

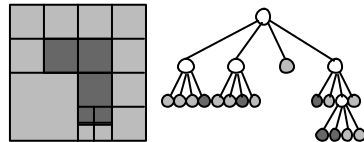
Image Representation

Quad Trees
Gaussian pyramids
Laplacian Pyramids
Wavelet Pyramids
Applications

Quad Trees

Quad tree image representation = a tree representation which represents recursive subdivisions of an image.

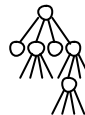
Example:
Quad tree representation of an image



Image



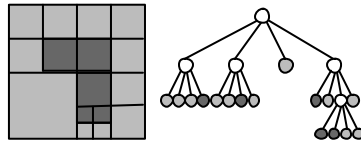
Quad Tree representation



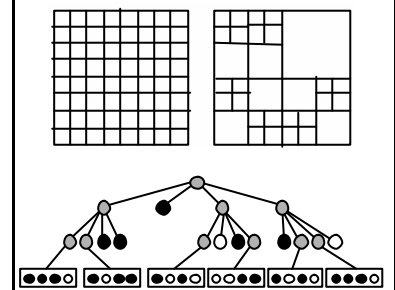
Quad Tree Applications:

- Compression
- Segmentation (Split & Merge)
- Smoothing
- Binary Image Operations ("And" "Or" "Not")

Quad Tree Representation - Example



Quad Tree Representation - Example



Quad Tree Representation

Original

Thresh = 0.20

Thresh = 0.40

Thresh = 0.55



Binary Operations Using Quad Trees

○ = 1 ● = 0

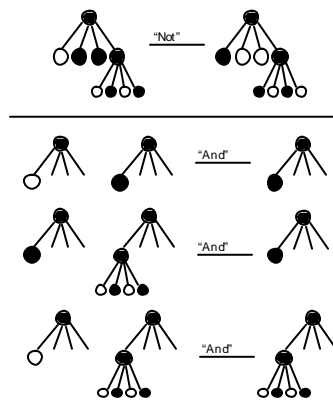


Image Pyramids

Image features at different scales require filters at different scales

Edges (derivatives):

$f(x)$

$f'(x)$



Objects (correlation):

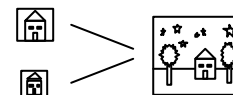
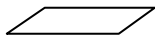


Image Pyramids

Image Pyramid = Hierarchical representation of an image

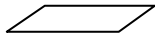
Image Pyramid = A collection of images at different resolutions.

Low
Resolution



No details in image-
(blurred image)
low frequencies

High
Resolution



Details in image-
low+high frequencies

Image Pyramid

Low resolution



High resolution

Image Pyramid Frequency Domain

Low resolution



High resolution



Image Blurring = low pass filtering

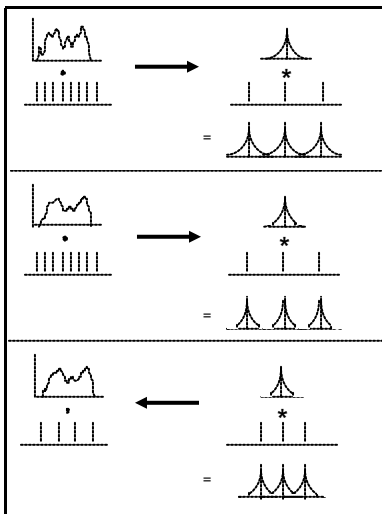
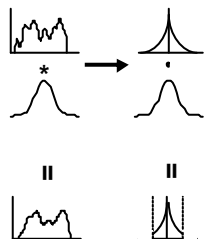
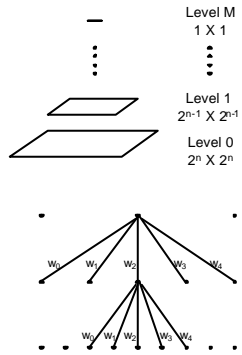


Image Pyramid

Low resolution

High resolution

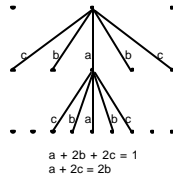
Gaussian Pyramid



Gaussian Pyramid

Burt & Adelson (1981)

Normalized: $\sum w_i = 1$
 Symmetry: $w_i = w_{-i}$
 Unimodal: $w_i \geq w_j$ for $0 < i < j$
 Equal Contribution: for all j $\sum w_{j+2i} = \text{constant}$



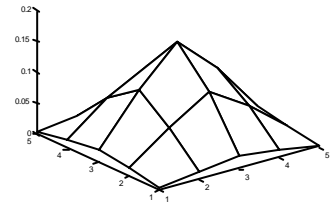
$$\begin{aligned} a &> 0.25 \\ b &= 0.25 \\ c &= 0.25 - a/2 \end{aligned}$$

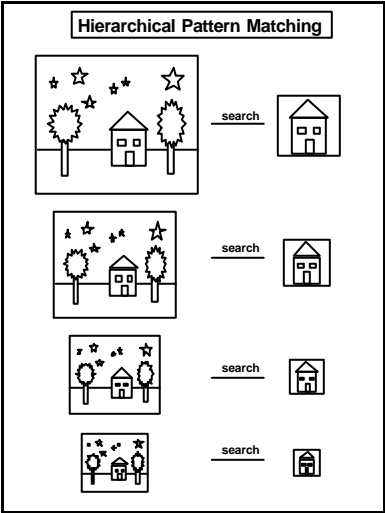
For $a = 0.4$ most similar to a Gaussian filter

$$g = [0.05 \ 0.25 \ 0.4 \ 0.25 \ 0.05]$$

low_pass_filter = $g * g'$ =

0.0025	0.0125	0.0200	0.0125	0.0025
0.0125	0.0625	0.1000	0.0625	0.0125
0.0200	0.1000	0.1600	0.1000	0.0200
0.0125	0.0625	0.1000	0.0625	0.0125
0.0025	0.0125	0.0200	0.0125	0.0025





