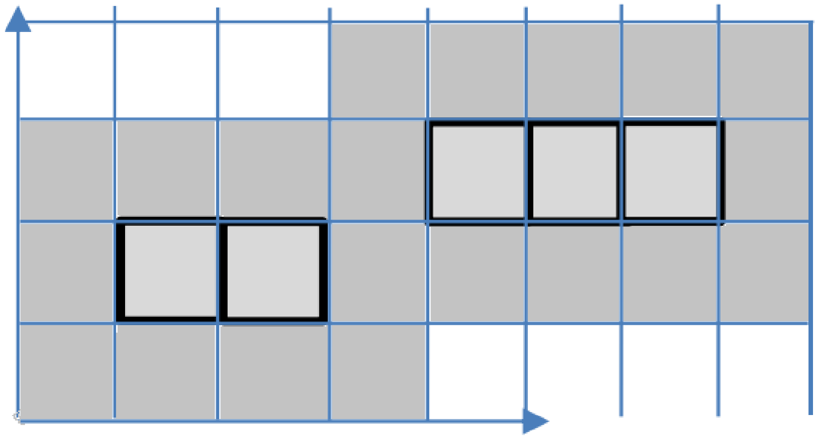
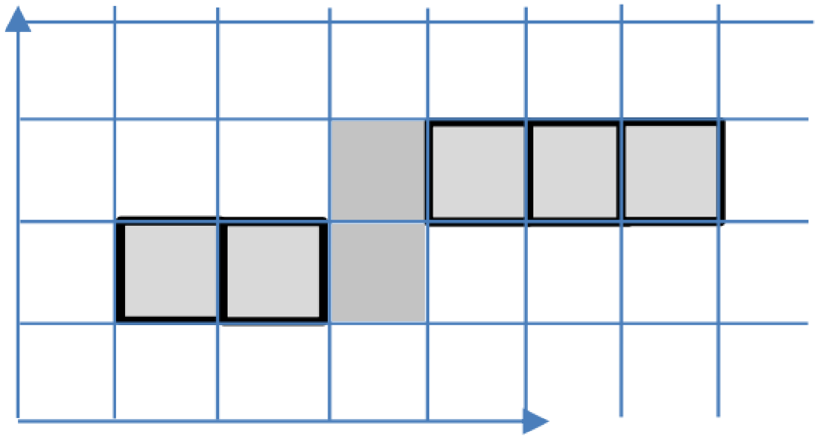
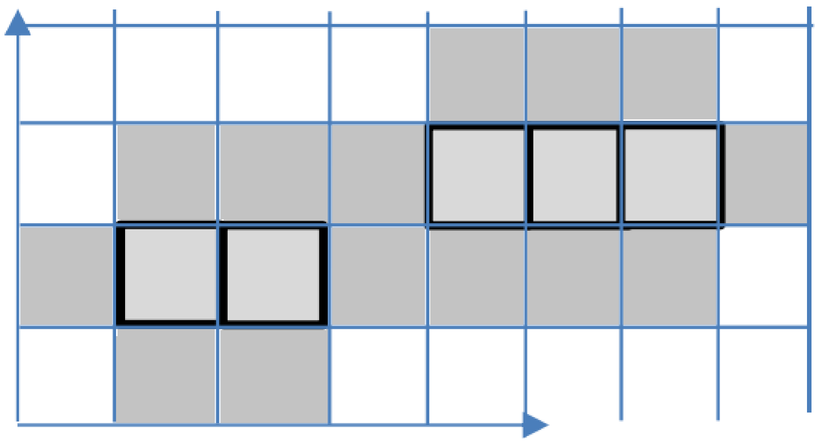
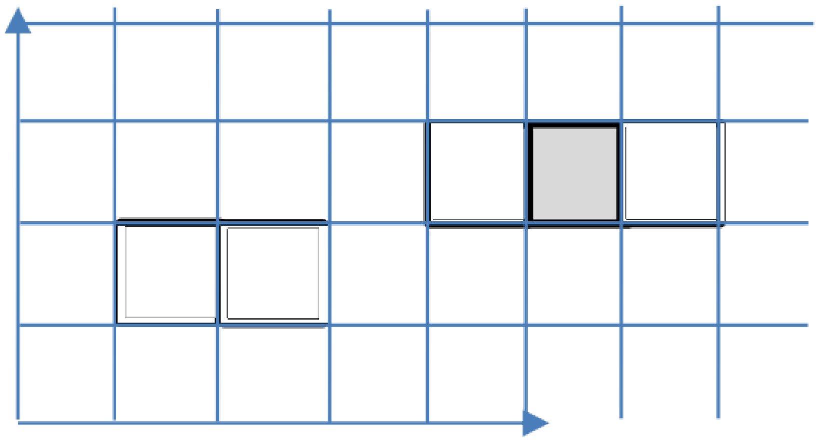
1] a





1] b





2] As per the definition of convolution,

$$g(s) \otimes h(s) = \int g(s)\, h(t-s)\, dt$$

$$= G(\omega) \times H(\omega)$$

where

$$g(s) \xleftrightarrow{F} G(\omega)$$

This is the fourier transform pair

$$G(\omega) = \int_{-\infty}^{\infty} g(s)\, e^{-j\omega s}\, ds$$

$$\therefore f(x) \otimes (g(x) \otimes h(x)) = F(\omega) \times (G(\omega) \times H(\omega))$$

