1] There are 4 possible connectivity relations
- 8 connected, if only one dimension changes.
In this case, neighbors are
$(x \pm 1, y, z, t), (x, y \pm 1, z, t), (x, y, z \pm 1, t)$ &
$(x, y, z, t \pm 1)$
- 32 connected, if one or two dimensions change. In this case neighbors are all from the 8-connected plus 24
$(x \pm 1, y \pm 1, z, t), (x, y \pm 1, z \pm 1, t), (x \pm 1, y, z \pm 1, t),$
$(x, y, z \pm 1, t \pm 1), (x \pm 1, y, z, t \pm 1),$
$(x, y \pm 1, z, t \pm 1)$

- 64 connected if three dimensions change
- 80 - connected if all dimensions can change.

2.

$$A = \left[\begin{array}{c:c} R & \vec{T} \\ \hdashline 0 & 1 \end{array}\right]$$

A is the transformation matrix.

$$A^{-1} = \left[\begin{array}{c:c} R^{-1} & -R^{-1}\vec{T} \\ \hdashline 0 & 1 \end{array}\right] = \left[\begin{array}{c:c} R^{-1} & yS_\theta - xS_\theta \\ & -yC_\theta - xS_\theta \\ \hdashline 0 & 1 \end{array}\right]$$

* A^{-1} forms a seperate hemogenous coordinate
* A point p in 2 different coordinate frames with different basis vector and defined by

$${}^{B}p = {}^{B}_{A}R\,{}^{A}p$$

Similarly $${}^{B}\vec{T} = {}^{B}_{A}R\,{}^{A}\vec{T}$$

Where ${}^{A}_{B}R$ is rotation from frame (B) to other frame (A)

3 a) the background and object are equally likely

$$\frac{k}{\sigma_o^2} e^{\frac{-k^2}{2\sigma_o^2}} = \frac{k}{\sigma_b^2} e^{-\frac{k^2}{\sigma_b^2}}$$

$$\frac{\sigma_b^2}{\sigma_o^2} = e^{\frac{k^2}{2\sigma_o^2} - \frac{k^2}{2\sigma_b^2}}$$

$$\ln \frac{\sigma_b^2}{\sigma_o^2} = \ln e^{\frac{k^2}{2\sigma_o^2} - \frac{k^2}{2\sigma_b^2}}$$

$$2\ln\left(\frac{\sigma_b}{\sigma_o}\right) = \frac{k^2}{2\sigma_o^2} - \frac{k^2}{2\sigma_b^2}$$

$$k^2 = \frac{4\ln\left(\frac{\sigma_b}{\sigma_o}\right)\sigma_o^2\sigma_b^2}{\sigma_b^2 - \sigma_o^2}$$

So the threshold should be

$$T = \sqrt{\frac{4\ln\left(\frac{\sigma_b}{\sigma_o}\right)\sigma_o^2\sigma_b^2}{\sigma_b^2 - \sigma_o^2}}$$

b) $N_0 P_0(k) = N_b P_b(k)$

So $$N_0 \frac{k}{\sigma_0^2} e^{\frac{-k^2}{2\sigma_0^2}} = N_b \frac{k}{\sigma_b^2} e^{\frac{-k^2}{2\sigma_b^2}}$$

$$\frac{N_0 \sigma_b^2}{N_b \sigma_0^2} = e^{\frac{k^2}{2\sigma_0^2} - \frac{k^2}{2\sigma_b^2}}$$

$$\ln\left(\frac{N_0 \sigma_b^2}{N_b \sigma_0^2}\right) = \frac{k^2}{2\sigma_0^2} - \frac{k^2}{2\sigma_b^2}$$

$$k^2 = \frac{2\sigma_0^2 \sigma_b^2 \ln\left(\frac{N_0 \sigma_b^2}{N_b \sigma_0^2}\right)}{\sigma_b^2 - \sigma_0^2}$$

$$k = \sqrt{\frac{2\sigma_0^2 \sigma_b^2 \ln\left(\frac{N_0 \sigma_b^2}{N_b \sigma_0^2}\right)}{\sigma_b^2 - \sigma_0^2}}$$