

Question 1)

Solution:

Given: After being tested positive, the patient tests positive again for a second test.

⇒ To find: $P(\text{cancer} | +, +)$

$$\begin{aligned}\text{We know from the previous problem that } P(\neg\text{cancer}) &= 1 - P(\text{cancer}) \\ &= 1 - 0.008 \\ &= 0.992\end{aligned}$$

$$\begin{aligned}P(\neg\text{cancer} | +) &= 1 - P(\text{cancer} | +) \\ &= 1 - 0.21 \\ &= 0.79\end{aligned}$$

$$P(+ | \neg\text{cancer}) = 0.3$$

Also, we found that $P(\text{cancer} | +) = 0.21$ { i.e, Probability of cancer given positive test results }

Now,

Since the first test has already returned positive, we only consider the cases where we are given that the test results are positive.

Therefore,

we substitute $P(\text{cancer} | +)$ and $P(\neg\text{cancer} | +)$ for $P(\text{cancer})$ and $P(\neg\text{cancer})$ respectively.

$$\begin{aligned}\text{Thus, } P(\text{cancer} | ++) &= \frac{P(+ | \text{cancer}) \times P(\text{cancer} | +)}{\{ P(+ | \text{cancer}) \times P(\text{cancer} | +) + P(+ | \neg\text{cancer}) \times P(\neg\text{cancer} | +) \}} \\ &= \frac{(0.98 \times 0.21)}{(.98 \times .21 + .03 \times .79)} = 0.91\end{aligned}$$

Hence, after the second positive test, we are a lot more certain that the patient has cancer.

Question 2)

Solution:

Correct predictions = 85

=> mispredictions = $100 - 85 = 15$

Thus, error = 0.15

For, 95% confidence interval for Error $D(h)$ =

$$= \text{Error } D(h) \pm [1.96 \times \sqrt{\text{Error } D(h) \times (1 - \text{Error } D(h))/n}]$$

$$= 0.15 \pm 1.96 \sqrt{(0.15 \times (1 - 0.15))/100}$$

$$= 0.15 \pm 0.0357$$

$a, b \rightarrow \text{inputs}$; $c \rightarrow \text{hidden layer unit}$; $d \rightarrow \text{o/p unit}$

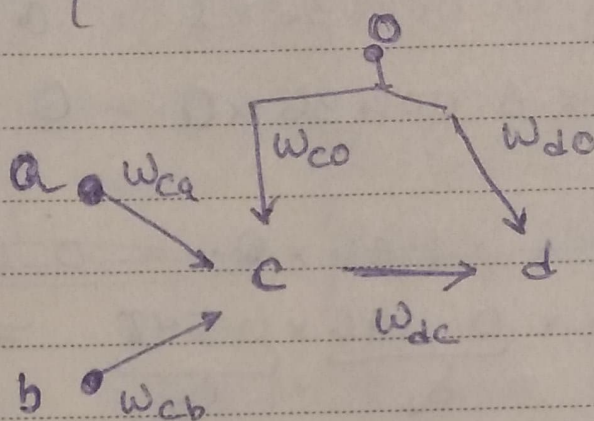
n/w has five weights ($w_{ca}, w_{cb}, w_{co}, w_{dc}, w_{do}$)

a	b	d
1	0	1
0	1	0

$w_{xo} = \text{threshold weight for unit } x.$

initialize to $(-1, -1, -1, -1, -1)$

learning rate, $\eta = 0.3$; momentum $\rho = 0.9$



$$\sigma(y) = \frac{1}{1 + e^{-y}}$$

Sigmoid fⁿ.

