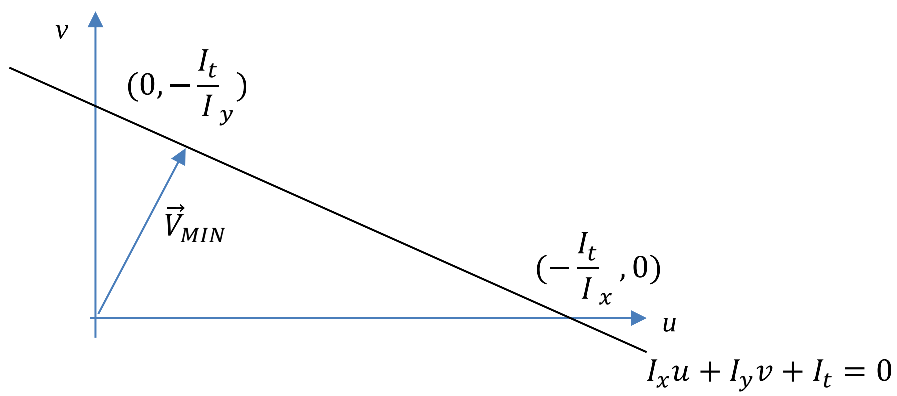
CS/RBE 549 Computer Vision Name:________________________________

Fall 2019 Due: Monday 4 Nov 2019

HW #8

1. **Smallest Optical Flow (4 pts):** What velocity $\vec{V}_{MIN}$ that satisfies the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ has the smallest magnitude $|\vec{V}|$? Hint: This can be solved geometrically as was outlined in class by considering the OFCE in u, v space.



2. **Moving Gaussian Blob (6 pts):** A Gaussian blob is observed over time to have brightness

$$I(x, y, t) = e^{-\frac{1}{2\sigma^2}\left(t^2 - 2\left(\frac{x}{k_1} + \frac{y}{k_2}\right)t + \left(\frac{x}{k_1} + \frac{y}{k_2}\right)^2\right)}$$

   a. What are I_x, I_y, and I_t? Hint: You should find that these derivatives have a simple form.

   b. The Optical Flow Constraint Equation is $I_x u + I_y v + I_t = 0$. Write this out using the results of Part a. and simplify it as much as possible. For example, you should be able to cancel terms that occur in each of I_x, I_y, and I_t.

3. **Quadratic Optical Flow (8 pts):** Suppose the image brightness is given by

$$I(x, y, t) = I_0 + \frac{1}{2}[(x - c_1 t)^2 + (y - c_2 t)^2]$$

   a. What are I_x, I_y, and I_t? Hint: You should find that these derivatives have a simple form.

   b. Express the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ in the simplest terms possible for this image sequence.

   c. The equation from b. must hold for all *x*, *y*, and *t*. Find a constant solution for *u* and *v* that makes this true, that is, such that *u* and *v* do not depend on *x*, *y*, and *t*.

4. **Iterative Optical Flow(8 pts):** We saw in class an iterative method for computing optical flow, where at each iteration, the optical flow $u(x,y), v(x,y)$ is updated according to

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}^{\text{new}} = \begin{bmatrix} \lambda I_x^2 + 4 & \lambda I_x I_y \\ \lambda I_x I_y & \lambda I_y^2 + 4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n \in \text{neighbors}(x,y)} u^{\text{old}}(n) - \lambda I_x I_t \\ \sum_{n \in \text{neighbors}(x,y)} v^{\text{old}}(n) - \lambda I_y I_t \end{bmatrix}$$

a. Show that this is equivalent to

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}^{\text{new}} = \frac{1}{4\lambda I_x^2 + 4\lambda I_y^2 + 16} \begin{bmatrix} \lambda I_y^2 + 4 & -\lambda I_x I_y \\ -\lambda I_x I_y & \lambda I_x^2 + 4 \end{bmatrix} \begin{bmatrix} \sum_{n \in \text{neighbors}(x,y)} u^{\text{old}}(n) - \lambda I_x I_t \\ \sum_{n \in \text{neighbors}(x,y)} v^{\text{old}}(n) - \lambda I_y I_t \end{bmatrix}$$

b. Show that this is equivalent to update equations

$$u^{\text{new}}(x,y) = \bar{u}^{\text{old}} - \frac{I_x}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x \bar{u}^{\text{old}} + I_y \bar{v}^{\text{old}} + I_t \right)$$

$$v^{\text{new}}(x,y) = \bar{v}^{\text{old}} - \frac{I_y}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x \bar{u}^{\text{old}} + I_y \bar{v}^{\text{old}} + I_t \right)$$

where $\bar{u}^{\text{old}}, \bar{v}^{\text{old}}$ are the averages of the 4 neighbors of $u(x,y), v(x,y)$. Hint: You only need to show this for u^{new} because v^{new} follows an identical derivation.

c. In the case that $\lambda = 0$, what do the update equations reduce to?