1. Chain code

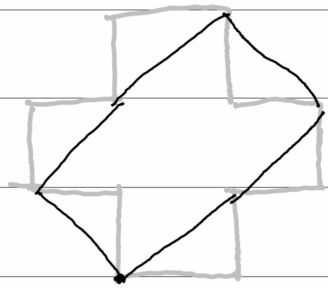
a) $1, j, -1, -j$ correspond to $0, 1, 2, 3$ in 4-connected chain code. The example can be represented as 0 1 0 0 1 2 2 2 3 3

b)

Since a shape is close and the grids are equispaced, a point has to go through the same number of movement upward and downward to go back to the same vertical level. This means $\sum c_{up} + \sum c_{down} = 0$. Similarly, in horizontal direction, $\sum c_{left} + \sum c_{right} = 0$.

So, $\sum_{i} c(i) = \sum c_{up} + \sum c_{down} + \sum c_{left} + \sum c_{right} = 0$

c) The smoothed image might be different from the original one. Here I define "reasonably" is keeping similar area, for example, at least 80% of the original area.



A shape can be reasonably smoothed.

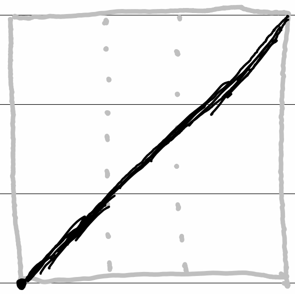
Original: $1, j, 1, j, -1, j, -1, -j, -1, -j, 1, -j$

Smoothed: $1+j,\ 1+j,\ -1+j,\ -1-j,\ -1-j,\ 1-j$

original area: 5 units

Smoothed area: $1 \cdot \sqrt{2} \cdot 2 \cdot \sqrt{2} = 4$ (units)

$\Rightarrow$ 80% of original area.



not reasonably smoothed.

Original: $3,\ 3j,\ -3,\ -3j$

smooth: $3+3j,\ -3-3j$

original area: 9 units

Smoothed area: 0

d)

$$C(0) = \sum_{i=0}^{N-1} c(i) e^{-2\pi j \frac{i \cdot 0}{N}} = \sum_{i=0}^{N-1} c(i) = 0$$

2.

a) Not invariant to translation, scale, or rotation

$$x(t) = \sum_{i=0}^{N} a_i t^i \qquad y(t) = \sum_{i=0}^{N} b_i t^i$$

So we know

$$x(t) = a_0 + a_1 t + a_2 t^2 + \cdots$$
$$y(t) = b_0 + b_1 t + b_2 t^2 + \cdots$$

would be different before and after translation $\Rightarrow$ not translation invariant.

Let $x(t') = k\,x(t) = ka_0 + ka_1 t^1 + ka_2 t^2 + \cdots$

$$\neq x(t)$$

$\Rightarrow$ not scale invariant

Let $x(t') = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sum a_i t^i \\ \sum b_i t^i \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta \sum_{i=0}^{N} a_i t^i + \sin\theta \sum_{i=0}^{N} b_i t^i \\ -\sin\theta \sum_{i=0}^{N} a_i t^i + \cos\theta \sum_{i=0}^{N} b_i t^i \\ 1 \end{bmatrix}$$

$\neq x(t) \Rightarrow$ not rotation invariant

b) $X(t) = \cos \pi t$

$$= 1 - \frac{\pi^2 t^2}{2!} + 0 + \frac{\pi^4 t^4}{4!} \cdots$$

$a_0 = 1 \quad a_1 = 0 \quad , a_2 = \frac{-\pi^2}{2!} \quad a_3 = 0$

$Y(t) = \sin \pi t$

$$= 0 + \frac{\pi t}{1!} + 0 - \frac{\pi^3 t^3}{3!} + \cdots\cdots$$

$b_0 = 0 \quad b_1 = \pi \quad b_2 = 0 \quad b_3 = -\frac{\pi^3}{3!}$

3. a.

