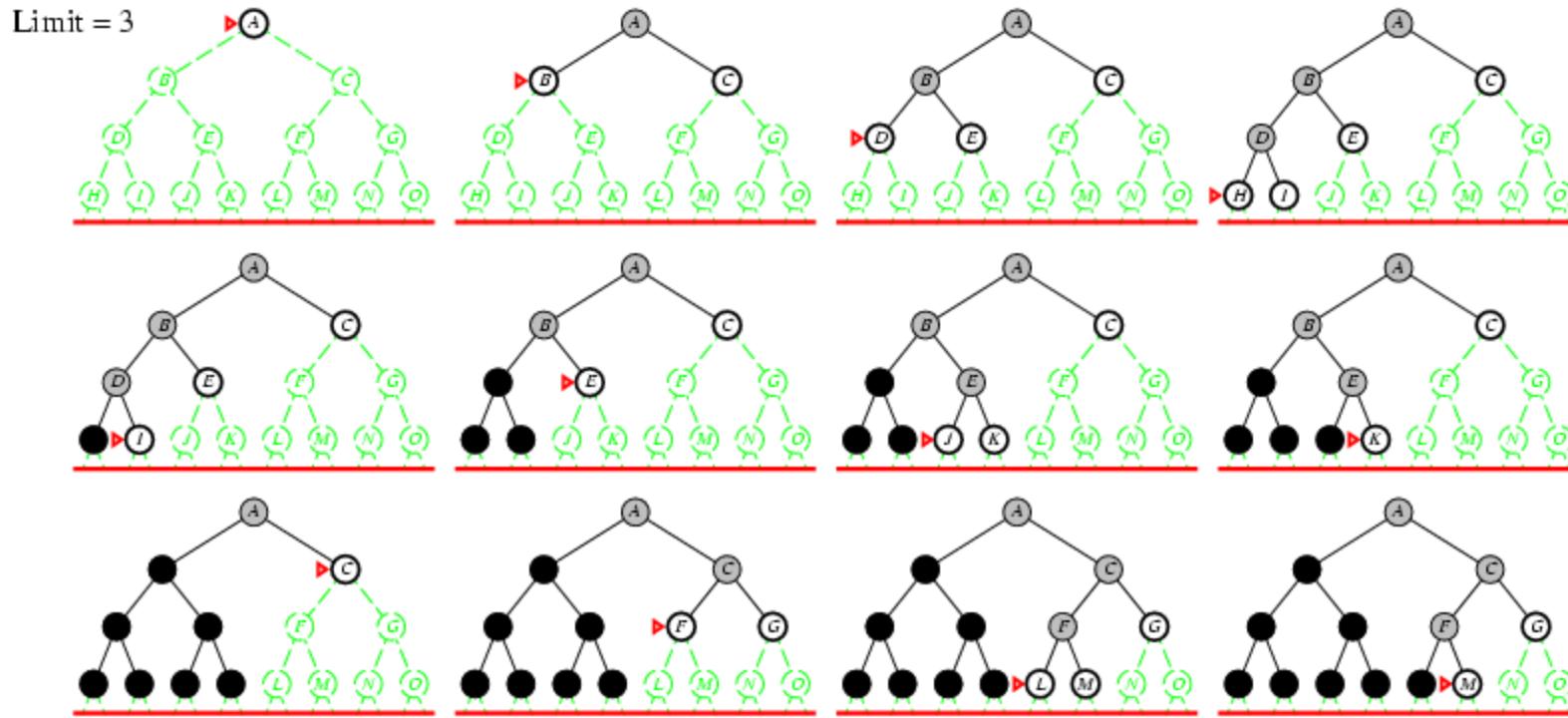


Iterative deepening DFS

Limit = 2



Iterative deepening DFS



Algorithm

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution
    inputs: problem, a problem
    for depth  $\leftarrow 0$  to  $\infty$  do
        result  $\leftarrow$  DEPTH-LIMITED-SEARCH(problem, depth)
        if result  $\neq$  cutoff then return result
    end
```

Time complexity analysis

- Number of nodes generated in a depth-limited search to depth d with branching factor b :

$$N_{DLS} = b^0 + b^1 + b^2 + \dots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth d with branching factor b :

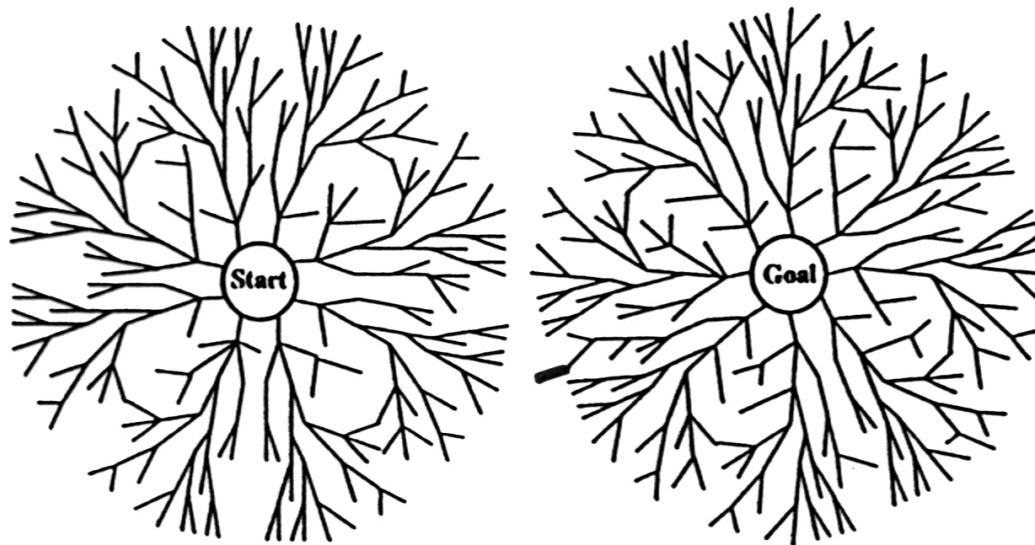
$$N_{IDS} = (d+1)b^0 + d b^1 + (d-1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10, d = 5$,
 - $N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111$
 - $N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456$
- Overhead = $(123,456 - 111,111)/111,111 = 11\%$

Measuring performance of iterative deepening DFS

- *Completeness*: Yes
- *Optimality*: Yes if step cost = 1
- *Time complexity*: $(d+1)b^0 + d b^1 + (d-1)b^2 + \dots + b^d = O(b^d)$
- *Space complexity*: $O(bd)$ — linear!!!

Bidirectional search



