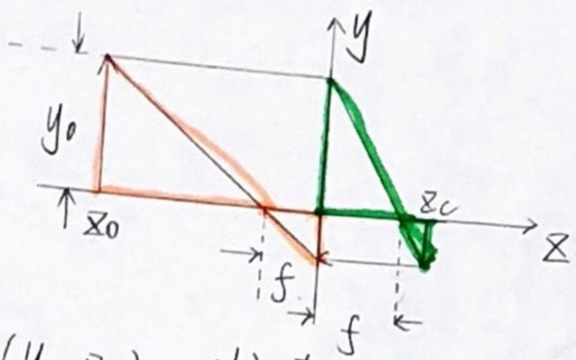
1



$\vec{x}_0 = (y_0, z_0)$ object

$\vec{x}_c = (y_c, z_c)$ image

Use the knowledge of similar triangles.

$$\frac{y_0}{-y_c} = \frac{-z_0 - f}{f}$$

$$\Rightarrow \frac{f}{z_c - f} = \frac{-z_0 - f}{f}$$

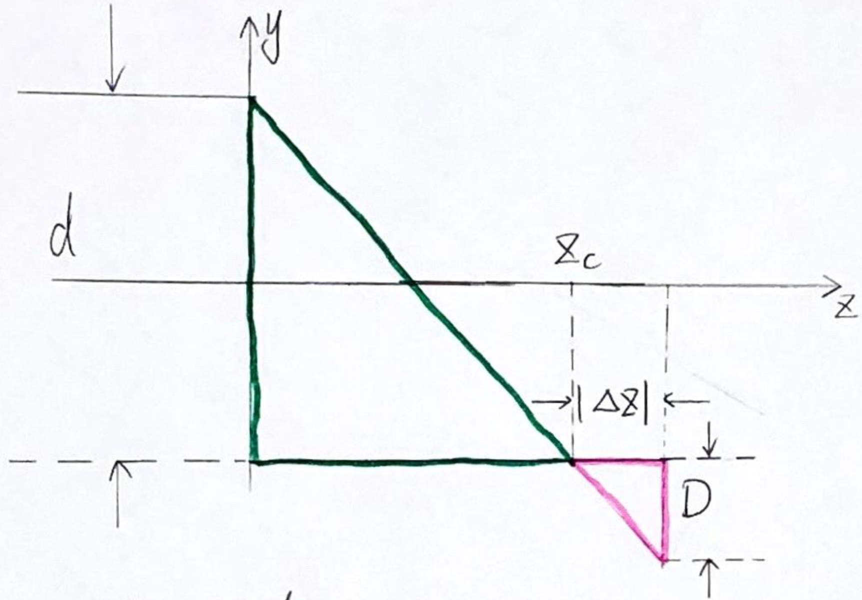
$$\frac{y_0}{-y_c} = \frac{f}{z_c - f}$$

After manipulating terms

$$\Rightarrow z_0 z_c = z_0 f - f z_c$$

Then divide both sides by $z_0 z_c f$

$$\boxed{\frac{1}{-z_0} + \frac{1}{z_c} = \frac{1}{f}}$$



Use the knowledge of similar triangles

$$\frac{d}{z_c} = \frac{D}{|\Delta z|}$$

D: the diameter of the blur circle

d: the lens diameter

$$D = d\frac{|\Delta z|}{z_c}$$

3] Diameter of eye = 24mm

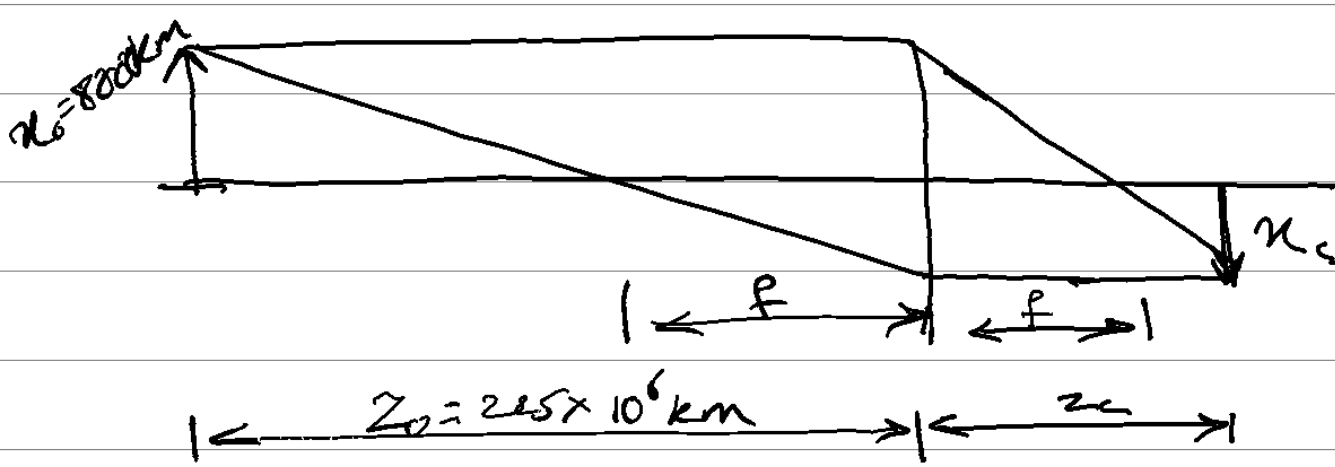
Area of hemisphere = $\frac{\pi d^2}{2} = \frac{3.14 \times 24^2}{2}$