Training Example 1:

o/p of two neurons, for $d = 1 \Rightarrow a = 1$ & $b = 0$

$$O_c = \sigma(0.1 \times 1 + 0.1 \times 0 + \textcircled{0.1}) = \sigma(0.2) = \textcircled{0.5498} = 0.5498$$

$$O_d = \sigma(0.1 \times \underline{0.5498} + 0.1 \times 1) = \sigma(0.15498) = 0.53867$$

Error terms for the two neurons noting that $d = 1$.

$$\delta_d = 0.53867 \times (1 - 0.53867) \times (1 - 0.53867) \xrightarrow{\text{from ①}} 0.1146$$

$$\delta_c = 0.5498 \times (1 - 0.5498) \times 0.1 \times 0.1146$$

No bird soars too high, if he soars with his own wings. - William Blake

$$= 0.002836$$

w_{dc}

from ②

New correction terms for $\eta=0.3, a=1, b=0$

$$\Delta w_{do} = 0.3 \times 0.1146 \times 1 = 0.0342$$

$$\Delta w_{co} = 0.3 \times 0.002836 \times 1 = 0.000849$$

$$\Delta w_{ca} = 0.3 \times 0.002836 \times 1 = 0.000849$$

$$\Delta w_{cb} = 0.3 \times 0.002836 \times 0 = 0$$

$$\Delta w_{dc} = 0.3 \times 0.1146 \times \cancel{0.6} = 0.0189$$
$$0.3 \times \underbrace{0.1146}_{\delta_1} \times \underbrace{0.548}_{o_c} = 0.0189$$

New weights

$$w'_{do} = 0.1 + 0.0342 = 0.1342$$

$$w'_{dc} = 0.1 + 0.0189 = 0.1189$$

$$w'_{co} = 0.1 + 0.000849 = 0.100849$$

$$w'_{ca} = 0.1 + 0.000849 = 0.100849$$

$$w'_{cb} = 0.1 + 0 = 0.1$$

Second Training Iteration:

$$a = 0 \quad b = 1 \quad d = 0$$

$$\begin{aligned} O_c &= \sigma(w_{ca} + w_{cb} \times b) \\ &= \sigma(0.100849 + 0 + 0.1 \times 1) = \frac{1}{1 + e^{-0.200849}} \\ &= \underline{0.55004} \end{aligned}$$

$$\begin{aligned} O_d &= \sigma(w_{do} + w_{dc} \times c) \\ &= \sigma(0.1342 + 0.1189 \times 0.55004) = \sigma(0.19965) \\ &= 0.54974 \end{aligned}$$

$$\delta_d = O_d \times (1 - O_d) \times (d - O_d) = (0.54974) \times (1 - 0.54974) \times (-1) = -0.136078552$$

$$\delta_c = O_c \times (1 - O_c) \times (w_{dc} \times \delta_d) = (0.55004) \times (1 - 0.55004) \times (0.1189 \times (-0.136078552))$$

$$\Delta w_{ca} = \eta \delta_c = -0.0041$$

$$\begin{aligned} \Delta' w_{ca} &= \eta \delta_c \times a + \alpha \Delta w_{ca} \times (2-1) \\ &= 0.3 (-0.0041) \times 0 + 0.9 \times 0.000849 \times 1 \\ &= 0.00076 \end{aligned}$$

$$\begin{aligned} \Delta' w_{cb} &= \eta \delta_c \times b + \alpha w_{cb} (2-1) = 0.3 (-0.0041) \times 1 + 0.9 \times 0.1 \\ &= -0.00123 \end{aligned}$$

$$\Delta' w_{co} = 0.3 \times 0.004 \times 1 + 0.9 \times 0.000849 \times 1 = 0.00043$$

$$\begin{aligned} \Delta' w_{dc} &= 0.3 \times (-0.136078) \times 0.55004 + 0.9 \times 0.1189 \\ &= -0.00593 \end{aligned}$$

Notes

$$\Delta'W_{do} = 0.3 \times -0.13607 \times 0 + 0.9 \times 0.03439 \\ = -0.00987$$

Final weights:

$$W_{ca} = W'_{ca} + \Delta'W_{ca} = 0.100849 + 0.00076 \\ = 0.101609$$

$$W_{cb} = W'_{cb} + \Delta'W_{cb} = 0.1 + 0.00123 \\ = \cancel{0.100} 0.101877$$

$$W_{cd} = W'_{cd} + \Delta'W_{cd} = 0.100849 - 0.0004 \\ = 0.1004$$

$$W_{dc} = W'_{dc} + \Delta'W_{dc} = 0.1189 + (+)0.00543 \\ = 0.11347$$

$$W_{do} = W'_{do} + \Delta'W_{do} = 0.1342 - 0.00987 \\ = 0.12433$$