A schematic view of a bidirectional breadth-first search that is about to succeed, when a branch from the start node meets a branch from the goal node.

Motivation: $b^{d/2} + b^{d/2} \ll b^d$

Summary of algorithms

Criterion	BFS	Uniform-Cost	DFS	Depth-Limited	Iterative Deepening DLS	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{1+C^*/\varepsilon})$	$O(b^m)$	$O(b^l)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+C^*/\varepsilon})$	$O(bm)$	$O(bl)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

[a] complete if b is finite

[b] complete if step costs $\geq \varepsilon$ for positive

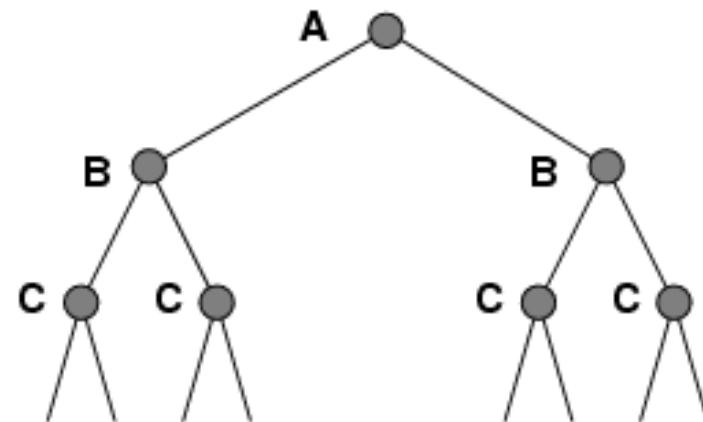
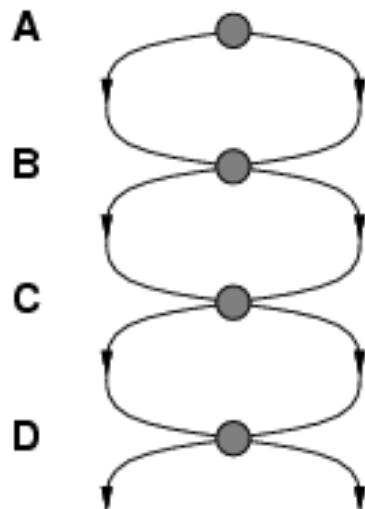
[c] optimal if step costs are all identical

(also if path cost non-decreasing function of depth only)

[d] if both directions use breadth-first search

↑
Generally the preferred
uninformed search strategy

Repeated states



Need to be able to detect repeated states in order to avoid exponential time complexity:

- Do not return to the state you just came from. Have the expand function (or the operator set) refuse to generate any successor that is the same state as the node's parent.