$$Z'(k) = \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s)$$

$$= \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} \left(z(s) + \Delta x + j\Delta y\right)$$

$$= \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s) + \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} (\Delta x + j\Delta y)$$

$$= Z(k) + (\Delta x + j\Delta y) S \delta_{0k}$$

$$Z'(k) = \begin{cases} Z(k) + (\Delta x + j\Delta y) S, & \text{if } k = 0 \\ Z(k), & \text{if } k \neq 0 \end{cases}$$

b) $Z(k) = \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s)$

$$Z'(k) = \sum_{s=0}^{S'-1} e^{-2\pi j \frac{ks}{S}} c\, z(\lfloor \tfrac{s}{c} \rfloor)$$

$$= c \sum_{s=0}^{cS-1} e^{-2\pi j \frac{ks}{cS}} z(\lfloor \tfrac{s}{c} \rfloor)$$

$$= c \left(\sum_{s=0}^{c-1} e^{-2\pi j \frac{ks}{cS}} z(0) + \sum_{s=c}^{2c-1} e^{-2\pi j \frac{ks}{cS}} z(1) + \cdots\cdots + \sum_{s=c\cdot(S-1)}^{cS-1} e^{-2\pi j \frac{ks}{cS}} z(S-1) \right)$$

$$= c \cdot \sum_{n=0}^{S-1} \sum_{s=0}^{c-1} e^{-2\pi j \frac{k(s+n\cdot c)}{cS}} z(n)$$

$$= c \sum_{n=0}^{S-1} \sum_{s=0}^{c-1} e^{-2\pi j \frac{ks}{cS}} e^{-2\pi j \frac{nc}{cS}} z(n)$$

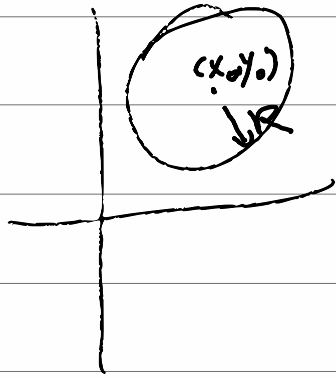
$$= c \sum_{n=0}^{S-1} e^{-2\pi j \frac{n\cancel{c}}{\cancel{c}S}} z(n) \sum_{s=0}^{c-1} e^{-2\pi j \frac{ks}{cS}}$$

$$= c \cdot Z(k) \cdot \underbrace{\sum_{s=0}^{c-1} e^{-2\pi j \frac{ks}{cS}}}_{\text{a period function}}$$

a period function which $= \begin{cases} c, & \text{when } k = n\cdot cS,\ n\in\mathbb{Z} \\ 0, & \text{when } k = n\cdot S,\ n\in\mathbb{Z} \end{cases}$

$$\Rightarrow Z'(k) = \begin{cases} c^2 Z(k), & \text{when } k \text{ is a multiple of } c\cdot S \\ 0, & \text{when } k \text{ is a multiple of } S \end{cases}$$

c)



A circle with radius R centered at (x_0, y_0)

d) $z(s) = \left[x_0 + R\cos\frac{2\pi s}{S}\right] + j\left[y_0 + R\sin\frac{2\pi s}{S}\right]$

$$z(s) = (x_0 + jy_0) + R\left(\cos\frac{2\pi s}{S} + j\sin\frac{2\pi s}{S}\right)$$

$$= (x_0 + jy_0) + R\left(e^{j\frac{2\pi s}{S}}\right)$$

$$Z(k) = R\sum_{s=0}^{S-1} e^{-2\pi j\frac{ks}{S}} \cdot e^{j\frac{2\pi s}{S}} + (x_0 + jy_0)\sum_{s=0}^{S-1} e^{-2\pi j\frac{sk}{S}}$$

$$= R \cdot \delta_{k1} + (x_0 + jy_0)\,\delta_{k0}$$

$$Z(0) = R \cdot 0 + (x_0 + jy_0) \cdot S$$

$$Z(1) = R \cdot S + 0$$

$Z(k) = 0$ for $k > 1$