CS/RBE 549 Computer Vision Name: **Solutions**

Fall 2015 Monday 12 Oct 2015

Exam #1

1. **Segmentation (15 pts).** An image has object and background pixels whose brightness values are distributed according to the exponential distribution with parameters α_o and α_b with $\alpha_o < \alpha_b$. α_o and α_b are each less than 0.98. The probability of a pixel having brightness *x* is given by

$$P_o(x) = (1 - \alpha_o){\alpha_o}^x \text{ and } P_b(x) = (1 - \alpha_b){\alpha_b}^x$$

It is desired to segment the image into object and background. Find the decision rule that maximizes the probability of a correct decision.

Decide that the pixel is object if $P_o(x) > P_b(x)$, otherwise it is background.

$$\boldsymbol{P_o(x) > P_b(x)}$$

$$\boldsymbol{(1 - \alpha_o){\alpha_o}^x > (1 - \alpha_b){\alpha_b}^x}$$

$$\boldsymbol{\frac{{\alpha_o}^x}{{\alpha_b}^x} > \frac{1 - \alpha_b}{1 - \alpha_o}}$$

$$\boldsymbol{x \ln\frac{\alpha_o}{\alpha_b} > \ln\frac{1 - \alpha_b}{1 - \alpha_o}}$$

$$\boldsymbol{x < \frac{\ln\frac{1 - \alpha_b}{1 - \alpha_o}}{\ln\frac{\alpha_o}{\alpha_b}}}$$

Note the change from > to < because $\alpha_o < \alpha_b$ makes the ln negative.

2. **Edge Detection (30 pts).** Ima Robot proposes an edge detector as follows:

Convolve image $f(\vec{x})$with $g_{\sigma_1} = \frac{1}{2\pi\sigma_1^2} e^{-\frac{1}{2}\frac{|\vec{x}|^2}{\sigma_1^2}}$ to form $h_1(\vec{x})$.

Convolve $f(\vec{x})$with g_{σ_2} to form $h_2(\vec{x})$.

Compute $h_3(\vec{x}) = \frac{h_2(\vec{x}) - h_1(\vec{x})}{\sigma_2 - \sigma_1}$.

Find zero-crossings of $h_3(\vec{x})$.

a. (10 pts) Describe how $h_3(\vec{x})$ can be computed by a single convolution with some kernel $g(\vec{x})$. What is the convolutional kernel $g(\vec{x})$?

Compute $h_3(\vec{x})$ by convolving $f(\vec{x})$ with the single kernel $g(\vec{x}) = \frac{g_{\sigma_2}(\vec{x}) - g_{\sigma_1}(\vec{x})}{\sigma_2 - \sigma_1}$

b. (5 pts) Sketch $g(\vec{x})$.

c. (10 pts) What is the Fourier transform $G(\vec{u})$ of $g(\vec{x})$?

$$\boldsymbol{G(\vec{u}) = \frac{G_{\sigma_2}(\vec{u}) - G_{\sigma_1}(\vec{u})}{\sigma_2 - \sigma_1} = \frac{e^{-\frac{1}{2}\sigma_2^2|\vec{u}|^2} - e^{-\frac{1}{2}\sigma_1^2|\vec{u}|^2}}{\sigma_2 - \sigma_1}}$$

d. (10 pts) As $\sigma_2 \to \sigma_1$, is this a good edge detector, that is, do zero-crossings of h_3 occur at edges? Why or why not? Hint: Consider $G(\vec{u})$ as $\sigma_2 \to \sigma_1$.

As $\boldsymbol{\sigma_2 \to \sigma_1,\ G(\vec{u}) \to \frac{\partial G_{\sigma_1}}{\partial \sigma} = \left(\frac{1}{\sigma}\right)\left(-\frac{1}{2}2\sigma|\vec{u}|^2 e^{-\frac{1}{2}\sigma^2|\vec{u}|^2}\right) = -\sigma|\vec{u}|^2 e^{-\frac{1}{2}\sigma^2|\vec{u}|^2}}$, which is the Fourier Transform of the Laplacian of a Gaussian $\boldsymbol{\nabla^2 g_\sigma}$. Zero-crossing of $\boldsymbol{h_3}$ will occur at zero-crossing of the Laplacian of a Gaussian. This is a good edge detector.

3. **Hough Transform.** In this problem *x*, *y*, *b*, and *m* may be positive or negative, integers or fractions. 2 lines in (*m*,*b*) space are given by

L1: $b = 1$

L2: $b = -2m + 2$

a. (10 pts) What are points P1 and P2 in (*x*,*y*) space corresponding to each of these lines?

P1: $\boldsymbol{b = m0 + 1 \Rightarrow 1 = -m0 + b \Rightarrow (x, y) = (0, 1)}$

P2: $\boldsymbol{b = -2m + 2 \Rightarrow 2 = m2 + b \Rightarrow (x, y) = (2, 2)}$

b. (10 pts) What equation describes the line passing through points P1 and P2?

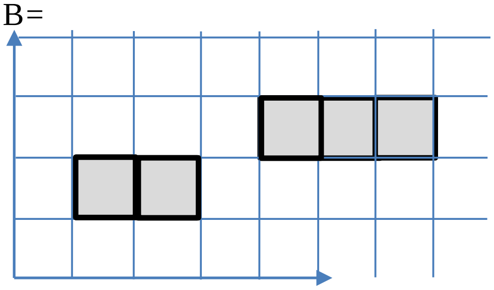
The intersection of L1 and L2: $\boldsymbol{(m, b) = (1/2, 1), y = \frac{m}{2} + 1}$

c. (10 pts) Line L3 in (*m*,*b*) space passes through (*m*,*b*) = (0,0). What is its corresponding P3 such that P3 lies along the line from part b?

Line L3 passes through (*m*,*b*) = (0,0) and $\boldsymbol{(m, b) = (1/2, 1)}$. $\boldsymbol{b = 2m \Rightarrow (x, y) = (-2, 0)}$

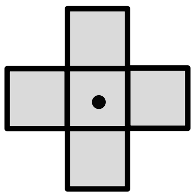
4. **Morphology.** The following binary image B

B=

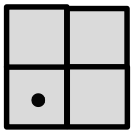


is operated on by structuring elements S1 and S2

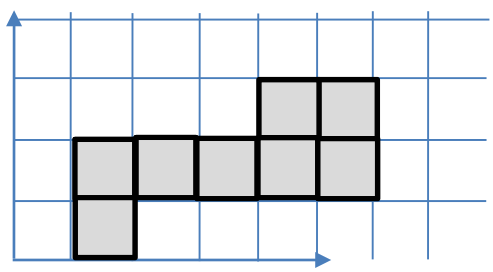
S1=



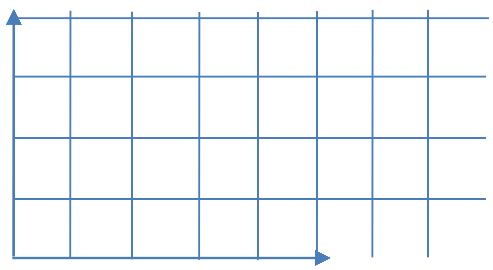
S2=



a. (10 pts) What is obtained by dilating B by S1 and then eroding that result by S2?



b. (10 pts) What is obtained by dilating B by S2 and then eroding that result by S1?



An empty image is obtained.