- Do not create paths with cycles in them. Have the expand function (or the operator set) refuse to generate any successor of a node that is the same as any of the node's ancestors.
- Do not generate any state that was ever generated before. This requires every state that is generated to be kept in memory, resulting in a space complexity of $O(b^d)$, potentially. It is better to think of this as $O(s)$, where s is the number of states in the entire state space.

Let's continue: Informed search

Material

- Reading Chapter 3 (Section 3.5), Chapter 4

Informed algorithms

- Best-first search
- Greedy best-first search
- A^{*} search
- RBFS
- SMA
- Heuristics
- Local search algorithms
- Hill-climbing search
- Simulated annealing search
- Local beam search
- Genetic algorithms
- AND-OR Search trees
- Searching with no observations
- Online search

Best-first search

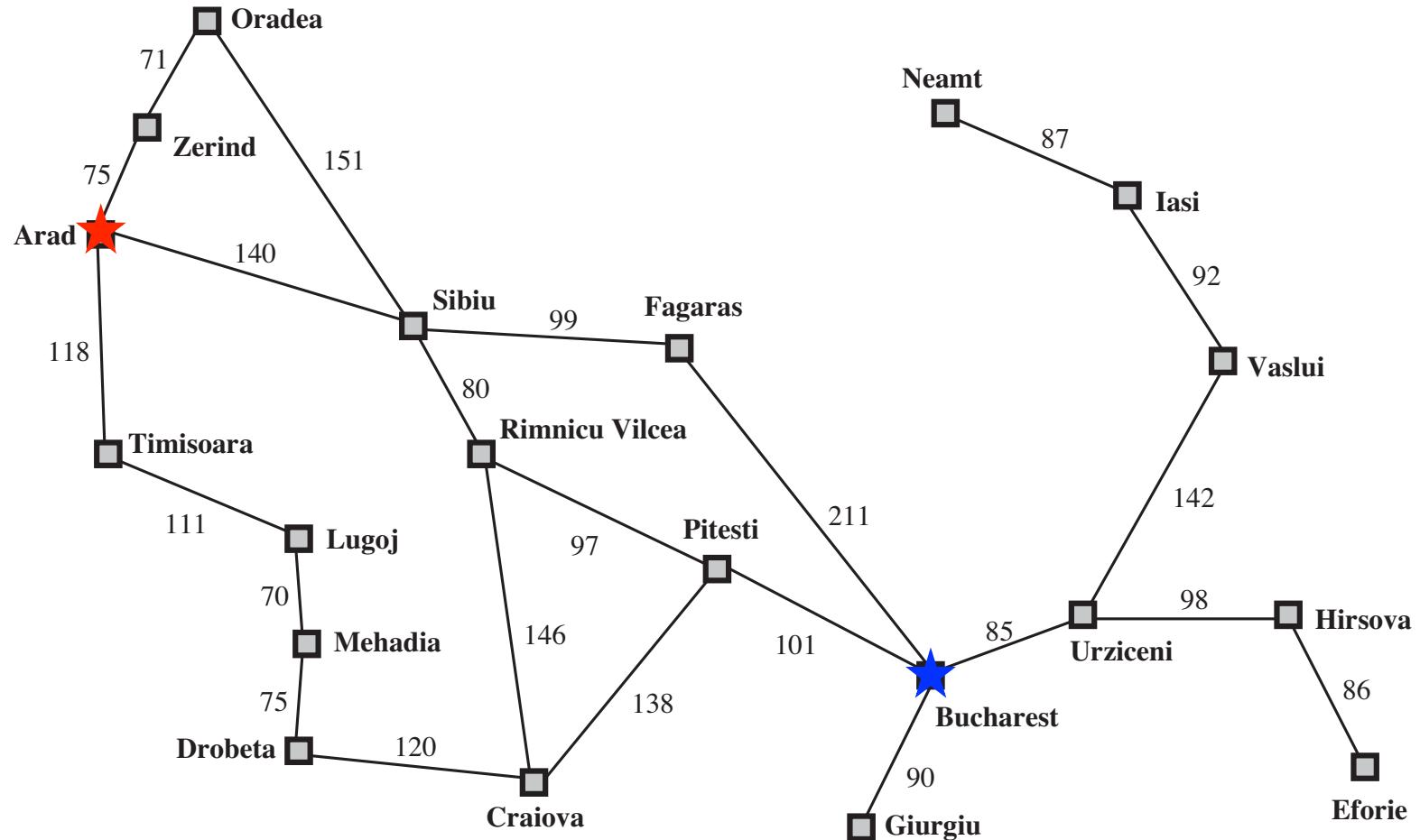
- Idea: use an **evaluation function** $f(n)$ for each node
 - Estimate of "desirability"
 - Expand most desirable unexpanded node
- Implementation:

Order the nodes in frontier in decreasing order of desirability
- Special cases:
 - Greedy best-first search
 - A* search

Greedy best-first search: Ideas

- Tries to expand the node that **appears** closest to the goal first, assuming that it leads to solution quickly
- Evaluation function $f(n) = h(n)$ (**heuristic**)
