Q1.

a) $$g(x) * h(x) = \int_{-\infty}^{\infty} g(x)\, h(z-x)\, dx$$

$$F\{g(x) * h(x)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x)\, h(z-x)\, dx\, e^{-jwz}\, dz$$

$$= \int_{-\infty}^{\infty} g(x) \int_{-\infty}^{\infty} h(z-x)\, e^{-jwz}\, dz\, dx$$

let : $y = z-x \quad dy = dz$

$$= \int_{-\infty}^{\infty} g(x) \int_{-\infty}^{\infty} h(y)\, e^{-jw(y+x)}\, dy\, dx$$

$$= \int_{-\infty}^{\infty} g(x) \int_{-\infty}^{\infty} h(y)\, e^{-jwy}\, dy\, e^{-jwx}\, dx$$

$$= \int_{-\infty}^{\infty} g(x)\; H(w)\; e^{-jwx}\, dx$$

$$= H(w) \int_{-\infty}^{\infty} g(x)\, e^{-jwx}\, dx$$

$$= H(w) \cdot G(w)$$

b) $F\left\{\frac{df(x)}{dx}\right\} = \int_{-\infty}^{\infty} \frac{df(x)}{dx} e^{-jwx} dx$

Let $u = f(x)$, $du = df(x) = \frac{df(x)}{dx} dx$

$v = e^{-jwx}$, $\frac{du}{dx} = -jwe^{-jwx}$, $du = -jwe^{-jwx} dx$

$\Rightarrow \int_{-\infty}^{\infty} v \cdot du$

$= uv - \int u\, dv$

$= f(x) \cdot e^{-jwx} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(x)\,(-jw)\, e^{-jwx} dx$

$\because f(x) \to 0$, when $x \to \pm\infty$

$= 0 - (-jw) \int_{-\infty}^{\infty} f(x)\, e^{-jwx} dx$

$= jw\, F(w)$

c) $\nabla^2 f(x,y) = \frac{\partial^2}{\partial x^2} f(x,y) + \frac{\partial^2}{\partial y^2} f(x,y)$

$$\mathcal{F}\{\nabla^2 f(x,y)\} = \mathcal{F}\left\{\frac{\partial^2}{\partial x^2} f(x,y)\right\} + \mathcal{F}\left\{\frac{\partial^2}{\partial y^2} f(x,y)\right\}$$

$$= \mathcal{F}\left\{\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f(x,y)\right)\right\} + \mathcal{F}\left\{\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} f(x,y)\right)\right\}$$

$\because$ part b)

$$= ju\,\mathcal{F}\left\{\frac{\partial}{\partial x} f(x,y)\right\} + jv\,\mathcal{F}\left\{\frac{\partial}{\partial y} f(x,y)\right\}$$

$$= (ju)(ju)\,\mathcal{F}\{f(x,y)\} + (jv)(jv)\,\mathcal{F}\{f(x,y)\}$$

$$= -u^2\,\mathcal{F}\{f(x,y)\} + -v^2\,\mathcal{F}\{f(x,y)\}$$

$$= -(u^2+v^2)\,F(u,v)$$

since $\vec{\omega} = \begin{bmatrix} u \\ v \end{bmatrix}$

$$\mathcal{F}\{\nabla^2\} = -|\vec{\omega}|$$

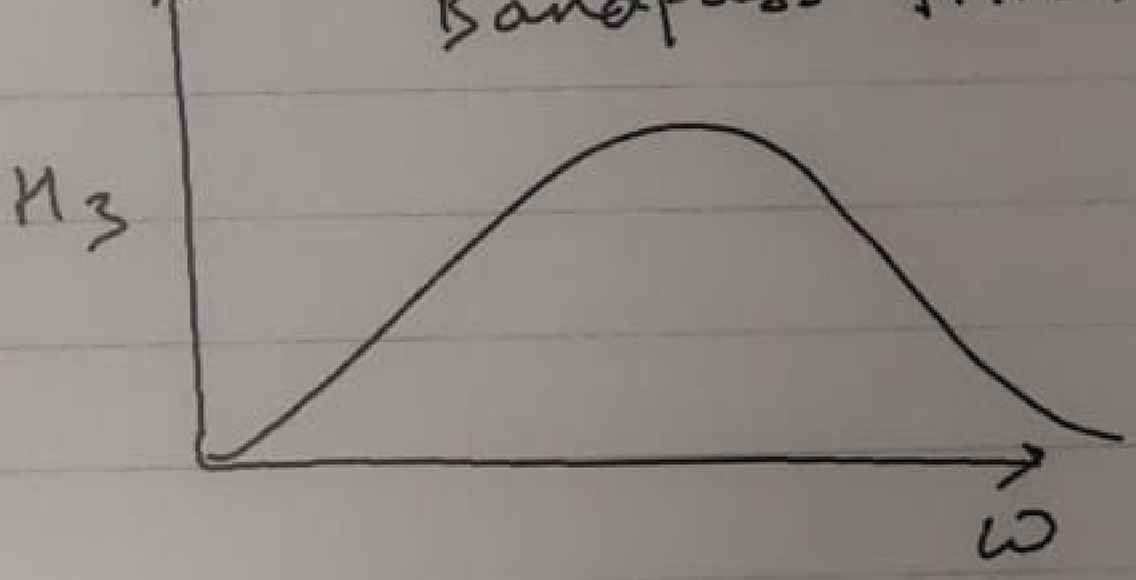
2]

