CS/RBE 549 Computer Vision Name:______________________________
Fall 2019 Due: Monday 25 Nov 2019

HW #11

1. **Inexact Match (7 pts):** In stereo imaging, if the rays defined by $\vec{X}_L$ and $\vec{X}_R$ do not intersect, we can find $\vec{X}^W$ anyway by minimizing an error measure. One way to do this is to project $\vec{X}^W$ into the left and right image planes to give $\vec{X}_L{}'$ and $\vec{X}_R{}'$. The error E is defined as the squared difference between the observed image points $\vec{X}_L$ and $\vec{X}_R$ and the projected image points $\vec{X}_L{}'$ and $\vec{X}_R{}'$.
   a. Show that for the simplified parallel optical axis camera geometry used in class where $\vec{f} \cdot \vec{B} = 0, R = I, \vec{T} = \pm\frac{\vec{B}}{2}$
   $$E \equiv \left|\vec{X}_L' - \vec{X}_L\right|^2 + \left|\vec{X}_R' - \vec{X}_R\right|^2$$
   $$= \left(\frac{f}{z^W}\left(x^W + \frac{b}{2}\right) - x_L\right)^2 + \left(\frac{f}{z^W}y^W - y_L\right)^2 + \left(\frac{f}{z^W}\left(x^W - \frac{b}{2}\right) - x_R\right)^2 + \left(\frac{f}{z^W}y^W - y_R\right)^2$$
   b. By differentiating E with respect to x^W and y^W, show that
   $$x^W = \frac{x_L+x_R}{2}\frac{z^W}{f} \text{ and } y^W = \frac{y_L+y_R}{2}\frac{z^W}{f}$$
   c. By differentiating E with respect to z^W and using the results of b., show that
   $$z^W = \frac{fb}{\Delta_x}$$
   and conclude that
   $$\vec{X}^W = \vec{X}_{AVG}\frac{\left|\vec{B}\right|^2}{\vec{B}\cdot\vec{\Delta}}$$

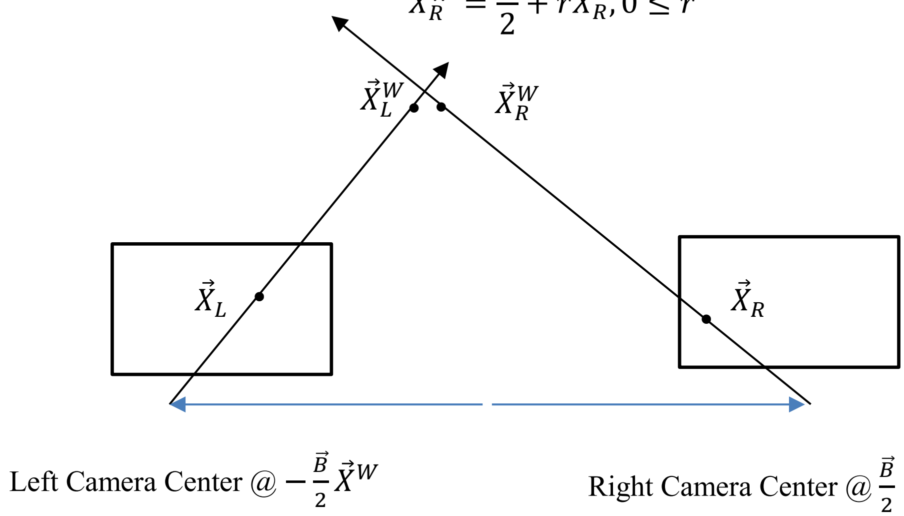
2. **Reference Image (4 pts):** Sometimes we treat one image in a stereo pair as a “reference image” aligned with world coordinates. The other image and disparity are taken with respect to the reference image. Suppose that the right image is the reference image. Then we have
   $$\vec{X}_R^C = \vec{X}^W \text{ and } \vec{X}_L^C = \vec{X}^W + \vec{B}$$
   Show that in this formulation, the world point is given by
   $$\vec{X}^W = \vec{X}_{AVG}\frac{\left|\vec{B}\right|^2}{\vec{B}\cdot\vec{\Delta}} - \frac{\vec{B}}{2}$$
   You may assume that the left and right image rays intersect, although as Problem 1. shows, they don’t have to.

3. **Inexact Match, Alternative Version (10 pts):** Another way to determine distance when $\vec{X}_L$ and $\vec{X}_R$ do not intersect is to find the point $\vec{X}^W$ closest to the rays defined by $\vec{X}_L$ and $\vec{X}_R$. The left image ray is defined by

$$\vec{X}_L^W = -\frac{\vec{B}}{2} + l\vec{X}_L, 0 \leq l$$

Similarly, the right image ray is defined by

$$\vec{X}_R^W = \frac{\vec{B}}{2} + r\vec{X}_R, 0 \leq r$$



The goal is to find points $\vec{X}_L^W$ and $\vec{X}_R^W$ on the left and right image rays that are closest to each other; the point equidistant between them is the desired $\vec{X}^W$. We can find these points by minimizing the following:

$$\min_{l,r} \left|\vec{X}_L^W - \vec{X}_R^W\right|^2$$

Show that the solution is given by

$$\vec{X}^W = \frac{1}{2}\frac{\left(\left|\vec{X}_R\right|^2(\vec{X}_L \cdot \vec{B}) - (\vec{X}_L \cdot \vec{X}_R)(\vec{X}_R \cdot \vec{B})\right)\vec{X}_L + \left((\vec{X}_L \cdot \vec{X}_R)(\vec{X}_L \cdot \vec{B}) - \left|\vec{X}_L\right|^2(\vec{X}_R \cdot \vec{B})\right)\vec{X}_R}{\left|\vec{X}_L\right|^2\left|\vec{X}_R\right|^2 - (\vec{X}_L \cdot \vec{X}_R)^2}$$