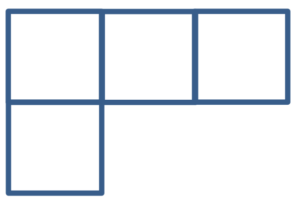
CS/RBE 549 Computer Vision Name:________________________________
Fall 2019 Due: Monday 11 Nov 2019

HW #9

1. **Chain Code (7 pts):** A shape may be represented in a similar fashion to a chain code by using real and imaginary numbers to represent consecutive edge segments in the same direction. A run of length 1 in the positive horizontal direction would be represented by 1, a run of length 1 in the positive vertical direction by $j = \sqrt{-1}$, and runs in the negative directions by −1 and −*j*. A run of length *n* is represented by *n*, *nj*, −*n*, or −*nj* as appropriate. For example the shape:



is represented by: [1, *j*, 2, *j*,−3,−2*j*]

a. Compare this code with the 4-connected chain code that takes on values from $\{0, \ldots, 3\}$.

b. Show that for any shape *S*, the corresponding code *C* has the property that

$$\sum_i c(i) = 0$$

c. We can consider smoothing this shape representation by combining adjacent short real and imaginary runs into a single complex run. For example,
[1, *j*, 2, *j*,−3,−2*j*] → [1+*j*, 2, *j*,−3,−2*j*] → [1+*j*, 2+*j*,−3,−2*j*] etc.
The more runs are combined, the more smoothing takes place. Give examples of:
• A shape that can be reasonably smoothed this way
• A shape that cannot be reasonably smoothed this way.
Be sure to define "reasonably" in this context.

d. Suppose one takes the Discrete Fourier Transform of this code according to

$$C(\omega) = \sum_{i=0}^{N-1} c(i) e^{-2\pi j \frac{i\omega}{N}}$$

What is $C(\omega = 0)$?

2. **Object Representation by Basis Functions (8 pts)::** Ima Robot proposes to represent shapes by functions $x(t)$ and $y(t)$ for $-1 \leq t \leq +1$. A shape begins at $t = -1$ and ends at

$t = +1$. In order to represent the shape more compactly, the functions $x(t)$ and $y(t)$ can be treated as Nth-degree polynomials.

$$x(t) = \sum_{i=0}^{N} a_i\, t^i \text{ and } y(t) = \sum_{i=0}^{N} b_i\, t^i$$

where a_i and b_i are coefficients.

a. In general, is this representation invariant to translation, scaling, and rotation? Explain.

b. For the unit circle, $x(t) = \cos \pi t$ and $y(t) = \sin \pi t$, what are the coefficients a_i and b_i for $0 \leq i \leq 3$? Hint: Consider a series expansion of sin and cos.

3. **Object Representation (10 pts):** We can represent an object by its boundary $(x(s), y(s)), 0 \leq s \leq S$ where S is the length of the object's boundary and s is distance along that boundary from some arbitrary starting point. We can combine x and y into a single complex function $z(s) = x(s) + jy(s)$. The Discrete Fourier Transform (DFT) of z is

$$Z(k) = \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s), 0 \leq k \leq S-1$$

We can use the coefficients $Z(k)$ to represent the object boundary. The limit on s is S-1 because for a closed contour $z(S) = z(0)$. The Inverse Discrete Fourier Transform is

$$z(s) = \frac{1}{S} \sum_{k=0}^{S-1} e^{+2\pi j \frac{ks}{S}} Z(k), 0 \leq k \leq S-1$$

a. Suppose that the object is translated by $(\Delta x, \Delta y)$, that is, $z'(s) = z(s) + \Delta x + j\Delta y$. How is z''s DFT $Z'(k)$ related to $Z(k)$?

b. Suppose that the object is scaled by integer constant c, that is, $z'(s) = cz(\lfloor s/c \rfloor)$, where $\lfloor \cdot \rfloor$ is the floor function with $\lfloor 1.5 \rfloor = 1$, etc. Note that the length of the scaled object $S' = cS$. How is z''s DFT $Z'(k)$ related to $Z(k)$?

c. What object has $z(s) = [x_0 + R \cos \frac{2\pi s}{S}] + j[y_0 + R \sin \frac{2\pi s}{S}]$? Sketch it.

d. What is $Z(k)$ corresponding to $z(s)$ from Part c? Hint: Most coefficients are 0.