= estimate of cost from n to *goal*
- Example:
 $h_{SLD}(n)$ = straight-line distance from n to Bucharest

Traveling in Hungary



Romania with step costs in km: Straight line distance to Bucharest

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

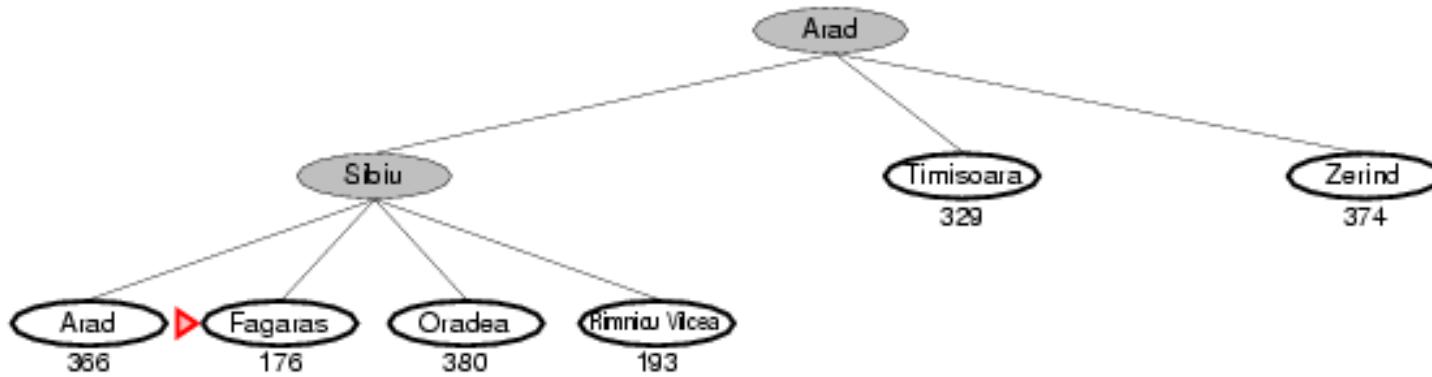
Greedy best-first search example: The initial state



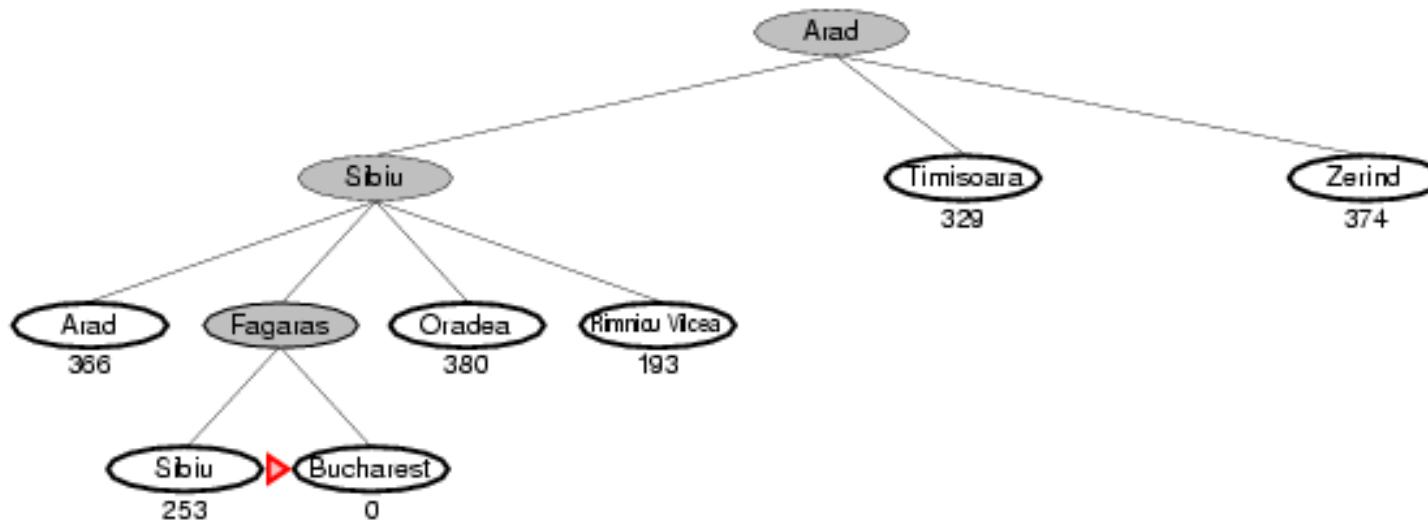
Greedy best-first search example: After expanding Arad



Greedy best-first search example: After expanding Sibiu



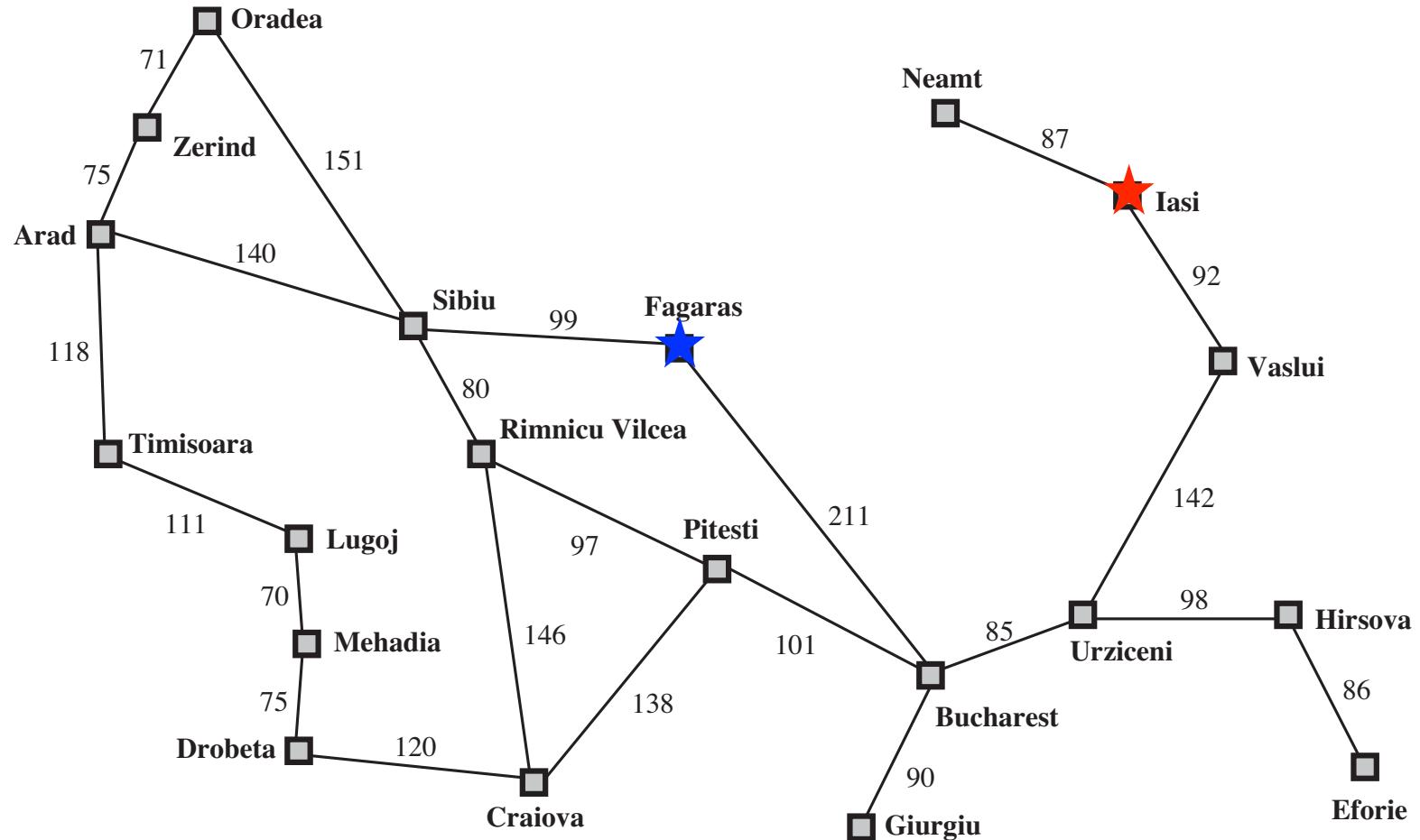
Greedy best-first search example: After expanding Fagaras



Measuring performance of greedy best-first search

- *Completeness*: No – can get stuck in loops, e.g.
