1. We accept anything reasonable.

a) We can represent this:

$$\vec{x}_i = \bar{\vec{x}} + \sum_{j=0}^{M-1} g_{ij} \vec{e}_j$$

where $\vec{x}_i$ is the pic.

$\bar{\vec{x}}$ is the average

g_{ij} is the weight

$\vec{e}_j$ is the basis function

b) we can calculate the covariance matrix

$$Q = \sum_i (\vec{x}_i - \bar{\vec{x}})(\vec{x}_i - \bar{\vec{x}})^T$$

and we choose $k \ll \mu$ basises with larger eigenvalues λ_i.

$$\vec{x}_i \approx \bar{\vec{x}} + \sum_{j=0}^{k-1} g_{ij} \vec{e}_j$$

c) we compute the error and find the pic having the least square error

$$|\vec{x}_u - \vec{x}_i|^2 = ||\sum_j (g_{uj} - g_{ij})\vec{e}_j||^2 = \sum (g_{uj} - g_{ij})^2$$

d) This method is invariant to translation Not to rotation and scale.

e) No, you can use the symmetric proper to recover the pic first.

2.

a) Tree size of matching model to image: 12^5

Tree size of matching image to model: 5^{12}

Since $12^5 << 5^{12}$, matching model to image edges is more preferable.

b) By adding one Null element to model, the tree sizes become $12^{(5+1)}$ and $(5+1)^{12}$.

So the answer doesn't change.

3.

For binary image, the true table of a pixel $(i_1 - i_2)$:

$i_1 - i_2$	i_1 1	0
i_2 1	0	-1
0	1	0

then, we can get $(i_1 - i_2)^2$

$(i_1 - i_2)^2$	i_1 1	0
i_2 1	0	1
0	1	0

which is just the true table of XOR. XOR means that the output is true when the two input are different. So that the sum of $(i_1 - i_2)^2_{jk}$ is the number of the pixels where $I_1 \neq I_2$.

Since $\|I\|^2 = \sum i^2_{jk}$, $\|I_1 - I_2\|^2 = \sum (i_1 - i_2)^2_{jk}$ = number of pixels when $I_1 \neq I_2$

4. let $\vec{\mu}_A = [x_a, y_a]$

$\vec{\mu}_B = [x_b, y_b]$

Assume that both classes are equally likely

$\Rightarrow (x-x_a)^2 + (y-y_a)^2 = (x-x_b)^2 + (y-y_b)^2$

we can get:

$$y = -\left(\frac{x_a - x_b}{y_a - y_b}\right)x + \frac{x_a^2 + y_a^2 - x_b^2 - y_b^2}{2(y_a - y_b)}$$

$$m_1 = \text{slop} = -\left(\frac{x_a - x_b}{y_a - y_b}\right)$$

m_2 = slop of the line between $\vec{\mu}_A, \vec{\mu}_B$

$$\Rightarrow \frac{y_b - y_a}{x_b - x_a}$$

$$-\left(\frac{x_a - x_b}{y_a - y_b}\right) \cdot \left(\frac{y_b - y_a}{x_b - x_a}\right) = -1$$

which means the line $(x, y) \perp$ the line

$\Rightarrow$ between $\vec{\mu}_A, \vec{\mu}_B$

the middle point $P(\frac{x_a+x_b}{2}, \frac{y_a+y_b}{2})$

is on the decision boundary

$$y = -\left(\frac{x_a-x_b}{y_a-y_b}\right) x + \frac{x_a^2+y_a^2-x_b^2-y_b^2}{2(y_a-y_b)}$$

$\Rightarrow$ pass through midpoint of $\vec{\mu}_A$ and $\vec{\mu}_B$