1.

a) $\vec{X}^W = \vec{X}_{AVG} \frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} \Rightarrow \vec{X}_{AVG} = \vec{X}^W \frac{\vec{B}\,\vec{\Delta}}{|\vec{B}|^2}$

$$\vec{v}^W = \frac{d\vec{X}^W}{dt}$$

$$= \frac{d}{dt}\left(\vec{X}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}}\right)$$

$$= \frac{d}{dt}\vec{X}_{AVG}\cdot\frac{|\vec{B}|^2}{\vec{B}\,\vec{\Delta}} + \vec{X}_{AVG}\frac{d}{dt}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}}$$

$$= \vec{V}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} + \vec{X}^W\frac{\vec{B}\cdot\vec{\Delta}}{|\vec{B}|^2}|\vec{B}|^2\frac{d}{dt}\left(\frac{1}{\vec{B}\cdot\vec{\Delta}}\right)$$

$$= \vec{V}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} + \vec{X}^W\frac{\vec{B}\cdot\vec{\Delta}}{|\vec{B}|^2}|\vec{B}|^2\frac{-\vec{B}}{(\vec{B}\cdot\vec{\Delta})^2}\frac{d}{dt}\vec{\Delta}$$

$$= \vec{V}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\,\vec{\Delta}} + \vec{X}^W\frac{-\vec{B}}{\vec{B}\,\vec{\Delta}}\frac{d}{dt}\vec{\Delta}$$

b) $\vec{X}_{AVG} = \dfrac{\vec{X}_L^I + \vec{X}_R^I}{2} = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$ $\qquad \vec{X}^W = \vec{X}_{AVG} \dfrac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}}$

$$\vec{V}_{AVG} = \frac{d}{dt}\vec{X}_{AVG} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and we know: $\vec{B} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$ $\quad \vec{F} = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$

$$\vec{\Delta} = \vec{X}_L^I - \vec{X}_R^I = \begin{bmatrix} \frac{c}{2t} \\ 0 \\ f \end{bmatrix} - \begin{bmatrix} -\frac{c}{2t} \\ 0 \\ f \end{bmatrix} = \begin{bmatrix} \frac{c}{t} \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{d\vec{\Delta}}{dt} = \begin{bmatrix} -\frac{c}{t^2} \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{V}^W = \vec{V}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} - \vec{X}^W \frac{\vec{B}\cdot\frac{d\vec{\Delta}}{dt}}{\vec{B}\cdot\vec{\Delta}} \quad \text{——— from (a)}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} - \vec{X}^W \frac{\vec{B}\,\frac{d}{dt}\vec{\Delta}}{\vec{B}\cdot\vec{\Delta}}$$

$$= -\vec{X}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} \frac{\vec{B}\cdot\frac{d}{dt}\vec{\Delta}}{\vec{B}\cdot\vec{\Delta}}$$

$$= -\begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix} \frac{\cancel{b}^{\cancel{2}}}{\frac{\cancel{b}c}{\cancel{t}}} \cdot \frac{-\frac{b\cancel{c}}{t^{\cancel{2}}}}{\frac{\cancel{b}\cancel{c}}{\cancel{t}}} = -\begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix} \cdot -\frac{b}{c}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \frac{bf}{c} \end{bmatrix}$$

$$= -\begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix} \frac{\begin{bmatrix} 0 \\ 0 \\ b^2 \end{bmatrix}}{\left(\begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \frac{c}{t} \\ 0 \\ 0 \end{bmatrix}\right)} \frac{\begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} -\frac{1}{ct^2} \\ 0 \\ 0 \end{bmatrix}}{\left(\begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \frac{c}{t} \\ 0 \\ 0 \end{bmatrix}\right)}$$

$$= -\begin{bmatrix} 0 \\ 0 \\ fb^2 \end{bmatrix} \frac{\begin{bmatrix} -\frac{b}{c\cancel{t^2}} \\ 0 \\ 0 \end{bmatrix}}{\begin{bmatrix} \frac{b^2c^2}{\cancel{t^2}} \\ 0 \\ 0 \end{bmatrix}}$$

$$= -\begin{bmatrix} 0 \\ 0 \\ fb^2 \end{bmatrix} \frac{-\frac{1}{c}}{bc^2}$$

$$= -\begin{bmatrix} 0 \\ 0 \\ fb^2 \end{bmatrix}$$

Q_2: $z_1 = x^2 + y^2$. $p(x,y) = \frac{dz}{dx} = 2x$ $q(x,y) = \frac{dz}{dy} = 2y$

$z_2 = 2xy$ $p(x,y) = 2x$ $q(x,y) = 2y$

for z_1: $R(p,q) = R(4x^2+4y^2)$ so z_1 and z_2 can give

for z_2: $R(p,q) = R(4x^2+4y^2)$ rise to same shading

Q_3 $$\nabla f_x = \frac{\partial f(x_{k-1}, u_k)}{\partial x_{k-1}} = \begin{bmatrix} \nabla f_{ux} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\nabla f_u = \frac{\partial f(x_{k-1}, u_k)}{\partial u_k} = \begin{bmatrix} \nabla f_u \\ 0 \end{bmatrix}$$

plugin these 2 into the equation, you will find it easy to prove. All is about math.