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CS 549
HW4

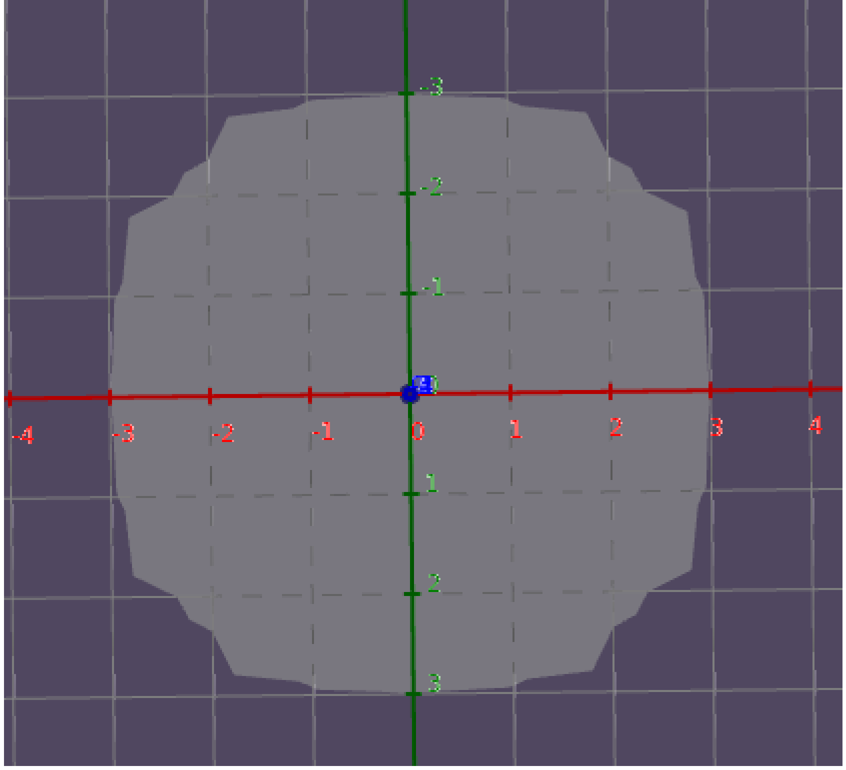
1.

   1. $F\{(g*h)(x)\} = F\{f(x)\}$, Given $f(\vec{x}) = g(\vec{x}) * h(\vec{x})$
      $F\{(g* h)(x)\}$ = 1/sqrt(2pi)* integral(inf,-inf, (g*h)(x) e^{-iwx})dx
      = 1/sqrt(2pi)* integral(inf,-inf, integral(inf,-inf, g(x)h(x-p))dp* e^{-iwx})dx
      = 1/sqrt(2pi)* integral(inf,-inf, integral(inf,-inf, g(x)h(x-p))* e^{-iwx})dx dp
      let q = x-p such that dq = dx and x = p+q therefore the above is equivalent to:
      $F\{(g*h)(x)\}$ = 1/sqrt(2pi)* integral(inf,-inf, integral(inf,-inf, g(p)h(q))* $e^{-iw(p+q)}$)dqdp
      = 1/sqrt(2pi) * integral(inf,-inf, g(p)*$e^{-iw(p)}$)dp * integral(inf,-inf, h(q)*$e^{-iw(q)}$)dq
      as integral(inf,-inf, g(p)*$e^{-iw(p)}$)dp = $F\{g(x)\}$ and integral(inf,-inf, h(q)*$e^{-iw(q)}$)dq = $F\{h(x)\}$
      $F\{(g*h)(x)\} = F\{g(x)\} * F\{h(x)\}$
   2. f(x)d/dx = f'(x)
      F(f'(w)) = jw*F(f(w))
      F(f'(x)) = integral(inf,-inf, e^{-jwx}*f'(x))dx
      Integrate by parts:
      u = e^{-jwx} , du = -jwx*e^{-jwx} dx
      v = f(x), dv = f'(x)dx
      = e^{-jwx} * f(t) – integral(inf,-inf, -jwx*e^{-jwx} *f(x)dx)
      =jx*F(f(x))
   3. F(x) = integral(inf,-inf,f(r)e^{jxr})dr
      assuming $r^2 \blacktriangledown f$ goes to 0 as r→ inf
      then using integration by parts (as shown in previous question):
      integral(inf,-inf, $\blacktriangledown^2$f(r)e^{jxr})dr = -integral(inf,-inf, $\blacktriangledown$f(r) * $\blacktriangledown$(e^{jxr}))
      again using integration by parts (as shown in previous question):
      = integral(inf,-inf,f(r) * $\blacktriangledown^2$(e^{jxr}))dr
      as $\blacktriangledown^2$(e^{jxr}) = (-$|x|^2$ *e^{ixr}) then
      integral(inf,-inf,f(r) * $\blacktriangledown^2$(e^{jxr}))dr = integral(inf,-inf,f(r) * (-$|x|^2$ *e^{ixr}))dr
      substituting for F(x) we then obtain our result
      = -$|x|^2$ * F(x)

2.

   1. As G(w) is the Fourier Transformed kernel g(x), we can simplify the proposed detector
      As H3(w) = H2(w)-H1(w)/s2-s1
      H2 = F(f(x))*F(g2(x)) = F(f ∗ g2)(x)
      H1 = F(f(x))*F(g1(x)) = F(f ∗ g1)(x)
      H3 = F(f * g2-g1)(x)/s2-s1
      h3 = (f * g2-g1)(x)/s2-s1
      Therefore the new kernel g(x) can be simplified to (g2-g1)(x)/s2-s1 where g(x) = $e^{-.5s^2*(u^2+v^2)}$

   2.



3. As s2 → s1 it becomes a less effective edge detector. As s2 → s1 then the difference between them becomes smaller and therefore more sensitive to minor changes. Although zero-crossings may happen at the edges, it is a result of being so sensitive and producing false positives.

3.

1. How are these operators related to the Sobel H and V operators?

These operators are spatially related to the [1,0,-1] vector of the Sobel operators.

2. Suggest two different ways in which to combine the NW and NE operators into a single measure of edge strength. What are the relative strengths and weaknesses of each?

Addition:
This combines the NW and NE operators such that is measures both edges in such a way that is rotationally invariant. But it being small (3x3) it is highly susceptible to noise but it is fast.

(NE x NW) * (NW x NE):
This again provides a kernel that is rotationally invariant, as a 5x5 kernel it is less susceptible to noise but slightly slower.

3. Express the NW operator as the convolution of two different 2×2 operators.

-1	-1
-1	-1

0	-1
1	0

4. Show that |NW*I| + |NE*I| = Max(|H*I|,|V*I|)

As |NW|*I + |NE| * I = (|NW|+|NE|)*I
We can eliminate I from the equation.

Then |NW|+|NE| = K =

1	2	1
2	0	2
1	2	1

For corners and center K = |H| = |V|
As for all other cells, K = 2. |H| = [0,0,2,2], |V| = [2,2,0,0]
Then K = Max(|H|,|V|)

Then K*I = Max(|H|,|V|)*I
= |NW*I| + |NE*I| = Max(|H*I|,|V*I|)

4.
   1. List the 3 criteria that his approach optimizes.
      1. Detection
      2. Localization
      3. Single Response
   2. Explain the drawback of using the Differences of Boxes (DoB) edge operator.
      The DoB edge operator is highly susceptible to noise. A noisy step, results in numerous “sharp maxima near the edge.” This causes the noise to be nearly indistinguishable from the actual edge.