CS/RBE 549 Computer Vision Name:________________________________
Fall 2016 Wednesday 07 Dec 2016

Exam #2

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Write your name at the top of this page now.

This examination is open book and open notes. Electronic devices are allowed to access stored information provided that they are operating in wireless mode.

No other electronic computing / communications devices are permitted. No wireless communication is allowed.

Write all your answers on the examination in the space provided. You may use the back of the examination for extra space. Partial credit will be given, but you must justify your work.

The examination will end exactly 90 minutes after it begins.
Good luck.

Problem 1: /30
Problem 2: /40
Problem 3: /30
Problem 4: /01

Total: /101

1. **Iterative Optical Flow (30 pts):** An iterative method for computing optical flow updates $u(x, y), v(x, y)$ at each iteration according to

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}^{\text{new}} = \begin{bmatrix} \lambda I_x^2 + 4 & \lambda I_x I_y \\ \lambda I_x I_y & \lambda I_y^2 + 4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n \in \text{neighbors}(x,y)} u^{\text{old}}(n) - \lambda I_x I_t \\ \sum_{n \in \text{neighbors}(x,y)} v^{\text{old}}(n) - \lambda I_y I_t \end{bmatrix}$$

Consider a local coordinate frame (x', y') where x' is aligned with the image gradient and y' is perpendicular to the image gradient. Likewise, $(u', v') = \left(\frac{dx'}{dt}, \frac{dy'}{dt}\right)$ are the image velocities in this frame. In this coordinate frame,

$$I_{x'} = \sqrt{I_x^2 + I_y^2} \text{ and } I_{y'} = 0$$

Show that the update equations

$$\begin{bmatrix} u'(x,y) \\ v'(x,y) \end{bmatrix}^{\text{new}} = \begin{bmatrix} \lambda I_{x'}^2 + 4 & \lambda I_{x'} I_{y'} \\ \lambda I_{x'} I_{y'} & \lambda I_{y'}^2 + 4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n \in \text{neighbors}} u'^{\text{old}}(n) - \lambda I_{x'} I_t \\ \sum_{n \in \text{neighbors}} v'^{\text{old}}(n) - \lambda I_{y'} I_t \end{bmatrix}$$

reduce to

$$u'^{\text{new}}(x,y) = \bar{u}'^{\text{old}} - \frac{{I_{x'}}^2 \overline{u}'^{\text{old}} + I_{x'} I_t}{I_{x'}^2 + \frac{4}{\lambda}}$$

$$v'^{\text{new}}(x,y) = \overline{v}'^{\text{old}}$$

Hint: This is not as hard as it looks.

2. **Object Representation (40 pts):** We can represent an object by its boundary $(x(s), y(s)), 0 \leq s \leq S$ where S is the length of the object's boundary and s is distance along that boundary from some arbitrary starting point. We can combine x and y into a single complex function $z(s) = x(s) + jy(s)$. The Discrete Fourier Transform (DFT) of z is

$$Z(k) = \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s), 0 \leq k \leq S-1$$

We can use the coefficients $Z(k)$ to represent the object boundary. The limit on s is S-1 because for a closed contour $z(s) = z(0)$. The Inverse Discrete Fourier Transform is

$$z(s) = \frac{1}{S} \sum_{k=0}^{S-1} e^{+2\pi j \frac{ks}{S}} Z(k), 0 \leq k \leq S-1$$

a. (10 pts) Suppose that the object is translated by $(\Delta x, \Delta y)$, that is, $z'(s) = z(s) + \Delta x + j\Delta y$. How is z''s DFT $Z'(k)$ related to $Z(k)$?

b. (15 pts) Suppose that the object is scaled by integer constant c, that is, $z'(s) = cz(s)$. For simplicity, assume that $S' = S$. How is z''s DFT $Z'(k)$ related to $Z(k)$?

c. (10 pts) What object has $z(s) = [x_0 + R\cos\frac{2\pi s}{S}] + j[y_0 + R\sin\frac{2\pi s}{S}]$? Sketch it.

d. (5 pts) What is $Z(k)$ corresponding to $z(s)$ from Part c? Hint: Most coefficients are 0.

3. **Chain Code (30 pts):** An object boundary is represented by an 8-connected chain code C with values $c_i \in \{0, \ldots, 7\}$. Each c_i represents a segment at 45° = π/4 radians, with c_i = 0 representing (1, 0), c_i = 1 representing (1, 1), c_i = 2 representing (0, 1), etc.

   Note that segments have length 1 when c_i is even, and length $\sqrt{2}$ when c_i is odd.

   Computer Vision researcher Ima Robot claims that the chain code for a closed contour of length N must obey Eq (1):

$$\sum_{i=0}^{N-1} e^{-j\frac{\pi}{4}c_i} = 0$$

   a) (15 pts) Show that Eq (1) holds for a square

$$C = [0, 0, 2, 2, 4, 4, 6, 6]$$

   and octagon

$$C = [0, 1, 2, 3, 4, 5, 6, 7]$$

   b) (15 pts) Show that Eq (1) holds for any closed contour or provide a counterexample.

4. **Teamwork (1 pt):** On a scale of 1 to 5, with 1 being the lowest, 3 is neutral and 5 being the highest, rate how well your project team is working together.