CS/RBE 549 Computer Vision Name:______________________

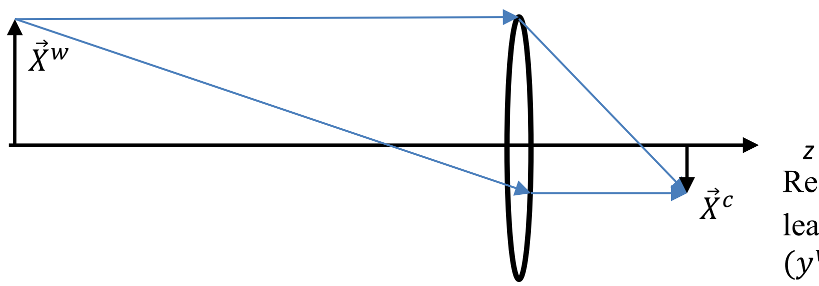
Fall 2019 Due: Wed 04 Sep 2019

HW #1

1. In addition to the similarities and difference from Class 01, list 5 other ways in which eyes and cameras are similar. List 5 other ways in which they are different.

2. Prove that for a thin lens, the image is in focus when

$$\frac{1}{-z^w} + \frac{1}{z^c} = \frac{1}{f}$$



Reason as follows: A ray leaving the object at $\vec{X}^w = (y^w, z^w)$ parallel to the z axis passes through the lens, then bends to pass through focal point $(0, f)$ before hitting the image plane at $\vec{X}^c = (y^c, z^c)$. If the image is in focus, then similarly, a ray leaving the object at $\vec{X}^w$ passing through the negative focal point $(0, -f)$ will be bent parallel to the z axis and hit the image plane at the same point $\vec{X}^c$.

Hint: As discussed in class, consider similar triangles from the lens to the focal point and the focal point to the image plane. There are 2 pairs of similar triangles, one for the positive and negative focal point. Then show that

$$-z^c f + z^w f = z^c z^w$$

3. Suppose that, in the imaging geometry above, the image plane is located distance $z^{c\prime} = z^c + \Delta z$ from the lens, so that the image is out of focus. Show that the blur circle has diameter $D = d\frac{|\Delta z|}{z^c}$, where d is the lens diameter.

Hint: Consider rays coming from the top and bottom of the lens that would be in focus at z^c. What happens when they hit the image plane at $z^{c\prime}$?

4. A typical human eyeball is 2.4 cm in diameter and contains roughly 150,000,000 receptors. Ignoring the fovea, assume that the receptors are uniformly distributed across a hemisphere (it is actually closer to 160°).
   a. How many receptors are there per mm^2?
   b. Mars has a diameter of 8,000 km and an average distance from Earth of 225,000,000 km. Using a value of f equal to the eye's diameter, on how many receptors does the image of Mars fall?

5. Show that a ray in the world projects to a line segment in the image as follows:
Define world ray $R^w = \{\vec{x}^w | \vec{x}^w = \vec{s}^w + \alpha \vec{t}^w, 0 \leq \alpha \leq \infty\}$. Show that it projects to camera line segment $L^c = \{\vec{x}^c | \vec{x}^c = (1-\beta)\vec{s}^c + \beta \vec{t}^c\}$ where $\vec{s}^c$ is the projection of $\vec{s}^w$ onto the image plane and $\vec{t}^c$ is the projection of ray R^w in the limit as $\alpha \rightarrow \infty$. You should find that β ranges from 0 to 1 and is related non-linearly to α.