lasi → Neamt → lasi → Neamt →
- *Optimality*: No, since there might be a shorter path
- *Time complexity*:
- *Space complexity*:

Traveling in Hungary



Measuring performance of greedy best-first search

- *Completeness*: No – can get stuck in loops, e.g.
lasi → Neamt → laси → Neamt →
- *Optimality*: No, since there might be a shorter path
Arad → Sibiu → Rimnicu Vilcea → Pitesti → Bucharest is 32 km shorter
- *Time complexity*:
- *Space complexity*:

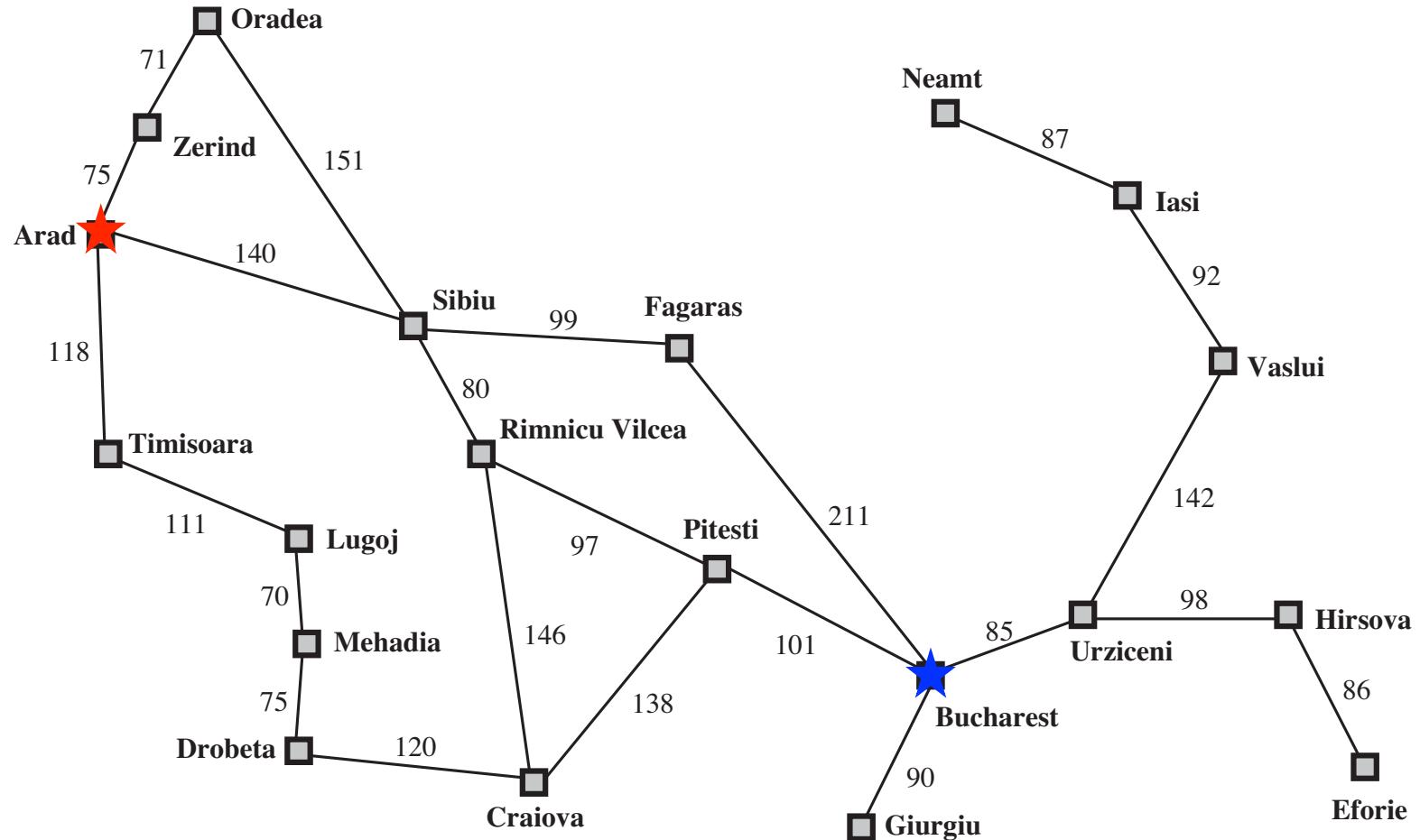
Measuring performance of greedy best-first search

- *Completeness*: No – can get stuck in loops, e.g.
