

Exam #1

DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE TOLD TO DO SO!

Write your name at the top of this page now.

This examination is open book and open notes.

No electronic computing / communications devices are permitted. No wireless communication is allowed.

Write all your answers on the examination in the space provided. You may use the back of the examination for extra space. Partial credit will be given, but you must justify your work.

The examination will end exactly 90 minutes after it begins.
Good luck.

Problem 1: /25
Problem 2: /20
Problem 3: /20
Problem 4: /10
Problem 5: /25
Total: /100

1. **Edge Detection (25 pts).** Ima Robot wants to convolve an image with the following convolutional kernel K :

$$K = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

- a) (5 pts) What derivative does K approximate?
- b) (5 pts) Express K as the convolution of 2 different, smaller kernels.
- c) (5 pts) What is the DFT of K ? Hint: Recall that the DFT of the x -direction first difference operator is

$$je^{-\left(j\frac{\pi k}{2}\right)} \sin \frac{\pi k}{2}$$

- d) (10 pts) What kernel results when Ima convolves K with the following blurring operator?

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

2. **Segmentation (20 pts).** An image has object and background pixels whose brightness values are distributed according to the Rayleigh distribution with parameters σ_o and σ_b with $0 < \sigma_b < \sigma_o$. The probability of a pixel having brightness k is given by

$$P_o(k) = \frac{k}{\sigma_o^2} e^{\frac{-k^2}{2\sigma_o^2}} \text{ and } P_b(k) = \frac{k}{\sigma_b^2} e^{\frac{-k^2}{2\sigma_b^2}}$$

Ima Robot wants to segment the image into object and background. Assuming that background and object pixels are equally likely, find the decision rule that maximizes the probability of a correct decision, that is, pick the greater of P_o and P_b .

3. **Morphology (20 pts).** Ima Robot dilates binary image B

$$B =$$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

with unknown structuring element S to yield output image O

$$O =$$

0	0	0	0	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

a) (10 pts) What is S such that $B \oplus S = O$?

b) (10 pts) Find another structuring element S_2 that is different from the one in a) that also satisfies $B \oplus S_2 = O$. Hint: There are several. Any one will do.

4. **Focus (10 pts).** Astronomer Stella Gazer notices that her telescope is out of focus. Every star, rather than being a point of light, appears as a circle with diameter D in the image. Assuming that the telescope has a lens with diameter d and focal length f , how far should she move the image plane to achieve perfect focus?

5. **Hough Transform (25 pts).** In this problem x , y , b , and m may be positive or negative, integers or fractions. 2 points in (m,b) space are given by

$$P1: m = \frac{1}{2}, b = 2$$

$$P2: m = 1, b = 0$$

Hint: It might help to construct the (x,y) and (m,b) spaces.

- a) (5 pts) What line L1 in (m,b) space passes through points P1 and P2?
- b) (10 pts) What point P3 in (x,y) space corresponds to line L1?
- c) (10 pts) Horizontal line L2 in (x,y) space passes through P3. What is its corresponding point P4 in (m,b) space?