CS/RBE 549 Computer Vision Name:_____
Fall 2019 Due: Monday 23 Sep 2019

HW #4

0. (Not to turn in, 0 pts) Use OpenCV or Matlab to compute the Sobel edges of an image. In addition, compute the Marr-Hildreth edges (zero-crossings of $\nabla^2 g_\sigma * f$) for various values of σ. Try 1, 2, 4, 8, 16. Do you get closed contours?

1. (9 pts) Show that
   a. $f(\vec{x}) = g(\vec{x}) * h(\vec{x})$ has Fourier Transform $F(\vec{\omega}) = G(\vec{\omega}) \times H(\vec{\omega})$. Hint: Write out the F.T. and change variables.

   b. In 1-D $df(x)/dx$ has Fourier Transform $j\omega F(\omega)$, assuming that $f(x) \to 0$ as $x \to \pm\infty$. Hint: Integrate by parts.

   c. In 2-D the Laplacian operator $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$ has Fourier Transform $-|\vec{\omega}|^2$. Hint: Use Part b. repeatedly and the fact that $\vec{\omega} = \begin{bmatrix} u \\ v \end{bmatrix}$.

2. (9 pts) Ima Robot proposes an edge detector as follows:

   Compute the Fourier Transform $F(\vec{\omega})$ of image $f(\vec{x})$.

   Multiply $F(\vec{\omega})$ by $G_1(\vec{\omega}) = e^{-\frac{1}{2}\sigma_1{}^2|\vec{\omega}|^2}$ to form $H_1(\vec{\omega})$.

   Multiply $F(\vec{\omega})$ by $G_2(\vec{\omega}) = e^{-\frac{1}{2}\sigma_2{}^2|\vec{\omega}|^2}$ to form $H_2(\vec{\omega})$.

   Compute $H_3(\vec{\omega}) = \frac{H_2(\vec{\omega}) - H_1(\vec{\omega})}{\sigma_2 - \sigma_1}$.

   Compute $h_3(\vec{x})$ as the Inverse Fourier Transform of $H_3(\vec{\omega})$.

   Find zero-crossings of $h_3(\vec{x})$.

   a. Describe how $h_3(\vec{x})$ can be computed by a single convolution with some kernel $g(\vec{x})$. What is the convolutional kernel $g(\vec{x})$?

b. If $F(\vec{\omega}) = 1$, that is, the image has a "flat" spectrum, sketch $H_3(\vec{\omega})$. Because $H_3(\vec{\omega})$ is rotationally symmetric, that is, $H_3(\vec{\omega}) = H_3(\rho)$, where $\rho = \sqrt{u^2 + v^2}$, you only need to show a slice through H_3.

c. As $\sigma_2 \to \sigma_1$, is this a good edge detector, that is, do zero-crossings of h_3 occur at edges? Why or why not? Hint: Consider $G(\vec{\omega})$ as $\sigma_2 \to \sigma_1$.

3. (10 pts) Ima Robot proposes the following operators to detect diagonally oriented edges:

1	1	0
1	0	-1
0	-1	-1

NE

0	1	1
-1	0	1
-1	-1	0

NW

a. How are these operators related to the Sobel H and V operators?

b. Suggest two different ways in which to combine the NW and NE operators into a single measure of edge strength. What are the relative strengths and weaknesses of each?

c. Express the NW operator as the convolution of two different 2×2 operators.

d. Show that |NW*I| + |NE*I| = Max(|H*I|,|V*I|)

4. (5 pts) Read Canny's PAMI article on Computational Edge Detection, available on Canvas.

a. List the 3 criteria that his approach optimizes.

b. Explain the drawback of using the Differences of Boxes edge operator.