1.

a) $B = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$

project x^w into camera frame

$$X_L' = RX^w + T$$

$$= X^w + \frac{B}{2}$$

then, project $X_L'^c$ into left image plane

$$X_L' = \frac{f}{Z^w} X_L'^c = \frac{f}{Z^w}\left(X^w + \frac{B}{2}\right)$$

similarly,

$$X_R' = \frac{f}{Z^w}\left(X^w - \frac{B}{2}\right)$$

So, $E \equiv |\vec{X}_L' - \vec{X}_L|^2 + |\vec{X}_R' - \vec{X}_R|^2$

$$= \left(\frac{f}{Z^w}\left(x^w + \frac{b}{2}\right) - x_L\right)^2 + \left(\frac{f}{Z^w} y^w - y_L\right)^2$$

$$+ \left(\frac{f}{Z^w}\left(x^w - \frac{b}{2}\right) - x_R\right)^2 + \left(\frac{f}{Z^w} y^w - y_R\right)^2$$

b) $$\frac{\partial E}{\partial x^W} = 2\left(\frac{f}{z^W}\left(x^W + \frac{b}{2}\right) - x_L\right)\frac{f}{z^W} + 2\left(\frac{f}{z^W}\left(x^W - \frac{b}{2}\right) - x_R\right)\frac{f}{z^W}$$

$$= \frac{2f}{z^W}\left(\frac{2f}{z^W}x^W - x_L - x_R\right)$$

When E reaches its minimum value

$$\frac{\partial E}{\partial x^W} = 0$$

So, $\frac{2f}{z^W}x^W - x_L - x_R = 0$

$$x^W = \frac{x_L + x_R}{2}\frac{z^W}{f}$$

Similarly,

$$y^W = \frac{y_L + y_R}{2}\frac{z^W}{f}$$

c)

$$\frac{\partial E}{\partial z^w} = 2\left(\frac{f}{z^w}\left(x^w + \frac{b}{2}\right) - x_L\right)\left(-\frac{f}{(z^w)^2}\left(x^w + \frac{b}{2}\right)\right)$$

$$+ 2\left(\frac{f}{z^w} y^w - y_L\right)\left(-\frac{f}{(z^w)^2} y^w\right)$$

$$+ 2\left(\frac{f}{z^w}\left(x^w + \frac{b}{2}\right) - x_R\right)\left(-\frac{f}{(z^w)^2}\left(x^w + \frac{b}{2}\right)\right)$$

$$+ 2\left(\frac{f}{z^w} y^w - y_R\right)\left(-\frac{f}{(z^w)^2} y^w\right)$$

$$= -\frac{2f}{(z^w)^2} x^w \left(\frac{2f}{z^w} - x_L - x_R\right)$$

$$+ b\left(\frac{bf}{z^w} + x_R - x_L\right) - \frac{2f}{(z^w)} y^w \left(\frac{2f}{z^w} - y_L - y_R\right)$$

$$= 0 + b\left(\frac{bf}{z^w} + x_R - x_L\right) + 0$$

$$= 0$$

So, $$z^w = \frac{fb}{x_R - x_L} = \frac{fb}{\Delta x}$$

$$X_{AVG} = \begin{bmatrix} \frac{x_L + x_R}{2} \\ \frac{y_L + y_R}{2} \\ f \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \Delta x \\ \Delta y \\ 0 \end{bmatrix}$$

So, $$X_{AVG} \frac{|B|^2}{B \cdot \Delta} = \begin{bmatrix} \frac{x_L + x_R}{2} \\ \frac{y_L + y_R}{2} \\ f \end{bmatrix} \frac{b^2}{\Delta x\, b}$$

$$= \begin{bmatrix} \frac{x_L + x_R}{2} \\ \frac{y_L + y_R}{2} \\ f \end{bmatrix} \frac{z^w}{f}$$

$$= \begin{bmatrix} \frac{x_L + x_R}{2} \\ \frac{y_L + y_R}{2} \\ z^W \end{bmatrix}$$

$$= \begin{bmatrix} x^w \\ y^w \\ z^w \end{bmatrix} = X^w$$

2. $\vec{x}^c_R = \vec{x}^w \qquad \vec{x}^c_L = \vec{x}^w + \vec{B}$

$$\vec{x}_L = \frac{f}{z^w}(\vec{x}^w + \vec{B}) \qquad \vec{x}_R = \frac{f}{z^w}(\vec{x}^w)$$

$$\vec{\Delta} = \vec{x}_L - \vec{x}_R = \frac{f}{z^w}(\vec{B}) = \frac{|\vec{f}|^2(\vec{B})}{\vec{f}\cdot\vec{x}^w}$$

$$\vec{x}_{avg} = \frac{f}{z^w}\frac{(2\vec{x}^w + \vec{B})}{2}$$

$$\vec{x}_{avg} = \frac{|f|^2\,\vec{x}^w}{\vec{f}\cdot\vec{x}^w} + \frac{|\vec{f}|^2\,\vec{B}}{2(\vec{f}\cdot\vec{x}^w)}$$

$$\vec{x}^w = \frac{x_{avg}\,|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} - \frac{\vec{B}}{2}$$

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$$\min \left| \vec{X_L}^{w} - \vec{X_R}^{w} \right|^2 = \min \left| l \cdot \vec{X_L} - r \vec{X_R} - \vec{B} \right|^2$$

Assume this expression is E

$$\begin{cases} \dfrac{dE}{dl} = 0 \\ \dfrac{dE}{dr} = 0 \end{cases} \quad \text{leads to} \quad \begin{array}{l} (l \vec{X_L} - r \vec{X_R} - \vec{B}) \times 2 \vec{X_L} = 0 \\ (l \vec{X_L} - r \vec{X_R} - \vec{B}) \times 2 \vec{X_R} = 0 \end{array}$$

$$\begin{cases} l \vec{X_L}^2 - r \vec{X_R} \vec{X_L} = \vec{B} \cdot \vec{X_L} \\ l \vec{X_L} \vec{X_R} - r \vec{X_R}^2 = \vec{B} \cdot \vec{X_R} \end{cases} \Rightarrow \begin{cases} l = \dfrac{|\vec{X_R}|^2 (\vec{B} \cdot \vec{X_L}) - (\vec{X_L} \cdot \vec{X_R})(\vec{B} \cdot \vec{X_R})}{|\vec{X_L}|^2 |\vec{X_R}|^2 - (\vec{X_L} \cdot \vec{X_R})^2} \\ r = \dfrac{(\vec{X_L} \cdot \vec{X_R})(\vec{B} \cdot \vec{X_L}) - |\vec{X_L}|^2 (\vec{B} \cdot \vec{X_R})}{|\vec{X_L}|^2 |\vec{X_R}|^2 - (\vec{X_L} \cdot \vec{X_R})^2} \end{cases}$$

so $\vec{X}^{w} = \frac{1}{2} (\vec{X_L}^{w} + \vec{X_R}^{w})$

$$= \frac{1}{2} \, \frac{\left(|\vec{X_R}|^2 (\vec{B} \cdot \vec{X_L}) - (\vec{X_L} \cdot \vec{X_R})(\vec{B} \cdot \vec{X_R}) \right) \cdot \vec{X_L} + \left((\vec{X_L} \cdot \vec{X_R})(\vec{B} \cdot \vec{X_L}) - (\vec{X_L})^2 (\vec{B} \cdot \vec{X_R}) \right) \cdot \vec{X_R}}{|\vec{X_L}|^2 \cdot |\vec{X_R}|^2 - (\vec{X_L} \cdot \vec{X_R})^2}$$