$= 904.32 mm^2$

$\approx 900 mm^2$

Receptors per mm^2 = $\frac{150,000,000}{900}$

$= 166,666$ receptors per mm^2



$x_c = \frac{f}{225 \times 10^6 \times 10^3} \times 8000 \times 10^3$

$= \frac{f}{28125}$

Area of image $= \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{24}{28125}\right)^2$

No. of receptors $= \frac{\pi}{4}\left(\frac{24}{28125}\right)^2 \times 166{,}166$

$= 0.095$

$\therefore$ Image of mars is not registered on any receptor.

4] $R^w = \{ \vec{x}^\omega \mid \vec{x}^\omega = \vec{s}^\omega + \alpha \vec{t}^\omega, 0 \leq \alpha \leq \infty \}$

$L^c = \{ \vec{x}^c \mid \vec{x}^c = (1-\beta)\vec{s}^c + \beta \vec{t}^c \}$

As per Camera Projection Equation

$$\vec{x}^c = \vec{x}^\omega \frac{|\vec{F}|^2}{\vec{x}^\omega \cdot \vec{F}}$$

At $\alpha = 0$, $\vec{x}^c = \frac{\vec{s}^\omega |\vec{F}|^2}{\vec{s}^\omega \cdot \vec{F}}$ which is $\vec{s}^c$

$\therefore \vec{s}^c = \frac{\vec{s}^\omega |\vec{F}|^2}{\vec{s}^\omega \cdot \vec{F}}$ or $\beta = 0$

At $\alpha = \infty$,

$$\vec{x}^c = \vec{t}^\omega \frac{|\vec{F}|^2}{\vec{t}^\omega \cdot \vec{F}} = \vec{t}^c$$

$\therefore \quad \beta = 1$ at $\alpha = \infty$

$$\vec{x}_c = (1-\beta)\vec{S}_c + \beta \vec{t}_c$$

$$= (1-\beta)\vec{S}^\omega \frac{|\vec{F}|^2}{\vec{S}^\omega \cdot \vec{F}} + \beta \, \vec{t}^\omega \frac{|\vec{F}|^2}{\vec{t}^\omega \cdot \vec{F}}$$

$$= \frac{\vec{S}^\omega |\vec{F}|^2}{\vec{S}^\omega \cdot \vec{F}} + \beta \left(\frac{\vec{t}^\omega |\vec{F}|^2}{\vec{t}^\omega \cdot \vec{F}} - \frac{\vec{S}^\omega |\vec{F}|^2}{\vec{S}^\omega \cdot \vec{F}} \right)$$

$$\vec{x}^\omega \frac{|\vec{F}|^2}{\vec{x}^\omega \cdot \vec{F}} = \frac{\vec{S}^\omega |\vec{F}|^2}{\vec{S}^\omega \cdot \vec{F}} + \beta \left(\vec{t}^\omega \frac{|\vec{F}|^2}{\vec{t}^\omega \cdot \vec{F}} - \frac{\vec{S}^\omega |F|^2}{\vec{S}^\omega \cdot \vec{F}} \right)$$

$$\frac{\vec{S}^\omega |F|^2 + \alpha \vec{t}^\omega |\vec{F}|^2}{\vec{S}^\omega \cdot \vec{F} + \alpha \vec{t}^\omega \cdot \vec{F}} = \frac{\vec{S}^\omega |F|^2}{\vec{S}^\omega \cdot \vec{F}} + \beta \left(\cdots \right)$$

Let $\vec{S}^{\omega} \cdot \vec{F} = A$

$\vec{t}^{\omega} \cdot \vec{F} = B$

$$\therefore \frac{\vec{S}^{\omega}|\vec{F}|^2 + \alpha \vec{t}^{\omega}|\vec{F}|^2}{A + \alpha B} = \frac{\vec{S}^{\omega}|\vec{F}|^2}{A} + \beta\left[\frac{\vec{t}^{\omega}|\vec{F}|^2}{B} - \frac{\vec{S}^{\omega}|\vec{F}|^2}{A}\right]$$

$$\therefore \beta = \left[\frac{\vec{S}^{\omega}|\vec{F}|^2 + \alpha \vec{t}^{\omega}|\vec{F}|^2}{A + \alpha B} - \frac{\vec{S}^{\omega}|\vec{F}|^2}{A}\right] \times \left[\frac{AB}{A\vec{t}^{\omega}|\vec{F}|^2 - B\vec{S}^{\omega}|\vec{F}|^2}\right]$$

thus α & β are non-linearly related