a) $g(\vec{x})$ must be inverse fourier transform of

$$\frac{G_2(\vec{\omega}) - G_1(\vec{\omega})}{\sigma_2 - \sigma_1}$$

i.e. $$g(\vec{x}) = \frac{1}{\sigma_2 - \sigma_1}\left(\frac{1}{2\pi\sigma_2^2} e^{-\frac{1}{2}\frac{|\vec{x}|^2}{\sigma_2^2}} - \frac{1}{2\pi\sigma_1^2} e^{-\frac{1}{2}\frac{|\vec{x}|^2}{\sigma_1^2}}\right)$$

b) Bandpass filter



c) As $\sigma_2 \to \sigma_1$,

$$G(\vec{\omega}) \to \frac{\partial G_{\sigma_1}}{\partial \sigma} = \left(\frac{1}{\sigma}\right)\left(-\frac{1}{2} 2\sigma |\vec{\omega}|^2 e^{-\frac{1}{2}\sigma^2|\vec{\omega}|^2}\right)$$

$$= -\sigma|\vec{\omega}|^2 e^{-\frac{1}{2}\sigma^2|\vec{\omega}|^2}$$

This is the Fourier Transform of the Laplacian of a Gaussian $\nabla^2 g_\sigma$. Zero-crossing of h_3 will occur at zero-crossings of the Laplacian of a Gaussian. This is a good edge detector.

3.

(a) Sobel $H = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$ $V = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

$$V+H = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & -2 \end{bmatrix} = 2\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}}_{NE}$$

$$NE = \frac{1}{2}(V+H)$$

$$V-H = \begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & 2 \\ -2 & -2 & 0 \end{bmatrix} = 2\underbrace{\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}}_{NW}$$

$$NW = \frac{1}{2}(V-H)$$

(b) As long as the solution is reasonable.

(c) Assume $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ $\text{conv2}(A, B) = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Q3 d) $eq^n \; ① = |NW * I| + |NE * I|$

from 3 a) we know

$NW = \frac{1}{2}(V - H)$

$NE = \frac{1}{2}(V + H)$

$\therefore eq^n \; ① = \left|\frac{1}{2}(V-H) * I\right| + \left|\frac{1}{2}(V+H) * I\right|$

$= \frac{1}{2}\left[|V*I - H*I| + |V*I + H*I|\right]$

case ① $\max(|H*I|, |V*I|) = |V*I|$

$eq^n \; ① = \frac{1}{2}\left[|V*I| - |H*I| + |V*I| + |H*I|\right]$

$= \frac{1}{2}\left[2|V*I|\right]$

$= |V*I|$ ---- same as case ①

case ② $\max(|H*I|, |V*I|) = |H*I|$

$eq^n \; ① = \frac{1}{2}\left[-|V*I| + |H*I| + |V*I| + |H*I|\right]$

$= \frac{1}{2}\left[2|H*I|\right]$

$= |H*I|$ ---- same as case ②

(i) The 3 criterios optimised by hi's approach :-

→ Good detection

→ Good localisation

→ Only one response to a single edge.

ii) Drawback of using differences of boxes edge operator :-

In the case of noisy step edges, the diff. of boxes edge opr tends to display multiple maxima, because of it's high bandwidth.

Extra edges, which are obviously undesirable, are acccomodated in the final output.