CS/RBE 549 Computer Vision Name: **Sol U. Tion**
Fall 2018 Wednesday 10 Oct 2018

Exam #1 Solutions

1. **Edge Detection (25 pts).** Ima Robot wants to convolve an image with the following convolutional kernel *K*:

K =

-1	1
1	-1

a) (5 pts) What derivative does *K* approximate? **The 2nd cross derivative** $\frac{\partial^2 I}{\partial x \partial y}$**.**

b) (5 pts) Express *K* as the convolution of 2 different, smaller kernels.

K =

-1
1

*

1	-1

c) (5 pts) What is the DFT of *K*?

DFT = $\boldsymbol{je^{-\left(j\frac{\pi k}{2}\right)}\sin\frac{\pi k}{2}\, je^{-\left(j\frac{\pi l}{2}\right)}\sin\frac{\pi l}{2} = -e^{-\left(j\frac{\pi}{2}\right)(k+l)}\sin\frac{\pi k}{2}\sin\frac{\pi l}{2}}$

d) (10 pts) What kernel results when Ima convolves *K* with the following blurring operator?

1	1
1	1

Answer:

-1	0	1
0	0	0
1	0	-1

2. **Segmentation (20 pts).** An image has object and background pixels whose brightness values are distributed according to the Rayleigh distribution with parameters σ_o and σ_b with $0 < \sigma_b < \sigma_o$. The probability of a pixel having brightness *k* is given by

$$P_o(k) = \frac{k}{{\sigma_o}^2} e^{-\frac{k^2}{2{\sigma_o}^2}} \text{ and } P_b(k) = \frac{k}{{\sigma_b}^2} e^{-\frac{k^2}{2{\sigma_b}^2}}$$

Ima Robot wants to segment the image into object and background. Assuming that background and object pixels are equally likely, find the decision rule that maximizes the probability of a correct decision, that is, pick the greater of P_o and P_b.

Label a pixel as object when

$$\frac{k}{\sigma_o{}^2} e^{-\frac{k^2}{2\sigma_o{}^2}} > \frac{k}{\sigma_b{}^2} e^{-\frac{k^2}{2\sigma_b{}^2}}$$

$$e^{\frac{k^2}{2\sigma_b{}^2}} e^{-\frac{k^2}{2\sigma_o{}^2}} > \frac{\sigma_o{}^2}{\sigma_b{}^2}$$

$$e^{k^2\frac{1}{2}\left(\frac{1}{\sigma_b^2}-\frac{1}{\sigma_o^2}\right)} > \frac{\sigma_o{}^2}{\sigma_b{}^2}$$

$$k^2\frac{1}{2}\left(\frac{1}{\sigma_b^2}-\frac{1}{\sigma_o^2}\right) > 2\ln\frac{\sigma_o}{\sigma_b}$$

$$k^2\left(\frac{\sigma_o^2-\sigma_b^2}{\sigma_b^2\sigma_o^2}\right) > 4\ln\frac{\sigma_o}{\sigma_b}$$

$$k > \sqrt{4\left(\frac{\sigma_b^2\sigma_o^2}{\sigma_o^2-\sigma_b^2}\right)\ln\frac{\sigma_o}{\sigma_b}} = 2\sigma_b\sigma_o\sqrt{\frac{\ln\sigma_o-\ln\sigma_b}{(\sigma_o^2-\sigma_b^2)}}$$

3. **Morphology (20 pts).** Ima Robot dilates binary image B with unknown structuring element S to yield output image O

$B =$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$O =$

0	0	0	0	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

a) (10 pts) What is S such that $B \oplus S = O$?

b) (10 pts) Find another structuring element S_2 that is different from the one in a) that also satisfies $B \oplus S_2 = O$. Hint: There are several. Any one will do.

a) and b) may be any two structuring elements of the following form, where x can be either 0 or 1, and • indicates the center pixel.

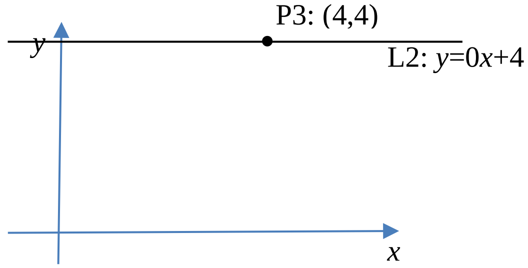
1	x	1
x	x•	x
1	x	1

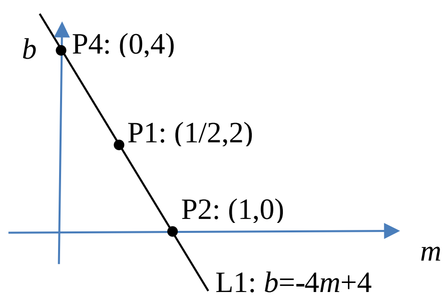
4. **Focus (10 pts).** Astronomer Stella Gazer notices that her telescope is out of focus. Every star, rather than being a point of light, appears as a circle with diameter D in the image. Assuming that the telescope has a lens with diameter d and focal length f, how far should she move the image plane to achieve perfect focus?

   For astronomy, the distance z_o to any object is effectively ∞. The focusing equation $\frac{1}{-z_o} + \frac{1}{z_c} = \frac{1}{f}$ lets us conclude that $z_c = f$. We know that the blur circle has diameter $D = d\frac{|\Delta z|}{z_c}$. Therefore, $|\Delta z| = \frac{fD}{d}$ is how far the image plane is out of focus, and is how far Stella must move it back to achieve perfect focus. Note that the direction to move the image plane cannot be determined from the information given.

5. **Hough Transform (25 pts).** In this problem x, y, b, and m may be positive or negative, integers or fractions. 2 points in (m,b) space are given by

   P1: $m = \frac{1}{2}, b = 2$; P2: $m = 1, b = 0$. Hint: It might help to construct (x,y) and (m,b) spaces.





a) (5 pts) What line L1 in (m,b) space passes through points P1 and P2?

P1 and P2 satisfy $b = -4m + 4$.

b) (10 pts) What point P3 in (x,y) space corresponds to line L1?

Because $y = mx + b$ and $4 = 4m + b$, deduce that $x = 4$ and $y = 4$. Alternatively, solve the pair of equations $y = \frac{1}{2}x + 2$ and $y = 1x + 0$ to get the same result.

c) (10 pts) Horizontal line L2 in (x,y) space passes through P3. What is its corresponding point P4 in (m,b) space?

P4 must lie along the line in (m,b) space from a), that is, $b = -4m + 4$. For a horizontal line, m=0; therefore b=4 and P4 is at (0,4). Alternatively, observe that L2 has equation $y = 0x + 4$ to get the same result.