Pattern matching using Pyramids - Example

image	pattern	correlation

Gaussian Pyramid - Computational Aspects

Memory:

$$2^N \times 2^N (1 + 1/4 + 1/16 + \dots) = 2^N \times 2^N * 4/3$$

Computation:

Level i can be computed with a single convolution with filter: $h_i = \underbrace{g * g * g}_{i \text{ times}} * \dots$

Example:

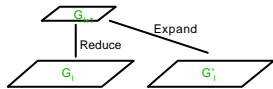
$h_2 = g * g =$

Laplacian Pyramid

Compression -
compression rates are higher for predictable values.
e.g. values around 0.

G_0, G_1, \dots = the levels of a Gaussian Pyramid.

Predict level G_i from level G_{i+1} by **Expanding** G_{i+1} to obtain G'_i

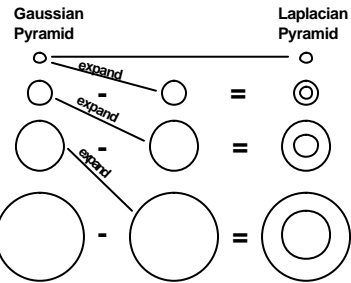


Denote by L_i the error in prediction:

$$L_i = G_i - G'_i$$

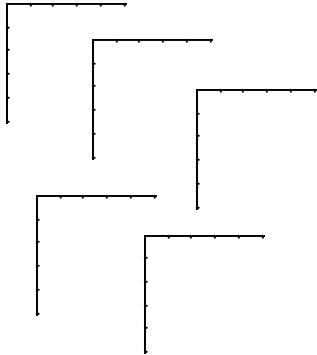
L_0, L_1, \dots = the levels of a **Laplacian Pyramid**.

Laplacian Pyramid

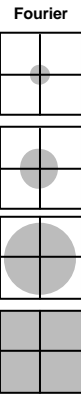


Laplacian Pyramid - Example

Laplacian Pyramid -
No scaling



Gaussian
Pyramid

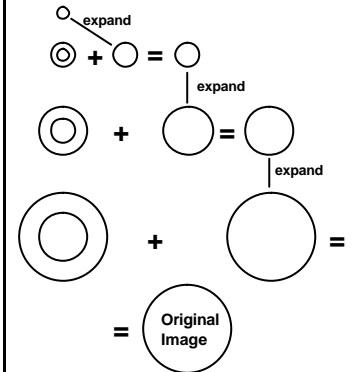


Laplacian
Pyramid



Reconstruction of the original image from the
Laplacian Pyramid

Laplacian
Pyramid



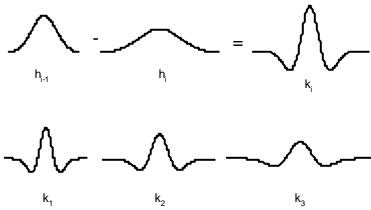
Laplacian Pyramid - Computational Aspects

Memory:

$2^N \times 2^N (1 + 1/4 + 1/16 + \dots) = 2^N \times 2^N * 4/3$
However coefficients are highly compressable.

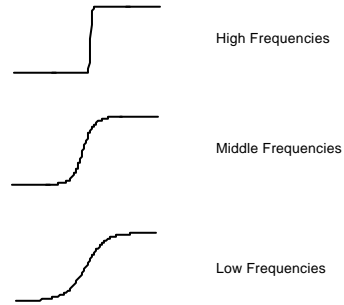
Computation:

L_i can be computed from G_0 with a single convolution with filter: $k_i = h_{i-1} - h_i$

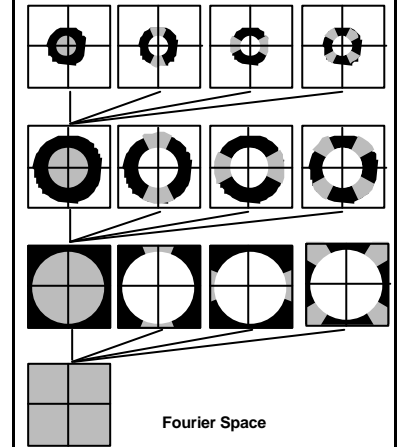


Multiresolution Spline

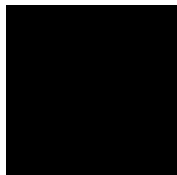
When splining two images, transition from one image to the other should behave:



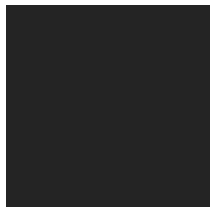
Wavelet Decomposition



Wavelet Transform - Example



Wavelet Transform - Example



Wavelet Transform - Example












Image Pyramids -Comparison			
Transform	Basis	Frequency	Characteristics
Fourier	 Sines+Cosines		Not localized in space Localized in Frequency
Gaussian Pyramid	 Gaussian Filters		Localized in space Not localized in Frequency
Laplacian Pyramid	 Laplacian Filters		Localized in space Not localized in Frequency
Wavelet Pyramid	 Wavelet Filters		Localized in space Localized in Frequency

Image Pyramids -Comparison


Image pyramid levels = Filter then sample.

Filters:

Gaussian Pyramid



Laplacian Pyramid



Wavelet Pyramid