lasi → Neamt → lași → Neamt →
- *Optimality*: No, since there might be a shorter path
Arad → Sibiu → Rimnicu Vilcea → Pitesti → Bucharest is 32 km shorter
- *Time complexity*: $O(b^m)$ — *might be big, but a good heuristic can give dramatic improvement*
- *Space complexity*: $O(b^m)$ — *keeps all nodes in memory*

A* search: Ideas

- Most widely known form of informed search
- A* tries to avoid expanding paths that are already too expensive
- Evaluation function $f(n) = g(n) + h(n)$
 - $g(n)$ = path cost from the start node to node n
 - $h(n)$ = estimated cost of the cheapest path from n to goal
 - $f(n)$ = estimated total cost of path through n to goal
- Algorithm is identical to Uniform-Cost-Search, except using $g(n) + h(n)$ instead of $g(n)$

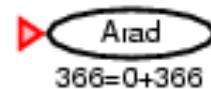
Traveling in Hungary



Romania with step costs in km: Straight line distance to Bucharest

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A* search example: Traveling in Hungary



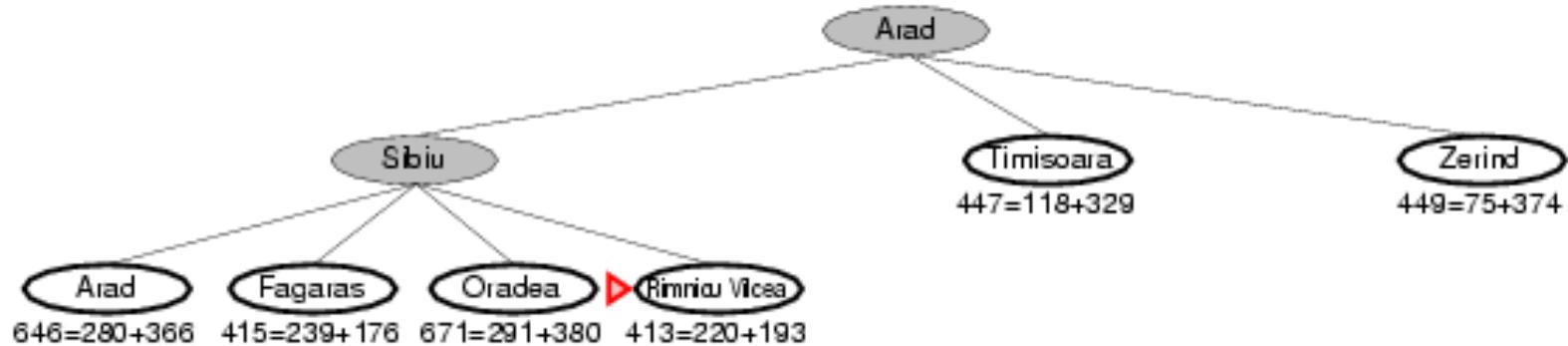
A* search example: Traveling in Hungary



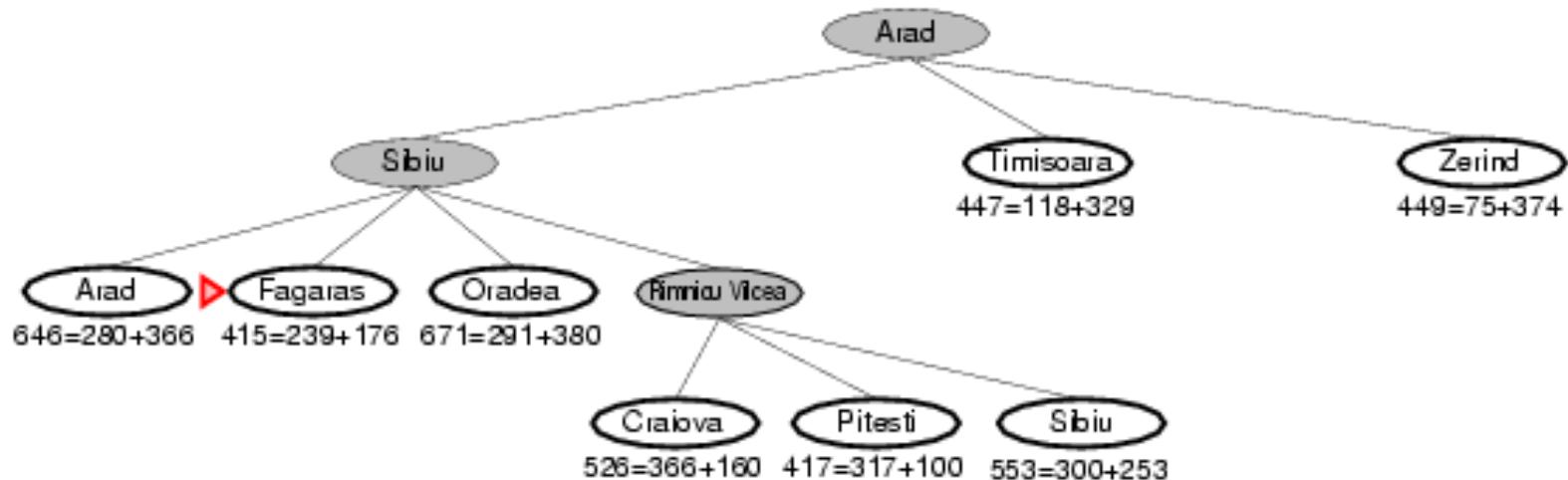
A* search example: Expanding Arad



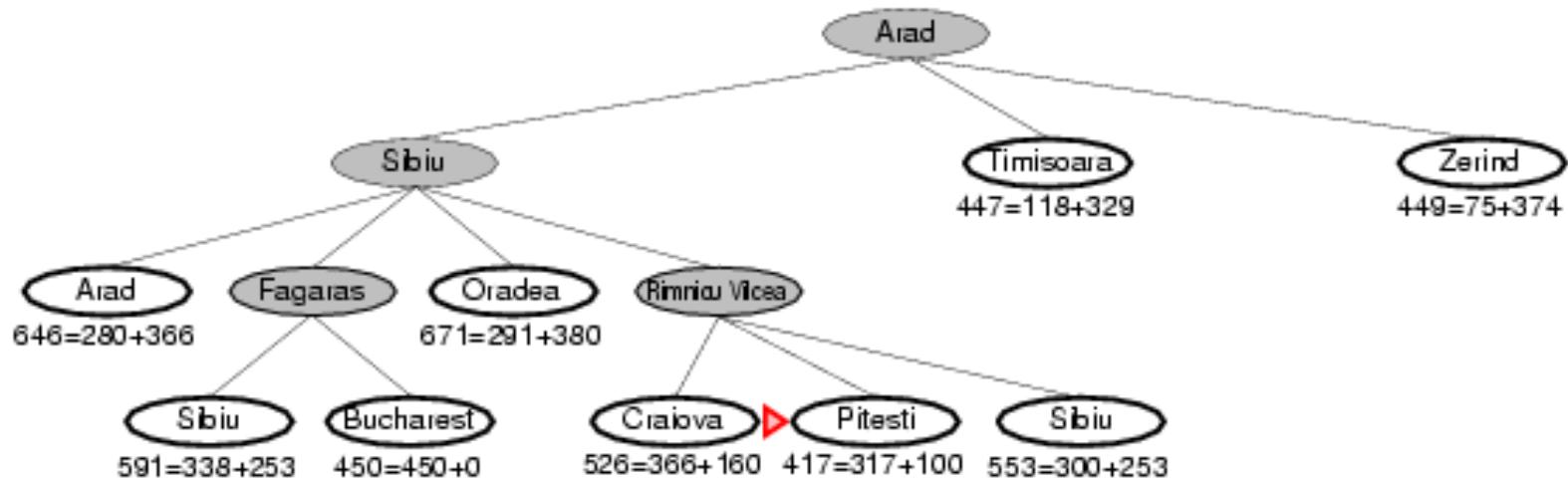
A* search example: Expanding Sibiu



A* search example: Expanding Rimnicu Vilcea



A* search example: Expanding Fagaras



A* search example: Expanding Pitesti