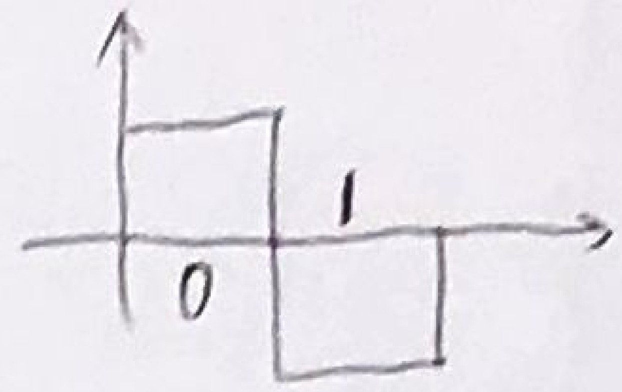
$$= (F(\omega) \times G(\omega)) \times H(\omega)$$

↑

because multiplication is assosiative

$$\therefore f(x) \otimes (g(x) \otimes h(x)) = (f(x) \otimes g(x)) \otimes h(x)$$

$$H(k) = \frac{1}{N}\sum_{x=0}^{N-1} h(x)\, e^{-j\frac{2\pi k x}{N}}$$



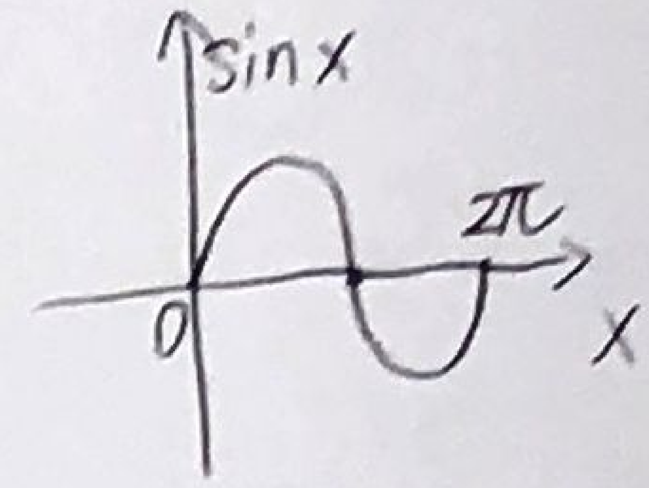
$$= \frac{1}{2}\left(1\, e^{-j\frac{2\pi k 0}{2}} - 1\, e^{-j\frac{2\pi k 1}{2}}\right)$$

$$h(x) = \begin{cases} 1 & x=0 \\ -1 & x=1 \\ 0 & \text{else} \end{cases}$$

$$= \frac{1}{2}\left(1 - e^{-j\pi k}\right)$$

Euler's formula

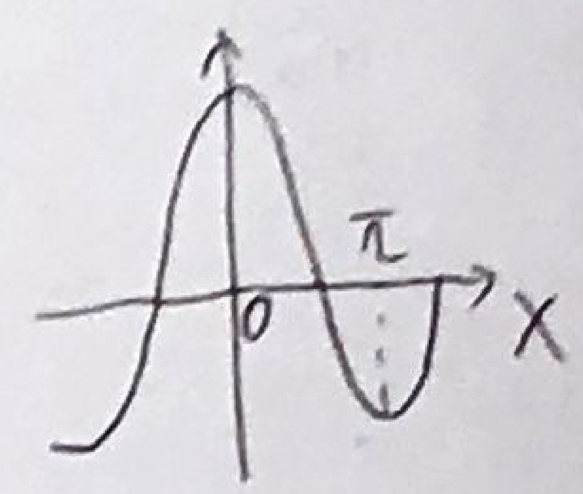
$$= \frac{1}{2}\left(1 - \left[\cos(-k\pi) + j\sin(-\pi k)\right]\right)$$



$$= \frac{1}{2}\left(1 - \cos(k\pi) + j\sin(k\pi)\right)$$

Trigonometric identities

$$= \frac{1}{2}\left(2\sin^2\left(\frac{k\pi}{2}\right) + j\, 2\sin\left(\frac{k\pi}{2}\right)\cos\left(\frac{k\pi}{2}\right)\right)$$



$$= \sin^2\left(\frac{k\pi}{2}\right) + j\sin\left(\frac{k\pi}{2}\right)\cos\left(\frac{k\pi}{2}\right)$$

$$= j\sin\frac{k\pi}{2}\left(-j\sin\frac{k\pi}{2} + \cos\frac{k\pi}{2}\right)$$

Euler's formula

$$= j\, e^{-\frac{\pi k}{2} j}\sin\left(\frac{k\pi}{2}\right) \qquad e^{\frac{\pi}{2} j(1-k)}\sin\frac{\pi k}{2}$$

costant: j

$e^{[\text{something}]}$: $e^{-\frac{\pi k}{2} j}$

$\sin[\text{something}]$: $\sin\left(\frac{k\pi}{2}\right)$