CS/RBE 549 Computer Vision   Name:________________________

Fall 2019   Due: Monday 09 Dec 2019

HW #12

Note that you have 2 weeks to complete this assignment to give you time to work on final projects.

1. **Stereo Motion (7 pts):** In stereo imaging we can compute a point's world coordinates from left and right images as

$$\vec{X}^W = \vec{X}_{AVG} \frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}}$$

   a. Show that if the world point is in motion, we can compute its velocity as

$$\vec{V}^W = \vec{V}_{AVG} \frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}} - \vec{X}_W \frac{\vec{B} \cdot \frac{d\vec{\Delta}}{dt}}{\vec{B} \cdot \vec{\Delta}}$$

   b. Suppose that a moving world point is imaged as

$$\vec{X}_L^I = \begin{bmatrix} \frac{c}{2t} \\ 0 \\ f \end{bmatrix}, \vec{X}_R^I = \begin{bmatrix} \frac{-c}{2t} \\ 0 \\ f \end{bmatrix}$$

   With the usual imaging geometry of $\vec{B} = [b \quad 0 \quad 0]^T, \vec{F} = [0 \quad 0 \quad f]^T$, what is $\vec{V}^W$? Express $\vec{V}^W$ in the simplest terms.

2. **Shading (6 pts):** Consider the two surfaces

$$z_1 = (x^2 + y^2) \text{ and } z_2 = 2xy$$

   a. Find $p(x, y)$ and $q(x, y)$ for both surfaces.
   b. Show that z_1 and z_2 give rise to the same shading when a rotationally symmetric reflectance map applies, that is, when $R(p, q) = R(p^2 + q^2)$.

3. **SLAM:** Following Bailey & Durrant-Whyte Part II, at time k the vehicle pose $\mathbf{x}_{v_k}$ and landmark locations $\mathbf{m}$ can be combined into a single state $\mathbf{x}_k$.

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{v_k} \\ \mathbf{m} \end{bmatrix}$$

   The covariance matrix of $\mathbf{x}_k$, denoted by $\mathbf{P}_{k|k}$, is partitioned into

$$\mathbf{P}_{k|k} = \begin{bmatrix} \mathbf{P}_{vv} & \mathbf{P}_{vm} \\ \mathbf{P}_{vm}^T & \mathbf{P}_{mm} \end{bmatrix}_{k|k}$$

Control input $\mathbf{u}_k$ only affects the vehicle pose, not the landmark locations, via $\mathbf{x}_{v_k} = \mathbf{f}_v(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k)$. Thus, the update equation for state $\mathbf{x}_k$ is

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k) = \begin{bmatrix} \mathbf{f}_v(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k) \\ \mathbf{m} \end{bmatrix}$$

When control $\mathbf{u}_k$ is applied, but before any landmark measurement updates are made, the covariance prediction $\mathbf{P}_{k|k-1}$ is given by

$$\mathbf{P}_{k|k-1} = \nabla\mathbf{f}_{\mathbf{x}} \mathbf{P}_{k-1|k-1} \nabla\mathbf{f}_{\mathbf{x}}^T + \nabla\mathbf{f}_{\mathbf{u}} \mathbf{U}_k \nabla\mathbf{f}_{\mathbf{u}}^T$$

where $\nabla\mathbf{f}_{\mathbf{x}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{k-1}}$, $\nabla\mathbf{f}_{\mathbf{u}} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}_k}$, and $\mathbf{U}_k$ is the control convariance. Show that the covariance prediction simplifies (!) to

$$\mathbf{P}_{k|k-1} = \begin{bmatrix} \nabla\mathbf{f}_{v_{\mathbf{x}}} \mathbf{P}_{vv,k-1|k-1} \nabla\mathbf{f}_{v_{\mathbf{x}}}^T + \nabla\mathbf{f}_{v_{\mathbf{u}}} \mathbf{U}_k \nabla\mathbf{f}_{v_{\mathbf{u}}}^T & \nabla\mathbf{f}_{v_{\mathbf{x}}} \mathbf{P}_{vm,k-1|k-1} \\ \mathbf{P}_{vm,k-1|k-1}^T \nabla\mathbf{f}_{v_{\mathbf{x}}}^T & \mathbf{P}_{mm,k-1|k-1} \end{bmatrix}$$

where $\nabla\mathbf{f}_{v_{\mathbf{x}}} = \frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_{v_{k-1}}}$ and $\nabla\mathbf{f}_{v_{\mathbf{u}}} = \frac{\partial \mathbf{f}_v}{\partial \mathbf{u}_k}$.

Hint: Expand $\nabla\mathbf{f}_{\mathbf{x}}$ and $\nabla\mathbf{f}_{\mathbf{u}}$**.** This problem is not as hard as it looks once you understand what is being asked.